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A

NEW TREATISE

ON THE

USE OF THE GLOBES,

AND

Practical Astronomy;

OR

A COMPREHENSIVE VIEW

OF

THE SYSTEM OF THE WORLD.

IN FOUR PARTS.

I. An extensive collection of Astronomical and other Definitions.

II. Problems performed by the TERRESTRIAL GLOBE, including those relative to Geography, Navigation, Dialling, &c. with many new and important problems and investigations, particularly useful to the Navigator and Practical Astronomer.

III. Problems performed by the CELESTIAL GLOBE, including those of finding the longitude at sea, new methods of finding the latitude, with only one altitude of the sun, or a star, at any given time, with the method of representing the spherical triangles on the globe, &c.

IV. A comprehensive account of the SOLAR SYSTEM, with the elementary principles, and most valuable modern discoveries in Astronomy to the present time. The nature and motion of COMETS, OF THE FIXED STARS, ECLIPSES, THE THEORY OF THE TIDES, LAWS OF MOTION, GRAVITY, &c. with DIAGRAMS elucidating the demonstrations.

The whole serving as an introduction to the higher Astronomy and Natural Philosophy, is illustrated with a variety of important notes, useful remarks, &c. and each problem with several examples. The necessary astronomical instruments are pointed out, and the most useful tables are inserted in the work.

DESIGNED FOR THE INSTRUCTION OF YOUTH,

AND PARTICULARLY ADAPTED TO THE UNITED STATES.

BY J. WALLACE,

Member of the New-York Literary Institution, &c.

Quid munus Reipublicæ majus aut melius afferre possimus, quam si Juventutem bene erudiamus? CICERO.

NEW-YORK:

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1812.

PREFACE.

MAN cannot but behold with gratitude and delight, the multiplied benefits and amazing objects which surround him on all sides, contributing equally to his wants and pleasure. This pleasure, however, is greatly increased in proportion as the nature, utility, and number of these objects are known and understood; and this knowledge is only attained from a cultivation of those noble powers with which the mind of man is gifted, and which so eminently distinguish him from the brute creation.

The savage that ranges our forests in common with the brute; that at the same fountain satisfies his thirst, and eats of nature's fare, whatever his taste or appetite craves; that seems no way distinguished from the animals with which he associates, than by the figure of his species; has still within him the seeds of those noble acquirements which exalt and dignify human nature. Yes, this same savage enjoying similar advantages with a Cicero, a Demosthenes, or a Newton, might become their rival; but those seeds, from a want of cultivation, must remain for ever buried in oblivion. Such is the picture of uncultivated man, whom, in his wild and savage state, the mines of Peru cannot enrich, or whose wants the most fertile regions of the earth cannot lessen. In the midst of profusion he is indigent, and in the unequal conflict with those animals, whose master he was destined to be, must often become a prey to their superior strength and ferocity.

It is evident, then, that an acquaintance with the elements of science is intimately connected with our necessities, no less than with our future progress, advancement, and eminence; and that in proportion as we neglect the acquirement of this knowledge, we approximate to the state of the rude, uncultivated savage. It is well known, that Great-Britain and France respectively owe more to the successful cultivation and application of the sciences, than they do to the valour of their armies, or to the strength of their marine.

Among all the branches of science within the compass of human acquirements, there are few that unite greater importance and utility, than that which exhibits and explains the phenomena of the earth, our destined habitation, and more pleasure, than that which traces the evolutions of those immense orbs that decorate the heavens, and investigates the unerring laws by which they are regulated and governed: for there is nothing which so much excites our attention and curiosity, which unites in itself so much grandeur and magnificence, and which produces in the soul so much sublimity and admiration, as the contemplation of those prodigies which that immense vault surrounding the habitation of man exhibits to our view. And if there be some in whom this grand spectacle excites no emotion, it is because they are too much absorbed in those artificial wants or necessities which they create to themselves; *veluti pecora,* as Sallust says, *quæ natura prona, atque ventri obedentia finxit.*

It is in the heavens that the Creator has chiefly manifested his greatness and majesty. It is here that the Sovereign Wisdom shines with the greatest lustre, and that the sublime ideas of order and harmony reign. In this immense host of celestial bodies all is prodigy and magnificence : all is regularity and proportion : all announce a power infinitely fertile in the production of beings, infinitely wise in their arrangement and destination.

But this magnificent spectacle is not thus exposed to our constant view, to be the object of an idle admiration, or a fruitless contemplation ; it is much more connected with the wants and advantages of the inhabitant of the earth. It is in the heavens that we have found the means of arresting time in the rapidity of its course : of regulating our seasons, and fixing those interesting epochs, from which the Historian and Chronologer date the most important events. The form, the extent, the exact position of the different parts of the earth we inhabit, and its situation in the immense expanse, is attained only by the assistance of Astronomy. If we now traverse the ocean with so much security and skill, it is principally owing to this science which has furnished the means of ascertaining our place, at any time, in this trackless element. Thus by the interposition of the heavens the most distant nations hold their correspondence : extensive deserts, immense oceans, seas, and unknown countries are explored, and their riches transported to other countries destitute of these resources. In a word, it is to this science that Columbus owed the greatest discovery that human ingenuity has ever made, and that he has been able to add a new world to the old.

It is not only in enlarging the sphere of human knowledge, and contributing to the wants and conveniencies of man that Astronomy is useful ; it has also dissipated the alarms occasioned by extraordinary celestial phenomena, and destroyed many of the errors arising from our true relation with nature. Such are the obligations we have to this science ; such the benefits which it has conferred on society ; such the services it has rendered the human mind. This sublime science then, claims a right to our esteem and respect, and without doubt, there is not among human sciences another, more worthy to engage our attention, and better calculated to occupy and amuse our leisure moments.

It is no objection to it that it has often been made the unwilling instrument of impiety in the hands of the impious, or of an absurd science in the hands of the Astrologer ; for the greatest benefits conferred on man are susceptible of abuse. To put a stop to these growing evils, Emperors have passed their edicts and enforced their decrees, to expel those impious pretenders from cities that became the scenes of their folly and impiety, and some who deserved a better fate were unhappily involved in their number. The irreligious Philosopher and the impious of the day, will ascribe many of these unhappy occurrences to the religious prejudices and ignorance of those times ; but with no more reason than those have, who charge this science with supporting impiety, though of all others the least calculated to afford it any support. If history has any truth in it, history affirms that it was in houses dedicated in those days to piety and religion, that the most precious remains of science were preserved, and that it is from them they have been principally handed down to the present time.

To trace this science to its origin, and point out the various alterations and improvements it has received, the long series of discoveries which it presents, and the illustrious authors who have contributed to them, would far exceed the limits of a preface. It will be sufficient to observe, that the origin of Astronomy commences its date with that of Agriculture and of Society itself. There is still an immense difference between the first view of the heavens, and the view by which, at present, we comprehend the past and future state of the system of the world. It is, however, to the improvements in the past and present age, that we are principally indebted for this developement of the most important and curious discoveries in this system; and such of those authors as have been most successful, and have particularly excelled in this respect, have been consulted in the following compendium. Their works have been also pointed out to direct the choice of the student, and exhibit their superior advantages and excellence.

Among the inconveniencies attending our public places of education, it is no small one, that many of those works which are the standard of elegance and perfection, are inaccessible both to the Student and Master, in consequence of the difficulty of procuring them from Europe, and their too great expense to be introduced into Schools. To remedy, in some measure, this inconvenience, the author of the present work has undertaken to draw up an entire course of Mathematics and Natural Philosophy (if his avocations will suffer him to continue) principally for the use of the Students belonging to the *New-York Literary Institution.* And conceiving that this course, undertaken more from necessity than choice, would asssist him no less than others occupied in the education of youth, he has been induced, principally from this motive, to make this *introduction* public.

The present treatise on the *Use of the Globes and Practical Astronomy* is complete in itself, and detached from the contemplated course, the author having immediate and urgent necessity for its use; and being a subject uniting extensive utility with pleasure and ornament, no pains have been spared, in calculating it for these important objects, as far as his hurry in drawing it up would allow.

Each problem is illustrated with several examples, and their demonstration or calculation, &c. given in notes at the bottom, in order to make it more fully answer the end of an elementary treatise on practical astronomy, and to adapt it to academies and places of public education in general, where this branch of science is now considered as one of the most entertaining and necessary.

Many new and important problems will be found in this, in addition to those found in other treatises; and which likewise are performed on the globes, by methods generally entirely new, and found in no other treatise; which cannot but render this work extremely interesting to those who are capable of relishing the beauties of science, and of appreciating its value. Many important Tables are inserted in the course of the work, as well as figures to illustrate the demonstrations, &c. and it is no small recommendation to it that these figures were cut by the celebrated Dr. Anderson. There are also given, besides a complete account of the Solar System, the elements and laws of the planet's motions, their phenomena, their principles, &c a full investigation of the nature and motion of Comets, the doctrine of Eclipses, the Tides, the General laws of Motion, Gravity, &c. enriched with many discoveries and late improvements from Herschel, Vince, Maskelyne, La Lande, Laplace, Delambre, &c.

The work being printed close, and the notes (which are of considerable length) being in small type, this treatise must contain more matter than any other of the size and nature in print; so that in one volume of moderate size, besides the Treatise on the Globes, an entire course of Astronomy is given, including both the calculations, and the geometrical and physical part; and the author does not believe that he has omitted any thing of importance, that has any particular relation to these subjects.

The teacher will immediately perceive that the work is calculated for three distinct classes of students. The first is, of those who are supposed to be unacquainted with the principles of Mathematics, and who may read the definitions and all the problems on the globes, contained in the 2d and 3d parts. The second class, who are supposed to have some knowledge of Geometry and Trigonometry, may read the notes on the definitions and problems on the globes, and perform the problems by calculation; they may also read some select parts of the 4th part, particularly those relative to the order and motion of the planets in the solar system. The third class, supposed to be somewhat acquainted with the elements of the Conic Sections, Algebra, and the first principles of Fluxions, may continue the 4th part This last class, by finishing the elements of Fluxions, will obtain any further knowledge in Physical Astronomy that may be necessary, being the most proper place for fully investigating this abstruse subject.

The author in presenting this work to the public, is equally regardless of its censure or praise, as his object is neither emolument nor celebrity. His whole aim in the undertaking was to lighten the burden of the Teacher and to improve the student If by comprising in a comparatively small compass all that is useful and necessary either on the Globes or in Astronomy, he succeed in this, his object will be fully attained.

Distance from the press and hurry in the execution, have produced some few errors, most of which are found in the errata at the end.

CONTENTS.

PRACTICAL ASTRONOMY, &c.

IN THE

DEFINITION AND USE OF THE GLOBES.

PART I.

DEFINITIONS, &c.

1. A GLOBE OF SPHERE, is a round solid body, having every part of its surface equally distant from a point within it, called the *centre*. It is formed by the revolution of a semicircle round its diameter, which remains fixed.

2. The *terrestrial globe** is an artificial representation of the earth, having the different countries, empires, kingdoms, chief towns, seas, rivers, &c. truly represented on it, according to their relative situations on the real globe of the earth.

* If a map of the world be accurately delineated on a spherical ball, its surface will represent the surface of the earth. For the highest hills are so inconsiderable with respect to the bulk of the earth, that they take off no more from its spherical figure, than grains of sand do from the spherical figure of an artificial globe. The diameter of the earth is about 7964 miles. Chimborazo, one of the Andes, considered the highest mountain in the world, is about 20,282 feet or nearly 4 miles high. The radius or semi-diameter of the earth is about 3982 miles, and that of an 18 inch globe 9 inches: hence we have this proportion 3982m : 3986m :: 9 in. : 9.009 in. Now by taking the radius of the artificial globe from this, the remainder $.009=\frac{9}{1000}=\frac{1}{111}$ of an inch, nearly, which is the elevation of the highest peak of the Andes on an 18 inch globe. That the globe of the earth is spherical, or nearly so, appears 1. From its casting a spherical shadow on the moon, whatever be its position, when it is eclipsed. 2. From our seeing the further, the higher we are elevated on its surface. 3. From our first seeing the tops of mountains, the masts of vessels, &c. when we advance towards them in any direction. 4. From its having been sailed round from east to west by several persons; and that in whatever direction a ship sails, the stars are elevated above the horizon as many degrees as the vessel sails towards them, and those behind depressed in like manner. Thus in sailing from the equator towards the north pole one degree, the pole star is elevated 1°; in sailing 2°, the pole is elevated 2°, &c. so that if there were a star exactly in the pole, its height would always indicate the number of degrees a place is from the equator or its latitude. This phenomenon could not possibly take place unless the globe was round. 5. From the length of pendulums vibrating in the same time in different parts of the world, being always as the force of gravity (Emerson's Tracts, part 1. prop. 27.) that is, as the distance from the earth's centre (Newton's Principia. b. 3, prop. 6.) But the increase of gravity or weight in passing from the equator to the poles is as the square of the sine of the lat. (Newton, b. 3, prop. 20.) so that the equator is something higher than the poles, the diameters being as

3. A *great circle* of a sphere is any circle on its surface, whose centre is the same as the centre of the sphere. Its plane divides the sphere into two equal segments called *hemispheres.*

Note. The plane of a circle is the surface included within its circumference.

4. A *lesser circle* is that whose centre is different from the centre of the sphere. Its plane divides the globe into two unequal segments.

5. The *axis* of a sphere is the fixed straight line about which the generating semicircle revolves. The *axis* of the *earth*, is an imaginary straight line passing through its centre, and upon which it is supposed to turn. The *axis* of the *artificial globe* is a line which passes through its centre from north to south, and is represented by the wire on which it turns.*

6. The *poles* of a great circle of the sphere, are the two points equally distant from any part of the circumference of that circle.—The *poles* of the *earth* are the extremities of its axis, at the earth's surface ; one of which is called the north or *arctic pole :* the other the south or *antartic pole.* The *celestial poles* are the imaginary points in the heavens corresponding to the terrestrial poles, or the extremities of the earth's axis produced to the heavens.†

7. The *diameter of a sphere* is any straight line which passes through the centre, and is terminated both ways by the surface of the sphere.

8. The *circumference* ‡ of a sphere is any great circle described on its surface.

230 to 229—And the pendulum indicates not only this small difference, but even the difference made in the height of mountains ; for a pendulum that vibrates seconds in a valley, will not vibrate seconds exactly when carried to the top of a mountain. Now if the semi-diameter of the equator be 3982, the polar semi-diameter will be 3964.6. For 230 : 229 :: 3982 : 3964.6 nearly. Hence the radius or semidiameter of the earth at the pole, is shorter than the semidiameter at the equator by $17\frac{1}{2}$ miles nearly. But this difference is so imperceptible on the largest globes, that it is not thicker than the paper and paste on the surface. For suppose the diameter of a globe at the equator be 18 inches, then 230 : 229 :: 9 : $8\frac{221}{230}$ the polar semidiameter ; therefore the difference is $\frac{9}{230}$ of an inch, the flatness of an 18 inch globe at each pole ; a difference less than the 23d part of an inch. Hence though the earth be not strictly speaking a globe, yet no other figure can give so exact an idea of its shape. And a lecturer who informs his hearers that it is in the form of a turnip or orange, gives a very false idea of its true figure. Though 7964 be generally assumed for the earth's diameter, it is however probably something less.

* The diurnal motion of the earth on its imaginary axis is from west to east, and is the cause of the apparent motion of the heavens from east to west. This phenomenon of the earth is not unlike that of a large vessel carried along the current of a river, in which the passengers imagine themselves at rest, and that the banks and objects on shore, which are at rest, are actually in motion.

† The poles of the earth are the same as those of the equator. The poles are 90° distant from the great circle to which they belong.

‡ The circumference of every circle is divided into 360 equal parts called degrees, each degree into 60 equal parts called minutes, each minute into

9. The *equator** is a great circle of the earth equidistant from the poles, which divides the globe into two equal *hemispheres*, *northern* and *southern*.

10. *Latitude of a place* † on the terrestrial globe, is its distance from the equator north or south.

60 equal parts called seconds, &c. The length of a degree is therefore different in different circles, and on the equator is 60 geographical or 69½ English miles nearly. It varies in the respective parallels of latitude towards each pole, in the direct proportion of the cosine of the latitude, or which is the same as the semidiameter of the respective circles. The utility of finding the length of a degree, in order to determine the magnitude and figure of the earth is apparent, and may be rendered familiar to a learner thus; suppose the latitude of New-York be 40° 43′, and that a person travels due north until the latitude be found 41° 43′, then he will have travelled a degree, and the distance between the two places will be its length. Mr. Richard Norwood in 1635 measured the distance between London and York, and found it equal 905751 feet London measure, and observing the difference of latitude to be 2° 28′ found that 1 degree was equal 367196 feet. M. Picard found by a trigonometrical survey, that the distance of the "Pavillon de Malvoisine" south of Paris, to the steeple of the cathedral of Amiens, reduced to the meridian, was 78907 toises. He found also by astronomical observation, that the distance of these places was 1° 22′ 58″; hence 1° 22′ 58″ : 78907 :: 1° : 57064 toises the length of a degree. The assumed distance (in the late French measures) from the equator to the north pole, established on the measure of a degree of the meridian equally distant from both, is 30794580 feet, which divided by 90 gives 342162 feet or 57027 toises. Now as 5280 feet make a mile, therefore 367196÷5280=69.54 (or 69½) miles nearly, which multiplied by 360 produces 25034 the circumference of the earth; but the circumference of a circle is to its diameter as 355 to 113; hence 355 : 113 :: 25034 : 7965 miles the earth's diameter according to Norwood's measure. Again; as 811 French feet are equal to 864 English feet, or 107 to 114 nearly, hence 107 F. f. : 114 E. f. :: 342162 F. f. : 364546 English feet, which divided by 5280 gives 69.04 English miles, the length of a degree, according to the late French measure. Now 342162×360=123178320 French feet the circumference of the earth, and 811 : 864 :: 123178320 : 131228188 English feet =24853.82 miles the circumference, and 355 : 113 :: 24853.82 : 7911.2, the diameter in English miles. According to Picard the circumference is 24871.5 miles, and diameter 7916.8 miles. It was Picard's measure that Sir Isaac Newton has followed in his principia, making the number of toises in a degree=57060 by taking the distance between Malvoisine and Amiens 1° 22′ 25″. See his principia book, 3 prop. 19.

* The equator, so called from its dividing the earth into two equal parts, is, when referred to the heavens, termed the equinoctial, because when the sun appears in it, the days and nights are equal all over the world, viz. 12 hours each. This circle is also by mariners called the line. On this line is found the rt. ascension, oblique ascension, oblique descension, ascensional difference, longitude of places, semidiurnal and nocturnal arches, planetary hour, distinction between north and south latitude of places, difference of longitude, most exact and equal measure of time, &c.

† Difference of latitude is the nearest distance between any two parallels of latitude shewing how far the one is to the north or south of the other, and difference of longitude is the nearest distance between any two meridians either east or west. If the latitude be in the northern hemisphere, it is called north latitude, if in the southern, south latitude. The greatest latitude that a place can have N. or S. is 90°, and the greatest longitude E. or W. is 180°.

11. *Longitude of a place*, is its distance from the first meridian, reckoned on the equator towards the east or west.

12. *Parallels of latitude*, are small circles drawn on the terrestrial globe, through every ten degrees of latitude parallel to the equator.

13. The *tropics** are two lesser circles parallel to the equator, at the distance of 23° 28′ from it; the northern is called the *tropic of Cancer*, the southern the *tropic of Capricorn*.

14. The *polar circles* are two lesser circles, parallel to the equator, at the distance of 66° 32′ from it, or 23° 28′ from each pole.

15. A *zone*† is a portion of the surface of the earth contained between two lesser circles, parallel to the equator; they are *five* in number, one *torrid*, two *temperate*, and two *frigid*.

16. The *torrid zone*‡ is the space contained between the two tropics, and is 46° 56′ broad.

17. The *temperate zones* § are the spaces between the tropics and polar circles, in both hemispheres. They are each 43° 4′ broad.

18. The *frigid zones* are the spaces included within the polar circles.

19. *Amphiscii* || are the inhabitants of the torrid zone, so called because they cast their shadows both north and south at different times of the year.

20. *Heteroscii* is a name given to the inhabitants of the temperate zones, because they cast their shadows at noon only one way.¶

21. *Periscii* are those people who inhabit the frigid zones, because their shadows, during a revolution of the earth on its axis, are directed towards every point of the compass.

* So called from the Greek word *trepo*, to turn, because when the sun comes to either tropic, it begins to return again towards the other.

† So called from *zone* or *zona*, a girdle, being extended round the globe in that form. It is similar to the term climate, for pointing out the situation of places on the earth, but less exact, as there are only five zones, whereas there are 60 climates, as will be seen in its proper place.

‡ This zone was called by the ancients Torrid, because they conceived that being exposed to the perpendicular or direct rays of the sun, the heat must be so great, and the country so barren and parched, as to render it entirely uninhabitable. But this idea has long since been refuted. The sun is perpendicular twice in the year to every part of this zone.

§ These zones were called temperate by the ancients, because, meeting the sun's rays obliquely, they enjoy a moderate degree of heat, the sun being never perpendicular to any part of them. The breadth of the temperate zones increases a little every year, whilst that of the torrid and frigid zones decrease in the same proportion, owing to the annual decrease of the obliquity of the ecliptic.

|| When the sun is vertical or in the zenith, which happens twice a year, they are then called *ascii*, or shadowless, because at that time they have no shadow.

¶ Thus the shadow of an inhabitant of the north temperate zone always falls to the north at noon, because the sun is then directly south; and an inhabitant of the south temperate zone casts his shadow towards the south at noon, because the sun is due north at that time. These distinctions are however rather trifling.

22. The *antœci* * are those who live under the same meridian, and in the same latitude, but on different sides of the equator.

23. *Periœci* † are those who live in the same latitude, but in opposite longitudes.

24. *Antipodes* ‡ are those inhabitants of the earth, who live diametrically opposite to each other.

25. *Meridians* § are great circles passing through the poles, and cutting the equator at right angles.

* The antœci have the same hours, but contrary seasons of the year; thus when it is noon with one, it is noon with the other, &c. But when it is summer with one, it is winter with the other, &c. consequently the length of the days with one, is equal to the length of the nights with the other; the sun when in the equinoctial rises and sets to the one at the same time that it rises and sets to the other, &c. Those who live at the equator have no antœci.

† The periœci have their seasons of the year at the same time, and also their days and nights of the same length with each other; but when it is noon with the one it is midnight with the other, and when the sun is in the equinoctial, he rises with one when he sets with the other. Those who live under the poles have no periœci. Their difference of longitude is 180°.

‡ The antipodes have both their latitude and longitude different, and consequently both their seasons and hours; so that when it is summer with one it is winter with the other; when it is twelve o'clock in the day with one it is twelve at night with the other. They have like seasons, and the same length of days and nights, but at different times. When they stand, their feet are towards one another, and their heads opposite. Hence that part of the heavens which appears over the head of one, seems to be beneath or under the feet of the other; and therefore, when we speak of *up* or *down*, we speak relatively and only with regard to ourselves; for no point, either in the heavens, or on the surface of the earth is *above* or *below*, but only with respect to ourselves. Upon whatsoever part of the earth we stand, our feet is always nearly directed towards the centre, and our head towards the sky; in the latter case we say *up*, in the former *down*.

§ These are so called from the Latin word meridies, midday, because when the sun is on any of these meridians, it is then noon or 12 o'clock, in all places under that meridian. Every place on the globe is supposed to have a meridian passing through it, though on most globes there are but 24, the deficiency being supplied by the brass meridian, which is therefore called the universal meridian. They are drawn through every 15° of the equinoctial, and are therefore sometimes called hour circles, the reason of which is evident; for if 360°, the number of degrees in a circle, be divided by 24, the hours in one day, the quotient 15° will give the number of degrees corresponding to each hour. Geographers assume one of these meridians as the first, commonly that which passes through the metropolis of their own country, but the general practice is, to reckon longitude from the meridian of Greenwich observatory in England. The brazen meridian is divided into 360 equal parts, called degrees, these are again supposed to be divided into 60 equal parts, called minutes, and these into 60 equal parts, called seconds, &c. to thirds, fourths, fifths, &c. On the globes, however, the degrees are seldom subdivided into fewer parts than quarters. In the upper semicircle of the brass meridian, the degrees are numbered from 0 to 90 from the equator towards the poles, and are generally used in finding the latitude of places. On the lower semicircle they are numbered from 0 to 90, reckoning from the poles towards the equator, and are principally used in elevating either of the poles to the latitude, &c.

26. The *brazen meridian* (or universal meridian) is the brass circle in which the artificial globe turns.

27. The *first meridian* is that from which geographers begin to count the longitude of places.

28. *Hour circles*,* or *horary circles*, are the same as the meridians; they are supplied by the brass meridian, the hour circle and its index.

29. The *hour circle* or index, is a small circle of brass fixed to the north pole, and on which the hours of the day are marked

30. The *ecliptic* † is that great circle in which the earth performs its annual motion round the sun, or in which the sun seems to move round the earth once in a year.

31. *Signs* of the ecliptic are the 12 equal parts into which it is divided. The signs and the days on which the sun enters them are

* These circles are drawn through every 15° of longitude reckoning from any meridian, for the reason given above, but on *Cary's globes* they are drawn through every 10°, as on a map, though without answering any useful purpose. As 15° correspond to an hour, 4 minutes of time must correspond to each degree, 2 minutes to half a degree, 1 minute to one quarter of a degree, &c. (see Keil's astr. lect. 18.) On some globes the index, which points out the hours, has two rows of figures on it, others but one. On Bardin's new British globes, there is an hour circle at each pole numbered with two rows of figures. On Cary's there is but one hour circle placed under the brass meridian at the north pole, marked with only one row of figures, and is therefore more convenient, as it answers every purpose to which a circle of this kind can be applied, without that confusion generally arising from two rows of figures. On *Adams'* common globes there is but one index; but on his improved globes the hours are counted by a brass wire with two indexes placed over the equator. On many of the globes fitted up by *Jones*, the hour circle is calculated to slide on the brass meridian, for the conveniency of pointing out the bearings of places, &c. These circles are however of little consequence, as the equator and quadrant of altitude will answer every purpose to which they can be applied.

† The ecliptic (so called, because the eclipses of the sun and moon can happen only in the plane of this circle) makes an angle of 23° 28′ with the equinoctional, one half being in the northern hemisphere, and the other in the southern. The *spring* and *autumn* signs being in the northern hemisphere, are therefore called *northern signs*; the other six, or the *summer* and *winter* signs, being in the southern, are for the same reason called *southern signs*. The spring and autumnal signs are likewise called *ascending* signs, because when the sun is in any of these signs, his declination is increasing; the summer and winter signs are called *descending* signs, because when the sun is in any of them, his declination is decreasing. Each of these signs is divided into 30°, &c. and in whatever sign and degree the sun is, that point is called the *sun's place*. The day of the month corresponding to the sun's place is likewise commonly marked on this circle. The equinoctial point *aries* is that point from which the sun's place or longitude is reckoned, without any regard to the constellations themselves, which, on account of the precession of the equinoctial points, are now a whole sign advanced from west to east, or according to the order in which the signs are reckoned. Besides the sun's place or longitude, his apparent and annual motion, stars longitude, poetical rising and setting, increase and decrease of days, culminating degree, eclipses of the sun and moon, distinction of north and south latitude of the stars, &c. are also found on this circle.

as follows, according as they are represented on Cary's globes. The beginning of each day is to be taken.

Spring signs.	*Summer signs.*
♈ *Aries*, the ram, 21st of March.	♋ *Cancer*, the crab, 21st of June.
♉ *Taurus*, the bull, 20th of April.	♌ *Leo*, the lion, 23d of July.
♊ *Gemini*, the twins, 21st of May.	♍ *Virgo*, the virgin, 23d of August.
Autumnal signs.	*Winter signs.*
♎ *Libra*, the balance, 23d of Sept.	♑ *Capricornus*, the goat, 22d of Dec.
♏ *Scorpio*, the scorpion, 23d of Oct.	♒ *Aquarius*, the water-bearer, 20th of January.
♐ *Sagittarius*, the archer, 22d of Nov.	♓ *Pisces*,, the fishes, 18th of Feb.

32. The *equinoctial points** are Aries and Libra, where the ecliptic cuts the equinoctial.

33. The *solstitial points*† are Cancer and Capricorn.

34. The *colures*‡ are the two meridians passing through the equinoctial and solstitial points. The one called the equinoctial, and the other the solstitial colure.

35. The *horizon* § is a great circle, which separates the visible half of the heavens from the invisible. It is distinguished into two kinds, the *sensible* and the *rational*.

* The point aries is called the *vernal equinox*, and the point libra the *autumnal equinox*. When the sun is in either of these points, the days and nights are equal on every part of the globe.

† When the sun is in or near these points, the variation in his meridian, or greatest altitude, is scarcely perceptible for several days, because the ecliptic, near these points, may be considered nearly parallel to the equinoctial, and hence in these points, the sun does not perceptibly vary his declination for some days. When the sun enters the beginning of *cancer*, all the inhabitants on the north side of the equator have their *longest day*, and those in the southern hemisphere their *shortest*. When he enters *capricorn*, the inhabitants of the northern hemisphere have their *shortest day*, and those in the southern their *longest*. The learner must notice, that when the sun enters cancer, all the inhabitants within the north polar circle have *constant day*, and those within the south polar circle *constant night*, but when the sun enters capricorn, the reverse happens. They are called *solstices* from the circumstance of the sun's *standing still*, or having no motion when he is in either of these points, hence said to be *stationary* (solis statio.)

‡ These colures divide the ecliptic into four equal parts, and mark the *four seasons* of the year. In the time of *Hipparchus* the equinoctial colure is supposed to have passed through the middle of the constellation aries.— Hipparchus was born at Nicæa, a town of Bythinia in Asia minor, about 75 miles S. E. of Constantinople, now called Isnic; he flourished between the 154th and 163d olympiads, or between 160 and 135 years before Christ. He foretold eclipses, and as Pliny remarks, was the first who dared to number the stars for posterity, and reduce them to a standard. He gave a catalogue of 1022 stars, and rendered many other important services to astronomy.

§ Horizon takes its name from the Greek word *orizon (finiens)* because it defines or bounds our view. The sensible horizon extends only a few miles; thus at the height of 6 feet, the utmost extent of our view on the earth, or sea, would be 2.42 miles; at 20 feet 4.4 geographical miles, &c. In general, if h be the height of the eye above the surface of the sea, and d the diameter of the earth in feet, then $\sqrt{\overline{d+h}\times h}$ will nearly shew the greatest extent to which a person can see, or the diameter of the sensible horizon, the centre being supposed at the eye. (Euclid, 36 prop. 3b.) This

36. The *sensible* or apparent horizon is that circle that terminates our view, where the sky, and the land or water, appear to meet.

37. The *rational* or real horizon, is an imaginary circle, whose plane passes through the centre of the earth, parallel to the plane of the sensible horizon.

38. The *wooden horizon* is that circular plane circumscribing the artificial globe, which represents the rational horizon on the real globe.

39. The cardinal points of the horizon, are the east, west, north and south points.*

40. The cardinal points in the heavens, are the zenith the nadir, and the points where the sun rises and sets.

41. The cardinal points of the ecliptic are the equinoctial and solstitial points, which mark out the four seasons of the year ; and the cardinal signs are ♈ Aries, ♋ Cancer, ♎ Libra, and ♑ Capricorn.

42. The Zenith is a point in the heavens exactly over our heads, and is the elevated pole of our horizon.

determines the apparent rising, setting, &c. of the sun, stars, planets, &c. The rational horizon determining their real rising, setting, &c. The wooden horizon respecting the rational horizon on the real globe of the earth, is divided into several concentric circles. On *Bardin's new British globes* they are arranged in the following order : the 1st circle marked *amplitude*, is numbered from the east towards the north and south, from 0 to 90°, and from the west towards the north and south in the same manner. The 2d circle marked *azimuth*, is numbered from the north and south points of the horizon towards the east and west from 0 to 90°. The 3d circle represents the 32 *points of the compass*. The degrees belonging to these may be found in the circle of amplitude. The 4th circle contains the *twelve signs of the Zodiac*. The 5th, the degrees corresponding to each sign, each comprehending 30°. The 6th contains the *day of the month* corresponding to each degree, &c. of the sun's place in the ecliptic. The 7th contains the *equation of time*, the sign + shews that the clock is faster than the dial by so many minutes, the sign — that it is slower, and the number of minutes in the difference is expressed opposite the corresponding days of the month. The 8th circle contains the *twelve calendar months* of the year, &c. These circles are in the same order on Cary's globes, except that of the equation of time, which is represented on a vacant part of the globe between the tropic's, nearly in the shape of the figure 8. The days of the month being marked in the curve of the figure, and the time or equation on a small scale drawn through that point where the curve of the figure intersects, in a direction parallel to the equator.

Though the rising and setting of the stars respect the rational horizon, and the place of observation reduced to the earth's centre, yet it holds true of the sensible horizon, the spectator being placed on the earth's surface, on account of the great distance of the fixt stars, the semidiameter of the earth being no more than a point at that immense distance.

* The *east* is that point of the horizon where the sun rises when in the equinoctial, and the *west* is the point directly opposite on the plane of the horizon, or where the sun sets when the days and nights are equal : the *south* is 90° distant from the east or west, and is that point towards which the sun appears at noon, to those situated in north latitude, and the *north* is that point of the horizon directly opposite to the south.

43. The *nadir* is a point in the heavens opposite to the zenith, or directly under our feet, and is the depressed pole of our horizon.

44. The *mariners compass* is a representation of the horizon, which is divided into 32 equal parts, and is so called from its being used to ascertain the course of a ship at sea.

45. The *variation of the compass** is the deviation of its points from the corresponding points in the heavens, or the angle formed between the true and magnetic meridian, and is reckoned towards the east or west.

46. *Azimuth* or *vertical circles* are imaginary circles passing through the zenith and nadir, cutting the horizon at right angles.†

47. The azimuth of any object in the heavens is an arch of the horizon, contained between a vertical circle passing through the object, and the north or south points of the horizon.

48. The *prime vertical* is that azimuth circle, which passes through the east and west points of the horizon.‡

49. The *altitude* of any object in the heavens is an arch of a vertical circle, contained between the centre of the object and the horizon.

50. The *zenith distance* of any celestial object is an arch of a vertical circle, intercepted between the centre of the object and the zenith.

51. The *meridian altitude*, or meridian zenith distance, is the altitude or zenith distance, when the object is on the meridian.

52. The *polar distance* of any celestial object, is an arch of the meridian, contained between the centre of that object and the pole of the equinoctial.

53. The *quadrant of altitude* is a thin slip of brass, one edge of which is divided into degrees, &c. equal to those of the equator, and is used to find the distances of places, &c. on the earth, and the distances, altitude, &c. of bodies in the heavens.

54. The *amplitude* of any object in the heavens, is an arch of the horizon contained between the centre of the object, when rising or setting, and the east or west points of the horizon. Or it is the number of degrees which the sun or a star rises from the east and sets from the west. §

* See the note to definition 54 and problems 49 and 50, part 2d.

† The altitudes of the heavenly bodies are measured on these circles; they may be represented by the quadrant of altitude screwed in the zenith of any place and moving the other end along the wooden horizon of the globe. These circles are always at right angles to the horizon.

‡ This is always at right angles both with the brazen meridian and horizon.

§ In our summer the sun rises to the north of the east and sets to the north of the west; and in the winter it rises to the south of the east and sets to the south of the west. The amplitude and azimuth are in point of utility, much the same; the amplitude shewing the bearing of any object when it rises or sets, from the east or west points of the horizon, and the azimuth the bearing of any object when it is above the horizon, either from the north or south points thereof. They are generally useful in determining

55. *Time** is that succession in the existence of beings, which have a beginning and will have an end, and is measured by the motion of some moving body. It is distinguished into years, months, weeks, days, hours, minutes, &c.

56. *Time* is either *absolute* and *relative*, *true* and *apparent*, or *mathematical* and *common*. *Absolute*, *true*, and *mathematical time*, of itself and from its own nature flows equably, without regard to any thing external, and by another name is called *duration ; relative*, *apparent*, and *common* time, is some sensible and external measure of duration, by means of motion, whether accurate or unequal, and is commonly used instead of true time.

57. The *equation of time* † is the difference between the absolute and relative time, or it is the difference of time shewn by a well regulated clock and a correct sun dial.

58. *Apparent noon* is the time when the sun comes to the meridian, viz. 12 o'clock, as shewn by a correct sun dial.

the variation of the magnetic needle. For if the observed and true amplitudes be both north or both south, their difference will be the variation ; but if one be north and the other south, their sum will be the variation. In like manner if the true and observed azimuth, be both east or both west, their difference will be the variation ; if otherwise, their sum will be the variation. The variation is easterly, when the true bearing is to the right hand of the magnetic bearing, but westerly when to the left hand ; the observer being supposed to look directly towards the point representing the magnetic bearing.

* What *time* is in itself, or what its physical essence is, no philosopher can fathom or define, but this we know, and it is the most important knowledge for us, if reflected on, that it hurries us to that *eternity* in which time has no existence, and that every moment may be the last—" *momentum a quo tota pendit æternitas.*" If then it be necessary to consider time, as it regulates our seasons, is it not more necessary to consider it, as it relates to an immortal existence towards which it imperceptibly hurries us. Truths of this nature are better calculated to expand our ideas, and point out to us that state which has no termination or limits, and in which we are destined to enjoy a dignified existence, than those whose objects are as fleeting as time itself; for as soon as futurity begins to expand its extensive prospects, then we see the vanity of what the world sets such a value on, and learn to value those things alone which are immortal.

† The equation of time arises from two principal causes, the sun's unequal motion in the ecliptic, describing the southern signs in less time than the northern, the difference amounting to about eight days; and from the obliquity of the plane of the ecliptic to that of the equator. For the space between two meridians, or hour lines on the ecliptic will not, in consequence, be always the same as the space between the same meridians on the equator, the difference being sometimes greater, sometimes equal, and sometimes less ; but as the sun in consequence of this difference, takes sometimes less, sometimes more than 24 hours, in revolving from any meridian, until his return to the same again, it thence follows that the hours shewn by a well regulated clock, must be different from those shewn by a true sun dial, and hence the equation of time. If the sun performed its annual revolution in the plane of the equator, there would be no equation except what arises from the difference in his annual motion. (see prob. 22, part 2d, Keil lect. 25, Ferguson, chap. 13, or Mayer's tables, published by Nevil Maskelyne, and note to prob. 8.)

59. *True* or *mean noon* is the middle of the day, or 12 o'clock, as shewn by a well regulated clock, adjusted to go 24 hours in a mean solar day.

60. An *hour* is a certain determined part of the day, and is either equal or unequal. An equal hour is the 24th part of a mean natural day, as shewn by well regulated clocks, &c. unequal hours are those measured by the returns of the sun to the meridian, or those shewn by a correct dial. Hours are divided into 60 equal parts called minutes, a minute into 60 equal parts called seconds, a second into 60 equal parts called thirds, &c.

61. A *true solar day*,* is the time from the sun's leaving the meridian of any place on any day, till it returns to the same meridian on the next day. Or it is the time elapsed from 12 o'clock at noon, on any day, to 12 o'clock at noon on the next day, as shewn by a correct sun dial.

62. A *mean solar day*,† is the *space* of time consisting of 24 hours, as measured by a clock or time-piece.

63. The *astronomical or natural day*,‡ is the time from noon to noon, as shewn by a correct dial, and also consists of 24 hours.

* A true solar day is subject to a continual variation, arising from the obliquity of the ecliptic and the unequal motion of the earth in its orbit; the duration thereof sometimes exceeds and sometimes falls short of 24 hours, as taken notice of in the note on the equation of time. The variation is the greatest about the 1st of November, when the solar day is 16′ 15″ less than 24 hours, as shewn by a well regulated clock.

† There are in the course of a year as many *mean solar days* as there are *true solar days*, the clock being as much faster than the sun dial on some days of the year, as the sun dial is faster than the clock on others, as may be seen by consulting the analemma or the circle on which the equation of time is marked on the globes. Thus the clock is faster than the sun dial from the 24th of December to the 15th of April, and from the 16th of June to the 31st of August; but from the 15th of April to the 16th of June, and from the 31st of August to the 24th of December, the sun dial is faster than the clock. When the clock is faster than the sun dial, the true solar day exceeds 24 hours; and when the sun dial is faster than the clock, the true solar day is less than 24 hours; but when the clock and sun dial agree, viz. about the 15th of April, 16th of June, 31st of August, and 24th of December the true solar day is exactly 24 hours. (See the table annexed to problem 21.)

‡ This is called a natural day, being of the same length in all latitudes. It begins at noon, because the increase and decrease of days, terminated by the horizon are very unequal among themselves; which inequality is likewise augmented by the inconstancy of the horizontal refractions (see § 183 Ferguson's Astronomy) and therefore the astronomer takes noon, or the moment when the sun's centre is on the meridian, for the beginning of the day. The hours are reckoned in numerical succession from 1 to 24. Navigators begin their computation at noon 24 hours before the commencement of the astronomical day, reckoning their hours from 1 to 12; the first 12 hours are marked A. M. (*ante meridiem*) or forenoon, the second P. M. (*post meridiem*) or afternoon. All the calculations in the nautical almanac are adapted to astronomical time. The declination, &c. there calculated, is adapted to the beginning of the astronomical day, or to the end of the sea day; it being at the end of the sea day, that mariners want the declination, to determine their latitudes.

64. The *artificial day*, is the time elapsed between the sun's rising and setting, and is variable according to the different latitudes of places. *Night*, is the time from sun setting to sun rising, and varies in like manner.

65. The *civil day*,* like the astronomical or natural day, consists of 24 hours, but begins differently, according to the customs of different nations.

66. A *sidereal day*, is the interval of time from the passage of any fixed star over the meridian, till it returns to it again; or it is the time which the earth takes to revolve once round its axis, and consists of 23 hours, 56 minutes, 4 seconds.

Note. *Though we suppose the earth to turn on its axis once in 24 hours from west to east, yet its exact revolution is as above, making about 366 revolutions in 365 days. But as the sun advances about 1° in its orbit daily,*[a] *which corresponds to about 4′ of time, the day is properly taken 24 hours, because the earth has to advance 1° more on its axis to have the sun over the same meridian as on the preceding day.*

* The ancient Babylonians, Persians, Syrians, and most of the eastern nations, began their day at sunrising, which custom is followed by the modern Greeks. The ancient Greeks, Jews, &c. began their day at sunsetting, and this custom is observed by the modern Austrians, Bohemians, Silesians, Italians, Chinese, &c. The Arabians begin their day at noon like the astronomers. The more ancient Jews, together with the ancient Egyptians, Romans, &c. began their day at midnight, and this custom is followed by the English, French, Germans, Dutch, Spanish, Portuguese and Americans. The famous astronomers Hipparchus, Copernicus, and some others, began their day in like manner from midnight. Those who begin their day at sun rising, have this advantage, that their hours tell them how much time is already past since sun rising; and they who reckon their hours from sun setting, know how long it is to sun setting; and hence they may proportion their journies and labours for that time. But both have this inconvenience, that their midday and midnight happen on different hours, according to the seasons of the year. The Babylonians, &c. reckoned 24 hours in order, from sun rising to its rising again, hence called *Babylonish hours*. In Italy, &c. where they reckon their day from sun setting, they likewise reckon 24 hours in order: these hours are hence called *Italian hours*. The Jews and Romans formerly divided the artificial days and nights each into 12 equal parts; these are termed *Jewish hours*, and are of different lengths, according to the seasons of the year. This method of computation is now in use among the *Turks*, and the hours are styled the *first hour*, *2d hour*, &c. of the day or night, so that midday always falls upon the 6th hour of the day. These hours were also called planetary hours, because in each of these hours one of the seven planets was supposed to preside over the world. The first hour after sun rising on Sunday was allotted to the *Sun*, the next to *Venus*, the 3d to *Mercury*, and the rest in order to the *Moon*, *Saturn*, *Jupiter* and *Mars*. By this means, on the first hour of the next day, the moon presided, and so gave the name to that day; and thus the seven days had names given to them respectively, from the planets that were supposed to govern on the first hour.

[a] The earth revolves round the sun in 365¼ days nearly, and the ecliptic consists of 360°, hence 365¼D : 360° :: 1D : 59′ 8″ 2, the daily mean motion of the earth in its orbit, or the apparent mean motion of the sun, in a day.

67. A *week*,* is a system of seven days, each of which is distinguished by a different name.

68. A *month*, is properly the space of time the moon takes to perform one revolution round the earth, and is either astronomical or civil.

69 The *astronomical month*, is the time in which the moon passes through the zodiac (or that zone in which are the 12 constellations or signs, through which the sun passes.) This month is either periodical or synodical.

70. The *periodical month*, is the time intervening, in a revolution of the moon, from her leaving any point until her return to the same, and is equal to 27 days, 7h. 43′ 5″.

71. The *synodic month*, or *lunation*, is the time between the moon's parting with the sun at a conjunction, until her return again, or the time between two new moons, and is equal to 29d. 12h. 44′ 3″.

72. The *civil months*, are those which are framed for the uses of civil life, and are different, as to their names, number of days, and times of beginning, in different countries.†

* A week is the most ancient collection of days that ever was, as is evident from the sacred writings. The Jews always made use of this collection, and every other nation since the establishment of christianity, wherever it has been received. All nations that have any notion of religion, set apart one day in seven for *public worship*. The *Jews* observe Saturday, or the seventh day of the week, for their *Sabbath* or day of rest, being that appointed in the 3d commandment under the law. But the day solemnized by christians, is *Sunday* or the first day of the week, being that in which our Saviour rose from the grave, and on which the apostles afterwards used more particularly to assemble together, to perform divine worship. The French had adopted a calendar entirely new, soon after the abolition of royalty in 1792 ; " But from the rapidity with which every thing is there returning to the ancient customs (says a late writer) it is probable that in a short time it will be discontinued." This has accordingly since taken place, which shews no less the truth of the assertion, than the futility of the project.

† Thus the first month of the Jewish year fell, according to the moon, in our August and September, old style; the 2d in September and October, &c. The first month of the Egyptian year began on the 29th of our August: the first month of the Arabic and Turkish year began the 16th of July: the first month of the Grecian year fell according to the moon in June and July; the 2d in July and August, &c. For a further account of these, see Ferguson's Astronomy, chapter 21.

The names, both of the weeks and months, which are now adopted in civil use, originated among our heathen ancestors, and, as well as the names of the constellations in the heavens, alluded to some part of their idolatrous worship, or to their gods. The months, as will be shewn in the following note, for the most part, took their names from some of these gods. The Jews, before the Babylonish captivity, reckoned their twelve months in numerical order, 1st. 2d. 3d. &c. Sometimes, however, besides these ordinal names, they distinguished some of their months by particular appellations, alluding to the season, &c. Thus the 1st month was called *Abib* (Exod. 13, v. 4.) the 2d. *Zius* (3 Kings 6, v. 1 & 37.) the 7th. *Ethanim* (3 Kings 8, v. 2.) the 8th. *Bul*, &c. Where the first *Abib* signifies the *month of new corn*, &c. This month answers to the *Nisan* of the ancient Syrians (whose months, with little variation, the Hebrews adopted after their captivity.) The *Phanemoth*

73. That space of time in which the sun describes one sign, or 30° of the ecliptic, is also called a *solar month*, and is about 30½ days.

74. A *year** is properly the space of time measured by a revolution of the sun in the ecliptic, and is of several kinds.

of the Chaldeans and Egyptians, the *Elaphebolion* of the Athenians, the *Xanthikos* of the Macedonians and other Grecian states, with the Maccabees and all Syria, the *Muharram* of the Arabs and Turks, the *Martius* of the Pagan Romans and our *March*. (See Censorinus, &c.) July and August, among the old Romans, were called Quintilis and Sextilis, and hence, except a few of their first months, they agreed with the ancient Jewish reckoning.

The days of the week were also called, by all the idolatrous nations, after the names of the planets, and these after the names of their pretended gods. Thus the first day was called Dies Solis (the sun being the principal luminary) the 2d. Dies Lunæ; the 3d. Dies Martis; the 4th. Dies Mercurii; the 5th. Dies Jovis; the 6th. Dies Veneris, and the 7th. Dies Saturni. The Saxons called the days of the week by the name of the idol which on that day they particularly worshipped. Thus the first day was called *Sunday*, from their worshipping the sun on that day; the 2d. *Monday*, from their worshipping Diana, or the moon; the 3d. *Tuesday*, from their idol Tuisco or Tew, the Saxon name of Mars; the 4th. *Wednesday*, from Woden or Odin, another of their idols; the 5th. *Thursday*, from their idol Thor, the Saxon name of Jupiter or Jove; the 6th. *Friday*, from Friga or Frigidag, supposed to be the Venus of the Saxons, and the 7th. *Saturday*, Saeter or Saetor, an idol by them then worshipped. For this reason some reject their names, but in general all the modern Europeans adopt them, to avoid that confusion in their calendar resulting from the introduction of new names. The Jews, however, called the days of the week Sabbaths. In St. Mark, c. 16, v. 2, the first day of the week is called *Una Sabbatorum*, and in v. 9, *Prima Sabbati*. In St. John, c. 20, v. 1, the first day of the week is called *Una Sabbati*. In St. Luke, c. 18, v. 12, twice in the week is called *Bis in Sabbato*, &c. The Latin church called the days of the week *Feriæ* (holidays or days of rest.) Thus Feria prima (Sunday) Feria secunda (Monday) &c. Feria being the same as Sabbath, which signifies rest or cessation. These latter denominations of the days of the week, evidently allude to the six days, in which God created the world, ordering the 7th to be a day of rest, from which the others, therefore, took their name. And hence this division of the week is anterior to all others.

* Or a year is that period of time in which all the variety of seasons return and afterwards begin anew. The ancient *Romans* divided the year into twelve calendar months, to which they gave particular names, as follows: January from *Janus*, the most ancient king of Italy, to whom the people dedicated this month. February from the latin word *Februo*, to purify. In this month the ancient Romans, particularly the priests of Pan, made use of purifications and sacrifices for the ghosts of the dead.— March from their god *Mars*, to whom this month was kept sacred. April from *Aperio*, to open or unfold, because in this month the spring begins to disclose all the beauties of the vegetable creation; or as some suppose from the Greek appellation of *Venus*. May from *Maia*, or *Maius*, a heathen goddess, to whom this month was kept sacred. June from the heathen goddess *Juno*, or as some say from *Juvenis* a youth, as nature, in this month, appears in the vigour and bloom of youth. July from *Julius Cæsar*, the Roman general. August from *Augustus Cæsar*, the first Roman emperor. September, October, November, and December, from *Septem, Octo, Novem*, and *Decem;* these months in the Roman calendar, being the 7th, 8th, 9th, and 10th months, their year beginning on March and ending on February. (See Adams Roman Antiquities, page 327.) April,

75. A *solar* or tropical year, is the time measured by the sun in the ecliptic, in passing from one equinox or solstice, until it returns to the same again, and is equal to 365 days, 5 hours, 48 minutes, 48 seconds.

76. A *sydereal year* is the space of time which the sun takes, in passing from any fixed star, till it returns to the same again, and consists of 365 days, 6 hours, 9 minutes, 8 seconds.

77. The *civil year* is the common or political year, established by the laws of a country, and is either *lunar* or *solar*.

June, September, and November, has each 30 days, February in common years has 28 days, but in leap year 29. Each of the rest has 31 days.—The year is also divided into four quarters, viz. *Spring*, *Summer*, *Autumn* and *Winter*. These quarters are properly made when the sun enters into the equinoctial and solstitial points of the ecliptic; but in civil uses they are differently reckoned, according to the customs of several countries.

The year used by the ancient *Grecians* and *Romans* did not exactly agree with the motion of the sun, and hence their Winters, Summers, and in general their seasons, were found every year to differ considerably, so that the same seasons did not uniformly happen in the same months, as in a well regulated calendar should be the case. They therefore proposed the rising and setting of the stars instead of this erroneous calendar, not doubting that the sun returning to the same place in respect of the same fixed star, did not come to the same place again in respect to the equinoxes or solstices; that is when the same star should rise or set cosmically, achronically, or heliacally, the same seasons should again return. And hence we find such use made by the antients, of what is called the *poetical* rising and setting of the stars. In time, however, this method was found erroneous, and is now almost entirely out of use, owing to the *precession*, or rather *recession* of the *equinoctial points*, not then taken notice of. The difference between a solar and sidereal year being 20° 24′, and as the sun returns to the equinox every year before it returns to the same point in the heavens, the equinoctial points have therefore a slow motion backwards. The sun apparently describing the whole ecliptic or 360° in a tropical year; hence we have this proportion, $365\frac{1}{4}d : 360° :: 1d : 59' 8'' 2$. the daily mean motion of the earth, or the apparent mean motion of the sun in a day, and therefore the sun in 20′ 24″ of time, describes $50\frac{1}{4}''$. For $1d : 59' 8'' :: 20' 24'' : 50\frac{1}{4}''$ the precession of the equinoxes, nearly corresponding to what *Newton* makes it, as derived from *physical causes.* (prop. 39, b. 3, of his principia.) This slow motion of the sun in receding from the equinoctial points, every year, is called the precession of the equinoxes, and is performed from east to west, contrary to the *order of the signs*, which is from west to east. Now $50\frac{1}{4}'' : 1yr. :: 360° :$ 25791 years, the time in which the equinoctial points would perform one revolution. Hence in 2149 years the stars would appear to recede 30° or 1 sign backwards. In the time of Hipparchus, the equinoctial points were fixed in Aries and Libra; but now these signs are 30° to the eastward, &c. so that Aries is now in taurus, taurus in gemini, &c. hence the rising and setting of the stars, at particular seasons of the year, as described by later writers, such as *Hesiod*, *Eudoxus*, *Pliny*, &c. do not answer their description.

The anticipations of the seasons is not however owing to the precession of the equinoctial and solstitial points in the heavens, which can only effect the apparent motions, places and declinations of the fixt stars; but to the difference between the civil and solar year, which is 11′ 12″ (commonly reckoned 11′ 3′.) From this difference it would happen that in 129 years

78. The *lunar year* is the time measured by twelve synodic revolutions of the moon, and consists of 354 days, 8 hours, 48 minutes, and 36 seconds.

there would be a difference of one day in the time of the equinoxes, and therefore a difference of 10 days in 1260 years, the time between the *Nicene Council*, A. D. 325 and 1585, when *Pope Gregory*, 13*th* reformed the Julian calendar. Hence the vernal equinox happening the 21st of March in the time of the council of Nice, in the year 1582 it was found to happen on the 11th. That the equinox might therefore be reduced to its former place, 10 days were suppressed in the month of October in the year 1582, and the 5th day was called the 15th, and thus the 11th day of the March following, being the time of the equinox, became the 21st as in the time of the council of Nice, which fixed the time of keeping *Easter*. At the time that Cæsar with the assistance of Sosigines reformed the calendar, the vernal equinox happened on the 25th of March. Besides the above, another correction was found necessary, that the same seasons might be kept to the same times of the year, as one day every fourth year, according to the Julian intercalation, was found too much. For this purpose the *bissextile day* in February, at the end of every century of years not divisible by 4, was to be omitted, and these years reckoned as common years (every 4th year according to the Julian or civil account being bissextile or leap year.) Thus the 17th, 18th, 19th centuries, or the years 1700, 1800, 1900, &c. which according to the Julian account would be leap years, were to be reckoned as common years, (17, 18, 19, &c. not being divisible by 4) and to retain the bissextile day at the end of those centuries divisible by 4, as the 16th, 20th, 24th, &c. or the years 1600, 2000, 2400, &c. By this correction the difference between the civil and solar accounts will differ no more than 2 hours in 400 years, and in less than 5082 years will not amount to a whole day, at the end of which time a new correction for this day will become necessary. Without these changes, the seasons in time would be entirely reversed with regard to the months of the year. It was Julias Cæsar who first ordained that one day should be added to February, every 4th year, by causing the 24th to be reckoned twice; and because this 24th day was the 6th (*sextilis*) before the *Kalends* of March, there were in this year two of these *sextiles*, and hence this year was called *bissextile*. This being corrected, was thence called the *Julian year*, afterwards the *Gregorian*, from the farther corrections of *Pope Gregory*. This Gregorian year is now received in almost every country where truth or exactness is regarded, and from hence is called the *Civil year*. The Civil year thus corrected took place in different countries of Europe at different times, and was not adopted in England until A. D. 1752, at which time a correction of 11 days became necessary, the 3d of September being called the 14th. —This is now called the *New Stile*, as the Julian is called *Old Stile*. The year 1700, happening between the time of the correction by *Pope Gregory*, and that made by the *British*, this year in the Gregorian account was considered as a common year, and thus a day was omitted, which in the Julian was not; and as the Gregorian account omitted 10 days in the beginning, the English omitted 11, to make their's agree with the former. And moreover, as the year 1800 was a common year, there is now 12 days difference between the old and the new style. It is almost needless to mention, that in 1752, the United States were British colonies, and hence the corrected account or new style was here adopted at the same time, and is the account now in use.

The beginning of the year in different countries is no less various than its form: but a further detail would be inconsistent with the plan of this

79. The civil *solar year*, or julian year, is a period of 365 days, 6 hours, but the common years contain only 365 days, and every 4th year or bissextile 366 days.

80. A *cycle*,* is a period of time, after which the same phenomena of the celestial bodies begin to occur again, in the same order.

abridged introduction. Those who wish to see more on this subject, may consult Gregory, Keil, Ferguson, Ewing, or Vince's astronomy, or vol. 3 of Ozinam's Mathematical Recreations.

* In the cycle of the sun the return of the days, &c. does not differ 1° in 100 years, and the leap years begin their course again with respect to the days of the week on which the days of the month fall. In the cycle of the moon the new and full moons return, &c. within $1\frac{1}{2}$ hours of the time in which they happened on the same days of the month, 19 years before; hence in 312 years this difference increases to a whole day, so that this cycle can only hold for that time, and hence for the next 312 years the golden number ought to be placed one day earlier in the calendar. This correction is however made at the end of whole centuries, and hence at the end of 300 years the new moon is advanced 1 day for 7 times successively, that is, during 2100 years. To account for the odd $12\frac{1}{2}$ years, they deferred putting the moon forward to the end of 400 years, making a period of $8\times312\frac{1}{2}=2500$ years. The indiction was established by Constantine in the year 312. The year of our Saviour's birth according to the vulgar era, was the 9th year of the solar cycle, or the 1st year of the lunar cycle; and the 312th year after his birth was the 1st year of the Roman indiction. Therefore to find the year of the solar cycle add 9 to any given year of Christ, and divide the sum by 28, the quotient will give the number of cycles elapsed since his birth, and the remainder the cycle for the given year; if nothing remains, the cycle is 28. To find the lunar cycle, add 1 to the given year of Christ and divide the sum by 19, the quotient is the number of cycles in the interval, and the remainder the cycle for the given year: if nothing remains, the cycle is 19. For the indiction, subtract 312 from the given year of Christ, divide the remainder by 15, the remainder after this division is the indiction for the present year.

The ancients formed the cycle of the moon, by taking any year for the cycle and observing all the days in which the new moon happened through the year, and placing the number 1 against each day; in the 2d year of the cycle they placed the number 2 against each day in which the new moon happened as before; the 3d year the number 3, &c. through the whole 19 years. These numbers corresponding to one cycle, were fitted to the calendar to point out the new moons in every future cycle, and from their great use were written in gold, and thence called *golden numbers*. The whole day gained in 312 years, which since the council of Nice in 325 has since been neglected, causes the golden numbers to be 5 days higher in the old style, or 7 days lower in the new, than they were at the abovementioned council, and ought to be so placed in the calendar. Since 1800 there are 12 days difference between the old and new style. The golden number is not, however, so well adapted to the Gregorian as the Julian calendar. The golden number is the same as the lunar cycle, and found in the same manner. Thus to find the golden number for 1812, $1812+1\div19=95$ and 8 over; hence 8 will be the golden number. Any other year will answer as well as the current year, by adding its own golden number to it, and proceeding as above with the difference between both years.

In the calendar it is usual to mark the seven days of every week with the first 7 letters of the alphabet, calling the first of January A, the 2d

81. The *cycle of the sun*, is a period of 28 years, which being completed, the days of the month return in the same order to the same days of the week; the sun's place to the same signs and degrees in the ecliptic, &c.

82. The *cycle of the moon*, or *metonick cycle*, called also the *golden number*, is a revolution of 19 years, which being completed, the new and full moons return to the same days of the month, &c.

B, the 3d C, the 4th D, the 5th E, the 6th F, the 7th G, the 8th A again, and so on through the year; and whatever letter corresponds to the first Sunday of January, will answer to every Sunday in a common year, and is therefore called the *dominical letter.*

A common year contains 52 weeks and one day, therefore the first and last days of a common year fall on the same day of the week; hence if any year begins on Sunday, the next will begin on Monday, &c. but this order is interrupted by the leap years; February having one day more than in common years: so that the dominical letter for March and the rest of the year, will be the letter preceding that which served for January to the 24th of February; leap years having therefore two dominical letters. The dominical letter is thus found: to the given year add $\frac{1}{4}$th of it, for the leap years contained in it (neglecting the fractions if any) and from the sum subtract 7 for the 18th century (or from 1800 inclusive to 1900) 8 for the 19th and 20th centuries, 9 for the 21st century, 10 for the 22d century, 11 for the 23d and 24th centuries; because the three years 2100, 2200, and 2300 will not be leap years, &c. divide the remainder by 7, and the remainder after this division will give the dominical letter, reckoning from the last G towards the first A. If 0 remains, the dominical letter will be A; if 1 remains, the dominical letter will be G; if 2 remains, the dominical letter will be F, &c. thus $\begin{matrix} A, & B, & C, & D, & E, & F, & G, \\ 0, & 6, & 5, & 4, & 3, & 2, & 1, \end{matrix}$ where the figures or remainders, correspond to the dominical letters above them. Hence to find the dominical letter for 1807, it will be $1807 + \frac{1807}{4} - 7 = 2251$ (rejecting the remainder) which being divided by 7, will leave a remainder of 4 corresponding to D the dominical letter required, and counting back from D to A, thus (D) Sunday, Saturday, Friday, Thursday, which corresponds to A, the day on which the year began. To find the dominical letter for 1812, proceeding as above, we shall find a remainder of 4, which corresponds to D, but as 1812 is a leap year, it has two dominical letters, that is E, the letter preceding D, counting from G, which answers for January and February to the 24th, and D the rest of the year. To find the dominical letter for 1910, we have $1910 + \frac{1910}{4} - 8 = 2379$, which divided by 7 leaves a remainder of 6 corresponding to B the dominical letter for that year, which therefore will begin on Saturday. The dominical letters for 1996, a leap year, are GF; this year will therefore begin on Monday, &c. (see a table of the dominical letter to the year 4000 in Ferguson's Astronomy, pa. 398, 8th ed.)

The difference between a solar and lunar year, which is 10 days 21h. 0′ 12″ (defs. 75 and 78) or nearly 11 days, constitutes the epact. When the solar and lunar years begin together, the epact for that year is 0 or 29; the 2d year the epact is 11; the 4th, 33: but when the epact exceeds 30, an intercalary month is added, making the lunar year consist of 13 months, and hence at the beginning of the 4th year the epact is 3, the 5th, 14, &c. all the varieties happening in 19 years, or one lunar cycle, except the corrections made at the end of centuries, &c. to allow for which the following rule must be observed in calculating the epact for any year from 1800

83. The *cycle of indiction*, is a period of 15 years, but has no reference to the celestial motions.

84. The *Dionysian period*, is the number of years that arises by multiplying the cycles of the sun and moon together, and consists of 532 years.

to 1900 Multiply the golden number for the given year by 11, and the product divide by 30, then subtract 11 from the remainder, the last remainder will be the epact. If 11 cannot be subtracted, 30 must be added to the remainder, and then 11 subtracted as before. Thus to find the epact for 1810; the golden number for 1810 is 6, this multiplied by 11 gives 66, which divided by 30, leaves a remainder of 6; hence $6+30-11=25$ the epact required. To find the epact for 1812, the golden number is 8; hence $8\times11\div30$ leaves a remainder of 28, and $28-11=17$ the epact required. The epact may be found thus, without the golden numbers. Divide the given year by 19, multiply the remainder by 11, the product will be the epact if it does not exceed 29, but if it exceeds 29, divide 30 into it, and the remainder will be the epact. Ozanam in his math. recreations, gives the following rule: multiply the golden number by 11, and take the number of days retrenched by the reformation of the calendar, from the product; that is, 11 days if the year be between 1700 and 1800, 12 if between 1800 and 1900, 13 if between 1900 and 2100, &c. divide the remainder by 30, the remainder after this division will be the epact. Between 1800 and 1900 this gives the epact 1 day less than the former methods, between 1900 and 2100 two days less, &c. The former is however used in the present calendar. But this method will sometimes differ from the true epact (which is the age of the moon for any year, on the 1st of January exclusively, or at the end of the preceding year; or the number of days since the last mean new moon) the annual epact being too great.

From the dominical letter, the day of the week on which any day of a given month falls, may also be found. When the days of the week are marked by the seven first letters of the alphabet, the letter A is always at the first day of January and October; B at the first of May; C at the first of August; D at the first of February, March and November; E at the first of June; F at the first of September and December; and G at the first of April and July. Hence each letter in the following order, A, D, D, G, B, E, G, C, F, A, D, F, marks the first day of each month in the year: and the same letters mark the 8th, 15th, 22d, and 29th days of the month. If the dominical letter be A, the first of January and October will be Sundays; the first of May marked B, will be Monday; the first of August marked C, will be Tuesday; the first of February, March, and November, being marked with D, will be Wednesdays; the first of June marked E, will be Thursday; the first of September and December marked F, will be Friday; and the 8th, 15th, 22d, and 29th of the month, will be on the same days of the week. In the same manner may these days be found when the dominical letter is any other besides A; and hence any day of the year. Tables for this purpose are given in most books of practical astronomy.

Besides the annual epacts, there are monthly epacts commonly called the *numbers of the months*, which are the moon's age on the first day of every month when the solar and lunar years begin together, and are thus found: divide the number of days between the first of January and the first day of any month by 29½, the remainder will be the number for that month. Thus the epact for January is 0, for February nearly 2, for March in common years 0, but in leap years 1, &c.

85. The *Julian period*, is the number of years that arises from the product of the cycles of the sun, moon and indiction, viz. $28\times19\times15=7980$ years.

86. *Positions of the sphere*, are its situations with respect to the horizon, and are principally three, right, parallel and oblique.

The number for the month being given, the *moon's age* on any day is thus found: to the epact for the year add the days of the month, and the number for the month, the sum, if it does not exceed thirty, is her age; but if it exceed thirty, take 30 from it, and the remainder is the moon's age, if the month has 31 days; but in months of 30, subtract only 29, except when it is leap year. Thus to find the moon's age on the 28th of January, 1811; here the epact $6+28=34$ and $34-30=4$ days the moon's age, one day less than in the nat. almanac. For the 28th of April, 1811, we have $6+2+28=36$ and $36-29=7$ agreeing with the nat. alm. For 1812, to find the moon's age on the 20th of April, we have epact $17+3+20=40$ and $40-30$ (the year being bissextile) $=10$ days, agreeing also with nat. alm. Whenever accuracy is required, recourse must however be had to Astronomical calculation.

If the moon's age be multiplied by 5 and divided by 6, the quotient is the hours, and the remainder multiplied by 12 the minutes, nearly, *when the moon comes to the meridian*, reckoning from noon; if to this be added the time of tide on the days of new and full moon at that place, the sum will give the time of high water there. The tides at any place happen always when the moon is in the same position with respect to the meridian of the place. Thus at London it is always high water when the moon is S. W. or 3 hours past the meridian; at New-York when she is S. E. or 3 hours before noon; at Sandy-Hook when she is E S. E. or $4\frac{1}{2}$ hours before noon, &c. (See the table pa. 142 of Hamilton Moore's Navigation, 10th ed.)

This rule is sufficiently exact for common use, and no rule of calculation can be given that will always produce an exact answer, as the time of high water depends so much on the winds, swell of the sea, &c. at the time. The mean motion of the moon from the sun in a day is 12° 11′ 26″ 7. For according to Mayer, the sun's daily mean motion is 59′ 8″ 3, and that of the moon 13° 10′ 35″ the difference of which is the above. Now 15° : 1h. or 60′ :: 12° 11′ 26″ 7 : 49′ 8. Hence if the moon's age be multiplied by 49.8 and divided by 60, or multiplied by 5 and divided by 6 as above ($\frac{49.8}{60}$ being nearly equal $\frac{5}{6}$) the quotient will give the moon's southing nearly. For more accurate methods see the note to ex. 8, prob. 18, part 3d, or prob. 39, part 3.

From the above rules the method of finding on what day *Easter Sunday* falls in any year, is very simple. At the Council of Nice, Easter Sunday was fixed on the first Sunday after the full moon, which happens on or next after the 21st of March, and therefore it must always fall between the 21st of March and 25th of April. The method is this; find the day of full moon on or next after the 21st of March, and then find what day of the week the full moon is on, and the next Sunday will be *Easter Sunday*. The moon's age being given, subtract it from the day of the month; or the day of the month increased by 30, the remainder will give the day on which *new moon* falls. If to this $7\frac{1}{2}$ days (or rather $29\frac{1}{2}\div4$) be added, the mean time of *first quarter* is given; add 15 days (or $29\frac{1}{2}\div2$) for mean full moon and $22\frac{1}{2}$ days nearly, for the third quarter. Thus to find *Easter Sunday* in the year 1811, the moon's age on the 21st of March was 27 days, hence $21+30-27=24$ the day of the month on which new moon takes place, and $24+15=39$ and $39-31=8$; hence full moon fell on the

87. A *right sphere** is that position of the earth, where the equator passes through the zenith and nadir, the poles being in the rational horizon.

88. A *parallel sphere*† is that position of the earth, where one pole is in the zenith and the other in the nadir; in which case the equator coincides with, and all its parallels are parallel to the horizon.

89. An *oblique sphere*‡ is when the rational horizon cuts the equator obliquely, which is the case with all parts of the earth, except those under the poles and the equinoctial.

8th of April, which being Monday, the Sunday following, or the 14th of April, was Easter Sunday. To find Easter in the year 1812. Here the moon's age on the 21st of March will be 9 days (this being leap year) and $21-9=12$ the day on which new moon will fall; hence $12+15=27$ the day on which full moon will fall (the Nautical Alm. gives it 16′ after 12 on the night of the 27th) which being Friday, the 29th of March will therefore be Easter Sunday, &c.

The Nautical Alm. gives the moon's age one day later than here, probably making the epact the age of the moon on the 1st of January inclusively, or astronomical time, the above calculation being adapted to civil time.

The feast of Easter regulates the moveable feasts of the whole year.—Thus the 1st Sunday after Easter is *Low Sunday; Rogation days* commence 35 days after Easter; *Ascension Thursday* is the Thursday following, or the 40th day after Easter; the feast of *Pentecost*, commonly called Whitsuntide, is 10 days after, or the 50th day after Easter; on the Sunday after, or 56 days after Easter, the feast of the *Holy Trinity* is celebrated; and the Thursday following, or 11 days after Pentecost, or 60 days after Easter, is the feast of *Corpus Christi*. The 9th Sunday before Easter, or 63 days before it, is called *Septuagesima*, the 8th or following Sunday, which is 56 days before Easter, *Sexagesima*, the 7th or 49 days before Easter is called *Quinquagesima*, and the Wednesday following, *Ash Wednesday*, the 1st Sunday of Lent is called *Quadragesima*, the 5th Sunday of Lent is called *Passion Sunday*, the 6th or the Sunday before Easter, *Palm Sunday*. The other Sundays in Lent and those after Easter are called by other names, as Reminescere, Lætare, Judica, Misericordia, Jubilate, &c. *Advent Sunday* does not depend on Easter, but on the feast of St. Andrew, which is on the 30th of November, being the nearest Sunday to this feast. *Christmas day* is always on the 25th of December. The first of January or new year's day, is the feast of the *Circumcision* of our Lord, the 6th of January is the feast of the *Epiphany*, or manifestation of Christ to the *Gentiles*, &c.

* The inhabitants who have this position of the sphere, live at the equator. It is called a right sphere, because all the parallels of latitude cut the equator at right angles, and the horizon divides them into two equal parts, making equal day and night.

† The inhabitants who have this position of the sphere (if there be any) live at the poles. It is called a parallel sphere, because all the parallels of latitude are parallel to the horizon. In this position of the sphere the sun appears constantly above the horizon for six months.

‡ So called from the parallels of latitude cutting the horizon obliquely. In this position of the sphere, the days and nights are of unequal lengths, the parallels of latitude being divided unequally by the rational horizon.

90. A *Climate** in a geographical sense, is a part of the surface of the earth, contained between two lesser circles, parallel to the equator; and of such a breadth, that the longest day in the parallel, nearest the pole, exceeds the longest in that nearest the equator, by half an hour in the torrid and temperate zones; or by one month in the frigid zones.

* There are therefore 24 climates between the equator and each polar circle, and 6 climates between each polar circle and its pole. The climates between the polar circles and the poles were, in a great measure, unknown to the ancient geographers; for Ptolomy does not give an exact computation of the parallels as far as the polar circles itself. They reckoned only seven climates north of the equator. The middle of the first northern climate they made to pass through *Meroe,* the metropolis of the Ethiopians, built by Cambyses, on an island in the Nile, of the same name, nearly under the tropic of cancer; the second through *Syene,* a city of Thebais, in Upper Egypt, near the cataracts of the Nile; the third through *Alexandria;* the fourth through *Rhodes;* the fifth through *Rome,* or the *Hellespont:* the sixth through the mouth of the Borysthenes, or Dnieper; and the seventh through the *Riphæan mountains,* supposed to be situated near the Tanais or Don river. The southern parts of the earth being in a great measure unknown, the climates received their names from the northern, and not from any particular places. Thus the climate which was supposed to be at the same distance southward, as Meroe was northward, was called *Antidiameroes,* or the opposite climate to Meroe; *Antidiasyenes,* was the opposite climate to Syenes, &c. The following table exhibits the climates from the equator to the poles, with their latitudes, breadth, &c. The twenty-four first are the climates between the equator and polar circles; the six last those between the polar circles and poles.

Climates.	*Length of the longest day.*	*Ends in latitude.*	*Breadths of the climates.*	*Climates.*	*Length of the longest day.*	*Ends in latitude.*	*Breadths of the climates.*	*Climates.*	*Length of the longest day.*	*Ends of latitude.*	*Breadths of the climates.*
1	12½h	8° 34′	8° 34′	11	17½h	56° 38′	2° 8′	21	22½h	66° 5′	17′
2	13	16 44	8 10	12	18	58 27	1 49	22	23	66 21	16′
3	13½	24 12	7 28	13	18½	59 59	1 32	23	23½	66 29	8′
4	14	30 48	6 36	14	19	61 18	1 19	24	24	66 32	3′
5	14½	36 31	5 43	15	19½	62 26	1 8	25	1mo	67° 18′	46′
6	15	41 24	4 53	16	20	63 22	56	26	2	69 33	2° 15′
7	15½	45 32	4 8	17	20½	64 10	48	27	3	73 5	3 32
8	16	49 2	3 30	18	21	64 50	40	28	4	77 40	4 35
9	16½	51 59	2 57	19	21½	65 22	32	29	5	82 59	5 19
10	17	54 30	2 31	20	22	65 48	26	30	6	90	7 1

For the method of constructing this table, see the note to problem 29, part 2d.

In tables of this kind, it is usual to give the names of the principal places situated in these respective climates, but these the learner may easily find on the globes (by prob. 3.) or without the globes on a map. Although it appears that all places situated in the same parallel of latitude are in the same climate, yet we must not infer from thence that they have the same atmospherical temperature. Large tracts of uncultivated lands, sandy deserts, elevated situations, woods, morasses, lakes, winds, &c. have a con-

91. The *right ascension* of the sun or a star, is an arch of the equinoctial between the first point of aries and the meridian, or circle of declination, which passes through the centre of the star, and is reckoned from west to east, round the globe. *Declination* is their distance from the equinoctial north, or south.

92. *Oblique ascension* is an arch of the equator between the beginning of aries and that point of the equinoctial which rises with the sun, or a star, in an oblique sphere, and is reckoned as the right ascension.

93. *Oblique descension*, is that degree of the equinoctial, which sets with the sun or a star.

siderable effect on the atmosphere. In New-Britain the climate, even about the mouth of Haye's river, between Lake Winipeg and Hudson's Bay, and in only lat. 56 or 57° N. is, during winter, so excessively cold, that the ice on the river is seven or eight feet thick. Port wine freezes into a solid mass, and even brandy coagulates, which only happens with a cold of —7° of Farenheit; and what is contrary to the ordinary course of nature, the cold seems to increase every year, in these northern regions. (See Martin's essay towards a natural and experimental history of the various degrees of heat in bodies.) This shews that the seasons owe much of their mildness to cultivation. The climate between Edinburgh and Aberdeen, in Scotland, is the same as the above, but no such extremes of heat and cold are perceived there. In Canada, in about the latitude of Paris, and the south of England, the winter is so severe from the latter end of November to April, that the St. Lawrence and other rivers are frozen over, and the snow all this time lies generally about 5 feet deep. During a great part of the summer, on the western coast of America, it is extremely hot, and what is more astonishing, and in which we cannot sufficiently admire the wise dispensations of Providence, is, that in the higher latitudes, such as 59 and 60 degrees, the heat of July is frequently greater than in lat. 51°, which heat seems necessary for the growth and maturity of corn, &c. during their short summer, &c. (See Winterbotham's America or Kerwin's ingenious work, entitled an estimate of the temperature of different latitudes.) The heat on the western coast of Africa, after the wind has passed over the sandy desert, is almost suffocating; but after the same current of air has passed over the Atlantic ocean, it is cool and refreshing to the inhabitants of the Caribbean or West-India islands. On the eastern coasts of America, and even beyond the Allegany mountains, the seasons are not so variable or subject to so great extremes of heat and cold as on the western. It cannot be doubted but mountains have a great effect on the temperature of the countries to which they belong, by stopping the course of certain winds (as the Allegany stops part of the trade winds, and probably increases the force of the N. W. and other winds reflected from their sides) by forming barriers to the clouds, by cooling the atmosphere from the snow on their summits, or by reflecting the sun's rays from their sides, and likewise by serving as elevated conductors to the electricity of the atmosphere. Hence on the Alps, the Andes, &c. the traveller experiences, even in summer, all the four seasons of the year. The climates in the United States are by late geographers divided into four principal regions, &c. (See Spafford's geog. ch. 6.) The winds having not only a considerable influence on the seasons, but also produce many other phenomena, as currents in the ocean, &c. A general theory of them deduced from facts would therefore be a desideratum. Our limits are too contracted to specify any in our present undertaking: in the philosophical part of our course, we shall consider this subject more fully. See a piece written by the author of this introduction, signed J. W. and published in the Mercantile

94. The *ascensional difference* is the difference between the right and oblique ascension or descension, and shews how long the sun rises or sets before or after the hour of six.

