RESEARCH PAPER

# Comparative Effectiveness of Example-based Instruction and van Hiele Teaching Phases on Mathematics Learning

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This article explores the effectiveness of the pedagogical practices associated with the cognitive load theory and the van Hiele theory, which are two theories from cognitivism and constructivism perspectives, respectively. Following a quasi-experiment, the quantitative analysis of 157 high school students' responses to pre, post, and retention tests revealed that the students taught with the van Hiele teaching phases performed significantly better at the post and retention tests. While the cognitive load theory intervention bridges the gap between low and high ability students, the van Hiele teaching phases is beneficial to both low and high performing students. These results have implications for mathematics teaching practices and learning.

The concern for higher mathematics students' outcomes and the application of mathematical knowledge to real-world scenarios is increasing across the globe. While some 15-year-old students' performance in mathematics is below the average competency level at the Programme for International Student Assessment (PISA), others at an average level or above could not transfer mathematical knowledge to solve practical problems around them (Organisation for Economic Co-operation and Development, 2019). This is similar to reports from African countries, particularly Nigeria (Omobude, 2014). However, one of the main facilitators of students' learning outcomes is teachers' pedagogical approach (Bolstad, 2021; Li & Schoenfeld, 2019), which is the focus of this investigation.

There are numerous studies that have applied several pedagogies generated from different learning theories (Ginga & Zakariya, 2022; Schneider et al., 2022). Of interest to this study are the worked example instruction – a popular instructional design, following the principles of the cognitive load theory (CLT) (Sweller, 2011)—and the van Hiele teaching phases (VHTP) (van Hiele, 1986)—which align with the constructivist approach and an element of the van Hiele theory. While available empirical findings have reported that both the CLT worked example instruction and VHTP is effective (Centre for Education Statistics and Evaluation, 2017), no study, either empirical or theoretical, has compared the effectiveness of the approaches. As several studies around the world have indicated that students struggle to solve complex algebraic equations with rich knowledge of concepts and procedures (Johari & Shahrill, 2020), which means students cannot transfer the acquired mathematical knowledge to solve real-life problems (Bolstad, 2021; Li & Schoenfeld, 2019), this study explores the CLT worked example instruction and the VHTP to determine their effectiveness for solving complex mathematical problems using the example of simultaneous equations. A predictor of students' levels of mathematical understandings is their mathematical ability (Ayebale et al., 2020). Consequently, this study examines the influence of students' ability levels on the effectiveness of these pedagogies. Specifically, this article answers the following research questions:

1. *How do students' learning outcomes in the cognitive load group differ from the van Hiele group?*
2. *What are the effects of each of the cognitive load theory intervention and the van Hiele teaching phases across the three time-points?*
3. *Are there differences in the learning outcomes of students in the cognitive load group and van Hiele group based on ability levels?*

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## Cognitive Load Theory

The CLT (Sweller, 2011) aims to improve mathematics and science teaching and learning by focusing on human cognitive architecture, which is characterised by a limited working memory and unlimited long-term memory. The working memory processes information while the long-term memory stores the processed information. CLT contends that for learning to occur, the cognitive resources required to learn a task must not exceed the available working memory resources (Sweller et al., 2019). Moreover, the working memory can only process four to five pieces of new information at a time, and such information may be missing if not properly rehearsed after 20 seconds (Miller, 1956). Based on the understanding of the principles for processing information in humans, CLT recommends several instructional designs that could facilitate effective learning. One popular instructional design that manages the working memory resources is worked example instruction. In this design, students are provided with worked examples to study and transfer their understanding to similarly structured problems. Several studies have established that the use of worked examples is effective because it imposes a relatively low cognitive load and does not interfere with learning (Ngu et al., 2019; Renkl, 2017; Richey & Nokes-Malach, 2013). However, these studies mainly focused on simple mathematical topics such as one-step and two-step equations. Moreover, most of the studies assessed students' responses based on the procedural steps leading to the final answer, without considering the quality of the students' responses. Furthermore, the long-term effect of this pedagogy has not been widely investigated, and no study has reported the effectiveness of the pedagogy in relation to students in Africa.