95. The *six o'clock hour line* is that great circle passing through the poles, which is 90° distant, on the equator, from the meridian or 12 o'clock hour circle *

96. *Culminating point* of the sun, or star, is that point of its orbit, which, on any given day, is the most elevated ; or that point in which it is at 12 o'clock, or when on the meridian.

97. *Angle of position*, between two places, on the terrestrial globe, is an angle at the zenith of one of the places, formed by the brass meridian and the quadrant of altitude, passing through the other place, and is measured on the horizon.

98. *Rhumbs* are the divisions of the horizon into 32 points, called the points of the compass. *A rhumb line* is the way a ship describes, while she sails on any point of the compass, and cuts all the meridians in the same angle.†

99. *Course* is the angle which the rhumb, or ship's way, makes with the meridian.

100. *Crepusculum*, or twilight, is that faint light which we perceive before the sun rises, and after it sets.‡

Advertiser of Nov. 1st, 1809, in New-York, where the theory of some important phenomena of this nature is experimentally illustrated. Observations, similar to that made by Dr. Franklin, on the course of a N. E. storm, (Philosophical letters, pa. 38.) would throw much light on this subject. Its velocity was 100 miles an hour.

* The sun and stars are on the eastern half of this circle 6 hours before they come to the meridian, and on the western half 6 hours after they have passed the meridian.

† A rhumb line, properly speaking, is a spiral curve drawn on the earth's surface, as above described, and which, if continued, will never return on itself so as to form a circle, except it happens to be due east or west, or due north and south; these can never be right lines on any map, except the meridians be parallel to each other, as in Mercator's and the plane chart, unless the parallels and meridians : hence it is only on these charts that the bearing can be easily found. By the compass if a place A bear due east from a place B, the place B will bear due west from A: but if the bearing on the globe should be measured by the quadrant of altitude, as some direct, this would not be the case, for the angle thus measured on the globe by the quadrant, is the *angle of position* between the places.

‡ The twilight is supposed to end in the evening or begin in the morning, when the sun is 18° below the horizon. The twilight in the morning and evening, arises both from the refraction and reflection of the sun's rays by the atmosphere. Some suppose that the reflection principally arises from the exhalations of various kinds, with which the lower parts of the atmosphere are charged, for the twilight lasts until the sun is further below the horizon in the evening than it is in the morning when it begins ; and in summer it is longer than in winter, which phenomena seem to confirm the above supposition. The greater heat also from the earth may have its share in producing this effect, by resisting the rays of light and changing their direction ; many phenomena and experiments might be adduced to prove this latter supposition : and it is to this medium of *heat* or *light*, that the particles of the atmosphere, very probably, owe

101. *Refraction* is that change in the elevation of any celestial object, caused by the earth's atmosphere.*

102. *Parallax*, is the difference between the altitude of any celestial object, seen from the earth's surface, and the altitude of the same object, seen at the same time from the earth's centre; or the angle under which the semidiameter of the earth would appear as seen from the object.

103. *Eclipse of the sun*, is an occultation of the whole or a part of the face of the sun, occasioned by the moon's interposition between it and the earth.

104. *Eclipse of the moon* is a privation of the light of the moon, caused by the earth's interposition between the sun and moon.

105. *Diurnal arch* is the arch described by the sun, moon or stars, from their rising to their setting. The sun's semidiurnal arch, is the arch described in half the length of the day.

106. *Nocturnal arch*, is the arch described by the sun, moon, or stars, from their setting to their rising.

107. *Circles of perpetual apparition*, are those in an oblique sphere as much distant from the elevated pole, as the place itself is from the equator. They are the greatest of all those that constantly appear, and are such that all the stars inclosed within them never set.

108. *Circles of perpetual occultation*, are those opposite the former, and within which all the stars that are contained never rise.

their elasticity. Its remaining in gross bodies in a latent state, may cause them to produce the same effects in proportion to its density, &c. These properties, if fully investigated, might lead to important discoveries.

* When a ray of light passes out of a vacuum into any medium (or fluid substance) it is found to deviate from its right line course towards a perpendicular to the surface of the medium, through which it enters; and if the medium be of different densities, the ray will continually deflect from its former direction, and describe a curve. From its being thus continually broken, it is said to be *refracted.* Now as the earth is surrounded by a body of air called the *atmosphere,* into which the rays of light enter from a vacuum, or at least a very rare medium, and as in approaching the earth's surface the density of the atmosphere continually increases, the rays entering it obliquely will therefore be refracted, and describe a curve; and hence the *apparent* place of the body from which the light proceeds, must differ from its true place. But where the ray enters perpendicularly, there can be no refraction, and the less oblique the ray is, the less will be the refraction; (from a principle in optics, that the angle of incidence is equal to that of reflection;) hence it happens, that at noon the refraction of the sun is the least, because it has then its greatest altitude, and the nearer the horizon it is, the greater will be its refraction. At the horizon the refraction is the greatest, and this is called the horizontal refraction: and hence also the refraction is least in the torrid zone and greatest at the poles. The property of refraction being to elevate the body from which the light proceeds, it must therefore be subtracted from the observed altitude. From this property it follows, that the sun and moon will sometimes appear of an oval figure near the horizon; for the lower limbs being more refracted than the upper, the

109. The *fixed stars** are so called from their being observed to keep, nearly, the same apparent distance, with respect to each other.

perpendicular diameters will be less than the horizontal, which is not affected by refraction: for the diameter of the sun being supposed 32′, then the mean refraction of the lower limb when it just touches the horizon, will be 33′, but the altitude of the upper limb being then 32′, its refraction is only 28′ 6″, the difference of which is 4′ 54″, the excess of the diameter parallel to the horizon above the vertical diameter. The refraction is also variable according to the different densities of the air, and hence we can sometimes see the tops of mountains, steeples, &c. which at other times are invisible, though we stand in the same place.

The ancients were not unacquainted with these effects of refraction. *Ptolemy* mentions a difference in the rising and setting of the stars in different states of the atmosphere, but makes no allowance for it in his computations. *Archimedes* observed, that in water the refraction was in proportion to the angle of incidence. *Alhasen*, an Arabian, in the 11th century, found the distance of a circumpolar star, from the pole, to be different when observed above and below the pole, and such as ought to arise from refraction. For suppose a circumpolar star passes through the zenith, and its distance from the pole be then observed, this will be its true distance, if its distance be again observed when on the meridian below the pole, this latter distance will be its distance affected by refraction, the difference between which and the former will be the refraction at the lower altitude. *Snellius* first observed the relation between the angles of incidence and refraction; but *Tyco Brahe* was the first who constructed a table, though incorrect, for that purpose. Cassini in 1660 published another more correct, and Mayer in his tables has given another much more accurate. Modern astronomers have bestowed much attention on this subject, the niceties in the present improved state of astronomy requiring the greatest accuracy.

* By star, in astronomy, is understood any body which shines in the heavens whether it emits or reflects light; the latter are called planets or wanderers, because they do not observe the same position among themselves: the former are called fixed, for the reason above given; though strictly speaking, they have several motions among themselves, which only a lapse of many ages can render preceptible (see part 4th) the principal is, that motion caused by the precession of the equinoctial points, their longitudes from thence increasing yearly 50¼″. This likewise causes a variation in their rt. ascentions and declinations: their latitudes are also subject to a small variation. The nutation or change of the earth's axis, the aberration of light, &c. have some effect in changing the places of the stars.

The fixed stars are divided into six classes, from their apparent various magnitudes. Those that appear largest (occasioned probably from their being nearer to us than the first) are called *stars of the 1st magnitude;* the next to them in lustre, *stars of the 2d magnitude;* and so on to the 6th, which are the smallest that are visible to the naked eye. Besides these, there are an inconceivable number which are not visible without the help of a telescope, and these are called *telescopic stars.* The distinction of stars into six classes or degrees of magnitude, is commonly received by astronomers: there is however no rule for classing the stars but by the estimation of the observer; for in reality there are as many orders of stars as there are stars, few of them being exactly of the same bigness and lustre; and hence some astronomers reckon those stars of the first magnitude, which others reckon to be of the second.

110. *The poetical rising and setting of the stars*, is that particular rising and setting of the stars referred to the sun by the ancient poets; whence called poetical. Thus, when a star rose at sun setting, or set with the sun, it was said to rise and set *archronically*: when a star rose with the sun, or set when the sun rose, it was said to rise and set *cosmically*: when a star first became visible in the morning, after having been so near the sun as to be hidden by the splendour of his rays, it was said to *rise heliacally*: and when a star first became invisible in the evening on account of its nearness to the sun, it was said to *set heliacally*.

111. A *Constellation** is a collection of stars in the heavens, represented on the surface of the celestial globe, and contained within the out lines of some assumed figure, as a *ram*, a *lion*, a *bear*, a *dragon*, &c.

112. The *Zodiac* is a space which extends about 8° on each side of the ecliptic, within which, the motion of all the planets (except Ceres and Pallas lately discovered) are performed. It is so called from the figure of the animals described in it, to represent the twelve signs, commonly called the 12 signs of the zodiac, (*zodion* in Greek signifying an animal.)

* The division of stars into constellations is of great antiquity, as Job makes mention of Orion, Arcturus and the Pleiades. In the writings of Homer, Hesiod, &c. many of the constellations are mentioned. The ancients took the figures which represent them from the fables of their religion, and the moderns still retain them to avoid the confusion resulting from introducing new ones; as this, or some similar division of the stars is necessary, in order to direct a person to any part of the heavens which he wants to point out, or in which any particular star is situated. The whole heavens is almost thus divided into constellations. Those stars which could not be brought into any particular constellation, were called *unformed stars*. These constellations are ranged in order on the surface of a celestial globe, and their names are to be learnt by inspection, as also their forms and disposition. On Bardin's globes all the figures, &c. are painted, but on Cary's there are only the boundaries or limits of the constellations given. The Revd. Mr. Wollaston has published in 1789 a general catalogue of the stars, arranged in zones of north polar distance, and adapted to January 1, 1790.

The following tables contain all the constellations on the *New British Globes*, with the number of stars, double stars, clusters, clusters and nebulæ, and nebulæ, according to the latest observations. The number of the stars in each constellation in the first column is taken from Flamstead, except those marked thus*. Those in the other columns are taken from Cary's celestial globe, according to their respective degrees of magnitude, &c. The constellations in the zodiac are 12. The northern constellations on Bardin's globe are 34, on Cary's 30. The southern constellations on Bardin's are 47, on Cary's 45: amounting in all, on Bardin's globe to 93; on Cary's to 87. The respective magnitudes are denoted by the numbers 1, 2, 3, &c. double stars by two dots, thus ·· ; clusters, thus ⁂ ; clusters and nebulæ, thus ⁂ ; nebulæ, thus ⁂. All those less than the 6th mag. are only visible with the telescope.

CONSTELLATIONS IN THE ZODIAC.	No. of stars fr. Flamstead, &c.	Number of stars, double stars, clusters, &c. from Cary's globes, with their respective magnitudes.												
		1	2	3	4	5	6	7	8	9	du ..	clu	c. n	neb
1. Aries. *The ram.*	66		1	1	2	6	22	15	4	6	11			
2. Taurus. *The bull.*	141	1	1	4	8	23	60	51	21	43	17	3		1
3. Gemini. *The twins.*	85	1	2	4	7	13	27	29	6	9	16	5		
4. Cancer. *The crab.*	83				8	11	48	30	5	11	5	2		
5. Leo. *The lion.*	95	2	2	6	15	12	47	17	14	7	11	1		15
6. Virgo. *The virgin.*	110	1		6	10	16	71	16	12	5	12	2		50
7. Libra. *The balance.*	51		1	3	12	4	27	6	12		4			2
8. Scorpio. *The scorpion.*	44		2	11	10	4	29	3			4	3		4
9. Sagittarius. *The archer.*	69			5	10	12	59	23	16	9	1	5		10
10. Capricornus. *The goat.*	51			3	3	7	44	8	15	8	5			1
11. Aquarius. *The water-bearer.*	108			4	7	28	59	9	8	1	6		1	1
12. Pisces. *The fishes.*	113			1	5	28	63	19	15	10	14			1
NORTHERN CONSTELLATIONS.														
1. Ursa minor. *The little bear.*	24		1	2	4	6	4	5			2			
2. Ursa major. *The great bear.*	87	1	3	7	13	31	37	13		1	7			8
3. Draco. *The dragon.*	80		4	7	12	25	32	3			9			
4. Cepheus.	35			3	6	13	22	3			10			
5. Cassiopeia.	55			5	6	8	38	5		1	8	2		
6. Camelopardalus. *The camelopard.*	58				6	25	42	9			10			
7. Auriga. *The charioteer.*	66	1	1		9	20	26	5	1		14	4		3
8. Lynx. *The lynx.*	44				3	15	25	12			8			
9. Leo minor. *The little lion.*	53			1	5	10	39	4						3
10. Canes venatici. *The greyhounds.*	25			1	1	7	15	2			3			1
11. Coma berenices. *Benerice's hair.*	43				13	13	17	6		1	7	1	1	21
12. Bootes.	54	1		7	10	18	30	7	13	1	14			2
13. Corona borealis. *The northern crown.*	21		1	1	5	9	5				7			1
14. Hercules.	113			9	19	36	46	12			18			2
15. Lyra. *The harp.*	21	1		2	2	6	12				10	1		
16. Cygnus. *The swan.*	81		1	6	11	16	49			1	23	3		1
17. Vulpecula et anser. *The fox and the goose.*	35				5	13	21				3	4		1
18. Sagitta. *The arrow.*	18				4		15				12	2		1
19. Delphinus. *The dolphin.*	18			5	1	2	11				3	1		2
20. Equuleus. *The little horse.*	10				4	1	5				4			
21. Pegasus. *The flying horse.*	89		3	3	9	14	51	11		1	8			2
22. Lacerta. *The lizard.*	16				3	7	7				3			1
23. Andromeda.	66		3	2	12	15	34	2			11			8
24. Triangulum. *The triangle.*	16				3	1	7	4			4			1
25.* Musca borealis. *The northern fly.*	4			1	2	1					2			
26. Perseus et Caput Medusæ. *Head of Medusa.*	59		2	4	10	14	31	9	1	1	16	1		
27. Serpens. *The serpent.*	64		1	9	5	3	40	9			4			1
28. Ophiucus vel serpentarius.	74		1	5	10	9	42	23	10	1	3	8	3	5
29. * Taurus Poniatouski. *Poniatouski's bull.*	16				3	1	12	1			5	2		
30. Aquila. *The eagle,* and Antinous.	71	1		9	7	14	38	3	2	1	15	5		3

SOUTHERN CONSTELLATIONS.	No. of stars fr. Flamstead, &c.	Number of stars, double stars, clusters, &c. from Cary's Globes, with their respective magnitudes.												
		1	2	3	4	5	6	7	8	9	du ..	clu ⁺⁺₊	c.n	neb
1. Cetus. *The whale.*	97		2	7	13	11	66	9	1	3	7	1		3
2. Eridanus. *The river Po.*	84	1	1	11	27	20	57	2	1		8	1		3
3. Orion.	78	2	4	3	15	18	36	2	2	1	28	6	2	1
4. Monoceros. *The unicorn.*	31				7	7	12	1			5	14		2
5. Canis minor. *The little dog.*	14	1		1		3	9	1		6	3			
6. Hydra.	60		1		13	16	45	2		1	9			3
7. Sextans. *The sextant.*	41				1	6	36	1	1	1		1		2
8. Crater et hydra. *The cup, &c.*	31				10	9	14	1			4		1	3
9. Corvus. *The crow.*	9			3	2	2	2				1			
10. Centaurus. *The centaur.*	35	2	1	6	10	14	100				1	1	1	5
11. Lupus. *The wolf.*	24			3	3	18	29							
12. * Norma. *The rule or square.*	12					3	26							
13. * Circinus. *The compasses.*	4				1	1	8	1						
14. * Triangulum australe. *The southern triangle.*	5		1	2		1	16							1
15. Ara. *The altar.*	9			3	3	1	30						1	1
16 * Telescopium. *The telescope.*	9				3	6	30					2		
17. Corona australis. *The south. crown.*	12					5	10							
18. Indus. *The indian.*	12			1	1	2	54							
19. * Microscopium. *The microscope.*	10					1	12							
20. Piscis australis. *The southern fish.*	24	1		2	5	9	19							
21. Grus. *The crane.*	13		1	2	2	6	41							
22. Toucana. *The American goose.*	9			1	2	5	58	1						
23. Phœnix.	13		1	1	3	7	63							
24. * Apparatus sculptoris. *The sculptor's apparatus.*	12					5	29	1						
25. * Fornax chemica. *The furnace.*	14					2	43							
26. * Horologium. *The clock.*	12					2	39					1		
27. * Cela sculptoria. *The engraver's tools.*	16					4	18							
28. Lepus. *The hare.*	19			3	7	3	13					2		2
29. Canis major. *The great dog.*	31	1	4	2	7	7	36	3	1	1	1	15	.	4
30. Columba. *The dove.*	10		1	1	2	4	53	1						
31. * Equuleus pictorius. *The painter's horse or easel.*	8				1		39							
32. Argo navis. *The ship Argo.*	64	2	4	9	12	37	289	6	2		1	11	2	2
33. * Pixis nautica. *The mariner's compass.*	4					2	13							1
34.* Antlia pneumatica. *The air pump.*	3					2	18							
35. * Crux. *The cross.*	5	1	2	1	1	1	12							
36.* Musca australis. *The southern fly.*	5				4		17							1
37. * Apus vel avis Indica. *The bird of Paradise.*	11					2	16							
38. * Pavo. *The peacock.*	14		1	2	3	4	80							
39. * Octans. *The octant.*	43			1		6	64							
40. Hydra, or the water snake.	10			2	3	2	38						1	
41. * Reticulus. *The net.*	10			1	2	5	17							
42. * Dorado. *The sword fish.*	6			1	1	4	24							
43. * Piscis volans. *The flying fish.*	8					6	8	3						
44. * Chameleon.	10					6	35							
45. * Mons mensæ formis. *The table mountain.*	30						31						1	1

Some of the stars in the above tables inserted in the column of double stars, are marked triple, quadruple, &c. on the globe, but this distinction was thought unnecessary in the tables, as the globes may be consulted.—These double stars, &c. are useful in trying the goodness and magnifying power of telescopes. The constellations on Bardin's globes, and not marked on Cary's, will be found among the following observations. Changeable stars not taken notice of in the following remarks, may be seen on the globe, and likewise those which have disappeared, as the letters indicating them are not inserted on the globe. See part 4th.

The following observations on the different constellations collected from various authors, may not be uninteresting.

The constellations in the zodiac appear to relate to the motion of the sun, or to refer to the climate and agriculture of those nations to whom the zodiac owes its origin, and are therefore Chaldean or Egyptian Hieroglyphics, intended to represent some remarkable occurrence in each month.—Thus the spring signs were distinguished for the production of those animals which were held in the greatest esteem, viz. the sheep, the black cattle, and the goats; the latter being the most prolific, were represented by the figure of *Gemini*, afterwards so called from the two brothers Castor and Pollux, placed in this constellation by the Greek philosophers. The retrograde motion of the sun in the tropic of Cancer, was represented by a *Crab*, which is said to go backwards. The heat that usually follows in the next month, is represented by the *Lion*, an animal remarkable for its fierceness, and which at this season was frequently impelled, through thirst, to leave the sandy deserts, and make its appearance on the bank of the Nile. The sun entered the 6th sign *Virgo*, about harvest time, which season was therefore represented by a virgin, or female reaper, with an ear of corn in her hand. When the sun enters *Libra*, the days and nights are equal all over the world, and seem to be an equilibrio like the arms of a *Balance*. Autumn, which produces fruits in great abundance, brings with it its variety of diseases. This season is represented by that venomous animal the *Scorpion*, which wounds with its sting in its tail as it recedes. The fall of the leaf was the season for hunting, and the stars which marked the sun's path at this time, were represented by a huntsman or *Archer*, with his arrows and weapons of destruction.

The *Goat*, which delights in ascending some mountain or precipice, is the emblem of the winter solstice, when the sun begins to ascend from the southern tropic, and gradually to increase in height for the ensuing half year.

Aquarius, or the water-bearer, is represented by the figure of a man, pouring out water from an urn, an emblem of the dreary and uncomfortable season of winter.

The last of the zodiacal constellations was *Pisces*, or a couple of fishes tied back to back, representing the fishing season. The severity of the winter is over, the flocks do not afford sustenance, but the seas and rivers are open and abound with fish.

Aries is thought by some to be the ram, whose fleece was of gold, that carried Phryxus and his sister Helle through the air on his back, when they fled to Colchis from the persecution of their step-mother Ino. The fable of the flight of Phryxus to Colchis on a ram, is explained by some, who observe, that the ship on which he embarked was called by that name, or carried on her prow the figure of that animal. The fleece of gold is explained by the immense treasures which he carried from Thebes. He was afterwards murdered by his father-in-law Ætes, who envied him this treasure, which gave rise to the famous Argonautic expedition under the command of Jason. (See Lempriere.) Others imagine that this constellation was first formed from Jupiter appearing to Hercules, or as some say to Bacchus, in the deserts of Lybia in Africa, in the form of a ram; and shewed him a fountain, when, with his army, he suffered extremely for want of water. Whence the temple of Jupiter Ammon was erected in this place. α Arietis, 2d mag. is the principal star in this constellation.

Taurus. Some say that this was the animal, under the figure of which Jupiter carried away Europa, daughter of Agenor, king of Phænicia, to the Island of Crete; from whom Europe, according to some, has derived its name. (See Lempriere or Chompre.) She is supposed to have lived 1552 years before the christian Æra. See Ovid's Met. lib. 8. The meaning of this allegory according to some, is, that the ship in which Europa was carried, was in the shape of this animal, or according to others, that the master was called Taurus, &c. &c. Aldebaran, 1 Mag. the Pleiades and the Hyades are in this constellation.

Gemini. In this constellation are two remarkable stars called Castor (1) or Apollo, and Pollux or Hercules (2) They were sons of Jupiter by Leda, the wife of Tyndarus, king of Lacedæmon. They accompanied Jason in his expedition to Colchis. See their history in Lempriere; also Ovid, lib. 6.

Cancer. Some say this was made a constellation by Juno, as he went by her order, and bit the foot of Hercules when he attacked the Lernean Hydra, and was killed by him.

Leo, is supposed to be the famous lion killed by Hercules on mount Cithæron. This huge monster preyed on the flocks of Amphitryon, his supposed father, and laid waste the adjacent country, until at length killed by this hero. Others suppose it to be the Nemean line killed by Hercules (which was his first labour) and which Juno placed among the stars.— Regulus (1) and β or Deneb, 2 mag. are in this constellation.

Virgo. This constellation, according to some, took its rise from *Astrea*, a daughter of Astreus, king of Arcadia; or according to others, of Titan, Saturn's brother by Aurora. Some make her daughter of Jupiter and Themis, and others consider her the same as Rhea, wife of Saturn. She was called *Justice*, of which virtue, according to some, she was the goddess. She lived upon the earth as the poets mention, during the golden age, which they often call the age of Astrea, but the wickedness and impiety of mankind drove her to heaven in the iron ages, and she was placed among the constellations of the zodiac under the name of *Virgo*. She is represented as a virgin, with a stern but magestic countenance, holding a pair of scales in one hand, and a sword in the other. The scales is the *Libra* in the next sign: others give the office to Themis. See *Libra*. Others, in fine, consider *Erigone* as the Virgo we here speak of. She was daughter of Icarius, an Athenian, who hung herself when she heard that her father was killed by some shepherds whom he had intoxicated, and was changed into the constellation Virgo. Icarius, as some say, was changed into Bootes, and the Dog Mæra, by which Erigone was led to discover where her father was buried, was changed into the star Canis or Sirius. Spica virginis and vindemiatrix 1 and 3 mag. are the principal stars in this constellation.

Libra, from Themis (filia Cæli et terræ) the goddess of Justice; she is also represented with a balance in one hand, and a bandage on her eyes, and sometimes with a sword in the other hand. Jupiter made her the goddess of law and peace, and placed her balance among the constellations. She had an oracle in Bæotia, near the river Cephisus. Others consider Astrea as the goddess of Justice, &c. See Virgo. Zuben-el-Chamali of the 2 mag. is the principal star.

Scorpio. According to Ovid, Orion died of the bite of a Scorpion, which the earth produced to punish his vanity in boasting that there was not on earth any animal which he could not conquer: on account of which, Jupiter placed the Scorpion in the heavens. See Ovid's Delp. pa. 42, notes, &c. Lempriere, Orion. When Orion sets, Scorpio rises. Antares, 1 m. is in this const.

Sagittarius, took its name from *Chiron*, the famous Centaur (half man and half horse) so called from his skill in chirurgery. He was the son of Philyra and Saturn, who changed himself into a horse to escape the inquiries of his wife Rhea. He was famous for his knowledge of music, medicine, and shooting. He was the master of Achilles, Æsculapius, Hercules, Jason, Peleus, Eneas, &c. Hercules, when in pursuit of the Centaurs, wounded him in the knee with a poisoned arrow. Chiron, on account of the excruciating pain, begged of Jupiter to deprive him of immortality: he was therefore placed by the god among the constellations under the name of Sagittarius. Hesiod in Scuto. Ovid, lib. 2, &c. See Newton's Chronology. Some take this to be *Crotus* a son of Pan and Eupheme, the nurse of the muses.

Capricornus. This is supposed to be the goat Amalthea which fed Jupiter with her milk, and with whose skin he afterwards covered his shield.—Jupiter placed this goat among the constellations, and gave his shield to Pallas, who placed upon it Medusa's head, which turned all those who fixed their eyes upon it into stones. Some maintain that it represented Pan, who changed himself into a goat, at the approach of Typhon. Pan was the god of shepherds, of huntsmen, and of all the inhabitants of the country. Homer makes him the son of Mercury by Dryope; some give him Jupiter and Calisto for parents. Lucian, Hyginus, &c. maintain that he was the son of Mercury and Penelope, the daughter of Icarius and wife of Ulysses, &c. Though extremely deformed, the multiplicity of his amours was little inferior to those of Jupiter, *and was therefore thought worthy to be ranked among the gods.* The worship, and the different functions of Pan, are derived from the mythology of the ancient Egyptians. This god was one of their eight great gods, who ranked before the other twelve whom the Romans called consentes. His statues represented him as a goat, which is the emblem of fecundity, and they looked upon him as the principle of all things; his horns represented the rays of the sun; his ruddy complexion and vivacity expressed the brightness of the heavens; the star which he wore on his breast was the symbol of the firmament, and his legs, feet, and tail, being those of a goat, denoted the inferior parts of the earth as woods, plants, &c. When the gods fled into Egypt, in their war with the giants, Pan changed himself into a goat, an example that was followed by all the rest. As Pan usually terrified the inhabitants of the neighbouring country, it is from him that kind of fear which is only ideal, has received the name of *panic fear.*

Aquarius. This is the famous Ganymede, a beautiful youth of Phrygia, son of Tros, King of Troy, or according to Lucian, son of Dardanus. He was taken up to heaven by Jupiter as he was tending his father's flocks on mount Ida, and he became the cup bearer of the gods in place of Hebe; hence made the constellation *Aquarius* or the water-bearer. The principal star in it is Scheat, 3 mag.

Pisces. These are thought to be the fishes which carried Venus and Cupid over the Euphrates, when they fled the pursuit of the giant Typhon; others think that they were the dolphins who carried Amphitrite to Neptune, and others that they were the dolphins that carried the famous musician Arion to Tænarus, having leaped into the sea when the sailors attempted to murder him for his riches. These two last opinions, however, rather relate to the constellation Delphinus.

THE NORTHERN CONSTELLATIONS.

Ursa Minor and *Ursa Major*, are said to be *Calisto* and her son *Arcas*. Calisto or Helice, was daughter of Lycaon, king of Arcadia, and one of Diana's attendants. She was seduced by Jupiter; and Juno in revenge changed her into a she bear, and her son Arcas into a little bear: but Jupiter, fearful of their being hurt by the huntsmen, made them constellations. Some consider Arcas the same as Bootes. (See Bootes.) The ancients also represented each of these constellations under the form of a wagon drawn by a team of horses; and the country people, at the present, sometimes call Ursa Major by the name of *Charles's Wain*: in some places it is called the plough, which it resembles. There are two remarkable stars in Ursa Major called the pointers, because an imaginary line drawn through them, will pass over the pole star in the tail of the little bear. These stars are the hindmost in the square of the wain, or dubhe and β. The great bear is also sometimes called Mænalis Ursa, from a mountain in Arcadia. In this constellation is the pole star, 2d mag.

Draco. There are various accounts given of this constellation; some represent it as the watchful dragon which guarded the golden apples in the garden of the Hesperides, near mount Atlas in Africa, and was slain by Hercules, being his eleventh labour. Juno, who presented these apples to Jupiter on the day of their nuptials, took Draco up to heaven, and made a constellation of it as a reward for its fidelity. These Hesperides who were three sisters, are considered by Varro as having immense flocks of sheep, and that the ambiguous Greek word *melon*, which signifies an apple and a sheep, gave rise to the fable, and that Draco was their shepherd. Others that this Draco, in the famous war with the giants, was brought into combat and opposed to Minerva, who seized it in her hands, and threw it, twisted as it was, into the heavens round the axis of the earth, before it had time to unwind its foldings. Others imagine that it was the dragon killed by Cadmus when in search of his sister Europa, and which he had slain by the assistance of Minerva. This was the dragon from whose teeth, sowed in a plain, armed men suddenly sprung from the ground, &c. Some suppose the dragon to be a king whom Cadmus conquered, and the teeth his soldiers. See Lempriere. The principal star is Rastaben, 2 mag.

Cepheus was a king of Ethiopia, father of Andromeda, by Cassiopeia. He was one of the Argonauts who accompanied Jason to Colchis in quest of the golden fleece. In this const. the principal star is Alderamin, 3 mag.

Cassiopeia was the wife of Cepheus and mother of Andromeda. She boasted herself to be fairer than Juno and the Nereides. Neptune at the request of these, punished the insolence and vanity of Cassiope by sending a huge sea monster to ravage Ethiopia. The wrath of Neptune could only be appeased by exposing Andromeda, tied to a rock, to be devoured by this monster; but Perseus mounted on Pegasus, with the head of Medusa, changed this monster into a rock, delivered Andromeda, and obtained of Jupiter that Cassiope might have a place among the stars. Some suppose Cetus (see Cetus) to be the monster sentto devour Andromeda. Shedir or Schedar, 3 mag. is the principal star.

Camelopardalus is one of the new constellations formed by Hevelius: this animal is described in various natural histories. (See Monoceros.)

Auriga is represented on the celestial globe by the figure of a man in a kneeling or sitting posture, with a goat and her kids in his left hand, and a bridle in his right. There are various accounts given of this constellation. Some suppose it to be Erichthonius, the fourth king of Athens, and son of Vulcan and Minerva, who is said to have invented chariots, and the manner of harnessing horses to draw them; some, however, take this Erichthonius to be Bootes. Others think that Auriga was Mirtilus, a son of Mercury and Phætusa. He was charioteer to Œnomaus, king of Pisa, in Elis, and so experienced in riding and in the managing of horses, that he rendered those of Œnomanus the swiftest in all Greece; his infidelity to his master proved fatal to him at last, but being a son of Mercury, he was made a constellation after his death. It has been supposed that the goat and her kids refer to Amalthæa, daughter of Melissus, king of Crete, who, in conjunction with her sister Melissa, fed Jupiter with goat's milk. It is moreover said, that Amalthæa was a goat called Olenia, from its residence at Olenus, a town of Peloponesus. Capella 1 mag. is the principal star.

Lynx, one of Hevelius' constellations, composed of the unformed stars of the ancients, between Auriga and Ursa Major.

Leo Minor, was formed by Hevelius out of the unformed stars of the ancients, and placed above Leo, the zodiacal constellation. Mythologists are not agreed whether the latter be the Nemæan lion slain by Hercules, as this constellation was among the Egyptian hieroglyphics long before this exploit of Hercules. Nemea was a town of Argolis, in Peloponnesus.

Canes Venatici, these are *Asterion et chara,* the two greyhounds held in a string by Bootes: they were formed by Hevelius out of the unformed stars. *Cor Caroli* a double star of the 3d magnitude is in this constellation.

Coma Berenices, is composed of the unformed stars between the lion's tail and Bootes. Berenice was the wife of Ptolemy Evergetes. When Ptolemy went on a dangerous expedition, she vowed to dedicate her hair to the goddess Venus if he returned in safety. Some time after Conon, an astronomer of Samos, to make his court to Ptolemy, publicly reported that the queen's locks were carried away by Jupiter, and were made a constellation. Conon, according to Lempriere, flourished 247 years before Christ. He was intimate with the celebrated Archimedes.

Evergetes was a surname signifying benefactor, which Ptolemy received from the Egyptians, on account of carrying back 2500 statues of their gods, which Cambyses had carried away into Persia when he conquered Egypt. This title was also given to Philip of Macedonia, to antigonus Doson, to the kings of Syria and Pontus, and to some of the Roman emperors.

Bootes, also called Bubulcus, is supposed to be Icarus the father of Erigone, who was killed by shepherds for inebriating them. Others maintain that it is Arcas or Arctophylers, son of Jupiter and Calisto. (See ursa minor, &c.) Bootes is represented as a man in a walking posture, grasping in his left hand a club, and having his right hand extended upwards, holding the cord of the two dogs Asterion and Chara, which seem to be barking at the great bear; hence Bootes is sometimes called the bear driver, and the office assigned him is to drive the two bears round the pole. Arcturus 1 and Mirach 2 mag. are the principal stars.

Corona borealis is a beautiful crown of seven stars, given by Bacchus, the son of Jupiter, to Ariadne, the daughter of Minos, second king of Crete and Pasiphæ. Bacchus is said to have married Ariadne after she was basely deserted on the Island of Naxos, by Theseus, king of Athens, whom she had delivered from the labyrinth of Crete, after having conquered the Minotaur. After her death the crown which Bacchus had given her, was made a constellation. This crown is called by Virgil Gnossia Stella, because Ariadne was born at Gnossus. The principal star in it is Alphecca 2 magnitude.

Hercules is represented on the celestial globe with a club in his right hand, the three headed dog Cerberus in his left, and the skin of the Nemæan lion, thrown over his shoulders. This Hercules was the son of Jupiter and Alcmena, and generally called the Theban. He was a scholar of Chiron the centaur, and accompanied Jason in the Argonautic expedition. Ninus, son of Belus, was however the Hercules of the Chaldeans. He founded the Assyrian monarchy, and was therefore the Jupiter of the Assyrians. He caused the same honours to be paid his father Belus in Babylon, as to the creator of the universe, and established his worship wherever he extended his conquests. The Grecians afterwards followed the example, and worshipped him under the title of Jupiter; and the heathen world formed to themselves no less than 300 gods of this name. The worship of Jupiter became universal. He was the Ammon of the Africans, the Belus of Babylon (for Ninus after his death was called Hercules, the son of Jupiter) the Osiris of Egypt, &c. An account of his worship is given in the sacred writings. See the remarks at the end of the constellations. Ras. Algethi 3 mag. is situated in the head of Hercules.

Lyra was at first a tortoise, afterwards a lyre, because the strings of the lyre were originally fixed to the shell of a tortoise. It is asserted that this is the lyre which Apollo or Mercury gave to Orpheus, and with which he descended the infernal regions in search of his wife Euridice. Orpheus after death received divine honours; the muses gave an honourable burial to his remains, and his lyre became one of the constellations. Pythagoras and his followers represent Apollo playing upon a harp of seven strings, by which is meant (as appears from Pliny, lib 2. c. 22, Macrobius 1. c. 19. and Censorinus c. 11.) the sun in conjunction with the seven planets, for they made him the leader of that septenary chorus and moderator of nature, and thought that by his attractive force he acted upon the planets in the harmonical ratio of their distances; and hence they called the sun Jupiter's prison, alluding to the force with which he retains the planets in their orbits. Pythagoras made this discovery from observing, as he passed by a smith's shop, that the sounds of the hammers were more acute or grave, in proportion to their weight, and from thence found that the sound of strings were as the weights suspended, &c. see Macrobius lib. 2. insomn. scip. c. 1. or Doctor Gregory's preface to his astronomy. Pythagoras was born at Samos; he travelled into Egypt and Chaldea, where he gained the confidence of the priests, and was initiated into their mysteries, learned their symbolic writing, the nature of their gods, &c. and acquired a knowledge of the true system of the world. Pythagoras distinguished himself by his discoveries in geometry, astronomy, and mathematics in general, and invented the demonstration of the 47 prop. of the 1st book of Euclid. He was the first who supported the doctrine of *Metempsychosis*, or the transmigration of souls into different bodies. He founded a sect in Italy called the *Italian*. The most learned and eloquent men of the age, the rulers and the legislators of all the principal towns of Greece, Sicily, and Italy, boasted in being the disciples of Pythagoras. See Lempriere. Lyra 1. and β a quadruple star of the 3 mag. are the most remarkable stars in this constellation.

Cygnus is fabled by the Greeks to be the swan, under the form of which Jupiter deceived Leda, or Nemesis, the wife of Tyndaris, king of Sparta. Leda was the mother of Pollux and Helena, who was the cause of the Trojan war; and also of Castor and Clytemnestra. The two former were deemed the offspring of Jupiter, and the others claimed Tyndarus as their father. Some, however, supposed that this constellation derived its name from Cycnus, a son of Mars by Pelopea, who was killed by Hercules. Others that it was Cycnus, whom Achilles smothered, his darts having no effect on him; but was immediately changed into a swan, &c. Arided 2. Albireo 3, and two stars that sometimes are invisible, at other times of the 3d mag. are the most remarkable stars.

Vulpecula and Anser was made by Hevelius out of the unformed stars.

Sagitta the arrow, is supposed to be one of the arrows of Hercules, with which he killed the eagle or vulture that perpetually knawed the liver of Prometheus, who was tied to a rock on Mount Caucusus by order of Jupiter.

Delphinus, the dolphin, was placed among the constellations by Neptune, because by means of a dolphin he obtained Amphitrite, his wife.

Equulus, the little horse, sometimes called *equisectio*, the horse's head, is supposed to be the brother of Pegasus; some take him to be the horse which Neptune struck out of the earth with his trident, when he disputed with Minerva for superiority, and which some confound with Pegasus.

Pegasus, a winged horse, sprung from the blood of Medusa after Perseus had cut off his head. He received his name from his being born, according to Hesiod, near the sources (Pege) of the ocean. According to Ovid, he fixed his residence on mount Helicon, where, by striking the earth with his foot, he produced a fountain called Hippocrane. He became the favourite of the muses; and being tamed by Neptune or Minerva, he was given to Bellerophon, son of Glaucus, king of Ephyre, to conquer the Chimæra, a hideous monster that continually vomited flames; it had three heads, that of a lion, a goat, and a dragon. The fore parts of its body were those of a lion, the middle that of a goat, and the hinder parts those of a dragon. It lived in Lycia in the reign of Jobates, by whose orders Bellerophon was sent to destroy it. The Chimæra is supposed to be a burning mountain in Lycia, whose top was the resort of lions, on account of its desolate wilderness; the middle, which was fruitful, was covered with goats, and at the bottom, the marshy ground abounded with serpents; and that Bellerophon was the first who made his habitation on it. Plutarch says that it was the captain of some pirates who adorned their ship with the images of a lion, a goat, and a dragon. After the destruction of this monster, Bellerophon attempted to fly to heaven. This presumption was punished by Jupiter, who sent an insect to torment Pegasus, which occasioned the melancholy fall of his rider.—Pegasus continued his flight up to heaven, and was placed by Jupiter among the constellations. From the Chimæa and Orthos, a dog with two heads which belonged to Geryon, and which Hercules killed, sprung the sphinx and lion of Nemea. In this const. are the stars Markab 2, Sheat Alperas 2, and Algenib, 3d mag.

Lacerta, the lizard, was added by Hevelius to the old constellations.

Andromeda is represented on the celestial globe by the figure of a woman almost naked, having her arms extended, and chained by the wrist of her right arm to a rock. She was the daughter of Cepheus, king of Ethiopia, by Cassiope. Cepheus, by the advice of the oracle of Jupiter Hammon, exposed Andromeda tied to a rock, near Joppa, now Jaffa in Judea (according to Pliny) to be devoured by a sea monster, to preserve his kingdom, but she was rescued by Perseus. (See Cassiopeia.) Pliny says that the skeleton of the huge sea monster, to which she had been exposed, was brought to Rome by Scaurus, and carefully preserved. The fable of Andromeda and the sea monster, is explained, by supposing that she was courted by the captain of a ship who attempted to carry her away, but was prevented by another more successful rival. The star marked α 2, Mirach 2, and Almaach 2, are the principal in this constellation.

Triangulum. A triangle is a well known figure in geometry; it was placed in the heavens in honour of the most fertile part of Egypt, being called the delta of the Nile, from its resemblance to the Greek letter of that name $\triangle$. The Nile, anciently called Ægyptus, flows through the middle of Egypt, in a northerly direction, and when it comes to the town of Cercassorum, it divides itself into several streams, and falls into the Mediterranean by seven channels or mouths; the island which these several streams form is called *Delta*. The invention of geometry is usually ascribed to the Egyptians, so called, as some think, from Ægyptus son of Belus and brother of Danaus, and it is asserted that the annual inundations of the Nile which swept away the bounds and landmarks of estates, gave rise to it, by obliging the Egyptians to consider the figure and quantity belonging to the several proprietors. The Nile yearly overflows the country, and it is to those regular inundations that the Egyptians are indebted for the fertile produce of their lands. It begins to rise in the month of May for 100 successive days, and then decreases gradually for the same number of days. If it does not rise as high as 16 cubits, a famine is generally expected; but if it exceeds this by many cubits, it is of the most dangerous consequences; houses are overturned, the cattle are drowned, and a great number of insects are produced from the mud, which destroy the fruits of the earth. As it very seldom rains in Egypt, the cause of the Niles overflowing, is the heavy rains which regularly fall in Ethiopia in the months of April and May, and which rush down in torrents upon the country, and lay it all under water.

Musca Borealis. This is a new constellation supposed to be formed in opposition to Musca australis, which see.

Perseus et Caput Mudusæ. Perseus is represented on the celestial globe with a sword in his right hand, the head of Medusa in his left, and wings at his ancles. Perseus was the son of Aerisius and Danae. He was no sooner born, than he was thrown into the sea, with his mother Danae, but being driven on the coast of the island of Seriphos, one of the Cyclades, they were found by one Dictys, a fisherman, and carried to Polydectes, the king of the place, who intrusted them to the care of the priests of Minerva's temple. Here he promised the king to bring him the head of Medusa. To equip him for this arduous task, Pluto, the god of the infernal regions, lent him his helmet, which had the power of rendering its bearer invisible. Minerva, the goddess of wisdom, furnished him with her buckler, which was as resplendent as glass; and he received from Mercury wings, and the telaria, with a short dagger made of diamonds; or, as some say, he received the telaria or *herpe* from Vulcan, which was in form like a scythe. Thus equipped he began his expedition, and traversed the air, conducted by the goddess Minerva. Having discovered Medusa, he cut off her head, and from the blood which dropped from it in his passage through the air, sprung innumerable serpents, which is said to have ever since infested the sandy deserts of Lybia: from the same blood sprung Chrysaor with his golden sword, and the horse Pegasus, which immediately flew to Mount Helicon, though Ovid says Perseus was mounted on him when he freed Andromeda. Medusa was one of the three Gorgons, who were daughters of Phorcys and Ceto; their names were Stheno, Euriale, and Medusa; all immortal except the latter. They were represented with snakes on their heads, instead of hair, with yellow wings and brazen hands; their bodies were also covered with impenetrable scales, and their very looks turned all those who beheld them into stones. Medusa, according to some, was celebrated for the beauty of her locks, but having, with Neptune, violated the temple of Minerva, that goddess changed her locks into serpents. Perseus in gratitude to Minerva, placed her head on her ægis or shield, where it retained the same petrifying power as before. It was afterwards placed among the constellations. (See Lempriere.) Diodorus and others suppose that the Gorgons were a warlike race of women near the Amazon, whom Perseus, with the help of a large army, totally destroyed. The principal stars in it are α 2, and Algol 2.

Serpens is also called Serpens Ophiuci, being grasped by the hands of Ophiucus. (See Serpentarius.)

Serpentarius, also called Ophiucus or Opheus, is supposed by some to be Hercules, who before he had completed his eighth month, squeezed two serpents to death, which Juno sent to devour him. Some, however, take him to be Æsculapius, son of Apollo by Coronis. He was taught the art of medicine by Chiron, and was physician to the Argonauts. He was considered so skilful in the medical power of plants, that he was called the inventor as well as the god of medicine. Æsculapius was represented with a large beard, holding in his hand a staff, round which was wreathed a serpent; his other hand being supported on the head of a serpent. Serpents were more particularly sacred to him, not only as the ancient physicians used them in their prescriptions, but because they were the symbols of prudence and foresight, so necessary in the medical profession. Cicero reckons three of this name. (de nat. Deor. 3, c. 22.) The most remarkable star in this constellation is Ras Alhagus.

Aquila et Antinous. Aquila is supposed to be Merops, a king of the island of Cos, one of the Cyclades; he was changed into an eagle (Ovid, met. 1.) and placed among the constellations. Antinous was a youth of Bithynia, in Asia Minor, a great favourite of the emperor Adrian, who erected a temple to his memory, and placed him among the constellations. Antinous was armed by Hevelius with a *bow* and *arrow*. *Altair* or *Atair* 1, is situated in this constellation.

Taurus Poniatowski was so called in honour of Count Poniatowski, a polish officer of great merit, who saved the life of Charles XII. of Sweden, at the battle of Pultowa, a town near the Dnieper, about 150 miles south east of Kiow ; and a second time at the island of Rugen, near the mouth of the river Oder.

Scutum Sobieski was so named by Hevelius, in honour of John Sobieski, king of Poland. Hevelius was a celebrated astronomer, born at Dantzick ; his catalogue of fixed stars was entitled *Firmamentum Sabieskianum,* and dedicated to Sobieski.

Mons Mænalus, the mountain Mænalus in Arcadia, was sacred to the god Pan, and much frequented by shepherds : it was covered with pine trees, whose echo and shade has been much celebrated by all the ancient poets : it received its name from Mænalus, a son of Lycaon. It was made a constellation, and placed by Hevelius under the feet of Bootes.

Cor Caroli is a star in the neck of Chara, and was so denominated by Sir Charles Scarborough, physician to king Charles II. in honour of king Charles I.

Triangulum Minus. This constellation was made by Hevelius, of the unformed stars between Triangulum Boreale and the head of Aries.

THE SOUTHERN CONSTELLATIONS.

Cetus is pretended by the Greeks to be the sea monster, which Neptune, brother to Juno, sent to devour Andromeda. In this are Menkar 2. Baten Kailos 3, and Mira, which is sometimes of the 2d. mag. and sometimes invisible. The period of its variations is 334 days.

Eridanus, the river Po, called by Virgil the king of rivers, was placed in the heavens for receiving Phæton, whom Jupiter struck with thunderbolts, when the earth was threatened with a general conflagration, through the ignorance of Phæton, who had presumed to be able to guide the chariot of the sun. According to the poets, while Phæton was unskilfully driving the chariot of his father, the blood of the Ethiopians was dried up, and their skins became black, a colour which is still preserved by the greater part of the inhabitants of the torrid zone. The territories of Libya were also parched up, according to the same tradition, on account of their too great vicinity to the sun, and ever since Africa, unable to recover her original verdure and fruitfulness, has exhibited a sandy country and uncultivated waste. Phæton, according to the Mythologists, was a Ligurian prince, who studied astronomy, and in whose age the neighbourhood of the Po was visited with uncommon heats. From his love of astronomy, he was called a son of the sun, or Phœbus and Clymene ; or as others say, of Aurora and Tithonus or Pausanias. The river Po is sometimes called Orion's river.—The most remarkable star in it is Achernar 1.

Orion is represented on the globe by the figure of a man with a sword in his belt, a club in his right hand, and the skin of a lion in his left ; he is said by some authors to be the son of Neptune and Euriale, and that he had received from his father the privilege and power of walking over the sea without wetting his feet. Others make him son of Terra, like the rest of the giants, according to Diodorus. Orion was a celebrated hunter, superior to the rest of mankind, by his strength and uncommon stature. He built the port of Zancle, and fortified the coast of Sicily against the frequent inundations of the sea, with a mound of earth called Pelorum, on which he built a temple to the gods of the sea. Others say that Jupiter, Neptune, and Mercury, as they travelled over Bæotia, met with great hospitality from Hyrieus, a peasant of the country, who was ignorant of their dignity and character. When Hyrieus had discovered that they were gods, he welcomed them by the voluntary sacrifice of an ox. Pleased with his piety, the gods promised to grant him whatever he required, and the old man who had lately lost his wife, and to whom he had made a promise never to marry again, desired them, that, as he was childless, they would give him a son without obliging him to break his promise.—The gods consented, and ordered him to bury in the ground the skin of the victim; nine months after he dug the skin, and found a beautiful child, which he called Orion, *ab urina, quia Dii urinam in pellem redderant, ex qua procreatus.* Ovid says that the name was changed from Urion to Orion. Orion was buried in the island of Delos, and after his death was made a constellation. According to the ancient poets, this constellation never rises or sets without great storms, and hence he is called *nimbosus Orion* by Virgil, and *tristis Orion* by Horace. Authors who explain this fable say, that Orion was a great astronomer and disciple of Atlas. The stars Betelgeux 1, and Rigel 1, are in this constellation.

Monoceros, the unicorn, was composed by Hevelius, according to most authors, of those stars which the ancients had not comprised within the outlines of the other constellations. According to Doctor Gregory (astr. b. 2, pr. 22) this constellation, together with the Camelopard, was first described by *Bartschius* on his globe of four feet diameter, and afterwards retained by Hevelius.

Canis Minor, according to the Greek fables, was one of Orion's hounds.... Some suppose it to refer to Anubis, an Egyptian god, with the head of a dog; others to Diana, the goddess of hunting; others to Acteon, who was changed by Diana into a stag and devoured by his own dogs, &c. Others are of opinion, that the Egyptians were the inventors of this constellation, and as it rises before the dog star Sirius, which in the dog-days was so much dreaded, it is properly represented as a little watchful creature, giving notice of the others approach; hence the Latins have called it Antecanis, the star before the dog. The most remarkable star in it is Procyon or Algomeiza 1.

Hydra is the water serpent which, according to the fable of the poets, infested the neighbourhood of the lake Lerna in Peleponesus. It had several heads, and as soon as one was cut off, two immediately grew in its place, if not prevented by fire. It was one of the labours of Hercules to kill this monster, which he effected with the assistance of Iolaus, king of Thessaly. It was in the gall of this Hydra that Hercules dipt his arrows, the wounds inflicted by which were incurable and mortal. The general opinion is, that the Hydra was a multitude of serpents which infested the marshes of Lerna. Cor Hydræ is in this constel. a triple star 2 mag. There is another constellation of the same name near the south pole.

Sextans, called also *Sextans Uraniæ*, is a mathematical instrument well known to mariners. This constellation was formed by Hevelius, of the unformed stars between Leo and Hydra. Urania was one of the muses, daughter of Jupiter and Mnemosyne, who presided over astronomy. She was represented as a young virgin, dressed in an azure coloured robe, crowned with stars, and holding a globe in her hands, and having many mathematical instruments placed around.

Crater, according to the mythologists, is the cup or pitcher of Aquarius. Alkes, 4 mag. is its principal star.

Corvus, according to the Greek fables, was made a constellation by Apollo. This god being jealous of Coronis, the daughter of Phlegyas and mother of Æsculapius, sent a crow to watch her behaviour; the bird perceived her criminal partiality to Ischys, the Thessalian, and acquainted Apollo with her conduct. Some think that this Corvus was the daughter of Coronæus, king of Phocis, changed into a crow by Minerva, when flying before Neptune.

Centaurus. The Centauri were a people of Thessaly, half men and half horses. The ancient people of Thessaly were famous for their skill in taming horses, and their appearance on horseback was so uncommon a sight to the neighbouring states, that at a distance they imagined the man and horse to be one animal. When the Spaniards landed in America, and appeared on horseback, the Mexicans had the same ideas ; and similar ideas were excited when they saw their vessels expand their wings and fly along the surface of the ocean. Plutarch and Pliny are however of opinion, that such monsters have really existed. The battle of the Centaurs with the Lapithæ is famous in history. This constellation is by some supposed to represent Chiron, the Centaur ; but as Sagittarius is likewise a Centaur which some contend to be Chiron, it is probable that Theseus is represented by this constellation. Among the principal stars in this constellation, the most remarkable is a double star marked α 1 of the 1st, and α 2 of the 4th mag.

Lupus is supposed to be Lycaon, king of Arcadia, celebrated for his cruelties. He was changed into a wolf by Jupiter, because he offered human victims on the altars of the god Pan. (See Ovid, met. 1.) Some suppose that this king, to try the divinity of Jupiter, who once visited Arcadia, served up human flesh on his table, and that it was Arcas, son of Calisto, who thus became the victim of his impiety, and was served up for Jupiter ; for which horrid crime Jupiter punished him by metamorphosing him into a wolf.

Norma, the square, a well known instrument, is a new constellation made of the unformed stars between Lupus and Ara.

Circinus, the compasses, an instrument known for its extensive utility, is a new constellation formed near the Centaur, in allusion to the neighbouring constellations.

Triangulum Australe is a new constellation formed near the constellations Circinus and Norma. The three foregoing constellations are placed near Ara, the altar, the two former being useful to the practical artist, and the latter being the foundation of many important sciences ; Euclid, as is well known, commencing his elements with this figure. The oracle of Apollo, at Delphos, being consulted about a raging pestilence which desolated Athens, answered, that the pestilence would cease, if his altar, which was of a cubical form, were doubled. To execute the orders of the oracle, a knowledge of the properties of such solid bodies became necessary, and this gave rise to a great part of the geometry of solids.

Ara, the altar, is supposed by some to be the altar on which the gods swore before their combat with the giants ; but from the observation on the last constellation, it is more probable that it was Apollo's altar at Delphos.

Telescopium, a well known optical instrument, is a new constellation formed near Ara.

Corona Australis, a new constellation formed near Sagittarius.

Indus, the indian, is a new constellation formed to commemorate the original inhabitants of the new world.

Microscopium, an optical instrument for distinctly viewing minute objects, is a new constellation formed between Sagittarius and Piscis Australis.

Piscis Australis, is supposed by the mythologists to be Venus, who transformed herself into a fish, to escape from the giant Typhon. (See Pisces.) Fomalhaut 1, is in this constel.

Grus, the Crane, a new constellation formed near Piscis Australis, in allusion, perhaps to the cranes, against which the Pygmies, a race of dwarfs said to be no more than one foot high, was accustomed yearly to make war; or perhaps in allusion to their princess Gerana; who was changed into a crane, for boasting herself fairer than Juno.

Toucana or Touchan, the American goose; a new constellation near Indus.

Phœnix, the Phenix, a new constellation near Eridanus. This was the name of a fabulous bird worshipped among the Egyptians.

Apparatus sculptoris, is a new constellation between Cetus and Phœnix.

Fornax chemica, a new constellation formed out of Eridanus.

Horologium, a recent constellation formed near Eridanus.

Cela sculptoria, Columba Noachi, or Noah's dove, *Equuleus pictorius, Pixis nautica,* and *Antlia pneumatica,* are all new constellations.

Lepus, the hare. This constellation is formed near the great dog, which, from the motion of the earth, seems continually to pursue it.

Canis major, according to the mythologists, is one of Orion's hounds. The Egyptians, who carefully watched the rising of this constellation, and by it formed their judgment of the swelling of the Nile, called the bright star Sirius, the centinel or watch of the year; and represented it, according to their hieroglyphic manner of writing, under the figure of a dog. The Egyptians called the Nile *Siris,* and hence, some are of opinion, they derived the name of their deity *Osiris.* This Osiris was son of Jupiter and Niobe, and king of Egypt, who is said not only to have civilized his own subjects, but also to have civilized and polished many other nations. To perform this task, he left his own kingdom, accompanied by his brother Apollo, and by Anubis, Macedo and Pan; and left the management of affairs to Isis or Io, his wife, and his faithful minister Hermes or Mercury; and the command of his troops at home to Hercules. At his return, he was murdered by his brother Typhon, and his body was thrown into the Nile. The Egyptians worship him under the titles of Apis, Serapis, &c. Historians take him to be *Mizraim,* eldest son of *Cham,* the third son of Noah, who, in the division of the world, received Africa for his lot. He was worshipped by the Egyptians under the title of *Hammon* or *Jupiter Hammon.* The dog star Sirius, is of the 1st mag. and the most remarkable, not only in this constellation, but in the heavens, being the largest and brightest, and therefore considered the nearest to us of all the fixed stars.

Argo Navis, the ship Argo, which carried Jason and his Argonauts to Colchis, in quest of the golden fleece. Some say that this vessel was built at Argos, from whence it derived its name; others derive it from one Argos, who first proposed the expedition; others because it carried Grecians, commonly called Argives; and others, from the Greek word *argos,* swift. She had 50 oars, and, according to some, a beam in her prow, cut in the forest of Dodona, which gave oracles to the Argonauts. After the expedition, she was drawn ashore at the isthmus of Corinth, and consecrated to the god of the sea; and afterwards made a constellation by the poets. Canopus 1 mag. is the principal star. There is another marked η remarkable only with the telescope, from its containing 9 other stars in the neighbourhood of a nebeculous cluster, &c.

Crux, the cross. There are four stars in this constellation forming a cross, by which mariners, sailing in the southern hemisphere, readily find the situation of the antartic pole, by means of the stars α and γ which nearly point in this direction. This is a new constellation, and formed, no doubt, in honour of that instrument on which the Son of God redeemed mankind. Venerable Bede gave christian names to the signs of the zodiac, and Julius Schillerus has followed the example in his *Cœlum Stellatum,* or Starry Heavens, published in 1627. But later astronomers think that the ancient names ought to be retained, to avoid confusion, and to preserve the ancient astronomy. They will also afford so many monuments of the folly and stupidity of the most enlightened nations, in the article of religion, when carried away by the current of their passions, and the violence of human depravity. In these constellations we find some of the gods of the heathens, it is true, *placed in the heavens*, to whose monstrous vices so much incense has been offered, under the veil of an absurd religion, but a veil, however, too transparent to hide their guilt. The most abandoned men became the most powerful divinities; worshipped with sacrifices, addressed with prayers and supplications, and their vices at length became the objects of adoration. The light of the gospel dispelled these dark shades of infidelity; the wisdom of the Son of God raised man from this state of abasement and folly; the arms which he made use of, was the cross on which he suffered; and wherever this is planted, these idols must fall. The humility of the cross is an antidote to the pride of man, the sufferings of the cross an antidote to his passions.

Musca australis, a new constellation, formed between Crux and Apus.

Apus, vel Avis Indica, a new constellation, near Pavo. This bird is a native of the Molucca Islands.

Pavo, the peacock, one of the new constellations near the south pole. This is often called *Junonia avis*, Juno's bird; this goddess being represented as drawn through the air in her chariot by these birds. Juno, being the wife of Jupiter, became queen of the gods, of the heavens, &c. and is famous for her severity to the mistresses and illegitimate children of her husband Jupiter, whom she at length abandoned on account of his debaucheries. This goddess employed Argus, who had an hundred eyes, two of which only slept at a time, to watch Io, one of Jupiter's mistresses. She afterwards put the eyes of Argus on the tail of the peacock.

Octans, the octant, a well known marine instrument, one of the new constellations surrounding the antartic pole.

Hydra, a new constellation near the south pole, in which the remarkable *nebecula minor* is situated.

Reticulus, the net, is a new constellation, formed between Horologium and Dorado. It is sometimes called Reticulus rhomboidalis.

Dorado, or Xiphias, the sword fish, a new constellation, formed round the pole of the ecliptic.

Piscis volans, the flying fish, a new constellation formed out of the ship Argo.

Chameleon, a new constellation near the south pole.

Mons mensi formis, or table mountain, a mountain at the Cape of Good Hope, well known to mariners. In this constellation the greatest of the nebula, called by sailors the Magellanic clouds, is situated.

Robur Caroli, or Charles' oak, was so called by Dr. Halley in honour of the tree in which Charles II. saved himself from his pursuers, after the battle of Worcester. It was made out of the unformed stars between the ship Argo and Centaur. This constellation is not on Cary's globe. Dr. Halley went to St. Hellena in the year 1676, to make a catalogue of such stars as do not rise above the horizon of London.

The reader may not be a little surprised to find us dwell so long upon these fables of the ancients, this masquerade of folly, which exhibits nothing but a chaos of fiction, without order or connection; where we find the same heroes presented under different names, the same actions at different times, and though sometimes true, at other times destitute of foundation, and worthy only of contempt. A slight acquaintance with these chimeras of the poets, and the mythology of the Pagans, was however deemed, not only necessary in understanding these authors, but also, being the foundation of much of the ancient astronomy, considered as not foreign to a work of this nature, and as useful in conveying an idea of the pretended wisdom of those ancient nations, who were the inventors of them, or who misapplied their original use and meaning.

The mystical or allegorical sense of these fables in a philosophical or historical view, conveyed an obscure explanation of some of the ordinary operations of nature, or the inventions or exploits of some of these pretended gods. In a religious sense, they served as a cloak for vice, and in a political sense, they served to keep a superstitious people in subjection, to those whose interest it was to conceal their mysteries.

The different parts of nature were portioned out to those whose knowledge was the greatest, or who were most successful in investigating the properties of these parts, and applying that knowledge to the advantage of mankind: and lest these persons, who were afterwards converted into deities, should be thought mortal, their names were changed, and others were given to them expressive of their rank among the gods. Uranus, Auranos, or the Heavens, was considered by them as the oldest of the gods; and Tithea, Tellus, Terra, or the Earth, his wife, by whom he had the Titans. The chief of these was Saturn or Time, who is said to have disputed superiority with his father; as these heathens probably thought nothing anterior to time. He married *Ops* or *Terra*, also called *Rhea* or *Cybele.* She was therefore called the mother of the gods, who were nothing else in reality but sons of the earth, or mortals. Saturn, considered then the most remote of the planets, was called after the name of this god; and hence the planet Herschel has, for the same reason, obtained the name of Uranus, from modern astronomers, being more remote than Saturn. Saturn is represented as a cruel god, who devoured his own children, in allusion to time, which at length destroys every thing; and hence human sacrifices were offered to him, by the ignorant and superstitious Pagans. Jupiter, however, the most illustrious of his offspring, escaped his fury, and afterwards dethroned him for attempting to take away his life, and thus became sole master of the empire of the world, which he divided with his brothers. He reserved the heavens and the earth for himself, which, according to the poets, he filled with his natural children, as he became a Proteus to gratify his passions. The empire of the air he gave to Juno, his wife, that of the sea to Neptune, and constituted Pluto king of the infernal regions. He was called Jupiter, or Jove, in allusion to the Jehovah of the Jews; as the Chaldeans, who were so recently descended from Noah's son, could not be entirely ignorant of the supreme being. The planet Jupiter is so called from this god, being the largest of the planets, and the next in order after Saturn. The three next planets, Mars, Venus and Mercury, the offspring of Jupiter, were emblems of war, pleasure and science, which this god was so famous for. The sun was called the prison of the gods, which shews that they had some idea of the force of gravity which retains the planets in their orbits; it was therefore an emblem of Jupiter, who held all the other deities in subjection. Science, which conveyed the mysteries of these gods and their pretended knowledge, was indicated by Mercury, who, from his rapid velocity in its orbit, represented their messenger, and hence he was painted with wings, &c. The goddess of love, or rather lust, was represented by Venus, from its beautiful appearance, and its remaining alone with the sun, the emblem of Jupiter, when all the other luminaries disappeared. War was represented by Mars, from his fiery or bloodlike appearance, &c.

That the Chaldeans were the first who under these fictitious titles deified their kings, their warriors, their philosophers, &c. is now universally allowed. It is well known that Cham, one of Noah's sons, received Africa for his portion, and made Egypt the chief seat of his residence. His name signifies *Calor* or *Niger*, and Chamo, signifies Terra Cham or Egyptus, so called therefore from Cham. His offspring Chanaan was cursed on account of his immodest behaviour to his father. Nemrod, his grandson, founded the Babylonish empire, and is supposed to be the Saturn described above, his cruel nature, agreeing with that of Saturn, who devoured his own children; as Nemrod hunted his subjects, as others hunt wild beasts. Uranus, or Cœlum, is supposed to allude to Cham, as these proud nations wished to derive their origin from heaven, and not to acknowledge their gods as the children of men. Belus, according to most authors, was the son of Nemrod, and second king of Babylon. Belus signifies *Dominans*, from Bel, which in the Syriac language signifies the sun, the ruler of the solar system. He was the first among the Chaldeans that cultivated astronomy, and hence was honoured with the title of Jupiter or Jove, from his superiority in the knowledge of this science, which obtained him so eminent a station in the heavens. His son Ninus set up his father's image, and caused his people to worship it. Ninus was therefore probably the first that attempted to pay those honours to a man (and an impious man too) that was only due to God himself. Other nations followed the example, each bestowing the same honours and marks of distinction on their founders, and ranking them in the number of the gods. Hence Ninus, the Hercules of the Chaldeans, being a great warrior, becomes the Jupiter of the Assyrians, whose empire he founded. The Greeks and Romans had likewise their Jupiters, their Hercules's, their Junos, their Venuses, their Mercuries or Minervas, &c. which the mythologists so often confound with each other, and hence the confusion in the accounts that we have of them. The tower of Babel, begun by Nemrod, was converted into a temple, in which Belus was worshipped. The priests of Belus applied themselves to the study of astronomy, and placed in the heavens, among the number of their deities, all those that distinguished themselves either by their valour, their knowledge, or their vices, or that supported them in their superstition. Hence arose the numerous gods of the Chaldeans, the Egyptians, the Grecians, &c. and hence the rapid increase of idolatry almost all over the world; every nation being desirous of claiming an alliance with, and of boasting their descent from the gods. I shall mention here one circumstance that will throw some light on the nature of these pretended deities, and account for their vast number and increase. The famous tower of Babel was composed of eight pyramidal towers raised one above another, in the highest of which was a magnificent bed, where the priests daily conducted a woman, who, as they said, was honoured with the company of the god. (For more particulars see Joseph. ant. Jud. 10. Herodot. 1. c. 181, &c. Strabo. 16. Arrian. 7. Diodorus 1, &c.) Hence so many sons of Jupiter, so many heroes, so many Gods, &c. The pretended worship of these priests, their religious ceremonies, &c. were all calculated to support and gratify their infamous passions; and there was no place, from which modesty was more industriously banished than from these ceremonies. They even gave every vice its own god, to support the worship of it. We need not, therefore, be surprised at the rapid increase of idolatry, or at the description given of this impious Babylon by different authors. (See Curtius. lib. 5. c. 5.) The sacred writings also point out its abominations, and exhibit it, as an example for posterity, of the folly of those who abandon their maker, and even their reason, to gratify their passions; and of the ridiculous pride of man in desiring to be honoured as God.

The history of these gods became at length so obscure, and the human mind so blind and corrupted, that the sun, moon, stars, &c. and at length serpents, crocodiles, onions, &c. became objects of veneration and worship. And this worship, extravagant as it may seem, became the worship of the learned as well as the ignorant, except among the few whom God selected from among these idolators, who retained a knowledge of him, an esteem for the dignity of human nature, a recollection of the glorious end to which the true religion points the hope of man, and a reverence for that being alone, who called all other beings out of nothing. This chosen people was Abraham and his posterity.

113. The *Galaxy*, *via Lactea*, or *Milky way*, is a whitish luminous tract, which seems to encompass the heavens, sometimes in a double, but generally in a single path, varying in breadth from about 4 to 25 degrees.*

114. *Bayer's Characters.* This is a useful invention of denoting the stars in every constellation, by the letters of the Greek and Roman alphabets; setting the first letter in the Greek alphabet α,

What a contrast would the true knowledge of God, handed down by these venerable patriarchs and prophets, and displayed in its full splendour in the gospel, afford, compared with the abominable mysteries of the Pagans (and the impious absurdities of those who in modern times have copied their example) traced to their origin, if modesty could permit the reading of them, or our contracted limits a more ample detail.

Who could believe that the most enlightened part of modern Europe, could afford anew, these scenes, so degrading to civilized man, and to human nature; that amid the acclamations of the *polite*, the *accomplished*, the *enlightened* citizens of Paris, the prostitutes of this city should be carried *nudis corporibus* on triumphal cars, together with several youths in the same savage degradation! That dressed and decorated *as became the solemnity of the occasion*, the citizens should march in *solemn* procession, accompanied with the greater part of the youth of the city, crowned with chaplets of flowers, emblematic of their being disciples of the *goddess of reason*, whom they conveyed in *such* pomp to be worshipped in *her temple*. (Sometimes the Cathedral church de notre dame!) When this age of science and of christianity, affords so humiliating a picture of the *abuse of reason*, and of man, under the dominion of *false philosophy*, have we not reason to exclaim with the philosophic Cicero, O tempora! O mores!!!

Every age then affords monuments as testimonies to the value of that religion, in which nothing is left to the vanity of human speculation, but by its own divine constitution conducts man to a greatness above his nature, to that dignified and immortal existence, alone worthy the nobility of a rational being, the greatness and hopes of an immortal soul—monuments that will for ever decide the question in favour of those amiable virtues emanating from the practice of the christian religion, and the spirit which it breathes; when contrasted with the folly of impious or philosophic man, adoring those idols which are the objects of his brutal passions, or overturning those laws which forbid the commission of his crimes.

* The milky way comes properly under the head of constellations, being composed of an infinite number of small stars, which causes that whiteness from which it derives its name. It passes through Cassiopeia, where it is nearest to the north pole, then through Perseus, Auriga, Taurus, the feet of Gemini, Orion's Club, Monoceros, part of Canis Major, the ship Argo, Robur Caroli, Crux, the feet of the Centaur, Musca Australis (where it approaches nearest to the south pole) Circinus, Norma, Ara, and Scorpio, where it divides into two parts. The eastern branch passes through the tail of Scorpio, the bow of Sagittarius, Scutum Sobieski, the feet of Antinous, Aquila, Sagitta, Vulpecula, and Cygnus. The western branch passes through the tail of Scorpio, the right side of Serpentarius, Taurus Poniatowski, Sagitta, Anser and Cygnus, where it meets the foregoing branch, and ends in Cassiopeia, where *Manilius* begins the description of it.

Manilius Caius was a celebrated mathematician and poet of Antioch, who wrote a poetical treatise on Astronomy, of which five books are extant, treating of the fixed stars. The age in which he lived is not known, though some suppose that he flourished in the augustan age.

There are other lesser divisions of the galaxy, which may be seen in Hevelius's firmament.

to the principal star in each constellation, β to the second in magnitude, and so on in order ; and when the Greek alphabet is finished, the first letters a, b, c, &c. of the Roman alphabet is used.*

115. *Nebulous* or *cloudy*, is a term applied to certain fixed stars, smaller than those of the 6th magnitude, which only shew a dim, hazy light, like little specks or clouds. *Nebulæ* is when several of these form a *Cluster*.†

116. The *Solar System*‡ is that part of the *universe* § which consists of the *sun*, *planets*, and *comets*.

* John Bayer, of Augsburg, in Swabia, published in 1603, an excellent work entitled *Uranometria*, being a complete celestial atlas of all the constellations, in which the stars are denoted as above. Succeeding astronomers have adopted this useful method of describing the stars, and enlarged it by adding the numbers 1, 2, 3, &c. in order, when any constellation contains more stars than can be marked by both alphabets. These figures are also sometimes placed above the Greek letter, especially where double stars occur ; for though many stars may appear single to the naked eye, yet, when viewed through a telescope of considerable magnifying power, they appear double, triple, &c. Thus in Dr. Zach's Tabulæ Motum Solis, we find *f* Tauri, β Tauri, γ Tauri, δ^1 Tauri, δ^2 Tauri, &c.

The following Greek alphabet is inserted for the use of those who are unacquainted with the letters ; the capitals are however seldom used.

	Name.	*Sound.*		*Name.*	*Sound.*
Α α	*Alpha*	a	Ν ν	*Nu*	n
Β β ϐ	*Beta*	b	Ξ ξ	*X*	x
Γ γ ϒ	*Gamma*	g	Ο ο	*Omicron*	o short.
Δ δ	*Delta*	d	Π π ϖ	*P*	p
Ε ε	*Epsilon*	e short.	Ρ ρ ϱ	*Rho*	r
Ζ ζ ς	*Zeta*	z	Σ σ ς	*Sigma*	s
Η η	*Eta*	e long.	Τ τ	*Tau*	t
Θ ϑ θ	*Theta*	th	Υ υ	*Upsilon*	u
Ι ι	*Iôta*	i	Φ φ	*Phi*	ph
Κ κ	*Kappa*	k	Χ χ	*Chi*	ch
Λ λ	*Lambda*	l	Ψ ψ	*Psi*	ps
Μ μ	*Mu*	m	Ω ω	*Omega*	o long.

† Dr. Herschel has discovered no less than 1250 of these nebulæ : there were only 103 known to former astronomers. He has shewn that the milky way is a continued nebulæ. There are two remarkable nebulæ near the south pole, called by sailors the magellanic clouds, which resemble in brightness the milky way. The number of stars in these nebulæ exceed conception. 70 stars have been reckoned in the *Pleiades*, no less than 2500 in the constellation Orion; and Herschel, in some of his observations on the milky way, found that by allowing 15′ for the diameter of his field of view, a belt of 15° long, and 2° broad, which he had often seen pass before his telescope in an hour's time, could not contain less than 50,000 stars, large enough to be distinctly numbered.

‡ By system is meant a lucid body, with some number of opake bodies situated within the sphere of its influence, and round which the others revolve.

§ By the universe we understand the whole material creation. The Greeks called it *Topan*, signifying every thing, and the Latins *Inane*, the void. See the fourth part of this work.

117. The *sun* is that lucid body, situated nearly in the centre of the solar system.

118. *Planets* are opake* bodies similar to our earth, which perform their motions round the sun, in certain periods of time. They are divided into *primary* and *secondary*.

119. The *primary planets* † are those which regard the sun as their centre of motion. There are 9 primary planets, distinguished by the following characters, and names, according to their proximity to the sun, viz. ☿ Mercury, ♀ Venus, ⊕ Earth, ♂ Mars, ⚵ Juno, + Pallas, ⚳ Ceres, ⚶ Vesta, ♃ Jupiter, ♄ Saturn, ♅ Herschel or Uranus.

120. The *secondary planets*, called *Satellites* or moons, are those bodies which are attendants on the primary planets, and regard them as the centres of their motion; as the moon which revolves round the earth, the Satellites of Jupiter which revolve round Jupiter, &c. There are 18 secondary planets, of which the Earth has *one*, Jupiter *four*, Saturn *seven*, and Uranus *six*.

121. The *orbit* of a planet is the imaginary path which it describes round the sun. The earth's orbit is the ecliptic. The real motion of all the planets in their orbits round the sun is from west to east, or according to the order of the signs on the ecliptic.

122. The *nodes* are the two opposite points, where the orbits of the primary planets cut the ecliptic, and where the orbits of the secondaries cut the orbits of their primaries. That node is called *ascending*, where the planet passes from the south to the

* Opake bodies are such bodies as do not shine by their own light, or which only reflect the light received from another body, as the planets which reflect the light received from the sun.

† The planets are so called from Planeta, a wanderer, because they change their positions in the heavens, with regard to the other bodies, which are called fixed, for a contrary reason. *Uranus, Juno, Pallas,* and *Ceres,* were recently discovered, and obtained their names in conformity to the names given to the other planets by the ancients.

Uranus was discovered by Dr. Herschel, in 1781. La Place, in B. 1. c. 9, vol. 1, of his Astronomy, observes, that Flamstead, at the end of the last century, and Mayer and Le Monnier, in this, had observed it as a small star.

On the 1st of January, 1801, M. Piazzi, astronomer royal at Palermo, in Sicily, discovered *Ceres*, generally called *Ceres Ferdinanda* (or rather *Fernandea*) the latter name being added in honour of Ferdinand IV. king of the Two Sicilies. It is of the 8th mag. and consequently invisible to the naked eye, nor is it confined within the ancient limits of the zodiac. It is called by some astronomers an asteriod.

On the 28th of March, 1802, Dr. Olbers, of Bremen, discovered *Pallas*, and on the 29th of March, 1807, at 21 min. after 8, mean time, he discovered another, which he called *Vesta*. This last, in size, appears like a star of the 5th mag.

On the first of September, 1804, Mr. Harding, of Lilienthal, in the duchy of Bremen, discovered the planet *Juno*. It appears like a star of the 8th mag. These four last satellites are all so nearly at equal distances from the sun, that it is not as yet ascertained, with certainty, which of them is nearest to or most remote from it.

north side of the ecliptic; and the opposite point, where the planet appears to descend from the north to the south, is called the *descending* or south node. The ascending node is marked thus ☊, and the descending node thus ☋. The straight line which joins the nodes is called the *line of the nodes.*

123. *Aspect* of the stars or planets, is their situation with respect to the sun or each other. There are five aspects, viz. ☌ *Conjunction*, when they have the same longitude, or are in the same sign and degree with the sun; ✱ *Sextile*, when they are two signs or a sixth part of a circle distant; □ *Quartile*, when they are distant three signs, or a fouth part of a circle; △ *Trine*, when they are four signs, or a third part of a circle from each other; and ☍ *Opposition*, when they are six signs, or half a circle from each other.

The Conjunction and Opposition, particularly of the moon, are called the *Syzygies*,* and the quartile aspect the *Quadrature.*

124. The apparent *motion* of the planets is either *Direct*, *Stationary*, or *Retrograde*. Direct is when a planet appears to a spectator on the earth to perform its motion from west to east, or according to the order of the signs. A planet is *Stationary* when, to an observer on the earth, it appears, some time, in the same point of the heavens; and *Retrograde* when it apparently goes backward or contrary to the order of the signs.

125. *Aphelion*, or Aphelium, is that point in the orbit of a planet which is furthest from the sun. This point is also called the higher *Apsis.*

126. *Perihelion*, or Perihelium, is that point in the orbit of a planet, which is nearest to the sun. This point is called the lower *Apsis.*

127. *Apogee*, or Apogæum, is that point in the orbit of a planet, the moon, &c. which is furthest from the earth.

128. *Perigee*, or Perigæum, is that point in the orbit of a planet, the moon, &c. which is nearest to the earth.

129. *Apsis* of an orbit, is either its aphelion or perihelion, apogee or perigee, and the straight line which joins the higher and lower apsis is called the *line of the Apsides.*

130. *Eccentricity* of the orbit of any planet, is the distance between the sun and the centre of the planet's orbit.

131. *Geocentric* latitudes and longitudes of the planets, are their latitudes and longitudes as seen from the earth.

132. *Heliocentric* latitudes and longitudes of the planets, are the latitudes and longitudes as they would appear to a spectator, situated in the sun.

133. *True Anomaly* of a planet is its angular distance at any time, from its aphelion or apogee. Mean Anomaly is the angular distance at the same time, and from the same point, if it had moved uniformly with its mean angular velocity.

* So called from the Greek word *Suzugia*, Conjunctio, *Zugos* signifying *jugum*, a *yoke*, or pair.

134. *Equation of the centre* is the difference between the *true* and *mean* anomaly; this is sometimes called the *prosthapheresis*.

135. The *mean* place of a body is the place where it would have been if it had moved with its mean angular velocity (on supposition that the body in motion does not move with an uniform angular velocity about the central body.) The *true* place of a body is the place where the body actually is at any time.

136. *Equations*, are corrections which are applied to the *mean* place of a body to get its *true* place.

137. *Argument*, is a term used to denote any quantity by which another required quantity may be found. Thus the argument of a planet's latitude is its distance from the node, because it is upon that the latitude depends.

138. The *elongation* of a planet from the sun, is its angular distance from the sun when seen from the earth; or the angle formed by two straight lines drawn from the earth, the one to the sun, and the other to the planet.

139. The *curtate distance* of a planet from the sun or earth, is the distance of the sun or earth from that point of the ecliptic where a perpendicular to it passes through the planet.

140. A *Digit*, is the twelfth part of the apparent diameter of the sun or moon.

141. *Disc*, is the face of the sun or moon, such as they appear to a spectator on the earth; the sun and moon appearing as circular planes, though they are in reality spherical bodies.

142. *Occultation* of a star or planet, is when they are hidden from the sight by the interposition of the moon or some other planet.

143. *Aberration*, is an apparent motion of the celestial bodies, occasioned by the earth's annual motion in its orbit, combined with the progressive motion of light.

PROBLEMS

PERFORMED BY THE

TERRESTRIAL GLOBE.

PART II.

PROB. I.

To find the latitude and longitude of any given place.

Rule. BRING the given place to the graduated side of the brazen meridian, which is numbered from the equator ;* the degree over the place is the latitude (definition 10.) and the degree on the equator, cut by the brass meridian, is the longitude (def. 11.)

Example 1. What is the latitude and longitude of Washington city ? †

Answer. Lat. 38° 53′ north. Longitude 77° 14½′ west.

* Whenever a place is brought to the brazen meridian, the graduated edge which is numbered from the equator towards the north or south pole is always understood, unless the contrary be mentioned.

† The latitude and longitude of this city is not, as yet, so correctly ascertained as might be expected, and therefore it cannot be made the basis of any accurate or important calculations or tables. This being therefore a point of such public utility, its importance must appear evident to every American citizen, who has the most superficial knowledge of these matters, and feels an interest, not merely for science, but for the reputation and growing importance of his country.

Various methods are given by different authors, for finding the latitudes and longitudes of places on the earth, the substance of which is given in the course of this work, with some new and important methods not published in any other treatise, together with the principles on which they are founded. See problems 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, &c. part 2d. and problems 19, 20, 21, 22, 23, 24, 25, 26, 27, &c. part 3d. with the notes, &c. to these respective problems. For finding a star or planet's transit over the meridian, see probs. 8 and 39, part 3d.

The principal difficulty in any of the methods for finding the *latitude*, is to find the correct altitude, and when necessary, the time of the body's transit over the meridian. To obtain these requisites, various instruments have been contrived ; the most useful of which on land, for want of a good horizon, are the astronomical and mural quadrants, and a good transit instrument. These are fully described in *Vince's* treatise on Practical Astronomy. But at sea, the best instruments are Godfrey's quadrant, commonly called Hadley's quadrant, and a good sextant, which is preferable to the foregoing instrument. These are sufficiently described in the various books on Navigation, particularly in M'Kay, Norie, Hamilton Moore, Bowditch's American Practical Navigator, &c.

There is also a circle of reflection or repeating circle (now sometimes called the astronomical circle) described in the latin part of Mayer's tables, published by Nevil Maskelyne, and after him by different authors, which

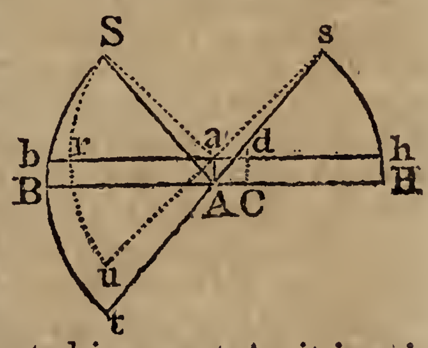
2. What is the latitude and longitude of New-York?
Ans. Lat. 40° 42′ 40″ N. and long. 74° 1′ W.

Note. The long. of New-York, from a solar eclipse observed in June, 1806, is found to be 74° 1′ west of Greenwich ob. See pa. 60 of the Nautical Almanac, 1811 or 1812, published in New-Brunswick, New-Jersey, under the direction of Mr. John Garnett.

3. Required the latitudes and longitudes of the following places:

Amsterdam,	Cape of Good Hope,	Lexington,
Aleppo,	Charlestown,	Lima,
Algiers,	Constantinople,	Lisbon,
Baltimore,	Copenhagen,	London,
Barcelona,	Dantzic,	Madrid,
Batavia,	Delhi,	Paris,
Bencoolen,	Dresden,	Pekin,
Berlin,	Dublin,	Petersburg,
Boston,	Edinburg,	Philadelphia,
Breslaw,	Fez,	Quito,
Buenos Ayres,	Funchal,	Rome,
Cadiz,	Greenwich, (obs.)	Stockholm,
Cairo, (Grand)	Halifax,	Tripoli,
Calcutta,	Hamburg,	Vienna.
Canton,	Ispahan,	

at sea is a very useful and accurate instrument, and by a method easily practised, may be used with equal advantage on land, being lately much improved and adapted for astronomical observations in general. The method consists in bringing the image of the sun, &c. (in the same manner as it is brought to the edge of the horizon at sea) to coincide with its image reflected from a basin of water, quicksilver, molasses, or any reflecting surface parallel to the horizon; half the sum of the angle thus found, will be the altitude required.



Thus let S represent the sun's place, A the reflecting surface, placed parallel to the horizon BH, C the place of the spectator, and Cd the height of his eye. Now it is evident that the arch SB, being described from the centre A, will be the altitude of S above the horizon. If the image of the sun be then brought by the instrument to the point B, the altitude will be pointed out by the index; but to have the image of the sun (now at B) to coincide with the reflected image at A, it is evident that it must be brought to the point t, where it meets with the reflected ray As; and an eye placed in any part of the line As, where the ray is reflected, will observe the image of the sun, brought by the instrument to t, in this direction. But as the angle of incidence SAB is equal to the angle of reflection sAH (prop. 9. cor. 2. Emerson's tracts, or his optics B. 1. prop. 10.) that is to BAt (15 Eucl. 1. B.) the arch St, which is the measure of the angle observed by the instrument, is double of SB; therefore half of St, without any allowance for the height of the eye, above the reflecting surface, will be the altitude required.

Though it be evident that no allowance is to be made for the height of the eye above the reflecting surface, as an eye placed in any part of the line As will observe both images coincide in the direction At, there is, however, rigorously speaking, an allowance to be made for the height of the specta-

2. Find all the places on the globe which have no latitude.
3. What is the greatest latitude a place can have?

tor or bason at A above the level of the sea, &c. or the true horizontal level; it being evident that the more elevated the point A is above this level, the less will the altitude be. For if a be taken as the reflecting surface, the arch Su, which is double of Sr, will be less than St.

But this will make so small a difference even on the tops of the highest mountains, that it may be always neglected except when very great accuracy is required. For in the sun's altitude the whole semidiameter of the earth will only make a difference of 8″ 83 even at the horizon; hence the semd. of the earth : the height of the mountains, &c. :: 8″ 83 : the correction of the horizontal alt. which will also diminish in proportion to the height of the object. And even for the altitude of the moon thus found, scarce any allowance must be made for the height of the mountain, &c. though its mean horizontal parallax amounts to 57′ 39″. For 3956 miles the earth's sem. diam. : 1 mile, the supposed height of a mountain :: 57′ 39″ : 0″ 82, not amounting to 1″ even at the horizon, when the height of the mountain is 1 mile; and this will diminish in the proportion of Rad : cosine apparent altitude. Hence this method may be successfully practised in any situation on land, care being taken that the reflecting surface be not agitated by the wind, &c. to prevent which several contrivances may be made use of, which the skill and ingenuity of the observer will suggest.

A good observer ought to be well acquainted with the elementary principles of Geometry, Astronomy, Mechanics, and Optics, to be able to adjust his instruments with skill, and in every circumstance to apply them to the best advantage, and allow for every defect or error, &c. that may take place. Hadley's quadrant or the sextant, will answer equally well on land, as the above instrument, when the altitude of the object does not exceed half the number of degrees marked on them. The learner must also take notice that the glass through which the sun's reflected image, in the water or on the reflecting surface, is observed, must be coloured to preserve the eye from injury, and to render the sun's image more distinct by destroying the effects of irradiation.

There are likewise various methods given for finding the *longitude*, the most useful of which are the following: 1st. Having the time of the moon's southing at Greenwich, or at any other place whose longitude is known, observe the time of the moon's southing at the place of observation, by help of a correct meridian line, and a good clock or time piece, exactly adjusted to mean time, or 24 hours in a day; (see notes to prob. 29 and 39, part 3d.) find the difference of these times, then say as the difference of the times of the southing of the sun and moon in 24 hours : this difference :: 360° : diff. longitude; this difference added to the longitude of the known place, if the time of southing there be later, or subtracted if sooner, gives the longitude of the place of observation. Where exactness is required, the motion of the moon from the sun in 24 hours, must be taken from the Nautical Almanac, or from correct astronomical tables, the latest and best of which are Mr. de Lambre's new tables of the sun, &c. and Mr. Burg's new tables of the moon, published in 1806 by the French board of longitude, and since translated and published by Vince. The difference of the times of the sun and moon's southing, at a medium, is about 48 minutes daily. The southing of any celestial body is found by a transit instrument, or by suspending two plummets in the meridian line, the lower end of each being immersed in a basin of water to prevent their swinging. When the object comes in a straight line with the threads of these plummets, viewed with the naked eye, a small hole made with a pin through a sheet of paper, or with a telescope, it is then on the meridian.

4. What is the greatest longitude a place can have?
5. Find all those places which have no longitude.

The longitude may be also found by the meridian transit of a fixed star, allowance being made for the sun's right ascension, &c.

Solar and lunar eclipses afford another method, by converting the difference of times between their beginning, middle, or ending, as observed in the place whose longitude is required, and the same times calculated or rather observed in any other, whose longitude is correctly ascertained; the difference of times converted into degrees, &c. of longitude, will give their difference of longitude, from which the required longitude is found.

Eclipses of Jupiter's satellites are however much preferable on land, because they happen almost every day, and the times of their happening are more correctly and easily found. This time should be found very accurately, as an error of one second in time, will produce an error of fifteen seconds in the longitude. (For 4 min. : 1° :: 1″ : 15″) The first satellite is the most proper for determining the longitude. Its emersions are not however visible from the time of Jupiter's conjunction with the sun to the time of his opposition, and its immersions are not visible from Jupiter's opposition to his next conjunction. The positions or configurations of Jupiter's satellites as they appear at Greenwich, are laid down in page 12th of the month in the Nautical Almanac, for every night when visible. The times of their eclipses happening at the meridian of Greenwich, are found in page 3d of the Nautical Almanac for every month. These eclipses must be observed with a good telescope, and a well adjusted pendulum clock, that beats seconds or half seconds. The telescopes proper for observing the eclipses of Jupiter's satellites, as Nevil Maskelyne remarks, are common refracting telescopes from 15 to 20 feet, reflecting telescopes of 18 inches or 2 feet, focal length, and telescopes of Mr. Dolland's construction, with two object glasses from 5 to 10 feet, or which are still more convenient, those of 46 inches focal length, and $3\frac{3}{4}$ inches aperture, constructed with three object glasses, which are as manageable as reflecting telescopes, and perform as much as those which he makes of 10 feet with two object glasses. The manner of adjusting the clock, and an explanation of the configurations of the satellites, will be given in part 4th.

At sea the principal method of finding the longitude, is by computing the distance between the moon and the sun, or some principal fixed star; from which their altitudes being given, the required longitude is found. The principles of this method are demonstrated in the notes to prob. 28 and 29, part 3d. The reader is also referred to M'Kay's treatise on the longitude, his treatise on Navigation, to Norie or Bowditch's improvements on Hamilton Moore, or to the Nautical Almanac for 1812, revised by John Garnet, New-Jersey. Mayer in the beginning of his tables gives also a method of finding the same under the title of *Methodus Longitudinum promota et Additamentum.* Mr. Delamar in Philadelphia, has likewise lately favoured the public with some very useful methods in the practice of this important problem.

Another method of finding the longitude is with a time piece. If the time piece could be depended on, this method would be by far the most easy and expeditious. For if set to mean time in any place whose longitude is known, it would point out the difference of times between this place and any other place to which a person arrives, which converted into degrees, &c. would be the difference of longitude required. The watch should be kept going, and not changed the whole time during the journey or voyage; or if changed, allowance should be made for it.

The last method I shall take notice of here is that of the variation chart, introduced by the celebrated Doctor Haley. On this chart are drawn curve lines which represent the variation in almost every degree of longitude.—

6. Find all places those which have neither latitude nor longitude.*

7. Find all those places that have the greatest longitude.

8. Find those places that have the greatest latitude and longitude.

9. Find those places that have all possible degrees of longitude, reckoned from the same meridian.

PROB. 2.

To find any place on the globe, the latitude and longitude of which are given.

Rule. Find the longitude of the given place on the equator,† (problem 1.) and bring it to the brazen meridian ; then under the given latitude found on the brass meridian is the place required. (Def. 10.)

As this chart is so familiar to mariners, it needs no further description here. This would also be an easy and expeditious method of finding the longitude, could the variation and its yearly change be once exactly ascertained in the different parts of the world.

An attempt has likewise been made of rendering the magnetic needle useful in finding the latitude. In the year 1580 it was discovered by one Robert Norman, a compass maker in England, that the needle had a certain inclination in a contrary direction to the inclination of the earth's axis ; this discovery being communicated to others, it was found that at the equator it has no inclination, being there parallel to the plane passing through the earth's axis or to the horizon ; but that it depresses one end if we recede from the equator towards either pole ; the north end, if we advance towards the north, and the south end if we go towards the south. This inclination was found to vary in proportion to the distance from the equator, and was therefore thought to correspond to the latitude : but the poles of the earth varying from those of the needle, and continually changing, besides many other latent causes, prevent its theory from being sufficiently known, and render the task of making the necessary experiments to ascertain the phenomena, rather discouraging in the present state of science, so that nothing conclusive can as yet be deduced from it. The reader will find more particulars on this subject in Cavallo's treatise on Magnetism.

* As all places lying under the equinoctial or on the equator have no latitude, and all places situated on the first meridian have no longitude, therefore that point on the globe where the first meridian intersects the equator, has neither latitude or longitude. Again, as the latitudes of places increase as their distance from the equator increases, and their longitudes increase as their distance from the first meridian increases, it follows that the greatest latitude a place can have is 90°, and the greatest longitude 180°, which being half the circumference of the globe, no two places can be at a greater distance from each other than 180°.

† On Adam's globes there are two rows of figures above the equator.—When the place lies to the right hand of the meridian of London, the longitude must be reckoned on the upper line ; when it lies to the left hand, it must be reckoned on the lower. On Cary's globes, on which are also two lines, the longitude being reckoned from the meridian of Greenwich, when the longitude is east, the upper line is used, but when west, the lower. The figures under the equator in this globe indicate the half hours and quarters, and the dots the minutes. There are two rows of hours on each

Example 1. Find that place whose latitude is 32° 25′ 40″ N. Longitude 63° 35′ 40″ west from Greenwich.

Ans. The most northerly part of the Island of Bermudas.

3. Find those places whose latitudes and longitudes are as follows:

Latitudes.	*Longitudes.*	*Latitudes.*	*Longitudes.*
52° 22′ 45″ N.	4° 45′ 30″ E.	42° 23′ 15″ N.	70° 58′ 0″ W.
31 11 20 N.	30 15 30 E.	4 56 10 N.	52 16 0 W.
52 32 30 N.	13 26 E.	41 0 0 N.	28 22 30 E.
30 2 30 N.	31 15 15 E.	51 28 39 N.	0 0 0
33 55 15 S.	18 29 E.	12 1 15 S.	76 55 30 W.
48 50 14 N.	2 19 E.	39 55 16 N.	116 21 30

PROB. 3.

To find all those places that are in the same latitude or longitude with any given place.

Rule. Bring the given place to the brazen meridian,* and mark the point over it; then all those places under the same edge of the meridian, between both poles, are in the same longitude (def. 11.) and all those places passing under the mark are in the same latitude. (def. 10.)

Example 1. Find all those places that have the same, or nearly the same latitude or longitude as New-York.

Ans. Montreal, Heneaga island, the western part of St. Domingo, St. Martha, Guamanga, La Conception, &c. in S. America, are nearly in the same longitude; and Madrid, Naples, Constantinople, Pekin, &c. nearly in the same latitude.

2. What places have the same or nearly the same latitude as the following places: London, Petersburg, Rome, Philadelphia, and Lima?

3. Find all those places that have nearly the same longitude as the following places: Paris, Archangel, Naples, Boston, and Mexico, in North America.

4. What inhabitants of the earth have their days of the same length as those of Philadelphia?

5. What inhabitants of the earth have the same seasons of the year as those of London?

side of the equator, reckoned all round from aries both ways; the one towards the right hand or east, is carried to XXIV, to this the minutes, &c. are adapted. The other, which is reckoned towards the left hand or west, is counted only to XII, and then begins I, II, III, &c. again. These lower lines are useful in finding the difference of times between any two meridians, or for shewing how much sooner or later the time is in one place than in another. One is fitted for astronomical, the other for civil time. Bardin's new British globes have also two rows of figures above the equator, but the lower line is always used in reckoning the longitude.

* It will answer equally as well to bring the given place to the horizon, and count the degrees from the east or west points, &c.

6. Find all those places that have their longest day the same as at Petersburg.

7. When it is noon at Baltimore, what inhabitants of the earth have the same hour?

8. When it is noon, midnight, or any other hour in Boston, find all those places that have the same hours respectively.

PROB. 4.

To find the difference of latitude and difference of longitude between any two places.

FOR THE DIFFERENCE OF LATITUDE.

Rule. FIND the latitude of both places (prob. 1.) and the number of degrees between them, reckoned on the brazen meridian, will be the difference of latitude. (Note to def. 10.)

Or, find both latitudes; (prob. 1.) then if they be of the same name, that is both north or both south, their difference is the difference of latitude; but if they be of different names, that is one north and the other south, their sum will be the difference of latitude.

FOR THE DIFFERENCE OF LONGITUDE.

Rule. Find the longitude of both places (prob. 1.) and the number of degrees between them, reckoned on the equator, will be their difference of longitude. (Note to def. 10.)

Or, find both longitudes as before, then if they be of the same name, that is both east or both west, their difference is the difference of longitude; but if they be of different names, their sum will give the difference of longitude.

Note. If this last sum should exceed 180°, take it from 360, and the remainder will be the difference of longitude.

For the difference of longitude in time. Bring one of the places to the brazen meridian, and set the hour index to 12; then bring the other place to the meridian, and the hours, &c. passed over by the index, will be the difference of longitude in time, as required. The same may be found more correctly on the equator, by taking the sum or difference of the times corresponding to the longitudes on the equator, instead of the longitudes themselves.

Example 1. What is the difference of latitude and difference of longitude between Philadelphia and Greenwich observatory?

Ans. Diff. lat. 11° 31′ 45″. Diff. long. 75° 8′ 45″.

Note 2. If one of the places have no latitude or no longitude, the latitude or longitude of the other will be the difference.

2. What is the difference of latitude and difference of longitude between Paris and Greenwich observatories?

Ans. Diff. lat. 2° 38′ 25″. Diff. long. 2° 19′.

3. What is the difference of longitude in time and degrees between Paris and Gottingen observatories?

Ans. Diff. of longitude in degrees 7° 32′ 45″. In time 30° 11′.

Note 3. For exactness in these problems, consult the table of latitudes and longitudes at the end of the work.

4. Find the greatest difference of latitude and difference of longitude between any two places.

5. Required the difference of latitude and longitude between the following places?

London and New-York,
Goa and Rome,
Constantinople and Quito,
Petersburg and Vienna,
Dublin and Boston,
Charlestown and Pekin,
Pekin and Lima,
Edinburgh and Baltimore,
Washington City and Jerusalem,
Hamburg and New-Orleans,
Cape of Good Hope and Canton,
Calcutta and Philadelphia,
Havanna and Gibraltar.

Note 4. The difference of lat. or difference of long. between two places being given, and if one of the places be also given, the other is given by adding or subtracting this difference, according as the place is north or south, east or west of the given place.

Thus, for the latitude. 1. If the latitude of Washington city be 38° 53′ N. and the difference of lat. between it and New-York be 1° 49′, then 38° 53′+1° 49′=40° 42′, the lat. of New-York, being northward of the given place.

2. The lat. of Washington being 38° 53′ N. and the diff. between it and that of St. Domingo 20° 33′, then 38° 53′—20° 33′=18° 20′, the lat. of St. Domingo, being southward of Washington.

3. The lat. of Washington being as above, and the diff. of lat. between it and Lima, in south lat. being 50° 54′, then 50° 54′—38° 53′=12° 1′, the lat. of Lima. The difference of lat. in this case being the sum of both latitudes, the lat. of either is evidently found by taking the lat of the other from the diff. of lat.

Again, for the longitude. 1. If the longitude of Washington city be 75° 14′ 22″ W. of Greenwich observatory, and the diff. of long. between it and New-York be 1° 13′ 22″, required the long. of New-York. Here 75° 14′ 22″ —1° 13′ 22″=74° 1′, New-York being eastward of Washington.

2. The longitude of Washington city being given as above, and the diff. of long. between it and New-Orleans being 14° 52′ 8″, then 75° 14′ 22″+14° 52′ 8″=90° 6′ 30″, the long. of New-Orleans, being west of Washington.

3. If the long. of Washington be as above, and the difference of longitude between it and Paris observatory be 79° 33′ 22″; then 79° 33′ 22″—77° 14′ 22″=2° 19′. Paris being situated in east long. and the sum of their longitudes from Greenwich being the difference of longitude.

PROB. 5.

To find the antæci, periœci, and antipodes of any place.

Rule. Bring the given place to the brass meridian, and find its latitude (prob. 1.) then under the same meridian, in the same degree of latitude, in the opposite hemisphere, you will find the antœci. (Def. 22.) The globe remaining in the same position, set the index to 12, and turn the globe on its axis until the other 12 comes to the meridian (or until the index points to it) then under the latitude of the given place you will find the periœci (def. 23.) and under the same meridian, in the same degree of latitude, but in the opposite hemisphere, you will find the antipodes. (def. 24.)

Or thus,

Place both poles in the horizon, and bring the given place to the eastern part of the horizon ; then, if the place be in north latitude, observe how many degrees it is to the northward of the east point of the horizon ; the same number of degrees reckoned to the southward of the same point will give the antœci ; an equal number of degrees counted from the west point of the horizon towards the north will shew the periœci ; and the same number of degrees counted towards the south from the west, will point out the antipodes.

If the place be in south latitude, the same rule will serve, by reading south for north, and the contrary. This method is the same in effect as the above.

Example 1. Required the antœci, periœci, and antipodes of Bermudas ?

Ans. The antœci is in Paraguay, a little S. E. of Cordova, or N. W. of Buenos Ayres ; the periœci is near Yongyong, N. W. of Nankin, in China ; and the antipodes is near Binning's land, in the S. W. part of New Holland.

2. Required the antœci, periœci, and antipodes of the following places : Constantinople, Rome, London, Cape of Good Hope, Quito, Buenos Ayres, Kingston, and Skalholt.

3. A person sailing in lat. 51½° south, and long. 180°. Where was his antipodes ?

Note. Those places situated on the equator have no antœci, and their periœci are their antipodes ; and those places at the poles have no periœci, and their antœci are their antipodes.

4. Required those places whose seasons are directly contrary to those of New-York (that is summer with one being winter with the other, &c) but whose hours are the same (that is mid-day with one being midnight with the other, &c.)

5. Required those places whose seasons are the same as those of Philadelphia, but hours contrary ?

6. Required those places whose seasons and hours are contrary to those of Washington city ?

For the three last problems see notes to definitions 22, 23, 24.

PROB. 6.

The hour of the day at any particular place being given, to find the corresponding hour (or what o'clock it is at that time) in any other place.

Rule. Bring the place where the time is given to the brass meridian, set the index to the given hour; then turn the globe till the other place comes to the meridian, and the index will point out the time required.

Or, Having brought the given place to the meridian, as before, set the index to 12 ; then bring the other place to the meridian, and the hours passed over by the index will be the difference of time between both places. If the place where the hour is sought be to the east of the other, the time there is so much later, if to the west, the time is so much earlier. Hence, in the former case, you add the diff. to the given time, in the latter you subtract. Thus a place 15° to the eastward of another, has the sun on its meridian an hour earlier than the latter place; therefore 12 o'clock in the former place is but 11 o'clock in the latter; and 12 o'clock in the latter place is 1 o'clock in the former, &c.

WITHOUT THE HOUR CIRCLE.

Find the difference of longitude between the two places (prob. 4.) and turn it into time by allowing 15° for every hour, and 4 minutes of time to every degree, &c. The difference of longitude in time will be the difference of time between the two places, with which proceed as above.

Example 1. When it is 7 o'clock in the morning at Philadelphia, what hour is it at London?

Ans. Twelve o'clock at noon; the difference of time being five hours nearly, and London to the east of Philadelphia.

Or, The difference of longitude between both places is 75° 13′. Now 75°÷15=5 hours and 13′×4=52, hence 5h. 0′ 52″+7h.= 12h. 0′ 52″, or 52″ after 12 o'clock.

Note. Degrees of longitude multiplied by 4 produce minutes of time, and minutes multiplied by 4 produce seconds of time, &c. and minutes and seconds of time divided by 4 give degrees and minutes of longitude, &c.

2. When it is 7 o'clock in the morning in London, what o'clock is it at Philadelphia?

Ans. 2 o'clock in the morning, or 7h.—5h. 0′ 52″=1h. 59′ 8″.

3. When it is 2 o'clock in the afternoon at Greenwich observatory, what o'clock is it at Baltimore?

Ans. 8h. 52′ 40″ in the morning. The difference of longitude is 76° 50′, which multiplied by 4=307′ 20″=5h. 7′ 20″. 12+2 =14; hence 14h.—5h. 7′ 20″=8h. 52′ 40″, or 52′ 40″ after 8 in the morning at Baltimore. The same answer will be found, if 5h. 7′ 20″, the difference of longitude in time, be counted backwards from 2 o'clock in the afternoon, as Baltimore is to the west of Greenwich observatory.

4. When it is noon at Paris, what hour is it at Quito?

5. When it is 10 o'clock in the morning at Kingston in Jamaica, what hour is it at Petersburg?

6. When it is 1 o'clock in the afternoon in Washington city, what o'clock is it in Canton?

7. When it is midnight in New-York, what o'clock is it in London, in Madrid, in Rome, in Vienna, in Calcutta, and in Botany Bay?

8. My watch being well regulated at Dublin, and when I arrived at Philadelphia it was 5 hours faster than the clocks there. I want to know whether it gained or lost during the voyage, and how much?

9. Are the clocks in Philadelphia faster or slower than those at Calcutta, and how much ?

10. Being at sea in lat 10° 45′ N my watch, which was adjusted for the meridian of Greenwich, was by observation found to be 4h. 4 minutes too slow. Required the place of observation and its longitude ?

Ans. Trinidad in the West-Indies.

11. Being at sea in the year 1806, on the 16th of June, I observed the beginning of an eclipse of the sun at 10h. 16′ 12″ in the forenoon, apparent time, and found that by an Almanac calculated for New-York, in longitude 74° 1′ W. from Greenwich, the beginning of the eclipse there, happened at 9h. 39′ A. M. app. time. The latitude of the place of observation was 32° 15′ N. required the place and its longitude ?

Ans. The place is near the western part of the Island of Bermudas, and its longitude 64° 43′ W. from Greenwich. See the following problem.

PROB. 7.

The hour of the day being given in any place, to find all the places on the globe where it is then noon, or any other given hour.

Rule. Bring the place to the meridian, and set the index to the given hour in that place ; turn the globe until the index points out any other given hour ; all the places that are then under the brazen meridian, are those places required.*

Note. This method is attended with some confusion, if there be more rows of figures than one on the hour circle ; to remedy which, the following methods are given. The same must be observed with respect to the preceding and some of the following problems.

* This rule is manifest from what is said in the preceding problem, from which, or from this prob. the following observations are evident.

1. If a ship set out from any port and sail round the earth eastward until she arrives at the same port again, the people in that ship will gain one entire day, in their reckoning, at their return. If they sail westward, they will lose one day or reckon one day less.

2. Hence if two ships sail from the same port, the one eastward, and the other westward, until they arrive again to the place from which they departed, they will differ two days in their reckoning ; the one reckoning one day less, the other one day more than those who remained in the port. If they sail twice, they will differ four days from each other, and two from those who remained in the port. If three times, six days, &c.

3. If the vessels sail the one northward and the other southward, no difference will appear in their reckoning, nor will they differ from those who reside at the port ; the difference in time being in proportion to the change made in their longitude east or west.

4. As the distance of meridians near the poles is very small, an inhabitant situated within 5 or 6 miles, &c. of either, may make the same changes in his actual account of time, as the ship mentioned in the first remark, or may keep pace with the sun during his apparent diurnal revolution round the earth. Whoever is curious to see more of these remarks, may consult a small mathematical miscellany, published by Samuel Fuller.

Rule 2. Bring the given place to the meridian, and set the index to 12; then, if the hour at the required places be earlier than the hour at the given place, turn the globe eastward until the index has passed over as many hours as are equal to the given difference of time; but if the hour at the required places be later than the hour at the given place, turn the globe westward until the index has passed over the given difference of time; and in each case all the places required will be found under the brass meridian.

WITHOUT THE HOUR CIRCLE.

Rule 3. Find the difference of longitude in time (prob. 4.) reduce it to minutes, &c. these minutes divided by 4 will give degrees of longitude; if there be a remainder after dividing by 4, reduce it to seconds, and add the seconds in the difference of longitude, if any: this sum again divided by 4, will give minutes or miles of longitude. Now, if the hour at the required places be earlier than the hour at the given place, the required places lie as many degrees to the westward as are equal to the difference of longitude; but if the hour at the required places be later than the hour at the given place, the required places lie as many degrees to the eastward of the given place, as are equal to the difference of longitude.

Note. Whenever we direct the globe to be turned, we mean on its axis, either east or west.

Examples. 1. When it is 12 o'clock in the day at London, where is it 8 o'clock in the morning, at that time?

Answer. If London be brought to the meridian, and the index set to 12 o'clock (or 12 o'clock brought under the meridian) the globe being then turned until 8 o'clock comes under the meridian, or until the index points to 8, all the required places will then be under the meridian; as, Cape Canso, Martinico, St. Lucia, Trinidad, &c. the mouth of the river Oronoko, a part of Amazonia, Paragay, &c. the Falkland Islands, &c.

Or, bring London to the brazen meridian, and set the index to 12 as before; turn the globe eastward until the 8 o'clock hour line comes under the meridian, or until the index has passed over 4 hours. Then under the brass meridian all the places required will be found as above. Or, (without the hour circle.) The difference of longitude between London and the required places, is 4 hours or 240 minutes, which divided by 4 gives 60° the difference of longitude. ($=4\times15$.) Now as the hour at the required places is earlier than at London, they lie 60° westward of it.—Hence all the places situated in 60° west longitude from London, are the places required and will be found by prob. 3, as above.

2. When it is 2 o'clock in the afternoon at London, where is it half past 5 in the afternoon?

Ans. The places sought will be found as above (in method 1st) to be the Caspian sea, western part of Novazembla, the island of Socotra, eastern part of Madagascar, &c.

Turning the globe westward (the time being later than at the given place) until the index has passed over $3\frac{1}{2}$ hours, London being brought to the brazen meridian, the places will be found as above by 2d method; or, by the 3d method, the difference of longitude in time being $3\frac{1}{2}$ hours or 52° 30′. The required places, therefore, lie so many degrees to the east of London.

3. When it is 5 o'clock in the afternoon at Madrid, where is it noon?

4. When it is half past 5 in the morning at Pekin, where is it noon?

5. When it is noon at Delhi, where is it 6 o'clock in the morning?

6. When it is 5 o'clock in the morning at Philadelphia, where is it 5 o'clock in the evening?

7. When it is noon at New-York, where is it midnight?

8. Being at sea in lat. 42° north, when it was 9 o'clock in the morning by the time piece, which shews the hour at Washington city (77° 43′ W. long.) and finding by a correct celestial observation, that it was 11 o'clock in the morning at the ship, in what longitude was the vessel?

9. When it is 10 o'clock in the morning in New-York, find all those places that have the same hour. (Prob. 3.)

Note. On Cary's globes the hours are marked on the equator to every minute, particularly on his large globes, and adapted to the meridian of Greenwich observatory. Hence any place being brought to the meridian, the hours, &c. on the equator will point out the time to the minute that the sun will come to the meridian of the place sooner or later than to the meridian of Greenwich. Any other place may be taken instead of Greenwich, and the hours will answer equally as well, by taking the difference of times in each place.

PROB. 8.

The day of the month being given to find the sun's place, or his longitude in the ecliptic, and his declination.

Rule. Look for the given day in the circle of months on the horizon, and opposite to it in the circle of signs, are the sign and degree which the sun is in that day. (Def. 31.) Find the same sign and degree in the ecliptic, on the surface of the globe, and bring the degree of the ecliptic thus found to that part of the brazen meridian which is numbered from the equator towards the poles, then that degree of the meridian which is over the sun's place, is the declination required. (Def. 91.)

OR BY THE ANALEMMA.*

Bring the analemma to the brass meridian, and the degree, cut on it, exactly above the day of the month, is the sun's declination;

* The Analemma is properly an orthographic projection of the sphere on the plane of the meridian, and is a useful invention for shewing by inspection the time of the sun's rising and setting, the lengths of days and nights, the points of the compass on which the sun rises and sets, the be-

turn the globe until the point of the ecliptic corresponding to the day passes under this degree of the sun's declination, that point will be the sun's place or longitude for the given day.

Note 1. If the sun's declination be north, and increasing, the sun's place will be between Aries and Cancer. If the declination be decreasing, his place will be between Cancer and Libra. If the declination be south, and increasing, it will be between Libra and Capricorn. If decreasing, between Capricorn and Aries.

The sun's longitude and declination are given in the 2d page of every month in the Nautical Almanac for every day in the month. The method of accurately calculating them for any time, is given in prob. 3d and 9th of Mayer's tables, published by Nevil Maskelyne.

The sun's place, &c. on the latest globes, viz. Bardin's and Cary's, is adapted to the year 1800.

Example 1. What is the sun's longitude and declination on the 22d of February?

Ans. The sun's place is $4\frac{1}{4}°$ in Pisces, declination 10° S.

2. What is the sun's place and declination on the 15th of April?

Ans. $25\frac{1}{2}°$ in ♈, declination 10° N.

3. Required the sun's place and declination for the first day of each month.

4. Required the sun's place on the following days:

January	10,	April	2,	July	12,	October	29,
February	13,	May	10,	August	5,	November	22,
March	15,	June	15,	September	30,	December	31.

ginning and end of twilight, &c. but the Analemma on the globe is a narrow slip of paper, the length of which is equal to the breadth of the torrid zone. It is pasted on some vacant place on the globe, between the two tropics, and is divided into months and days of the month, corresponding to the sun's declination for every day in the year. It is divided into two parts; the right hand part begins at the winter solstice or December 22d. and is reckoned upwards towards the summer solstice or June 21st. where the left hand part begins, which is reckoned downwards in a similar manner, or towards the winter solstice. On Cary's globes the Analemma somewhat resembles the figure 8, being drawn in this shape for the convenience of shewing the equation of time by means of a straight line which passes through the middle of it. It begins at the tropic of Cancer with the 24th of December, at which time there is no equation of time, thence towards the opposite tropic January, February, &c. during which months the clock is faster than the sun, to the 15th of April, at which time the clock and sun are equal, and therefore no equation of time; from thence it continues April, May, &c. during which the clock is slower than the sun to the 16th of June, nearly, at the tropic, at which time the clock and dial are again equal, thence returning in the order of the months from July to the 31st of August, the clock is too fast; at the 31st of August the equation is again nothing from thence to the 24th of December, reckoning towards the southern tropic, the clock is too slow, &c. And if any day on the Analemma be brought to the brass meridian, the degree cut on the line which crosses the middle of the Analemma, will show how much the clock is fast or slow. The equation of time is placed on the horizon of Bardin's globes corresponding to the respective days of the month. (See note to def. 57, 61, and 62, and also, prob. 22.)

Note 2. The sun's place being given, the day of the month corresponding is found in the outer circle or calendar of months, &c. on the horizon. On Cary's globes the days are likewise marked on the ecliptic.

Note 3. The declination being given, the corresponding months and days are found by observing the two points of the ecliptic that come under the declination, which will be the sun's place corresponding.

5. On what day of the month does the sun enter each of the signs? (See def. 31.)

Note 4. The earth's place, as seen from the sun, among the fixed stars, is always in the sign and degree opposite the sun's place. Thus when the sun is 10° in aries, the earth is 10° in libra, and so of any other. The sun in reality having no motion (at least to produce this phenomenon) but the earth by revolving on its axis every 24 hours from west to east, causes an apparent diurnal motion of all the heavenly bodies from east to west: (Dr. Kiel, lect. 26.) In like manner by revolving round the sun in a year, the sun seems to pass over the same signs in the heavens which the earth has passed, and in the same direction. But the sun being in the centre, it is plain that in whatever sign the earth is, as seen from the sun, the sun must be in the sign diametrically opposite as observed from the earth. The physical causes, &c. of these phenomena, is given by Newton in his principia, and after him by the writers on physical astronomy and the laws of centrepetal forces, as Dr. Gregory in his astronomy, McLaurin in his fluxions and view of Newton's philosophy, Emerson in his fluxions, tracts, and in his astronomy, Simpson, and others; and lately in France, the celebrated La Grange, De la Place, La Lande, De Lambre, &c. In Germany, Mayer in his theory of the moon, &c. Mr. Burg, of Vienna, has lately in his tables constructed principally on the observations of Nevil Maskelyne, much improved this subject.

Note 5. The declination for every day is given in page 2 of the month in the Nautical Almanac, or it may be found by having the sun's meridian alt. given, see note to prob. 42, from which and the obliquity of the ecliptic, the sun's place or longitude is found by Napier's rule, thus: As sine greatest decl. : sine present decl. :: Rad. : sine longitude from aries. This longitude is likewise given in page 2 of the N. A. as also the sun's rt. ascension, the equation of time, &c. (For the sun's greatest declination or obliquity of the ecliptic, see note to prob. 49.) To find the sun's longitude at any time different from noon, say as 24h. is to the hour from noon reckoned by the meridian of Greenwich, so is the daily variation of the sun's longitude to a fourth number, which added to the longitude at noon, gives the longitude for the given time; if the time be that of a place differing in longitude from Greenwich, it must be reduced to it. In like manner proportion may be made for any of the other articles in the Nautical Almanac.

PROB. 9.

To rectify the globe for the latitude, zenith, and sun's place; and to place it agreeably to the corresponding situation of the earth, or the four quarters of the world.

1. For *the latitude.* If the latitude be north, elevate the north pole as many degrees above the horizon as are equal to the latitude*

* The reason of this method is evident from the latitude of the place being always equal to the height of the elevated pole above the horizon. (D. Gregory's astr. b. 2, prob. 7.) On any part of the earth we shall always see one half of the heavens, or 90° from the vertex to the horizon in every di-

but if the lat. be south, elevate the south pole, until the degrees upon the meridian below the pole cuts the horizon, and then the globe is rectified for the latitude.

2. *For the zenith.* Having elevated the pole to the latitude, the same number of degrees reckoned from the equator towards the elevated pole will give the zenith or vertex of the place. (To this point the graduated edge of the quadrant of altitude is fixed.)

3. Bring the sun's place in the ecliptic (prob. 8.) to the meridian, and set the hour index to 12 at noon, or bring the upper 12 to the graduated edge of the brazen meridian, and the globe is then rectified for the sun's place.

4. Lastly, by means of the mariners' compass attached to the globe, let the intersection of the planes of the meridian and horizon be placed in the meridian line by the compass (allowing for variation, if necessary) so that the elevated pole of the globe may point towards the elevated pole of the world. Then the different points of the compass on the globe will point to the corresponding bearings on the earth, &c.

Note. The same method will answer for the Celestial globe.

Example 1. On the 10th of May it is required to rectify the globe for the lat. 40°, the sun's place, zenith, &c. to fix the quadrant of alt. and to place the meridian north and south, as on the globe of the earth.

2. Rectify the globe for the lat. of Washington city, the zenith, and sun's place, on the 1st of June, &c.

PROB. 10.

The month and day of the month being given, to find those places on the globe to which the sun will be vertical, or in the zenith, on that day.

Rule. FIND the sun's declination for the given day (prob. 8.) and mark it on the brass meridian; then the globe being turned on its axis, all those places which pass under this mark will have the sun vertical on that day.*

rection, if our view be not intercepted by hills, &c. Therefore to an observer on the equator, the poles of the heavens would appear in his horizon; and if he advance from the equator towards either of the poles, he will see that pole towards which he advances rise as many degrees above the horizon as he advances towards it from the equator: so that to an inhabitant at the poles, the corresponding pole would appear in his vertex.

The 2d rule is evident; for the height of the pole is equal to the distance of the equator from the vertex, both being equal to the complement of the latitude, or what the latitude wants of 90°. In applying the 4th rule, the globe must be placed on a plane parallel to the horizon.

* The reason of this rule is evident, for the distance of the sun from the equinoctial or his declination, is equal to the distance of those places from the same or their latitude, and therefore the sun, on that day, must pass over the parallel of latitude passing through those places.

OR BY THE ANALEMMA.

Bring the analemma to the brass meridian, then the degree over the given day is the sun's declination, with which proceed as above.

Example 1. Find all the places on the earth to which the sun will be vertical on the 15th of April.

Ans. It will be nearly vertical to Carthage, Porto Bello, Carora, Barcelona, &c. in South-America; to the island of Trinidad; to all that part of Africa under the parallel of 10°; the northern extremity of the island of Ceylon; the mouths of the Cambodia or Japanese river (nearly) Parago, Negros, &c. in the Philippine islands, &c.

2. Find all those places where the sun is vertical on the 9th of May.

Ans. St. Anthony, one of the Cape Verd islands, Antigua, St. Kitts in the West-Indies, Acapulco, Anatajan in the Ladrone islands, Manilla in the Philippines, the southern parts of Pegu, Golconda, the southern parts of the great desert in Africa, &c.

Note. In solving this problem, it is more natural to turn the globe from west to east, as in the last example, because those places to the eastward have the sun first on their meridian, and thence in order towards the west.

3. Find all the places to which the sun will be vertical on the following days, viz. 21st of March, 21st of June, 23d of September, and 22d of December.

4. Find all the places of the earth where the inhabitants have no shadow when the sun is on their meridian on the 1st of May. (See note to def. 19.)

PROB. 11.

A place being given in the torrid zone, to find those two days of the year on which the sun will be vertical there.

Rule. Bring the given place to the brass meridian, and mark the degree of latitude that is exactly over it; turn the globe on its axis, and observe what two points of the ecliptic pass under that latitude: These points will be the sun's place corresponding to

By finding where the sun was vertical on any day, the limits of the torrid zone were discovered by the ancient geographers. For, knowing that an object will project no shadow where the sun is vertical, they observed the most northerly place where objects cast no shadow when the sun's declination north was greatest; the distance of which place from the equator, gave them the limits of the northern tropic, and consequently half the breadth of the torrid zone. But in accurately determining the aforesaid place, though their method was correct, they found themselves, notwithstanding, considerably embarrassed, as on the same day no shadow was cast for a space of no less than 300 stadia; the reason of which is, that the apparent diameter of the sun being about 31½′ of a degree at this time, seemed to extend itself over as much of the surface of the earth, and to be vertical to every place within that space. But this difficulty might be easily overcome by taking the middle of the space in which objects were found to project no shadow, as this would give the place where the sun's centre was then vertical, and consequently the tropic required.

the two days required, which days are found on the horizon exactly opposite to the sun's place. (See notes 2d and 3d. prob. 8.)

OR BY THE ANALEMMA.

Bring the analemma to the brass meridian, upon which, exactly under the latitude of the given place (found by prob. 1.) will be the two days required.

Example 1. On what two days of the year will the sun be vertical at Batavia, in the island of Java?

Ans. On the 4th of March, and on the 8th of October.

2. On what two days of the year will the sun be vertical at the following places:

Kingston in Jamaica,	St. Helena,	Borneo,
St. Domingo,	Gondar in Abyssinia,	Manilla,
Fort Royal in Martinico,	Goa,	Otaheite,
Barbadoes,	Columbo in Ceylon,	Owhyhee,
St. Antonio, Cape verd islands,	Achen,	Lima.

3. If the sun be vertical at a certain place on the 15th of April, how many days will elapse before he is vertical a second time at that place?

Note. The sun's declination on those days is equal the latitude of the respective places. (See note to prob. 1.)

PROB. 12.

The day of the month and the hour at any place being given, to find where the sun is vertical at that hour.

Rule. Find the sun's declination (prob. 8.) bring the given place to the brass meridian, and set the index to the given hour; then turn the globe westward if the hour be given in the forenoon, or eastward if the hour be given in the afternoon, until the index points to 12; the place then exactly under the sun's declination is that required.

Or: Having found the sun's decl. as before, bring the given place to the brass meridian, and set the index of the hour circle to 12 (or bring 12 to the meridian.) Then turn the globe as above directed, as many hours as the given time is from noon, and the place under the sun's declination will have the sun that moment in the zenith.

Or: Find the longitude of the given place (by prob. 1.) and reckon, on the equator (eastward if the time be given in the forenoon, or westward if the time be given in the afternoon) as many degrees as are equal to the given time from noon, converted into degrees (see the note to prob. 6.) this will give the longitude of the place required. Then having the latitude of the place (being equal to the sun's declination) and the longitude, the place is given by prob. 2.

Example 1. When it is 15 minutes after 8 in the morning at New-York, on the 30th of April, where is the sun, at that time, vertical?

Ans. Cape Verd. Here the globe must be turned towards the west, the time being given in the forenoon.

Or by the last method. The given time before 12 o'clock is 3 hours 45 minutes, which converted into degrees (by note to prop. 6.) gives 56° 15′, the difference of longitude, or the number of degrees the place is eastward from New-York, the hour being given in the forenoon. This difference therefore, subtracted from the longitude of New-York, which is 74° 1′ W. (see note 4, prob 4.) gives 17° 46′, the longitude of the place required; and under the sun's declination for the present day is the latitude. (Note to prob. 10.) Hence the place is given, by prob. 2, and corresponds to Cape Verd nearly.*

2. When it is 4 o'clock in the afternoon at London, on the 18th of August, where is the sun vertical?

Ans. Here the given time is 4 hours past noon; hence the globe must be turned eastward until the index has passed over 4 hours, then under the sun's declination you will find Barbadoes, the place required.

3. When it is half past one o'clock at the Cape of Good Hope, on the 5th of February, where is the sun vertical?

Ans. At St. Helena.

4. When it is 20 minutes past 5 o'clock in the afternoon at Philadelphia, on the 18th of May, where is the sun vertical?

5. When it is 8 minutes past 8 in the morning at Petersburg, on the 6th of June, where is the sun vertical?

PROB. 13.

To find the time of the sun's rising and setting, and the length of the day and night at any place.

Rule.† Elevate the north or south pole to the latitude of the place (according as it is north or south) by prob 9. Bring the sun's place for the given day (found by prob. 10.) to the brass me-

* If the time in Example 1st. was given, 8 hours, 13 minutes, 32 seconds, the longitude of Cape Verd would come out 17° 33′, as it ought. But the hour circles, in general, are not divided into parts less than a quarter of an hour, and therefore such exactness was unnecessary. On Cary's large globes, however, the index is divided into parts, each corresponding to 5 minutes. The index or hour circle on these globes having but one row of figures, and being placed under the brass meridian, renders them much more convenient and less liable to perplex beginners. When more exactness is required, the hours, and the degrees of longitude corresponding, &c. should be found on the equator, on which every quarter of a degree corresponds to one minute of time. It is upon a 21 inch globe of Cary's, that most of these problems have been tried.

† The reason of elevating the pole thus, may be seen in the note to prob. 9. The reason of bringing the sun's place to the meridian, and of setting the index to twelve, is because the sun is always on the meridian, or north and south (see note to def. 25.) at 12 o'clock. The reason of bringing the sun's place to the eastern part of the horizon, to find his rising, is because the sun rises towards the east, and the contrary reason

ridian, and set the hour circle (or the index) to twelve ; then turn the globe *eastward* until the sun's place comes to the eastern part of the horizon, and the index will shew the time of sun rising. In like manner bring the sun's place to the *western* part of the horizon, and the index will shew the time of sun setting ; then double the time of sun rising, it will give the length of the night ; and double the time of sun setting, it will give the length of the day ; or, the time of sun rising taken from twelve, will give the time of sun setting, and *vice versa.* And the length of the night taken from twenty-four, will give the length of the day, and the contrary, &c. Also, half the length of the day, gives the time of sun setting, and half the length of the night, the time of sun rising. (See the note at the bottom.)

By the same rule the length of the longest day in all places not in the frigid zones may be found. For in north latitudes, the longest day is when the sun is in the beginning of cancer, that is on the 21st of June ; and in south latitudes, the longest day is when the sun enters capricorn, which is on the 22d December.* Therefore to find the longest day, in the northern hemisphere, not exceeding 24 hours, bring cancer to the meridian, and proceed as in the rule.

OR,

Rule 2. Find the sun's declination (prob. 8.) and elevate the north or south pole, according as the latitude is north or south, as many degrees above the horizon as are equal to this declination ; bring the given place to the brass meridian, and set the index to twelve ; turn the globe eastward until the given place comes to the eastern part of the horizon, and the number of hours passed over by the index will be the time of sun setting ; whence the time of sun rising, and the length of the day and night is found as above.

Note. The time of sun rising and setting may be found independent of the globe by the following rule : To the tangent of the latitude add the tangent of the declination, the sum rejecting radius will be the log. co. sine of an arch, which reduced to time, will be the time of sun rising, the lat. and decl. being of the same name, or the time of its setting if of different names. (See note to prob. 8, part 3d. where the demonstration of this rule is given.)

holds for bringing it to the western part, because the sun sets there. It is likewise evident that the index passes over as many hours as the time from 12 o'clock to the sun's rising or setting. Now as the sun rises as many hours before twelve as it sets after twelve, it is evident that the time of sun rising subtracted from 12, must give the time of sun setting, and *vice versa.* Again, as the hours are reckoned from 12 o'clock in the night or *midnight*, it is plain that the hour indicating sun rising must also indicate half the length of the night, and that therefore its double must be the length of the night ; for the same reason the hour indicating sun setting must also indicate half the day, as the hours are reckoned from 12 o'clock or *midday*, and therefore its double must be the length of the day.

* The longest day in both hemispheres, is placed on the 21st of June and 22d of December. But as the sun varies from the equinoxes about

OR BY THE ANALEMMA.

Elevate the pole to the latitude of the given place, as above; bring the middle of the analemma* or the 16th of June, or 25th of December, corresponding to it, to the brass meridian, and set the index of the hour circle to twelve; turn the globe westward until the day of the month on the analemma comes to the western part of the horizon, and the number of hours passed over by the index will be the time of the sun's setting, &c.; which being given, the rest is easily found as above. On Cary's globes the given day must be brought to the brass meridian, and not the middle of the analemma except on the above days.

Note. If the day of the month on the analemma be brought to the eastern or western part of the horizon respectively; those hour circles placed under the brass meridian, and with only one row of figures, will always point out the hour of sun rising or setting, the prob. being performed as in the last rule.

Examples. 1. What time does the sun rise and set at New-York on the 10th of May, and what is the length of the day and night?

Ans. The sun rises at 4h. 56m. or 56 minutes after 4 o'clock, and sets (12h.—4h. 56m.=) 7h. 4m. or 4 minutes after seven, and therefore the length of the day (7h. 4m.×2) is 14 hours, 8 minutes, and hence 24h.—14h. 8m. (=4h. 56m.×2) =9h. 52m.

2. What time does the sun rise and set at Dublin on the 11th of March, and what is the length of the day and night?

Ans. Sun rises 6h. 20min. sets 5h. 40min. length of the day is 11h. 20min. and length of the night 12h. 40min.

3. What time does the sun rise and set at New-York on the 21st of June, and what is the length of the day and night? (On this day the sun enters Cancer, and makes the longest day in the northern hemisphere, &c.)

Ans. Sun rises 4h. 32m. sets 7h. 28m. longest day 14h. 56m. shortest night 9h. 4m.

4. At what time does the sun rise and set at the following places, and what is the length of the day and night, on the respective days mentioned?

Washington City, 1st of May,	Cape Horn, 1st of June,
London, 17th of July,	Petersburg, 20th of Oct.
Pekin, 10th of May,	Constantinople, 1st of Jan.

$50\frac{1}{4}''$ yearly, or 1° in 71.6 years (see note to def. 74, &c.) in the course of some time the longest day will not happen on these days, the sun receding backwards from cancer and capricorn $50\frac{1}{4}''$, as above, every year. Now as the sun's mean motion in the ecliptic is 59′ 8″.2, we have this proportion, 1° : 71.6 years :: 59′ 8″.2 : 70.56 years, or 70 years 6 months 22 days, the time in which the equinoctional points will recede one day from their present place in the ecliptic.

* One of the meridians passes through the middle of the analemma on Cary's globes, and this meridian passes through the 16th of June and 24th of December, at which times the clock and sun are equal. (See the remark on the analemma prob. 8, part 2d.)

5. At what time does the sun rise and set at every place on the surface of the globe on the 21st of March, and likewise on the 23d of September ?

6. Required the length of the longest day and shortest night at the following places :

Washington,	Paris,	Botany Bay,
London,	Vienna,	Boston,
Dublin,	Madrid,	Charleston,
Edinburgh,	Prague,	Buenos Ayres,
Petersburg,	Copenhagen,	Cape of Good Hope.

7. At what hour does the sun rise and set, at any time of the year to all the inhabitants of the equator, and what is the length of the day and night ?

8. Required the length of the shortest day and longest night, at the following places :

Philadelphia,	Quito,	Rome,
London,	Mexico,	Baltimore,
Quebec,	St. Helena,	Georgetown, near
Lima,	Lisbon,	Washington City.

9. How much longer is the 21st of June at Halifax than at Mexico ?

10. What is the difference between the 22d of December at Boston, and Cape Horn ?

11. At what time does the sun rise and set at the South Cape, in Spitzbergen, on the 31st of March, and 30th of April ?

Note. On the 30th of April in the last example, the learner will easily perceive that the sun does not set at all on that day, as his place, during an entire revolution of the globe on its axis, remains the whole time above the horizon.

PROB. 14.

The month and day of the month being given, to find those places where the sun does not set, and likewise where he does not rise on the given day ; or to find where the sun begins to shine constantly without setting, and also where he begins to be totally absent.

*Rule.** Find the sun's declination for the given day. (prob. 8.) Count the same number of degrees towards the equator from the

* The reason of these rules is very clear. For on the 21st of March and 23d of September the sun is on the equinoctial, and therefore enlightens the globe exactly from pole to pole : hence as the earth turns round its axis, which terminates in the poles, every place on the surface of the globe will equally go through the light and the dark, and thus make equal day and night in every part of the earth. But as the sun declines from the equator towards either pole, he will enlighten as many degrees round that pole as are equal to his declination from the equator, so that no place within that distance of the pole will then go through any part of the dark, and consequently the sun will not set to any part of this space. Now as the sun's declination is northward from the 20th of March to the 23d of September, he must constantly shine round the north pole during that time, and from thence

north and south poles, then all those places that pass under the degree where the reckoning ends, are the places required. If the declination be north, then to those places near the north pole and under the declination, the sun will not set, and to those places at the same distance from the south pole, the sun will not rise, and the contrary if the declination be south.

Or: The globe may be elevated according to the sun's declination; then, when turned on its axis, to those places which do not descend below the horizon, in that frigid zone near the elevated pole, the sun does not set on the given day, and to those which do not ascend above the horizon in that frigid zone adjoining to the depressed pole, the sun does not rise on the given day.

Note 1. Both these methods are the same in effect; the latter, however, seems to be more natural, the former more convenient. The learner will also observe, that when the decl. of the sun becomes equal to the complement of the lat. (or what it wants of 90°) and they are both of the same name, the sun does not descend below the horizon, but at midnight passes the meridian again, so as to touch the horizon exactly at the north or south (according to the lat.) and thus continues to circulate, gradually rising higher above the horizon (in proportion as his decl. exceeds the comp. of the lat.) until he arrives to his greatest declination, from which he continues to descend in like manner, until he again reaches the horizon; that is, when his decl. becomes equal to the comp. of lat. as before. But when the decl. becomes less than this, he will descend below the horizon in proportion, &c.

This problem is performed by the analemma in the same manner, the only difference being to find the sun's declination by it, and then proceed as above.

Examples. 1. Find all those places in the north frigid zone, where the sun does not set on the 20th of May, and those places in the south frigid zone, where he does not rise on the same day.

gradually to that space included between the pole and the arctic circle in proportion as his declination increases, so that on the day in which he is in Cancer, his declination being then 23° 28′, he will shine so many degrees beyond the pole, or to the polar circle, and therefore to all that space within the north polar circle the sun will not set during that day (21st June.) In like manner, from the 23d of September to the 21st of March inclusively, the sun constantly shines round the south pole, &c. And hence the same phenomenon will take place at this pole as at the north. So that when the sun is on the equator, it evidently shining as far as both poles, will here make the days equal to those in any other part of the hemisphere then enlightened, or equal to 12 hours, for the reason given above (or because the sun being over the equator, all the parallels from the equator to the poles are divided equally, and must therefore be 12 hours enlightened, and 12 hours in the dark, as the earth performs her revolution in 24 hours.) Now the sun advancing towards the south until his declination is supposed 10°, it is evident that the space within 10° of the south polar circle must be enlightened the whole time of the sun's revolution; and equally as evident that the space within 10° of the north pole must be in the dark: so that when it is day in the one, it is night in the other, and the contrary. But as the sun takes half a year from its crossing the equator until its return again, therefore at each pole the day and night must each be half a year long. (We do not here speak of natural days of 24 hours, but of artificial days, or the time which the sun remains above the horizon;) and from the poles to the polar circles, the length of their day is proportional to the sun's declination.

Ans. All places within 20° of the north pole (or within the lat. 70°) will have constant day; and those (if any) within 20° of the south pole, will have constant night.

2. Whether does the sun shine over the north or south pole on the 30th of September, and where is there constant day and constant night on that day?

3. What inhabitants of the earth have their shadows directed to every point of the compass during a revolution of the earth on its axis, on the 10th of June. (See def. 21.)

4. How far does the sun shine over the south pole on the 2d of February, and what places have perpetual darkness on that day?

Note 2. By perpetual darkness is only meant the absence of the sun, and not of that faint light called twilight, Aurora Borealis, &c. With regard to perpetual darkness, &c. the learner will easily observe, that every part of the world partakes of an equal share, and consequently of an equal share of day-light. The wisdom of the Creator is here displayed in a wonderful manner, by causing the twilight, Aurora Borealis, &c. to supply the absence of the sun during the long winter nights near the poles (as will be seen hereafter) and thus enabling the inhabitants to carry on their work, which they would otherwise be unable to perform during this gloomy season.

PROB. 15.

The month and day of the month being given at any place (not in the frigid zones) to find what other day of the year is of the same length.

Rule. Find the sun's place in the ecliptic for the given day, (by prob. 8.) bring it to the brass meridian, and observe the degree over it; turn the globe on its axis, until some other point of the ecliptic comes under the same degree of declination, and the day of the month corresponding, found on the horizon, will be the day required.

OR BY THE ANALEMMA,

Look for the given day of the month, and opposite to it will be the day required.

OR WITHOUT A GLOBE,

Find how many days the given day is before the longest day, the same number of days will the required day be after it, and the contrary. The same may be found by counting the number of days between the 21st of March and the given day, and reckoning the same number from the 22d of September backwards, and on the same side of the equator; the day on which the reckoning ends, is that required, and the contrary if the given day be after the longest.*

* The reason of this rule is evident, as any two days of the year which are of the same length, will be equally distant from the longest or shortest day, or from the days corresponding to the sun's entry into Aries and Libra; the sun's declination, to which the length of the day is proportional, being equal on both these days.

Example 1. What day of the year is of the same length as the 15th of April ?

Ans. The 27th of August.

2. What day of the year is of the same length as the 20th of August ?

3. If the sun rises at 4h. 20m. in the morning at Dublin, on the 9th of May, on what other day of the year will it rise at the same hour ?

4. If the sun set at seven o'clock in the evening at London, on the 24th of August, on what other day of the year will the sun set at the same hour ?

5. If the sun's meridian altitude be 90° at Barbadoes, on the 24th of April, on what other day of the year will the meridian altitude be the same ?

6. If the sun's meridian altitude be 51° 35′ at London, on the 25th of April, on what other day of the year will the meridian altitude be the same ?

PROB. 16.

The length of the day at any place being given to find the sun's declination and day of the month.

Rule 1. Bring the given place to the brass meridian, and set the index to twelve, turn the globe on its axis until the index has passed over as many hours as are equal to half the length of the day, keep the globe from revolving on its axis, and elevate or depress one of the poles until the given place exactly coincides with the horizon ; then the distance of the elevated pole from the horizon will be the sun's declination ; this declination being marked on the brass meridian, the two points of the ecliptic, which pass under it, correspond to the days required, and may be found on the circle of months on the horizon.

Note. It is more convenient to turn the globe eastward, as the brazen meridian is graduated on that side, and as the learner should generally stand at that side in performing his problems.

OR,

Rule 2. Bring the meridian passing through Aries* to the brass meridian, elevate the pole to the latitude of the given place, and set the index to twelve ; turn the globe eastward until the index has passed over as many hours as are equal to half the length of the day, and mark where the meridian passing through Aries is cut by the eastern part of the horizon ; bring this mark to the brass meridian, and the degree over it is the sun's declination, with which proceed as above.

* Any meridian will answer the purpose as well as this ; but as this on Cary's and most globes is graduated like the brass meridian, the point cut by the horizon will be the sun's declination, and therefore there is no necessity of bringing it to the brass meridian. The meridians passing through Libra, Cancer, and Capricorn, are also marked on Cary's globes, and may therefore be used in the same manner.

THE SAME BY THE ANALEMMA.

Bring the middle, or the meridian* passing through the middle, of the analemma to the brass meridian, elevate the pole to the latitude, and set the index to twelve; turn the globe eastward until the index has passed over as many hours as are equal to half the length of the day, then observe the point in the middle (or in the brass meridian, passing through the middle) of the analemma, that is cut by the horizon, the days opposite to it are those required, and if the point be brought to the brass meridian, the degree over it will be the sun's declination.

Example 1. What two days of the year are each fourteen hours long at New-York?

Ans. The 6th of May and the 6th of August.

2. What two days of the year are each 16 hours long at London?

3. What two days of the year are each 9 hours long at Boston?

4. On what two days of the year does the sun set at 7 o'clock at Copenhagen?

Note. Having the sun's rising or setting, the length of the day, &c. is given by prob. 13.

5. On what two days of the year does the sun rise at 4 o'clock at Petersburg?

6. What is the sun's declination when the sun rises at 5 o'clock in Washington city?

7. What two nights of the year are each 10 hours long at Amsterdam?

8. Required the sun's declination and day of the month, when the length of the day is 14 hours, 44 min. at Georgetown, District of Columbia, latitude 38° 55′ north?

PROB. 17.

The length of the longest day at any place, not within the polar circles, being given, to find the latitude of that place; or which is the same, to find in what latitude the longest day is of any given length less than 24 hours.

Rule. BRING the beginning of cancer or capricorn to the meridian (according as the latitude is north or south) and set the index to twelve; turn the globe westward on its axis, until the index has passed over half the number of hours given; then elevate or depress the pole until the sun's place (viz. cancer or capricorn)

* On Cary's globes, the analemma resembling the figure 8, the meridian passes through the point of intersection, as also through those days at the top and bottom on which the equation of time is nothing. But on other globes, the analemma being a narrow slip of paper, is drawn parallel to the meridian, and therefore either of its sides, or rather the line passing through the middle, will answer. The days cut by the horizon (as Keith says in his treatise on the globes) are not the days required, but those days corresponding to the point cut on this line, and opposite to each other, as is evident, unless the middle of the analemma on Bardin's globes be made use of.

comes to the horizon, and that elevation of the pole will shew the latitude. This method will answer for any other day, the sun's place being used instead of cancer or capricorn.

OR BY THE ANALEMMA.

Bring the analemma to the brass meridian, as before directed, and set the index to twelve ; turn the globe westward until the index has passed over half the number of hours, the day of the month being made to coincide with the horizon, by elevating or depressing the pole, this elevation will then shew the latitude.

Example 1. In what degree of north latitude, and at what places is the length of the longest day 16 hours?

Ans. In latitude 49°, and in all other places that have this lat. the day will be of the same length.

2. In what degree of south lat. is the longest day 17 hours?

3. In what lat. north does the sun set at 5 o'clock on the 10th of April?

4. There is a town in Norway, where the longest day is five times the length of the shortest night, what is its name?

5. In what latitude north is the 20th of May 16 hours long?

6. In what lat. north is the night of the 15th of August 10 hours long?

A TABLE, *shewing the length of the longest day in almost every degree of latitude from the equator to the pole.*

Latitude.	Longest day.	Latitude.	Longest day.	Latitude.	Longest day.	Latitude.	Longest day.
	H. M.		H. M.		H. M.		D. H. M.
0	12 0	42	15 4	59	18 10	74	96 17 0
6	12 20	43	15 12	60	18 30	75	104 1 4
12	12 42	44	15 18	61	18 54	76	110 7 27
16	12 58	45	15 26	62	19 20	77	116 14 22
20	13 12	46	15 34	63	19 50	78	122 17 6
24	13 26	47	15 42	64	20 24	79	127 9 55
27	13 42	48	15 52	65	21 10	80	134 4 58
30	13 56	49	16	66	22 18	81	139 1 36
32	14 6	50	16 10	66½	24	82	145 6 43
34	14 16	51	16 20		D. H. M.	83	152 2 6
35	14 22	52	16 30	67	24 0 0	84	156 3 3
36	14 28	53	16 42	68	42 1 16	85	161 5 23
37	14 34	54	16 54	69	54 16 25	86	166 11 23
38	14 38	55	17 8	70	64 13 46	87	171 21 47
39	14 44	56	17 22	71	74 0 0	88	176 5 29
40	14 52	57	17 36	72	82 6 36	89	181 21 58
41	14 58	58	17 52	73	89 4 58	90	187 6 39

The length of the longest day from the equator to the polar circles is found by the following proportion:

As radius : to tangent 23° 28′ :: so is tangent latitude : to sine of the ascensional difference, which converted into time, will give the time the sun rises or sets before or after six o'clock, from which the length of the longest day is given by prob. 13. For the reason of this rule see note to prob. 49.

The length of the longest day from the polar circles to the poles, may be found thus :

Find the complement of the latitude, which consider as the sun's declination, find the sun's places corresponding to this declination (north and south, as directed, prob. 19.) from a Nautical Almanac, with which proceed as directed in that prob. or without the Almanac the sun's longitude may be thus found :

As sine of the greatest declination 23° 28′,
To sine of the present declination,
So is Radius,
To sine of the sun's longitude.

The day corresponding to this longitude may be found in the Nautical Almanac, and the hours, minutes, &c. by allowing 59′ 8″ 3 for the sun's daily motion, 2′ 27″ 8 for every hour, 2″ 5 for a minute, &c.—or rather by taking the difference between the corresponding and preceding day, from the Nautical Almanac for the sun's daily motion, and then allowing proportional parts.

The longitude given by the preceding rule may be reckoned from Aries the contrary way, and also from Libra both ways, to find the four places of the sun required in prob. 19. By applying these principles, the above table taken from Fuller's treatise on the globes, may be rendered more correct, it being rather old. This author and some others have given this table, without any principles of calculation.

PROB. 18.

*The latitude and day of the month being given, to find how much the sun's declination must increase or decrease, to make the day an hour longer or shorter than the given day.**

Rule. Elevate the pole to the given latitude ; bring the sun's place, for the given day, to the brass meridian, and set the hour index to twelve ; turn the globe westward until the sun's place comes to the horizon, and observe the hours passed over by the index ; then if the days be increasing, turn the globe westward until the index has passed over half an hour more ; the point of the ecliptic then cut by the horizon, will correspond to the sun's place, where the day is an hour longer, &c. and hence the decl. is found. (prob. 8.) But if the days be decreasing, turn the globe eastward until the index has passed over half an hour, the point of the eclip-

* Note. The prob. may become general for any time corresponding to the length of the longest day in any place, if instead of half an hour, the globe be turned until the index passes over half the time that the required day is to be longer or shorter than the given day, and then proceeding as before. The latitude of the place, on the longest or shortest day, must admit of the given increase or decrease in the day required, otherwise the rules will not hold. Those places within the polar circles are not here considered.

tic then cut by the horizon, will shew the sun's place when the day is an hour shorter than the given day.

OR,

Find the sun's declination for the given day, and elevate the pole to that declination; bring the given place to the brass meridian, and set the index to twelve, turn the globe eastward until the given place comes to the horizon; then if the days be increasing, continue the motion of the globe eastward, but if decreasing, westward, until the place comes a second time to the horizon, the last elevation of the pole will shew the sun's declination required.

OR BY THE ANALEMMA,

Proceed as above, only instead of the sun's place, bring the analemma to the brass meridian, and use the day of the month on the analemma instead of the sun's place.

Example 1. How much must the sun's declination vary, that the day at New-York may be increased one hour from the 13th of March, 1810.

Ans. On the 13th of March the sun's decl. is 3° south, and the sun sets at 50 minutes past 5. Now when the sun sets at 20 min. after 6, his declination will be found to be about 5° 40′ north, nearly, answering to the 4th of April. Hence the sun must cross the equator, and make his declination 5° 40′ N. and in 22 days the day has increased one hour.

2. How much must the sun's decl. vary that the day at London may decrease one hour, in length, from the 26th of July?

Ans. The sun's decl. on the 26th of July (1807 or 1811, &c.) is 19° 38′ north, and the sun sets at 49′ past seven (see note 1 to prob. 14.) When the sun sets at 19′ after 7, his declination will be found to be 14° 43′ north, answering to the 13th of Aug. Hence the declination has decreased 5° 55′, and the days have decreased 1 hour in 18 days.

3. How much must the sun's declination vary from the first of Oct. that the day at Petersburg may decrease one hour?

4. How much must the sun's decl. vary from the 10th of April, that the day at Skalholt may increase two hours. (See the note at the bottom of page 77.)

PROB. 19.

A place being given in the north frigid zone, to find the length of the longest day and longest night there, or (which is the same) to find what number of days of 24 hours each the sun constantly shines upon it, how long he is absent; likewise the first and last day of his appearance, and the number of days of 24 hours each that he will there rise and set.

Rule. FIND the complement of the latitude, or what it wants of 90°, and reckon an equal number of degrees from the equator on the brass meridian north and south, and mark the points where the reckoning ends; then bring the first quadrant of the ecliptic, or that from aries to cancer, to the brass meridian, and observe what point of it passes under the above mark; this point will give the sun's place when the longest day commences, or the first day on which the sun will constantly shine without setting. The globe being then turned westward until some point in the second quarter of the ecliptic coincides with or falls under the same mark, this point will give the sun's place when the longest day ends, and the day corresponding to it will be the last day on which the sun will constantly shine without setting; the number of natural days between these two, will be the length of the longest day in the given place. The motion of the globe being continued westward, mark the next point of the ecliptic, in the 3d quadrant that comes under the mark on the brazen meridian, south of the equator; this point will give the sun's place corresponding to the last day of his appearance above the horizon or the beginning of the longest night; next find that point in the 4th quadrant of the ecliptic that comes under the mark south of the equator, and it will be the sun's place when the longest night ends; lastly, the number of days between the end of the longest day and the beginning of the longest night, together with the number of days from the end of the longest night to the beginning of the longest day, will be those days on which the sun will rise and set alternately every 24 hours.

Note. Though it be more natural to have the globes rectified for the latitude, and that the points of the ecliptic cut the meridian at the horizon in the north and south points, in the same order as above, yet the above method is more convenient in practice. The learner will easily perceive that both methods are the same in effect, but the reason of the rule will appear more evident from the latter method, as the rising, setting, &c. of the sun, will be seen on the horizon of the globe in the same manner as in the horizon on the earth, corresponding to the place whose latitude is given. The application of this method is left to the learner. The problem is not applied to the south frigid zone, this zone being uninhabited (at least we know of none) however the rule is general, reading south for north, &c. and proceeding as above.

The time when the longest day or night begins being known, their end may be found, as the beginning and end of either are equally distant from the solstice that intervenes, that is, the beginning of the longest day is the same number of natural days from the succeeding solstice that the end of the longest day is after it, &c. The number of days which the sun alternately rises and sets, is also found by adding the length of the longest day and longest night together, and taking their sum from 365 days.

The reason of reckoning the complement of the latitude from the equator is evident, as it must always be equal to the sun's declination, when the longest day commences and ends there. For when there is no declination, then the longest day commences or ends at the pole. When there is 10° north declination, then the longest day commences or ends in the parallel of 80° distant 10° from the pole, because the whole of that parallel will then be in the illuminated hemisphere, &c.

OR BY THE ANALEMMA.

If the place be in the north frigid zone, the two days on the analemma, that pass under the complement of the latitude north of the equator on the brass meridian, will be the beginning and end of the longest day, and those two days that pass under the complement of the latitude south of the equator, will be the beginning and end of the longest night; from which the rest is given as above. The contrary will answer for the south pole.

Example 1 What is the length of the longest day and longest night at the North-cape, in the island of Maggeroe, latitude 71° 10′ north; the first and last day of his appearance, and the number of days that he rises and sets there?

Ans. The complement of the lat or what it wants of 90°, is 18° 50′, this being marked on both sides of the equator on the brass meridian, the four points of the ecliptic that pass under it will correspond to the 15th of May, 28th of July, 14th of November, and 26th of January. Consequently the longest day begins on the 15th of May, and ends on the 28th of July, and is therefore 74 natural days long (that is the sun does not set during 74 revolutions of the earth on its axis.) The longest night commences on the 14th of November, and ends on the 26th of January, and is therefore 73 days long. The sun will rise and set alternately from the 26th of January to the 15th of May, which is 109 days from the end of the longest night to the beginning of the longest day; and also from 28th of July to the 14th of November, which is also 109 days from the end of the longest day to the beginning of the longest night. The learner will observe, that on the 26th of January the upper edge of the sun will just touch the horizon, and again descend below it; the next day it will advance a little higher, &c. increasing the day by little and little, until the sun crosses the equator, when the day and night will be exactly equal; then after crossing the equator, the day will become longer than the night, and will continue increasing, in proportion to the sun's declination until the 14th of May, at which time the day will be exactly 24 hours. The same observation will hold, *vice versa*, with regard to those days on which the sun rises and sets from the 28th of July to the 14th of November. The length, therefore, of the longest day is 74 days, of the longest night 73*, and the number of days that the sun rises and sets is 218, making in all 365 days.