## The van Hiele Theory

The van Hiele theory, formulated by Pierre van Hiele, originated from the difficulties students encountered in learning geometry. He proposed a developmental framework that requires teachers to understand how students' geometric thinking progresses in levels, known as the van Hiele levels of thinking. In a joint effort with his wife, Dina van Hiele, they prescribed five sequential teaching phases for developing students' cognitive reasoning through the levels, called the van Hiele teaching phases (VHTP) (van Hiele, 1986). This was based on their belief that students' cognitive progression from one level to the next is dependent on instruction rather than maturity and age. They claimed that learning from real-life scenarios enhances life-long learning, and they recommended student-centred activities for learning. The five teaching phases are information, directed orientation, explication, free orientation, and integration. Upon successful completion of these teaching phases, students' thinking is moved to the next level and the phases are repeated. The van Hiele teaching phases align with the constructivist perspective and emphasise that students construct their own mathematical knowledge in their own unique way by exploring the learning environment, seeking clarification, and developing initiatives for problem-solving (Serow et al., 2019). This pedagogy acknowledges the changing roles of teachers and students during learning and emphasises language development and building new knowledge on pre-existing information. Moreover, this pedagogical lens serves as a tool for guiding teachers in designing relevant activities for a lesson (for further information see van Hiele [1986] and Serow et al. [2019]).

As the van Hiele theory was formulated to improve performance in geometry, several studies across the world have reported the effectiveness of the phase-based pedagogy for geometry teaching, learning, and curriculum (Alex & Mammen, 2016; Machisi & Feza, 2021; Serow & Inglis, 2010). There is, however, need to transfer the lens of van Hiele theory to other areas of mathematics (Colignatus, 2014; Vojkuvkova, 2012). Since then, some attempts have been made to investigate the effectiveness of the van Hiele teaching phases in other aspects of mathematics, but results have been inconsistent (Nisawa, 2018; Walsh, 2015). Thus, there is the need for further research in this area.

Notably, both pedagogies considered in this paper rely on schema from other people to learn and emphasise the contribution of prior knowledge to learning. However, while VHTP stresses that students are to construct their knowledge by exploring their environment and developing crisis in thinking, CLT claims that these activities may overload students' working memory and thus result in no learning. Furthermore, unlike the VHTP, the CLT-associated pedagogy does not encourage social interaction such as peer discussion.

## Method

This quasi-experimental design followed a pre-, intervention, post-, and retention test sequence, which involved two experimental groups—one for the CLT and the other for VHTP. Each group comprised one intact class of first-year senior school students (ages 14 to 15 years) from two government schools in Nigeria. A total of 157 male and female students was involved: CLT group *(n* = 72) and VHTP group (*n* = 85). The groups were equivalent in terms of mathematical content coverage, access to materials and human resources, English language competencies, and geographical locations. Due to the limitations in contact occasioned by the COVID-19 pandemic, the regular teachers of each group implemented the interventions after undergoing training from the researchers. The students completed three similar tests and were exposed to the interventions across eight weeks. Initially, students completed an open-ended pre-test to determine their current knowledge about solving simultaneous equations. The groups were then exposed to eight (40-minute) carefully sequenced lessons, with one group receiving the worked examples instruction and the other receiving the VHTP instruction. The students then completed a post-test. Three weeks after the post-test, a retention test was administered to the students to establish the lasting effects of the interventions. Students were required to solve the mathematical problems in the tests and provide an explanation for their responses. Generally, the study followed the research ethics standard and was approved by the University of New England (Approval number HE20-224). Rasch analysis was employed to ascertain the degree to which the data (items and persons) fit the model. The Rasch model is suitable because of its significant role in considering both items and persons as connected constructs, the acknowledgment of unequal intervals within the functioning of the items, and the non-assumption that all items are of equal difficulty (Bond & Fox, 2013). The model fitness is reported by four statistical parameters: outfit, infit, separation index, and reliability. According to Linacre (2013), infit and outfit values ideally range between 0.5 and 1.5. Hence, when the infit mean square estimate is close to 1, it indicates that the set of items and persons perfectly fit the Rasch model.

## Scoring

Students' responses (procedural steps and explanation of the procedures) to the tests were classified into increasing levels of thinking and scored following the rubrics of the structure of the observed learning outcomes (SOLO) model (Biggs & Collis, 2014). SOLO is considered appropriate because it examines both the quality and quantity of students' responses in the evaluation process. Six levels of response were identified: prestructural = 0, unistructural = 1, multistructural = 2, relational = 3, formal mode 1 = 4, and formal mode 2 = 5.

## Results

Table 1 presents the Rasch results. The item reliability (I) indices (> 0.9) across time indicate that a large range of item measures are adequate for stable item estimates, which implies that the sample size can be used to establish a reproducible item difficulty hierarchy.