2. What is the length of the longest day and longest night in the northeast land in Spitzbergen, under the parallel of 80°; when

* Here there is a difference of one day between the longest day and longest night, owing to the obliquity of the ecliptic and the eccentricity of the earth's orbit. (See notes to def. 57 and 61.) Fuller and Keith in their respective treatises on the globes, make this difference amount to 4 days, and each reckons the lat. of the north cape 71° 30—But accuracy in these matters cannot be obtained on globes; the learner, however, should be cautioned against errors, and not to trust too much to instruments where calculation, &c. is not applied. The above prob. has been tried on one of Cary's globes of nearly two feet diameter. Some authors make the lat. of this cape 71° 38′. (Vyse's Geogr. pa. 47 of the introduction, where he makes a difference of 7 days between the longest day and longest night.) If more accuracy be required in this problem, tables of the sun's motion, &c. may be consulted. See *Vince's Tables lately published.*

do they begin and end, and how many days does the sun rise and set there?

Ans. The longest day begins on the 15th of April, and ends on the 27th of August. The longest night begins on the 18th of October, and ends on the 22d of February. Hence the rest is easily found as above.

3. What is the length of the longest day and longest night at Ice-cape in Novazembla, lat. $76\frac{1}{2}$° N. and how many days does the sun rise and set there?

4. What is the length of the longest day and longest night at the north and south poles; when they do commence, and what is the difference between the length of the summer and winter half year at both poles?

PROB. 20

Any number of days not exceeding $186\frac{1}{2}$ in north, or $178\frac{1}{2}$ in south latitude, being given, to find in what latitude the sun does not set during that time.

Rule. Count half the number of days from the 21st of June, or the 22d of December (according as the place is in north or south lat.) eastward or westward on the horizon, and find the sun's declination corresponding to the days where the reckoning ends (by prob 8.) the same number of degrees reckoned from either pole, on the brass meridian, will give the latitude required.

OR BY THE ANALEMMA.

Count half the number of days from the 21st of June or 22d of December, &c. towards the equator, the sun's declination corresponding to the day on which the reckoning ends, will be the complement of the latitude.

Note. In the same manner can be found the latitude, in which the sun does not rise for any time less than 178 natural days, in north, or 187 in south latitude (the longest absence of the sun at the poles consisting of so many days) by reckoning half the number of natural days from the 22d of December, in north, or 21st of June, in south latitude, and proceeding as above. If the end of the longest night and beginning of the longest day be given, and the number of days between them be found, reckon half that number from aries on either side, and the sun's declination, corresponding to the place where the reckoning ends, will be the complement of the latitude where the sun will rise and set before and after the vernal equinox only so many days. In the same manner we may proceed reckoning from libra for the autumnal equinox, the end of the longest day, and the beginning of the longest night being given. The longest day at the north pole reckoning from the 21st of March inclusive to the 23d of September, is 187 days; but on account of refraction the sun will remain longer visible above the horizon.

Example 1. In what degree of north latitude, and in what places, does the sun continue above the horizon during 134 natural days?

Ans. Half the number of days being 67, which reckoned towards the east from the 21st of June, will answer to the 15th of April, or reckoned towards the west, to the 27th of August; on either of which days the sun's declination is 10° north; consequently the latitude is 80° north, and the places all those passing under this parallel of latitude.

2. Where is the longest day 74 days or 1776 hours?

3 In what degree of north lat. does the sun continue above the horizon 90 days or 2160 hours?

4. In what degree of north lat. is the longest night 1752 hours long? See the last note for this and the following prob.

5. In what degree of north lat. does the sun alternately rise and set no more than $54\frac{1}{2}$ days, both before and after the vernal equinox?

PROB. 21.

To find how much any number of days in one month, is longer or shorter than the same number in another month.

Rule. FIND the sun's place for the beginning and ending of the given days in one month, bring these places to the brazen meridian, and mark the corresponding degrees on the equator cut by the brass meridian; the two points, on the equator, will be the sun's right ascension, and the number of degrees between these points converted into time (by the note to prob. 6.) will give the length of the days in that month. In the same manner find the right ascension of the sun for the beginning and ending of the given days in the other month, and convert it into time as before; if this time agrees with the former, the given days in one month are equal to the same number in the other; but if not, their difference in time (or in degrees converted into time) will show the difference.

Example 1. How much longer is the number of days from the 1st to the 20th of August inclusively, than the same number from the 1st to the 20th of September?

Ans. The right ascensions corresponding to the beginning and 20th of August, are $130\frac{3}{4}°$ and $149\frac{3}{4}°$ the difference of which is 19°, and the right ascensions corresponding to the beginning and 20th of September, are $159\frac{3}{4}°$ and $177\frac{3}{4}°$ respectively, the difference of which is 18°. Now the difference between this and the former is 1°, which in time is 4 minutes, the excess of the given number of days in August above those in September.

Note. The right ascension is here reckoned from the beginning of the 1st day in each month, or from the last day in the preceding month.

2. Find how much longer or shorter the month of January is than that of May?

3. How much longer or shorter is the month of September than that of November?

PROB. 22.

*To find the part of the equation of time which depends on the obliquity of the ecliptic, and also the true equation.**

Rule. FIND the sun's place in the ecliptic, and bring it to the brass meridian; count the number of degrees from aries both on the equator, and the ecliptic to the brass meridian, the difference converted into time is the equation depending on the obliquity of the ecliptic. If the number of degrees on the ecliptic exceed those

* The difference between a well regulated clock and a good sun dial, will always give the equation of time; it is therefore necessary we should point out the manner of regulating these instruments and the principles on which this difference depends, the equation of time being necessary, not only in civil affairs, but also in almost every part of practical astronomy, and absolutely necessary in the important problem of determining the longitude, &c.

A pendulum clock is the best measure of time as yet discovered; but from the expansion or contraction of the materials by heat or cold, which varies the length of the pendulum, from which, as well as from the imperfection of the workmanship and other accidental causes, the time indicated by the best clocks must be subject to irregularity. Hence it becomes necessary that we should be able, at any time, to ascertain how much it is too fast or too slow, and at what rate it gains or loses; and for this purpose we must compare it with some motion which is uniform, or whose variation, if it be not uniform, can be ascertained. Now as the earth revolves uniformly on its axis, the apparent diurnal motion of the fixed stars must be uniform, and is therefore considered as most proper to ascertain the variation above mentioned. If a clock be therefore adjusted to go 24 hours from the passage of any fixed star over the meridian, until it returns to the same meridian again, it is said to be adjusted to *sidereal time*, and its rate of going may, at any time, be determined, by comparing it with the transit of any fixed star, and observing whether the interval be exactly 24 hours; if not, the difference will be what it has gained or lost during that time; or if the apparent right ascension of a known star when it passes the meridian, be observed, and this right ascension be compared with the right ascension shewn by the clock, the difference will be the error of the clock. In this latter case the clock must begin its motion from 0h. 0′ 0″ at the moment that the first point of aries is on the meridian; then, when any star comes to the meridian, the clock will shew the apparent right ascension of the star, allowing 15° for 1h. because, when subject to no error, it will then shew how far the point aries is from the meridian. The error, if any, is found and allowed for as follows: let the apparent right ascension of *aldebaran*, for example, be 4h. 23′ 50″ at the same time that its transit over the meridian is observed by the clock to be 4h. 23′ 52″, then the error of the clock is 2″ more than it ought to be. If similar observations be made with other stars, and the mean error taken, the error, at the mean time of all the observations, will be more accurately found. These observations being repeated every day, we shall get the rate of the clocks going, or how fast it gains or loses. It may not be unworthy of remark here, though not its proper place, that the error of the clock, and the rate of its going being thus ascertained, if the time of the true transit of any star or planet be observed, and the error of the clock, for the time, corrected, the right ascension of the star or planet will be given; this being the method by which the right ascension of the sun, moon and planets are regularly found in observatories.

If we adjust a clock with the sun, or to go 24 hours from the time the sun leaves the meridian on any day until he returns to the same meridian again, which interval is a *true solar day*, the clock will soon vary from the sun, on the equator, the sun is faster than the clock; if equal, the clock and sun agree; if less, the clock is faster than the sun.

TO FIND THE TRUE EQUATION.

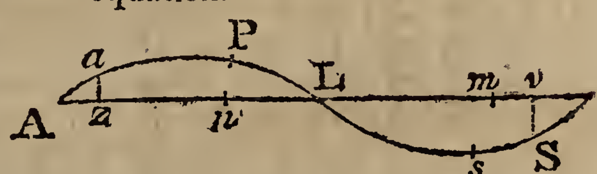
Look on the horizon, in Barden's globes, and corresponding to the day of the month, you will find the equation required. On Ca-

even in the supposition that it is good and goes uniformly, and will not indicate 12 when the sun comes on the meridian. This inequality depends on the *two* following causes. The 1st arises from the obliquity of the ecliptic, or its inclination to the equator. Let the sun and an imaginary star commence their motion at *aries*, and move through equal spaces in equal times, the sun in the ecliptic, and the imaginary star in the equinoctial; at the end of every degree let both bodies be brought under the brass meridian on the globe, and it will be found that they will never come to the meridian together, except at the time of the equinoxes and on the longest and shortest days, or the solstices; and that from the equinox to the next tropic or solstice, the *apparent* time, or the time shewn by the sun, precedes the *true*, or the time shewn by the imaginary star, or a clock regulated to mean solar time, because then the degrees in the ecliptic exceed those on the equator (at the equinox, the ratio is that of radius to cos. of the obliquity of the ecliptic) and that from the solstices to the equinoxes, the true time precedes the apparent (for a contrary reason) the proportion of the degrees on the equator to those on the ecliptic being as R. to cos. obl. at the solstices. The 2d cause on which this inequality depends, is the unequal motion of the earth in its orbit, being slower in its aphelion or greatest distance from the sun, and quicker in its perihelion or least distance. This part of the equation of time is found from the distance of the sun at any time from the apogee, reckoning round with the sun (or from the point where it commences its motion, and not to this point) being its *mean anomaly*. For as the earth describes equal areas in equal times by lines drawn from the sun to its orbit (see prob. 1. sec. 2. Newton's prin. B. I. Emers, centerp. forces prob. 11. Greg. astr. prob. 11 or part 4th of this treatise) the sun's motion is therefore slower in its apogee, and increases in velocity to its perigree where it is swiftest, and from thence decreases until it comes to its apogee again. If we now suppose the sun to revolve round the earth, instead of the earth round the sun, the effect being here the same, and the explanation easier on this supposition; and that the sun departs from its apogee or aphelion, whilst an imaginary star departs from thence at the same time with the mean angular velocity of the sun (or to perform its motion in the equinoctional, supposed here to coincide with the ecliptic, as its obliquity is not considered) so as to describe an equal arch every 24 hours, it will then be evident that the imaginary star will gain on the real sun, and every day advance more to the east, and therefore that the sun will come to the meridian first; and hence that the apparent time will precede the true. But before the sun comes to its perihelion, or perigee, it is plain, from the above general law, that it will move quicker than the imaginary star, but will not be able to overtake it until they are both in conjunction, which will take place in the perigee after the sun has performed half its revolution, and hence *from the apogee to the perigee, the apparent time will precede the true.* Now the sun and star departing together from the perigee, the sun's velocity will be greater than the star's, so as to advance more to the eastward, and therefore to come later to the meridian, until they are both in conjunction again, or in the apogee, the point from whence their motion commenced, and hence *from perigee to apogee the true time will precede the apparent.* There are then but two points in which their motions will be equal between the apogee and perigee (see Emerson's centr. forces cor. 9 prop. 16 sect. 2d.) and but two points in which they will

.ry's globes, bring the day of the month on the analemma to the brass meridian, under which on the scale drawn through the analemma parallel to the equator, you will find the number of minutes, &c. required. The scale indicates whether the equation be fast or slow.

come to the meridian together, viz. at the perigee and apogee ; and hence while the sun is describing the first 6 signs of the *anomaly*, the imaginary star, which shews the mean time or the time by the clock, being more to the east than the sun (which shews the apparent time, or time shewn by a sun dial) comes to the meridian later, and shews the apparent time greater than the mean. Hence in these 6 signs, to find the equation depending on this cause, the difference between the mean and apparent time must be taken, which subtracted from the apparent time, will give the mean time, or added to the mean time, will give the apparent. While the sun is in the last 6 signs of its anamoly, the mean noon precedes the apparent, for the reason given above, and hence the difference between the motion of the imaginary star and the sun, or the difference between the sun's mean and true motion, converted into time, must be added to the apparent time to give the mean, and *vice versa*.

Now as both the above causes conspire to make the inequality before noticed, or the *equation of time*, it is evident that when both are faster or slower, their sum is the true equation of time ; but when one is faster and the other slower, their difference is the true equation.



To compute this equation, let APLS be the ecliptic, ALm the equator, A the beginning of aries, P the sun's apogee, S any given place of the sun ; draw Sv perpendicular to the equator, and take An=AP. Now when the sun departs from P, let the imaginary star depart from n with the sun's mean motion in longitude or in right ascension, or at the rate of $59' \, 8'' \, 2$ in a day ($365\frac{1}{4}$d. : 1d. :: $360°$: $59' \, 8'' \, 2$) and when n passes the meridian, let the clock be adjusted to 12 as described above ; take nm=Ps, and when the star comes to m, the sun if it moved uniformly with its mean motion, would be at s, but let S be the sun's place at that time, and let S, and consequently v, be on the meridian, then as the imaginary star at that instant is at m, mv is the equation of time. Let a be the *mean* equinox, or the point where the equinox would have been if it moved uniformly backwards with its *mean* velocity, and draw az perpendicular to AL ; then z on the equator would have coincided with a if the equinox had moved uniformly ; therefore the *mean* right ascension from z must be reckoned. Now mv=Av—Am ; but Am=Az+zm and Az=Aa×cos. aAz for the small triangle aAz being considered as right lined, it will be rad. : cos. aAz ($23° \, 28'$) :: Aa : Az or using the nat. sines, &c. 1 : .9172919 :: Aa : Az, hence Az=Aa×.9172919 or cos. aAz, but .9172, &c.=$\frac{11}{12}$ nearly, whence Am=$\frac{11}{12}$ aAz+m ; and therefore mv= Av—zm—$\frac{11}{12}$ Aa. Now Av is the sun's true right ascension, zm the mean right ascension, or mean longitude, and $\frac{11}{12}$ Aa (Az) is the equation of the equinoxes in right ascension, hence *the equation of time is equal to the difference of the sun's true right ascension and its mean longitude corrected by the equation of the equinoxes in right ascension.* When Am is less than Av, *mean* time precedes the apparent, but the *apparent* precedes the *mean* when Am is greater. For as the earth revolves on its axis in the direction Av, or in the order of right ascension, that body whose right ascension is least, comes to the meridian first ; that is, when the sun's true right ascension is *greater* than its mean longitude corrected as above, the equation of time must be *added* to the apparent, to get the mean time, and when it is *less*, it must be

Example 1. What is that part of the equation of time which depends on the obliquity of the ecliptic on the 17th of July, and what is the true equation?

Ans. The degrees on the ecliptic is less than those on the equator by two nearly, which in time is 8 minutes, and hence the sun,

subtracted. But to convert mean time into apparent, we must *subtract* in the *former* case, and *add* in the *latter.*

The following tables were constructed on the above principles. The first gives the equation resulting from the obliquity of the ecliptic alone, the 2d the equation depending on the eccentricity of the earth's orbit or the sun's mean anomaly, and the 3d the true equation, or the equation resulting from both these causes.

TABLE I.

Sun faster than the clock in					*Correction of Ta.* 1.		
Deg	♈. 0. ♎. 6.	♉. 1. ♏. 7.	♊. 2. ♐. 8.	12 32	Sun's place	Cor.	Sun's place
0	0′ 0″ 0	8′ 22″ 6	8′ 44″ 7	30	♈ ♎		♎ ♈
1	0 19 8	8 33 0	8 34 5	29	0	0″000	0
2	0 39 7	8 42 9	8 23 6	28	3	0 001	27
3	0 59 5	8 52 2	8 12 1	27	6	0 003	24
4	1 19 2	9 0 9	8 0 0	26	9	0 004	21
5	1 38 8	9 8 9	7 47 1	25	12	0 005	18
6	1 58 3	9 16 3	7 33 7	24	15	0 007	15
7	2 17 7	9 23 1	7 19 6	23	18	0 008	12
8	2 37 0	9 29 2	7 5 6	22	21	0 009	9
9	2 56 1	9 34 6	6 49 8	21	24	0 010	6
10	3 15 0	9 39 4	6 34 1	20	27	0 011	3
11	3 33 7	9 43 5	6 18 0	19	♉ ♏	——	♍ ♓
12	3 52 1	9 46 8	6 1 2	18	0	0 012	0
13	4 10 3	9 49 5	5 43 9	17	3	0 013	27
14	4 28 2	9 51 5	5 26 2	16	6	0 013	24
15	4 45 9	9 52 7	5 8 1	15	9	0 014	21
16	5 3 2	9 53 3	4 49 5	14	12	0 014	18
17	5 20 2	9 53 1	4 30 5	13	15	0 014	15
18	5 36 8	9 52 1	4 11 1	12	18	0 014	12
19	5 53 1	9 50 5	3 51 4	11	21	0 014	9
20	6 9 0	9 48 1	3 31 4	10	24	0 014	6
21	6 24 5	9 45 0	3 11 1	9	27	0 014	3
22	6 39 6	9 41 2	2 50 6	8	♊ ♐	——	♌ ♒
23	6 54 2	9 36 6	2 29 8	7	0	0 013	0
24	7 8 3	9 31 3	2 8 8	6	3	0 012	27
25	7 22 0	9 25 3	1 47 6	5	6	0 011	24
26	7 35 2	9 18 6	1 26 2	4	9	0 010	21
27	7 47 9	9 11 2	1 4 8	3	12	0 009	18
28	8 0 1	9 3 0	0 43 2	2	15	0 008	15
29	8 11 8	8 54 2	0 21 6	1	18	0 006	12
30	8 22 6	8 44 7	0 0 0	0	21	0 005	9
					24	0 003	6
22	♍. 5.	♌. 4.	♋. 3.	Deg.	27	0 002	3
42	♓ 11.	♒ 10.	♑ 9.		0	0 000	0
					♋ ♑	——	♋ ♑
Sun slower than the clock in							

as depending on the obliquity of the ecliptic is 8′, or rather 7′, 49″, slower than the clock The true equation is 5 min. 41 seconds slower: hence the equation depending on the sun's mean anomaly or the sun's distance from the apogee is 7′, 49″,—5′, 41″,=3′, 8″, sun slower, &c

2. On what days of the year is the true equation of time nothing, and also the equation depending on the obliquity of the ecliptic.

TABLE II.

Sun faster than the clock if his anomaly be

Deg.	0s.—	1s.—	2s.—	3s.—	4s.—	5s.—	
0	0′ 0″0	3′ 46″8	6′ 35″7	7′ 41″7	6′ 44″1	3′ 55″2	30
1	0 7 9	3 53 6	6 39 7	7 41 8	6 40 1	3 48 1	29
2	0 15 8	4 0 4	6 43 7	7 41 7	6 36 0	3 40 9	28
3	0 23 7	4 7 1	6 47 5	7 41 5	6 31 7	3 33 6	27
4	0 31 6	4 13 8	6 51 2	7 41 2	6 27 3	3 26 3	26
5	0 39 4	4 20 4	6 54 8	7 40 8	6 22 8	3 18 9	25
6	0 47 3	4 26 9	6 58 2	7 40 2	6 18 2	3 11 5	24
7	0 55 1	4 33 3	7 1 5	7 39 4	6 13 5	3 4 0	23
8	1 3 0	4 39 6	7 4 7	7 38 5	6 8 6	2 56 4	22
9	1 10 8	4 45 9	7 7 8	7 37 5	6 3 6	2 48 8	21
10	1 18 6	4 52 1	7 10 8	7 36 3	5 58 5	2 41 1	20
11	1 26 4	4 58 2	7 13 6	7 35 0	5 53 3	2 33 4	19
12	1 34 1	5 4 2	7 16 3	7 33 6	5 48 0	3 25 6	18
13	1 41 8	5 10 1	7 18 8	7 32 0	5 42 6	2 17 8	17
14	1 49 4	5 16 0	7 21 3	7 30 3	5 37 1	2 9 9	16
15	1 57 1	5 21 7	7 23 5	7 28 4	5 31 4	2 2 0	15
16	2 4 7	5 27 4	7 25 7	7 26 4	5 25 7	1 54 0	14
17	2 12 4	5 32 9	7 27 7	7 24 2	5 19 8	1 46 0	13
18	2 19 9	5 38 4	7 29 6	7 21 9	5 13 9	1 38 0	12
19	2 27 4	5 43 7	7 31 4	7 19 5	5 7 8	1 30 0	11
20	2 34 9	5 49 0	7 33 0	7 17 0	5 1 7	1 21 9	10
21	2 42 3	5 54 1	7 34 5	7 14 3	4 55 4	1 13 8	9
22	2 49 7	5 59 2	7 35 9	7 11 5	4 49 1	1 5 6	8
23	2 57 1	6 4 1	7 37 1	7 8 5	4 42 6	0 57 5	7
24	3 4 3	6 8 9	7 38 1	7 5 4	4 36 1	0 49 3	6
25	3 11 5	6 13 7	7 39 1	7 2 2	4 29 5	0 41 1	5
26	3 18 6	6 18 3	7 39 9	6 58 8	4 22 8	0 32 9	4
27	3 25 8	6 22 8	7 40 5	6 55 3	4 16 0	0 24 7	3
28	3 32 8	6 27 2	7 41 0	6 51 7	4 9 1	0 16 5	2
29	3 39 8	6 31 5	7 41 4	6 48 0	4 2 2	0 8 2	1
30	3 46 8	6 35 7	7 41 7	6 44 1	3 55 2	0 0 0	0
	11s +	10s +	9s +	8s +	7s +	6s +	Deg.

Sun slower than the clock if his anomaly be

3. What is the equation of time depending on the obliquity of the ecliptic and the eccentricity of the earth's orbit, respectively, on the 27th of October?

Note. Here as before, the equation depending on the ecliptic, and likewise the true equation being found, their difference will give the equation depending on the earth's eccentricity, &c.

4. What is the equation of time when the sun is in the beginning of taurus?

TABLE III.

Days.	*Jan.*	*Feb.*	*March.*	*April.*	*May.*	*June.*	*July.*	*Aug.*
	Add	*Add*	*Add*	*Add*	*Sub.*	*Sub.*	*Add*	*Add*
1	3′ 32″ 9	13′ 51″ 3	12′ 36″ 6	3′ 55″ 7	3′ 6″ 3	2′ 36″ 5	3′ 21″ 1	5′ 56″ 1
2	4 1 3	13 59 3	12 24 2	3 37 4	3 13 8	2 27 4	3 32 6	5 52 4
3	4 29 4	14 6 6	12 11 3	3 19 3	3 20 6	2 18 0	3 43 8	5 48 1
4	4 57 1	14 13 0	11 58 0	3 1 3	3 26 9	2 8 1	3 54 8	5 43 2
5	5 24 5	14 18 6	11 44 3	2 43 5	3 32 6	1 57 8	4 5 5	5 37 7
6	5 51 5	14 23 5	11 30 1	2 25 9	3 37 7	1 47 2	4 15 8	5 31 6
7	6 18 0	14 27 5	11 15 6	2 8 6	3 42 3	1 36 3	4 25 8	5 25 0
8	6 44 1	14 30 8	11 0 6	1 51 5	3 46 2	1 25 0	4 35 4	5 17 8
9	7 9 7	14 33 3	10 46 3	1 34 6	3 49 6	1 13 5	4 44 7	5 10 0
10	7 34 8	14 34 9	10 29 7	1 18 0	3 52 4	1 1 7	4 53 6	5 1 6
11	7 59 3	14 35 8	10 13 7	1 1 7	3 54 6	0 49 7	5 2 0	4 52 6
12	8 23 2	14 36 0	9 57 5	0 45 7	3 56 2	0 37 5	5 9 9	4 43 0
13	8 46 6	14 35 3	9 40 9	0 29 9	3 57 2	0 25 2	5 17 4	4 32 9
14	9 9 4	14 33 9	9 24 1	0 14 5	3 57 7	0 12 7	5 24 4	4 22 2
15	9 31 5	14 31 8	9 7 0	*sub* 0 6	3 57 7	*add* 0 0	5 30 9	4 10 9
16	9 52 9	14 28 9	8 49 7	0 15 3	3 57 0	0 12 8	5 36 9	3 59 1
17	10 13 6	14 25 2	8 32 1	0 29 7	3 55 8	0 25 6	5 42 4	3 46 8
18	10 33 6	14 20 8	8 14 4	0 43 7	3 54 1	0 38 5	5 47 3	3 33 9
19	10 52 9	14 15 7	7 56 4	0 57 3	3 51 9	0 51 4	5 51 6	3 20 5
20	11 11 4	14 9 9	7 38 3	1 10 6	3 49 1	1 4 3	5 55 4	3 6 7
21	11 29 2	14 3 4	7 20 0	1 23 5	3 45 8	1 17 2	5 58 6	2 52 3
22	11 46 1	13 56 2	7 1 6	1 35 9	3 42 0	1 30 0	6 1 3	2 37 5
23	12 2 3	13 48 3	6 43 1	1 47 9	3 37 7	1 42 8	6 3 4	2 22 2
24	12 17 7	13 39 9	6 24 6	1 59 4	3 32 8	1 55 6	6 4 9	2 6 5
25	12 32 2	13 30 8	6 5 9	2 10 5	3 27 4	2 8 2	6 5 8	1 50 4
26	12 45 9	13 21 1	5 47 2	2 21 1	3 21 6	2 20 8	6 6 1	1 34 0
27	12 58 8	13 10 8	5 28 5	2 31 2	3 15 3	2 33 2	6 5 9	1 17 1
28	13 11 0	12 59 9	5 9 8	2 40 8	3 8 5	2 45 4	6 5 1	0 59 9
29	13 22 3	12 48 5	4 51 2	2 49 9	3 1 2	2 57 5	6 3 7	0 42 3
30	13 32 8		4 32 6	2 58 4	2 53 4	3 9 4	6 1 7	0 24 4
31	13 42 4		4 14 1		2 45 2		5 59 2	0 6 2

In the 1st table the signs of the 1st and 3d quarters of the ecliptic are at the top, and the degrees at the left hand. The signs of the 2d and 4th quarters are at the bottom, and the degrees at the right hand. When the sun is in the former signs, it is faster than the clock, but when in the latter, slower. Thus when the sun is in 15° of ♉ or ♏ it is 9′ 52″ 7 faster than the clock, or the apparent time is faster than the mean; but when the sun is in 20° of ♋ or ♑ it is 6′ 34″ 1 slower than the clock. The 2d table is applied in like manner, according to the respective sign and degree of the sun's anomaly. Thus when his anomaly is 2 signs 13°, or when the sun is

PROB. 23.

To shew at one view the length of day and night, in all places upon the earth, at any given time; and to explain how the vicissitudes of day and night are really made, by the motion of the earth on its axis, in 24 hours, the sun standing still.

The sun being at an immense distance from our globe, the rays of light emitted from it may therefore be considered as parallel, and hence it will always illuminate one half of the globe, or that

TABLE III.—*Continued.*

Days.	*Sept.*	*Oct.*	*Nov.*	*Dec.*
	Sub.	*Sub.*	*Sub.*	*Sub*
1	0′ 12″ 3	10′ 21″ 4	16′ 14″ 8	10′ 38″ 2
2	0 31 1	10 40 2	16 15 4	10 15 0
3	0 50 1	10 58 6	16 15 3	9 51 2
4	1 9 3	11 16 7	16 14 3	9 26 7
5	1 28 8	11 34 4	16 12 5	9 1 7
6	1 48 6	11 51 8	16 9 9	8 36 2
7	2 8 5	12 8 7	16 6 4	8 10 1
8	2 28 7	12 25 3	16 2 1	7 43 6
9	2 49 0	12 41 4	15 57 0	7 16 7
10	3 9 5	12 57 1	15 51 1	6 49 3
11	3 30 1	13 12 4	15 44 3	6 21 6
12	3 50 9	13 27 2	15 36 7	5 53 5
13	4 11 7	13 41 5	15 28 3	5 25 1
14	4 32 7	13 55 3	15 19 0	4 56 3
15	4 53 7	14 8 5	15 9 0	4 27 3
16	5 14 8	14 21 2	14 58 1	3 58 1
17	5 35 9	14 33 4	14 46 3	3 28 8
18	5 57 0	14 45 0	14 33 8	2 59 2
19	6 18 1	14 55 9	14 20 4	2 29 5
20	6 39 2	15 6 2	14 6 2	1 59 6
21	7 0 2	15 15 9	13 51 2	1 29 7
22	7 21 1	15 24 9	13 35 4	0 59 8
23	7 41 9	15 33 3	13 18 7	0 29 8
24	8 2 6	15 40 9	13 1 3	*add* 0 2
25	8 23 1	15 47 8	12 43 1	0 30 2
26	8 43 4	15 54 0	12 24 1	1 0 1
27	9 3 5	15 59 4	12 4 4	1 29 8
28	9 23 4	16 4 0	11 43 9	1 59 4
29	9 43 0	16 7 9	11 22 7	2 28 9
30	10 2 3	16 11 0	11 0 8	2 58 1
31		16 13 3		3 27 1

2s. 13° distant from his apogee, the equation of time depending on this cause is 7m. 18″ 8, when in 9s. 29° the equation is 6m. 39″ 7, &c.

The 3d table is constructed from the two first, and contains the equation of time for leap years, but particularly calculated for the year 1812. From this however the equation of time may be nearly found for common years as follows: 1st. When the equation increases and the clock is faster than the sun, or the sign is add. take the difference between the equation of the given and preceding days: then add ¾ of this difference for the 1st year after leap year, ½ for the 2d. and ¼ for the 3d. from the 1st of January until the equation begins to decrease, but at other times of the year, in this case subtract ¼ of the difference for the 1st. ½ for the 2d. and ¾ for the 3d year after leap year.

2d. When the clock is faster or the equat. to be added, and the equation decreases; take the difference of the equations for the given and preceding days as before, then add ¼ this difference for the 1st. ½ for the 2d. and ¾ for the 3d. year after leap year.

3d. When the equation increases and the clock is slower, or the equat. to be subtracted; take ¾ of the difference, &c. found as above, and subtract it for the 1st year, ½ for the 2d. and ¾ for the 3d. after leap year. 4th. and last case, when the equation decreases, and the clock is slow (or the equat. to be subtracted) add ¼ of the difference, &c. for the 1st year, ½ for the 2d. and ¾ for the 3d. after leap year. Thus to find the equat. of time on the 14th of January, 1811, being the 3d. after leap year: from table 3d. above the diff. between the equat. on the 13th and 14th days, is 22″ 8, ¼ of which is 5″ 7, which in this case is to be added to 9′ 9″ 4, the sum is 9′ 15″ 1 the equation for the 14th of Jan. 1811, agreeing to the decimal with that given in the hemisphere turned towards it, while the other will remain in darkness. If the gobe be therefore elevated according to the sun's declination, it is evident that the sun will illuminate all that hemisphere which is above the horizon, that the wooden horizon itself will be the circle terminating light and darkness; and that all those places below it are wholly deprived of the solar light. The globe being fixed in this position, those arches of the parallels of latitude which are above the horizon, are the diurnal arches (def. 105) and shew the length of the day in all those latitudes, at that time of the year, corresponding to the declination for which the globe was rectified; the remaining parts of those parallels, which are below the horizon, are the nocturnal arches (def. 106) which shew the length of the night in those places at the same time. (The length of the diurnal arches may be found by reckoning how many hours are contained between any two meridians or any two places cutting the same parallel of latitude, in the eastern and western parts of the horizon. Or if these two places be brought to the brazen meridian respectively, and marking the two points then cut on the equator by the meridian, the number of degrees between

nautical almanac for 1811. Again to find the equat. for the 23d of June, 1811. Here the differ. is 12″ 8, $\frac{3}{4}$ of which is 9″ 6, hence 1′ 42″ 8—9″ 6=1′ 33″ 2 the eq. required; that given in the naut. alm. is 1′ 33″ 8. These methods are therefore sufficiently exact for almost any purpose. The other rules are applied in the same manner.

The rule given by Mc. Kay in his complete Navigator in the explanation of table 29th. holds only in a few particular cases, though given as general. It is therefore only calculated to lead into innumerable errors.

The equation of time being applied to the apparent time (or time shewn by a dial, &c.) according to its title in the table, will give mean time, or the time shewn by a watch or clock; but the contrary method is to be used to turn mean time into apparent. Thus, on the 20th of March the equation is 7′ 38″ 3 *additive* to the apparent time, which shews that the sun or dial is slower than the clock or watch, and therefore 12 o'clock by the dial is 12h. 7′ 38″ 3, by the watch; on the contrary, 12 o'clock by the watch is 11h. 52′ 21″ 7, by the sun or dial; and it is worthy of remark, that this is the instant when the sun's *meridian altitude* ought to be observed, in order to find the latitude.

The best tables of the equation of time will not hold for many years. For the sun's apogee has a progressive motion, the equinoctial points a regressive motion, the obliquity of the ecliptic continually varies, and even the sun's longitude at noon, at the same place, is different for the same days on different years, and as it is for apparent noon the equation is computed, it must therefore be computed anew every year, when great exactness is required.

The sun's apogee, according to Mayer, in the year 1716, was 8° of cancer, and in 1771, 9° of cancer, which gives its motion equal to 1° in 55 years, or nearly 1′ 5″ 45, every year. Delambre (tab. 1.) makes the sun's apogee, in 1800, 3s. 9° 29′ 3″, and in 1820, 3s, 9° 49′ 46″, the difference between which is 20′ 43″, the mot. of the apogee in 20 years, or 1′ 2″ 15, yearly. For more information on the equation of time, consult Delambre's tables, pa. 14, 15, 16, 17, and pa. 30, 31, 32, &c. notes; and the articles there referred to in La Lande's astronomy. It may be necessary to add, that the 1st table is calculated for the obliquity 23° 27′ 54″, and that the these two points, on the equator, converted into time (note to prob. 6) will give the length of the day, and the number of degrees between the same points reckoned on the other part of the equator, converted into time in like manner, will give the length of the night. The globe being again fixed in the same position as before, all those places that are in the western semicircle of the horizon will have the sun rising. (For the sun standing still in the zenith or vextex, or over that degree of the brazen meridian corresponding to its declination, appears easterly, and 90° distant from the zenith of all those places that are in the western semicircle of the horizon (definitions 42 and 50) and consequently is then rising in those places.) If we now take any particular place on the globe and bring it to the meridian, and then bring 12 marked on the hour circle to the meridian (the index must be set to the lower 12, if it be fixed on the outside of the brass meridian) the globe being then turned on its axis, until the aforesaid place comes to the western side of the horizon, the index will then shew the time of sun rising in that place. Turn the globe again from west to east, and the index will shew the progress made in the day, every hour, and in every place on the globe, by the real motion of the earth on its axis. When they come under the brass meridian they have their noon, and the sun has then its greatest altitude be-

correction annexed to this table is calculated for a second in the variation of this obliquity; and that the 2d table is calculated for the year 1800, and may be adapted to any other year, by diminishing 1″ 2 for every 100 years, according to Delambre. Nevil Maskelyne, in the nautical alm. for 1813, assumes the mean obliquity of the ecliptic for the beginning of that year 23° 27′ 51″ 3, and its mean secular diminution 42″ 6.

All the elements, &c. from which these tables are calculated, such as the sun's mean longitude, mean anomaly, obliquity of the ecliptic, true right ascension, &c. are easily found from *Mayer's* tables, or rather from *Delambre's*, translated by *Vince*. These tables, rendering these calculations, and that of the nautical almanac itself, a mere arithmetical or mechanical operation, which any schoolboy can become acquainted with in a few weeks. The learner must however notice, that, as the equation of time is computed for apparent noon, or when the sun is on the meridian, and as the time of apparent noon in mean solar time, for which we compute, can only be known by knowing the equation of time, it follows, that to compute the equation on any day, the equation must be assumed the same as it was *four* years before on that day, from which it will differ but very little. This will give the apparent time sufficiently exact for the purpose of computing the equation. Where great exactness is required, the operation may be repeated. Thus, if it be required to find the equation for the 10th of March, 1816; the equation for the 10th of March, 1812, being 10′ 29″ 7, to be added to apparent noon, to give the corresponding mean time; hence the computation must be made from the tables for March 10th, at 0h. 10′ 29″ 7. In the month of January and February, in leap year, *one* day must be taken from the given time before the computation is made. When the equation 4 years before is not given, the equation may be computed accurately enough for noon mean time, particularly if the operation be repeated.

That the learner may understand the reason of the above, he must observe, that as a meridian of the earth, when it leaves *m* (in the foregoing

ing equal to the number of degrees between the place and the horizon, reckoned the nearest way. The motion of the globe being still continued easterly, the sun will seem to decline westward, until, as the places successively come to the eastern part of the horizon, the sun appears to set in the western part.*

Example 1. To find the length of the day and night in all places on the earth on the 16th of April, and to shew how they are caused, &c.

fig.) returns to it in 24 hours, it may be considered, when it leaves that point as approaching a point 360° distant from it, at which it arrives in 24 hours The relative velocity, therefore, with which a meridian accedes to or recedes from *m*, is at the rate of 15° an hour, and consequently when the meridian passes through *v*, the arc *vm* reduced into time at the rate of 15° to an hour, will give the equation at that instant. The equation of time is therefore computed for the instant of *apparent* noon, or when the sun is on the meridian.

If the place be situated *east* or *west* from the meridian of Greenwich, allowance must be made for the difference of longitude in time.

The above note, &c. though rather long, must not, however, be unwelcome to students who desire accuracy on that subject, and wish to penetrate deeper than the *surface* of those branches of science which they make their study, in order to become useful to the community and to themselves.

* From the foregoing solution it will appear, that all those places upon the earth which differ in latitude, have their days of different lengths, except when the sun is in the equinoctial, being longer or shorter, in proportion to that part of the parallel that is above the horizon; if the entire of the parallel be above the horizon, it is evident that there is constant day, but if no part appear above the horizon, that there is constant night. Those places that are in the same latitude have their days of the same length, but commencing sooner or later, according as the places differ in longitude. It will likewise appear, that the arches of those parallels which are above the horizon in north latitude, are equal to those below the horizon in south latitude, and therefore when the inhabitants of north latitude have the longest day, those in south latitude have the longest night, and *vice versa*, the arches of the parallels in south latitude which are above the horizon being equal to those in north latitude which are below. In this problem, as in all others where the pole is elevated to the sun's declination, the sun is supposed to be fixed, and the earth to revolve on its axis from west to east. If a small brass ball fixed upon a strong wire, be contrived to screw on the brazen meridian like the quadrant of altitude, this ball placed over the sun's declination at a considerable distance from the globe, will represent the real sun, and assist the young student in more easily comprehending the problem. Such contrivances, however, to boys of genius, who at one glance can comprehend such problems, will appear childish and unnecessary, but the teacher, from experience, will find that there are few who does not stand in need of such. The learner will perceive that the 2d problem above is calculated to shew the positions of the earth with regard to the sun, that are most remarkable, such as the *equinoxes* and *solstices*, from which will be seen, at one view, when the days and nights are equal all over the world, and the comparative lengths of the longest and shortest. When the sun is in the equinoxes, the poles are placed in the horizon, the sun having then no declination, but when the sun is in either solstice, the pole is elevated 23° 28′, the north pole if it be the summer solstice, and the south pole if the winter solstice.

Ans. On the 16th of April the sun's declination is 10° north, the north pole being therefore elevated 10°, the sun will then illuminate all those places above the horizon, &c. being fixed over the meridian at 10° of declination. The globe being then fixed in this position, the arches of the respective parallels of latitude or the diurnal arches will be as follow: In the parallel of 20° N. there are $12\frac{1}{2}$ meridians on Bardin's, or $18\frac{4}{5}$ on Cary's globes, which answer to 188° on the equator, or to 12 hours 32 minutes. In the parallel of 40° N. there are $197\frac{1}{2}$°, which are equal to $19\frac{3}{4}$ meridians of Cary's, or $13\frac{1}{6}$ of Bardin's globes, or 13 hours and 10 minutes. In the parallel of 60° N. there are 211°, which correspond to $21\frac{1}{10}$ on Cary's, or $14\frac{1}{15}$ on Bardin's globes, or 14 hours 4 minutes; and so on, for any other latitude. It is plain, also, that if the above degrees be taken from 360°, the meridians on Cary's globes taken from 36, or those on Bardin's, or the hours, from 24, the remainders will give the degrees, meridians, or hours respectively, corresponding to the length of the night in each of the above respective latitudes. In the same manner the length of the day is found in any parallel of south latitude; thus in the parallel of 60° south, there are 145°=$14\frac{1}{2}$ mer. on Cary's, or $9\frac{2}{3}$ on Bardin's=9 h. 40 min. &c. If we now bring any particular place, as Washington, to the brazen meridian, then all those places in the eastern horizon will have the sun setting when it is noon at Washington, all those in the western will have the sun then rising, and all those places under the brass meridian will have noon; and the height of the sun in these respective places under the meridian, will be equal to their distance from the nearest horizon; thus, in latitude 10° N. the sun is vertical; in any latitude less than 10° N. the sun appears north; in 20°, 40°, and 60° north, the meridian altitude is 80°, 60°, and 40° respectively, the sun appearing due south; and in 60° south latitude, the meridian altitude is 20°, &c. The index being now set to 12, and Washington brought to the western part of the horizon, the hours past over by the index, or the hours pointed out by it taken from 12, will shew the time of sun rising there, namely, half past five nearly; the globe being then turned eastward, the index will shew the progress of day and night in each place. Thus, when the index has passed over two hours, to all those places that were at the western horizon with Washington, it will be then two hours after sun rise, and to those places that were at the eastern horizon, it will be two hours after sun set, &c.; when Washington comes to the eastern horizon, the sun will then set at half past six, nearly. The length of the night will be pointed out by the index, if the motion of the earth be continued eastward, until Washington again appears in the western part of the horizon, &c.

Note. If the meridian be drawn through every 15° of the equator, twice the number reckoned from the horizon to the brass meridian will give the length of the day, &c.

2. Required the length of the day and night, &c. as above, in all places on the globe, on the following days ; 10th of March, 21st do. 19th of May, 21st of June, 23d of July, 23d of September, and 21st of December ? (See the next prob.)

PROB. 24.

To explain in general the alteration of seasons, or length of the days and nights, in every part of the earth, caused by the earth's annual motion in the ecliptic.

Rule. RECTIFY the globe for every degree of the sun's declination from the equinoxes (or any other point of the ecliptic) until the sun returns to the same point again, the different portions of the parallels of latitude, which are above the horizon, corresponding to each degree of elevation, will give the length of the day in each respective latitude, as in the last problem.

Note. The last problem is only a particular case of this ; what is required here in general being there required only for one day, and the method of performing this, therefore, differs in nothing from that given in the last problem but in the different elevations of the pole. We shall here therefore more particularly solve the problem for the most remarkable positions, the *equinoxes* and *solstices*, as the method of performing the problem for any other point in the ecliptic, corresponding to any day in the year, or to any degree of the sun's declination, is the same as in the last problem, only observing, that when the declination is south, the south pole must be elevated.

For the Equinoxes. At this time the sun having no declination, the two poles of the globe must be placed in the horizon ; then the point aries on the equator being brought to the eastern part of the horizon, the point libra will be in the western point, and the sun will appear setting to the inhabitants of Greenwich, and to all the places under the same meridian (if the first meridian pass through Greenwich) from this position let the globe be gradually turned on its axis towards the east, the sun will then appear to move towards the west, and to be setting, as the different places successively enter the dark hemisphere ; the motion of the globe being continued until Greenwich comes to the western edge of the horizon, the moment it emerges above the horizon, the sun will then appear to be rising in the east. If the motion of the globe be continued eastward, the sun will appear to rise higher and higher, and to move towards the west ; when Greenwich comes to the brass meridian, the sun will appear at its greatest height, and after Greenwich has passed the meridian, the sun will continue its apparent motion westward, and gradually diminish in altitude, until Greenwich comes to the eastern part of the horizon, when the sun will again be setting. During the revolution of the earth on its axis, every place on its surface has been twelve hours in the dark, and twelve hours in the enlightened hemisphere, and therefore the days and nights are equal all over the world. For all the parallels of latitude are divided into two equal parts by the horizon, and in every degree of latitude there are 90° between the eastern part of the horizon and the brass meridian, or nine meridians on Cary's, which are equal to six on Bardin's globes, and correspond to six hours, which is half the length of the diurnal arch; and hence the length of the day in every latitude, when the sun is in the equinoxes, is twelve hours. The meridian altitude of each place, found as in the foregoing problem, will be exactly equal to the complement of their latitudes. Thus the meridian altitude of the sun at Greenwich will be 38° 31′ 21″, at Philadelphia 50° 3′ 6″, at Boston 47° 36′ 45″, at Washington city 51° 7′, (in each of the above places the sun will appear south, when on the meridian) at Quito 89° 46′ 43″, at the Cape of Good Hope (the town) 56° 4′ 45″. In both these places the sun will appear north when on the meridian. At the equator the sun's altitude is 90°, and is there consequently vertical. But at the poles the sun having no altitude, will therefore appear in the horizon, and as its altitude varies very little during the space of 24 hours, it will appear to glide the whole day along the edge of the horizon, until it comes to the same point again; but as its declination increases, it will rise gradually above the horizon, describing a kind of spiral in the heavens, until it reaches its greatest altitude 23° 28′, from which it returns in the same gradation until it appears again in the horizon at the next equinox. At the contrary pole it will descend below the horizon in the same manner, and at the equinoxes the sun will rise and set at six o'clock to all the inhabitants of the earth except at the poles.

As the sun now advances from aries towards the next tropic, or *summer solstice*, if we gradually elevate the north pole, according to the progressive alterations made in the sun's declination, by his motion in the ecliptic, we shall find the diurnal arches of all those parallels that are in the northern hemisphere, continually increase; and those in the southern hemisphere continually decrease, in the same proportion as the days increase and decrease in those respective places. We shall likewise see the entire of those parallels where constant day begins round the north pole, gradually elevating above the horizon, whilst those round the south pole, where constant night commences, are depressed in the same manner.— Let us for example observe the sun when his declination is 10° north, the same phenomena will take place as described in the solution of example the 1st. in the foregoing problem. Moreover, the globe remaining in this position, the meridian altitude of the sun in all those places whose latitude is north, will be equal to the complement of the latitude, or what it wants of 90°, added to the sun's declination, this being their distance from the nearest horizon; and the meridian altitude of the sun in all those places having south latitude, will be the complement of the latitude made less by the sun's declination. To those in 10° north latitude, the sun will appear vertical, and to the southward of those whose latitude is more than 10° north. But to those in south latitude, the sun will appear to the northward, and likewise to those within the parallel of 10° north. Thus the meridian altitude of the sun at Greenwich will now be 48° 31′ 21″. at Philadelphia 60° 3′ 6″, at Washington 61° 7′, at Quito 79° 46′ 43″, at the Cape Town 46° 4′ 45″. Hence it appears that as the sun's declination increases northward, the meridian altitude of the sun, to those in north latitude, increases, and to those in south latitude lessens in the same proportion. Thus when his declination is 20° north, the meridian altitude at Greenwich is 58° 31′ 21″, but at Quito is 69° 46′ 43″. Now as the sun's greatest declination cannot exceed 23° 28′, his greatest altitude at Greenwich cannot exceed 61° 59′ 21″, at Philadelphia 70° 31′ 6″, &c. On the contrary, the least meridian altitude at Quito cannot be less than 66° 18′ 43″, nor at the town of the Cape of Good Hope less than 32° 36′ 45″, &c. The contrary rule must be observed when the declination is south. From what has been here said, it appears how the sun's meridian altitude may be found at any place, on any given day, by having his altitude on some preceding day, and a correct table of the sun's declination. But to proceed : The globe remaining still in the same position, we shall find that the lower part of the 80th parallel of longitude just touches the horizon, and that therefore all the space between this and the pole is in the illuminated hemisphere, or has constant day, the beginning of constant day light being then at this parallel. From this parallel to the equator, and from thence to the 80th parallel of south latitude, the days gradually shorten ; the upper part of this parallel just touching the horizon, therefore total darkness commences there, and all the places between this and the south pole have constant night. It holds likewise universally, that whatever be the length of the day in north latitude, the night will be equally long in the same latitude south, *vice versa*, and that at the equator the days and nights are always equal. In the same manner we may reason with regard to any other degree of the sun's declination until he is advanced to the tropic of cancer.

For the summer solstice. The summer solstice, in north latitude, happens on the 21st of June. On this day the sun enters *cancer*, at which time his declination is greatest, being 23° 28′. The globe being elevated to this declination, bring cancer to the brass meridian, over which, in the point where cancer intersects it, let the sun be supposed to be fixed at a considerable distance from the globe, whilst the globe remains in this position, the equinoctial point aries will appear in the western part of the horizon, and the opposite point libra in the eastern . hence, the equator being divided into two equal parts, the one half in the illuminated and the other half in the darkened hemisphere, it will therefore appear that the day and night at the equator is of the same length, that is, 12 hours long each. From the equator to the arctic circle, the diurnal arches will exceed the nocturnal, or the days will be longer than the nights. All the parallels of latitude within this north polar circle, will be above the horizon, and therefore all the inhabitants within it will have no night. From the equator to the antarctic or south polar circle, the nocturnal arches will exceed the diurnal, or the night will be longer than the day. All the parallels of lat. within the south polar circle will now be below the horizon, and the inhabitants, if any, will have twilight or dark night. If instead of cancer, aries or the first meridian be brought to the brass meridian, the meridians passing through the respective places at the horizon, will (on the equator on which the hours are marked) point out half the length of the day. If no meridian on the globe passes through any given place, its meridian may be found by bringing the place to the brass meridian. The globe remaining in this position, the sun will have its greatest or least meridian altitudes according as the places are situated N or S. of the tropic of cancer in the enlightened hemisphere ; this altitude may be found as before. From this position, if the north pole be now continually depressed as the sun's declination lessens, until both poles are again in the horizon, the days and nights decrease or lengthen until the sun arrives at the equinoctial again, in the same gradations as before, from aries to capricorn, &c.

From the autumnal equinox to the *winter solstice* (which to the inhabitants of north latitude happens on the 22d of December, at which time the sun enters capricorn) the same alteration of seasons, of day and night, &c. will take place, and in the same gradation, to all the inhabitants of the southern hemisphere, as was observed before to have taken place, while the sun performed his apparent motion in the ecliptic, from aries to cancer. By now elevating the south pole in the same manner for the sun's declination as before the north pole was elevated, the same phenomena will appear in succession, until the sun advances to capricorn. Here as at the summer solstice, the days at the equator will be twelve hours long, but the equinoctial point aries will now be in the eastern part of the horizon, and libra in the western. From the equator to the south pole, the seasons, &c. will be as before in the summer solstice, in the northern hemisphere (exclusive of the variation made by the earth's distance from the sun being now at her nearest) the greatest meridian altitudes of the sun will now be in south latitude, and the least in north ; the reverse of what they were when the sun was in cancer. Thus at Greenwich the sun's greatest altitude will be 15° 2′ 21″, instead of 61° 59′ 21″, &c. Hence it appears, that the difference between the sun's greatest and least meridian altitudes at any place in the temperate zone, is equal to the breadth of the torrid zone, viz. 46° 56′. The difference between the sun's greatest and least altitude at the poles, is equal to the sun's greatest declination ; his altitude or depression at either pole being always equal to the declination. Moreover the sun's altitude at the pole never varies, as in other latitudes, for at the poles the sun while visible is always on the meridian, &c. &c.

PROB. 25.

*To place the globe in the same situation with respect to the poles of the equinoctial, as our earth is to any of its inhabitants, so as to shew at one view the length of the days and nights in any particular place, at all times of the year.**

Rule. Rectify the globe according to the latitude of the place; then those parts of the parallels of declination which are above the horizon, are the diurnal arches, and those parts which are below the horizon are the nocturnal arches. Hence the length of the days and nights at any time of the year may be determined, as in the preceding problems, by finding the number of hours contained between the two extreme meridians, which cut any parallel of declination, in the eastern and western parts of the horizon.

We shall exemplify this rule, by placing the globe in its more remarkable positions, such as the *right*, *oblique* and *parallel*. (See definitions 87, 88 and 89.)

For the right sphere. Here the given place must be on the equator, and the globe rectified for 0° of latitude, or both poles of the globe must be placed in the horizon, then the north pole on the globe will correspond to the north pole of the heavens, and all the heavenly bodies will appear to revolve round the earth from east to west, in circles parallel to the equinoctial, according to their different declinations.† When the sun is in the equinoctial, he will

* In this problem and in all others, where the pole is elevated to the latitude of a given place, the earth is supposed to be fixed, and the sun to move round it from east to west. When the given place is brought to the brass meridian, the wooden horizon is the true rational horizon of that place, but it does not separate the enlightened part from the dark, as in the two preceding problems; however, there is nothing unnatural in elevating the pole to the latitude of the place on the earth. For, as Keith remarks, in the note to prob. 22d of his treatise on the globes, this is placing the globe in its true situation respecting the heavens and the fixed stars. The pupil who wishes to make himself master of the globes, must endeavour to comprehend why he sometimes elevates the pole to the latitude of the place, and at other times to the sun's declination. A little perseverance and diligence will soon remove every difficulty, and he should be well convinced, that, notwithstanding the exertions of the most eminent masters, without close application and attention on his side, nothing but a superficial knowledge of any subject can be obtained.

† The ecliptic being drawn on the terrestrial globe, young students are often led to imagine that the daily apparent motion of the sun round the earth is performed in the same oblique manner. To correct this false principle, we must suppose the ecliptic to be transferred to the heavens, where it properly points out the sun's apparent annual path among the fixed stars. As the sun, in receding from or advancing towards the equinoctial every day, alters, a little, his declination; if we therefore suppose all the torrid zone to be filled up with a spiral line or thread, having as many turns, or a screw having as many threads as the sun is days in going from one tropic to another, and these threads at the same distance from one another on the globe as the sun alters his declination, in one day, in all those places over which it passes; this spiral line or screw will represent the apparent paths described by the sun round the earth every day, in passing from one tropic to another. Thus,

be vertical to all the inhabitants situated upon the equator, and his apparent diurnal path will be over that line: when the sun has any declination, as 10° N. for example, his apparent diurnal path will be from east to west, nearly along that parallel. When he comes to the tropic of cancer, his diurnal path in the heavens will be along that line, and he will be vertical to all the inhabitants on the earth in latitude 23° 28′ north. The inhabitants on the equator will always have 12 hours day and 12 hours night, notwithstanding the variation of the sun's declination, from north to south, or from south to north, because the parallel of latitude, which the sun apparently describes for any day, will always be cut into two equal parts by the horizon; all the stars will here be 12 hours above the horizon from rising, and 12 hours below it from their setting; and in the course of a year an inhabitant on the equator may see all the stars in the heavens. Those which are at or very near the pole, will always nearly remain in the same point of the heavens, and the circles which the stars describe in their apparent diurnal revolutions, will be greater in proportion as they are distant from the poles, or approach nearer the equator. The sun's greatest meridian altitude at the equator will be 90°, and the least 66° 32′, the altitude of any celestial body being here always equal to the complement of its declination, or its distance from the equinoctial. Hence the stars that are situated in the equinoctial, will be always vertical to the inhabitants of the equator, and the meridian altitude of any others will be always equal to the complement of the parallel of latitude where they are vertical, or to the complement of their declination. Those at the pole, in a right sphere, will have no altitude, for the same reason, and will therefore appear in the horizon. (Here we do not consider the effects of refraction, parallax, &c.) During one half of the year an inhabitant of the equator will see the sun due north at noon, and during the other half it will be due south when on the meridian.

*For the oblique sphere.** If from the right position we gradually elevate the pole (the north for example) according to the dif-

if the thread be fastened at the point capricorn, and wound round the globe towards the right hand or the equator, by turning the globe from east to west, until we arrive at cancer, it will point out the paths described by the sun daily, from the winter to the summer solstice. But if the thread be wound towards the left hand from cancer, until we come to capricorn again, it will describe the sun's path from the summer to the winter solstice, or the remaining half year. But, as the inclinations of those threads to one another are but small, especially near the tropics, we may suppose each diurnal path to be one of the parallels of latitude drawn, or supposed to be drawn upon the globe, as above.

* Every inhabitant of the earth, except those who live upon the equator and at the poles, has an oblique sphere, and hence the globe must be rectified for every latitude accordingly. But by elevating and depressing the poles, for every latitude according to the situation of the places which are given, the student may imagine that the earth's axis moves northward or southward just as the pole is elevated or depressed. This is however a mistake, as the earth's axis has no motion, except a kind of libratory motion,

ferent latitudes from the equator towards the elevated poles, the lengths of the diurnal arches will continually increase, until we come to a parallel of latitude as far distant from the equator as the place itself is from the pole. This parallel will just touch the horizon, and all the celestial bodies that are between the parallel of declination corresponding to this in the heavens and the pole of the equinoctial, never descend below the horizon. While we thus gradually elevate the globe to the different latitudes, the diurnal arches in the southern hemisphere continually diminish in the same proportion, that those in the northern hemisphere increased, until we come to that parallel which is so far distant from the equinoctial southerly, as the place itself is from the north pole. The upper part of this parallel in the heavens, just touches the horizon, and all the stars that are between it and the south pole, never appear above the horizon. Here all the nocturnal arches of the southern parallels are exactly of the same length as the diurnal arches of the corresponding parallels north ; and hence every place on the surface of the earth equally enjoys the benefit of the sun, in respect of time, the days at one time of the year being exactly equal to the nights at the opposite season.

Thus, the latitude of Washington city being 38° 53′ N. if Washington be brought to the meridian, and the north pole elevated 38° 53′ above the horizon, then the wooden horizon will be

which is called its *nutation,* and which has no relation to that conceived above, nor can it be represented by elevating or depressing the pole. During the earth's annual revolution round the sun, the axis always remains parallel to itself, and the poles always point to the same star or point in the heavens, the whole semidiameter of the earth's orbit causing no sensible change or deviation in the earth's axis. Dr. Bradley having found from a series of accurate observations on the star γ draconis, that its annual parallax or the angle under which the semidiameter of the earth's orbit would appear as seen from γ draconis, did not amount to a single second. The precession of the equinoxes, the aberration of light, &c. causes also some small change in the earth's axis, &c.; but nothing compared to that motion conceived in elevating the poles, &c. In going from the equator towards either pole, our horizon varies ; thus when we are on the equator, both poles are in the horizon, the northern point of the horizon representing the north pole, being opposite the north pole in the heavens, &c. If we advance 10°, for example, northward, the north point of our horizon is 10° below the pole, &c. Now the wooden horizon on the terrestrial globe is immoveable, otherwise it ought to be elevated or depressed, and not the poles ; but whether the pole be elevated or the horizon depressed, the appearance will be exactly the same. Though the wooden horizon be the true horizon of the place for which the pole is elevated, it does not however separate the enlightened hemisphere from the dark. For instance, when the sun is in aries, and Washington at the meridian, all the places on the globe above the horizon beyond those meridians which pass through the east and west points thereof, reckoning towards the north, are in darkness, although they are above the horizon, and all places below the horizon between the same meridians and the southern point of the horizon, have day light, notwithstanding they are below the horizon of Washington. Thus the meridians passing through 14° E. and 166° W. longitude from Greenwich, will be the boundaries of light and darkness.

the true horizon of Washington: and if the artificial globe be placed north and south, by a mariner's compass or a meridian line, it will have exactly the same position, with respect to its axis, as the real globe has in the heavens Now if we imagine lines to be drawn through every degree of the sun's place parallel to the equator, or rather through every 59′ 8″ 3, the sun's apparent daily mean motion, these lines will give the sun's diurnal path on any given day.* By comparing these diurnal paths with each other, they will be found to increase in length from the equator northward, and to decrease from the equator southward, therefore when the sun is north of the equator, the days are increasing in length, and when south of the equator, decreasing. When the sun is in the tropic of cancer, the day is nearly 14 h. 40 min. at the equator 12, and when the sun is at the tropic of capricorn, the day has decreased to 9 hours 20 min. nearly. The meridian altitude of the sun, for any day, may be found by reckoning the number of degrees from the parallel in which the sun is on that day, towards the horizon, on the brass meridian. Thus when the sun is in that parallel which is 10° north of the equator, his meridian altitude at Washington will be 61° 7′, and is equal to the complement of the latitude added to the sun's declination. If the declination be south, it must be subtracted from the complement of the latitude to find the sun's meridian altitude. The lower part of that parallel of declination which is 51° 7′ from the equinoctial northerly, just touches the horizon; and all the stars between this parallel and the north pole, never set at Washington. In like manner the upper part of the southern parallel of 51° 7′, just touches the horizon, and all the stars that lie between this parallel and the south pole are never visible in this latitude. If we now rectify the globe for the latitude 66° 32′ north, we shall find, that when the sun is in cancer, he just touches the horizon on that day, without setting to the inhabitants of the arctic circle, remaining 24 hours complete above the horizon; and when he is in capricorn, his centre would just appear in the horizon were it not elevated on account of refraction, about an entire diameter above the horizon, and would not rise again for the space of 24 hours. When the sun is in any other point of the ecliptic, the days are longer or shorter according to his distance from the tropics. All the stars that lie between the tropic of cancer and the north pole, never set in this latitude; and those between the tropic of cancer and the south pole never rise. If we elevate the globe still higher, the circle of *perpetual apparition* (see def. 107) will be nearer the equator on one side, as will that of *perpetual oculation* (see def. 108) on the other. If for example the globe be rectified for the lat. 80° N. the sun's declination being 10° N. he will then begin to revolve above the hori-

* On Adams' globes such lines are drawn through every degree of the meridian within the torrid zone, parallel to the equator; these will nearly represent the sun's diurnal path on any given day.

zon without setting, in an oblique direction, just touching it in the north point, and at the south being elevated 20° above it; during his progress from this point in the ecliptic to the tropic of ♋, and his return again to the same degree of declination, he never sets. In like manner when his declination is 10° S. he is just seen at noon in the south point of the horizon, and during his progress from this point to the tropic of capricorn, and his return again to 10° S. decl. he remains below the horizon; the time of his being invisible or below the horizon, being as long as the time he appeared visible at the opposite season of the year.

For the parallel sphere. The north pole being now elevated 90° above the horizon or to the zenith, then the equator or equinoctial will coincide with the horizon, and the parallels of latitude will consequently be parallel to it; those in the northern hemisphere being above, and those in the southern hemisphere below it. When the sun enters aries, that is on the 21st of March, he will be seen by the inhabitants of the north pole (if there be any) to glide along the edge of the horizon, and as his declination increases, he will increase in altitude until he comes to the tropic of ♋, forming a kind of spiral as before described, from this tropic until his return to the autumnal equinox on the 23d of September, his altitude will again also gradually decrease in proportion as his declination decreases. From the vernal to the autumnal equinox, or during the summer half year, the sun will therefore appear above the horizon, and make constant day; and consequently the stars and planets will be invisible during that time. The sun's altitude at any time, or at any hour of the day, will be always equal to his declination, and his greatest altitude cannot exceed 23° 28′, at which time he will have arrived to the tropic of cancer. When the sun just enters the sign libra, he will again appear to glide along the edge of the horizon, after which he will entirely disappear until his arrival again at aries or the vernal equinox; hence during six months, from the autumnal to the vernal equinox, there will be constant night at the north pole. But though the inhabitants at this pole will lose sight of the sun at the autumnal equinox (or a short time after, on account of refraction, &c. which is very great near the poles) yet the twilight will continue for nearly two months, for the sun will not be 18° below the horizon until he enters the 20th of scorpio (as may be seen by observing on the globe the sun's place corresponding to 18° on the brass meridian below the horizon, as the quadrant of altitude cannot be conveniently screwed over the pole) so that dark night will only continue from the 12th of November to the 29th of January, during which time the sun will be more than 18° below the horizon; and even then the light of the moon and of the aurora borealis, increased by the reflection from the snow, supplies, in a great measure, the absence of the sun in this inclement region. The inhabitants at the north pole can, at any time of the year, only see those stars that are situated in the northern hemisphere, and the greater part of these will be invisible except from about the 12th of November until the 29th of January, during which time the sun will be 18° or more below the horizon. The planets when they are in any of the northern signs, will also be visible, and together with the stars will appear to have a diurnal revolution round the earth from east to west, as the sun appeared to have when above the horizon.

The moon is likewise above the horizon during fourteen revolutions of the earth on its axis or half a lunation, and at every full moon which happens from the autumnal to the vernal equinox, the moon is in some of the northern signs, and therefore visible at the north pole; for the moon being in that sign which is diametrically opposite to the sun, at the time of full moon, and the sun while in the southern signs being below the horizon, the moon must therefore be above the horizon at this time, while in any of the northern signs. When the sun is at his greatest depression below the horizon, which happens when he is in capricorn, the moon is then *full* at *cancer*. The new moon being in capricorn, her *first quarter* will be in *aries*, and the *third* in *libra*. Now the beginning of aries being the rising point, cancer the highest, and libra the setting point, the moon rises at her *first quarter* in aries, has her greatest height when *full* in cancer, and sets in her *last quarter* in libra, being visible for fourteen days, or during her passage from aries to libra. Thus the north pole is supplied one half of the winter time with constant moon light in the sun's absence, and the inhabitants there are only deprived of her light from her 3d to her 1st quarter, while she gives but little light, and can therefore be but of little or no service to them.

Many other useful observations might be added here, were not these three last problems already rather long, and therefore discouraging to beginners. But their utility in giving a general idea of the seasons, &c. in every part of the world, deserve their particular attention, as they will here learn, in the most easy and entertaining manner, how these happen from the regular motion of the earth and its various positions. The following observations, deduced as collaries from the preceding problems, may be no less worthy the readers perusal.

1. When the north pole was in the zenith, the equator just touched the horizon, and as the pole was depressed, the equator was raised the same number of degrees above the horizon, whence it follows that the elevation of the equator above the horizon is always equal to the complement of the latitude or what it wants of 90°.

2. The sensible horizon of a place changes as often as we change the place itself.

3. Every place on the earth, in respect to time, equally enjoys the benefit of the sun's light, and is equally deprived of it: that is, the whole time that the sun is above the horizon of any place, is equal to all the time taken together, that he is above the horizon of any other, being about 6 months annually. The same may be

said of the time that he is below the horizon, and the days at one time is equal to the nights at the opposite season in any place. Here the effect of refraction, twilight, aurora borealis, &c. is not considered, nor the difference of time in which the sun is passing through the northern and southern signs, being *seven days* longer in the former than in the latter. These latter causes being considered, the inhabitants at or near the north pole, have in consequence more light in the course of a year than any other inhabitants on the earth. But what it gains in duration, it loses in the intensity of the sun's *light* or *heat*, from the same causes and the obliquity of the sun's rays, and the quantity and the density of the atmosphere through which they have to pass. (See Simpson's Fluxions, vol. 2. prob. 32 and 33.) For the article aurora borealis, &c. see Ree's Cyclopedia, a new edition of which is now printed in New-York, or the Encyclopedia. Simpson makes the proportion of the heat received at the equator, to that received at the pole during one year, as 17 to 7, nearly.

4. In all places of the earth, except under the poles, the days and nights are each 12 hours long at the equinoxes, that is on the 21st of March and 23d of September, at which time the sun has no declination.

5. In all places situated on the equator, the days and nights are always equal, viz. 12 hours each.

6. In all places between the equator and the poles, the days and nights are never equal, but when the sun enters the equinoctial points ♈ and ♎.

7. In all places lying under the same parallel of latitude, the days and nights at any particular time of the year are always equal; that is, the days in one place are equal to the days in the other, at the same time, &c.

8. The nearer any place is to the equator, the less is the difference between the days and nights, and the more remote the greater. The increase of the longest days does not however bear any regular proportion to the increase of the latitude. For if the longest days increase equally, that is half an hour, an hour, &c. the latitudes increase unequally, as is evident from consulting a table of climates. (See the table in the note to def. 90.)

9. The twilight is shortest at the equator, and increases from there to the poles, where it continues the longest.

10. To all places situated within the torrid zone, the sun is vertical twice a year, to those under each tropic once, but to those in the temperate and frigid zones it is never vertical.

11. In all places between the equator and polar circles, the sun rises and sets alternately every twenty-four hours.

12. At all places between the polar circles and the poles, the sun appears a certain number of natural days without setting, and at the opposite season of the year disappears for nearly the same length of time; and the nearer the place is to the pole, the longer the sun continues without setting, and the contrary.

13. Between the end of the longest day and beginning of the longest night, in the frigid zones, and between the end of the longest night and beginning of the longest day, the sun rises and sets alternately every 24 hours, as at other places on the earth.

14. At all places situated exactly at the polar circles, the sun when he is in the nearest tropic, appears 24 hours without setting, but when in the opposite tropic, he does not rise for the same length of time ; but at all other times of the year rises and sets as in cther places.

15. In all places situated in the northern hemisphere, the longest day and shortest night take place when the sun is in the northern tropic ; and the shortest day and longest night when the sun is in the southern tropic. The contrary must be observed with respect to those situated in the southern hemisphere.

16. All places situated under the same meridian as far as the globe is enlightened, have noon or any other hour at the same time, and those situated on the same parallel of latitude have the same seasons and are in the same climate.

17. At the north pole none of the stars ever rise or set, but move round it in circles parallel to the horizon, and have therefore always the same altitude. (The small yearly variation of about 50″, owing to the procession of the equinoxes, is not here taken notice of.)

18. At all places on the earth, except the poles, all the points of the compass may be distinguished in their horizon : but from the north pole every place is south, and from the south pole every place north. Consequently there is no distinction of noon or meridian at the poles, or rather the sun is constantly on the meridian during six months in each. And although the winds in any other place may blow from any point of the compass, at the poles they can only blow from one ; that is, at the north pole from the south, and at the south pole from the north.

19. When the sun's declination is greater than the latitude of any place, the sun will come twice to the same azimuth or point of the compass in the forenoon, and twice to a like azimuth in the afternoon, at that place : that is, the sun will go back twice every day while his declination continues to be greater than the latitude, which can only happen between the tropics, or in the torrid zone. Thus suppose the globe rectified for the lat. of Port Royal, in Jamaica, which is 18° north, and the sun in any point of the ecliptic between 21° of taurus and 9° of leo, suppose the beginning of gemini, and the quadrant be set to any degree between 12° and 21° from the east northward on the horizon, at 18° for example, the globe being then turned westward on its axis, the sun will rise in the horizon about 3½° north of the quadrant, and thence ascending will cross it towards the south at an elevation of about 11°, and thence advancing until his azimuth be about 80° nearly, from the north, from which azimuth circle it will return again towards the north, until, at an elevation of about 82°, and consequently before

it comes to the meridian, it will again cross the quadrant, and pass over the meridian about 2° north of Port Royal. In like manner if the quadrant be set about 18° north of the west, the sun will pass over the edge of it twice as it descends from the meridian towards the horizon, in the afternoon.

20. At all places situated on the equator, the shadow at noon, of any object placed perpendicular to the horizon, falls towards the north for one half of the year, and towards the south the other half. The nearer any place is to the torrid zone, the shorter the meridian shadows of objects will be. When the sun is perpendicular, there is no shadow, and when his altitude is 45°, the shadow of any perpendicular object is equal to its height (Euclid, 6 prob. 1 B.) as the sun inclines towards the horizon, the shadows lengthen, &c.

21. At the equator the sun always rises in the east, and sets in the west points of the horizon; but the more distant any place (situated in the temperate or torrid zones) is from the equator, the greater will be the rising and setting amplitude of the sun, or his distance from the east or west points of the compass. At the poles the sun always performs his revolutions round the horizon as before remarked.*

* The utility of such general observations as the foregoing, will be readily perceived by those who retain a relish for the study of history, geography, &c. and take a pleasure in contemplating the wisdom of the Creator in all the phenomena which nature exhibits. They might be rendered much more extensive and interesting, but these few remarks will enable the ingenious student to pursue them at his leisure. I shall however make here one more remark, which will open an extensive field for speculations of this nature, to those who have a taste or inclination for them. That in all the primary planets similar phenomena take place, as they have almost all been found to revolve on their axis, and to have an atmosphere the same as the earth; but the axis of Mars and Jupiter are not inclined to the planes of their orbits, and hence in these the seasons will be always the same, that is, a constant spring. In all the others the seasons, &c. will vary as on the earth. The terrestrial globe with a few additional circles, or even a moveable ecliptic, might be contrived so as to exhibit the greater part of their phenomena; but the instrument best calculated for this purpose is an *orrery*, the use of which is so generally known, that it is unnecessary for me, in a treatise calculated for the globes alone, to enter into any description of it. (See its description in the Philadelphia edition of the Encyclopedia, article Orrery or Astronomy, in Low's Encyclopedia, printed in New-York, or Ree's Cyclopedia; also in Ferguson's Astronomy or Fuller's Treatise on the Globes.) The famous Rittenhouse, of Philadelphia, has considerably improved this useful instrument. Those made by Messrs. Wm. & S. Jones, London, being on a small scale, are recommended for cheapness and utility. Besides the appearances of the superior planets, the stationary and retrograde appearances of the inferior planets are neatly illustrated by them. A learner who is but slightly acquainted with the elementary principles of mathematics, will, however, have little or no use for such instruments, calculated only to help the conceptions of beginners, as at one glance he can conceive infinitely more than such machines can represent, and calculate the phenomena which they exhibit to a degree of exactness at which he can with no instrument ever arrive. Such readers are referred to the 2d vol. of Dr. Gregory's astronomy, where the elements of comparative astronomy are given by this

PROB. 26.

The day of the month being given, to find when the morning and evening twilight begins, its duration and end, at any place on the globe.*

Rule. RECTIFY the globe for the latitude, zenith, and sun's place (prob 9.) and screw the quadrant of altitude upon the brass meridian over the given degree of latitude, and set the hour index

able master. It is from principles alone, and not from any machinery, that a learner can obtain a complete or general knowledge of any branch of science.

* This phenomenon is caused by the reflection of the sun's rays which fall on the higher parts of the atmosphere after sun setting or before he rises. If there were no atmosphere, the sun would shine immediately before his setting as bright as at noon, but the moment after his setting, we should have as great darkness as at midnight. This is one of the innumerable instances in which the wisdom of the Creator appears in providing for the conveniences of man, in this element alone. The height at which the atmosphere is supposed capable of reflecting the sun's light, so as to render it visible to us, is, at a medium, about 49 or 50 miles. Now, if a straight line drawn from an object, situated at this height, to the sun, just touches the surface of the earth, the sun at that instant will be 18° below the horizon (see the demonstration in Keil's astronomy, lect 20, or in Keith's treatise on the globes, note, pa. 107) which is the limit of the sun's depression below the horizon to have any of its light reflected to us. This particle of the sun's light in passing from that part of the atmosphere where it is first reflected, will be continually bent from the right line in which it would otherwise proceed, were the atmosphere equally dense, as is fully demonstrated by the writers on optics, and this property, called the refraction of light, increases the twilight, as the property of refraction is always to elevate the object from which the light is reflected; and whether it be the light of the sun, moon, or stars, whether it be native or reflected, intense or weak, &c. the refraction is the same, provided the medium through which it passes remain the same. But as the atmosphere continually varies, particularly towards the poles, the limits given above will also vary in the same proportion. And the variation, even during one day, and in the same place, is so sensible, that the evening twilight is found to continue longer than the morning twilight, owing to the expansion of the atmosphere during the day, and consequently to its greater height. The more oblique the sun's rays, the greater the refraction or reflection. When the rays fall perpendicular, then there is no refraction, because the rays, if reflected at all, are reflected back in the same direction. All these properties are accounted for from mechanical principles, and may be easily applied to the motion of light in the atmosphere with the assistance of a thermometer and barometer, the law of its expansion being given. Dr. Coles has demonstrated that if the altitudes of the air be taken in arithmetical proportion, its rarity will be in geometrical proportion. But to enter into any investigation of these principles, would far exceed our prescribed limits in this introduction (see Mayer's tables, prob. 13, and Scholia.) we shall however give an example which shews how much the refraction is affected by the density of the atmosphere. In the year 1682, the Dutch navigators who wintered in Nova-zembla, in lat. about 75° north, saw the sun 17 days before he could have been seen were there no atmosphere, or were it not endowed with this refractive power. It is owing to this property in the atmosphere, of reflecting the sun's light, that the sky is always illuminated while the sun

to twelve, then turn the globe westward until the sun's place comes to the western edge of the horizon, the hours passed over by the index will give the time of sun setting or the beginning of the evening twilight ; continue the motion of the globe westward until the sun's place coincides with 18° on the quadrant of altitude below the horizon, or the opposite point be 18° above the horizon in the eastern part of it, the time passed over by the hour circle after sun setting, will be the duration of evening twilight, and the index will point out the time of its ending. In like manner, if the sun's place be brought to the eastern horizon, the beginning, duration and ending of morning twilight may be found, its beginning being when the sun is 18° below the horizon, and ending the same as sun rising.

Note. When the sun's place does not extend 18° below the horizon, or the opposite point in the ecliptic, 18° above it, the twilight will continue the whole night.

OR THUS,

Find the sun's declination for the given day (prob. 8) and elevate the north or south pole, according as the declination is north or south, to this declination ; screw the quadrant of altitude in the

shines, for without this property, or were there no atmosphere, the whole heavens, except that part in which the sun appeared, would be as dark as at midnight, and the smallest stars, as in a clear night, would be visible ; nor would our artificial lights be of any service to us during the absence of the sun. M. De Saussure when on the top of Mount Blanc, which is elevated 5101 yards above the level of the sea, and where the atmosphere must therefore be more rare than on the surface of the earth, says, that the moon shone with the brightest splendour in the midst of a sky as *black* as ebony. (Append. vol. 74, Monthly Review.) The sun's atmosphere likewise shines after the sun is set, and increases the light reflected by our atmosphere. In the northern regions, the sun, when visible, rises and sets with a large cone of yellowish light, the stars appear of a fiery redness, owing to the density of the atmosphere, and the aurora borealis spreads a thousand different lights and colours over the whole firmament. Taking 18° at a medium for the limits, beyond which the sun being depressed below the horizon, there is no twilight, the prob. may be solved by spherical trigonometry, thus : The comp. of the lat. the compl. of the sun's declination, and the arch formed by the quadrant of alt. between the sun's place, and the zenith (being always equal $90°+18°=108$) form a triangle, the three sides of which are given to find the angle included by the meridian passing through the zenith, and the meridian passing through the sun's place depressed below the horizon as above, which converted into time, will give the end of evening twilight, reckoning from 12 o'clock. The time of sun setting (found by prob. 13 or the annexed note) taken from the above, will give the duration of evening twilight, the rest is found in the same manner. See the proportions for calculating this and similar probs. investigated in lect. 20th of Keil's astronomy, or more correctly in article 2206 and 2241 of La Land's astronomy, 3d edit. See also Gregory's astronomy, b. 2. probs. 39 and 41 and Scholium. The reader is also referred to P. S. Laplace's astronomy, b. 1, ch. 14, vol. 1, where he will find many important and interesting observations on this subject. This useful work, together with the same author's *Celestial Mechanics*, are translated into English, by J. Pond, F. R. S. now Astronomer Royal, and published in London, in 1809.

zenith, bring the given place to the brass meridian, and set the index to 12; turn the globe eastward until the given place comes to the horizon, and the hours passed over by the index will shew the time of sun setting, or the beginning of evening twilight; continue the motion of the globe eastward until the given place coincides with 18° on the quadrant of altitude below the horizon, or until the opposite point of the ecliptic be 18° above the western part of the horizon, the time passed over by the index, from sun setting, will be the duration of evening twilight, &c. The morning twilight is nearly of the same length, and found in the same manner.

OR BY THE ANALEMMA.

Elevate the pole to the latitude as before, and screw the quadrant of altitude in the zenith; bring the middle of the analemma (corresponding to the 16th of June, &c. on Cary's globes) to the brass meridian, and set the index to 12; turn the globe westward until the given day of the month on the analemma comes to the western part of the horizon, and the index will shew the beginning of the evening twilight; continue the motion of the globe westward until the day of the month coincides with 18° on the quadrant, as before, and the index will point out when twilight ends, the time between the beginning and ending of which is the duration. To find the morning twilight, bring the day of the month to the eastern part of the horizon, and proceed as before.

Example 1. Required the beginning, end, and duration of morning and evening twilight at Washington city, on the 21st of March?

Ans. The sun sets at 6 o'clock, and rises at 6. The evening twilight ends at half past seven, and the morning at half past four, its duration being therefore one hour and a half.

2. What is the duration of twilight at London on the 19th of April; what time does dark night begin, and what time does day break in the morning?

Ans. The sun sets at 2 minutes past 7, and rises 58 min. after four; the duration of twilight is 2 hours 17 min. and hence evening twilight ends at 19 min. past nine, and morning twilight begins, or day breaks, at 41 min. past two.

3. Required the beginning, end, and duration of morning and evening twilight at Philadelphia, on the 1st of August?

4. Required the beginning, end, and duration of morning and evening twilight at Buenos Ayres on the 10th of March?

5. Required the same as above, in the following places, on the 1st of January, New-York, Lima, Cape of Good Hope, and Canton?

6. Required the beginning and end of morning and evening twilight at the north pole on the 1st of March, and likewise on the 2d of February?

PROB. 27.

*To find the beginning, end, and duration of constant day or twilight at any place.**

Rule. If the complement of the latitude be greater than 18°, subtract 18° from it, and the remainder will be the sun's declination (north if the place be in the northern hemisphere, &c.) when total darkness ceases. But if the complement of the latitude be less than 18°, their difference will be the sun's declination, of a contrary name with the latitude, when the twilight begins to continue all night. Observe what two points on the ecliptic correspond to this declination, the day of the month corresponding to that point in which the sun's declination is increasing, will be that on which constant twilight commences, and the day corresponding to that point in which the sun's declination is decreasing, will be the last day or end of constant twilight.

Note 1. When the sun has 18° south declination, constant twilight commences, &c. at the north pole, as is plain from the above, the days corresponding to which are found as in the rule.

Note 2. If after subtracting 18°, the remainder be greater than 23° 28′, the sun's greatest declination, there can be no constant twilight at that place, as is evident. Hence between the latitude 48° 32′ and the equator, there can be no constant twilight.

Examples. 1. When do the inhabitants of London begin to have constant day or twilight, and how long does it continue ?

Ans. The latitude of London being 51° 31′ N. hence 90°—51° 31′—18°=20° 29′ the sun's declination, the two days corresponding to which (found by note 3, prob. 8.) are the 23d of May and 20th of July. So that on the 23d of May constant twilight begins, and on the 20th of July it ends ; hence its duration is nearly two months.

2. What is the duration of twilight at the north pole, and also the duration of dark night there ?

Ans. The two points corresponding to the sun's declination 18° south (see note 1.) are the 12th of November and 29th of January, between which days the sun is 18° below the horizon ; hence the duration of total darkness is 78 days ; the twilight continues from the 23d of September (the time of the autumnal equinox when the sun first disappears) to the 12th of November, the beginning of total darkness, being 50 days ; and from the 29th of January (the last day on which total darkness ceases) to the 21st of March (the vernal equinox, when the sun again begins to appear and the longest day commences) being 51 days. Hence there are 186 days constant day, 101 days twilight, and only 78 days dark night at the north pole, and even during this short period, the moon and aurora borealis shine with uncommon splendour, as before remarked.

* See the Trigonometrical Solutions of this prob. in lect. 20th, Keil's Astronomy. See also Vince's Astronomy, 8vo. articles 94, 96, 97 and 98.

3. Can there be constant day or twilight at Washington city at any time of the year?

Ans. The lat. of Washington city being 38° 53′ N. hence 90°—38° 53′=51° 7′ the complement, and 51° 7′—18°=33° 7′, which being greater than 23° 28′, there can never be constant day or twilight in this latitude.

4. When do the inhabitants of Petersburg cease to have constant day or twilight, and how long does their dark night continue?

5. How long do the inhabitants of the north cape in Lapland, enjoy the benefit of constant twilight, and how long does their dark night continue?

6. When does constant twilight begin and end, and what is its duration in Ice Cape, the most northern part of Nova Zembla?

PROB. 28.

The month, day, and hour of the day at any place being given, to find all those places on the earth, where the sun is then rising, setting, where it is noon, that particular place where the sun is vertical, where it is daylight, twilight, darknight, midnight, where the twilight then begins and where it ends, the height of the sun in any part of the illuminated hemisphere, also his depression in the obscure hemisphere.

Rule. ELEVATE the north or south pole to the sun's declination for the given time, according as it is N. or S. (probs. 1 and 8) bring the given place to the brass meridian, and set the index to 12; then if the given time be before noon, turn the globe westward, if in the afternoon, eastward as many hours as the given time precedes or is after noon: the globe being kept in this position; then all those places along the eastern edge have the sun setting, those under the brass meridian above the horizon, have noon, that particular place under the sun's declination on the brass meridian, has the sun vertical, all those places within 18° below the western edge of the horizon, have morning twilight, those within 18° below the eastern edge of the horizon, have evening twilight. In the former, twilight begins 18° below the horizon, and ends at the horizon in clear day: in the latter twilight begins at the horizon, and ends 18° below it in dark night. The sun's altitude in any place in the enlightened hemisphere, is equal to the height of that place above the horizon, reckoned on the brass meridian, on which its meridian altitude is found, or on the quadrant of altitude screwed in the zenith. Its depression below the horizon, is equal to the depression of the place, or the altitude of its antipodes; to those between the eastern part of the horizon and the meridian, the sun will appear westward, having crossed their meridian; to those between the meridian and the western part, the sun will appear towards the eastward, not having as yet passed their meridian.

This problem may be solved by taking the globe out of the frame, and fastening a strong thread to the latitude of the place on the brass meridian ; then suspending it in the sun shine, and bringing the place to the brass meridian or to the zenith, and fixing the meridian north and south, by a meridian line or compass, the elevated pole of the globe will then point to the elevated pole in the heavens, and the whole globe will correspond, in every respect, to the position of the earth itself, in respect of the sun.

If then a pin, or bit of wire be erected perpendicularly (on a hollow basis of wood, cork, wax, &c.) in the middle of the enlightened hemisphere, it will project no shadow, which shews that the sun is vertical to that place If this place be brought under the brass meridian, the degree over it will be its latitude, and the sun's declination at the given hour. Then all those places under the brass meridian have noon or midnight, according as they are in the illuminated or dark hemisphere.* Those on the westward (or right hand when our face is turned towards the sun if situated N. of the place where it is vertical) have their morning, for with them the sun is ascending from the east, and those in the semicircle bounding light and darkness westward, will have the sun rising, those towards the eastward have evening, for with them the sun is descending towards the west, and those situated between the enlightened and dark half of the globe eastward, have the sun setting. All those countries within the sun shine have day, all those in the shade have night or twilight. On the east side of the globe is seen those places where night comes on, and on the west where the darkness is dispelled by the approaching day. In those places round the elevated pole, where it is sun shine while the globe is turned round, there is constant day until the sun decreases in declination. At the opposite pole within the polar circles, where it is dark while the globe is revolved on its axis, there will be constant night until the sun decreases in declination. The number of degrees that the sun shine reaches beyond either pole, will be his declination N. or S. The difference of longitude between any place situated in the semicircle, separating the enlightened from the darkened hemisphere, and any other place on the globe, reduced to time, will give the time before or after sun rising or sun setting, according to the situation of the place. If any place be

* When we here speak of dark hemisphere, we mean that in the shade or on which none of the sun's rays fall but by reflection ; it being evident that no part of the globe, thus suspended in the sun shine, can be in the dark, which evidently shews the great utility of the reflecting principle in the atmosphere : for otherwise that part of the artificial globe on which the direct rays of the sun do not fall, would be as dark as if placed in a dungeon where no ray of light could have access : but the twilight on a small globe, for this reason, cannot be ascertained by this method, and the limits of light and shade is very doubtful. The experiment succeeds best in a dark room, where the sun's rays are admitted through a hole in the window shutters. The best time for performing the prob. is when the sun is rising or setting, or on the meridian, as the shadow of the brazen meridian will not then prevent the light of the sun from illuminating the hemisphere over which it is perpendicular. Noon is however preferable.

brought to the brass meridian, the number of degrees between it and the circle bounding light and darkness, will be the sun's meridian altitude that day; if the index be set to 12, and the globe then turned on its axis until any given hour comes under the meridian, the nearest distance in degrees between the given place and the shaded hemisphere, will give the sun's altitude for that hour. If pins be erected perpendicularly on different parts of the globe, their shadows will be projected the same way as the shadows of the inhabitants of those respective places; some pointing to the north, some to the south, some to the west, others to the east, &c. and some projecting no shadow at all.

If a narrow slip of paper be placed round the equator, and divided into twice 12 hours, beginning at the meridian of your place, and counting westward: the 6 o'clock mark being brought under the brass meridian, the sun at noon will then shine on this meridian, and the hours marked on the paper at the east or west part of the equator, or at the circle bounding light and shade, will indicate the time of the day in that place: thus at 12 o'clock the two 12's will be in the circles bounding light and shade; at one, the two one's, &c. In the evening, or at night, it may be seen in the same manner, if the moon shines, what nations are illuminated by her light, where she is rising and setting, the various projections of her shadow, where she is vertical, &c. and to which of the poles she does not set that night. Her meridian alt. or alt. for any given hour, may be found in the same manner as the sun's found above, with this difference, that when the given place is brought to the brass meridian, the index must be set to the time of her passing the meridian that night.

Example 1. When it was 15 minutes after 5 o'clock in the morning, at Washington city on the 16th of April, 1811, where was the sun then rising, setting, &c. &c.

Ans. On the 16th of April the sun's declination was 9° 58′ 42″ N. or 10° nearly, therefore elevate the north pole 10° above the horizon, and as the given time is 5 hours 15 minutes, in the morning, 12 h.—5 h. 15 m.=6 h. 45 m. what it wants of noon; hence the globe must be turned westward until the index has passed over 6 h. 45 m. The globe being fixed in this position, then,

The sun is rising from the north west to the south east parts of Hudson's Bay, at Burlington, N. Jersey, the eastern part of the Island of St. Domingo, near Porto Cabello in S. America, St. Christopher on the Amazon R. near Sta. Cruz. Buenos Ayres, east of the Falkland Islands, &c.

Setting, between the Lena and Indighirka rivers in Siberia, near Chynian in Chinese Tartary, the mouth of the Blue River east of Nanking, the eastern part of the Islands of Borneo and Java, the western extremity of N. Holland, &c.

Noon, at the eastern part of Spitzbergen, North Cape, the northern extremity of the Gulf of Bothnia, Revel on the Gulf of Finland, the eastern part of Prussia, Gallicia, Hungary, &c. the western part of the Archipelago, the middle of Negropont, east of Athens, the middle of the desert of Lybia, Bornou, Mossel Bay in Caffraria, east of the Cape of Good Hope, &c.

Vertical, in lat. 10° N. long. 24° E. about the middle of the Ethiopic mountains.

Morning twilight, at Prince William's sound, near Beering's bay, part of the Stony Mountains, Gulf of Mexico, Merida in New-Spain, Gulf of Papagaya, along the Pacific ocean, &c.

Evening twilight, at Gore's island, south of Beering's strait, Beering's island, east of the Japonese islands, the western part of New-Guinea, Van Diemen's and Nuyt's land in New Holland, the Southern ocean, &c.

Midnight, at the western extremity of N. America, Owhyhee Island, Pacific ocean, west of the Society islands, &c.

Day, in all Europe, Africa, and Asia, except a small portion of the eastern part: in Labrador, Newfoundland, Nova-Scotia, New-Brunswick, part of Canada, and the New-England states in N. America, all that part of the West-Indies and South America, comprehended between the eastern part of St. Domingo and Buenos Ayres, &c. towards the east, Sandwich land, &c.

Night. The remaining part of North and South America below the circle of twilight, Kamtschatka, the Carolinas, New-Guinea, New-Britain, &c. the eastern part of New-Holland, New-Hebrides, New-Zealand, Sandwich islands, the greater part of the Pacific ocean, &c.

The sun's alt. at Petersburg is nearly 40°, at Cairo 69°, at Calcutta 28¼°, at London 44½°, &c.

Those inhabitants situated at the northern extremity of the Island of Ceylon, at Cochin, &c. will see the sun due west: those in the same parallel of lat. west of the brass meridian to the horizon, will see it due east; from Tartary, Persia, &c. it will appear towards the S. W. from Madagascar towards the north west; from the western part of Candia island due south; from Mossel Bay in Caffraria due north; from Lisbon, Fez, &c. towards the S. E. from St. Helena, &c. towards the N. E. &c.

2. When it is four o'clock in the afternoon at Washington on the 21st of January, where is the sun rising, setting, &c. &c.

Ans. The sun's declination being nearly 20° south, the south pole must be therefore elevated 20° above the horizon: and as the given time is 4 o'clock in the afternoon, the given place being brought to the brass meridian, and the index set to 12, the globe must be turned eastward 4 hours. Then the sun will be *rising* at Berring's straits, Berring's island, Van Diemen's land, &c.—*setting* in Hudson's Bay, James' island, the western extremity of Nova-Scotia, the eastern part of South America, &c. *Noon* at the eastern extremity of King G. 3d's Archipellago, east of Marquesas' island, &c. vertical lat. 20° south, long. 136 W. near Whitsunday, island of Wallis, &c. The other places are easily found by following the directions in the rule.

3. When it is 6 o'clock in the morning at London, on the longest day, where is the sun then rising, setting, &c.

4. When it is 12 o'clock at Philadelphia, on the 10th of December, where is the sun then rising, setting, &c.

5. When it is 10 o'clock in the afternoon at Cape Horn, on the 21st of March, where is the sun then rising, setting, &c.

6. When it is midnight at Washington on the 4th of July, where is the sun rising, setting, &c.

PROB. 29.

*To find in what climate any given place on the globe is situated.**

Rule. If the place be not in the frigid zone, find the length of the longest day in that place (prob. 13, or the rule annexed) from which subtract twelve hours; the number of half hours in the remainder will shew the climate. (Def. 90.)

* This problem may be calculated by the following proportion:

As tangent of the sun's greatest declination
To radius or sine of 90°,
So is sine of the sun's ascensional difference
To tangent of the latitude not within the polar circles.

Thus suppose the ascensional diff.=3° 45′

As tangent of 23° 28′	-	9 63761
To radius	- - - - - -	10
So is sine of 3° 45′	- -	8 81560
To tangent lat. 8° 45′	-	9 17793

As the ascensional difference converted into time always shows how much before or after six the sun rises or sets, and as at the end of the 1st climate the sun rises a quarter of an hour before 6, or sets ¼ after 6; and in every climate forward to the polar circles the sun rises ¼ of an hour earlier and sets ¼ later than in the preceding: and moreover, as the longest day is found by doubling the time of sun setting, it will therefore follow, that if 6 hours be taken from half the length of the longest day, the remainder converted into degrees will give the ascensional difference. Hence the ascensional difference for the first climate, or where the day is 12½ hours long, is 15 minutes of time (before and after 6, which makes the half hour) equal to 3° 45′; for the 2d. climate 30 minutes =7° 30′; for the 3d. 45 min.=11° 15′; for the 4th. 1 hour =15°, &c. Hence the reason of taking 3° 45′ above. From these principles the climates from the equator to the polar circles in the table annexed to def. 90 were calculated, the remaining part of which corresponding to rule 2, above, may be constructed as follows:

The beginning and end of the longest day being equally distant from the solstice intervening (see note prob. 19.) reckoning half the number of days which the sun shines constantly without setting, from the 21st of June, both before and after it; find the sun's declination corresponding to those two days in the Nautical Almanac, or in a table of the sun's declination, half the sum of which taken from 90°, will give the latitude. The reason of which is plain, as the complement of the latitude is always equal to the sun's declination when the longest day begins or ends within the polar circles. (See note to prob. 19.) And as this declination is equally distant from the point cancer, in which the sun is on the 21st of June, the method is evident. From the

2. If the place be within the polar circles, find the length of the longest day at the given place (by prob. 19) and if that be less than 1 month or 30 days, the place is in the twenty-fifth climate or the first within the polar circle; if more than 1 month and less than 2 months or 60 days, the place is in the 26th climate or 2d within the polar circles, &c.

Example 1. In what climate is Washington city, and what other remarkable places are situated in the same climate?

Ans. The longest day at Washington city is 14 hours 44 min. hence 14 h. 44 m.—12 h.=2 h. 44 m. which multiplied by 2= 5 h. 28 m. or 6 half hours nearly: hence Washington is in the 6th climate north of the equator. And as the breadth of this climate extends from latitude 36° 31′ to 41° 24′ N. all those places within these two parallels are in the same climate, viz. in the United States, Richmond, Baltimore, Philadelphia, Lexington, Frankfort, Trenton, New-York, New-Haven, &c. In Europe, Lisbon, Madrid, most of the islands in the Mediterranean, Naples, Ancient Greece, the islands in the Archipellago, Constantinople, &c. In Asia, Bursa, Smyrna, the southern parts of the Caspian Sea, Samarcand, Pekin in China, the southern parts of the Japan Isles, &c.

2. In what climate is the North Cape in the island of Maggeroe, latitude 71° 10′ north?

Ans. The length of the longest day is 74 natural days, which divided by 30, gives 2 months 14 days for the quotient, and hence the place is in the 3d climate within the polar circle, or the 27th climate reckoning from the equator. As the breadth of this climate extends from 69° 33′ to 73° 5′ (see the note and table annexed to definition 90) the space contained within these parallels in Greenland, Baffin's Bay, the northern part of Siberia, the southern part of Novazembla, &c. is in the same climate.

3. In what climate is Dublin, and what other places are situated in the same climate?

variation of the sun's declination, it is plain that no table can answer exactly for every year, as the declination for that year ought to be taken from the Nautical Almanac, or from tables constructed for leap years and the three following years. A mean of these declinations has been taken in constructing the 2d part of the table alluded to, in order to have it correspond to every year as near as the nature of the problem can admit. Ricciolus (an Italian astronomer and mathematician, born at Ferrara, in 1598) in his *Astronomia Reformata*, published in 1665, makes an allowance for the refraction of the atmosphere, in his tables of climates. He reckons the increase of days by half hours from 12 to 16; by hours from 16 to 20; by 2 hours from 20 to 24; and by months in the frigid zones; making the number of days in each month in the north frigid zone something more than those in the south. But the refraction of the atmosphere is so variable, as to render such a table of no material advantage. In fact the division of places by their parallels of latitude, and length of their longest days, &c. being the most accurate and useful, renders all others, such as zones, climates, &c. of little comparative advantage.

4. In what climate is that part of N. E. land in Spitzbergen, situated in 80° lat. N. ?
5. In what climate is Cape South, in New Zealand ?
6. In what climate is Cape Horn situated ?

PROB. 30.

To find the breadths of the several climates from the equator to the poles.

Rule. 1. For the northern hemisphere, elevate the north pole 23° 28′ above the horizon, bring cancer to the meridian, and set the index to 12 ; turn the globe eastward on its axis until the index has passed over a quarter of an hour ; observe that particular point of the meridian passing through libra which is cut by the horizon, and at the point of intersection make a mark with a pencil ; continue the motion of the globe eastward until the index has passed over another quarter of an hour, and make a second mark as before ; proceed in this manner until the meridian passing through libra will no longer cut the horizon, the several marks * brought to the brass meridian will point out the latitudes where each climate ends, from the equator to the polar circles ; the difference of which will give the breadth of each climate.

2. For the climates from the polar circles to the poles. Find the latitude corresponding to the length of the longest day in each climate, namely, one month, two months, &c. (by prob. 20.) These will be the latitudes where each climate ends, and hence their difference will be the breadth of each climate. (See note to def. 90.)

Example 1. What is the breadth of the 6th climate, and what remarkable places are situated within it ?

Ans. The 6th climate extends from 36° 31′ to 41° 24′ N. the difference of which is 4° 53′, which is the breadth required, and all places situated within this space, are in the same climate. See Example 1 of the preceding prob.

2. What is the breadth of the 27th climate ?

Ans. The 27th climate is situated between 69° 33′ and 73° 5′, hence its breadth is 3° 32′.

3. What is the breadth of the 2d, 5th, 9th, 23d, 25th, and 30th climates respectively, and what remarkable places are situated within each of them ?

* On Cary's and Adam's globes the meridian passing through libra being divided into degrees, &c. in the same manner as the brass meridian ; the horizon will therefore cut this meridian in the several degrees answering to the end of each climate, and hence on these globes the above marks become unnecessary.

PROB. 31.

To find the distance between any two places on the globe.

Rule. As the shortest distance between any two places on the earth is an arch of a great circle contained between the two places* (the earth being considered a sphere) therefore, lay the graduated edge of the quadrant of altitude over the two places so that the division marked 0 may be on one of them, the degrees on the quadrant between the two places will give their distance. If these degrees be multiplied by 60, the product will give the distance in geographical miles: or by 69½ † the product will give the distance in English or American miles.

Or, Extend a pair of compasses between any two places; this extent applied to the equator will give the number of degrees between them.

If the distance between the places should exceed the number of degrees on the quadrant, stretch a piece of thread or narrow ribband over them; this extent applied to the equator, from the first meridian, will shew the number of degrees between them, or both places may be brought to the horizon, and the degrees on the horizon will give their distance. This method will answer when their distance is greater or less than 90°.

* See Emerson's Trigonometry, cor. 2. prop. 13. b. 3.

† According to the late French adopted measures, the length of a degree in English miles is 69.04 or $69\frac{1}{25}$ miles. See note to definition 8. To give the learner some idea of the variation in the length of a degree on the earth's surface, the following tables, &c. are inserted.

Mean latitudes.	*Toises*	*Observer's names and year.*	*Places.*
0° 0′ S.	56750	Condamine, Bouguer and Godin, in 1736 and 1743.	Near Quito.
33 18 S.	57037	De la Caille, 1752.	Near the Cape of Good Hope.
39 12 N.	56888	Mason & Dixon, 1764 & 1768.	In North America.
43 N.	56979	Boscowich & Le Maire, 1755.	In Italy.
44 44 N.	57069	Beccarius, 1768.	Lombardy.
45 N.	57028	Cassini, the Father, 1739 and 1740.	From Collioure to Paris observatory, thence to Dunkirk, distance in all 8° 31′ 11″⅚.
47 40 N.	57091	Liesganig, 1768.	Germany, near Vienna.
49 23 N.	57074	Maupertuis and Cassini, 1739 and 1740.	
52 44	57300	Norwood, 1635.	Between London and York, diff. lat. obs. 2°28′.
66 20	57422	Maupertuis, Camus, Clairaut, 1736 and 1737.	At the extremity of the Gulf of Bothnia.

Example 1. What is the nearest distance between Bermudas and Ferro Island in the Canaries ?

Distance in degrees $= 39\frac{3}{4}$
60

$\frac{60\times3}{4} =$ 2340
45

Geographical miles 2385

$39\frac{3}{4}\times69=2742\frac{3}{4}$ English miles.

$39\frac{3}{4}$
$69\frac{1}{2}$

$19\frac{7}{8}$
351
234
$\frac{69}{2}=34\frac{1}{2}$
$\frac{69}{4}=17\frac{1}{4}$

$2762\frac{5}{8}$ Eng. miles.

or, $39\frac{3}{4}$
70

$\frac{70\times3}{4} =$ 2730
$52\frac{1}{2}$

$2782\frac{1}{2}$
$\frac{39\frac{3}{4}}{2} =$ $19\frac{7}{8}$

$2762\frac{5}{8}$

Length of a degree from the theory of gravity. Newton's principia, prop. 20, *b.* 3.

Lat.	*Length of a deg. on the merid.*	*Lat.*	*Length of a deg. on the merid.*
deg.	*toises.*	*deg.*	*toises.*
0	56637	47	57022
5	56642	46	57035
10	56659	48	57048
15	56687	49	57061
20	56724	50	57074
25	56769	55	57137
30	56823	60	57196
35	56882	65	57250
40	56945	70	57295
41	56958	75	57332
43	56971	80	57360
43	56984	85	57377
44	56997	90	57382
45	57010		

M. Bouguer, an ingenious mathematician of France, thus corrects the foregoing table.

Lat.	*Toises.*
0°	56753
10	56776
20	56843
30	56946
40	57072
45	57139
50	57206
60	57332
70	57435
80	57530
90	57525

Newton takes it for granted in forming his table, that from lat. $48\frac{1}{2}$° to $49\frac{1}{2}$°, is 57060 toises, according to Picard ; he found that in Paris the length of a pendulum to vibrate seconds is 3 feet $8\frac{1}{2}$ lines, or rather $8\frac{5}{9}$ lines, and that under the equator a pendulum vibrating in the same time will be 1,087 lines shorter. But the length of pendulums being as the force of gravity, that is as the versed sine of double the lat. or as the square of the right sine of the same, the construction of the table is manifest. This is on supposition that the earth is homogeneous, but that it is not is well known. (See Clairault's treatise on the figure of the earth.) Other observations were made by the following persons : By Eratosthenes, of Cyrene, in Egypt, 270 years before Christ, who makes a deg.=250900 stadia, or 66493 toises ; by Hipparchus, of Rhodes, 140 years A. C. who makes it 275000 stadia, or 73142 toises ; Possidonius, of Alexandria, in Egypt, 60 years A. C. 240000 stadia, or 63833 ; Strabo and Ptolemy, in Egypt, in the year 135, makes it 180000 stadia=47875 toises ; Almaimon, an Arabian king, with his mathematicians, in the plains of Mesopotamia, in 800, makes it 43279 toises ; Fernell, from Paris to Amiens, in 1550, makes it 56746 ; Snell, in Holland, makes it 55021 toises ; Ricciolus and Grimaldus, in Italy, in 1661, make it 629000 toises ; Picard, in France, in 1670, makes it 57060 toises ; Cassini the younger, in 1700, makes it 57292 toises ; La Place, from an arch measured at the equator, and another between Dunkirk and Mountjoy, determines that the polar diameters of the earth is less than the equatorial, by $\frac{1}{334}$ part of the latter, and that a fourth part of the ecliptic meridian=5130740

2. What is the nearest distance between the island of Barbadoes and St. Helena?

Ans. The distance in degrees is $60\frac{1}{2}°$: hence $60\frac{1}{2}\times 60=3630$ geographical miles, or $60\frac{1}{2}\times 70-\frac{60\frac{1}{2}}{2}=4204\frac{3}{4}$ English miles. The reason of this last method is evident, as $70-\frac{1}{2}=69\frac{1}{2}$. For although $69\frac{1}{2}$ be not correct, this number is however generally used for the length of a degree. 69 should however be always used in preference. See the note to def. 8, and prob. 35.

toises. The toise being used for the measure in Peru, and reduced to a temperature of $16\frac{1}{4}°$ of a mercurial thermometer, divided into 100° from the freezing point to that of water boiling, under a pressure equivalent to a column of mercury, 76 centimetres or 30 inches English measure in height. Within these few years an arch has been measured extending from Dunkirk to Barcelona, and the degree whose middle is lat. 45° has by this means been found =57029 toises. At the equator likewise some of the members of the academy of sciences has found a deg. of the meridian =56753 toises, and in Lapland, about the lat. 66° 20′, they found it to be 57458. From all which it is evident, that the degrees of the meridian gradually increase from the equator to the poles.

From these latter data and the rules of mensuration, the equatorial diameter is found equal 3271267, and the polar 3261461 toises, the difference being 9769 toises=58614 French feet, equal $56647\frac{13}{149}$ English feet, or something less than $10\frac{3}{4}$ English miles. (3 feet English being equal to 3 feet $1\frac{1}{4}$ inches French, and 6 feet equal one toise, or 12 inches English or American equal 12 inches 9 lines $3\frac{3}{4}$ points French measure, 12 points being equal to a line, 12 lines an inch, &c.)

From the difference observed in the length of a degree of the meridian in the above tables, it is evident that the surface of the earth is of no regular form. It is known that there are rivers on its surface, which run from their sources from 2000 to 3000 miles and upwards, before they discharge themselves into the sea, and that frequently these rivers have cataracts or water falls of considerable height; (the falls of Niagara, for example, is 273 feet perpendicular height, including 65 feet fall in the chasm, and 58 for the half mile above the cataract.) Therefore it must be admitted that the part of the earth at the fountain head of such rivers is considerably higher than where they discharge themselves into the ocean; because if the earth were a perfect plane, or a perfect sphere, the water at the fountain head of rivers could not possibly flow to any other place, or in any direction in preference to another, except westward from the motion of the earth on its axis. Therefore to ascertain its figure to as much exactness as the nature of the thing can admit, the exact length of a degree in the various parallels of latitude, as well as a degree on the meridian, the elevation of countries above the level of the sea, the motion and direction of rivers, of currents in the ocean, &c. should likewise be ascertained. And yet if it be admitted that at the equator the earth is higher than at the poles, we may ask how it happens that the greatest rivers in the world flow towards or parallel to the equator, and generally in an easterly direction, while many others have their direction towards the poles. Thus the Amazon flows towards or parallel to the equator, while the La Plata flows towards the south pole. The Oronoke in some places directs its course westward, then northward, and afterwards eastward, declining in each winding from the equator. The Mississippi and all the rivers in the United States, and within that latitude as far as the southern ocean, flow almost universally towards the equator, while the St. Lawrence, and others direct their course towards the north. In the eastern

3. What is the nearest distance between the island of Barbadoes and Bermudas, in Geographical and English miles ?

4. What is the shortest distance between Washington city and London ?

hemisphere, the Senegal and Gambia flow westward, the Niger takes its course in an easterly direction parallel to the equator, while the Nile, the source of which is nearer the equator, flows towards the north ; on the contrary, the Euphrates, the Indus, the Ganges, &c. and all the rivers in India beyond the Ganges, direct their course towards the equator. Those in China, &c. eastward. The Danube, Dniper, Don, Volgo, &c. likewise flow towards the equator, whilst the Oby, Enissey, Lena, &c. which have their sources further south, direct their courses towards the north. Hence the great irregularity in the surface of the land indicated by these rivers. Nor is the sea without a similar or perhaps greater, as any one that considers the phenomenon of the gulf stream alone, may clearly perceive. First, it directs its course towards the north as far as the banks of Newfoundland, thence eastward towards the Azores, again it directs its course towards the equator, changes its direction about the Cape Verd islands, where, impelled by the trade winds, in a direction parallel to the equator west, it is forced again into the Gulf of Mexico, its mean velocity being about 3 miles an hour. From these phenomena it is evident, that the earth can have no regular figure, and as it deviates but little from a circular figure (see note to def. 1) our calculations are not the less certain when we consider it as such. St. Pierre in his Studies of Nature shews that the above measures of a degree, &c. tend as much to prove the excess of the polar diameter above the equatorial as the contrary. Hence, simple as this problem may appear in theory, on a superficial view, yet when applied to practice, the difficulties which occur are almost insurmountable. And granting even that the earth is a perfect sphere, much of the difficulty would still remain. For in sailing across the ocean, or in travelling through extensive and unknown countries, without any other guide than the compass, with such a guide it is plain that we cannot take the shortest rout, as measured by the quadrant of altitude (an arch of a great circle being the shortest distance between two places on the globe, as before observed) because the rhumb lines must always cut the meridians in the same angles, and this cannot happen in sailing or travelling by the compass, unless the places be situated directly north and south of each other, or upon the equator.

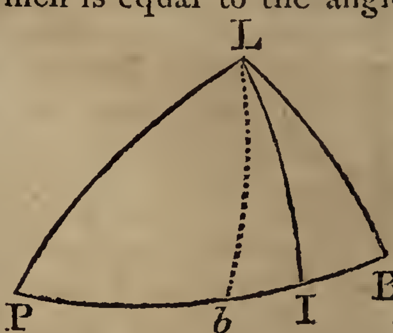
To render these observations more intelligible to the young student, 1st. Let two places be situated in lat. 50° N. and differing in longitude 48° 50′, which will nearly correspond with the lands end and the eastern coast of Newfoundland. Now there are given the complement of the lat. =40°, and the angle formed by the two meridians passing through both places equal 48° 50′, to find the distance between both places in an arch of a great circle ; but as the triangle formed by both complements of lat. which are equal, and the required arch, is therefore Isosceles, a perpendicular let fall from the given angle bisects the base, and also the vertical angle (Emerson's Trigonometry, prob. 14. cor. 2. B. 2.) Hence there is given the hypotenuse =40° and angle at the vertex =24° 25′ to find the base or half the distance ; and therefore by Baron Napier's rule (see Simson's Trig. at the end of his Euclid. Emerson's, prob. 28. B. 3. or Keith's.) Rad. × Sine of $\frac{1}{2}$ the distance = Sine 40° × Sine 24° 25′ (being the cosines of the opposite extremes.) And therefore rad. : sine 40° :: sine 24° 25′ : sine base or half the distance =15° 24′ 33″ 3, hence the whole distance=30° 49′ 6″ 6=1849.11 geographical miles, or 2127.7 English (allowing 69.04 miles to a deg. See note to def. 8) But if a ship steer from the lands end directly westward in lat. 50° N. until her difference of longitude be 48° 50′,

5. What is the shortest distance between Washington city and the junction between the Mississippi and Missouri?

6. What is the extent of Europe in English miles from Cape Matapan in the Morea, lat. 36° 35′ N. to the north cape in Lap-

then by parallel sailing, rad. : co. sine 50° :: diff. longitude 2930 miles : the distance = 1883.4 geographical, or 2167.16 English miles, which last distance is greater than the former, on the arch of a great circle by 34.29 geographical, or 39.46 English miles. Those who are acquainted with Spherical Trigonometry and the principles of Navigation, particularly great circle sailing, know that it is impossible to conduct a ship exactly on the arch of a great circle, except, as before observed, on the equator or meridian; for in this example, she must be steered through all the different angles from N. 70° 49′ 30″ W. to 90°; and continue sailing from thence through all the same variety of angles until she arrives at the intended place, where the angle will become 70° 49′ 30″, as at first. For as the complements of 50°, together with the distance, form an Isosceles triangle, as before observed, the angles at the base being equal, is found by Napier's rule thus: rad. × sine co. 40° = tangt. co. 24° 25′ × tangt. co. course. Hence co. tangt. 24° 25′ : co. sine 40° :: rad. : co. tangt. of the angle at the base = 70° 49′ 30″ as above. But as this is the angle which the ship's way makes with the meridian, it is equal to the course required. In the same manner may the course be found for any other point in the parallel between the land's end and the meridian which bisects the distance. Thus, if instead of 24° 25′, the distance from the vessel to this perpendicular be 18° (for ex.) the course will then be 76° 1′ 22″, &c.

2. Suppose it were required to find the shortest distance between the Lizard, lat. 49° 57′ N. lon. 5° 21′ W. and the Island of Bermudas, lat. 32° 25′ N. and lon. 63° 35′ W. Here there are given the complements of both latitudes, and the difference of longitude 58° 14′ (which is equal to the angle formed by the two meridians passing through the two given places) to find the distance in an arch of a great circle. Let PL = co. lat. Lizard = 40° 3′, PB = co. lat. Bermudas = 57° 35′ the angle LPB = 58° 14′ (P being the pole) to find the distance LB. Draw LI perp. to PB, then by Napier's rule rad. × co. sine LPI = co. tang. PL × tang. PI. Hence co. tang. 40° 3′ : rad. :: co. sine 58° 14′ : tang. PI = 23° 52′ 15″, therefore IB = PB—PI = 33° 42′ 45″.



The sides LI, LB may be found in like manner or shorter, thus: Co. s. PI 23° 52′ 15″ : co. sine BI 33° 42′ 45″ :: co. sine PL 40° 3′ : co. sine LB (Emerson's Trig. cor. 2. prop. 28. b. 3) = 45° 52′ 5″ the shortest distance between the two places, equal $2752\frac{1}{12}$ geographical or 3166.73 English miles (allowing 69.04 miles to a deg.) Now for a ship to sail on this arch, between the Lizard and Bermudas, she must sail from the Lizard S. 89° 10′ 46″ W. (being the angle which the ship's way LB makes with the meridian PL, or the angle PLB) and gradually lessen this course so as to arrive at Bermudas on the rhumb bearing S. 50° 46′ 2″ W. which is the angle that the ship's way makes with the meridian PB passing through Bermudas, or the angle PBL. But this, though true in theory, is impracticable, and therefore the course and distance must be calculated by *Mercator's sailing*. The direct course by the compass being S. 67° 59′ 43″ W. and the distance upon that course 2807.68 geographical or 3230.7 English miles, which is greater than the former by 55.6 geographical or 63.97 English miles.

The shortest distance between any two places, the lat. and long. of which are given, may be found in the same manner as LB has been calculated above, and that whether the perpendicular falls within or without LB, L*b*.

land, lat. 71° 10′ N. the places being situated nearly due north and south ?

Note. 1. Here the difference of lat. is nearly equal the distance. See the notes at the bottom.

7. What is the shortest distance between Quito, in longitude 77° 55′ W. and Macapa, in longitude 51° 20′ W. both situated nearly under the equator ?

Note. 2. Here the difference of longitude is nearly the distance.

8. What is the shortest distance between the town of St. Domingo and Cape Horn ?

9. What is the breadth of North-America from Sandy-Hook, in lat. 40° N. and that part of the coast of New-Albion in the Pacific ocean in the same parallel ? (See the notes and table to prob. 35.)

10. Suppose the track of a ship to Batavia be from New-York to Bermudas, thence to St. Anthony, one of the Cape Verd islands, thence to St. Helena, thence to the Cape of Good Hope, thence to the Isle of France or Mauritius, thence to the headland westward of Bantam, thence to Batavia ; how many English miles from New-York to Batavia, on these different courses ?

PROB. 32.

*A place being given on the globe to find all those places that are at the same distance from it as any other given place.**

Rule. EXTEND the quadrant of altitude between both places, so that the division marked 0 may be on the given place from

although it be more convenient to have the perp. always fall on the longest side as in the above calculation. But when we want to find the distance between any two places whose lat. and long. are known, in order to travel or sail from one place to the other, on a direct course by the compass, the following methods must be used :

1. If the places be situated on the same meridian, or have the same longitude, their difference of latitude (found by prob. 4) will be the *nearest* distance between them in degrees, and the places will be exactly north and south of each other.

2. If the places be situated on the equator, their difference of longitude will be the *nearest* distance in degrees, and the places will be exactly east and west of each other.

3. If the places be situated in the same parallel of lat. they will be directly east and west of each other, and their difference of longitude (found by prob. 4) multiplied by the number of miles which make a degree in the given lat. (see the table annexed to prob. 35) will give the distance.

4. If the places differ both in their latitudes and longitudes, the distance between them, and the point of the compass on which a person must travel, or a vessel sail from the one to the other, must be found by *Mercator's sailing*, as in navigation.

* A general solution to this prob. may be obtained from prob. 2d. sect. 1. vol. 2. of Simpson's Fluxions. Thus let $\dot{A}$ = fluxion of the angle of position between the given place and that required. $\dot{B}$ = flux. of the diff. long. and $\dot{F}$ = flux. of the complement of the lat. of the place required. D = comp. lat. of the given place, C = the angle of position between the re-

which the distance is reckoned, move the quadrant round, keeping 0 on the quadrant in its first position, all those places that pass under the degree of distance, observed to stand over the other place, will be the places required. The globe may be also rectified for the given lat. and the quadrant screwed in the zenith, &c.

Or, With the extent between both places, describe, with a pair of compasses, a circle, the centre of which is the first given place, all those places in the circumference of this circle, will be those required.

If the extent between both places should exceed the length of the quadrant, or the extent of a pair of compasses, stretch a piece of thread over both places, with which describe a circle as before. Or both places being brought to the horizon, turn the globe on the notch in the direction of the meridian, and the horizon will point out the places required.

Example 1. Find all those places that are at the same (or nearly the same) distance from Washington city, as Port au Prince in St. Domingo.

Ans. Kingston in Jamaica, Cape Catouche in New-Spain, Antonio in Mexico, the mouth of Haye's river, the island and strait of Bell Isle, at the mouth of the river St. Lawrence, St. John's in Newfoundland, &c.

2. Required all those places that are at the same distance from Paris as London ?

3. Find all those places that are at the same distance from London as Paris ?

4. Find all those places that are at the same distance from Constantinople as Naples ?

quired place and the given place ; then, co. sec. D : sin. B :: $\dot{A}$: $\dot{F}$ and sin. F : co. tang. C :: $\dot{F}$: $\dot{B}$; that is 1st. as the secant of the given latitude, to sine of the difference of longitude (assumed at pleasure) so is the alteration in the angle of position between the given place, and that required, to the alteration in the complement of lat. of the place required. 2d. As the co. sine of the lat. required, to co. tangent of its angle of position with the given place, so is the alteration in the lat. of the place required, to the alteration in the difference of longitude. From either of which theorems the required places may be found. If the difference of longitude be assumed, and any degree of position, the difference of lat. is given, and therefore the place itself. (Theo. 1.) If the diff. of lat. be assumed and any degree of position, the diff. of long. is found. (Theo. 2.) The difference of latitude N. or S. or the diff. of long. E. or W. can never exceed the distance between the two given places. The section alluded to in Simpson's Fluxions, which treats of the fluxions of spherical triangles, is extremely useful in a great variety of cases in Practical Astronomy, Geography, and Navigation, which are calculated with much more labour by other methods.

PROB. 33.

*Given the latitude of a place and its distance from a given place, to find that place the latitude of which is given.**

Rule. If the distance be given in English or geographical miles, reduce them to degrees (by allowing 60 geographical, or $69\frac{1}{2}$ English miles† to a degree) then place 0 on the quadrant of alt. upon the given place, and move the other end eastward or westward, according to the position of the place east or west, until the degrees of distance cut the given parallel of lat. under the point of intersection is the place required.

Or, Having taken the degrees of distance from the equator with a pair of compasses, with this extent, and one foot of the compasses on the given place, with the other intersect the given parallels, the point of intersection will be the place required, and will be east or west as before. If none of the parallels on the globe pass through the given place, you may describe one with a fine pencil, by holding it over the lat. and turning the globe on its axis. This prob. may be also performed by means of the horizon.

Example 1. A place in lat. 32° 25′ N is $2752\frac{1}{2}$ geographical miles westward from the Lizard in England, required the place?

Ans. $2752\frac{1}{2} \div 60 = 45° 52\frac{1}{2}'$; this on the quadrant will extend from the Lizard to Bermudas in the given parallel west; hence Bermudas is the place required.

2. A place in W. long. and 13° N. lat. is 3660 geographical miles from London, required the place?

3. A place in lat. 60° N. is $1320\frac{1}{2}$ English miles from London, and is situated in east longitude; required the place?

4. A place in lat. 53° 34′ N. is distant from Washington city 4002 Eng. miles; required the place?

PROB. 34.

Given the longitude of a place and its distance from a given place to find that place, the longitude of which is given.

Rule. Convert the degrees into minutes as before, and apply 0 on the quadrant of alt. to the given place, as in the foregoing prob. move the other end northward or southward (according as the required place lies north or south of the given place) until the degrees of distance cut the given longitude, and under the point of intersection you will find the place required.

Or, Bring the longitude of the place required to the brass meridian, then take the degrees of distance from the equator with a pair

* The reason of this and the following problem is too evident to need any explanation.

† Though we sometimes make use of $69\frac{1}{2}$ Eng. miles to a degree, yet the learner is advised to make use of 69, or where greater exactness is required, of 69.04, in preference. See note to def. 8, or notes to prob. 35 and 36 following.

of compasses, and with one foot in the given place, under the point where the other cuts the brass meridian, you will find the place required.

If the given place be west of the place required, the meridian passing through the place required, instead of the brass meridian, must be used ; but if no meridian pass through it, one may be described with a fine pencil by bringing its longitude under the brass meridian.

Example 1. A place in north lat. and long. 70° 58′ W. is 817 English miles north eastward of Charlestown in South Carolina ; required the place ?

Ans. 817÷69½ or 1634÷139=11¾° nearly ; hence the place is Boston.

2. A place in north lat. and 77° 10′ west long. is 3675 Eng. miles towards the south west from Greenwich observatory ; required the place ?

3. A place in longitude 5° 49′ W. is distant from Barbadoes 3630 geographical miles south eastward ; required the place ?

4. A place in longitude 63° 36′ W. is distant from Ferro island 2762⅝ English miles, and lies in a north westerly direction from it ; required the place ?

PROB. 35.

To find how many miles make a degree of longitude in any given parallel of latitude.

Rule. Lay the quadrant of alt. parallel to the equator between any two meridians in the given lat. which differ in longitude 15° for Bardin's, or 20° for Cary's globe, the number of degrees intercepted between them multiplied by 4 for Bardin's, or by 3 for Cary's, will give the length of a degree in geographical miles.

Or, Take the distance between two meridians which differ in long. 15° for Bardin's, or 20° for Cary's globes, in the same parallel of lat. with a pair of compasses ; apply this distance to the equator, and observe how many degrees it makes, with which proceed as before. The distance between 10° on Cary's globe mult. by 6, will likewise give the miles required.

Example 1. How many geographical and American miles make a degree in the latitude of Philadelphia ?

Ans. The lat. of Philadelphia is nearly 40° N. The distance between two meridians, in that lat. which differ 15° in long. is 11½°, or 15⅓° if the meridians differ 20°. Now 11½×4 or 15⅓×3=46 geographical miles for the length of a degree of longitude in the latitude of Philadelphia, which multiplied by 1.16 (because 60 : 69½ :: 1 : 1.16 nearly) gives 53.36 English miles, the length of a degree ; or 15° : 11½° :: 69½ : 53.36.*

* The reason of this is evident from this principle, that the distance or num. of degrees contained between any two meridians on the equator, is to a similar arch or distance between the same meridians in any parallel of lat.

2. How many miles make a degree in the parallels where the following places are situated :

Boston, Washington, Savannah, Kingston in Jamaica, London, Paris, Petersburgh, Skalholt, North Cape, and the most northern part of Spitzbergen ?

as the length of one degree on the equator, to the length of a degree in this parallel. For similar arches have the same proportion to the whole circumferences (Lemma 2. Simson's trig. or Emerson's geom. cor. to prop. 19. b. 4.) and therefore to one another. (Eucl. b. 5. prop. 15.) Thus in the above lat. of Phil. two places which differ 15° on the equator, differ only $11\frac{1}{2}$ such degrees in this parallel ; hence 15° : $11\frac{1}{2}$° :: 60′ : 46′, or 15° : 60m. :: $11\frac{1}{2}$° : 46, but 15 : 60 :: 1 : 4, hence the reason of multiplying $11\frac{1}{2}$ by 4, &c. If instead of 15° we take 20°, then 20 : 60 :: 1 : 3, or 10 : 60 :: 1 : 6, &c. But since the quadrant of alt. will measure no arch truly but that of a great circle, and that a pair of compasses will only measure the chord of the arch, and not the arch itself, it follows that the preceding rule is not mathematically true, though sufficiently correct for practical purposes. When greater exactness is required, recourse must be had to calculation or the following table.

Deg. Lat.	*Geog. miles.*	*Eng. miles.*	*Deg. Lat.*	*Geog. miles.*	*Eng. miles.*	*Deg. Lat.*	*Geog. miles.*	*Eng. miles.*	*Deg. Lat.*	*Geog. miles.*	*Eng. miles.*
0	60 00	69 07	23	55 23	63 51	46	41 68	47 93	69	21 50	24 73
1	59 99	69 06	24	54 81	63 03	47	40 92	47 06	70	20 52	23 60
2	59 96	69 03	25	54 38	62 53	48	40 15	46 16	71	19 53	22 47
3	59 92	68 97	26	53 93	62 02	49	39 36	45 26	72	18 54	21 32
4	59 85	68 90	27	53 46	61 48	50	38 57	44 35	73	17 54	20 17
5	59 77	68 81	28	52 97	60 93	51	37 76	43 42	74	16 54	19 02
6	59 67	68 62	29	52 48	60 35	52	36 94	42 48	75	15 53	17 86
7	59 55	68 48	30	51 96	59 75	53	36 11	41 53	76	14 52	16 70
8	59 42	68 31	31	51 43	59 13	54	35 27	40 56	77	13 50	15 52
9	59 26	68 15	32	50 88	58 51	55	34 41	39 58	78	12 48	14 35
10	59 09	67 95	33	50 32	57 87	56	33 55	38 58	79	11 45	13 17
11	59 89	67 73	34	49 74	57 20	57	32 68	37 58	80	10 42	11 98
12	58 69	67 48	35	49 15	56 51	58	31 79	36 57	81	9 38	10 79
13	58 46	67 21	36	48 54	55 81	59	30 90	35 54	82	8 35	9 59
14	58 22	66 95	37	47 92	55 10	60	30 00	34 50	83	7 31	8 41
15	57 95	66 65	38	47 28	54 37	61	29 09	33 45	84	6 27	7 21
16	57 67	66 31	39	46 63	53 62	62	28 17	32 40	85	5 22	6 00
17	57 38	65 98	40	45 96	52 85	63	27 24	31 33	86	4 18	4 81
18	57 06	65 62	41	45 28	52 07	64	26 30	30 24	87	3 14	3 61
19	56 73	65 24	42	44 59	51 27	65	25 36	29 15	88	2 09	2 41
20	56 38	64 84	43	43 88	50 46	66	24 40	28 06	89	1 05	1 21
21	56 01	64 42	44	43 16	49 63	67	23 45	26 96	90	0 00	0 00
22	55 63	63 97	45	42 43	48 78	68	22 48	25 85			

The length of a degree is here given=69.07 English miles. But as has been shewn in the note to def. 8, 69.04 is more correct ; moreover an arch about the lat. 45°, which is a mean of that which has been lately measured from Dunkirk to Barcelona, gives the length of a degree equal 57029 toises =342174 French feet, or 364559.2 English feet (see note def. 8) =69.04 English miles as above ; and as 45° is a mean between the lat. at the equator and at the poles, 69.04 E. miles is properly taken as the mean length of a degree. This table having been found ready calculated in Keith's Trea-

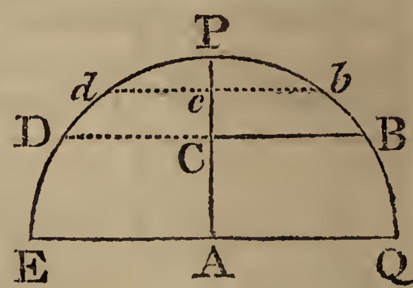
PROB. 36.

To find at what rate per hour the inhabitants of any given place are carried from west to east, by the revolution of the earth on its axis.

Rule. Find how many miles make a degree of longitude in the given latitude (by the preceding prob. or the table annexed) which multiplied by 15 for the answer.*

tise on the globes, pa. 173, it was thought unnecessary, for so trifling a difference, to repeat the calculation, as any one may perform it at pleasure. Thus in the lat. 60° a degree =30 geog. miles, or $\frac{30\times 69.04}{60}=\frac{69.04}{2}=34.52$ English miles, differing only $\frac{1}{100}$ of a mile from that given in the table, for there it ought to be 34.53. Hence also appears, that in practice we may consider 69 English miles in a degree, which will make the calculations much easier; for $\frac{30\times 69.5}{60}=34.5$, differing only $\frac{1}{50}$ miles from the truth. If we make use of 69½ miles 30 geog. miles will equal $\frac{30\times 69.5}{60}=\frac{69.5}{2}=34.75$ E. miles, &c.

The above table is thus calculated, radius : co. sine lat. of any parallel :: any given portion of the equator, as 1° : to a similar portion of the given parallel. For let EQ represent the equator, P the pole, B any given place on the meridian QP; then the arch BQ is the lat. of B, and BP its complement, and BC drawn perpendicular to the semidiameter PA, will be the co. sine of the lat. of B, to the radius AQ. Now as similar arches are as the radii of the circles of which they are arches (Emerson's geom. b. 4. prop. 8.) Therefore AQ : CB :: as any part of the circumference EQ : to a similar part of the parallel DB.



All the properties of parallel sailing will follow from the same principle (the earth being considered as a perfect sphere.) Thus the difference of longitude between any two places, in an arch of the equator, and the distance between these places, if under the same parallel, is a similar portion of that parallel; hence as rad. : co. s. lat. :: diff. long. : distance; and by inversion co. s. lat. : R. :: dist. : diff. long. Also as diff. long. : dist. :: R. : co. s. lat. whence it likewise follows, that co. s. of any given lat. : co. s. of any other lat. :: as any given position of the first parallel : any given portion of the second; and lastly, any portion of a given parallel : a similar portion of any other : co. s. lat. of the 1st : co. s. of the 2d.

By considering the three first terms of any of the above proportions as given, the 4th is given, and can be performed on the globes in the same manner as the above. The learner can therefore here find an agreeable exercise in performing all the cases in parallel sailing by help of the globe alone. To give the problems here in detail, would be contrary to our intended brevity in this introduction.

* The reason of this rule is evident; for if n = the number of miles in a deg. then 24 hours : 360° $\times$ n :: 1 hour : $\frac{360°\times n}{24}=15\,n$, the rule as above. This rule is on a supposition that the earth turns on its axis from west to east in 24 hours; but it has been observed in def. 66, that it revolves on its axis in 23 h. 56 m. 4 s. but this trifling diff. is scarce worth observing. The learner must not however forget, that it takes exactly 24 hours, for any place on the earth's surface, to perform its diurnal revolution from under any meridian, or any point in the heavens, until it comes exactly under that mer. or point again. (Note to def. 66.)

From the following tables the length of a degree given in the preceding table, may be reduced to the measures of other countries, which will often

Example 1. At what rate per hour are the inhabitants of Philadelphia carried from west to east by the revolution of the earth on its axis?

be of use to the learner, particularly in Geography, History, constructing Maps, &c. The first of these tables is collected from Dunn's Atlas; the second from Danville as given by Ozanam, vol. 3. of his Mathematical recreations; the third from vol. 1st. of Ozanam. (Montucla's Edition.)

TABLE I.

Length of a degree.		*Length of a degree.*	
60	Geographical miles.	$21\frac{1}{2}$	Turkish agash.
$69\frac{1}{2}$	Or rather $69\frac{1}{25}$ Eng. miles.	$18\frac{3}{4}$	Parasangs of Persia.
20	Marine leagues.	$56\frac{1}{4}$	Arabian miles.
50	Scotch miles.	50	do. in some places.
$54\frac{1}{2}$	Miles 14 poles, Irish.	105	Wersts of Russia.
15	Dutch miles or 20 Marine.	$37\frac{1}{2}$	Indian coss's.
15	Common leagues of Germany.	250	Common lis of China.
20	Common miles of Lithuania.	33	Jeribi. great measured coss of India.
18	Statute miles of Prussia.	34	Japanese leagues.
$13\frac{1}{2}$	Hungarian miles.	75	Ancient Roman miles.
25	Common French leagues.	80	Grecian miles.
12	Leagues of Switzerland.	671	Stadia of Herodotus.
60	Italian miles.	533	Egyptian stadia.
$17\frac{1}{2}$	Common Spanish leagues.	$22\frac{1}{2}$	Italian or 30 Arabian travelling leagues.
$87\frac{1}{2}$	Turkish miles.	60,000	Geometrical paces.
66	do. Berri.		
50	Common miles of Piedmont.		
65	Venetian miles.		

TABLE II.

Ancient and modern measures.	Toises.	*Ancient and modern measures.*	Toises.
Olympic stadia (Ancient Greece.)	$94\frac{1}{2}$	English mile or 1760 yards	826
Lesser stadia (Ancient Greece.)	$75\frac{1}{2}$	Scotch mile	1147
Least do. (Ancient Greece.)	$50\frac{1}{4}$	Irish mile	1052
Egyptian schene	3024	Spanish league of 5000 vars	2147
Parasang of Persia	2268	Do. common $17\frac{1}{2}$ to a deg.	3261
Roman mile (milliare)	756	Italian or Roman mile	768
Stadia of Judea or Rez.	76	The mile of Lombardy	$848\frac{2}{3}$
Mile do. (or Berath)	$569\frac{1}{2}$	Venetian mile	992
League of ancient Gaul	1134	The league of Poland	2850
German league (rast.)	2268	The ancient werst of Russia	656
Arabian mile	1084	Modern do.	547
French mile	1000	Turkish agash	2536
Fr. small league 30 to a deg.	1902	The little coss of India	1342
Do. mean 25 to a degree	2283	The great coss	1542
Do. great 20 to a degree	2853	The gau of Malabar	6000
German miles $12\frac{1}{2}$ to a deg.	4536	The Nari or Nali do.	900
Do. 15 to a degree	3800	The lis of China	295
Swedish mile	5483	The pu equal 10 lis	2950
Danish mile	3930		

Ans. The lat. of Philadelphia is nearly 40°, in which parallel a degree of long. = 46 geogr. or 53,36 English miles (prob. 35) hence $46 \times 15 = 690$ and $53.36 \times 15 = 800$ 4 therefore the inhabitants of Philadelphia are carried 690 geographical or 800 English miles per hour.

Note. The geometrical pace is 5 Roman feet, 1000 of which = 1 mile = 8 stadia = 400 cubits of 1 foot $9\frac{8}{9}$ inches English; 30 stadia = a parasang = $2188\frac{4}{5}$ English feet. The toise is 6 French feet. But as the foot differs in various countries, the following table will give the learner an idea of this variation. As the length of the toise is given in Paris feet, we shall compare all the other feet to it. It is divided into 12 inches, each inch into 12 lines, and each line into 10 parts; hence the foot will be 1440 of these parts. We shall therefore consider the foot of other countries under both these denominations, that is, in parts, and in inches, lines, &c.

ANCIENT FEET.

	Parts.	*Ft.*	*in.*	*li.*	*pts.*
The ancient Roman foot =	1306	= 0	10	10	6
Grecian and Ptolemaic	1364		11	4	4
Grecian Phyleterien	1577	1	1	1	7
Archimedes, or probably that of Syracuse & Sicily	986		8	2	6
The Drusian	1473	1	0	3	3
Macedonian	1567	1	1	0	7
Egyptian	1920	1	4	0	0
Hebrew	1637	1	1	7	7
Natural	1100		9	2	0
Arabic	1480	1	0	4	0
Babylonic	1546	1	0	10	6
or	1534	1	0	9	4

MODERN FEET.

	Parts.	*Ft.*	*in.*	*li.*	*pts.*
The foot of Paris	1440	1	0	0	0
Amsterdam	1253		10	5	3
Antona and Eccles. states	1732	1	2	5	2
Altorf (Underwald)	1047		8	8	7
Anvers or Antwerp	1270		10	7	0
Augsburg	1313		10	11	3
Avignon and Arles	1200		10	0	0
Aquileia (Venice)	1524	1	0	8	4
Basle	1276		10	7	6
Barcelona	1340		11	2	0
Bologne	1682	1	2	0	2
Bourg. (Bress and Bugey, Switz'd.)	1392		11	7	2
Berlin	1340		11	2	0
Bremen	1290		10	9	0
Bergame	1933	1	4	1	3
Besançon	1372		11	5	2
Brescia	2108	1	5	6	8
Bruges	1013		8	5	3
Brussels	1219		10	1	9
Breslaw	1520	1	0	8	0
China, tribunal of mathematics,	1523	1	0	8	3
The imperial foot	1420		11	10	0
Cologne	1220		10	2	0
Chambery (and Savoy)	1496	1	0	5	6
Copenhagen	1448		11	9	8
Constantinople	2966	2	0	8	6
	1575	1	1	1	5

By the table. In lat 40° a degree of long. = 45.96 geogr. or 52.85 English miles. Hence 45.96×15=689.4 and 52.85×15 = 792.75, consequently the inhabitants in this parallel are carried 689½ geogr. or 793 English miles per hour, by the earth's revolution on its axis. The latter result is the most correct.

MODERN FEET.

	Parts.	*Ft.*	*in.*	*li.*	*pts.*
The foot of Cracow =	1580	= 1	1	2	0
Dantzic	1247		10	4	7
Dijon	1392		11	7	2
Delft (Holland)	739		6	1	9
Denmark	1415		11	9	5
Dordrecht	1042		8	8	2
Edinburgh	1485	1	0	4	5
Ferrara (Italy)	1779	1	2	9	9
Florence	1345		11	2	5
Francfort on the Maine	1260		10	6	0
Franche Comte	1483	1	1	2	3
Genoa (Le Palme)	1098		9	1	8
Geneva	2592	1	9	7	2
Grenoble and Dauphin	1512	1	0	7	2
Halle (on the Elbe, Up. Sax.)	1320		11	0	0
Harlem	1267		10	6	7
Hambourg	1260		10	6	0
Heidelberg (Palat.)	1220		10	2	0
Inspruck (Cap. of Tyrol)	1488	1	0	4	8
Leyden	1382		11	6	2
Leipsic	1397		11	7	7
Liege	1276		10	7	6
Lisbon	1287		10	8	7
Leghorn	1340		11	2	0
Lombardy, &c.	1926	1	4	0	6
London	1351⅔		11	3	1
Lubeck (Holstein)	1260		10	6	0
Lucca (Italy)	2615	1	9	9	5
Lyons and Lyonnois, Ferez, &c.	1512	1	0	7	2
Lorraine	1292		10	9	2
Madrid	1237		10	3	7
Malta (Le Palme)	1207		10	0	7
Marseilles and Provence	1100		9	2	0
Malines	1017		8	5	7
Mentz	1335		11	1	5
Mastricht (on the Meuse) and the low countries	1238		10	3	8
Milan { pied decimal	1155		9	7	5
Milan { pied aliprand	1926	1	4	0	6
Modena	2812	1	11	5	2
Monaco	1042		8	8	2
Montpellier (Le Pan)	1050		8	9	0
Moscow	1255		10	5	5
Mantua (La Brasse)	2055	1	5	1	5
Munich	1280		10	8	0
Naples (Le Palme)	1164		9	8	4
Nuremberg { city	1346		11	2	6
Nuremberg { country	1226		10	2	6
Padua	1899	1	3	9	9

2. At what rate per hour are the inhabitants of the following places carried from west to east, by the earth's revolution on its axis ?

Boston, New-York, Washington city, Quito, Cape Horn, Madrid, London, Petersburg, Skalholt, Spitzbergen.

Note. If the velocity per hour be multiplied by any given number of hours, the velocity for that time is given.

MODERN FEET.

		Parts.		Ft.	in.	li.	pts.
The foot of Parma	=	2526	=	1	9	0	6
Pavia		2080		1	5	4	0
Prague		1336			11	1	6
Palermo		1010			8	5	0
The Rhine or Rhinland		1382			11	6	2
Riga		1260			10	6	0
Rome (Le Palme)		990			8	3	0
Rouen (as Paris)		1440		1	0	0	0
Seville (Andalusia)		1340			11	2	0
Stetin (in Pomerania)		1654		1	1	9	4
Stockholm		1450		1	0	1	0
Strasbourg { city		1292			10	9	2
Strasbourg { country		1309			10	10	9
Sien (common foot)		1674		1	1	11	4
Toledo		1237			10	3	7
Turin (Piedmont)		2265		1	6	10	5
Trent		1622		1	1	6	2
Valladolid		1227			10	2	7
Warsaw		1580		1	1	2	0
Venice		1537		1	0	9	7
Verona		1510		1	0	7	0
Vienna, in Austria,		1400			11	8	0
Vienne (Dauphine)		1430			11	11	0
Vicenza (Estates of Venice)		1535		1	0	9	5
Wessel		1042			8	8	2
Ulm (Swabia)		1117			9	3	7
Urbino (Italy)		1570		1	1	1	0
Utrecht		1001			8	4	1
Zurich		1323			11	0	3

Here the learner sees that there are few countries but differ in the length of their foot; and as almost all of them reckon 12 inches (digits) to their foot, the inch must be no less variable. Thus 1440 : $1351\frac{2}{3}$ (or 864 : 811) :: 12 inches : 11 in. 3 li. $1\frac{2}{3}$ pts. Paris measure, the length of an English or American foot. On the contrary $1351\frac{2}{3}$: 1440 :: 12 in. : 12 in. 9 li. 4 pts. the length of a Paris foot in English measure, and so on for any other country. Hence the English inch, foot, &c. is $\frac{811}{864}$ of the Paris, and the Paris inch, foot, &c. is $\frac{864}{811}$ of the English, &c. According to the late French measures, the old French foot =12.78933 English inches, and their metre equal 0,513074 toises =39.371 English inches, or 3.281 feet. 5130740 toises being the length of 90° or a quarter of the meridian. The metre is also equal to 443.296 lines, or 3 feet 11.296 lines, or 0.841 aunes of Paris. See tables of the comparison between the ancient and modern measures in France, published in Paris, by order of the Minister of the Interior, &c. &c. or La Place's System of the World, vol. 1. b. 1 ch. 12. where this subject is handled with that accuracy for which this author is

3. How many geographical miles are the inhabitants of Madrid carried, in 24 hours, more than those of Petersburg, by the earth's revolution on its axis?

4. At what rate per hour are the inhabitants of the north pole (if any) carried, by the earth's diurnal motion on its axis?

PROB. 37.

To find the bearing of one place from another.

Rule. If the places be situated on the same rhumb line, that rhumb line is their bearing; but if not, lay the quadrant of altitude over both places, and the rhumb line, that is the nearest of being parallel to the quadrant, is their bearing.

Note 1. As the parallels of lat. are real east or west, and the meridians north and south rhumb lines, hence those places situated in the same parallel are east or west, and those on the same meridian north and south, from each other.

Or, If the globe have no rhumb lines* drawn on it, make a small mariner's compass, and apply the centre of it to any given

particularly remarkable. The treatise on arithmetic by Theveneau, or L'arithmetique par le Cen. Prevost-Saint-Lucian may be consulted. It remains only to add, that the new French linear measures are the millimetre, centimetre, decimetre, metre, decametre, hecatometre, chilometre, myrcometre, each of which is 10 times the value of the foregoing; the millimetre being 0.039371 English miles. The centimetre is printed wrong in the table in the beginning of the English edition of La Place's Astronomy, 9.39371 being given for 0.39371. The decimetre =0.30784, the centimetre 0.36941 inches, and the millimetre 0.44330 lines, all the other measures being in proportion. The length of a degree in the above tables should be corrected by these measures, in the same manner as 69½ English miles was changed to 69.04, &c. Thus 69½ : 69.04 :: 50 Scotch m. : 49.669 m. The length of the metre above given was found from the arch of the meridian contained between Dunkirk and Barcelona, the length of which is 9° 40′ 25″.6, equal to 551584.70 of the iron toise used at the equator at the temperature 16¼°, the quadrant being divided into 100°, compared with the arch measured in Peru. The above arch from Dunkirk to Barcelona in Spain, is the result of Delambre and Mechain's measures; the astronomical and trigonometrical observations being made with a repeating circle, which gives great precision in the measure of angles. The above 9° 40′ 25′6, is given 6° 40′ 26″2, in the English translation of La Place. Hence, whoever uses this work should take the precaution of making the calculations over again, as there are many errors in the English measures. The other different measures, &c. of the respective countries will be given in the Treatise of Arithmetic, at present nearly ready for the press. The utility of the above table in a mercantile view, no less than in a geographical, will atone for its uncommon length, considering our contracted limits. The length of the note to prob. 31st precluded the tables from being inserted there as their proper place.

* Neither Cary's or Bardin's globes have any rhumb lines on them. On Adams' globes there are two compasses drawn on the equator, one on a vacant place in the Pacific ocean, between America and New Holland; the other in the Atlantic, between Africa and South America. Each point of either of these compasses will serve as rhumb lines. The compass may be easily made by describing a circle on a sheet of paper, or on a

place, so that the north and south points may coincide with some meridian, the other points will shew the bearings of all the circumjacent places, to the distance of more than 1000 miles, if the central place be not far distant from the equator.

Note 2. The latitudes and longitudes of two places being given, the course and distance are given, by this and prob. 31.

Example 1. What is the bearing between the Lizard and the island of Madeira ?

Ans. S. S. W.

card, &c. with a radius of any convenient length, and then dividing its circumference into 32, or each quadrant or 4th part into 8 equal parts, and annexing to each part its appropriate name found on the horizon of the globes. Any two lines drawn through the centre, at right angles to each other, may be first considered the E. W. N. & S. lines. These points may be again divided into halves, quarters, &c. or each quadrant into 90°, &c. The bearing is however found much more correct from *Mercator's sailing*, by the following proportion; Meridional difference of latitude : radius :: difference of longitude : tangent course. Here the diff. lat. and diff. long. are both considered as given. The meridional parts are ready calculated in books on navigation, in tables constructed for that purpose. See McKay's Treatise on Navigation, Hamilton Moore's, improved by Bowditch, Norey, requisite tables, or Robertson's Navigation. When no table of meridional parts is at hand, the defect may be supplied by the following rules: The length of the meridian line for the given lat. according to Wright's projection is equal to $7915.705 \times \log. \frac{\text{radius}}{\text{tang. } \frac{1}{2} \text{ com. lat.}}$ (Emerson's Math. prin. of Geography, Art. Navigation, cor. 3. prop. 4.) Or thus, To half the given lat. add 45°, and find the logarithmic tangent of the sum, and divide it by 7915.7, the quotient will be the meridional parts required, for the sphere. If the meridional parts for the spheriod be required, they may be found thus; As rad : sine lat. :: 30 : x; then this number subtracted from the meridional parts for the sphere, will give the meridional parts answering to the spheriod. (Simpson's Fluxions, vol. 2. sect. 11. prob. 29, and cor.) But the course may be found thus independent of the meridional parts. In the Philosophical Transactions, No 219, it is demonstrated, that the meridional line on Mercator's chart, is a scale of the logarithmic tangents of the half complements of the latitudes; that these logarithmic tangents, of Mr. *Briggs*' form, are a scale of the differences of longitude, upon the rhumb which makes an angle of 51° 38′ 9″ with the meridian; and that the differences of longitude on different rhumbs, are to one another, as the tangents of the angles which these rhumbs make with the meridian. Hence,

As the difference of the log. tangts. of the ½ complements of the latitudes of any two places,
To the difference of longitude between these places,
So is the tangt. of 51° 38′ 9″ (log. 10.1015104)
To the tangent of the course.

All the cases in *Mercator's sailing* may be solved by this rule. See Ewing's Synopsis, 4th ed. pa. 243. In his Mercator's sailing there are given other methods for finding the meridional parts, taken from Robertson's Navigation, 2d ed. pa. 532. In Emerson's treatise above quoted, there are given a variety of examples in sailing on the principles both of the sphere and spheriod, with the investigation of all the rules, &c.

The learner will remark, that Bowditch's Treatise on Navigation has been lately improved and published in England.

2. What is the bearing between the Lizard and St. Mary's, one of the western islands?

Ans. S. 47° 54′ W.

3. Required the bearing between Washington city and any of the following places:

Amsterdam, Annapolis, Berlin, Boston, Brussels, Charlestown, Copenhagen, Dublin, Edinburgh, Lisbon, Madrid, Paris, Philadelphia, Rome, Stockholm, Vienna, and New-York?

4. Required the bearing between Philadelphia and Madrid?

5. Required the bearing between Lima and Washington city?

PROB. 38.

To find the angle of position between any two places.

Rule. Rectify the globe for the lat. of one of the given places (prob. 9) bring that place to the brass meridian, and screw the quadrant of alt. in the zenith, or degree over the given place; then extend the graduated edge of the quadrant over the other given place, and the degrees on the horizon, between the graduated edge of the quadrant and the brass meridian, reckoning towards the elevated pole,* will be the angle of position between that place, for which the globe was rectified, and the other given place.

Note. All those places that are under the edge of the quadrant, have the same angle of position.

* Some choose to reckon the angle of position towards the nearest part of the brass meridian, as in the 2d example above, where 89° is given in place of 91°. It is of little consequence which method is used.

The angle of position between two places is a different thing from their bearing, the latter being determined by a *spiral line* or *loxodromic* (from *loxos,* oblique, and *dromos,* a course) called a *rhumb line,* which makes equal angles with all the meridians through which it passes; but the former by a great circle passing through the zenith of a given place, and another whose position from the former is required, in the same manner as the azimuth is found in astronomy. That the angle of position is therefore essentially different from the bearing, except when the places are on the equator, or upon the same meridian, is sufficiently manifest. But simple as it may appear, it has been the cause of various disputes among writers on the globes; some contending that the angle of position between two places is very different from their bearings, while others suppose that they are the same. Hence a further illustration of this matter may be deemed necessary for young students. By attending to the method given in the above rule of finding the angle of position, we shall find that the part of the quadrant of alt. included between both places, forms the base of a spherical triangle, the two sides of which are the distance of both places from the elevated pole, or the complements of the latitudes of the two places, when they are on the same side of the equator, or the complement of one of the places and the latitude of the other added to 90° (or its complement taken from 180°) when the places are on different sides of the equator; and that the vertical angle, included by both sides, and formed at the elevated poles, is their difference of longitude, the angles at the base of the triangle, being the angles of position between the two places respectively, which are therefore easily calculated by the method given in the note to prob. 31. Thus, *Example* 1. What is the angle of position between Washington city and Dublin ?

Ans. Nearly 48° from the north towards the east. The following places have nearly the same angle of position from Washington, viz. Harwick in England, Antwerp, Cologne, Temeswar, Aleppo, &c.

2. What is the angle of position between London and Prague ?

Ans. Nearly 91° reckoning from the north towards the east, or 89°, reckoning from the south towards the east. The southern

1. *When the two places are situated on the same parallel of latitude.*

Let the two places be the Land's end, and the eastern coast of Newfoundland (as given in the note referred to above) the lat. of which is nearly 50° N. and diff. of long. 48° 50′; the complement of both latitudes is therefore 40°, and hence the triangle is Isosceles, the sides being each 40°, and the vertical angle 48° 50′, from which the base or the arch of nearest distance measured by the quadrant of alt. will be =30° 49′ 6″6, and the angles at the base, or the angles of position each =70° 49′ 30″ (the triangle being Isosceles.) If we now take the middle point in the arch of distance, on the quadrant of alt. its distance from the elevated pole will be 37° 23′, and hence its lat. =52° 37′ N. and the meridian passing through this point will be at right angles to the arch of distance between the two places. An indefinite number of points being now taken along the edge of the quadrant of alt. between the two places, the angle of position between the Land's end and each of these points, will be 70° 49′ 30″ from the north westward. But if it were possible for a ship to sail by the compass on the arch of a great circle passing through these places, indicated by the edge of the quadrant of alt. her latitude would continually increase from the Land's end until she had sailed half her distance to the other place, or from 50° to 52° 37′ N. and her course would vary from 70° 49′ 30″ to 90°. But in sailing the other half of the distance to the eastern coast of Newfoundland, her lat. would continually decrease from 52° 37′ N. to 50° N. and her course must vary from 90° to 70° 49′ 30″ westward. But were the ship to sail along the parallel of lat. passing through both places, her course would then be invariably due west.—Hence it follows, that when the places are situated in the same parallel of lat. their angles of position can never represent their true bearing by the compass, unless when they are on the equator, where their angle of position would be invariably 90 degrees.