Table 1
*Rasch Summary Statistics for Items (I) and Persons (P) estimates*

| Tests | | Separation index (I) | Separation index (P) | Infit (I) | Infit (P) | Outfit (I) | Outfit (P) | Reliability (I) | Reliability (P) |
|---|---|---|---|---|---|---|---|---|---|
| Test1 | CLT | 7.08 | 1.31 | 0.93 | 0.83 | 1.70 | 1.23 | 0.98 | 0.63 |
| | VHTP | 5.87 | 1.46 | 1.01 | 0.87 | 0.95 | 0.95 | 0.97 | 0.68 |
| Test2 | CLT | 5.79 | 1.52 | 1.04 | 0.98 | 0.99 | 0.99 | 0.97 | 0.70 |
| | VHTP | 4.57 | 1.30 | 1.02 | 1.07 | 0.94 | 0.94 | 0.95 | 0.63 |
| Test3 | CLT | 4.98 | 0.92 | 1.02 | 1.04 | 0.95 | 0.95 | 0.96 | 0.46 |
| | VHTP | 4.25 | 1.15 | 0.97 | 0.83 | 0.86 | 0.86 | 0.95 | 0.58 |

With regard to the person estimates, most of the person reliability (P) and separation indices for both groups were greater than 0.5 and 1, respectively. This means that the Rasch model identified more than one level of ability within the participants. Correspondingly, the participants were classified into low and high ability levels. The infit and outfit for both items and persons ranged between 0.5 and 1.5, except for the item outfit of CLT, which was 1.70. The outfit measure of 1.70 may be a result of a few random responses by the low-performing students. Furthermore, the high item separation indices (> 3) for the two groups indicate that the samples for each group were large enough to identify the item difficulty of the test instrument.

Additionally, the Wright (variable) map in Figure 1 indicates the relationship between the ranking of person abilities and item difficulties before the intervention. The figure shows that the persons' abilities range between -5 and 3 logits while the item difficulties range between -3 (easiest) and 1.5 (most difficult) logits. The most difficult items were Questions 5, 8, and 9, which were located between 1 and 2 logits, while Question 1, the easiest question, was located at -3 logits. Since the question difficulties ranged between the logits of persons' abilities, the items of the test instrument were adequate for the targeted students. Thus, it was concluded that the test items fit the Rasch model, have a good range of difficulty, have high reliabilities, and are appropriate for the cohort of participants for whom it is targeted. This has the potential for significant productive measurement and results.

The data analysed and presented here were part of a robust investigation that sought to explore two pedagogical practices. The person estimates, measured in logits, from the Rasch measurement of 157 participants were exported to the Statistical Package for Social Sciences (SPSS). An independent *t*-test was performed to test the equality of the effectiveness of the CLT and VHTP pedagogical interventions. Initially, there was a weak difference between the two groups at the pre-test, in favour of the CLT. A further analysis of the immediate and long-term effects of the interventions yielded statistically significant differences between the two groups. For both the post-test and retention test, the van Hiele group significantly outperformed the CLT group with large effect sizes ($t_{(138.63)} = -6.15$, $p = 0.00$, $d = 1.01$ and $t_{(154)} = -9.76$, $p = 0.00$, $d = 1.57$ at 95% confidence interval), as shown in Table 2.

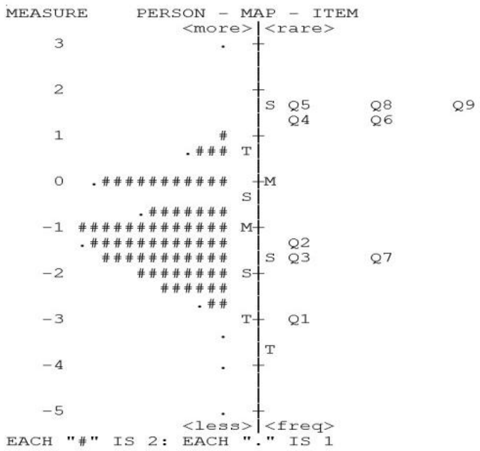


*Figure 1.* A Wright map showing the person abilities and item difficulties.