2. *If the two places differ both in latitudes and longitudes.*

Let L represent a place in lat. 50° N. B a place in lat. 13° 30′ N. (see the fig. in note to prob. 31) and let their difference of longitude BPL= 52° 58′, the angle of position between L and B will be found by Spher. Trigonometry =S.68° 57′ W. and the angle of position between B and L will be N 38° 5′ E.; whereas the direct course by the compass from L to B, by *Mercator's sailing*, is S. 50° 6′ W. and from B to L it is N. 50° 6′ E. If any number of points be taken in the arch LB, the angle of position between L and each of these points will be invariable, being each=68° 57′, while the angles of position between each of these places and L are continually diminishing. If a ship were, therefore, to sail from L in a S. 68° 57′ W. course by the compass, she would never arrive at B; and were she to sail from B on a N. 38° 6′ E. course by the compass, she would never arrive at L.

Hence the angle of position, in any case, can never represent the bearing, except, as before remarked, the places be on the equator, or on the meridian.

extremity of the Caspian sea, the mouths of the Indus, Bombay, the southern extremity of the island of Ceylon, &c. have nearly the same angle of position.

3. What is the angle of position between Dublin and Washington city?

Ans. 78° nearly, reckoning from the north westward.

4. Required the angles of position between New-York and the following places:

Petersburgh, Copenhagen, London, Paris, Constantinople, Cairo, Cape Verd, Bermudas, St. Domingo, Cape Nicholas do. New Orleans, Mexico, and the East Cape in Bhering's strait.

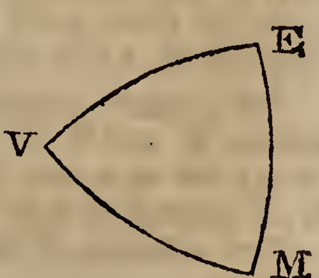
PROB. 39.

The distance of two places situated on the same meridian, and their angles of position with a third place, being given, to find that place, with its nearest distance from each of the other two.

Rule. Rectify the globe for the lat. of the first place, and screw the quadrant of alt. in the zenith, bring the given place to the meridian, extend the quadrant to the degree on the horizon which is equal to its position from the third, draw a line along the graduated edge of the quadrant; then elevate the pole to the lat. of the second place, bring it to the meridian, and screw the quadrant over it, which extend, as before, to the degree on the horizon which is equal to its position from the third; the intersection of the quadrant with the line drawn before, will give the third place required; the distance of which from the former two is found as in prob. 31st.

Example 1. The distance between Madrid and Edinburgh, situated on the same meridian, is 15° 33′=933 geographical miles; and the angle of position with a third is 53½° nearly, and of the latter 66° from the south nearly; required the place, with its nearest distance from each of the former?

Ans. The required place is Vienna, its distance from Madrid =922, and from Edinburgh 847 geographical miles, nearly.*



* This prob. may be calculated as follows: Let M represent Madrid, E Edinburgh, and V the required place; then there are given the distance ME between the given places = 933 miles, the angle MEV = the angle of position between Edinburgh and the place required = 66°, and EMV the angle of position between Madrid and the place V = 53½°, to find the sides VE, VM, the distance of the required place from each of the former. Hence by spherical trigonometry, we have as *cosine* of half the sum of the two angles of position : *cosine* of half their difference :: *tangt.* of half the distance of the given places E and M : *tangt.* of half the sum of the sides VE, VM (the required distances.) And *sine* of half the sum of the angles of position : *sine* of half their difference :: *tangt.* of half the given distance (EM) : *tangt.* of half the difference of the required distances; then to half the sum add half the difference, and

2. The distance between Halifax, in lat. 44° 40′ N. and the north-east part of Margarita island, in the West-Indies, lat. 11° 10′ N. both situated nearly under the same meridian, being equal to 33° 36′ or 2016 miles, and the angle of position of the former with a third being equal 62° from the south, and of the latter with the same place 75½° ; required the place, with its nearest distance from each of the foregoing places ?

PROB. 40.

Given the course and distance, to find the latitude and longitude come to, the place left being known.

Rule. MARK the given rhumb in the lat. of the place left, bring that mark to the meridian which passes through the long. left, convert the distance sailed into degrees, take one degree from the equator, in a pair of compasses, and turn it over on the rhumb as often as there are degrees in the given distance, and where the reckoning ends will be the place required, whose lat. and long. is found as in prob. 1.

Note. If the rhumb does not pass through the given place, find the longitude of the place where the reckoning ends, and the number of degrees between this and the longitude of the first mark on the rhumb line, will be the difference of longitude, whence the long. come to is found by note 4. prob. 4. Where no rhumb lines are given, a small mariner's compass made on paper, will answer.*

Example 1. A ship from Cape Clear, in lat. 51° 18′ N. and long. 11° 15′ West, sails S. E. ¼ S. 480 miles; required the lat. and long. come to?

Ans. The place required is in lat. 45° 22′ N. and long. 3° 9′ W.

2. A ship from New-York or Sandy-Hook light-house, in lat. 40° 28′ N. long. 74° 7′ W. sails E. N. E. 1200 geographical miles ; required the lat. and longitude the ship is in ?

you have the side opposite the greater angle of position given or the side VM, and from half the sum, take half the difference, and you have the side opposite the lesser angle given, or the side VE.

Note. That half the sum of the required sides will be of the same affection as half the sum of the given angles, and the contrary.

In the above prob. the bearing may be made use of instead of the angle of position, when the given distance is small.

* If a small compass, made of paper, be used, it may be always easily placed N. and S. by the meridians on the globe, or rather the brass meridian ; but as it may be difficult to place the centre on the given place exactly, a quarter or half of the compass will answer better.

The solution of the prob. on the principles of Mercator's sailing, is as follows :

Rad. : cos. course :: distance : difference of latitude, and

Rad. : tangt. course :: meridional diff. of lat. : difference of longitude.

The difference of latitude and difference of longitude being thus given, the latitude and longitude arrived at may be found by the method given in note 4. prob. 4th. part 2.

To enter into the investigation of the principles on which the above proportions are founded, would be foreign to our intended plan.

3. A ship from the Lizard, in lat. 49° 57′ N. long. 5° 21′ W. sails S. 47° 51′ W. 1162 miles; required the lat. and long. of the place the ship is in?

PROB. 41.

*Both latitudes and course given, to find their distance and difference of longitude.**

Rule. TURN the globe on its axis until the given rhumb cuts the brazen meridian in the lat. left, there mark the rhumb under the given degree of lat. and observe the degree of the equator cut by the brass meridian; then turn the globe until the same rhumb cuts the meridian in the lat. come to, under which on the rhumb make a mark as before; the number of degrees between these two marks, reckoned on the equator, will give their difference of longitude; and the distance is found by taking a degree of the equator in a pair of compasses, and extending it on the rhumb, between the two marks, as often as possible, the number of degrees, thus measured, being converted into miles, will give the distance required.

Note. If the globe has no rhumb lines described on it, a compass made of paper may be used as in the foregoing problems; in which case the rhumb will cut the meridian passing through the given place, or the lat. left, without first turning the globe. The shorter the radius of such a compass is, the more correct will the distance be; in which case it will be often necessary to find different centres in the same rhumb, &c.

Example 1. A ship from the Lizard, in lat. 49° 57′ N. makes her course S. 39° W. and then by observation is in lat 45° 31′ N. required her distance run and longitude in?

Ans. The difference of longitude being 5° 21′ W. and the long. of the Lizard equal 5° 15′ W. hence the long. is 10° 36′ W. and the distance is 342 miles.

2. A ship from Bayonne, in lat. 43° 29′ N. and long. 1° 30′ W. sails N. W. ½ N. until by observation she is in lat. 51° 31′ N. required the distance run and longitude come to.

* The proportions for calculating this prob. are as follow:
Rad. : secant course :: diff. lat. : distance,
Rad. : tang. course :: meridional diff. lat. : diff. long.
In the same manner may other problems in navigation be performed on the globes, the above being well understood.
For the sake of readers not in the habit of using the Nautical Almanac, it may not be improper to remark, that in pa. 96 of the Naut. Alm. for 1813, revised by John Garnett, there is a table for correcting the *middle latitude*, which renders the calculation by this method more expeditious, and as accurate as in Mercator's sailing. There is also in pa. 168 of the same almanacs, for 1812 and 1813, a table, shewing, very nearly, the difference between a ship's direct course in a great circle, and that found by Mercator's, or mid. lat. sailing. (See the note to prob. 38, part 2.)

PROB. 42.

To find the meridian altitude of the sun, on any day, at any given place.

Rule. ELEVATE the pole to the latitude of the given place, bring the sun's place to the meridian, and the degree over it will be the declination, the number of degrees reckoned from which to the horizon, will give the meridian altitude required.*

Or, Elevate the pole to the sun's declination, bring the given place to the brass meridian, and the number of degrees between it and the horizon, will be the meridian altitude required.†

OR BY THE ANALEMMA.

The globe being rectified to the latitude, bring the given day found on the analemma to the brass meridian, the number of degrees between which and the horizon will be the alt. required.

Example 1. What is the sun's meridian altitude at New-York, on the 10th of May?

Ans. 66° 56′.

* See note to prob. 1. and prob. 25. † See prob. 24.

If the learner be not accustomed to use the quadrant or sextant of reflection, and yet wish to perform this prob by observation, he may use a common quadrant with a plummet. These are to be had at the instrument makers, with lines, sometimes drawn on them, for finding the hour of the day, the sun's azimuth, &c. or they may be easily made of wood or slate sufficiently correct, where exactness is not required. It would however be better to have a quadrant, or rather semicircle, immoveable in the place of the meridian, and divided into degrees and their lesser parts, according to art (using either the nonius, as in Hadley's quadrant, or a scale divided diagonally) and having an index moveable on its centre, furnished with telescopic sights. But whoever wishes to use *the improved astronomical circle,* will have, with a good telescope and watch, all the astronomical apparatus necessary.

Note. The complement of the lat. added to the sun's declination, where they are of the same name (that is both north or both south) or subtracted when they are of different names (that is one north and the other south) will give the sun's meridian altitude.

The declination being a necessary requisite for solving this prob. is found in the 2d page of every month in the Nautical Almanac. (See the Nautical Almanac, published with important additions, under the direction of *John Garnett,* New-Brunswick, New-Jersey, where the daily difference of declination is given to reduce it to the meridian of Greenwich, &c.) The declination may be obtained by knowing the meridian altitude and latitude of the place, for as the co. lat. + sun's decl. = sun's mer. alt. (when the decl. and lat. are of the same name) hence in this case, sun's decl. = mer. alt. — co. lat. Again, when the decl. and lat. are of different names, co. lat. — decl. = mer. alt. hence declination = co. lat. — mer. alt. This latter only takes place when the complement of the lat. is greater than the declination; when less, the contrary sign must be used. The decl. and mer alt. being given, the lat. may be found from the same equations; thus, in the 1st equation, co. lat. = mer. alt. — sun's decl. &c. In the same manner from various other problems, a variety of conclusions may be drawn, with only a slight knowledge of the nature of equations in Algebra.

The Nautical Almanac is also lately published by E. M. Blunt, in New-York.

2. What is the sun's meridian altitude at Washington city, on the 21st of June?

Ans. 74° 35′.

3. What is the sun's meridian altitude at Philadelphia, when the days and nights are equal?

4. What is the sun's greatest altitude at New-York?

5. What is the sun's meridian alt. at Quito, on the 22d of December?

6. What is the sun's greatest meridian alt. at Cape Horn?

PROB. 43.

To find the sun's altitude by placing the globe in the sunshine.

Rule. MAKE the plane of the horizon on the globe truly level or horizontal, then erect a needle perpendicularly over the north pole, or in the direction of the axis of the globe, and having turned the pole towards the sun, move the brass meridian until the needle casts no shadow; then the arch of the meridian between the pole and the horizon, will give the sun's altitude. (See prob. 28.)

Or in general, Turn the north or south pole towards the sun, erect a needle, as before directed, towards the earth's centre on that part of the brass meridian where it will cast no shadow, and the degrees between it and the horizon will be the altitude required.

PROB. 44.

To find the sun's altitude for any time at any given place, independent of the foregoing method.

Rule. RECTIFY the globe for the latitude, screw the quadrant of altitude in the zenith, bring the sun's place for the given time to the brazen meridian, and set the index to 12; turn the globe on its axis until the index points out the given hour, extend the graduated edge of the quadrant of altitude over the sun's place, and the degree cut on it will be the sun's altitude.*

Or, Elevate the pole to the sun's declination, screw the quadrant of alt. in the zenith, bring the given place to the brass meridian, and set the index to twelve; then if the given hour be in the forenoon, turn the globe westward, but if in the afternoon, eastward, as many hours as the time is before or after twelve; extend the quadrant of alt. over the given place, and the degree cut on it will be the sun's altitude.†

* The reason of this method is evident from what is said in prob. 25.

† The reason of this rule is clear from what is delivered in prob. 24.

The prob. may be solved in numbers thus; the lat. day, and hour being given.

1 Rule. Here, to find the altitude, there are given the complement of the lat. the hour angle (or the angle formed between the brass meridian and the meridian passing through the sun's place) and the complement of the sun's declination. The learner will perceive that with these the comple-

Example 1. What is the sun's alt. at New-York, on the 10th of May, at 6 o'clock in the morning?

Ans. 11½° nearly.

2. What is the sun's alt. at Washington city, on the 21st of June, at 3 o'clock in the afternoon?

Ans. 51½°.

3. What is the sun's altitude at Philadelphia, on the 21st of March, at 10 o'clock in the morning?

4. What is the sun's altitude at Quito, on the 1st of January, at 1 o'clock in the afternoon?

PROB. 45.

To find all those places where the sun has the same altitude as any given place, at any given time.

Rule. Find where the sun is vertical at the given time (by prob. 12) mark this place, and find its distance from the given place (by prob. 31) find all those that are at the same distance from it as the given place (by prob. 32) these will be the places required.

Example 1. When it is 15 minutes after 8 in the morning at New-York, on the 30th of April, required all those places where the sun, at that moment, will have the same altitude as in New-York?

Ans. The place where the sun is then vertical being Cape Verd, those places that are at the same distance from it as New-York are Quebec, Moskitto Cove in west Greenland, the middle of the gulf of Bothnia and Finland, near Mecca, the middle of Abyssinia, Cape Volta in Caffraria, the western part of St. Domingo, Cumberland Harbour in Cuba, St. Salvador in the West Indies, &c.

ment of the sun's altitude will form a triangle, whose two sides and the included angle are given, to find the base, or the complement of the sun's altitude. Hence rad : cos. hour angle :: cot. latitude : tang. x, the segment between the pole, and a perpendicular from the zenith on the meridian passing through the sun's place; then cos. x : s. lat. :: cos. remaining segment (comp. decl. less x) to sine altitude required. (See Emerson's Trig. b. 2. sec. 4. case 8. or Simson's Euclid general, prop. case 4, &c. Sp. Trig.)

2 Rule. Here the brass mer. the meridian passing through the given place, and the quadrant of alt. form a spherical triangle, the two equal sides of which, or the complement of the decl. and co. lat. and the included angle (or hour angle) are given, to find the third side, or complement of the sun's altitude, which is found exactly as above. The hour angle is converted into degrees by allowing 15° for every hour.

* As a ray of light from the sun, conceived at an infinite distance from the earth, will make equal angles with the tangent touching the globe at each of the above places, which represents their horizon, and that the altitude of the sun is its height above the horizon, hence the reason of the rule is evident. The same reasoning is applicable to several other problems where the sun's alt. is required.

2. When it is 4 o'clock in the afternoon at London on the 18th of August, find all those places where the sun will then have the same altitude as in London?

3. Find all those places where the sun will have the same altitude as at Philadelphia, at 12 o'clock the 21st of March?

PROB. 46.

To find the sun's altitude at any place in the north frigid zone, where the sun does not descend below the horizon, when it is midnight at any place in the temperate or torrid zones, on the same meridian.

Rule. Elevate the pole to the lat. of the place in the frigid zone, bring the sun's place to the brass meridian, and set the index to twelve; turn the globe on its axis until the other twelve comes to the meridian, and the number of degrees between the sun's place and the horizon, counted on the brass meridian towards the elevated pole, will be the altitude required.

Or, Elevate the pole to the sun's declination for the given day; bring the place in the frigid zone to that part of the brass meridian which is numbered from the pole towards the equator, and the number of degrees between it and the horizon will be the sun's altitude.

Example 1. What is the sun's alt. at the South Cape in Spitzbergen, in lat. $76\frac{1}{2}°$ N. when it is midnight at Naples, on the 10th of May?

Ans. 4 degrees.

2. What is the sun's altitude on the 21st of June at the North Cape in Lapland, when it is midnight at Adrianople in Turkey in Europe?

3. What is the sun's altitude at the northwest part of Spitzbergen, latitude nearly 80°, when it is midnight at Cagliari in Sardinia?

PROB. 47.

To place the terrestrial globe in the sunshine, so as to represent the natural position of the earth.

Rule. Place the globe north and south by the mariner's compass (allowing for variation, if any, see note to prob. 49) or by a meridian line,* bring the place where you are situated to the meridian, and elevate the pole to its latitude; then the globe will correspond in every respect to the situation of the earth itself. All the circles, &c. on the globe will correspond to the same imaginary circles, &c. in the heavens; and each town, kingdom, state, &c. will point out the position of the real one which it represents, &c. See probs. 24 and 25.

* The method of drawing a meridian line is shewn in several of the following problems, but more particularly in prob. 75, part 2.

PROB. 48.

The latitude and day of the month being given, to find the hour of the day when the sun shines.

Rule. 1. PLACE the wooden horizon of the globe truly level or parallel to the horizon of the place, and the brazen meridian due north and south ; elevate the pole to the lat. bring the sun's place to the brass meridian, and set the index to 12 ; fix a needle perpendicularly over the sun's place in the ecliptic, turn the globe on its axis until the needle casts no shadow, and the index will point out the hour.

Or, The globe being placed horizontally, due north and south, and rectified for the lat. as before ; then if a long pin be fixed perpendicularly on the brass meridian, in the direction of the axis, and in the centre of the hour circle, and 12 on the hour circle be brought to the meridian, the shadow of this pin will point out the hour of the day.

Note. If the place be in north lat. and the decl. be N. the sun will shine over the north pole ; but if the declination be more than 10° south (nearly the radius of the hour circle) the sun will not shine upon the hour circle at the north pole.

Or, The equator being divided into 24 equal parts from the point aries, on which place the number 6, and then westward on the other points 7, 8, 9, 10, 11, 12, 1, 2, &c. to 6, which will fall on the point libra, 7, 8, &c. to 12, then again 1, 2, &c. to 6 ;* then place the globe horizontal, north and south, and rectify as before ; bring aries to the meridian ; observe the circle which is the boundary between light and darkness, if westward of the brass meridian, and it will intersect the equator in the given hour in the morning ; but if eastward, it will intersect the equator in the given hour in the afternoon.

Or, Having placed the globe as before, and the point aries being brought to the meridian ; tie a small string round the elevated pole, stretch its other end beyond the globes, and move it so that the shadow of the string may fall upon the depressed pole ; its shadow on the equator will then give the hour.

* The antartic circle on Adams' globes is thus divided, by which the problem may therefore be solved.

The altitude of the sun (which is equal to the number of degrees between the needle placed as above when it casts no shadow and the horizon, reckoning on a verticle circle) and the lat. and day of the month being given, the solution by spherics may be as follows :

$$\text{Cos. decl.} \times \text{cos. lat.} : R^2 :: \text{sine } \tfrac{1}{2} \times \overline{\text{co. decl.} + \text{co. lat.} + \text{co. alt.}} \times \text{sine } \tfrac{1}{2} \times \underline{\text{co. decl.} + \text{co. lat.} - \text{co. alt.}} : \text{cos. } \tfrac{1}{2} h^2$$

(h being the hour angle) which converted into time, will give the time from *apparent* noon. (See the note to prob. 11. part 3.)

PROB. 49.

*The latitude of the place and day of the month being given, to find the sun's amplitude, right ascension, oblique ascension, oblique descension, ascensional difference, and time of rising and setting.**

Rule. ELEVATE the pole to the given latitude ; bring the sun's place to the brass meridian, and the degree cut on the equator, reckoned from aries eastward, will be the sun's right ascension. The globe being then turned on its axis, until the sun's place comes to the eastern part of the horizon, the degree of the equinoctial cut by the horizon, reckoning from aries as before, will be the sun's oblique ascension, and the degree cut on the horizon, reckoning from the east, will be the sun's amplitude at rising. The globe being now turned again on its axis, until the sun's place comes to the western part of the horizon, the degree on the equinoctional cut by the horizon, reckoning from aries eastward, as before, will be the sun's oblique descension, the degree cut on

* To perform this prob. by calculation, the learner will first perceive that the sign and degree of the sun's place reckoned from aries, or the sun's longitude, the obliquity of the ecliptic (or its inclination with the equator) or the sun's declination, are requisite to find the sun's rt. ascension. The sun's rt. ascension, longitude, and declination, forming a right angled spherical triangle. Now the obliquity of the ecliptic may be found thus : let the sun's least distance from the vertex about the summer (or winter) solstice be observed ; this distance subtracted from the lat. of the place, when the place is nearer to the pole than the sun is, or added when the sun is nearer, will give the greatest declination of the sun, or the obliquity required, allowance being made for refraction, &c. particularly if the observation be made at the winter solstice. If the solstice should not take place when the sun is on the meridian (as it generally happens) allowance must be made. The error, however, is not worth observing here, as it never arises to more than 4″ when greatest, that is, when the solstice happens at midnight, being equal to what the sun's declination, 12 hours before or after the solstice, wants of its greatest declination. Professor Mayer in his Solar and Lunar Tables, gives a method of calculating this obliquity, having found from observations made with an excellent mural quadrant, at both solstices, in 1756, 57, and 58, that the mean obliquity of the ecliptic in the beginning of 1756, was 23° 28′ 16″, and the decrease in 100 years is about 46″; whence the mean obliquity for any other year, month, or day, may be easily found by proportion. Thus the mean obliquity for the beginning of 1811 is 23° 27′ 50″ 7 ; now to find the true obliquity, the nutation, &c. must be found as directed in prob. 4 of Mayer's, which is here =—9″ 6 ; so that the true or apparent obliquity for the beginning of 1811 was 23° 27′ 41″1, agreeing nearly with the Nautical Almanac for 1811. The greatest nutation according to Mayer is 9″6. From a like calculation it will be found, that the obliquity varies considerably in the space of one year. For on the first of January, 1811, according to the Nautical Almanac, the obliquity was 23° 27′ 41″8; on the 1st of April 23° 27′ 42″7 ; on the 1st of July 23° 27′ 41″8 ; on the 1st of October 23° 27′ 42″7 ; and on the 31st of December 23° 27′ 41″9. (See prop. 34 of Emerson's Centripetal forces. The reader is also referred to La Grange or De La Place's Physical Theories, or to Mayer's, printed in London in 1770, under the direction of Nevil Maskelyne, A. R.) For the beginning of 1811, N. Maskelyne makes the mean obliquity 23° 27′ 50″9, and corrects it by his folio tables 31 and 32. For the beginning of 1813, he makes it 23° 27′ 51″3, and makes the secular variation 42″6. For more information on this subject, &c.

the horizon, reckoning from the west point of it, will be the sun's amplitude at setting, and the difference between the sun's right ascension and oblique ascension, or descension, or which is the same, the time between the index at either of these positions and the hour of six, is the ascensional difference, which in the former case must be converted into time (by prob. 6) then if the sun's declination and the lat. of the place be both of the same name, that is both north or both south, the sun rises before, or sets after six, by a space of time equal to the ascensional difference; but if the latitude and sun's declination be of contrary names, that is one north and the other south, the ascensional difference will shew how long the sun rises after six, or sets before six.

Note 1. The ascensional difference reduced into time, and added to or subtracted from 6 o'clock, gives the length of half the day or *semi-diurnal arch*, the complement of which to a semicircle, or to 12 hours, will give the length of half the night or *semi-nocturnal arch*: or the time of the sun's continuance above the horizon, may be found by reckoning the number of hours on the upper part of the hour circle between the places where the index pointed when the sun's place was at the eastern and western parts of the horizon, or by prob. 13. See also probs. 23, 24, 25.

Note 2. From this prob. and prob. 8, the learner will observe that the method of finding the sun's right ascension and declination in the heavens, is the same as finding the latitude and longitude of a place on the earth, with this difference, that the rt. as. is reckoned quite round the globe.

consult Mason's Tables of 1780, Wargentine's Tables published at the end of the Nautical Almanac for 1779, La Land's Astronomy, 3d edition, for 1792, where accurate tables of the sun, moon, and planets, and of the eclipses of Jupiter's satellites are given; these being constructed principally by Delambre on the best observations, and on the Physical Theories of M. La Grange and M. De La Place, founded on Newton's Theory of Gravity. But the late lunar tables of Mr. Burg of Vienna, constructed principally on the observations of Maskelyne, is looked on by this astronomer as the most correct. Mayer's tables and precepts of calculation are given in the Philadelphia edition of the Encyclopedia. These observations being useful to direct the study and choice of the young astronomer, we think it necessary to caution him, at the same time, against several remarks found in some of these works, tending to favour impiety, and impose on superficial minds. We shall make it a particular study in our intended course, to point out the dangerous tendency and falsehood of such principles, assumed, for the most part, without a shadow of proof. And thus we hope to be able to present our young students with the most valuable observations and improvements of past ages, without any danger to the more valuable deposit which, as christians, enlightened by truths far more important, more consoling and sublime, they are in possession of. For truth is always consistent with itself. But to proceed.

Having now obtained the obliquity of the ecliptic, or the sun's greatest declination, and the present declination being obtained by note to prob. 42 (see prop. 8.) the rt. ascension is found by this proportion, Rad. : co. t. sun's greatest declination :: tangt. present decl. : sine rt. ascension. (Napier's rules.) Now to find the oblique ascension, amplitude, &c. the learner will observe, that the globe, being placed as above directed for finding the amplitude, &c. the amplitude reckoned on the horizon, the sun's declination, and the ascensional difference, form a right angled spherical triangle, and the inclination of the plane of the equator with the horizon being equal to the complement of the lat. is also equal to the opposite vertical angle (Emer-

Example 1. Required the sun's amplitude, right ascension, oblique ascension, oblique descension, ascensional difference, and time of rising and setting at New-York, in lat. 40° 42′ 40″, on the 21st of June?

Ans. The sun's amplitude at rising and setting is $30\frac{1}{4}$°, the right ascension is 90°, oblique ascension 68°, ascensional difference 22°, or 1h. 28′. Hence 6 — 1h. 28′ = 4h. 32′ = time of sun rising, and 6 + 1h. 28′ = 7h. 28′, time of sun setting.

son's Trig. cor. 2. prop. 3. b. 3) or the angle formed by the arches expressing the amplitude and the ascensional difference; hence we have these proportions; Rad. : tangt. lat. :: tangt. decl. : sine of the ascensional difference; which subtracted from the right ascension, when the declination is north, or added when the decl. is south, will give the oblique ascension when the place is in north lat. but when the declination is north, the as. diff. must be added, &c. when the lat. is south. The oblique descension, &c. is found in like manner. Again, to find the amplitude, it will be cos. lat. : sine decl. :: rad. : sine ampl. (Napier's rule, as above.)

It may not be improper to remark here, that the point of the compass on which the sun rises and sets being known, the magnetic amplitude is given, being equal to the distance from this point to the east or west points of the horizon respectively; and that the difference between this magnetic amplitude and the true amplitude found above, is the *variation of the compass,* if both be of the same name, that is both north or both south; but if they be of different names, that is one north and the other south, their sun is the variation. To know whether the variation be east or west, this rule must be observed; the observer's face being turned towards the sun; then if the true amplitude be to the right hand of the magnetic, the variation is easterly, but if to the left hand, westerly. The variation may be also found by taking the sun's alt. in the morning, and at the same time its bearing, and likewise in the afternoon when its alt. is the same; the middle point will be the *meridian,* the difference between which and the N. and S. points of the compass, will be the variation. If in place of taking equal altitudes of the sun, the points of the compass on which it rises and sets be observed, then half the difference will be the variation as before. The instrument calculated to make this observation with, though not generally very exact, is an *azimuth compass,* for the description and use of which the reader is referred to McKay's Complete Navigator, or the Encyclopedia. The astronomical circle answers the purpose of an azimuth compass, transit instrument, theodolite in surveying, &c. and is extremely exact. In New-York the variation for 1810 was about 3° west. (See definition 45.) In Washington city the variation for 1811 is nearly 0; along the coast of the United States the variation is decreasing. As the declination of the sun at rising or setting differs from his declination at noon, found in the Nautical Almanac, and in the former is used in finding the amplitude by calculation, the following proportion is necessary; as 24 hours is to the hours from sun rising, so is the daily variation of declination to a fourth number, which must be added or subtracted according as the declination is increasing or decreasing, &c. In the same manner proportion may be made for the right ascension, &c. Allowance must also be made if the meridian differ from that of Greenwich.

The longitude of the sun is easily found by prob. 3, 8 or 10 of Mayer's tables, &c. his hourly motion by prob. 6, rt. ascension by prob. 7, declination by prob. 9, sun's parallax by prob. 12, and refraction by prob. 13; the two last articles being necessary in finding the correct alt. of the sun. However the learner is desired to make use of *Burg* and *Delambre's* tables translated and corrected by Vince, and lately published in England, being the most valuable now extant. These are the tables, at present, principally used in calculating the Nautical Almanac.

2. What is the sun's amplitude, right ascension, oblique ascension and descension, ascensional difference, and time of rising and setting at Washington city, on the 10th of May?

3. On the 21st of December, what is the sun's amplitude, right ascension and declination, oblique ascension and descension, sun's rising and setting, and length of the day and night at London?

Note. At the vernal equinox the sun has no amplitude, rt. ascension or declination, no oblique ascension or descension, and therefore no ascensional difference; it rises and sets at six, making the days and nights each equal 12 hours all over the world.

PROB. 50.

The latitude, day, and hour being given, to find the sun's azimuth and his altitude.

Rule. RECTIFY the globe for the lat. zenith, and sun's place (prob. 9) then the number of degrees between the sun's place and the vertex is the sun's meridional altitude. The index being then set to 12, turn the globe eastward* if the time be in the forenoon, or westward if the time be in the afternoon, as many hours as the time is before or after 12 o'clock; the quadrant of altitude being then extended over the sun's place, the degrees cut by it on the horizon, reckoning from north to south, will give the azimuth, and the degrees from the horizon to the sun's place, reckoned on the quadrant of alt. will give the sun's altitude.

OR BY THE ANALEMMA.

Rectify the globe as before; bring the middle of the analemma to the brass meridian, and set the hour circle to 12; then the globe being turned as before, bring the graduated edge of the quadrant of alt. to coincide with the day of the month on the analemma, and the number of degrees on the horizon, cut by the quadrant, as before, will be the azimuth, and the number of degrees from the horizon, where the day of the month cuts the quadrant, will be the altitude.

Example 1. What is the sun's altitude and his azimuth at New-York, on the 10th of May, at 9 o'clock in the morning?

Ans. The alt. is $45\frac{1}{2}°$, and the azimuth $107\frac{1}{2}°$ from the north, or $72\frac{1}{2}°$ from the south.

2. What is the sun's altitude and azimuth at Boston, on the 10th of June, at 6 o'clock in the morning, and also his meridian altitude?

* Whenever the pole is rectified for the lat. the proper motion of the globe is from east to west, and the sun is on the east side of the brass meridian in the morning, and on the west in the afternoon; but when the pole is elevated for the sun's declination, the motion is from west to east, the place being on the west side of the meridian in the morning, and on the east side in the afternoon.

3. What is the sun's azimuth and altitude at St. Domingo, at 7 o'clock in the morning, and also at a quarter past 10, on the 10th of June ?*

4. Required the time of the sun's appearing twice on the same azimuth, both in the forenoon and in the afternoon, at Barbadoes, on the 20th of May ?

5. Being at sea, in lat. 57° N. on the 13th of August, I observed that the azimuth of the sun was 40° 14′ from the south, at half past 8 o'clock in the morning, what was the sun's alt. his true azimuth, and the variation of the compass? (See the notes.)

6. On the 14th of January, in lat. 33° 52′ S. at half past three o'clock in the afternoon, the sun's magnetic azimuth was observed to be 63° 51′ from the north ; required the true azimuth, variation of the compass, and the sun's altitude ?

PROB. 51.

Given the latitude, day and hour, as in the last prob. to find the depression of the sun below the horizon, and his azimuth at any hour of the night.

Rule. RECTIFY the globe for the latitude, zenith, and sun's place, as before ; take that point in the ecliptic exactly opposite to

* Whenever the declination of the sun exceeds the lat. and both are of the same name, the sun will appear twice in the forenoon, and twice in the afternoon, on the same point of the compass, at all places in the torrid zone; and will cause the shadow of an azimuth dial, to go back several degrees. In this example the sun's azimuth at 6 is N. 68° E.; at 7, N. 71° E.; at ½ past 7, 72° from the north ; at 8, 73° ; at 8½, 73½° ; at 9, 73½° ; at 9½, 72½° ; at 10, 71¼° ; at 10¼, 71° ; at 11, 61½° ; and at 11½, 39° from the north.

To perform the prob. by calculation, the learner will observe on the globe, that the complement of the latitude reckoned on the brazen meridian, the complement of the altitude reckoned on the quadrant of alt. and the complement of the sun's declination reckoned from where the quadrant cuts the ecliptic in the sun's place, to the pole (the globe being rectified, &c. as above, and turned until the index points at the given hour) form a spherical triangle ; that the angle formed by two of these sides, *i. e.* the comp. of the lat. and the comp. of the decl. is the hour from noon converted into degrees, &c. and that the azimuth is the angle formed by the comp. of the lat. and complement of the altitude. This being premised, the azimuth is found as follows : conceive a perpendicular arch to be drawn from the sun's place on the brazen meridian (the globe being rectified, &c. as above) then will rad. : cos. hour angle :: co. tangt. decl. :: tangt. x, a fourth arch or segment of the base (or base produced) between the pole and perpendicular on the mer. which being therefore given, the remaining segment, between the zenith and perpendicular, is given, which call y; then sine x : sine y :: co. tangt. of the hour angle : co. tangt. of the azimuth south ; which if reckoned from the north, is greater or less than a quadrant or 90°, according as the perpendicular falls north or south of the zenith. (See Emerson's Trig. b. 3. part 4, case 7.) To find the altitude. Sine azim. : s. 90° ± decl. :: s. hour angle : cos. altitude. (See prob. 9. part 2.) From this prob. the variation of the compass may be obtained, being the difference between the true azim. and the magnetic, or azim. observed by a compass.

the sun's place, and find its altitude and azimuth as in the preceding prob. and these will be the depression and azimuth required.

Example. What is the sun's depression and azimuth at New-York, on the 12th of November, at 9 o'clock at night?

Ans. The alt. will be the same as in ex. 1, of the preceding, and the azimuth likewise the same; but reckoned contrary, that is, $72\frac{1}{2}°$ from the north, or $107\frac{1}{2}°$ from the south. In the same manner may any of the examples in the foregoing problem be changed and performed.

PROB. 52.

Given the latitude, the sun's place and altitude, to find the sun's azimuth and the hour of the day.

Rule. Rectify the globe for the lat. zenith, and sun's place, and set the index to twelve; turn the globe eastward or westward (according as the altitude is given in the forenoon or afternoon) until the sun's place coincides with the given degree of altitude on the quadrant; then the hours passed over by the index, will shew the time from noon, and the quadrant will point out the azimuth on the horizon, as before.

OR BY THE ANALEMMA.

The pole being elevated for the lat. and the quadrant screwed in the zenith as before; bring the middle of the analemma to the brass meridian, and set the index to 12; turn the globe as before, moving the quadrant, at the same time, until the day of the month coincides with the given altitude; the hours passed over by the index will give the time, and the azimuth will be found on the horizon as before.

Example 1. At what hour of the day in the forenoon of the 21st of June, is the sun's altitude 30° at New-York, and what is his azimuth?

Ans. The time from noon is 7 hours 20 minutes, and the azimuth $83\frac{1}{4}°$ from the north towards the east.

Note 1. This prob. is performed more accurately with the hours on the equator than with the hour circle. On Cary's twenty-one inch globes the hours, quarters and single minutes are marked on the equator, and the half minutes may be also distinctly pointed out. In performing the problem, the learner should make the 0 on the quadrant coincide with the horizon, by drawing the end of the quadrant tight with one hand, adjusting it at the same time to the lat. and turning the globe with the other.

2. At what hour on the 21st of March, in the afternoon, is the sun's altitude $22\frac{1}{4}°$, and what is his azimuth?

3. On the 10th of May the sun's altitude at Washington city was observed 40° 25′; required the hour of the day and sun's azimuth, the observation being made in the forenoon?

4. In New-York, on the 10th of March, having observed that the shadow of a perpendicular object was exactly equal to its height, it is required from hence, to find the hour of the day when the observation was made, supposing it to have been made in the

morning, the point of the compass on which the shadow was projected, and the sun's azimuth?

Note 2. The length of the shadow of perpendicular objects is equal to their heights when the sun's alt. is 45°, as appears from the 6th and 13th problems of the 1st book of Euclid; and the point of the compass is shewn by the quadrant of alt. or azimuth circle, for though in reality this be the sun's position, yet in the small compass of our horizon, it agrees accurately enough with the bearing.

PROB. 53.

Given the latitude, the sun's place and his azimuth, to find his altitude and the hour of the day.

Rule. RECTIFY the globe for the lat. screw the quadrant of alt. in the zenith, bring the sun's place to the brass meridian, and set the index to twelve; then the quadrant being set to the azimuth on the horizon, turn the globe until its graduated edge meets the sun's place, the degree cut on the quadrant will be the altitude, and the index will point out the hour.*

* Here the complement of the altitude, the complement of the latitude, and the sun's declination subtracted from or added to 90°, according as it is of the same or of a different name from the latitude, will form a triangle, and the acute angle included between the brass meridian and the quadrant will be the azimuth; if this fall within the triangle, it will be the angle included between the comp. of the alt. and comp. of the lat. but if it be without the triangle, its supplement, or what it wants of 180°, will be the angle included by the above sides. There are therefore given two sides, and the angle opposite one of them, to find the third side, which is the comp. of the alt. and the angle opposite to it, or the hour angle included between the brazen meridian, and the meridian passing through the sun's place, which may be thus found, first letting fall a perpendicular from the pole on the quadrant of altitude produced, if necessary; Rad. : cos. azimuth :: tangt. co. lat. : tangt. of a 4th arch, which call x; then sine lat. : cos. x :: 90° $\pm$ decl. : cosine of another arch which call y, then the difference between x and y will be equal to the complement of the alt. when the perpendicular falls without the triangle, and their sum when the perpendicular falls within. Moreover, if 90° $\pm$ decl. and the angle formed by the arch x and co. latitude be of the same affection (that is each less or each greater than 90°) y will be less than a quadrant or 90°; but if these angles be of different affections, that is one less and the other greater than a quadrant, y will be greater than 90°; all which the learner will easily understand on the globe. Now to find the hour angle it will be sine 90° $\pm$ decl. : s. azimuth :: sine co. altitude : sine hour angle from 12.

Thus, in the 1st example above, Rad. : cos. azim. 65° :: co. tan. lat. 40° 43′ : tang. x 26° 9′; and s. lat. 40° 43′ : cos. 90° — 23° 28′ = 66° 32′ : cos. y 56° 43′, hence $y - x$ = 56° 43′ — 26° 9′ = 30° 34′, the complement of the alt. and therefore the alt. is 59° 26′, as above. Now to find the hour angle we have sine 66° 32′ : sine az. 65° :: cos. alt. 59° 26′ : sine hour angle from 12 = 30° 9′ or 2h. 0m. 36 seconds; hence 12h. — 2h. 0′ 36″ = 9h. 59′ 24″, or 59 min. 24 seconds after 9 in the morning.

The learner must take notice that the decl. is added to 90° when in the triangle, the opposite angle is greater than 90° or the supplement of the azimuth, but subtracted if the op. angle be less than 90°, or equal to the azimuth, or according as the arch of the meridian between the pole and the sun's place, is greater or less than 90°.

This prob. and the following may be performed by the analemma, nearly in the same manner as the foregoing.

Example 1. On the 21st of June, in lat. 40° 43′ N. the sun's azimuth in the morning was 65° from the south ; required his alt. and the hour of the day, when the observation was made ?

Ans. Alt. 59° 26′, and the time 9h 59′ 24″.

2. On the 4th of July, in lat. 38° 53′ N. the sun's azimuth in the morning was 70° from the south ; required the alt. and hour ?

3. In lat. 51½° N. the sun's azimuth from the south, in the evening, was 40°, on the 22d of Dec. required the alt. and hour ?

PROB. 54.

Given the sun's altitude and azimuth, to find the sun's place and the hour of the day, the latitude being known.

Rule. Rectify the globe for the latitude, screw the quadrant of altitude in the zenith, and set the graduated edge of the quadrant to the given azimuth on the horizon ; then turning the globe on its axis, that point of the ecliptic which cuts the altitude will be the sun's place, the quadrant being kept in the same position ; bring the sun's place to the brazen meridian, and set the index to twelve, then turn the globe again until the sun's place cuts the quadrant of alt. and the index will point out the given hour.*

Example 1. In lat. 40° 43′ N. the sun's altitude in the forenoon, being 59° 26′, and his azimuth from the south 65° ; required the sun's place, and the hour of the day ?

Ans. The sun's place is the beginning of cancer, and the hour nearly 10 o'clock.

* To perform this prob. by calculation, there are given the complement of the latitude on the brass meridian, the complement of the altitude on the quadrant of alt. and the angle included by these sides, which is equal to the sun's azimuth if acute or less than 90°, or its supplement if obtuse or greater than 90°, to find the opposite or third side which is always = 90° ∓ the declination (from which the declination will be given) and the hour angle or the angle included at the pole, between the brass meridian and the meridian passing through the sun's place ; the declination being therefore given, and the obliquity of the ecliptic, or sun's greatest declination, the sun's place is given by note 5, prob. 8.

Now to find the declination we have these proportions, having let fall a perpendicular as in the preceding prob. then, rad. : cos. azim. :: tang. co. lat. : tang. x, and the difference between the complement of the alt. and $x = y$. Whence cos. x : s. lat. :: cos. y. : cos. 90° ± decl. Then as the compl. alt. + x, and the angle included by the given sides are of the same or different affection, 90° ± decl. is greater or less than a quadrant. The declination being from thence given, we have sine obliq. of the ecliptic or greatest decl. : s. present decl. :: rad. : s. longitude from aries ; if the sun's place be nearer libra, the result will be the same, reckoning the degrees from libra, or taking the supplement of what the above proportion gives.

The hour angle is found as in the preceding problem.

2. In lat. $51\frac{1}{2}$ N. the sun's alt. in the forenoon was 40°, and his azimuth 60° from the south; required the sun's place, and the hour of the day?

3. In latitude 60° N. the sun's alt. in the morning was 9°, and his azimuth 70° from the north; required the sun's place, and hour of the day when the observation was made?

Note. In these and similar problems, there are two days of the year which will answer these conditions, both equally distant from the longest or shortest day.

4. In lat. 30° S. the sun's alt. in the morning was 28°, his azimuth being 80° from the south; required the sun's place, and the hour of the day?

5. In lat. 30° N. required the two days of the year in which the sun's altitude in the afternoon will be 30°, and his azimuth 79° from the north, and the hour when the observation is to be made?

PROB. 55.

The day of the month being given, to find the sun's altitude, azimuth, the latitude of the place, and hour of the day, by placing the globe in the sunshine.

Rule. PLACE the globe upon a truly horizontal plane, in a north and south direction, by the compass (or a good meridian line) fix a needle perpendicularly over the sun's place in the ecliptic for the given day (found by prob. 8.) bring it to the brass meridian, and set the index to twelve, move the globe until the index casts no shadow, in any direction; then the degree of the brass meridian cut by the horizon is the latitude, the index will point at the hour, and the quadrant of alt. being applied to the zenith and extended over the sun's place, the degree then cut by the sun's place will be the altitude, and the azimuth will be found on the horizon as before.

PROB. 56.

The latitude of the place being given, to find the sun's declination, his place in the ecliptic, his altitude, azimuth, and hour of the day, by placing the globe in the sunshine, as above.

Rule. PLACE the globe horizontally, and also north and south, as above, and elevate the pole to the given latitude; then the number of degrees which the sun shines beyond the north pole, is his declination north. If the sun do not shine beyond the north pole, his declination is as many degrees south as the enlightened part is distant from the pole; if the sun shine exactly as far as the pole, the sun is then on the equinoctial line, and consequently has no declination. The sun's declination being thus found, his longitude is given, and the day of the month corresponding (by prob. 8. note 3.) next fix a needle perpendicularly in the parallel of the sun's declination for the given day, and turn the globe on its axis until the needle casts no shadow; the globe being then fixed in this position, screw the quadrant of alt in the zenith, bring the graduated edge to coincide with the sun's place, or the point where the needle is fixed, the degree cut by the needle will be the sun's altitude, and the degree on the horizon will give the azimuth. The hour may be found as in the preceding prob.

PROB. 57.

The latitude of the place and the day of the month being given, to find when the sun is due east or west.

Rule. ELEVATE the pole to the given lat. screw the quadrant of alt. in the zenith, bring the sun's place, for the given day, to the brass meridian, and set the index to twelve, move the quadrant of alt until 0 on it coincides with the east point of the horizon; the quadrant being held in this position, turn the globe on its axis until the sun's place comes to the graduated edge of the quadrant; the hours passed over by the index will be the time from noon when the sun is due east, and at the same time from noon he will be due west.*

Or, This may be performed by the analemma in the same manner, only instead of bringing the sun's place to the meridian, you bring the analemma there, and then the day of the month on the analemma to the graduated edge of the quadrant.

Example 1. On the 21st of June, in latitude 40° 43′, required when the sun is due east or west?

Ans The sun is due east at 41 min. 8 seconds after 8 in the morning, and due west at 18 min. 52 sec. after 3 in the afternoon.

2. In latitude $51\frac{1}{2}$° on the 19th of May, at what hour will the sun be due east and also due west?

Ans. The hour angle from 12 is 4h. 54m. the time that the sun is west; hence 12h. — 4h. 54m = 7h. 6m. the time that the sun is due east. The alt. may be found at the same time as in prob. 59. Here it is 25° 26′.

3. At what hours will the sun be due east and west at Washington city, on the 21st of June and 22d of December, and what will his alt. be at the same time, on the 21st of June?

* Here the brass meridian, quadrant of alt. and the meridian passing through the sun's place, form a right angled triangle, two sides of which are given, viz. the complement of the latitude, and the distance from the elevated pole to the sun's place, or 90° ± decl. (For the day of the month being given, the declination is given prob. 8.) to find the included angle or hour angle, which converted into time, will give the hour from noon, at which the sun is due east or west. Hence from Napier's rule, we have this proportion; Rad. : co. tan. lat. :: co. tangt. 90° ± decl. : cosine hour angle from noon. Thus, in ex. 1. Rad. : co. tan. 40° 43′ :: co. tang. 90° — 23° 28′ = 66° 32′ : cosine hour angle = 59° 43′ = 3h. 18m. 52 seconds, the time when the sun is west; and therefore 12h. — 3h. 18m. 52s. = 8h. 41m. 8s. when the sun is due east.

The alt. may be found by this proportion; Rad. : sine 90° ± decl. :: sine hour from noon : cosine alt. Thus in ex. 1. Rad. : s. 66° 32′ :: s. 59° 43′ : cos. 37° 37′, the alt. required.

4. At what hours will the sun be due east and west, at every place on the surface of the globe, on the 21st of March and 23d of September?

5. At what hours is the sun due east and west at Lima, on the 22d of December?

PROB. 58.

*The declination and meridian altitude of the sun being given, to find the latitude of the place.**

Rule. MARK the declination on the brazen meridian; then count as many degrees from this mark on the brass meridian, as is equal to the given latitude, reckoning towards the south, if the sun was south of the observer, or towards the north, if the sun was towards the north; bring the degree where the reckoning ends, to coincide with the horizon, and the number of degrees the elevated pole is from the horizon, will be the latitude required.

Or, The latitude may be thus found without a globe: subtract the altitude of the sun's centre (corrected for dip, or height of the eye, and refraction, if necessary†) from 90°, the remainder is the zenith

* The reason of this prob. is evident from the operation.

† The height of the eye above the level of the horizon, or the sea, and refraction, both tend to elevate the sun above its true height, and therefore the sum of both must be subtracted from the observed altitude. The following table will answer the learner's purpose sufficiently. If more exactness be required, McKay's Treatise on Navigation, Mayer, or other authors may be consulted.

Feet.	*dip.*	*Alt.*	*refr.*	*alt.*	*refr.*	*alt.*	*refr.*	*alt.*	*refr.*	*Paral. in alt.*		*Sun's semid.*	
1	1′ 0	0° 0′	33′ 0″	3°½	13′ 6″	17°	3′ 4″	37°	1′16″	0°	9″	Jan. 1	16′18″
2	1 4	0 5	32 10	3¾	12 27	18	2 54	38	1 13	4	9	25	16 16
3	1 7	0 10	31 22	4	11 51	19	2 44	39	1 10	8	9	Feb. 1	16 15
4	1 9	0 15	30 35	4¼	11 18	20	2 35	40	1 8	12	9	25	16 11
5	2 1	0 20	29 50	4½	10 48	21	2 27	41	1 5	16	8	Mar.13	16 7
7	2 5	0 25	29 6	4¾	10 20	22	2 20	42	1 3	20	8	Apr. 1	16 2
9	2 9	0 30	28 23	5	9 54	23	2 14	43	1 1	24	8	25	16 55
12	3 3	0 35	27 41	5½	9 8	24	2 8	44	0 59	28	8	May 1	15 54
15	3 7	0 40	27 0	6	8 28	25	2 2	45	0 57	32	7	25	15 49
18	4 1	0 50	25 42	6½	7 51	26	1 56	46	0 55	36	7	June13	15 46
21	4 4	1 0	24 29	7	7 20	27	1 51	48	0 51	40	6	July 25	15 48
25	4 8	1 15	22 47	8	6 29	28	1 47	50	0 48	44	6	Aug 13	15 50
30	5 2	1 30	21 15	9	5 48	29	1 42	55	0 40	48	6	Sept. 1	15 54
35	5 6	1 45	19 51	10	5 15	30	1 38	60	0 33	52	5	25	16 0
40	6 0	2 0	18 35	11	4 47	31	1 35	65	0 26	56	5	Oct. 1	16 2
50	6 7	2 15	17 26	12	4 23	32	1 31	70	0 21	60	4	25	16 8
60	7 4	2 30	16 24	13	4 3	33	1 28	75	0 15	64	4	Nov. 1	16 10
70	8 0	2 45	15 27	14	3 45	34	1 24	80	0 10	68	3	25	16 15
80	8 5	3 0	14 36	15	3 30	35	1 21	85	0 5	80	2	Dec. 1	16 16
90	9 0	3 15	13 49	16	3 17	36	1 18	90	0 0	90	0	25	16 18

The learner will observe that there are here four tables, separated from each other by the double lines. The 1st contains the dip of the horizon in distance, which is north, if the zenith be north of the sun, or south, if the zenith be south; take the sun's declination out of the Nautical Almanac, or any good table, for the time and place, and observe whether it be north or south; then if the zenith distance and declination be both north or both south, add them together; but if the one be north and the other south, subtract the less from the greater, and the sum or difference will be the latitude, of the same name with the greater.

Note 1. If the alt. be taken by reflection from a basin of water, &c. allowance must be made for refraction. (See note to prob. 1.)

Example 1. On the 17th of October, 1805, the meridian alt. of the sun's centre was 28° 51′, the observer being north of the sun; required the lat. of the place of observation?

Ans. Here the declination is 9° 15′ south, which being marked on the meridian, and 28° 51′ reckoned from this mark towards the south, the reckoning will end at 38° 6′, which being brought to the horizon, the north pole will then be elevated 51° 54′, which shews the lat. to be so many degrees north.

BY CALCULATION.

90° — 28° 51′ S. (the sun's alt. at noon) = 61° 9′ N. the zenith distance, from which the sun's declination 9° 15′ S. being subtracted, leaves 51° 54′ N. the lat. required.

minutes and decimal parts, for the feet in the 1st column corresponding to the height of the eye. The *dip* is a vertical angle contained between a horizontal plane passing through the eye of an observer, and a line from his eye to the visible unobstructed horizon. As this increases the alt. it must be subtracted; but added, if a back observation with a Hadley's quadrant or sextant be used.

The 2d table contains the refraction in alt. of any celestial body corresponding to the degrees and min. of altitude given in the table. It is adapted to 29.6 inches of the barometer, and 50° of Fahrenheit's thermometer; as this increases the alt. of objects, it must likewise be subtracted. It also affects the distances of the sun and moon, or stars, and must therefore be allowed for. If the atmosphere, &c. should vary, allowance is to be made when great precision is necessary. (See tab. 32 of Mayer, or Delambre in his tables annexed to La Land's Astronomy, where the hor. refr. is 6″2 less than in Mayer, Delambre making it 32′53″8.

The 3d table contains the sun's parallax in alt. that is, the difference between the sun's places as seen from the surface, and the centre of the earth at the same time. This table, except the two last numbers, is calculated to every 4th degree. As the parallax always diminishes the apparent altitude, it must be added to the observed alt. to find the true, or the alt. observed from the earth's centre.

The 4th table contains the sun's semidiameter in minutes and seconds, corresponding to the days of the month opposite, the semidiameter being the angle under which it appears, as seen from the earth, is necessary to reduce the observed alt. of the sun's upper or lower limb to that of its centre. It is also useful to astronomers to ascertain the exactness of the scale of their micrometers, by comparison with the measure of the sun's horizontal diameter. This is practised principally in solar eclipses, when the distance of the cusps, or the versed sine of the uneclipsed part, has been measured with the micrometer. It is likewise used in finding the distance of the sun and moon's centres, when their nearer limbs are brought in contact, &c. When great accuracy is required, proportional parts for the dip, refraction, parallax, and semidiameter, may be taken.

2. On the 30th of May, 1808, the meridian alt. of the sun's centre was observed to be 49° 25′, the observer being south of the sun; required the latitude? *Ans.* 18° 45′ S.

BY CALCULATION.

90° — 49° 25′ S. = 40° 35′ S. the zenith distance, the difference between which and the sun's declination 21° 50′ N. is 18° 45′ S. the lat. sought.

Note 2. The table of the sun's declination, and its change for periods of four years, is given before the table of the lat. of places at the end of the book.

3. On the 10th of May, 1808, the sun's meridian alt. was observed to be 40° south of the observer; required the latitude?

4. On the 12th of July, 1810, the sun's meridian alt. was observed to be 50° 30′ north of the observer; what was his latitude?

5. On the 24th of February, 1809, the meridian alt. of the sun's lower limb was 38° 40′, the observer being north of the sun, and height of his eye equal 18 feet; required the latitude of the place of observation?

By help of the foregoing table, this may be performed thus:

Obs. alt. sun's lower limb	38° 40′ S.	90°—38° 51′=51° 9′ N. zenith dist.	
Sun's semidiameter	+ 16	Sun's declin. 24th Feb.	9° 30′ S.
Dip for 18 feet	— 4	Zenith distance	51 9 N.
Refraction	— 1		
		Latitude	41° 39 N.
True alt. sun's centre	38° 51′		

6. On the 10th of December, 1810, the upper limb of the sun was observed appearing in the south part of the horizon, height of the eye 16 feet; required the latitude?

BY CALCULATION.

Obs. alt. sun's upper limb	0° 0′ S.	90° + 53′ = 90° 53′ N. zenith distance
Semidiameter	— 16	22 54 S. declination
Dip for 16 feet	— 4	
Refraction	— 33	67° 59′ N. latitude.
Depression of the sun's cen.	0° 53′ S.	

7. May 10th, 1808, in longitude 60° W. the meridian alt. of the sun's lower limb, by a back observation, was 40° 10′, the observer being north of the sun, and height of the eye 27 feet; required the latitude?

BY CALCULATION.

Obs. alt. sun's up. limb	40° 10′	Sun's decl. 10th May	17° 39′ N.
Semidiameter	— 16	Variation of decl.*	+ 3
Dip	+ 5		
Refraction	— 1	Reduced declin.	17 42 N.
		Zenith dist. 90° — 39° 58′ =	50 2 N.
True alt. sun's centre	39° 58′		
		Latitude	67° 44′ N.

* When the longitude is different from that of Greenwich observatory, the difference of longitude must be converted into time, and reduced to that of Greenwich (by prob. 6) the variation of declination, during this time, may then be found by this rule; as 24 hours : hour from noon

8. At a certain place where the clocks are three hours slower than at *Greenwich*, the meridian alt. of the sun's lower limb on the 21st of March, was observed to be 32° 15′, the observer being north of the sun, and the height of his eye 17 feet; required the place?

9. Suppose that on the 4th of June, 1812, a ship in longitude 53° E. was distant about three quarters of a mile from land, at noon, the lower limb of the sun being brought down to the line of separation between the sea and land, the alt. was 46° 19′, the observer being south of the sun, and height of his eye 20 feet; required the latitude?

BY CALCULATION.

Obs. alt. sun's lower limb	46° 19′	Sun's declination	22° 27′ N.
Semidiameter	+ 16	Variation of decl.	0 1
Dip*	— 15		
Refraction	— 1	Reduced decl.	22 26 N.
		Zenith dist.	43 41 S.
True alt. sun's centre	46° 19′		
		Latitude	21° 15′ S.

reckoned by the meridian of Greenwich :: the daily variation of the sun's declination : a fourth number, which must be added to, or subtracted from the decl. for the given day at Greenwich, according as the reduced time is before or after twelve, and the declination increasing or decreasing. If the time be in the forenoon, and the decl. increasing, the variation must be subtracted; but if the time be in the afternoon, the variation must be added; again, if the reduced time be in the forenoon, and the declination decreasing, it must be added, but the contrary, if the reduced time be in the afternoon. Thus in ex. 7. the difference of long = 60° = 4 hours, and as the place is W. of Greenwich, the time reduced to the meridian of Greenwich is 4 o'clock in the afternoon. Now the decl. for the 10th of May is 17° 39′, and for the 11th 17° 54′, their diff. is 15′ increasing; hence 24 h. : 4 h. :: 15′ : 2′ 30″ which must be added, because the time is in the afternoon and the decl. increasing. (see table 26 in M'Kay's Navigator.) When great exactness is required, the decl. must be taken from the Nautical Almanac, where its daily variation is also given.

* If the land intervenes, and the sun's limb be brought in contact with the line of separation of the sea and land, the dip will be considerably increased, and will become greater in proportion as the land is approached. In this case the distance to the water's edge is to be found; with this distance and the height of the eye above the level of the water, the dip is found from the annexed table, or it may be calculated as follows:

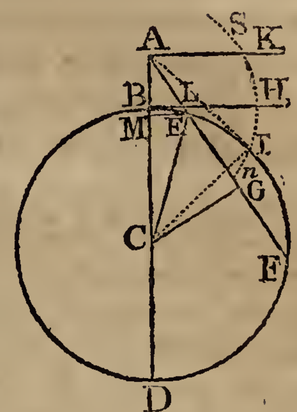
In the annexed figure, let A represent the place of the observer, AB the height of his eye above the level of the horizon, BD the diameter of the earth = 7911.2 Eng. miles or 4171136 feet, (note to def. 8) E

Dist. of land in sea mil.	Height above the sea in feet. 5	10	15	20	25	30	35	40
¼	11′	22′	34′	45′	56′	68′	79′	90′
½	6	11	17	22	28	34	39	45
¾	4	8	12	15	19	23	27	30
1 0	4	6	9	12	15	17	20	23
1 ¼	3	5	7	9	12	14	16	19
1 ½	3	4	6	8	10	11	14	15
2 0	2	3	5	6	8	10	11	12
2 ½	2	3	5	6	7	8	9	10
3 0	2	3	4	5	6	7	8	8
3 ½	2	3	4	5	6	6	7	7
4 0	2	3	4	4	5	6	7	7
5 0	2	3	4	4	5	5	6	6
6 0	2	3	4	4	5	5	6	6

PROB. 59.

The length of the longest day at any place, not within the polar circles, being given, to find the latitude of the place.

Rule. Bring the beginning of cancer or capricorn to the brass meridian (according as the place is in N or S. latitude) and set the index to twelve ; turn the globe westward until the index has



the place where the sea and land are separated, AE (or BE, which is nearly equal to it) the distance from the observer to E ; draw BH from B perpendicular to BD, to represent the horizon, and AK parallel to it. Then the angle HLF = KAE (29. E. 1.) is the dip or depression of the object below the horizon, and is equal ACG, each of the last being complements of CAG to 90°; AE being produced to F, and from C the centre CG being drawn perpendicular to EF. Again AB × AD = AE × AF (Eucl. cor. prop. 36. b. 3) and therefore AE : AB :: AD : AF; (16 E. 6.) hence EF = AF — AE is given, and as CG bisects EF in G (3 E. 3) AG = AE + EG is given. Now in the right angled triangle ACG, the two sides AC, AG are given, to find the angle ACG, the dip required. Thus AC : AG :: Rad. : sine ACG. The following method is no less accurate, and in practice extremely simple ; AL differing very little from BE, the distance of land in miles, and the angle ALB = KAE or HLF, that is equal the dip required, we have this proportion, AL : AB :: Rad. : sine of the dip. Thus let the observer's distance BE or AL = 2 sea or geographical miles = 12151 feet nearly (a sea or geographical mile being $= \frac{5280 \times 69.04}{60} = 88 \times 69.04 = 6075.52$ feet, allowing 69.04 Eng. miles to a deg. See notes to def. 8 and prob. 35) and height of his eye AB = 20 feet. Then,

As AL	12151 - -	4.0846120	*This rule in words is, as the distance from the observer to where the sun touches the horizon, is to the height of his eye, so is radius, to the sine of the angle of depression or dip.*
To AB	20 - -	1.3010300	
So is radius,		10.0000000	
To sine dip.	5′ 40″	7.2164180	

The dip may be also found thus : join CE, then in the triangle ACE there are given the three sides AC, AE (= BE nearly) and EC to find the angle BAE, whose complement is the depression required.

But the most correct manner of calculating the dip is as follows : Let EM be drawn perpendicular to AC. Now as the length of the arch BE, which is the measure of the angle ACE is given, in geographical or sea miles, each of which corresponds to a minute of a degree, the angle ACE is therefore given, and hence CM, ME are each given, and as CA is given, AM is therefore given ; hence in the triangle AME the angle AEM, which is the true dip, is likewise given.

The difference between AM and AB, or BM, is what the surface of the earth at E falls below the horizontal or true level at B ; and hence this latter solution may be useful to the practical surveyor, engineer, &c. This level varies as the square of the distance nearly. (See quest. 4 in the Diary for 1795 published by Hutton.)

If from the centre B, with the distance BI, a circle SHIn be described, and S be the sun's place, then SH will represent his true altitude, HI the dip, when the visible horizon is unobstructed, as in the 1st table, Hn the dip when the sun's limb is brought in contact with any other part of the visible horizon, in a vertical circle, as at E : when Hn = HS the dip is equal to the true altitude, in which case if the sun's image be made to coincide with its reflected image from the water or any horizontal reflect-passed over as many hours as are equal to half the length of the day ; elevate or depress that pole until the sun's place (cancer or capricorn) comes to the horizon ; the elevation of the pole will then shew the latitude.*

Note 1. The prob. may be performed in the same manner for any other day, by bringing the sun's place to the meridian, and proceeding as above.

Or, Bring the middle of the analemma to the brass meridian, and set the index to 12 ; turn the globe westward until the index points out the hours, &c. as before ; elevate or depress the pole until the day of the month coincides with the horizon ; this elevation will give the lat. required.

ing surface, the angle measured on the quadrant or sextant will be double the true alt. of the sun ; the angle of incidence being equal to the angle of reflection, or the angle formed by a line from S. to E. and ME produced, will be equal the angle AEM, &c. See note to prob. 1. Hence we have here several methods of finding the lat. on land near the sea shore, a river, lake, &c. or from any reflecting surface.

If no land intervenes, then the angle ACI is the dip, as given in the 1st table, AI being drawn to the extremity of the visible unobstructed horizon, in which case EF will vanish, E will coincide with I, CG will become = CI = the semid. of the earth at right angles to the tangent AI at the point I. (cor. 16. Eucl. 3.) whence we have this proportion; AC : CI :: Rad. : sine ACI the dip required ; or, as AC : CI :: Rad. : sine CAI the complement of which is the required depression.

More useful observations might be made here, but our contracted limits would not permit : what we have said is however sufficient to give the learner an idea how useful geometrical principles are in inquiries of this nature, and how necessary their study is for those who wish to be more than superficially acquainted with the nature and foundation of the most useful arts and inventions in general.

* This prob. is calculated thus : the complement of the sun's declination, the lat. reckoned from the elevated pole to the horizon, and the included angle, or the supplement of the hour angle or half the length of the day, form a right angled spherical triangle, the circular parts being the sun's declination, the comp. of the included angle between the brass meridian and the meridian passing through the sun's place, and latitude, of which this angle is the middle part (see Simson's Trig. at the end of his Euclid, pa. 26) then by Napier's first rule, rad. $\times$ co. sine of the included angle at the pole = tangt. decl. $\times$ tang. latitude, whence (16 E. 6.) tangt. sun's decl : rad. :: co. sine of the angle included between the meridian passing through the sun's place and brass meridian : tangt. latitude. Thus in ex. 1. the hour angle = 7 h. 30 min. = 112° 30′ ; the supplement of this = 180°.—112° 30′ = 67° 30′ = the angle included between the meridian passing through cancer and the brass meridian, the sun's declination being here greatest = 23° 28′; hence tangt. 23° 28′ : rad. :: co. sine 67° 30′ : tang. latitude 41° 24′ required.

In the same manner may the lat. be calculated for any other time, taking the sun's declination for the given day, instead of his greatest declination 23° 28′.

The reason of the rule is evident ; for when the globe is rectified to the lat. and the sun's place brought to the meridian, if then the globe be turned on its axis until the sun's place coincide with the horizon, the index in this revolution will pass over half the length of the day ; hence, *vice versa*, if the distance between the brass mer. and where the sun cuts the horizon be made equal to half the length of the diurnal arch, by elevating or depressing the pole, the elevation thus found must be the latitude. The same reasoning will answer for any other prob. performed in a similar manner.

Example 1. In what degree of north lat. and at what places is the length of the longest day 15 hours?

Ans. In lat. 41° 42′, and at all those places situated on or near that parallel.

Note 2. The prob. may be performed much more correctly by help of the equator than of the hour circle.

2. In what degree of south lat. and at what places is the longest day $13\frac{1}{2}$ hours?

3 In what degree of north lat. is the length of the longest day twice the length of the shortest night?

4. In what degree of south lat. is the longest day three times the length of the shortest night?

5. In what lat. is the 10th of May 14 hours 15 min. long?

Note 3. The minutes are marked on the equator of Cary's large globes.—The lat. may be here N. or S.

6. In what lat. north does the sun set at 7 o'clock, on the 10th of August?

7. In what lat. south does the sun rise at 5 o'clock, on the 31st of January?

8. In what lat. N. is the night of the 4th of July 9 hours long?

PROB. 60.

Given the sun's declination and amplitude, to find the latitude.

Rule. ELEVATE the north pole to the complement of the amplitude, screw the quadrant of alt. in the zenith, and bring the beginning of aries to the brass meridian; then bring the degree on the quadrant of alt. which is equal to the declination, to coincide with the equator, and the degree cut on the equator, reckoned from aries, will be the latitude required.*

* Here the complement of the decl. on the quadrant, the complement of the altitude reckoned on the brass meridian, and the lat. reckoned on the equator, form a right angled triangle, and hence the lat. is thus found; S. ampl. : rad. :: s. decl. : cos. latitude; thus, in the 1st ex. S. ampl. 32° : rad. :: s. decl. 23° 28′ : cos. lat. 41° 1′.

To understand how this rule was formed, let the globe be rectified to any given latitude, and the sun's place brought to the horizon; then the angle formed by the amplitude, sun's declination, and that part of the equator intercepted between the horizon and the meridian passing through the sun's place, form a right angled sp. triangle, and the angle included between the equator and amplitude, in this triangle, is equal to the complement of the latitude, whence in this triangle it will be S. ampl. : R. :: s. decl. : cos. lat. the same as above. If we now conceive the sides of the triangle which represent the decl. and ampl. to be produced to the brass meridian, another triangle formed from the compl. of the decl. the compl. of the amplitude, and the latitude, reckoned from the pole to the horizon, will be delineated on the globe, from which the first rule above is formed. The second rule is manifest from the globe being rectified to the lat. &c.

From the triangle formed at the horizon by rectifying the globe, &c. other methods may be deduced of solving the prob. Thus, bring the beginning of aries to the brass meridian; from aries on the equator reckon as many degrees as are equal to the decl. then with the amplitude in the compasses, and one foot in this point, cross the equinoctional colure, and mark the de-

OR THUS,

Find the sun's place corresponding to the given declination, and bring it to the eastern or western part of the horizon (according as the eastern or western amplitude is given) elevate or depress the pole until the sun's place coincides with the given amplitude on the horizon ; the elevation of the pole will be the lat. sought.

Example 1. The sun's amplitude was observed to be 32° from the east towards the north on the 21st of June ; required the lat. ?

Ans. 41° N. nearly.

2. In May when the sun's declination was 18° north, the rising amplitude was observed to be $30\frac{1}{2}$° from the east towards the north ; required the lat. ?

Ans. $51\frac{1}{2}$° N.

3. When the sun's declination was 10° south, his setting amplitude was 13° from the west southward ; required the latitude ?

PROB. 61.

Given two observed altitudes of the sun, the time between them, and the sun's declination, to find the latitude.

Rule. TAKE the complement of the first alt. from the equator (or any great circle on the globe which is divided into degrees, &c.) in your compasses, and with one foot in the sun's place, and a fine pencil, or a pen with ink, in the other, describe an arc on the surface of the globe ; then bring the sun's place to the meridian, and set the index to the hour at which the first altitude was taken, or mark the degree of the equator under the brass meridian ; turn the globe eastward until the index has passed over as many hours as are equal to the time between the two observations, or until the equator has passed over as many degrees, &c. as the elapsed time converted into degrees ;* (by prob. 6) then under the sun's de-

gree cut on it ; elevate this pole to the degree, and screw the quadrant of alt. in the zenith ; extend the quadrant over the degree of decl. marked on the equator, and the degree then cut on the horizon, reckoned from the nearest pole, will be the complement of the latitude, as is evident. Or if the decl. be marked on the colure, from aries, and with the ampl. in the compasses as before, and with one foot on the colure where the reckoning ends, cross the equator with the other, and mark the degree thus cut ; bring this degree to the brass meridian; screw the quadrant over it, and bring both poles to the horizon ; extend the quadrant over the degree of decl. on the colure; the degree then cut on the horizon will be the complement of the lat. as before. This method follows from the same principle.

* The elapsed time is found to a minute of time on Cary's large globes, on the equator. Any two points in the parallel of the sun's declination for the time, distant from each other by the interval of the elapsed time, may be taken. If the declination varies much during the elapsed time, the complement of the second alt. must be set off from the parallel in which the sun is at that time, the variation of the declination being found by the proportion in the latter part of the note to prob. 49.

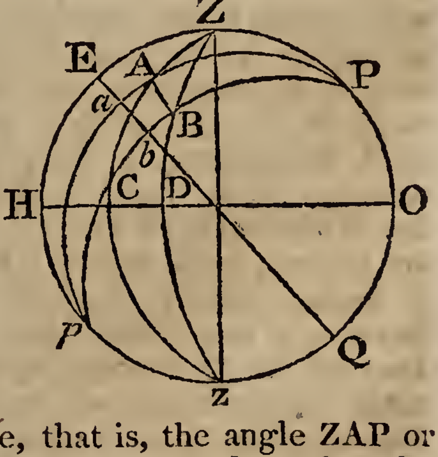
To perform this prob. by calculation, join the zenith, or point where the two circles representing the complements of the alt. intersect each other, and the apparent places of the sun at each alt. represented on the parallel of clination on the brass meridian mark the place on the globe; take the complement of the second altitude in your compasses, and with one foot in this mark, describe another circle intersecting the former; the point of intersection will be the zenith of the place, which being brought to that part of the brazen meridian which is numbered from the equator towards the poles, will give the latitude required.

Note. The respective altitudes may be corrected in this prob. as in the foregoing problems, when necessary.

Example 1. June 4, 1810, in north latitude, the corrected altitude of the sun at 29 minutes past 10 in the forenoon, was 65° 24′, and at 31 min. past 12, the correct alt. was 74° 8′; required the latitude?

Ans. The sun's declination was 22° 27′ N. the elapsed time was 2h. 2m. = 30° 30′; the complement of the first alt. was 24° 36′, of the 2d, 15° 52′, and the lat. sought 36° 57′ N.

declination, through each of the sun's places, and the zenith, let meridians be drawn with a fine pencil, by means of the brass meridian, then will the figure be projected on the surface of the globe. In this figure it will be seen that the complement of the latitude and the sun's declination at each alt. added to or subtracted from 90°, form with the complements of the altitudes, two triangles respectively, and that the angles at the pole, included by the com. of the sun's decl. and com. of the lat. reckoned on the meridian passing through the zenith, are the hour angles counted from 12. From either of which triangles, two sides and an angle being given, the third side, which is the complement of the latitude, may be found by the following proportions, first letting fall a perpendicular on the meridian passing through the zenith or given lat. from the sun's place at either of the altitudes. Rad. : cos. hour angle before or after 12 :: tangent 90° ± the sun's decl. : tangt. x or distance between the pole and the perpendicular; and sine 90° ± the decl. : cos. x :: sine alt. : y the distance between the zenith and perpendicular; then $x - y$ = complement of the lat. required. (See Emerson's Trig. b. 3. sect. 4. case 3, oblique spher. tri.) To know whether the declination is to be added or subtracted; when the lat. and decl. are of the same name, add; when they are of different names, subtract. In the above solution either of the latitudes above will answer. (See prob. 61.) If the watch be not adjusted so as to give the time at which each of the observations was made, though sufficiently correct to measure the elapsed time, then the following method will answer. In the annexed figure, let P represent the pole, Z the zenith, EQ the equator, HO the horizon, A and B the two places of the sun when the altitudes were taken; then in the triangle BPA there are given AP, BP the complement of the sun's decl. and the angle BPA the elapsed time, or the time between the two observations converted into degrees to find the side AB, and the angles ABP or BAP. In the triangle AZB there are given AZ the complement of the first alt. BZ the compl. of the second alt. and the side AB, to find the angles ABZ or BAZ, and from thence the angles ZAP or ZBP; then in the triangle ZPA or ZPB, two sides, and the included angle, that is, the angle ZAP or ZBP, are given to find ZP, the complement of the lat. required. When the altitudes are equal, or that the sun is on the equinoctial, the calculations is more simple. If the altitudes be equal, or AC = BD, and the sun's declina-



2. Given the sun's declination 12° 16′ N. his alt. in the forenoon at 10h. 24m. was 49° 9′, and at 1h. 14m. in the afternoon his alt. was 51° 59′; required the latitude ?

Ans. 47° 20′ north.

3. The sun's declination being given 11° 7′ N the alt. at 10h. 2m. in the forenoon was 46° 55′, and at 11h. 27m. in the forenoon the second alt was 54° 9′; required the latitude ?

Ans. 46° 27′ north.

4. Being at sea when the sun was on the equator, I observed that at 1 o'clock, P. M. the correct alt. of the sun's centre was 28° 53′, and at 3 o'clock, P. M. the alt. was 20° 42′; required the latitude ?

Ans. The lat. was 60° north.

5. If on February 24, 1811, at 38m. past 12 in the afternoon, the correct alt. of the sun's centre be observed 36° 5′, and 46m. past 2, the alt. be 24° 19′, the lat. is required ?

6. Oct. 17, 1820, at 32m past 12, the alt. of the sun's lower limb being supposed equal to 28° 32′, and at 41m. after 2, the 2d alt. equal 19° 25′, height of the eye 12 feet; required the latitude ? *Ans.* 51° 31′ N.

PROB 62.

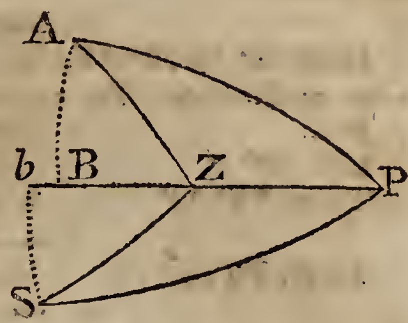
Given the sun's declination, his altitude, and the hour of the day, to find the latitude.

Rule. Find the sun's place in the ecliptic; from this place with the complement of the alt. describe an arc (as in the last prob.) bring the sun's place to the brass meridian, and set the index to 12; then if the time be in the forenoon, turn the globe eastward, but if in the afternoon, westward, as many hours as the given hour is before or after twelve; the degree on the brazen meridian, cut by the arc before described, will be the latitude required.

The examples given in the foregoing prob. will answer this, taking one of the altitudes and time corresponding, instead of both altitudes. If the lat. found by making use of both altitudes sepa-

tion remain nearly equal ($aA = bB$) then the middle time between the observations is the time of his being on the meridian. If this prob. be performed on shipboard, and that the ship is under sail, an allowance must be made for the alteration in lat. (See McKay, Blunt, Norey, Moore, or Robinson's Navigation, or the principles of navigation in Emerson's Math. Princ. of Geog. prop. 17, or Citizen Dulague's Lessons on Navigation, revised by Cit. Prudhomme of Rouen.) McKay remarks, that Hues in his Treatise on the Globes, published in 1594, solved this prob. on the globes; the substance of Keith's solution in prob. 52 of his Treatise on the Globes is the same as the above, and differs little from Fuller's solution of the same prob. in his Treatise on the Globes, published in Dublin, in 1732, prob. 35 astron. See other authors mentioned by McKay in pa. 158 of his Navigator, Amer. edit. The methods given in most of the books on Navigation, are but approximations, and consequently the answers obtained by such methods generally differ something from those found by the above, and it is very seldom that the altitudes and times are given correct, the examples mostly given at random, being seldom truly limited.

rately in the same example, agree with each other, the question is truly stated, otherwise not.