Table 2
*Independent t-test of Students' Learning Outcomes Across the Three Time-points*

| Tests | Intervention | *N* | *M* | *SD* | *t* | *df* | Sig | *D* |
|---|---|---|---|---|---|---|---|---|
| Pre-test | CLT | 72 | -0.75 | 1.15 | 2.72 | 155 | 0.00* | 0.44 |
| | VHTP | 85 | -1.26 | 1.16 | | | | |
| Post-test | CLT | 72 | -0.14 | 1.19 | -6.15 | 138.63 | 0.00* | 1.01 |
| | VHTP | 85 | 0.95 | 0.99 | | | | |
| Retention test | CLT | 72 | -0.31 | 0.87 | -9.76 | 154 | 0.00* | 1.57 |
| | VHTP | 84 | 1.23 | 1.06 | | | | |

*$p < 0.05$

A summary of the mean scores of the two groups from pre-test to post-test then to retention test is shown in Figure 2. The data indicate that while the learning outcomes of students in both groups at the pre-test were relatively close, they slightly favoured the CLT group. However, after the intervention, post-test scores of the VHTP group were significantly better than those of the CLT group, and at the retention test, the difference in the sizes of the effect was larger.

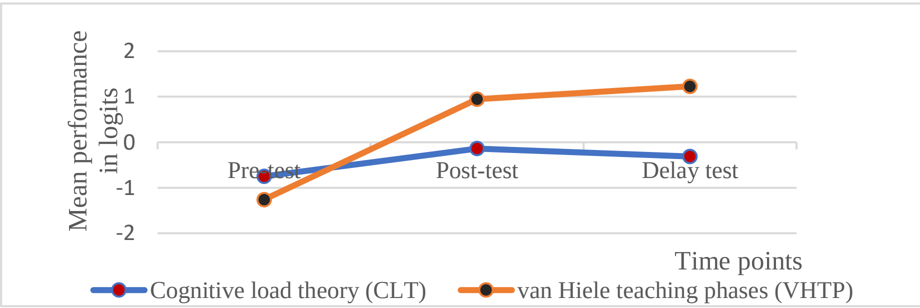


*Figure 2.* Line graph showing a summary of students' learning outcomes at the pre, post, and retention test.

To test the hypothesis of equal mean across the three tests for each interventional group, a repeated measure analysis of variance was conducted to explore the within-subject effects on students' learning outcomes. The Mauchly's test of sphericity was significant [$\chi^2$ (2) = 6.50,

$p$ = 0.04] for CLT group and not significant for VHTP group [$\chi^2$ (2) = 0.24, $p$ = 0.89]. Hence, the assumption on sphericity was considered differently. The results indicated that there was a significant medium effect of the CLT on students' learning outcomes [$F_{(1.84,\ 130.45)}$ = 8.88, $p$ = 0.00, $\eta_p^2$ = 0.11] and a significant large effect of VHTP on student learning outcomes [$F_{(2,\ 160)}$ = 260.93, $p$ = 0.00, $\eta_p^2$ = 0.76]. A Post hoc comparison using the Bonferroni adjustment revealed that for the CLT group, significant difference existed between the pre-test and post-test but no significant difference in the means of the post-test and retention test. Similarly, there is significant difference between the pre-test and post of the VHTP group, however, the significant difference between the post-test and retention test only existed at 90% confidence interval.

Table 3
*Mean, Standard Deviation and Repeated Measures Analysis of Variance of Students' Learning Outcomes*

| | Pre-test | | Post-test | | Retention test | | | |
|---|---|---|---|---|---|---|---|---|
| Intervention | *M* | *SD* | *M* | *SD* | *M* | *SD* | F | $\eta_p^2$ |
| CLT | -0.75 | 1.15 | -0.14 | 1.19 | -0.31 | 0.87 | (1.84,130.45) = 8.88* | 0.11 |
| VHTP | -1.26 | 1.16 | 0.95 | 1.00 | 1.23 | 1.06 | (2, 166) = 260.93* | 0.76 |

*$p$ < 0.05

An analysis of the influence of students' ability levels on the effectiveness of the interventions is shown in Table 4. The result of the CLT group indicated a large significant difference between the learning outcomes of low and high ability students at the pre-test ($t_{(70)}$ = -11.90, $p$ = 0.00, $d$ = 2.86). After the intervention, the difference between low and high ability students was still significant but with moderate size ($t_{(69.93)}$ = -3.20, $p$ = 0.00, $d$ = 0.72). During the retention test, no significant difference was found between low and high students ($t_{(70)}$ = -0.09, $p$ = 0.93, $d$ = 0.02), suggesting that the CLT intervention favours the low ability students than the high ability students. For the VHTP group, the strong differences observed between the low and high ability students at the pre-test ($t_{(39.55)}$ = -7.26, $p$ = 0.00, $d$ = 1.92) continues at the post-test ($t_{(37.65)}$ = -3.91, $p$ = 0.00, $d$ = 1.05) and the retention test ($t_{(82)}$ = -4.94, $p$ = 0.00, $d$ = 1.12). These results suggest that the VHTP is beneficial to both the low and high ability students.