In ex. 2, prob. 61, the lat. is found from both altitudes separately, thus: let P, in the annexed figure represent the pole, A and S, the sun's ap. places at each of the altitudes, PZB the meridian passing through the zenith Z or the given place, AZ the complement of the first altitude = 40° 51′, APZ the hour from noon = 12h.—10h. 24m. = 1h. 36m. = 24°, and AP the complement of the sun's declination = 77° 44′. For the second alt. SZ = its comp. = 38° 1′, SP comp. decl. = 77° 44′, and SPZ hour angle after noon = 1h. 14m. = 18° 30′.

Rad.	10.0000000	Cos. PA 77° 44′	9.3272811
Cos. APZ 24°	9.9607302	Cos. PB 76° 37′	9.3644852
Tangt. PA 77° 44′	10.6626887	Cos. AZ 40° 51′	9.8787656
Tang. PB 76° 37′	10.6234189		19.2432508
		Cos. BZ 34° 30′	9.9159797

PB—BZ = ZP = 42° 7′ . . 90°—42° 7′ = 47° 53′, the lat. for 2d observation.

Rad.	10.0000000	Cos. PS 77° 44′	9.3272811
Cos. SPZ 18° 30′	9.9769566	Cos. P*b* 77° 5′	9.3493429
Tang. PS 77° 44′	10.6626887	Cos. SZ 38° 1′	9.8964334
Tan. P*b* 77° 5′	9.6396473		19.2458763
		Cos. *b*Z 34°	9.9185952

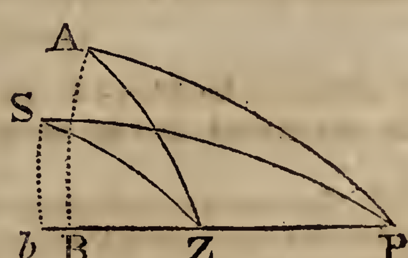
P*b*—*b*Z = 43° 5′ and 90°—43° 5′ = 46° 55′.

The examples given to illustrate the last problem, were selected from McKay and Hamilton Moore's Treatises on Navigation. To point out the errors unnoticed in most of these, calculated as directed in those authors, the preceding calculation was given. Mariners and others will therefore judge for themselves, whether the common method of double altitudes, attended with so much uncertainty, so much labour and tedious calculations, be of any real advantage. The method given in this prob. for finding the lat. will answer every purpose of the double altitudes, with only a single altitude and the time being given.

For a further illustration of this valuable problem, the following examples are given.

In north latitude, when the sun's declination was 13° 45′ N. his alt. at 8h. 39′ 33″ in the forenoon was 36° 53′, and at 9h. 39′ 33″, or one hour after, the alt. was 45° 53′; the lat. by either of the observations independent of the other, is required?

Latitude from the 1st altitude.



Here AP = 76° 15′, AZ = 53° 7′, the hour angle, APB = 12h.—8h. 39′ 33″ = 3h. 20m. 27s. = 50° 7′ nearly; hence,

Rad.	10.0000000	Cos. 76° 15′	9.3760034
Cos. 50° 7′	9.8070114	Cos. PB 69° 7′	9.5520184
Tang. 76° 15′	10.6113688	Cos. 53° 7′	9.7782870
Tang. PB 69° 7′	10.4183802		19.3303054
		Cos. BZ 25° 49′	9.9543020

Hence 69° 7′ — 25° 49′ = 43° 18′ and 90° — 43° 18′ = 46° 42′, the latitude required.

Latitude from the 2d altitude.

Here SP = 76° 15′, SZ = 44° 7′, the hour angle SP*b* = 12h. — 9h. 39m. 33s. = 2h. 20m. 27s. = 35° 7′ nearly.

Rad.	10.0000000	Cos. 76° 15′	9.3760034
Cos. 35° 7′	9.9127440	Cos. P*b* 73° 21′	9.4571618
Tang. 76° 15′	10.6113688	Cos. SZ 44° 7′	9.8560784
Tang. P*b* 73° 21′	10.5241128		19.3132402
		Cos. Z*b* 30° 3′	9.9372368

Hence 73° 21′ — 30° 3′ = 43° 18′ and 90° — ZP 43° 18′ = 46° 42′, the latitude required the same as above.

This last example is taken from Dulague's Lessons on Navigation, revised by Prudhomme (pa. 196) the answer by the double alt. being there given as above. The learner will there find more examples where the time is given to seconds, without which this prob. by any method, will seldom succeed, as 30″ of time, if rejected, correspond to 7½′ of a degree, and this is the principal reason why the examples in McKay, Moore and others, will not agree with accurate calculation. The learner will perceive that the lat. is here found by either of the altitudes, whereas the common method requires both. Whence the principal advantages of this method are, 1st. That there is nothing to do with elapsed time or variation of declination, except in reducing the declination to the meridian of the place. 2d. That there is no alteration of latitude from the vessel's sailing during the time between the observations. 3d. That the operation consists in the two simple proportions given above (which become more simple when the sun is due east or west, when on the equator, when the observation is made at 6 o'clock, &c. or by tables which may be easily adapted to it.) 4th. That the observation may be made at any time of the day, &c. The same method will answer by having the time and the altitude of any star when the declination is known, the time of its passage over the meridian being also given, which is found by prob. 8, part 3d. for the difference of time will be the hour angle APB or SP*b*, in the above fig. A or S being the star's place.

The principal difficulty in the practice of this prob. consists in determining the time exactly, and hence the observer ought to be provided with a good time piece well regulated; when this is not the case, the method by double altitudes with the intermediate time, will then become useful, as the watch requires no regulation to measure the elapsed time. See the foregoing problem.

PROB. 63.

*Given the sun's amplitude and ascensional difference, to find the latitude and sun's declination.**

Rule. ELEVATE the pole as many degrees above the horizon as are equal to the ascensional difference; screw the quadrant of

* The learner will observe, that when the prob. is performed by the first rule, the ascensional difference reckoned on the brass meridian, the amplitude on the quadrant, and the declination on the equator, form a right angled triangle; and that the angle included between the quadrant of alt. and brass meridian, or between the amplitude and as. diff. is equal to the complement of the latitude: hence, by Napier's rules, we have these proportions; Rad. : co. tangt. amplitude :: tang. ascen. diff. : sine latitude. And for the decl. Cosine as. diff. : cosine amplitude :: rad. : cosine declination.

alt. in the zenith, and bring the beginning of aries to the brass meridian; then number on the quadrant of altitude from 0 the complement of the sun's amplitude, and move the quadrant until that number cuts the equator; the degree then cut on the horizon, reckoning from the east or west points, will be the latitude, and the degree cut on the equator will be the sun's declination.

Or, If the day of the month be given, find the sun's place in the ecliptic (by prob. 8) bring this place to the brass meridian, and then mark the sun's right ascension on the equator; reckon as many degrees from this on the equator as are equal to the ascensional difference, and elevate or depress the pole until this point comes to the horizon; the elevation of the pole will then be the latitude required. The decl. will be found as in prob. 8.

Note. In north lat. when the sun is in the first and second quarter of the ecliptic, the oblique is less than the right ascension, in which case the as. diff. is to be subtracted from the right; but when the sun is in the 3d and 4th quarters, the oblique as. is greater, and hence the as. diff. is to be added to the right as. to find the oblique. When the sun has north decl. he is in the 1st or 2d quarters of the ecliptic, but in the 3d or 4th when he has south. The ascensional difference is always equal to the time the sun rises or sets before or after 6 o'clock, converted into degrees.

Example 1. On the 10th of May, the sun's amplitude at rising was 23° 45′, and the ascensional difference 16°; required the latitude and declination?

Ans. Lat. 40° 40′ N. decl. 17° 47′ nearly.

2. On the 4th of April, the sun's amplitude was 8°, and the ascensional difference $4\frac{3}{4}$°; required the latitude and declination?

3. On the 21st of June, the sun's amplitude was 39°, and the ascensional difference $31\frac{1}{4}$°; required the latitude?

4. On the 10th of August, the sun's amplitude was $22\frac{1}{2}$°, and he rose 3 minutes before 5; required the latitude?

5. On the 20th of October, the sun sets 17 minutes after 5, his setting amplitude being 16°; required the latitude?

PROB. 64.

Given the sun's declination and hour at east, to find the latitude.

Rule. Elevate the pole to the sun's declination, and screw the quadrant of alt. in the zenith; then reduce the time after 6, of the sun's being due east, into degrees and minutes, and reckon the same number on the horizon from the *east* towards the south; bring the quadrant of altitude to that degree on the horizon, and

If we now perform the prob. by the second rule, it will be seen that when the sun's place is made to coincide with the horizon, and likewise the oblique ascension, by elevating or depressing the pole, &c. that the amplitude, ascensional difference, and sun's declination, form a right angled triangle, similar to that before described, and that the inclination of the equator with the horizon, or the angle formed by the amplitude and ascen. diff. is equal to the complement of the lat. hence the calculation will be exactly the same as above. From this last method the reason of the foregoing method is therefore manifest.

the degree then cut on the quadrant by the equator, will be the complement of the latitude required.*

Note. Unless the sun's decl. be of the same name with the latitude, he cannot be seen in the east.

Example 1. On the 10th of May, the sun was observed in the east at 7 o'clock in the morning; required the latitude?

Ans. 51° N. nearly.

2. On the 4th of July, I observed the sun due east at 8 o'clock in the morning; required the latitude?

3. On the 21st of June, the sun was observed due east at 7 hours 30 minutes in the morning; required the latitude?

PROB. 65.

Given the sun's declination and azimuth at six, to find the latitude and altitude.

Rule. Elevate the pole above the horizon as many degrees as are equal to the complement of the given azimuth, screw the quadrant of altitude in the zenith, bring the beginning of aries to the brass meridian; then from 0 on the quadrant reckon the complement of the sun's declination, and bring that degree to the equator; the degree then cut on the horizon by the quadrant, reckoned from the N. or S. will be the complement of the latitude, and the degree of the equator cut by the quadrant will be the sun's altitude at six.†

* Here the learner will observe, that the complement of the alt. on the quadrant, the degrees corresponding to the time after the sun was due east, and that part of the equator between the edge of the quadrant and the horizon, form a right angled spherical triangle, and that the angle formed by the equator and horizon, is the complement of the declination: hence to find the lat. we have, from Napier's rule, this proportion; Tang. sun's decl.: Rad. :: sine hour after 6: Co. Tan. lat. Thus in ex. 1. Tan. decl. 17° 39′: R. :: sine 15°: Co. T. lat = 50° 52′.

Or thus, If we suppose the globe elevated to the lat. the quadrant of alt. screwed in the zenith, and extended over the sun's place so as to cut the east point of the horizon; then the angle formed by the brazen mer and the merid. passing through the sun's place, is equal to the time from the sun's being due east to 12, converted into degrees, the complement of which to 90° is the distance from the meridian passing through the sun's place, to the east point of the horizon, reckoned on the equator, or the time from 6 until the sun is due east. this time being therefore given, and likewise the sun's decl. the angle formed by the equator and the quadrant of alt. at the east point, being equal to the latitude, is likewise given. The right angled triangle thus formed is solved by the above proportion, which shews the truth of the method.

† The globe, quadrant, &c. being placed as directed in the rule, it will then be seen, that the complement of the azimuth, reckoned on the brass meridian from the equator, the sun's decl. reckoned from the zenith on the quadrant, and the sun's alt. reckoned from aries on the equator, to the point where it is intersected by the quadrant, form a right angled triangle; and that the angle formed by the complement of the azimuth reckoned on the brass meridian, and the decl. reckoned on the quadrant, is equal to the latitude: Whence by Napier's theorems we have these proportions; Rad. : co. tang. decl. :: co. tang. azim. : cos. latitude. And

Example 1. The sun's azimuth at 6 o'clock, on the 21st of June, was 71° 30′ from the north; required the latitude and sun's altitude?

Ans. Lat. 40° N. and the sun's altitude is 15° nearly.

2. The sun's azimuth on the 10th of May, at 6 o'clock in the morning, was 78½° from the north; required the latitude and altitude?

3. The sun's azimuth at 6 o'clock in the evening, on the 20th of April, was 82½° from the north; required the latitude and altitude?

PROB. 66.

Given the sun's declination and altitude at east, to find the latitude.

Rule. ELEVATE the pole to the complement of the sun's given altitude, screw the quadrant in the zenith; bring the beginning of aries to the brass meridian; reckon on the quadrant of altitude from the horizon upwards, or from 0, as many degrees as are equal to the sun's declination; bring the point where the reckoning ends to the equator, and the degree cut on the equator will be the complement of the latitude sought.*

Sine azim. : rad. :: cos. decl. : cosine altitude. Thus in the first example above, Rad. : co. t. 23° 28′ :: co. t. az. 71° 30′ : cos. lat. = 39° 35′. Again S. az. 71° 30′ : rad. :: co. s. decl. 23° 28′ : cos. alt. 14° 42′.

If we suppose, as in the foregoing problem, the globe to be elevated to the latitude, the sun's place being then brought to the meridian, the index set to 12, and the quadrant of altitude screwed in the zenith, and that the globe is turned on its axis eastward, until the index points to six; and that the quadrant is extended over the sun's place, we shall then have the azimuth on the horizon, the complement of which with the sun's declination, and altitude marked on the quadrant, will form a right angled spherical triangle as before. And as the meridian passing through the sun's place always coincides with the east point of the horizon at 6, the angle formed by the sun's declination and complement of the azimuth, on the horizon, will be always equal to the latitude. The triangle thus formed being in every respect equal to that described above, is therefore calculated in the same manner, and shews no less the truth of the rule than the method of forming it.

The learner may also observe, that in the triangle formed according to the above rule, of the sun's decl. azimuth at 6, altitude at 6, or the latitude of the place, any two being given, the rest are given, and may be found by the globe in the same manner as the above. Thus the sun's decl. and altitude at six being given to find the latitude, we have this proportion; Sine decl. : rad. :: sine alt. : sine latitude. As these right angled spherical triangles are solved by Napier's Theorems, being the most general and the simplest that has ever been discovered, the learner should be well acquainted with their use, and also with the practice of the 16th proposition of the 6th book of Euclid, in alternating, inversing, &c. the different proportions, as with very little labour many new and useful conclusions will thus arise, and on which the learner may try his own invention, in exercising them on the globes.

* Here the quadrant of alt. the brass meridian, and the equator, form a right angled spherical triangle, the three sides of which are the complement of the sun's alt. on the brass meridian, the complement of the declination on the quadrant, and the complement of the latitude on the equa-

Or, With a pair of compasses take as many degrees as are equal to the given alt. from the equator ; bring the beginning of aries to the horizon ; with one foot of the compasses in the point aries extend the other towards the zenith ; then elevate or depress the pole until the other point of the compass extends to the parallel of the sun's declination ; the elevation of the pole will then be the latitude required ; which will be of the same name with the sun's declinatian. Or the quadrant of alt. being extended from aries (or the east point of the horizon) through the point where the compass cuts the parallel of decl. will point out the lat. on the equator.

tor ; to find the latter of which, we have this proportion ; Sine altitude : Rad :: sine decl. : sine lat. Thus in ex. 1, sine 29° : Rad :: s 17° 39′ : s. lat. =38° 43′.

To understand how the rule has been formed, the pole must be elevated to the supposed latitude, the quadrant of alt. screwed in the zenith, and then brought to coincide with the east point of the horizon ; the sun's place corresponding to his declination being then brought to coincide with the graduated edge of the quadrant, it will be seen that the meridian passing through the sun's place or the sun's declination, the number of degrees from where this meridian cuts the equator to the east point of the horizon, and the sun's alt. form a right angled triangle, and that the angle contained by the equator and quadrant is equal to the lat. Hence we have this proportion ; s. alt. : Rad. :: s. decl. : s. lat. being exactly the same as the above. Moreover the sides of this latter triangle, representing the declination and altitude, being produced to the brass meridian, will form another right angled triangle, whose sides will be the complement of the altitude, the compl. of the decl. and the compl. of the latitude ; this latter triangle being that formed by the above rule, shews how the rule itself was formed. What is here said will be useful in assisting the learner's invention in investigating new rules or problems, and the teacher will find the advantage of thus teaching *Practical Astronomy* on the globes.

The second rule is but representing the triangle, found by the latter method on the globe. It may likewise be represented on the brass meridian, the equator, and quadrant of alt. thus ; from the point aries count on the meridian passing through it, as many degrees as are equal to the decl. from the point where the reckoning ends, with the number of degrees equal to the alt. in the compasses, extend the other leg to the equator, and the same triangle as above will be formed ; and the angle formed by the equator and decl. will be the lat. as before. Hence we have another method of solving the prob. by the globe as follows : bring both poles to the horizon, screw the quadrant of alt. over the point aries ; turn the globe westward until the point cut on the equator by the compasses (with the extent of the alt) comes to the brass meridian ; extend the quadrant over the sun's decl. marked on the equinoctial colure, and the degree cut by the quadrant on the horizon, reckoning from the nearest pole, will be the lat. required. The decl. might also be set off from aries on the equator, and with one foot of the compasses in this point, intersect the equinoctial colure with the other (the compasses being extended to the lat.) the triangle thus formed will be the same as the foregoing, and the angle formed by the colure and sun's alt. will be equal to the lat. If then the pole be elevated to the degree of the colure cut by the compasses, the quadrant screwed in the zenith and extended over the decl. marked on the equator (aries being brought to the brass mer.) the degree then cut on the horizon, reckoned from the nearest pole, will be the lat. required, which will always be of the same name with the decl.

Example 1. The sun's altitude when east was observed equal to 29°, and his declination 17° 39′ N. required the latitude?

Ans. Lat. 38° 43′ north.

2. On the 21st of June, the sun's alt. when east was observed equal to the sun's declination; required the latitude?

3. On the 20th of April, the sun's alt. when west was 10°; required the latitude?

4. The sun's altitude when east, on the 20th of October, was observed equal to 15°; required the latitude?

PROB. 67.

Given the sun's azimuth and altitude at six, to find the latitude and sun's declination.

Rule. Reckon on the equinoctial colure, from the equator, as many degrees as are equal to the altitude, and on the equator from aries as many degrees as are equal to the complement of the azimuth; bring the point on the equator where the reckoning ends to the brass meridian; bring both poles to the horizon; screw the quadrant of alt. in the zenith, and extend it over the degree marked on the colure; then the degree cut on the horizon reckoned from the west, will be the lat. and the number of degrees on the quadrant, between the colure and the equator, will give the declination.

Note. If the degrees be reckoned on the equator from aries westward, then the degree cut on the horizon, reckoned from the east, will be the latitude. This is the most convenient manner of performing the prob.

Or, Reckon the degrees equal to the compl. of the azimuth on the colure as before, and the degrees equal to the alt. on the equator from aries eastward; elevate the pole to the given amplitude; screw the quadrant in the zenith, and extend it over the degree marked on the equator; the degree then cut on the horizon, reckoning from the brass meridian, will give the latitude; and the number of degrees on the quadrant between the colure and equator, will be the declination as above.*

Example 1. The sun's altitude at 6, being equal 12°, and his azimuth from the south 76½°; required the latitude and sun's declination? *Ans.* 42° 19′ N.

Note. While the sun is in the northern signs, it always rises before 6 in northern latitudes, and does not rise until after 6 in southern latitudes; the contrary is to be observed when he is in the southern signs; hence it is always known whether the latitude be north or south.

* To understand the above rules, let the globe be elevated to the supposed latitude; screw the quadrant in the zenith, and bring its graduated edge to coincide with the azimuth on the horizon; then the alt. on the quadrant will cut the parallel of the sun's decl. for the given time; and as the mer. passing through this point coincides with the east or west point of the horizon, its inclination with the horizon will be the lat. required. Hence the alt. com. of the azimuth, and declination, form a right angled triangle, and the angle formed by the decl. and co. azimuth, is the alt. required. This triangle transferred to the equator and brazen meridian, gives the above rules. The proportion for calculating the lat. is the following; Tang. alt. : Rad. :: Co Sine azimuth : Co. Tangt. latitude. Thus

2. The sun's altitude at 6, was observed equal 17¼°, and his azimuth 74° from the north; required the latitude and sun's declination?

3. Required the latitude, when the sun's observed alt. at six was 13°, and his azimuth from the north 70½°?

PROB. 68.

Given the sun's declination, altitude, and azimuth, to find the latitude and the hour of the day.

Rule. ELEVATE the pole to the given altitude, screw the quadrant of altitude in the zenith, and bring it to coincide with the given azimuth on the horizon; then turn the globe on its axis until the sun's place comes to the graduated edge of the quadrant, and the degree cut on it will be the latitude required.*

Example 1. On the 10th of May, the sun's alt. being 44°, when his azimuth from the south was 75°; required the latitude?

Ans. The sun's declination for the given day being 17° 39′, corresponding to 20° of taurus, the latitude is therefore 40° 27′ N.

2. On the 21st of June, the sun's alt. was 50°, when his azimuth from the south was 63°; required the latitude?

3. The sun's alt. on the 20th of October, was 36°, and his azimuth from the south 26°; required the latitude?

PROB. 69.

Given the sun's declination and amplitude, to find the latitude.

Rule. COUNT from the beginning of aries on the equator as many degrees as are equal to the sun's declination, and mark the point where the reckoning ends; then with the sun's amplitude in

in ex. 1, Tang. alt. 12°: Rad. :: Co. S. azimuth 76° 30′: Co. Tangt. lat. 42° 19′. The declination may also be found thus; Rad. : cos. alt. :: s. azimuth : cos. decl. In ex. 1, R. : cos. 12° :: s. 76½° : cos. decl. 17° 59′.

* The reason of this rule is thus shewn; having found the latitude as above, elevate the pole to this latitude; screw the quadrant of altitude in the zenith, and set it to the given azimuth on the horizon; then the compl. of the latitude on the equator, the complement of the altitude on the quadrant, and the distance between the sun's place and the elevated pole, form an oblique spherical triangle, and the angle included by the compl. of the lat. and compl. alt. is the given azimuth, or its supplement. Now the triangle formed by the above rule being similar and equal to this in every respect, with this difference, that the complement of the lat. is reckoned on the quadrant, and the complement of the alt. on the brass meridian, shews the truth of the rule, and whence it is derived. The method of calculating the prob. is as follows: let fall a perpendicular from the pole on the co. lat. or quadrant of alt. produced, &c. then R. : cos. azimuth :: co. tang. alt. : tang. x, and sine alt. : cos. x :: cos. $90° \pm$ decl. : cos. y. Then if the perpendicular falls within, the difference between $x + y$ will be the compl. lat. But if $x + y$ be greater than 180°, $x + y =$ compl. lat. See note to prob. 53.

Thus in ex. 1. Rad. : cos. 75° :: co. tang. 44° : tang. $x = 15° 35'$; and sine alt. 44° : cos. x 15° 35′ :: cos. $90° - 17° 39' =$ cos. 72° 21′ : cos. $y = 65° 8'$. Hence $y - x = 49° 33' =$ compl. lat. and therefore the latitude is 40° 27′, as required.

the compasses and one foot on this mark, intersect the equinoctial colure with the other; elevate the pole to the degree cut on the colure; screw the quadrant of alt. in the zenith, and extend it over the degree marked on the equator; the degree then cut on the equator, reckoning from the nearest pole, will be the complement of the latitude required.*

Or, Reckon on the equinoctial colure from aries, as many degrees as are equal to the declination, with the sun's amplitude in the compasses, and one foot on the colure where the reckoning ends, intersect the equator with the other; then bring both poles to the horizon; screw the quadrant in the zenith, and extend it over the degree cut on the colure; the degree then cut on the horizon by the quadrant, will be the latitude required.

Example 1. On the 31st of May, the sun's rising amplitude from the east towards the north, was 30°; required the latitude?

Ans. The sun's declination being 21° 56′, the latitude is found equal 41° 40′ N.

2. When the sun's declination was 23° 28′ N. his rising ampl. was 40° from the east towards the north; required the latitude?

3. When the sun's declination was 20° south, his rising amplitude was 23° 30′ from the east towards the south; required the latitude?

PROB. 70.

The day and hour being given, when a solar eclipse will happen, to find where it will be visible.

Rule. Find the place where the sun is vertical at the given hour, by prob. 12; then at all places within about 35°† of this place, the eclipse *may* be visible, especially if it be a total eclipse.

Note. Where exactnes is required, the centre of the penumbra or shade should be found; then if from this centre with the distance or number of degrees corresponding to the semidiameter of the penumbra, a circle be described on the globe, all those places within this circle will have the eclipse visible at that time; the nearer they are to the centre of the penumbra, the greater will the eclipse be.

* This and the following rule is found in the same manner as the rules given in the foregoing problems, thus; elevate the pole to the supposed latitude, or the lat. found as above, and bring the sun's place to the horizon; then the sun's declination, amplitude, and that part of the equator included between the horizon and the meridian passing through the sun's declination, will form a right angled triangle, similar and equal in every respect to that found by the above rules, &c.; and the angle formed by the equator at the horizon, and the sun's amplitude, will be the complement of the latitude. Hence this proportion, Sine ampl. : Rad :: sine decl. : cosine latitude. Thus in ex. 1, Sine 30° : Rad. :: sine decl. 21° 56′ : Co. s. lat. 41° 40′.

To know whether the lat. be N. or S. if the sun's amplitude increase when in the northern signs by elevating the north pole, or decrease when in the southern signs, the lat. is N. If the contrary take place, the latitude is south.

† Keith in his Treatise on the Globes, in solving this problem, says, that all places within 70° of the place where the sun is vertical, may have the

Example 1. On the 3d of April, 1791, there was an eclipse of the sun; its beginning was at 0h. 17m. middle at 1h. 46m. and end at 3h. 9m. as observed in Greenwich. Where was this eclipse visible?

Ans. It was visible in every part of Europe, a great part of Asia, Africa and America. It was annular along the central track of the penumbra, as in Iceland, for example, at their 12 o'clock. It was no where total, because the sun's apparent diameter exceeded the moon's at that time.

If the central track of the penumbra be represented on the globe, and lines be drawn at each side of it at the distance of half the diameter of the penumbra, the portion of the earth involved in the shadow, during the eclipse, will be represented on the globe.

2. On the 17th of September, 1811, there will be an eclipse of the sun, its beginning will be at 12h. 35m. greatest obscuration at 2h. 17m. and end at 3h. 51m. apparent time at New-York; where will this eclipse be visible?

For more examples, consult the Nautical Almanacs or Ephemerides. See also part 4th. where the subject is more fully treated.

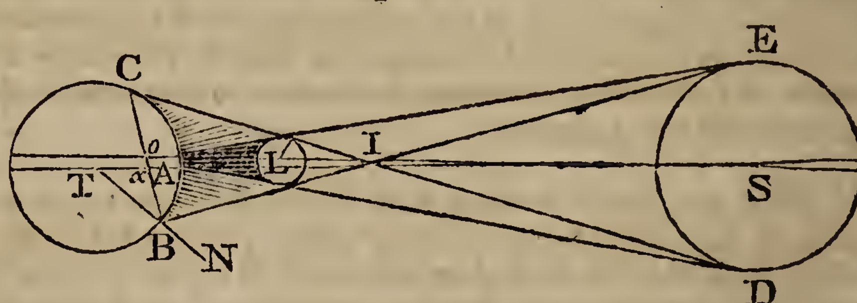
PROB. 71.

The day and hour being given when a lunar eclipse will happen, to find all those places on the globe to which the same will be visible.

Rule. Find the sun's place for the given time, elevate the pole which is most remote from the sun to this declination, bring the place where the hour is given to the brass meridian, and set the

eclipse visible if it be total; but this is evidently false, as the semidiameter of the penumbra or distance from the centre of the shade, is only about 35°. This may be shewn as follows:

Let the mean apparent diameter of the sun be taken equal 32′ 3″, the mean apparent diameter of the moon equal 31′ 7″. Let BAC be the earth, L the moon, AIB half the angle of the cone (supposing the moon to be in its node, and the centres of the sun, moon and earth to be in a straight line) this will be



equal to the semidiameter of the sun = 16′ 1″ nearly. Now the semidiameter of the moon being about .2692 semidiameters of the earth, we have Sine 16′ 1″ : rad. :: .2692 : 57.781 semidiameter of the earth = LI. The mean distance of the moon from the earth is also given (see part 4) equal 60.3 of the earth's semidiameters; hence TI = 57.781 + 60.3 = 118.081. But TB = 1 : TI, 118.08 :: sine TIB, 16′ 1″ (7.6682967) : TBI or IBN = 33° 17′ 23″. But as IBN=ITB+TIB; hence 33° 17′ 23″=16′ 1″=33° 1′ 22″. the double of which is the arch CB=66° 2′ 44″, the portion of the earth's surface covered by the penumbra. When the sun is nearest the earth, and the moon in her apogeon or greatest distance from the earth, this arch is then about 70° 50′, therefore the truth of the above remark is evident. If the centre of the sun, moon, and earth be not in a right line, or if the moon be not in its node, then the shadow will fall obliquely on the earth, as represented by index to 12; then if the given time be in the forenoon, turn the globe westward, but if in the afternoon, eastward, as many hours as the time is before or after noon; then the place exactly under the sun's declination will be the antipodes of that place where the moon is vertically eclipsed. The globe being kept in this position, set the index to 12, and turn the globe until the index has passed over 12 hours; then all those places above the horizon will have the eclipse visible, to those places along the western edge of the horizon the moon will rise eclipsed, to those along the eastern edge, she will set eclipsed,* and to that place directly under the zenith or that deg. on the brass mer. 90° from the horizon, the moon will be vertically eclipsed.†

Note. As lunar eclipses continue for a considerable time, they may be visible in more places than one hemisphere of the earth; for, owing to the earth's motion on its axis during the time of the eclipse, the moon will rise in several places after the eclipse began; hence if the prob. be performed for the beginning and ending of the eclipse, the limits of those places where the eclipse will be visible, will be determined.

Example 1. On the 10th of March, 1811, the beginning of an eclipse of the moon, at New-York, was at 13m. after 12 at night, and the end 48 min. after 2 in the morning, apparent time; where was it visible?

Ans. The sun's decl. being 4° 26′ S. to which the north pole being elevated, and New-York brought to the meridian; then the time being taken 11h. 47m. before noon, and the prob. performed as directed in the rule, it will be found that the eclipse will then be visible in all America, almost the whole of Europe, and about one half of Africa. The moon will rise eclipsed in the beginning, near Bhering's strait, the Fox islands, and west of Sandwich islands. She will set eclipsed between Wardhus and the North Cape, in Revel, Riga, the middle of Prussia, and Hungary, in Cephalonia, the middle of Congo in Africa, &c. and she will be vertical near Santa Fe in New Granada, 4° 26′ north from the equator. If the prob. be performed for 2h. 48′ in the morning, or 9h. 12′ before noon, we shall find that the moon will rise eclipsed in the eastern part of Siberia, the sea of Jesso, east of the Ladrone islands, near the middle of the New Carolinas, between Solomon's isles and New Zealand, &c. That she will set eclipsed at the eastern part of Iceland, eastward of the Azores, at St. Antonio, in the Cape

the plane CB, in which case the distance of the sun from the moon being found, the rest can be easily calculated. When the moon is not in her node, then SL*o* passing through the moon's centre will differ from S*x*T passing through the earth's, as in the figure.

The reader is referred to part 4th for a fuller elucidation of these properties in the doctrine of eclipses.

* The rising and setting of the moon eclipsed is for the given particular hour. See the answer to ex. 1st.

† Keith in his solution to this prob. says, that the moon will be vertically eclipsed under the sun's declination, which is evidently an error, as it ought to be under the degree of declination which is equal to that of the sun, reckoned on the brass meridian on the contrary side of the equator. This mistake would produce an error equal double the sun's declination.

Verd islands, between Sandwich land and the island discovered by La Roche. The eclipse will be vertical in about 117° W. long. and in lat. 4° 26′ N. And hence the eclipse will be visible in all that space between where the sun rose eclipsed at the end of the eclipse, and where she appeared setting at the beginning of the eclipse, which space is considerably more than a hemisphere. In all those places between where she rose eclipsed at the beginning and ending of the eclipse, she will successively rise eclipsed; in those places between where she appeared eclipsed at setting in the beginning and end of the eclipse, she will set successively eclipsed, and consequently in almost all Europe, &c. In the parallel 4° 26′ N. of the equator, and from long. 74° W. to long. 117° W. she will be successively vertically eclipsed.

2. On the 22d of August, 1812, the beginning of an eclipse of the moon will be at 1h. 10′, the middle at 2h. 58½′, and the end at 4h. 47′, Greenwich apparent astronomical time; where will the eclipse be visible, &c. at each of these times?

More examples may be found in the Naut. Alm. or in common almanacs.

PROB. 72.

To find when an eclipse of the sun or moon is likely to happen in any year.

Rule 1. Find the place of the moon's nodes, the time of *new moon*, and the sun's place at that time by the Naut. Alm. or an ephemeris;* then if the sun be within 17° 21′ 27″, or nearly 17° of the moon's node, there will be an *eclipse of the sun.*

2. Find the place of the moon's nodes, the time of *full moon*, and the sun's place or longitude at that time, by the Naut. Alm. or any good ephemeris; then if the sun's longitude be within 11° 34′ of the moon's node, there will be an *eclipse of the moon.*

Example 1. In 1812, on the 26th of February, at 17 hours 51 minutes astronomical time, or at 51m. after 5 in the morning of the 27th civil apparent time at Greenwich, there will be full moon, at which time the place of the moon's node will be 5s. 8° 9′, and the sun's longitude 11s. 7° 33′, or ♓ 7° 33′; will there be an eclipse of the moon at that time?

* This prob. may be solved, though not very accurately without an ephemeris, thus: If the place of the moon's nodes be given for any particular year, its place for any other year may be easily calculated; the mean annual variation according to Mayer, being 19° 19′ 43″1, or according to La Land, its diurnal motion being 3′ 10″ 638603696. The time of *new* and *full* moon may be found by the note to definition 80, and the sun's place for the given time may be found on the globe.

Dr. Halley remarks, that in the period of 223 lunations, there are 18 years, 10 or 11 days (according as there are 5 or 4 leap years) 7h. 43⅗′: that if we add this time to the middle of any eclipse observed, we shall have the return of a corresponding one, certainly, within 1h. 30′; and that, by the help of a few equations, the like series may be found for several periods: hence the time when an eclipse may be expected, can be easily found by this rule. For other methods see part 4th.

Ans. The moon's node being 5s. 8° 9′, the opposite node will be 11s. 8° 9′; hence 11s. 8° 9′ — 11s. 7° 33′ = 36′, the distance of the sun from this node; hence there will be a total eclipse; for when the sun is in one of the moon's nodes at the time of full moon, the moon is in the opposite node, and the earth is directly between them, in which position the shadow of the earth will fall directly on the moon, and produce a total and central eclipse.

2. On the 3d of November, 1808, there was a full moon, at which time the place of the moon's nodes was 7s. 12° 18′, and the sun's longitude 7s. 10° 55′; was there an eclipse of the moon at that time?

3. On the 17th of September, 1811, there will be a new moon at 57m. after 6 in the afternoon, Greenwich time, at which time the place of the moon's node will be 5s. 16° 27′, and the sun's longitude 5s. 23° 56′ 39″; will there be an eclipse of the sun at that time?

Ans. The distance of the sun from the node being 7° 29′ 39″, there will therefore be an eclipse.

4. On the 5th of September, 1812, at 22m. after 7 in the afternoon at Greenwich, there will be a new moon, at which time the place of the moon's node will be 4s. 28° 1′, and the sun's longitude 5s. 13° 0′ 35″; will there be an eclipse of the sun at that time?

5. On the 27th of March, 1812, at 12h. 16m. astronomical time at Greenwich, there will be a full moon, at which time the place of the moon's node will be 5s. 6° 35′, and the sun's longitude 7° 11′ 14″; will there be an eclipse of the moon at that time?

PROB. 73.

The time of an eclipse of any of the satellites of Jupiter being given, to find those places on the earth where it will be visible.

Rule. Find the place where the sun is vertical at the time of the eclipse; (found by prob. 12, part 2.) bring this place to the brass meridian, elevate the pole to the place where the sun is vertical, and set the hour circle to 12; then,

1. *If Jupiter be in consequentia or rise after the sun,** draw a line with a black lead pencil, or with ink, along the *eastern edge* of the horizon; this line will pass over all those places where the sun is setting at the given time; take the difference between the right ascension of the sun and Jupiter, and turn the globe westward on its axis, until as many degrees of the equator pass under the brass meridian as are equal to the difference; keep the globe from turning on its axis, and raise or depress the pole until its elevation be equal to Jupiter's declination; the globe being again fixed in this position, draw a line with a pencil along the eastern edge of the horizon; the eclipse will then be visible to every place between these lines, that is from sun setting until the time of Jupiter's setting.

* Jupiter rises after the sun, or is an evening star when his longitude is greater than the sun's longitude.

2. *If Jupiter be in antecedentia or rise before the sun.** Having rectified the globe as before, &c. draw a line along the *western edge* of the horizon; this line will pass over all those places where the sun is rising at the given hour; then elevate the pole according to Jupiter's declination, and turn the globe eastward on its axis, until as many degrees of the equator have passed under the brass meridian, as are equal to the difference between the sun's and Jupiter's right ascension; the globe being fixed in this position, draw a line along the western edge of the horizon; then the space contained between this and the former line, will comprehend all those places upon the earth where the eclipse will be visible, between the time of Jupiter's rising and the rising of the sun.

Example 1. On the 1st of May, 1812, there will be an emersion† of the first satellite of Jupiter at 14′ 33″ past 6 in the afternoon, at Greenwich; where will the eclipse be visible?

Ans. In this ex. the longitude of Jupiter will exceed that of the sun, and therefore Jupiter will rise after the sun, or be an evening star. His declination will be 23° 30′ north, and his longitude 3 signs 3° 18′ by the Nautical Almanac; the sun's longitude will be 1s. 11° 9′ 43″, and his rt. as. 38° 43′ 48″, and decl. 15° 11′ 24″ N. Jupiter's rt. as. may be found by the note to prob. 2, part 3, or more easily, but not so correctly, by the Globe. For if his longitude in the ecliptic be brought to the brass meridian, his place will be under the degree of his declination nearly,‡ and his right ascension will be found on the equator. In this ex. Jupiter's rt. as. will be found by the globes nearly $93\frac{1}{2}°$, his lat. being 4′ north. The eclipse will be visible in all those places in Europe, Asia and Africa, eastward of a great circle passing through those places where the sun sets at the given time, at Cagliari, Florence, Prague, Dant-

* Jupiter rises before the sun, or is a morning star, when his longitude is less than the sun's longitude.

† The emersion and immersion of a satellite, are terms used principally in the Nautical Almanac, to signify the appearance or disappearance of the satellites of Jupiter, &c. The immersion is the instant of the disappearance of a satellite by entering into the shadow of its primary planet, and the emersion is the instant of its appearance or emerging from the shadow. They generally happen when the satellite is at some distance from the body of Jupiter, except near the opposition of Jupiter to the sun, when the satellite approaches nearer to his body. The immersions and emersions take place on the west side of Jupiter, before his opposition to the sun, but on the east side after the opposition. If a telescope be used which reverses objects, this appearance will be directly contrary. Before the opposition the immersions only of the first satellite are visible, and after the opposition the emersions only. The same is generally the case with regard to the 2d satellite; both the phenomena of the same eclipse are frequently observable in the two outer satellites. The *longitude* from Greenwich is found, by taking the difference between the observed time and that found in the ephemeris, and converting it into degrees, &c. In part 4th this subject will be more fully entered into. The eclipses of Jupiter's satellites are set down in the lower part of pa. 3, in the Naut. Almanac.

‡ This is on supposition that Jupiter performs his motion in the ecliptic, and as he deviates but little from it, the solution by this method, on the globe, will be sufficiently accurate.

gic, Revel, &c. to another great circle passing through the western part of Madagascar, the eastern part of Arabia, Little Thibet, the middle of Siberia, &c. nearly, and therefore not visible in Greenwich, &c.*

2. On the 18th of January, 1811, there was an emersion of the 1st satellite of Jupiter at 32′ 10″ after 5 o'clock in the evening; where will it be visible? Jupiter's longitude being 1s. 21° 24′, lat. 52′ S. and decl 17° 17′ N. and the sun's right ascen. 299° 54′ 33″, and decl 20° 37′ 15″ 5 south.

3. On the 19th of August, 1812, there will be an immersion of Jupiter's 1st satellite at 40′ 26″ after nine o'clock in the morning, Jupiter's longitude being then 3s. 26° 40′, lat. 15′ N. and decl. 21° 5′, and the sun's right ascension 148° 22′ 45″, and decl. 12° 47′ 32″ north; where will it be visible?

4. On the 21st of December, 1812, at 31′ 34″ after one in the morning, there will be an immersion of Jupiter's 2d satellite, his longitude being then 4s. 8°, lat. 36′ N. and decl. 18° 51′ N. and the sun's right ascension 269° 2′ 24″, and decl. 23° 27′ 21″; where will this immersion be visible?

PROB 74.

To explain the phenomenon of the Harvest Moon.

Definition 1. In north latitude, the full moon which happens at, or is nearest to, the autumnal equinox, is called the *harvest* moon.†

* To know if an eclipse of any of the satellites of Jupiter will be visible at any place, the Naut. Alm. directs to find whether Jupiter be 8° above the horizon of the place, and the sun as much below it. This observation may be easily applied to the above examples.

† As the sun is in virgo, and libra in our autumnal months, and as the moon can never be full but when she is opposite to the sun, therefore the moon is never full in the opposite signs pisces and aries, but in these two months. This remarkable rising of the moon is not observed but in *harvest*, or in September and October, when the moon is in pisces and aries. For although the moon is in these signs twelve times in the year, it is only about the autumnal equinox that her orbit is nearly parallel to the horizon, so that there is very little difference in her rising for several nights. In winter these signs rise at noon, at which time the moon is in her first quarter, being only a quarter of a circle distant from the sun, so that when the sun is above the horizon, the moon's rising is neither perceived or regarded. In spring these signs rise with the sun, because the sun's place is then in them; but as the moon changes, or is new moon in them at that time, she is therefore invisible. In summer these signs rise about midnight, and the sun being then three signs or a quarter of a circle before them, the moon is in them about her third quarter, at which time she rises so late, and gives so little light, that her rising passes unobserved. In autumn these signs being opposite the sun's place, rise when the sun sets; and as the moon is then in opposition or at the full, her rising becomes very remarkable.

This phenomenon becomes more remarkable the farther the place is from the equator, if not beyond the polar circles; for in this case the angle which the ecliptic makes with the horizon, gradually diminishes when pisces and aries rise.

Def. 2. The harvest moon in south latitude, is the full moon which happens at or near the vernal equinox.*

Rule 1. *For north latitude.* Elevate the north pole to the latitude of the place; mark the point aries in the ecliptic, and also every 12°† on each side of that point, until there be 12 or 13 marks; bring that mark nearest to pisces to the eastern part of the horizon, and set the index to 12; the globe being then turned westward on its axis, until the other marks come to the horizon successively; then the intervals of time between the marks coming to the horizon, will shew the difference of time between the moon's rising every day. These marks being brought to the western edge of the horizon in the same manner, the diurnal difference of time between the moon's setting may be found.

Thus in New-York, the diurnal difference of time in the moon's rising, in pisces, aries, &c. is 26′, 27′, 28′, 29′, &c. and the diurnal difference in her setting is 60′, 61′, 62′, 63′, &c The cause of this difference arises from the different angles which the ecliptic makes with the horizon; for those parts or signs which rise with the smallest angles, set with the greatest, and the contrary. Thus in the above case, the point aries makes only an angle of 25½° with the horizon, when it rises, but when it sets, it makes an angle of 72½°. In equal times whenever this angle is least, a greater portion of the ecliptic rises than when the angle is larger; therefore when the moon is in those signs which rise or set with the smallest angles, she rises or sets with the least difference of time, and with the greatest difference in those signs which rise or set with the greatest angles, from which the whole is evident.‡

2. *For south latitude.* Elevate the south pole to the latitude of the given place; mark the point libra and every 12 degrees of the

* In northern latitudes the autumnal full moons are in pisces and aries, and the vernal full moons in virgo and libra; but in southern latitudes the reverse takes place, the seasons being contrary. Now as virgo and libra rise at as small angles with the horizon, in southern latitudes, as pisces and aries in northern latitudes, the harvest moons are therefore as regular on one side of the equator as on the other, only that they happen at contrary seasons of the year.

† The reason that 12° is marked, is because the moon gains nearly 12° on the sun every day; for the moon's daily mean motion is 13° 10′ 35″, and the sun's 59′ 8″ 3, the difference of which is 12° 11′ 26″ 7. The solution is on supposition that the moon remains constantly in the ecliptic, which is accurate enough for illustrating the prob. otherwise the moon's place may be marked on the globe at the time of full moon, and a few days before and after it, by having her lat. and long. or rt. ascen. and decl. given; which may be found from the Nautical Alm. or any good ephemeris. See the investigation of this prob. given in page 128 of Vince's Astronomy, 8vo. or in ch. 16, pa. 203 of Ferguson's Astronomy, 8th ed.

‡ As there is a complete revolution in the moon's nodes in about 18 years 8 months, all the varieties of the intervals of the rising and setting of the moon will happen within that time. The following table extracted from pa. 216 of Ferguson's Astron. will shew in what years the harvest moon's are the least, and the most beneficial with regard to the times of their rising, from 1811 to 1861. The columns of years under L, are those in which the harvest moon are the least beneficial, because they fall about the

ecliptic, preceding and following that point, as before; bring that mark which is nearest to virgo to the eastern edge of the horizon, and set the index to 12; then turn the globe westward until the other marks come to the horizon successively, and observe the hours passed over by the index (or rather on the equator) the intervals of time between the marks coming to the horizon, will be the diurnal difference of time between the moon's rising, &c. If these marks be brought to the western edge of the horizon in like manner, the diurnal difference of the moon's setting may be found, &c. as in the preceding part of the problem.

PROB. 75.

To draw a meridian line upon a horizontal plane.

Rule 1. FROM a correct altitude of the sun find the time of the day (by the note to prob. 48, or 52, part 2d) and set a well regulated watch to that time; then suspend a plumb line so that the sha-

descending node; and those under M are the most beneficial, because they fall about the ascending node.

L	L	L	L	M	M	M	M	M
1811	1827	1833	1848	1816	1822	1837	1843	1858
1812	1828	1834	1849	1817	1823	1838	1853	1859
1813	1829	1844	1850	1818	1824	1839	1854	1860
1814	1830	1845	1851	1819	1825	1840	1855	1861
1815	1831	1846	1852	1820	1835	1841	1856	
1826	1832	1847		1821	1836	1842	1857	

In this instance of the harvest moon, as in innumerable others, which astronomy points out, the beneficence and wisdom of that intelligent being who presides over the universe, discover themselves. Here we see the wandering course of the moon so ordered, as to bestow more or less light on those parts of the earth, where their circumstances and seasons render it more or less necessary and serviceable. Wherever we cast our eyes, we see that all is formed and regulated with design, that every thing has its particular use, that every thing proclaims the boundless wisdom, and witnesses the most attentive kindness of the maker. If at times (says De Feller, in his Philosophical Catechism) those visible beings occasion some physical evil, the reason and understanding given to man, supply him with means to escape the evil or to remedy it; and besides, what are those physical evils compared with the benefits attending them, the services they render, and the virtues they occasion? Even Voltaire himself, the great champion of every error, acknowledges, that "In the system that admits of a God, there are only *difficulties* to get over; but in all other systems there are *absurdities* to swallow."—"What idea, says De Feller, could make amends for that of God; an idea so vast in itself, and so rich, that begets and fosters so many others, that of duty, that of justice, that of charity! And what shall we say of the great, the exalted sentiments that flow from those ideas; the voice of conscience, the study of the law of God, the knowledge in detail, and upon principle of his commandments, &c. of the many obligations of a good christian, of the pious practices that occupy the soul, and with unction ineffable, render it happy in every situation of life. Heavens! what a void must not the loss of all this leave behind it, in the soul and the life of man! and is it not quite natural, we should become triflers and fools, in the same proportion as we become irreligious?"

dow of it may fall on the plane, and when the hour hand of the watch is at 12, the shadow of the plumb line will be the true meridian.*

Rule 2. Describe several concentric circles on a horizontal plane (as a board, &c.) In the centre of these circles fix a pin or straight wire perpendicular to the plane; observe, in the forenoon, when the extremity of the shadow exactly touches the respective circles, beginning with the outermost, and mark these respective points on the circles. In the afternoon mark where the extremity of the shadows cuts the same circles as before, beginning with the innermost; then with a pair of compasses bisect the arch between the two marks on any of the circles; a line drawn from the centre to that point, will be a true meridian line.† If the pin be not perpendicular, let the circles be described from the top of it, and proceed as before.‡

Rule 3. Find when the pole star and the star alioth in the great bear are in the same plumb line, or have the same azimuth, by means of two plummets suspended at a considerable distance, with their ends in vessels of water, to keep them steady; the line between the two plummets or this line produced, will be the meridian, sufficiently exact. Any two stars that have the same right ascension, will answer the same purpose.

Rule 4. Having the time of the northing of the star alioth, or the northing or southing of any other star or planet from astrono-

* The most proper time of the year to perform both this and the following rule, is about the solstices, because then the sun's declination does not sensibly vary for several days.

As there are various methods of finding the hour of the day (most of which are given in parts 2d and 3d of this work, particularly in the notes) and consequently of regulating the watch; hence there are as many different methods of performing this prob. Moreover from the true and magnetic azimuth or amplitude being given, the variation of the compass may be found, and hence a meridian line may be traced out by the compass. This prob. being necessary for fixing dials, is therefore given in this place.

† The board may be a foot or more in breadth, and the circles about a quarter of an inch or ½ an inch from each other. The pin or wire ought to be about ⅛ of an inch thick, with a round blunt point, or well defined head like the head of a pin, and of such a length that the shadow may fall within the innermost circle, at least four hours, in the middle of the day. This method is not, however, capable of very great accuracy, as the shadow is scarcely ever well defined. If however the mean of the several meridians, so found, be taken, the meridian thus found will be sufficiently accurate for all common purposes.

‡ In describing these circles, one end of a wooden ruler may be placed on the top of the wire, and with a sharp pointed iron pin, or wire, in the other end of the ruler, circles may be described. The same prob. may be performed by means of a small hole in a window-shutter, through which the sun shines, circles being described on the floor, with the hole as their common centre. Or if the casement of a window on which the sun shines at noon be perpendicular to the horizon, the shadow cast by it, on the floor, will trace out the meridian. Various other methods and contrivances will easily present themselves.

mical tables (see prob. 8. and 39, part 3d.) A line drawn from the observer towards the respective star or planet, at the time of its northing or southing, will trace out the meridian required. Or when the pole star and alioth, or any two stars having the same or opposite rt. ascensions, be in the same azimuth, set the watch to the time of their northing or southing, according as they are north or south of the observer, and next day at 12 o'clock, by the same watch or clock, draw a meridian line by the shadow of a plumb line hung in the sun.*

GNOMONICAL OR DIALLING PROBLEMS

SOLVED BY THE GLOBES.

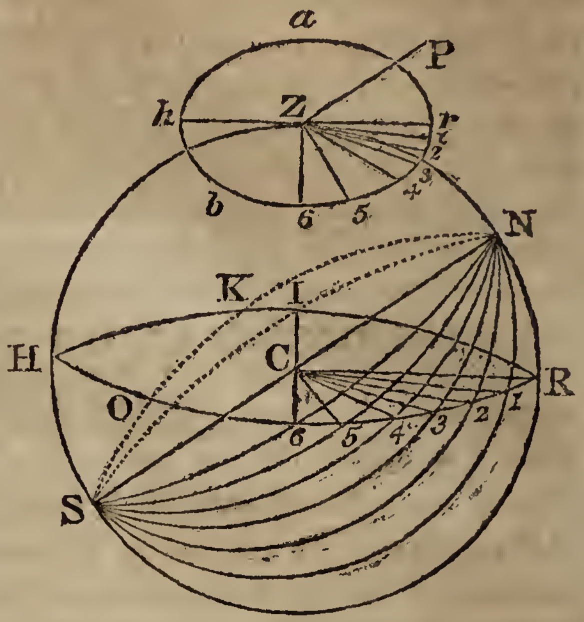
Fundamental principles, observations, &c.

DIALLING, or *the art of making dials*, is founded entirely on astronomy. For the lines on a dial which shew the hours, are the intersections of the respective circles with the plane of the dial; and the projection of these hour lines, is the same as the projection of the sphere upon the plane of the dial. And hence the construction of dials depends on the projection of the sphere, particularly the gnomonic projection, where the circles are projected into right lines. The principles of this projection Emerson has given at large, in his treatise on the projection of the sphere (see his Tracts.) As the art of measuring time is of the greatest importance, so the art of dialling, until the invention of clocks and watches, was held in the greatest estimation. And although at present we are furnished with these machines, yet as the best of them are often out of order, and that in general, they are liable to stop and go wrong, that unerring instrument, a true sun dial, will be always useful to correct and regulate them.

Suppose the globe of the earth as represented in the annexed figure, to be transparent, with hour circles or meridians, &c. drawn upon it, and that it revolves round a real axis NS, which is opake, and casts a shadow; then it is evident that the shadow of this axis will fall upon every particular meridian or hour line, when the sun comes to the plane of the opposite meridian, and will therefore shew the time in all those places on that meridian. Now if any

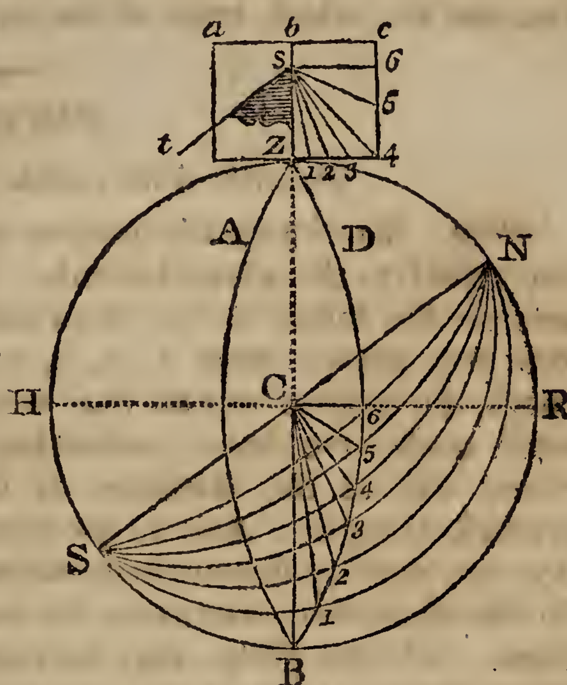
* To take away the star's rays, look through a small hole in a thin plate, or piece of paper. Any line drawn parallel to the meridians found as above, will also be meridians; hence when those found above will not answer, others may be easily drawn. When the meridian line is intended to be the basis of any nice astronomical calculations or observations, it must be traced out, very accurately, by the 1st rule, then two poles may be erected at a considerable distance from each other, with marks to render them visible, &c. For making these observations, the astronomical circle or circle of reflection is the most proper instrument. An artificial horizon may be made with molasses, quicksilver, &c. When this circle cannot be had, a Hadley's quadrant, or rather sextant, will answer. In performing the observations a calm place must be selected.

opake plane be imagined to pass through the centre of this transparent globe, the shadow of half the axis NC will fall upon either side of this intersecting plane.



Let HORI represent the plane of the horizon, RN the elevation of the pole or lat. of the place; then while the sun is above the horizon, the shadow of the axis CN will fall upon the upper side of the plane HOR. When the edge of the plane of any hour circle as K, I, &c. points directly to the sun, the shadow of the axis, being coincident with this plane, will mark the respective hour lines on the plane of the horizon HOR; and hence the hour line on the horizontal plane, is a line drawn from its centre, to the point where this plane intersects the meridian, opposite to that on which the sun shines. Now as the sun's apparent motion about the earth's axis is at the rate of 15°, an hour (nearly) let the shadow of the earth's axis be supposed to be projected into the meridian opposite to that in which the sun is, and then this meridian will move at the rate of 15° an hour. (See Emerson's Dialling, sect. 1. prop. 1. and cor. &c.) Let ZNRSH represent a meridian on the surface of the earth, SCN the earth's axis, Z the place of the spectator, being also the pole of the horizon HORI; let the meridians N1S, N2S, &c. be drawn so as to make angles with the mer. NRS of 15°, 30°, &c. respectively; then supposing NR the meridian into which the shadow of CN is projected at 12 o'clock, N1, N2, N3, &c. are the meridians into which the shadow is projected at 1, 2, 3, &c. of the clock, and these shadows will be projected on the plane HORI into the lines CR, C1, C2, C3, &c. and the angles NC1, NC2, &c. will be the angles between the 12 o'clock line CR, and the hour lines of 1, 2, 3, &c. Hence in the right angled triangle NR1, we have NR the lat. of the place (See the last note to problem 19, part 3d. art. 1.) and the angle RN1 = 15°, and therefore, by *Napier's rule*, *Co. tang.* 15° *(or hour angle)* : *rad.* :: *s. lat.* NR : *tang.* R1 the hour arch, which is the measure of the angle RCI; or *Rad* : *sine* NR *(the lat.)* :: *tang.* 15° : *tang.* R1 *; or tang.* 30° : *tang* R2 *or angle* RC2, *&c.* (Simson's Spher Trig. annexed to his Euclid prop. 17.) in the same manner by either of these proportions, the arch R3, R4, &c. may be found, and hence a table of the hour angles for any lat. may be easily calculated. In the foregoing elucidations we made the earth's axis the *gnomon*, and considered the shadow as projected upon the plane HORI. But as it is the same thing whether a dial be drawn upon any given plane, or upon the plane of the great circle of the sphere which is parallel to it, (Emerson's

Dialling, cor. 3. prop. 1. sect. 1.) Let the plane *arbh* be drawn parallel to HORI at Z, this plane will represent the *sensible* horizon, on which let ZP be drawn parallel to CN, and let its shadow be projected on the plane *arbh* in the same manner as the shadow of CN is projected on the *rational* horizon HORI, and the hour angles *r*1, *r*2, &c. be calculated in the same manner; this will be an *horizontal* dial, ZP will be its gnomon, and the lines Z1, Z2, &c. the hour lines required. For at the immense distance of the sun, ZP may be considered as coinciding with CN. If at the other side of *r*, arches *r*11, *r*10, *r*9, &c. be made equal to *r*1, *r*2, *r*3, &c. respectively, and lines be drawn from Z to these points; these will be the hour lines for the forenoon. Moreover, the hour lines may be continued from 6 towards H as far as may be necessary, by laying off from 6 to 7, the same distance as from 6 to 5, &c and also from I towards H, by producing 5C, 4C, 3C, &c. The reason of which see in Emerson's Dialling.



Again, Let ABD be an opake vertical plane, or great circle, perpendicular to the plane HBNZ (and therefore to the horizon at Z) and passing through the centre of the globe. Here the globe being supposed transparent as before, while it revolves on its axis SN, it is evident that the shadow of the part of the south end of the axis CS, considered as opake, will always fall on the plane ABD, and mark out the hour as in the horizontal dial Then for the same reason as before, if the angles BS1, BS2, BS3, &c. be 15°, 30°, 45°, &c. the shadow of SC will be projected into the lines C1, C2, C3, &c at the hours of 1, 2, 3, &c. of the day; and the angles BC1, BC2, &c. will be measured by the arches B1, B2, &c. Hence in the right angled triangle BS1, BS = ZN co. lat. or dist. of Z from the pole N, and the angle BS1 = 15°; therefore by Napier's rule as before, we have *Co. tang.* 15° *(or hour angle)* : *rad.* :: *cos. lat.* :: *tang.* B1 *the hour arch.* Or by Simson's Spher. prop. 17, *Rad.* : *sine* SB, *the co. lat.* :: *tang.* 15° : *tang.* B1. In the same manner B2, B3, &c. may be found, making use of 30°, 45°, &c. respectively, in place of 15°. Now if Z*abc* be a place coinciding with the plane ABD, and *st* be parallel to CS, *st* will project its shadow on the plane Z*abc* in the same manner as CS on the plane ABD, for the same reason as for the horizontal dial; hence the hour angles from the 12 o'clock line, are computed by the same proportion. This is a *vertical south dial.* Hence also it appears that the surface of every dial whatever, is parallel to the horizon of some place or other upon the earth, in which place it would become a horizontal dial; and that if a dial be taken to any other latitude different from that for which it was made, it will indicate the apparent time truly, if placed parallel to its former situation. Moreover as the angle SCB = *is*Z (29 Eucl. 1.) = the arch ZN = co. lat. and that the lat. of H or R is = co. lat. of Z, it follows that the elevation of the stile or gnomon above the dial's surface, when it faces the *south*, is always equal to co. lat. of the place, or equal to the lat. of the place whose horizon is parallel to its surface. In the same manner the hour lines may be calculated, when the shadow is projected upon a plane, in any other position. See sect. 1 of Emerson's Dialling.

Remark. It appears from the above observations, as the whole earth is but a point in comparison of the heavens, that if a small sphere of glass be placed on any part of the earth's surface, having an opake axis parallel to the axis of the earth, and such lines upon it, and such a plane within it as above described; it will shew the hour of the day as correctly as if it were placed at the earth's centre, and the whole body of the earth were as transparent as crystal.

PROB. 76.

To make a horizontal dial for any latitude.

Rule. ELEVATE the pole as many degrees above the horizon as are equal to the given latitude; bring aries to the brass meridian, and set the index to 12; then turn the globe westward until the index has passed over 1, 2, 3, 4, 5 and 6 hours successively, and mark the degree cut on the horizon by the equinoctial colure, at each respective hour, reckoning from the north or south points, these will be the distances of the hour lines from noon until 6 o'clock at night: And as the hour of 1 and 11, 2 and 10, 3 and 9, &c. are equally distant from noon, the hour arches for 1, 2, 3, &c. in the afternoon, will serve for those of 11, 10, 9, &c. in the forenoon. Or the globe may be turned eastward until the index has passed over 11, 10, 9, &c. on the index, and the degrees cut by the colure on the horizon marked as before. The hours on the equator will answer rather better than those on the hour index. If the half hours, quarters, &c. be brought to the meridian, in the same manner, the colure will mark the hour arches, &c. corresponding on the horizon.*

* The reason of this rule is evident from what is said in the preceding observations. For the latitude of the place, the hour arch on the horizon, and the arc of the colure between the elevated pole and the horizon, form a right angled triangle, similar to that described in the observations on the horizontal dial, from which the rule is derived.

From the above rule it appears, that there is no *absolute* necessity of having meridians drawn through every 15° on *Cary's* globes, as the hour index, or the hours marked on the equator, are *abundantly* sufficient, though some late authors imagine the contrary, owing to their manner of solving the problem. If however the meridians be drawn through every 15°, the whole of the hour arches may be seen at one view on the horizon, without any motion of the globe on its axis, which is much more convenient.

Example 1. To make a horizontal dial for New-York in latitude 40° 43′ N.

The pole being elevated to the lat. and the point aries brought to the meridian; then the hour arches from 12 will be 9° 55′, 20° 38′, 33° 7′, 48° 29′, 67° 40′ and 90° for the hours I, II, &c to VI respectively; or reckoning from the east towards the south, they will be 22° 20′, 41° 31′, 56° 53′, 69° 22′, 80° 5′ (the compl. of the former) for the hours VI, V, &c. to I, reckoning from VI o'clock backwards to XII.*

2. To make a horizontal dial for London in lat. 51½° N.

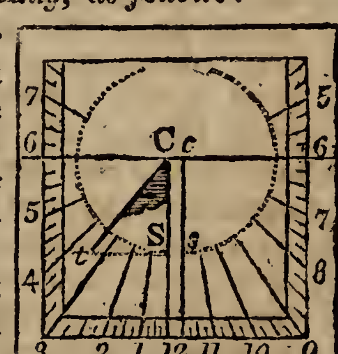
3. To make a horizontal dial for Philadelphia in lat. 39° 57′ N.

* It is not necessary in the above solution to give the distances further than VI, for the distances from XII to VI in the forenoon, are the same as from XII to VI in the afternoon; and if the hour lines be continued through the centre of the dial, they will point out the opposite hours.

The following table calculated for the lat. 40° 43′ by the following proportion, *Rad.* : *sine lat.* :: *tang. hour angle* : *tang. hour arch* (see the foregoing principles, &c.) contains the hour arches, halves, quarters, &c. from XII to VI.

Hours.	Hour Angles.	Hour Arches.	Hours.	Hour Angles	Hour Arches.	Hours.	Hour Angles.	Hour Arches.
12¼	3° 45′	2° 27′	2¼	33° 45′	23° 33′	4¼	63° 45′	52° 54′
12½	7 30	4 54	2½	37 30	26 35	4½	67 30	57 35
12¾	11 15	7 24	2¾	41 15	29 46	4¾	71 15	62 30
I	15 0	9 55	III	45 0	33 7	V	75 0	67 40
1¼	18 45	12 29	3¼	48 45	36 38	5¼	78 45	73 2
1½	22 30	15 7	3½	52 30	40 22	5½	82 30	78 35
1¾	26 15	17 50	3¾	56 15	44 19	5¾	86 15	84 15
II	30 0	20 38	IV	60 0	48 29	VI	90 0	90 0

The horizontal dial may be constructed geometrically, as follows:



Take any point C for the centre, and draw CS or C12 for the meridian or 12 o'clock hour line, C6 at right angles to it, for the 6 o'clock hour line; take Cc equal the thickness of the *stile* or *gnomon*, and draw *cs* parallel to CS. Now from the centre C or *c* with the *radius* of some *line of chords*, describe a circle; then on the arch of this circle lay off the hour arches 9° 55′, 20° 38′, &c. on each side of the 12 o'clock hour line, together with the half hours and quarters, as in the above table. Lastly, make the angle SC*t* = the lat. = 40° 43′ for the stile, which is to be placed perpendicular to the plane of the dial, the dial is then finished. To erect it, the line C12 must be placed in the meridian (found by the last prob.) so that the 12 may point towards the north; then the gnomon C*t* will exactly point to the north pole. The dial plane must be placed horizontal with a level, and then the dial is fit for use.

If the edge of the stile has no considerable breadth, and is in the same place with the *substile* CS, no allowance is necessary, so that C may be taken as the centre of both semicircles, &c. Various hours, ornaments, tables of the equation of time, &c. may be inserted on those dials, and different in-

PROB. 77.

To make an erect south dial for any latitude.

Rule. ELEVATE the pole to the complement of the latitude (the south pole if the lat. be north, &c.) bring the point aries to the brass meridian, and set the index to 12; then turn the globe eastward or westward until each hour, &c. on the equator or on the hour circle, comes successively to the meridian; then the *colure* passing through aries will point out, on the horizon, the distance of the respective hour arches, or hour lines from the meridian.

Or, If the meridian be drawn through every 15°, as on Bardin's globes, aries being brought to the brass mer. the meridians passing through every 15° will point out the hour arches on the horizon.*

Example 1. To make an erect south or vertical dial for New-York, lat. 40° 43′ N.

The south pole being elevated 49° 17′, and aries brought to the mer. then the globe being turned on its axis, the *colure* will intersect the horizon in the following degrees: 11° 29′, 23° 38′, 37° 10′, 52° 42′, 70° 32′, and 90° for the hours I, II, &c. to VI; or XI, X, &c. to VI; or if you count from the east towards the south, they will be 0°, 19° 28′, 37° 18′, 52° 50′, 66° 22′, and 78° 31′, for the hours VI, V, &c. to I; or VI, VII, &c. to XI.†

2. To make an erect south dial for Washington city, lat. 38° 53′ north.

struments, scales, &c. used in their construction, for which the learner is referred to Emerson's Dialling, Ferguson's Lectures, Ozanam's Mathematical Recreations, Jones's Dialling, &c.

Emerson in his Dialling (schol. to prob. 7, sect. 2) remarks, that if a horizontal dial be made for a place in the torrid zone, to shew the hour by the top of the perpendicular stile *t*S, whenever the sun's declination exceeds the lat. of the place, the shadow of the gnomon will *go back* twice in the day, once in the forenoon, and once in the afternoon. (See the note to ex. 3. prob. 51. part 2.) The greater the difference between the lat. and the sun's decl. the further will the shadow go back. The same will take place with respect to any star, &c. the declination of which is greater than our latitude.

In the 38th chap. of Isaias, it is related, that the shadow on the dial of *Achaz* was brought back ten lines or degrees, in confirmation of a promise made by *Isaias* to *Ezechias*, king of Juda, that his life should be prolonged 15 years. This was truly, as it was then considered, a *miracle*, being contrary to the established laws of nature; as *Jerusalem*, where the dial was erected, was not in the torrid zone, and therefore the shadow could not possibly go back from any natural cause. Whatever *incredulity* may object to this, it is certain that that Being who framed the laws of nature, can suspend their operations, or change them at his pleasure; as in the present case, to reward the piety of a virtuous prince, and exhibit to the world the efficacy of fervent and humble prayer.

* The reason of both these methods will appear from the observations in the beginning.

† The arches for the half hours, quarters, &c. may be found by the set method above in the same manner as the hour lines. The following table contains the hour arches, halves and quarters, from XII to VI. It is calcu-

PROB. 78.

To find the hour lines on the plane of any dial, by one position of the globe.

Rule. If the meridians do not pass through every 15°, draw them with a black lead pencil or ink, by bringing every 15° on the equator to the brass meridian, &c. then elevate the pole as many degrees above the horizon as are equal to the latitude; bring aries to the brass meridian; all the other meridians will then cut any circle representing the plane of a dial, in the number of degrees on that circle, that each respective hour line is distant from the 12 o'clock hour lines passing through the same circle.

PROB. 79.

To find in what part of the earth any dial plane, which is not horizontal in a given latitude, will become horizontal.

As every plane, whatever be its situation, is parallel to the horizon of some place on the earth, hence a dial, though not horizontal in one place may become so in another, and the horizontal dial made for the latitude of this place, will be the same as the former; thus, *For an erect direct south or north dial.** Find the co. lat. of

lated by the following proportion; *R : Cos. lat. :: tang. hour angle : tang. hour arch.* The hour angles are omitted, being the same as those in the table in the note of the foregoing prob. The reason of the rule is given in the preceding observations.

Hours.	*Hour Arches.*	*Hours.*	*Hour Arches.*	*Hours.*	*Hour Arches.*	*Hours.*	*Hour Arches.*
12¼	2° 50′	1¾	20° 30′	3¼	40° 50′	4¾	65° 52′
12½	5 42	II	23 38	3½	44 39	V	70 32
12¾	8 34	2¼	26 52	3¾	48 36	5¼	75 17
I	11 29	2½	30 11	IV	52 42	5½	80 9
1¼	14 26	2¾	33 37	4¼	56 57	5¾	85 3
1½	17 26	III	37 10	4½	61 20	VI	90 0

The *geometrical construction* of this dial being the same as for a horizontal dial, made for the complement of the lat. or in ex. 1, for lat. 49° 17′, the method given for constructing the horizontal dial (note to prob. 76) will answer for this. The point C, or the centre, may be taken near the top of the plane of the dial. The forenoon hours are to be numbered towards the west, and the afternoon hours towards the east; and the angle for the stile must be made = co. lat. See the preliminary remarks. The two dials described in this and the foregoing prob. are the most useful, and therefore the most common.

To find whether a wall be due south for a vertical south dial, erect a gnomon perpendicularly to it, and hang a plumb line from it; then the watch being adjusted to apparent time, if when it points out 12 o'clock the shadow of the gnomon coincides with the plumb line, the wall is due south.

* An *erect vertical dial* is that which is drawn on a plane perpendicular to the horizon; but a *direct dial* faces the east, west, north, or south points of the horizon.

the given place, then the horizontal dial made for any place in this lat. having its longitude the same as the given place, will be the erect direct north or south dial required.

Thus an erect dial under the pole will be an horizontal dial under the equinoctial; an erect dial in lat. 30° will be a horizontal dial in lat. 60°, &c. and the contrary.

For an erect decliner * The globe being rectified to the given latitude, bring the given place to the brass meridian; reckon the declination on the horizon, from the north or south; that place on the globe, opposite the point where the reckoning ends, will be the place required. Thus an erect decliner, which in New-York declines 60° from the south towards the east, will be horizontal in lat. 22½° S. long. 3° W. If it declines 90° from the south towards the *east* or *west*, it will become a horizontal dial on the equator in long. 18° E. or 162° W. &c. If the plane declines eastward, the sun will come later to the meridian of it, than to the meridian of the place where it becomes a horizontal dial; or sooner if the plane declines westward, by as many degrees or by as many hours, minutes, &c. as are equal to the difference of longitude.

For a *direct recliner*,† the complement of the plane's reclination will be the latitude, where it becomes a horizontal dial in the same longitude.

For a *declining recliner*. Rectify the globe for the given lat. bring the given place to the brass meridian; screw the quadrant of alt. in the zenith, bring the quadrant to coincide with the degree of the plane's declination on the horizon, and count from the horizon on the quadrant the degrees of the plane's reclination, under which mark the place on the globe, this will be the place required, where the declining recliner will become a horizontal dial, and its latitude and longitude may be found by the globe. The difference of longitude changed into time, will give the difference that the sun makes between the two meridians.

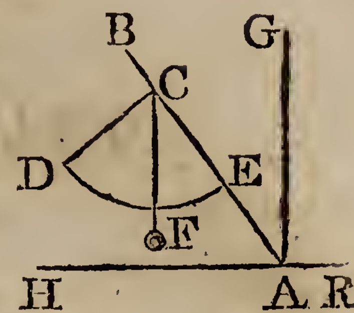
The learner can have no difficulty in applying the above principles when understood, to any kind of dial and for any latitude. For more information, the treatises on dialling before referred to, may be consulted. The demonstration of the above properties are given in sect. 1. Emerson's Dialling.

* A *declining dial* is a dial that faces none of the cardinal points, but declines towards the *east* or *west*, and an *inclining dial* is that whose plane makes oblique angles with the horizon; the *inclination* is the angle which it makes with the horizon.

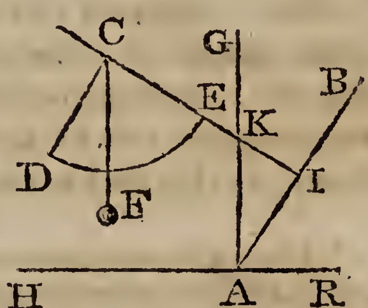
† The *reclination* of a plane is the angle it makes with a vertical plane, or the number of degrees it leans from you, being the plane's distance from the zenith; and the *declination* of a plane is an arch of the horizon contained between the plane and the *prime vertical*, or between the meridian and plane perpendicular to the dial plane, and is always reckoned from the south or north.

The inclination, reclination and declination of a plane, may be thus found.

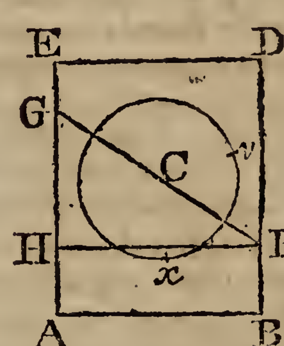
1. *To find the inclination of a plane.* Let AB be a plane inclined to the horizon HR; to this plane let the quadrant CDE be applied so that the plummet CF may touch its edge, or the surface of the quadrant; then the arc DF will be the measure of the plane's inclination or the angle BAH. Draw AG perpendicular to HR, then because CF is parallel to AG, the angle ACF = CAG (29 E. 1.) but DCE = GAH, both being right angles, therefore the remainders are equal or DCF = BAH. *Q. E. D.*



2. *To find the reclination of a plane.* Let AB be the reclining plane, and AG perpendicular to HR, the hor. will represent the prime vertical; then the angle BAG will be the plane's reclination. Draw IC perp. to AB, and apply the quadrant CDE to this perp. then will the arc DF, or the angle DCF, be the measure of the reclination. For in the rt. angled △ AIK the angles AKI + IAK = 90° = DCE; but because CF is parallel to GA, the angle ECF = AKI (29 E. 1) therefore DCF = KAI. *Q. E. D.*



3. *To find the declination of any plane.* Let ABDE be a piece of board, &c. whose surface is a rt. angled parallellogram, on which let the circle *xv* be described. On the centre C let a perpendicular pin or wire be erected, and place the plane AD in a horizontal position, with the side AB applied to the dial plane; observe when the shadow of the top of the pin is at *v* in the forenoon, and at *x* in the afternoon, on the same day, (see prob. 75) let the diameter FG bisect *xv*, and draw FH parallel to AB, it is evident that the angle GFH will be the angle of declination required.



Or, If the square box of an azimuth compass be applied to the dial plane (i. e. the north side of the box to a south plane, and the south side to a north plane) so that the box be kept horizontal; then the needle will point out the plane's declination, regard being had to the variation. For other methods see Emerson, &c. When great exactness is required, the problems on the azimuth, &c. will afford the learner various astronomical methods, &c.

PROBLEMS

PERFORMED BY THE

CELESTIAL GLOBE.

PART III.

PROB. 1.

*To find the right ascension and declination of the sun or a star.**

Rule. BRING the sun's place in the ecliptic, or the star, to the brass meridian; then the degree which is over the sun's place, or the star, on the meridian, is the declination, and the degree of the equinoctial, cut by the brass meridian, reckoning from aries eastward, is the right ascension.