Table 4
*Analysis of Students' Learning Outcomes Based on Ability Levels Across the Three Time-points*

| Groups | Tests | Level | *N* | *M* | *SD* | *t* | *df* | *Sig* | *D* |
|---|---|---|---|---|---|---|---|---|---|
| CLT | Pre-test | Low | 29 | -1.89 | 0.81 | -11.9 | 70 | 0.00* | 2.86 |
| | | High | 43 | 0.01 | 0.54 | | | | |
| | Post-test | Low | 29 | -0.62 | 0.88 | -3.20 | 69.93 | 0.00* | 0.72 |
| | | High | 43 | 0.19 | 1.27 | | | | |
| | Retention test | Low | 29 | -0.32 | 0.81 | -0.09 | 70 | 0.93 | 0.02 |
| | | High | 43 | -0.30 | 0.92 | | | | |
| VHTP | Pre-test | Low | 54 | -1.85 | 0.61 | -7.26 | 39.55 | 0.00* | 1.92 |
| | | High | 31 | -0.22 | 1.17 | | | | |
| | Post-test | Low | 54 | 0.61 | 0.59 | -3.91 | 37.65 | 0.00* | 1.05 |
| | | High | 31 | 1.55 | 1.26 | | | | |
| | Retention test | Low | 53 | 0.84 | 0.90 | -4.94 | 82 | 0.00* | 1.12 |
| | | High | 31 | 1.89 | 1.01 | | | | |

*$p$ < 0.05

## Discussion

As it is often claimed that pedagogical practices are essential in achieving the key goals of mathematics curriculum across the globe, this study examined the effectiveness of two pedagogical practices that have their roots in cognitivism and constructivism approaches. The results indicated that both the CLT and VHTP interventions were observed to have short-term effects on students' learning outcomes; however, the learning outcomes of students in the VHTP continued to increase at the retention test, while the CLT group experienced a waning effect after the post-test. This pattern of the results may be attributed to many factors, including the nature of the instruction and forgetfulness. Specifically, CLT recommends instructional designs that require students to acquire schema with minimal cognitive effort (Sweller, 2011). The worked examples instruction utilised in this study provided a step-by-step guide to solving a problem, emphasising more procedural knowledge than conceptual knowledge, and students do not experience interference with learning. Conversely, one of the main principles underlying the movement of students' thinking from one level to the next in VHTP is the crisis in thinking during learning (Serow et al., 2019), that is often experienced by students in the fourth teaching phase and allows them to investigate various thinking paths, identify correct reasoning for the domain of thought, and develop a strong perception of the mathematical ideas, which is observed to last for a long time. Therefore, the VHTP seems to offer more conceptual mathematical knowledge than procedural knowledge.

Furthermore, students' learning outcomes in the VHTP group were observed to be significantly better than the CLT group regardless of their ability levels. While the CLT is more favourable to the low ability students by bridging the gap between the low and high ability students, the VHTP appears to improve both the low and high ability students in similar magnitude. These findings seem to support and advocate that the major attributes of VHTP—an exploration of learning materials, sequential development of students' thinking, thinking crisis, students' active participation, language development, and discourse—are essential for learning complex mathematics topics. Another practical implication of these findings could be that pedagogies that strongly focus more on conceptual knowledge tend to have more lasting effect than the reverse. The results of this study, which was conducted in an African context, are consistent with several other studies around the globe (Kalyuga et al., 2001; Machisi & Feza, 2021; Renkl, 2017; Walsh, 2015). However, the appropriate use of VHTP requires extra commitment from teachers. Lastly, the researchers acknowledge the interference of noise from the natural setting, where this experiment was carried out, as a limitation of this study.

## Conclusion

The findings presented in this paper suggest that the pedagogical practices employed by teachers significantly affect students' learning outcomes and long-term knowledge retention. This study highlights that VHTP allows students to demonstrate ownership of mathematical ideas, and the crisis in thinking has a significant effect on students' achievement in mathematics. The VHTP students demonstrated higher achievement in the short and long term than their peers who learned through the CLT worked example instruction. The findings from this study extend existing evidence on the application of CLT and VHTP in mathematics learning. It also contributes to the growing evidence on effective teaching practices in mathematics education. Lastly, the study is significant for its methodological (SOLO model and Rasch model for scoring and analysis), empirical, and contextual contribution to the improvement of mathematics learning and retention.

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