* The right ascensions and declinations of the moon and the planets, must be found from astronomical tables or from a good ephemeris, as they cannot be represented on the globe on account of their continually changing their places in the heavens. In the 4th page of the month in the Nautical Almanac, their longitude, latitude, declination, and passage over the meridian, are given; and hence as their longitude is given, their rt. ascension may be easily found, by calculation. The declination of the sun or a star may be thus observed; when the sun is nearer to the equator than the place, the difference between the complement of the altitude (or the zenith distance) of the sun or star will give the declination. (See notes to prob. 8 and 48, part 2.) The declination being given, the proportion for determining the sun's rt. ascension is given in the note to prob. 49, part II.

To find the right ascension of a star by observation. With a good pendulum clock adjusted, that the hand may run through the 24 hours in the time that a star leaving the meridian will come to the same meridian again (which time is equal to 23h. 56′ 4″ 1, taking for unity the mean astronomical day, being less than the natural day by the space the sun moves through in the mean time eastward.) The clock being thus adjusted, when the sun is in the meridian, set the hand or index to 12; observe when the star comes to the meridian, and then observe the time shewn by the clock; this time, or the hours, minutes and seconds described by the index, turned into degrees and minutes of the equator, will give the difference between the right ascension of the sun and star; this difference added to the right ascension of the sun, will give the right ascension of the star.

If the dial plate on the clock, instead of being divided into 24 hours, be divided into 360°, and their sexagesimal parts, and if at the moment the sun is on the meridian, the index be placed to the number of degrees and minutes the sun's rt. ascension then consists of, the index will then point out the right ascension of the star, when it comes to the meridian. By knowing the right ascension of one star, we may from it find the rt. ascensions of all the others which are visible, by finding the difference of the time of their coming to the meridian, which converted into degrees and minutes of the equator, will give the difference of their right ascensions. Or the declination and right ascension of one star being given to find the right ascension of another whose distance from the former and its declination are given. As the

Or, Place both poles in the horizon, bring the sun's place, or star, to the eastern part of the horizon; then the degree cut on the horizon, from the east, northward or southward, will be the declination, north or south, and the degree on the equinoctial, from aries to the horizon, reckoning as before, will be the right ascension.

Example 1. Required the right ascension and declination of Aldebaran in Taurus?

Ans. Right ascension 66° 6′ 45″, declination 16° 5′ 50″.

2. Required the right ascension and declination of the following stars?

α *Acherner* in Eridanus	α *Belelgeux* in Orion
α *Alioth* in the Great Bear	γ *Bellatrix* in Orion
α *Arcturus* in Bootes	α *Capella* in Auriga
γ *Algenib* in Pegasus	α *Menkar* in Cetus
γ *Algorab* in the Crow	α *Procyon* in the Little Dog
α *Antares* in Scorpio	α *Regulus* in Leo
α *Atair* in the Eagle	α *Syrius* in Canis Major

PROB. 2.

*The right ascension and declination of the sun, a star, the moon, a planet, or a comet being given, to find its place on the globe.**

Rule. Bring the given right ascension to the brass meridian, and under the given declination, you will find the place of the planet or star required.

complements of the declinations and the given distance between the two stars form a spherical triangle, and that the angle at the pole included by the circles of declination passing through both stars, is equal to the difference of their right ascensions; this angle being therefore given, the difference of the star's right ascensions is given, and must be added to, or subtracted from, the right ascension of the given star, according as the circle of declin. passing through it, is east or west of the circle passing through the other. If it cannot be subtracted, 360 must be added to the other, and then the difference of rt. ascensions must be subtracted from the sum.

The rt. ascension and decl. of a star may be also thus found; having the latitude of the place, the hour from noon, and the sun's right ascension, together with the altitude and azimuth of the star given. For there are given the complement of the alt. the complement of the latitude, and the angle included by these sides; being the star's azimuth, or what it wants of 180°. Hence the star's decl. and the arch of the equator between the brass meridian and the circle of decl. passing through the star, is given, by the rule in the note to prob. 54, part 2; and as the hour of the day is given, the arch of the equator intercepted between the meridian and circle of decl. passing through the sun, is likewise given. The sum of these arches if the sun and star be on different sides of the meridian, or their difference if on the same side, will give the difference of the right ascensions of the sun and star, from which the right ascension of the star is known. The decl. is found as in the note prob. 54. Many other methods could be given, but our contracted limits would not permit. (See Gregory's Astronomy, b. 2. sect. 5. Keil's Astronomy, Lecture 19th. Vince's Astronomy, or notes to prob. 8th. part 2d.)

* As the latitudes and longitudes of the planets are given in pa. 4 of the month in the Nautical Almanac, their right ascensions and declinations from

This may be performed on the horizon as the foregoing.

Note. The star's right ascension may be given in time, or in degrees, both being marked on the equinoctial.

Example 1. Required the star whose declination is 30° 40′ 39″ S. and right ascension 22h 46m. 33s. or 341° 38′ 15″ ?

Ans. Fomalhaut in the southern fish.

2 On the 21st of April, 1811, the moon's right ascension was 9° 58′ 9″, and her declination 2° 53′ north ; required her place on the globe for that time ?

Ans. In piscés between the star δ and the equator.

thence may be thus found ; the complement of the lat. (or the lat. + 90°) the compl. of the decl. and the distance between the poles of the equator and ecliptic form a spherical triangle, two of the sides of which are given, viz. the distance of both poles, and the compl. of the lat. &c. and likewise the angle included by them at the pole of the ecliptic, being equal to the distance of the given planet in longitude, from the colure, reckoning on the ecliptic. Hence to find the decl. Rad. : cos. angle at the pole of the ecliptic :: tangt. co. lat. (or 90° + lat.) : tangt. x from the pole of the ecliptic ; then the sum or difference of x and 23° 28′ = y, and cos. x : cos. y :: sine lat. : sine decl. The declination being then given, the right ascension is thus found ; Co. sine decl. : sine angle at the pole of the ecliptic, or distance of the star in longitude from the solstitial colure :: cos. lat. or sine 90° + lat. : sine angle at the pole of the equinoctial, or the angle formed by the colure and compl. of the decl. This angle being equal to the distance, in right ascension, from the given planet to the colure, whence the right ascension is easily found. (See the note to the following problem.)

The declinations of the planets are given in pa. 4 of the Nautical Almanac, but not their right ascensions. In the same manner may the right ascensions and declinations of any of the fixed stars be obtained, from their latitudes and longitudes being given.

On the 1st of May, 1811, the latitude of Jupiter, as seen from the earth, was 31′ south, and his longitude 2s. 5° 49′ or 65° 49′, from which his rt. ascension and decl. is thus found ; Rad. : cos. angle at the pole of the ecliptic, 90° — 65° 49′ = 24° 11′ :: tang. 90° 31′ or 89° 29′ : tang. x, 90° 34′ ; and cos. x 90° 34′ or 89° 26′ : cos. y, 90° 34′ — 23° 28′ = 67° 6′ :: sine lat. 31′ : sine decl. 20° 47′ N. agreeing with the Nautical Almanac. Again, cos. decl. 20° 47′ : sine 90° — 65° 49′ = 24° 11′ :: sine 90° 31′ or 89° 29′ : cos. right ascension 64° 1′.

The annual variation of the stars in right ascension and decl. owing to the precession of the equinoxes, which, according to La Place, is 50″ 1 annually, and also to the nutation of the earth's axis, is not considered in these problems. See La Place's Astronomy, vol. 1. b. 1. ch. 11. or his treatise of *Celestial Mechanics*, where the laws of these phenomena are investigated, and agree with observation, as Nevil Maskelyne remarks, to surprising exactness.

The learner will also find all the problems relative to the latitude, longitude, right ascension and declination of the planets or stars, solved from accurate tables in Mayer. In the 3d edition of La Land's Astronomy, accurate tables of the places of the planets, and of Jupiter's satellites, are given.

In the above calculations where greater exactness is required, the seconds, &c. must be used, for which purpose *Taylor's* tables of logarithmic sines and tangents will be useful, as they are calculated to seconds ; Gardner's or Hutton's may also answer.

For the annual alteration of declination and right ascension of a fixt star through the precession of the equinox, or alteration of longitude, see the theorems in Simpson's Fluxions, vol. 2. sect. 1. prop. 2.

3. Required those stars, whose right ascensions and declinations are as follow ?

RIGHT ASCENSIONS.							DECLINATIONS.			
in time.				*in degrees.*						
12h.	15m.	38s.	=	183°	54′	30″	69°	59′	26″	S.
13	14	40	=	198	55	0	10	6	43	S.
16	17	9	=	244	17	15	25	58	23	S.
18	30	10	=	277	32	30	38	36	25	N.
20	34	36	=	308	39	0	44	34	21	N.
22	46	33	=	141	38	15	30	40	39	S.
22	54	5	=	343	6	15	27	0	8	N.
22	54	48	=	343	42	0	14	8	3	N.
	2	57	=		44	15	13	57	40	N.
	52	8	=	13	2	0	89	9	52	N.

4. On the first of December, 1810, the moon's right ascension was 320° 28′, and her declination 11° 45′ S. ; required her place on the globe ?

5. On the 1st of May, 1805, the declination of Venus was 11° 41′ N. and her right ascension 31° 30′ ; find her place on the globe ?

6. On the 19th of January, 1805, the declination of Jupiter was 19° 29′ S. and his right ascension 238° ; required his place on the globe ?

PROB. 3.

*To find the latitude and longitude of a given star.**

Rule. BRING the north or south pole of the ecliptic to the meridian (according as the star is on the north or south side of the ecliptic) elevate the pole 66½° above the horizon ; screw the

* The latitudes and longitudes of the planets must be found from astronomical tables, from the Nautical Almanac, or from any good ephemeris ; or their right ascensions and declinations being given (see the table at the end of this work) their latitudes and longitudes may be found as follows : Bring the star to the brass meridian, and draw with a pen and ink, or rather with a fine pencil, a circle of declination from the pole of the equator through it ; then if no circle of latitude pass through the star, draw by the help of the quadrant of altitude a circle of latitude from the pole of the ecliptic through it, as before, and intersecting the former at the given star ; then the complement of the declination, the compl. of the latitude of the star (or the lat. + 90°) and the distance between the pole of the equator and ecliptic, which is equal to the sun's greatest declination, or the obliquity of the ecliptic, will form a spherical triangle, two sides of which, viz. the comp. of the decl. of the star, and the distance of both poles are given, together with the angle formed at the pole of the equinoctial between the arch of the solstitial colure passing through both poles, and the arch representing the compl. of the decl. being equal to the distance of the star in right ascension from the colure or its supplement ; from which the complement of the latitude is found thus : Conceive a perpendicular arch let fall from the given star to the colure ; then it will be Rad. : cos. angle at the pole of the equinoctial formed by the colure, and compl. decl. :: co. tang. decl. : tang. x, the distance between the pole of the equinoctial and the perpendicular, the sum or

quadrant of altitude in the zenith, keep the globe from revolving on its axis, and move the quadrant until its graduated edge comes over the given star; then the degree on the quadrant, cut by the star, will be its latitude, and the sign and degree cut by the quadrant, on the ecliptic, will be its longitude.

Or, Place one end of the quadrant on the given pole of the ecliptic, and move the other end until the star comes to its graduated edge; then the number of degrees reckoned on the quadrant, between the ecliptic and the star, will be the latitude, and the number of degrees on the ecliptic, reckoning eastward from the point aries to the quadrant, will be the longitude.

Example 1. Required the latitude and longitude of Aldebaran, in Taurus?

Ans. Lat. 5° 29′ S. longitude 2 signs 7° or 7° in Gemini.

2. Required the latitudes and longitudes of the following stars?

α, *Altair* in the Eagle	γ, *Rastaben* in Draco
β, *Scheat* in Pegasus	α, *Arcturus* in Bootes
α, *Fomalhaut* in S. Fish	β, *Rigel* in Orion

A table of the longitudes of the nine principal fixed stars made use of in the Nautical Almanac, for determining the longitude, calculated for the beginning of the year 1809, *with their latitudes for the mid. of the same year.*

	Longitude beg. of 1809.	*Annual increase.*	*Latitude middle of* 1809.	*Annual variation.*
α Arietis	1s. 4° 59′ 31″ 0	50″ 271	9° 57′ 37″ 5 N.	+0″ 180
Aldebaran	2 7 7 10 4	50 204	5 28 48 7 S.	−0 317
Pollux	3 20 34 44 5	49 470	6 40 15 7 N.	+0 280
Regulus	4 27 10 27 8	50 004	0 27 35 5 N.	+0 200
Spica virg.	6 21 10 31 7	50 059	2 2 13 8 S.	+0 080
Antares	8 7 5 44 1	50 141	4 32 22 3 S.	+0 167
α Aquilæ	9 29 4 55 8	50 870	29 18 59 4 N.	+0 372
Fomalhaut	11 1 10 21 6	50 717	21 6 26 7 S.	+0 013
α Pegasi	11 20 49 33 6	50 133	19 24 46 9 N.	+0 163

difference of which, and the distance between both poles, will give y, the distance between the pole of the ecliptic and the same perpendicular; then cos. x : cos. y :: sine decl. : sine latitude.

Now to find the longitude, we have this proportion; Cos. lat. of the star or sine 90° + lat. : sine angle at the pole of the equinoctial :: co. sine decl. : sine angle at the pole of the ecliptic, formed by the solstitial colure and compl. lat which will give the distance in longitude reckoned on the ecliptic from the circle of lat. passing through the given star to the next solstitial colure, if the angle be greater than 90°, but its supplement if less; from which the longitude of the star is given. This will appear plainer by having the globe with the figure delineated on it as directed above; the following example will render the method more evident. The right ascension of Aldebaran being given, 66° 6′ 45″ or nearly 66° 7′, and his declination 16° 5′ 50″ or 16° 6′ nearly, for the year 1800; his latitude and longitude are required. Here we have Rad. : cos. 90° − 66° 7′ = 23° 53′ :: co. tang. decl.

PROB. 4.

The latitude and longitude of the moon, a star, or a planet, being given, to find its place on the globe

Rule. Bring the north or south pole of the ecliptic to the brass meridian, according as the latitude is north or south, elevate the pole 66½°; screw the quadrant in the zenith, over the elevated pole, and extend it over the given longitude in the ecliptic; then under the given latitude, on the graduated edge of the quadrant, you will find the star, or the place of the moon or planet.

Example 1. Required the star whose longitude is 3s. 22° 56″, and latitude 15° 58′ south?

Ans. Procyon in the little dog.

2. On the 1st of May, 1811, at noon, the moon's longitude was 4s. 20° 35′ 30″, and her latitude 2° 57′ 20″; required her place on the globe?

3. Required the stars which have the following longitudes and latitudes?

Longitudes	*Latitudes.*	*Longitudes.*	*Latitudes.*
3s. 11° 14′	39° 33′ S.	3s. 17° 22′	10° 4′ N.
6 20 57	2 3 S.	11 25	25 41 N.
9 28 51	29 18 N.	2 6 53	2 59 S.

4. On the 1st of December, 1811, the longitudes and latitudes of the planets will be as follows; required their places on the globe?

	Longitudes.			*Latitudes.*
☿ Mercury	8s.	15°	23′	1° 35′ S.
♀ Venus	8	21	1	0 35′ S.
♂ Mars	10	10	31	1 24′ S.
♃ Jupiter	3	4	2	0 16′ S.
♄ Saturn	8	26	43	0 55′ N.
♅ Herschel	7	20	21	0 18′ N.

Note. In the above and in most other examples, the geocentric places of the planets are made use of, or their places as seen from the earth's centre, being more convenient for a spectator on the earth than their heliocentric or true places, as seen from the centre of the sun. The learner must also take notice, that the planets' places are given for the meridian of Greenwich.

16° 6′ :: tangt. $x = 72° 29′$. In this case, therefore, $y = 72° 29′ + 23° 28′ = 95° 57′$. Whence Cos. x 72° 29′ : cos. y 95° 57′ or its suppl. 84° 3′ :: sine decl. 16° 6′ : sine lat. 5° 29′ as above. Again, for the longitude it will be, sine $90° + 5° 29′ = 95° 29′$ or sine suppl. 84° 31′ : sine angle at the pole of the equinoctial $180° - 23° 53′ = 156° 7′$ or its sup. 23° 53′ :: cosine decl. 16° 6′ : sine angle at the pole of the ecliptic $= 23°$. Now as the right ascension is in the first quadrant from aries, the longitude is in the same; hence $90° - 23° = 67°$ = the longitude of Aldebaran agreeing with that given in the Nautical Almanac, reduced to the year 1800.

In this manner the places of the stars in general are calculated, and a catalogue of them is made. In like manner the latitude and longitude being given, the right ascension and declination are found. See the note annexed to the foregoing problem.

PROB. 5.

*The latitude of a place being given, to find the amplitude of any star, its oblique ascension, and descension, its ascensional difference, and time of its continuance above the horizon.**

Rule. ELEVATE the pole to the given latitude, bring the given star to the eastern part of the horizon; then the degrees between the star and the east point of the horizon will be its *rising amplitude*, and the degree of the equinoctial cut by the horizon will be the *oblique ascension.* The globe being kept in this position, set the hour index to 12; then turn the globe westward, until the given star comes to the brass meridian, and the hours passed over by the index will be the star's *semidiurnal arch*, or *half the time of its continuance above the horizon;* the degree cut on the equinoctial by the brass meridian, will be the star's *right ascension*, the difference between which and the oblique ascension is the *ascensional difference.* The *setting amplitude*, and *oblique descension*, are found by continuing the motion of the globe, until the star comes to the western part of the horizon, &c.

Example 1. Required the rising and setting amplitude of Procyon, its oblique ascension and descension, ascensional difference, and diurnal arch, at New-York?

Ans. The rising ampl. is 7° to the north of the east, the setting ampl. 7° north of the west; oblique ascension $107\frac{1}{4}$°, oblique descension 117°; right ascension being 112° 12′, the ascensional diff is therefore 5° nearly; the semidiurnal arch is 6h. 20m. and hence the time of its continuance above the horizon is 12 hours 40 minutes.

2. Required the rising and setting amplitude of Sirius at Philadelphia, also its oblique ascension and descension, ascensional and descensional difference, and the time of its continuance above the horizon?

3. Required the rising and setting amplitudes of Aldebaran, Arcturus, Rigel, Regulus and Deneb; together with their oblique ascensions and descensions, ascensional differences, and their semidiurnal arches at London?

PROB. 6.

The latitude, day of the month, and hour being given; to place the globe in such a position as to represent the heavens, at that time, as seen from the given place; in order to find out the relative situations and names of the constellations and visible stars.

Rule. ELEVATE the pole to the given latitude; place it north and south (by the compass, allowing for variation, if any, or by a meridian line) bring the sun's place in the ecliptic to the brass meridian, and set the index to 12; then if the time be in the af-

* For the method of calculating this problem, see the notes to problem 49, part 2.

ternoon, turn the globe westward, but if in the morning, eastward, as many hours as the given time is after or before noon, the globe being fixed in this position ; then every star on the globe will correspond to the same star in the heavens, and a perpendicular erected over any of them, will point out the same star in the heavens.

By this means the constellations and remarkable stars may be easily known. All those stars which are on the eastern side of the horizon are then rising ; all those on the western side are setting ; all those under the brazen meridian are on the meridian of the place at the given hour ; those stars between the south point of the horizon and the north pole, have their greatest altitude, if the latitude be north, but those stars between the north point of the horizon and the south pole, are at their greatest altitude if the latitude be south. That star in the zenith, if any, is vertical, and if the sun's place be brought to the brass meridian, below the horizon, all those stars above the horizon whose declinations are equal to the given latitude, will be vertical successively, and visible in the given place.

Note. The globe should be taken into the open air or near a large window, on a clear night, where the view on the surrounding horizon is not intercepted by different objects ; a small observatory erected on the top of a house where the roof is flat, or nearly so, would best answer the purpose.

PROB. 7.

The latitude of a place, day of the month, and hour being given ; to find what stars are rising, setting, on the meridian, &c.

Rule. Rectify the globe for the given latitude ; bring the sun's place to the meridian, and set the index to 12 ; then if the time be in the forenoon, turn the globe eastward, but if in the afternoon, westward, until the index has passed over as many hours as the time is before or after noon ; then all the stars at the eastern semicircle of the horizon will be rising, those at the western semicircle will be setting, those under the graduated edge of the brass meridian, above the horizon, will be culminating or on the meridian ; all those that are above the horizon will be visible, and those below it invisible, at the given time and place ; if the globe be turned on its axis from east to west, those stars that do not descend below the horizon, never set at the given place, and those which do not come above the horizon, never rise. These circles of perpetual apparition or occultation may be found by describing circles on the globe, parallel to the equinoctial, at a distance from it equal to the complement of the latitude.

Example 1. At 10 o'clock in the evening in New-York on the 10th of May, required those stars that are rising, setting, on the meridian, &c. ?

Ans. Altair in the eagle is rising ; Spica in virgo, the two stars mizar and alcor in the tail of the great bear, and δ in Cassiopeia, are nearly on the meridian ; Procyon is about 6° above the western point of the horizon, the star marked γ in gemini, is nearly setting, &c.

2 On the 16th of November, at 4 o'clock in the morning, at New-York, what stars are rising, setting, on the meridian, &c.

Ans. Arcturus is after rising about 5° above the E. N. E part of the horizon, Procyon is on the meridian, Pollux is near the meridian, and Castor after passing it ; α in Andromeda, and mirac in Cetus, are near the western part of the horizon, &c.

3. On the 9th of February, when it is 9 o'clock in the evening at London, what stars are rising, on the meridian, setting, &c.?

4. Required those stars that never set in the latitude of New-York, and at what distance from the equinoctial is the circle of perpetual apparition ?

5. Required those stars that never rise at Cape Horn, and those that never set at Copenhagen ?

6. Required those stars that are always above the horizon at the north pole, and also those that cannot be seen there ?

7. How far must an inhabitant of New-York travel southward to lose sight of Arcturus ?

8. In what parallel of latitude do those reside to whom Sirius is never visible but when in their horizon ?

9. In what latitude do those reside to whom Aldebaran is always vertical when on their meridian ?

Note. When the decl. of the star is equal to the lat. of the place, the star will be always vertical in that lat. when on the meridian.

PROB. 8.

*To find at what hour any star or planet will rise, come to the meridian, and set at any given place.**

Rule. Elevate the pole to the given latitude, bring the sun's place in the ecliptic to the brass meridian, and set the index to 12 ; then bring the star or planet's place to the eastern part of the hori-

* The apparent time of the transit of any star over the meridian is thus found ; subtract the sun's right ascension in time at noon from the star's right ascension in time, increased by 24 hours, if necessary ; the remainder is the apparent time of the star's passing the meridian nearly ; from which the proportional part of the daily increase of the sun's right ascension, for this apparent time from noon (corrected by the longitude you are in or difference of longitude from Greenwich) being subtracted, the remainder will be the correct time of the star's passing the meridian.

The apparent time of a star's rising or setting is found by applying its semidiurnal arch answering to its declination, and the latitude of the place, by subtraction or addition, to the time of its transit over the meridian. The semidiurnal arch is thus found ; the complement of the latitude of the place, the complement of the star's declination, and the distance from the vertex to the point where the star rises or sets (which is always equal 90°) forming a quadrantal triangle, are given, to find the angle at the pole of the equinoctial formed by the brass meridian, or meridian of the place, and the circle of declination passing through the star, which will be the semidiurnal arch required. Or the decl. of the star, its zon, and the index will point out the time of the star's rising; turn the globe westward until the star, or planet's place comes to the brass meridian, and the index will shew the time of the star's coming to the meridian of the place; continue the motion of the globe westward until the star or planet's place comes to the western part of the horizon, and the index will shew the time of its setting.

Note 1. The time may be more accurately found on the equator, always reckoning the hours between the meridian passing through the sun's place and the brass meridian, for the time before or after noon when the star or planet rises, sets, &c.

If the sun's place be to the east of the brass meridian, the star or planet will rise before noon, but if to the west, the star or planet will rise in the afternoon.

Example 1. At what time will Sirius rise, come to the meridian, and set at New-York, on the 2d of November?

Ans. It will rise at 9 o'clock in the evening, come to the meridian at 2 in the morning, and set at 7 in the morning.

2 On the 13th of May, 1811, the longitude of Jupiter was 2 signs 8° 31′, and his latitude 30′ south; at what time did he rise, culminate, and set at Greenwich, and whether was he a morning or an evening star?

Ans. He rose at 5 o'clock in the morning, came to the meridian at 5 min. after 1 in the afternoon, and set about 10 minutes after 9 at night. Jupiter was here an evening star, because he set after the sun.

amplitude, and what the semidiurnal arch reckoned on the equinoctial exceeds or wants of 90° (according as the decl. is of the same or a different name with the lat.) and the angle in this triangle included by the equinoctional and amplitude, or the inclination of the equinoctial to the horizon, is the comp. of the lat. of the place; whence by Napier's rule, Rad. : tangt. lat. :: tang. decl. : sine of an angle which added to or subtracted from 90°, according as the star's decl. is of the same or a different name from the latitude. (This gives the investigation of the note in prob. 13, part 2d.)

The apparent time of a planet's passing the meridian may be found thus; let the planet's right ascension, the preceding noon or midnight, be calculated from its longitude and latitude (by note to prob. 2) and turned into time; subtract the sun's right ascension in time, the same noon or midnight from it, the remainder will be the time of the planet's passing the meridian nearly, which call x; take the difference of the sun's daily or half daily variations in right ascension in time, if the planet be progressive in right ascension, or the sum if it be retrograde, which call y; then say as 24h. $\pm y$ or 12h. $\pm y$: 24h. or 12h. (according as the daily or half daily variation is used) :: x : to the time of the planet's passing the meridian. The sign + is to be used if the planet's progressive motion in right ascension be greater than the sun's; in any other case the sign — is to be made use of. Where accuracy is required, the 2d differences of the right ascension should be allowed for, and the difference of longitude, if for any other meridian different from that of Greenwich. See the method of allowing for these differences at the end of the Nautical Almanacs for 1811, 1812 or 1813, published by Mr. John Garnett. See also Emerson's differential method in his Conic Sections.

Note 2. When a planet rises or sets after the sun, it is then an evening star, but when it rises or sets before the sun, it is a morning star.

3. At what time does Aldebaran rise, set, and come to the meridian of Philadelphia, on the 4th of July?

4. On the 1st of October, 1811, the longitude of Venus will be 6s. 4° 36′, and her latitude 1° 22′ N. at what time will she rise, set, and come on the meridian of Greenwich, and whether will she be a morning or an evening star?

5. On the first of June, 1812, the longitude of Mars will be 2s. 27° 37′, and his latitude 54′ north; required the time of his rising, coming on the meridian, and setting at Greenwich?

6. The longitude of Saturn on the first of November, 1813, will be 9s. 14° 24′, and his latitude 6′ N.; required the time of his rising, culminating, and setting at Greenwich?

PROB. 9.

To find on what day of the year a given star will be upon the meridian, at any given hour.

Rule. Bring the given star to the meridian, and set the index to 12; then turn the globe westward or eastward, according as the time is in the forenoon or afternoon, as many hours as the given time is from noon; the brass meridian will then cut the ecliptic in the sun's place corresponding to the time required, which may be found on the horizon.

Example 1. On what day of the month does Sirius come to the meridian of New-York, at 4 o'clock in the morning?

Ans. The time being 8 hours before noon, the globe must therefore be turned 8 hours towards the west, the point of the ecliptic then intersected by the brass meridian, will be 12° of Scorpio, answering nearly to the 4th of November.

2. At what time of the year will Regulus in Leo come to the meridian of Philadelphia, at 9 o'clock at night?

Here the time being 9 hours after noon, the globe must therefore be turned 9 hours towards the east; then the ecliptic will be intersected by the brass meridian in 15½° of Aries, corresponding to the 5th of April, nearly.

3. At what time of the year does Procyon come to the meridian of London, at 4 o'clock in the afternoon?

4. At what time of the year does Arcturus come to the meridian of Dublin, at 10 o'clock at night?

5. At what time of the year does Alcyone in the Pleiades come to the meridian of Washington city at noon,* or when the sun is on the meridian?

* If the given star comes to the meridian at noon, the sun's place will be found under the brass meridian without turning the globe; if the star comes to the meridian at midnight, the globe may be turned eastward or westward until the index has passed over 12 hours. When the time is given for the meridian of any other place, it must be reduced to that of the given place by prob. 6, part 2.

6. At what time of the year does Lyra in the harp, come to the meridian of Boston, at midnight ?

7. On what day of the month, and in what month, does Spica in Virgo come to the meridian of New-York, when the sun is on the meridian of Constantinople ?

8. Having observed Aldebaran in Taurus pass the meridian of George Town College, on the Potomac, when it was 8 o'clock in the morning, by a time piece set to the meridian of Greenwich observatory ; required the month and day when the observation was made ?

PROB. 10.

*Given the latitude, day of the month, and hour, to find the altitude and azimuth of any given star.**

Rule. RECTIFY the globe for the given latitude, screw the quadrant in the zenith, bring the sun's place for the given day to the brass meridian, and set the index to 12 ; then if the given

The reason of the rule is evident, as the sun always comes to the meridian at 12, and that it is distant from the meridian, on which the sun is at the given time, as many hours as are equal to the time that the star culminates before or after noon. This time is equal to the difference of the sun and star's right ascensions ; when their right ascensions are equal, they are on the meridian at the same time, that is at 12 o'clock. Hence the problem may be easily solved by calculation.

* Here the day of the month being given, the sun and stars right ascensions and declinations are given (by prob. 1. part 2.) Moreover the complement of the latitude of the place, the compl. of the star's altitude, and the compl. of his declination, or his distance from the elevated pole of the equinoctial, form a spherical triangle. Now the sun and star's right ascensions being given, their difference is given, or the distance between the sun and star reckoned on the equator ; and as the distance of the sun from the meridian is given in time, and consequently in degrees, being equal to the hour from noon, the distance of the star from the same meridian is also given, being equal to the remainder of the difference of their right ascensions, or equal to the angle formed at the pole by the brass meridian, or meridian of the place, and the circle of declination passing through the star. Therefore in the above triangle there are given two sides, viz. the comp. of the latitude, and comp. of the declination of the star, and the angle included by these sides, or the distance of the star in right ascension from the meridian, to find the third side or compl. of the alt. And as the angle included by the compl. of the lat. of the place and the compl. of the alt. of the star, is the star's azimuth or its supplement, according as the north or south point of the horizon, from which the azimuth is reckoned, is of the same or a different name from the elevated pole. Hence to find the azimuth we have the following proportions ; Rad. : cos. angle at the pole, or dist. of star in rt. ascension from the mer. :: tangent dist. of the star from the pole, or $90° \pm$ decl. : tangt. x, the distance from the pole of the equinoctional to the perpendicular let fall from the star to the meridian of the place, the sum or difference between which and the distance between the zenith and pole (according as the perpendicular falls towards the zenith or in a contrary direction from the pole) call y ; then sine x : sine y :: co. tang. angle at the pole, or distance of the star in right ascension from the meridian : co. tang. azimuth. Again, sine azim. : sine distance of the star from the pole, or $90° \pm$ decl. :: sine angle at the pole : cos. altitude.

time be in the morning, turn the globe eastward, but if in the afternoon, westward, as many hours as the time is before or after noon ; keep the globe in this position, and move the quadrant of altitude until its graduated edge coincides with the centre of the given star ; the degree then cut on the quadrant, reckoned from the horizon, will be the altitude, and the degree on the horizon, cut by the quadrant, reckoning from the north or south, will be the azimuth required.

Example 1. Required the altitude and azimuth of α Arietis at Philadelphia, when it is 5 o'clock in the morning of the 23d. of September ?

Ans. The alt. is 47°, and the azimuth nearly 78½° from the south towards the west.

2. Required the altitude and azimuth of Altair in the Eagle, at New-York, when it is 9 o'clock in the evening of the 21st of June ?

Ans. The alt. is 20° 22′, and azimuth 83° 23′ from the south towards the east.

3. Required the altitude and azimuth of Lyra in the harp, at Washington city, at 3 o'clock in the morning of the 21st of March ?

4. Required the altitude and azimuth of Procyon on the 10th of February, at 9 o'clock in the evening at London ?

5. On what point of the compass does the star Algol in Perseus bear at New-York, on the 10th of August, at 10 o'clock in the afternoon, and what is his altitude ?

Note. The points of the compass are reckoned the same way as the azimuth, allowing 11° 15′ to each.

PROB. 11.

The latitude, day of the month, and the altitude of any known star being given, to find the hour of the night, and the star's azimuth.

Rule. RECTIFY the globe for the latitude, screw the quadrant of altitude in the zenith, bring the sun's place for the given day to the brass meridian, and set the hour index to 12 ; bring the

Thus in ex. 2. the sun's right ascension = 90° his distance from the meridian or noon = 9 hours = 135°. Altair's right ascension in time = 19h. 41′ 1″ = 295° 15′ nearly. Whence Altair's distance from the meridian in right ascen. $= 295° 15' - \overline{90° + 135°} = 70° 15'$, the angle at the pole formed by the co. lat. and circle of declination passing through the star, and the star's declination is 8° 21′ 8″ N. or 8° 21′ nearly ; hence rad. : cos. 70° 15′ :: tang. 90°—8° 21′ =81° 39′ or cot. 8° 21′ : tang. x 66° 31′. Whence 66° 31′ — 49° 17′ (co. lat.) = 17° 14′ = y ; then s. x 66° 31′ : s. y 17° 14′ :: cot. 70° 15′ : co. tang. azimuth 83° 23′. Again, s. azim. 83° 23′ : s. dist. of the star from the pole, or cos. decl. 8° 21′ :: s. 70° 15′ : cos. alt. 20° 22½′ nearly.

The learner will observe, that the places of the stars on our newest globes are calculated for the year 1800, to which we have therefore adapted most of our calculations, &c.

The calculation of prob. 50, part 2d. is performed in the same manner as the above. The variation of the compass may be obtained from this prob. in the same manner as in prob. 50 above alluded to.

quadrant of altitude to the side of the brass meridian, east or west on which the star was situated when observed, turn the globe *westward* until the centre of the star cuts the given altitude on the quadrant; then the hours which the index has passed over, will shew the time from noon when the star has the given altitude, and the quadrant will intersect the horizon in the required azimuth.*

Example 1. The star Altair in the Eagle on the 21st of June, at New-York, was observed to be 20° 22½′ above the horizon, and east of the meridian; required the hour of the night and the star's azimuth?

Ans. The sun's place being brought to the meridian, and the globe turned westward until the star cuts 20° 22′ east of the meridian, the index will then have passed over 9 hours, and the star's azimuth, indicated by the quadrant, on the horizon, will be 83° 23′ from the south towards the east.

2. The altitude of α Arietis was observed 47° at Philadelphia, on the 23d of September, the star being west of the meridian; required the hour and the star's azimuth?

Ans. Here the globe being turned westward until the star cuts the given altitude on the quadrant, west of the meridian, the index will have passed over 17 hours corresponding to 5 o'clock in the morn. and the azim. from the south towards the west is 79° nearly.

* The prob. being performed as directed in the rule, the complement of the latitude of the place, the complement of the star's altitude, and the complement of its declination, will form a spherical triangle, and as the three sides are given, the angles are therefore given. Now the angle formed by the quadrant of alt. and the brass meridian is equal to the star's azimuth, and the angle formed at the pole, by the circle of declination passing through the star and the brass meridian, is the distance of the star from the same meridian; and as the sun and star's right ascensions are given, their difference is therefore given, from which, if the distance of the star from the meridian be taken, the remainder is the distance of the sun from the meridian, which converted into time, will give the hour required. Thus in example 1. The comp. of the lat. = 49° 17′, the comp. of the alt. = 69° 38′, and the comp. of the star's decl. = 81° 39′. Hence by spherical trigonometry we shall have this proportion, tang. ½ co. lat. 24° 38′ : tang. of half the sum of the com. of the decl. (81° 39′) and com. of the alt. (69° 37½′) = 75° 38′ :: tang. of half the difference of the co. of the decl. and co. of the alt. = 6° 0¾′ : tang. x = 41° 53′. (See Emerson's Trig. b. 3, sect. 4. case 11.) Hence 41° 53′ + 24° 38′ (half the co. lat. nearly) = 66° 31′; then by Napier's 1st rule, R : tang. 66° 31′ :: tang. decl of the star 8° 21′ : cos. 70° 15′, the angle formed at the pole by the brass meridian and circle of declination passing through the star, or the star's distance from the meridian. Now the sun's right ascen. is 90°, and the star's 295° 15′, the diff. is therefore 205° 15′, from which the distance of the star from the meridian being taken, the remainder 135° = dist. of the sun from the mer. = 9 hours; hence the time is 9 o'clock in the evening.

To find the azimuth, the sines of the sides of spherical triangle being as the sines of the angles opposite to them, it will be sine co. alt. 69° 37½′ : sine co. decl. 81° 39′ :: sine 70° 15′ : sine azimuth 83° 23½′ nearly, as required. The azimuth or its supplement will be found by this latter proportion.

3. The altitude of Lyra at Washington city, on the 21st of March, was observed 50° east of the meridian; required his azimuth and the hour?

4. The altitude of Deneb in the Lion's Tail, on the 28th of December, at London, was observed 40° when east of the meridian; required its azimuth and the hour?

PROB. 12.

The latitude, day of the month, and azimuth of a star being given, to find the hour of the night and the star's altitude.

Rule. ELEVATE the pole to the given latitude, screw the quadrant in the zenith, bring the sun's place in the ecliptic for the given day to the brass meridian, and set the hour index to 12; bring the graduated edge of the quadrant to coincide with the given azimuth on the horizon, and keep the quadrant in this position; turn the globe westward until the given star comes to the graduated edge of the quadrant, then the hours passed over by the index will be the time from noon, and the degrees on the quadrant, reckoning from the horizon to the star, will be the altitude.*

Example 1. On the 21st of June at New-York, the azimuth of Atair in the Eagle was observed to be 83° 23½', from the south towards the east; required the hour of the night and the star's altitude?

Ans. The globe being turned on its axis, the index will pass over 9 hours, corresponding to 9 o'clock in the evening, and the star's altitude will be 20° 22'.

2. On the 23d of September at Philadelphia, the azimuth of α Arietis was 79° from the south towards the west; required the hour of the night and the star's altitude?

3. On the 8th of October, the azimuth of the star marked β in the shoulder of Auriga, was 49° from the north towards the east; required its altitude at London, and the hour of the night?

* In this, as in the two preceding problems, the compl. of the lat. the compl. of the star's alt. and compl. of his decl. form a spherical triangle, two sides of which, viz. the co. lat. and the co. decl. are given, and the angle opposite the co. decl. is the azimuth or its supplement; hence the other parts of the triangle may be found, by case 2, b. 3, sect. 4, Emerson's Trig. Thus in ex. 1. s. co. decl. 81° 39' : s. co. lat. 49° 17' :: s. azim. 83° 23½' s. x 49° 33' = the angle opposite the co. lat.; then s. $\frac{81° 39' - 49° 17'}{2} = 16° 11'$: s. $\frac{81° 39' + 49° 17'}{2} = 65° 28'$ :: tang. $\frac{96° 37' - 49° 33'}{2} = 23° 32'$: co. tang. half the hour angle, or half the distance of the star from the meridian 35° 8'; hence the whole distance is 70° 16', and therefore the hour of the night is found as in the last note. 96° 37' is the supplement of the azimuth nearly, or 180°—83° 23' = 96° 37' = the angle opposite the co. decl. Now to find the altitude, it will be s. x. 49° 33' : s. 70° 16' :: s. co. lat. 49° 17' : s. co. alt. 69° 38'; hence the alt. is 20° 22'. The results in this and the foregoing notes would exactly agree if the seconds were retained, but as the calculations are given only to illustrate the prob. such nicety was considered unnecessary.

PROB. 13.

*The latitude, day of the month, and two star's having the same azimuth, being given, to find the hour of the night.**

Rule. ELEVATE the pole to the given latitude, screw the quadrant of altitude in the zenith, bring the sun's place in the ecliptic to the brass meridian, and set the hour index to 12; turn the globe westward on its axis, until the two given stars coincide with the graduated edge of the quadrant of altitude; and the hours passed over by the index, will be the time from noon. The common azimuth will be found on the horizon.

Example 1. At what hour at New-York, on the 22d of September, will Capella in Auriga and Castor in Gemini, have the same azimuth, and what will that azimuth be?

Ans. In turning the globe westward, &c. the index will pass over $13\frac{1}{4}$ hours before the stars coincide with the quadrant, they will therefore have the same aximuth at a quarter past one in the morning, and the azimuth will be $63\frac{1}{2}°$ from the north towards the east.

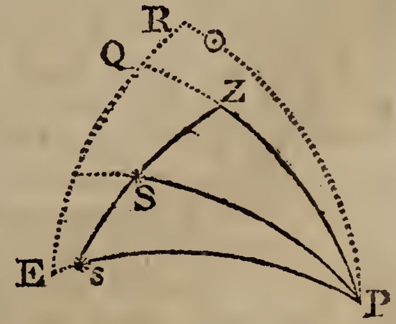
2. At what hour at London on the 1st of May will Altair in the Eagle, and Vega in the Harp, have the same azimuth, and what will that azimuth be?

3. At what hour will Arcturus and Spica Virginis, have the same azimuth at Paris, on the 20th of April?

4. At what hour will Arcturus and α Zuben el C. of Libra, have the same azimuth at Boston, on the 21st of June?

5. At what hour at Philadelphia will Procyon and Sirius have the same azimuth, on the 21st of March?

Note. When the two stars have the same right ascension, they will have the same azimuth when on the meridian, and as the star's passing the meridian is found by prob. 8, the hour is therefore given. If a correct table of those remarkable stars which have the same right ascension were given, and the times of their passing the meridian of any remarkable place, as that of Greenwich or Paris observatory, this would afford an easy method of finding the hour of the night, as every star is on the meridian of any place at the same hour. It would also afford a method of finding a meridian line, &c.



* This prob. may be thus calculated; let S, *s* be the two stars, P the pole, Z the zenith, EQR a portion of the equator, and ☉ the sun's place. In the triangle SP*s* there are given SP, *s*P the complements of the star's declinations, and the angle SP*s*, the difference of the star's right ascension; hence the angle at S and *s*, and the side S*s*, are given, and therefore the angle ZSP, the supplement of PS*s*, is given. Now in the triangle P*s*Z or PSZ, there are given *s*P or SP, the angles at *s* or S, and ZP the complement of the latitude; hence the angle PZS, =the suppl. of the azimuth, and therefore the angle QZS the azimuth are given, and likewise the angle *s*PZ or SPZ, the distance of the stars from the meridian PQ is given; but as the angle *s*PR or SPR, the difference between the sun and the stars right ascensions respectively, are given, therefore

PROB. 14.

*The latitude, day of the month, and two stars that have the same altitude, being given, to find the hour of the night.**

Rule. RECTIFY the globe for the latitude, zenith and sun's place, (prob 9 part 1) turn the globe westward until the two given stars coincide with the given altitude on the quadrant, or until the two stars be at the same distance from the horizon, if the altitude be not given; then the hours passed over by the index will be the time from noon, when the two stars will have that altitude.

Example 1. At what hour at New-York, on the 20th of July, will Belelgeux in Orion, and Castor in Gemini, have each 5° of altitude?

Ans. At 45 min after 3 in the morning.

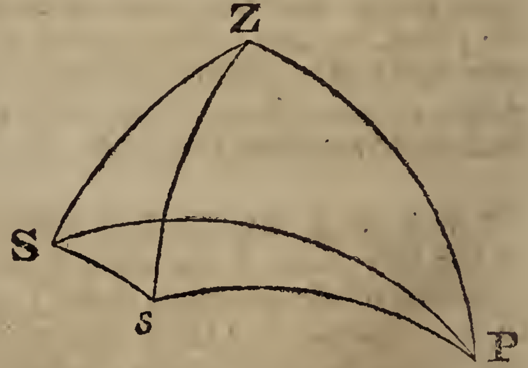
2. At what hour at London on the 2d of September, will Markab in Pegasus, and α in the head of Andromeda, have each 30° of alt.?

3. At what hour at Philadelphia on the 18th of January, will Altair in the Eagle, and Fomalhaut in the southern Fish, have each 12° of altitude?

4 At what hour at Dublin, on the 15th of May, will η Benetnach in the tail of the Great Bear and γ, in the shoulder of Bootes, have each 56° of altitude?

the remaining angle QPR, the distance of the sun from the meridian is given, which converted into time, will give the hour required.

In like manner when two stars in one azimuth are given, and the altitude of either being given, the latitude of the place may be easily found on the globe, or by calculation thus; if the altitude of S be given, its complement SZ is given, and in the triangle SP*s*, SP, *s*P, and the angle SP*s* are given, hence the angle PS*s* and its suppl. PSZ are given. Again, in the triangle ZSP, SP, SZ, and the angle at S are given, hence ZP, which is the compl. of the latitude, is given. See Emerson's Algebra, prob. 160. page 448.



* The prob. may be thus solved by Trigonometry, the altitude being given; let S, *s* be the two stars, P the pole, and Z the zenith; then in the triangle SPZ or *s*PZ, the three sides are given, and the angle ZPS, or ZP*s*, the distance of the stars S, or *s*, respectively, from the meridian, from which the hour may be found as in the notes to the last problems. From the solution it is evident, that the alt. of one star alone is sufficient to determine the hour when the lat. is given. When both altitudes are given, the lat. and hour may be found thus; in the triangle SP*s*, SP, *s*P, the complements of the stars decl. and the angle SP*s*, the difference of their right ascensions are given, and hence the side S*s*, and the angle PS*s*, are given. Again, in the triangle ZS*s*, the three sides are given, and therefore the angle ZS*s* is given, consequently the angle ZSP is given, and therefore ZS the co. alt. and SP being given, ZP the co lat. is also given. When neither of the altitudes are given, the solution becomes rather tedious and troublesome.

5. At what hour, at George Town on the Potomac, will Aldebaran, and Algol in Perseus, have each $17\frac{1}{4}$° of altitude, on the 31st of March ?

PROB. 15.

*Given the azimuth of a known star, the latitude of the place, and the hour ; to find the star's altitude, and the day of the month.**

Rule. RECTIFY the globe for the latitude, screw the quadrant of altitude in the zenith, bring the graduated edge of the quadrant to the given azimuth on the horizon, turn the globe until the star coincides with the quadrant, and set the index to 12 ; then if the time be in the forenoon, turn the globe westward, but if in the afternoon, eastward, until the index has passed over as many hours as the given time is from noon ; the degree then cut on the ecliptic by the brass meridian will correspond, on the horizon, to the day of the month required. The altitude of the star when brought to the graduated edge of the quadrant, will be the degree on it, cut by the centre of the star.

Example 1. At Washington city at 9 o'clock at night, the azimuth of Aldebaran was by observation 89° from the south towards the west ; required its altitude and day of the month ?

Ans. Its altitude is 26°, and the day is the 21st of March ; as the time is 9 hours past noon, the globe must be turned as many hours towards the east, &c.

2. At Philadelphia at 5 o'clock in the morning, the azimuth of α Arietis was 79° from the south towards the west ; required its altitude and the month and day when the observation was made ?

Ans. As the time wants 7 hours of noon, the globe must be turned 7 hours westward ; the altitude of the star will be found 47°, and the time the 23d of September.

3. At London, at 10 o'clock at night, the azimuth of Spica was observed 40° from the south towards the west ; required its altitude and the day of the month ?

* Here the compl. of the lat. the co. of the decl. of the star, and the co. of its altitude, form a spherical triangle, two sides of which, viz. the co. lat. and co. decl. of the star, and an angle opposite one of them, that is the angle opposite the co. decl. being the azimuth of the star or its supplement, or what it wants of 180° ; from which the distance of the star from the meridian will be found exactly as in the note to prob. 12 of the preceding. And as the hour is given, the distance of the sun from the meridian, or the angle formed by the circle of declination passing through the sun, and the brass meridian, is given, being equal to the time from noon converted into degrees ; hence the distance of the star from the meridian being added to that of the sun, will give the difference of their rt. ascensions, and as the rt. as. of the star is given, the right ascension of the sun will be therefore given. Now as the obliquity of the ecliptic is given, the sun's longitude may be easily found by Napier's rule, and hence the corresponding day may be found from an ephemeris or the globe. The application of these remarks is left as an exercise for the learner, in calculating the above examples.

4. At Dublin at 2 o'clock in the morning, the azimuth of β Pegasus or scheat, was 70° from the north towards the east; required its altitude and day of the month?

PROB. 16.

The latitude of the place, day of the month, and hour of the day being given, to find the Nonagesimal degree* *of the ecliptic, its altitude and azimuth, and the Medium Cœli, &c.*

Rule. Rectify the globe for the latitude, zenith and sun's place (by prob. 9.) then if the given time be in the forenoon, turn

* The *nonagesimal degree* of the ecliptic, so called from its being the 90th degree reckoning from the horizon on the ecliptic, is the most elevated point of the ecliptic above the horizon, and is measured by the angle which the ecliptic makes with the horizon at any elevation of the pole, and is equal to the distance between the zenith of the place and the pole of the ecliptic. It is frequently made use of in the calculation of eclipses. The *medium cœli*, or midheaven, is that point of the ecliptic which is on the meridian.

From the 22d of December to the 21st of June, the nonagesimal degree of the ecliptic is east of the meridian; and from the 21st of June to the 22d of December, it is west of the meridian.

The globe being rectified as above, then the day of the month being given, the sun's right ascension for the given time, may be found in the Nautical Almanac (see notes to problems 42 and 49) and therefore its distance from the equinoctial point, which is above the horizon, is given; moreover, as the hour is given, the sun's distance from the meridian of the place, or the brass meridian, is given, and hence the distance from this meridian to the next equinoctial point is given. Now as the obliquity of the ecliptic is given (note to prob. 49) the degree cut on the ecliptic by the brass meridian, or the *medium cœli* or *midheaven* will be given (by Napier's rule.) Again, as the number of degrees from the elevated equinoctial point to the brass meridian is given, its complement, or the distance from the equinoctional to the horizon, on the equator, is given, and the inclination of the equator with the horizon is the complement of the latitude. Hence in the spherical triangle formed by the equinoctial or equator, the ecliptic, and the horizon; two angles, viz. the obliquity of the ecliptic, and the co. lat. and one side, that is the dist. on the equinoctial to the horizon from the elevated equinoctial point, and therefore the angle opposite the given side, is given (by Napier's rules, or by case 10, s. 4, b. 3, Emerson's Trig.) the suppl. of which, or the inclination of the ecliptic with the horizon is the nonagesimal degree required. Thus in ex. 1, the sun's rt. ascension is 90°, and is 90° distant from libra, the elevated eq. point; and as the hour from noon is 3h. 45′ = 56° 15′, its compl. 33° 45′, is the distance of the meridian from the point libra. Hence by Napier's rule, Tang. 33° 45′ : r. :: cos. obl. eclip. 23° 28′ : co. tang. 36° 4′, the distance from libra to the medium cœli, reckoning backwards, which therefore corresponds with 23° 56′ of leo. Again, 90° — 33° 45′ = 56° 15′ dist. from libra on the equator to the horizon, and 90° — 51½° = 38½° co. lat.; hence, letting fall a perpendicular from the point libra, on the horizon, it will be R. : tang. 38½° :: cos. 56° 15′ : 66° 10′, the angle formed at libra by the equinoctial and perpendicular; from which the obliquity of the eclip. being taken, the rem. 42° 42′ is the angle formed at libra by the ecliptic and perpendicular; then S. 66° : s. 42° 42′ :: cos. 38½° : cos. 54° 32′, the inclination of the ecliptic to the horizon, or the nonagesimal degree required.

the globe eastward, but if in the afternoon, westward, until the index has passed over as many hours as the time is before or after noon ; reckon 90° on the ecliptic from the horizon, eastward or westward, the point where the reckoning ends will be the nonagesimal degree, and the degree of the ecliptic, cut by the brass meridian, will be the medium cœli : the graduated edge of the quadrant being brought over the nonagesimal degree, will point out its altitude, and its azimuth will be then seen on the horizon.

Example 1. On the 21st of June, at 45 minutes past 3 o'clock in the afternoon at London, required the point of the ecliptic which is the nonagesimal degree, its altitude and azimuth ; the longitude of the medium cœli, and its altitude, &c.

Ans. The nonagesimal degree is 10° in Leo, its altitude is $54\frac{1}{2}$°, and its azimuth 22° 30′ from the south towards the west or S. S. W. The midheaven or point of the ecliptic under the brass meridian is nearly 24° in leo, and its altitude above the horizon is 52°. The right ascension of it is 146°. The rising point of the ecliptic is 10° in scorpio, and the setting point 10° in taurus. If the quadrant of alt. be extended over the sun's place, the sun's alt. will be found equal $38\frac{3}{4}$°, and his azimuth $78\frac{1}{2}$° from the south towards the west, or W. by S. nearly.

2. At New-York on the 10th of May, at 10 o'clock at night, required the point of the ecliptic, which is the nonagesimal degree, its altitude and azimuth ; the point of the ecliptic, which is the midheaven, &c. &c.

3. At Philadelphia on the 25th of October, at 4 o'clock in the morning, required as in the last example, &c.

4. At George Town College on the Potomac, in lat. 38° 55′ N. required the nonagesimal degree of the ecliptic, the medium cœli, &c.

PROB. 17.

*Given the latitude, day, and hour, together with the altitude and azimuth of a star, to find the star.**

Rule. Rectify the globe for the latitude, zenith and sun's place, as before, and turn the globe eastward or westward (according as the time is in forenoon or afternoon, as many hours as the time is from noon ; keep the globe in this position, and bring the graduated edge of the quadrant to the given azimuth on the hori-

* As the star is given, when its right ascension and declination are given; hence the right asc. and decl. may be thus calculated. The prob. being performed as directed in the rule, it will be found that the comp. of the lat. the comp. of the star's decl. and the comp. of its alt. form a spherical triangle, two sides of which, viz. the co. lat. and co. alt. are given, and the included angle being the star's azimuth or its supplement, from whence the other side, or the co. decl. and the angle included by this and the brass meridian, or the distance of the star from the meridian passing through the zenith of the place, will be given ; now as the hour angle is given, the distance of the sun from the meridian is given, and hence the distance from the sun to the star, reckoning on the equator, or the

zon ; then under the given altitude on the quadrant, you will find the star required.

Example 1. At New-York on the 21st of June, at 9 o'clock in the afternoon, the altitude of a star was 20° 22′, and its azimuth 83° 23′ from the south towards the east ; required the star ?

Ans. Altair in the Eagle.

2. At Washington city on the 21st of March, at 9 o'clock at night, the altitude of a star was 26°, and its azimuth 89° from the south towards the west ; required the star ?

3. At Philadelphia at 5 o'clock in the morning of the 23d of September, the altitude of a star was 47°, and its azimuth 79° from the south towards the west ; required the star ?

4. At London, on the 22d of December at 4 o'clock in the morning, the altitude of a star was 50°, and its azimuth 37° from the south towards the east ; required the star's name ?

PROB. 18.

The latitude and day of the month being given, to find the meridian altitude of any star or planet.*

Rule. Rectify the globe for the given latitude ; then,

For a star. Bring the given star to the meridian, and the degrees between the star and the horizon will be the altitude required.

For the moon or a planet. Find the latitude and longitude, or the right ascension and declination of the planet, for the given time, in the Nautical Almanac, a good ephemeris, or from astronomical tables, and mark its place on the globe (as in prob. 4th or 2d) bring this place to the brass meridian, and the number of degrees between the point on the meridian over it, and the horizon, will be the altitude required.

OR WITHOUT THE GLOBE.

The declination of the star or planet at the time of its passing the meridian, added to or subtracted from the complement of the lat. according as they are of the same or a different name, will give the meridian altitude required.

Example 1. What is the meridian altitude of Aldebaran in taurus, at the New-York Literary Institution, York or Manhattan Island, lat. 40° 46′ N. ?

difference of their right ascensions is given, and as the sun's right ascension is given, therefore the star's right ascension is also given. The application of these principles to the above examples, must now be familiar to the learner, and is left for his exercise.

* The day of the month need not be attended to when the meridian alt. of a star is required, as the meridian altitudes of the stars on the globe, are invariable in the same latitude. Their places may be taken out of the ephemeris for noon without any sensible error. Their annual variation in decl. &c. should, however, be allowed for, where accuracy is required ; and the right asc. and decl. of the planets, reduced to the given time and place, in the same manner as the moon's, in the following notes.

Ans. 65° 20′. Or comp. of the lat. is 49° 14′, decl. of Aldebaran for 1800 was 16° 5′ 43″ or 16° 6′ nearly; hence the comp. of the lat. + 16° 6′ = 65° 20′, as before.

2. What is the meridian altitude of Procyon at London?

3. What is the meridian altitude of Arcturus in Bootes, at Washington city?

4. On the 1st of June, 1812, the longitude of Jupiter will be 3s. 9° 20′, and latitude 7′ north, or his declination will be 23° 15′ north; what will his meridian altitude be at Philadelphia?

5. On the 1st of August, 1811, the longitude of Mars was 8 signs 0° 8′, and his latitude 3° 3′ south, or declination 23° 10′ south; required the meridian alt. at Greenwich?

6. On the 1st of April, 1810, the longitude of saturn, was 8s. 15° 17′, and lat. 1° 45′ north; what was his meridian altitude at Paris observatory?

7. On the 22d of December, 1812, at the time of the moon's passing over the meridian of Greenwich, her right ascension will be 142° 53′ 13″, and declination 14° 37′ north; required her meridian altitude at Greenwich?*

Ans. Comp. lat. = 38° 31′ 20″ N. + decl. 14° 37′ N. = 53° 8′ 20″ mer. alt. required.

8. What will the moon's meridian altitude be at New-York, in longitude 74° 0′ 45″ W. from Greenwich, lat. 40° 42′ 40″ N. on the 16th of October, 1812; her right ascension at the time of passing the meridian† being 339° 37′ 56″, and declination 9° 33′ south?

Ans. The com. lat. = 49° 17′ 20′ N. — decl. = 9° 33′ S. = 39° 44′ 20″ mer. alt. required.

* By the Nautical Almanac the moon will pass over the meridian of Greenwich obs. on the 22d of December, 1812, at 30 min. after 3 in the morning (or on the 21st of December, 15h. 30m. astronomical time.)

140° 59′ 1″	☽'s rt. asc. at midnight,	Dec. 21st.	Decl.	15° 4′ N.
147 30 35	do. at noon,	Dec. 32d.	Decl.	13 32 N.
6 31 34	increase in 12 hours.		Decrease in 12h.	1 32

As 12h. : 3h. 30m. :: 6° 31′ 34″ : 1° 54′ 12″; hence 140° 59′ 1″ + 1° 54′ 12″ = 142° 53′ 13″ moon's rt. asc. at 15h. 30m.

Again, 12h. : 3h. 30m. :: 1° 32′ : 27′; hence 15° 4′ — 27′ = 14° 37′, the moon's decl. at 15h. 30m.

If greater accuracy be required, consult the theorems at the end of the Nautical Almanac for 1812 or 1813, published by J. Garnett.

† To find the *time of the moon's passing the meridian* of a given place, different from that of Greenwich. Take the difference between the time of the moon's passing over the meridian of Greenwich on the given day, and the day preceding or following, according as the place is to the east or west of Greenwich; then say, as 24 hours is to this difference, so is the difference of longitude in time, to a number of minutes and seconds, which must be added to the time of the moon's passage over the meridian of Greenwich, if the place be west, or subtracted if east of Greenwich; as the moon in the latter case will come to the meridian sooner than in the former. Thus on the 16th of Oct. 1812 (exam. 8) the moon will come to the meridian of Greenwich at 9 hours, and on the 17th at 9h. 55m. the

PROB. 19.

The meridian altitude of a known star or planet being given to find the latitude.

Rule. Bring the given star, or the place of the planet,* to the brass meridian, count the number of degrees in the given altitude (corrected†) on the brass meridian, from the star or planet's place, towards the south part of the horizon if the latitude be north, or towards the north part of the horizon if the latitude be south, and mark where the reckoning ends; elevate or depress the pole until this mark coincides with that part of the horizon towards which the altitude was reckoned; then the elevation of the pole above the horizon will be the latitude required.‡

difference is 55m. Hence 24h. : 55m. :: 4h. 56′ 3″ (the difference of longitude in time) : 11′ 18″ 4, which as the given place is west of Greenwich, must be added to 9 hours; whence 9h. 11m. 18.4 seconds, is the time the moon will come to the meridian of New-York on the 16th of Oct. 1812. Now to find her right ascension and decl. corresponding to this time; first reduce it to that of Greenwich, and proceed as above. Thus 9h. 11′ 18″ 4 + 4h. 56′ 3″ = 14h. 7′ 21″ 4, the time at Greenwich; then,

338° 23′ 20″	☽'s rt. asc. at midn. Oct. 16th.		Decl. 9° 56′ S.
345 25 4	do. at noon, Oct. 17th.		Decl. 7 46 S.
7 1 44	increase in 12 hours.	Decrease in 12h.	2 10

As 12h. : 2h. 7′ 21″ 4 :: 7° 1′ 44″ : 1° 14′ 35″ 9; hence 338° 23′ 20″ + 1° 14′ 35″ 9 = 339° 37′ 55″ 9, moon's rt. as. at 14h. 7′ 21″ 4 at Greenwich.

Again, 12h. : 2h. 7′ 21″ 4 :: 2° 10′ · 22′ 59″ 6; hence 9° 56′ — 22′ 59″ 6 = 9° 33′ 0″ 4, the moon's decl. corresponding to 2h. 7′ 21″ 4 after midnight.

The above method of calculating the time of the moon's coming to the meridian is a sufficiently near approximation; more accurate methods are given in the Nautical Almanacs for 1812 and 1813, revised by John Garnett, New-Brunswick.

Having the time of the moon's coming to the meridian or southing, the hour of the night by the moon shining on a *sun dial*, may be found thus; count how many hours and minutes the shadow on the dial wants of 12 o'clock, subtract them from the time of her southing for the hour of the night. But if the shadow be after 12, add these hours and minutes on the dial to the time of her southing, rejecting 12 if it exceed it, and you have the hour of the night.

Accurate methods are also given in the Nautical Almanacs for 1812 and 1813, for finding the moon's decl. rising and setting. The longitude from her meridional distance, &c. See the note to prob. 19.

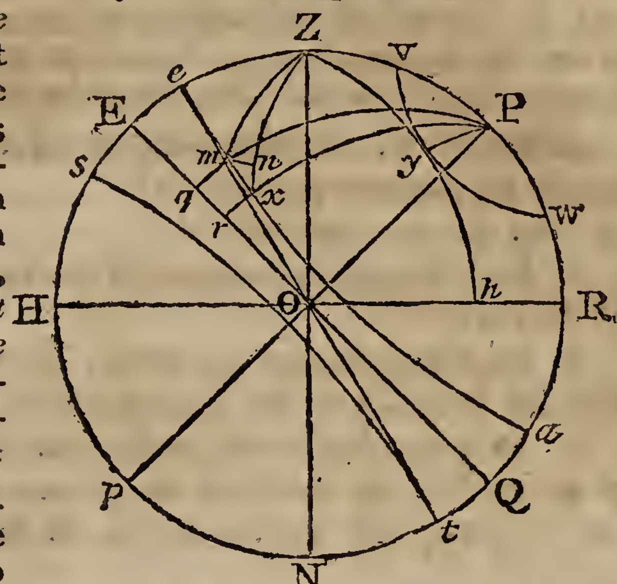
* The places of the planets when on the meridian may be calculated from the Nautical Almanac, when accuracy is required, in the same manner as that of the moon in the notes to prob. 18. But as their places vary less than that of the moon, they may be taken from the almanac for noon, without any sensible error, by only taking proportional parts for the daily variation.

† The observed altitude of a star may be corrected for dip and refraction, by the tables given in the note to prob. 58, part 2.

‡ It is evident that when the star is brought to the brass meridian, and the pole elevated or depressed until the star is at the same distance from the nearest part of the horizon as its altitude was observed to be, the globe will then be in the same position with regard to the horizon, as the earth itself; the height of the pole above the artificial horizon of the globe, being equal to the height of the real pole above the horizon of the earth; but *the*

Or, The declination of the star being given, or that of the planet (from the Nautical Almanac) reduced to the time and meridian of the place of observation (by note to prob. 18) then the sum or

height of the pole above the horizon, is always equal to the latitude, therefore the truth of the rule is evident. The demonstration of this property with the others that follow, may not be unworthy the readers perusal.



1. *The height of the pole above the horizon is equal to the latitude.* Let HR represent the horizon, EQ the equator, Z the zenith, and P the pole; then ZE = the lat. and PE = ZR being each equal 90°; hence ZP, which is common to both, being taken away, EZ will remain = PR. Again, 2. *Half the sum of the greatest and least alt. of a circumpolar star will give the latitude.* For if vw be the circle described by the star, v being the greatest, and w the least alt. then as Pv = Pw, PR = $\overline{\text{Rv} + \text{Rw}} \times \frac{1}{2}$. 3. *The lat. may be also thus found;* Let *e*O*t* be the ecliptic; then when the sun comes to *e*, *e*H will be its greatest meridian alt. its decl. being then greatest; but when the sun comes to *t*, *ts* being the parallel described on that day, *s*H will then be the least mer. alt. Now E*e* being = E*s*, $\overline{\text{H}e + \text{H}s} \times \frac{1}{2}$ = HE the co. lat. 4. *The inclination of the ecliptic to the equator, is equal to half the diff. of the sun's greatest and least meridian altitudes.* For EO*e* the obliq. of the ecliptic = $\frac{1}{2} \times \overline{\text{H}e - \text{H}s}$, or $\frac{1}{2}$ *se* = E*e*. 5. *The alt. of that point of the equator which is on the meridian, or the angle which the equator makes with the horizon, is equal to the compl. of the latitude.* For EH is the measure of the angle EOH, because EO and HO are each = 90°, and EH is the compl. of EZ, which is equal the lat. 6. As the *apparent time* is generally found by the alt. of some celestial body, hence if this latitude be wrong, the time must also be wrong. Now *the error in alt. being given, the error in time may be thus found;* let *mn* be parallel to the horizon, and *nx* represent the error in alt. then the body being supposed at *m* instead of *x*, as the time is calculated on supposition that there is no error in the declination, the angle *m*P*x*, or the arc *qr*, measures the error in time. Now the triangle *nmx* being small, the sides may be considered as right lines; then by trig. it will be *nx* : *xm* :: sine *nmx* : R. and *xm* : *qr* :: cos. *rx* : R. (by note to prob. 35, part 2.) hence multiplying the corresponding terms, and cancelling *xm* from the two first, *nx* : *qr* :: sine *nmx* × cos. *rx* : R^2 (Emerson's Geom. propor. prop. 18.) Therefore $qr = \frac{nx \times \text{R}^2}{\text{sine } nmx \times \text{cos. } rx}$; but the angle P*x*Z = *nmx*, *nxm* being the compl. to both; also sin. P*x*Z or *nmx* : sin. PZ :: sin. *x*ZP : sin. P*x* or cos. *rx*, (Emerson's Trig. b. 3, prop. 29) hence sin. *nmx* × cos. *rx* = sin. PZ × sin. *x*ZP; and therefore $qr = nx \times \text{R}^2 \div$ sine PZ × sin. *x*ZP = $nx \times \text{R} \div$ cos. lat. × sin. azim.

The error is therefore least at the *prime vertical,* or the vertical circle which cuts the meridian at right angles; and hence all altitudes for the purpose of obtaining the time, ought to be taken as near this circle as possible. The following ex. will illustrate this latter rule. In lat. 40° 43′, if the error in alt. at an azimuth 50° be 2′, then $qr = 2' \times 1^2 \div ,7579 \times ,766 = 3'\,445$ of a degree = 13″ 78 in time. The perp. ascent of a body is likewise quickest when on the prime vertical; for *nx* varies as the sine of the azim. when *qr* and the lat. are given, and the azimuth is then 90°.

difference of the zenith distance and decl. of the star or planet, according as they are of the same or a contrary denomination, will be the latitude required.

Note. The true alt. taken from 90° gives the zenith distance, which is north, if the observer be north of the star or planet, otherwise south. If the object be in the opposite meridian, or between the elevated pole and the horizon, at the time of observation, then the sum of the true alt. and the compl. of the declination or polar distance, will be the latitude. If the alt. be negative, or the centre of the object be below the horizon, it must be subtracted from the polar distance to find the latitude.

Example 1. In what degree of north latitude is the meridian altitude of Aldebaran 65° 20′ ?

Ans. In lat. 40° 46′ N.

2. In what degree of north lat. is the meridian altitude of Mirach in Bootes 70° ?

3. In what degree of north lat. is Procyon 90° above the horizon, or vertical when it culminates ?

4. In what degree of north lat. will the meridian altitude of Jupiter be 58° on the first of June, 1813, its longitude being then 4s. 5° 19′, and lat. 42′ north, or declination 19° 38′ north ?

7. The time *when the appar. diurnal motion is perp. to the horizon* is thus found; let vw be the parallel described by the star, and let the vertical circle *Zh* touch it at *y*; then when the star comes to *y*, its motion will be perpendicular to the horizon. Now P*y*Z being a rt. angle, we have by Napier's rule, R. × cos. ZP*y* = tang. P*y* × cot. PZ, or R × cos. distance of the star from the meridian = cot. decl. × tang. lat. and hence R. : cot. decl. :: tang. lat. : cos. hour angle, or star's distance from the meridian. The time of the star's coming to the meridian being known, the time required will from thence be given. 8. As the time of the sun's semidiameter passing the meridian serves to reduce an observation of a transit of the preceding or subsequent limb over the mer. to that of the centre, when only one limb was observed, the following method of finding the time in which the sun passes the meridian or horizontal wire of the telescope, may have its use. Let *mx* be the diameter of the sun in seconds = d'' estimated on the arch of a great circle; then the seconds in *mx* considered as a lesser circle, must be increased in proportion as the radius is diminished, the angle being inversely as the radius where the arc is given; hence P*x* or cos. decl. *rx* : R. :: d'' the seconds in *mx* of a great circle : to the seconds in *mx* of a lesser circle *eu* = the seconds in *qr*, or in the angle *q*P*r*; therefore *q*P*r* = d'' ÷ cos. decl. (rad. being 1.) = d'' × sec. decl. = the time of the sun's passing over a space equal to its diameter, or of passing the mer. Hence 15″ in space (being 1″ in time) : d'' × sec. decl. in space :: 1″ in time : the seconds of passing the mer. in time = d'' × sec. decl. ÷ 15″. The sun's diameter in rt. ascen. being = *qr* will be = d'' × sec. decl. If it be taken = 32′ or 1920″, and his decl. = 20°, its diam. in rt. as. = 1920″ × 1,064 = 34′ 2″,88. The same will hold for the moon if d'' be its diam. If *nx* (in the foregoing part of the note) = d'' the sun's diam. *qr* = d'' × R^2 ÷ cos. lat. × sin. azim. hence time of describing *qr* or of the sun's ascending or descending perpendicularly a space = its diam. will be $\frac{d''}{15''} \times \frac{R^2}{\text{cos. lat.} \times \text{sin. azim.}}$ If d'' = 33′ = 1980″, the horizontal refraction, then 1980″ ÷ 15″ = 132″; hence 132″ × R^2 ÷ cos. lat. × sine azim. is the acceleration of the sun by refraction, at rising, &c. (See Vince's Astron. 8vo.) Other useful principles could be deduced from the foregoing, but our limits would not permit their insertion.

5. In what degree of latitude will the meridian altitude of the moon be 53° 8′ 20″ south of the observer, on the 22d of December, 1812, astronomical time ?

PROB. 20.

Given the day of the month and the hour when any known star rises or sets, to find the latitude of the place.

Rule. BRING the sun's place in the ecliptic for the given day to the brass meridian, and set the hour index to 12 ; then turn the globe eastward or westward, according as the time is in the forenoon or afternoon, as many hours as the time is from noon ; elevate or depress the pole until the centre of the star coincides with the horizon ; then the elevation of the pole will be the latitude required.*

Example 1. In what latitude does Altair in the Eagle rise at 10 o'clock in the evening, on the 10th of May ?

Ans. 41° 35′.

2. In what latitude does Mirach in Bootes rise at half past 12 o'clock at night, on the 10th of December ?

3. In what latitude does β Rigel in Orion set at 4 o'clock in the morning, on the 21st of December ?

4. In what latitude does β Capricorni set at 11 o'clock at night, on the 10th of October ?

* The prob. being performed as directed in the rule, then the co. lat. co. decl. of the star, and its distance from the vertex (=90°) form a quadrantal spherical triangle. Now as the sun and star's right ascensions are given, their difference is given, and as the hour is given, the sun's distance from the meridian is given, therefore the star's distance from the meridian or the angle formed, at the pole, by the co. lat. and the co. decl. of the star, is likewise given ; and as the star's decl. is given, and the dist. of the star from the zen. = 90°, there are two sides, and the angle opposite one of them given to find the third, which is the co. lat. required. This may be solved more easily as follows ; the co decl. of the star (or the decl. + 90°) the lat. reckoning on the brass meridian, from the elevated pole to the horizon, by producing one of the sides of the former triangle, and the arc of the horizon, between the star and the point of the horizon, north or south, corresponding to the elevated pole, form a right angled sp. triangle, the hyp. of which, viz. the co. decl. of the star (or the decl. + 90°) and the angle formed at the pole between this side and the brass mer. (being the supplement of the distance of the star from the meridian) are given to find the lat. Thus in ex. 1. at 10 o'clock in the evening, the sun is 10 hours or 150° distant from the mer. Now the sun's right ascension on the 10th of May (suppose 1812) will be 3h. 8′ 38″ 4, and Altair's rt. as. 19h. 41′ 38″ 3, the difference of which is nearly 16h. 33′ ; hence 16h. 33′ — 10h. = 6h. 33′ = 97° 30′ 45″ Altair's distance from the mer. the supplement of which is 180°=97° 30′ 45″ = 82° 29′ 15″, the angle formed at the pole, and as the star's decl. will then be 8° 23′, the com. of which is 81° 37′, hence the comp. of the angle at the pole or 7° 30′ 45″ being middle part, and the decl. or 8° 23′, and the lat. adjacent extremes, we have by Napier's rule, r. × sine. 7° 30′ 45″ = tang. decl. 8° 23′ × tang. lat. therefore tang. decl. 8° 23′ : R :: sine. 7° 30′ 45″ : tan. lat. 41° 35′.

PROB. 21.

The altitudes of two known stars being given, to find the latitude of the place.

Rule. WITH one foot of a pair of compasses extended on the equinoctial to the complement of each star's altitude successively, and placed in the centre of each star respectively, describe arches on the globe with a black lead pencil, fixed in the other leg of the compasses; these arches will intersect each other in the zenith; the zenith being then brought to the brass meridian, the degree over it will be the latitude required.*

Example 1. Being at sea, I observed the altitude of Aldebaran to be 51° 45′, and at the same time that of Castor in Gemini equal 76° 40′; required the latitude?

Ans. With an extent of $38\frac{1}{4}°$ ($=90°— 51\frac{3}{4}°$) taken from the equinoctial (or any great circle on the globes which is divided into degrees) and one foot of the compasses in the centre of Aldebaran, describe an arc towards the north; then with 13° 20′ (90°—76° 40′) in the compasses, and one foot in the centre of Castor, describe another arc crossing the former; the point of intersection will be the zenith of the place, which being brought to the brass meridian, will give the latitude 42° nearly.

2. The altitude of Capella being observed 30°, and at the same time that of Aldebaran 35°, the latitude being north; required the latitude?

3. The altitude of Markab in Pegasus, was 30°, and that of Altair in the Eagle at the same time was 65°; required the latitude supposing it north?

4. In north latitude the altitude of Procyon was observed to be 50°, and that of Betelgeux in Orion, at the same time was 58°; required the latitude?

5. In south latitude the altitude of Betelgeux was $67\frac{1}{4}°$, and that of Aldebaran $60\frac{3}{4}°$; required the latitude?

PROB. 22.

Two known stars being observed, the one on the meridian, and the other on the east or west part of the horizon, to find the latitude of the place.

Rule. BRING the star which was observed on the meridian of the place, to the brass meridian; keep the globe from turning on its axis, and elevate or depress the pole until the other star comes

* Let Z be the zenith, P the pole, S, *s* the places of the star (see the fig. in note to prob. 14) then in the triangle *s*PS, there are given the sides *s*P, SP the co. declinations, and the angle *s*PS, the diff. of rt. ascensions; hence S*s* the distance of the stars, and the angle *s*SP are given. And in the triangle ZS*s*, all the sides are given to find the angle ZS*s*; hence the angle PSZ is given. Then in the triangle ZSP two sides ZS and SP, and the included angle are given, and therefore ZP is given, which is the co. lat. required.

to the eastern or western part of the horizon; the elevation of the pole will then be the latitude required.*

Example 1. When Procyon was on the meridian, Arcturus in Bootes was rising; required the latitude?

Ans. 23° north.

2. When the two pointers of the Great Bear marked α and β, or Dubhe and β were on the meridian; Vega in Lyra was rising; required the latitude?

3. When δ Leonis was on the meridian, the Pleiades were setting; required the latitude?

4. When the star marked β in Gemini was in the meridian, ε in the shoulder of Andromeda was setting; required the latitude?

5. In what latitude is α or Canopus, in the ship Argo, rising, when α in Phœnix is on the meridian?

6. In what latitude is Achernar in Eridanus on the meridian, when Procyon is rising?

PROB. 23.

Given two altitudes of a star, and the time between them, to find the latitude.

Rule. TAKE the complement of the first altitude in a pair of compasses, from the equinoctial (or any other great circle on the globes which is divided into degrees, &c.) and with one foot on the given star, describe an arch with the other (having a pencil fixed in it) in a contrary direction to that in which the star was observed; then bring the star to the brass meridian, and set the index to 12 (or any other hour) or mark the point on the equinoctial cut by the brass meridian; turn the globe *eastward* on its axis until the index, or the point marked on the equinoctial, has passed over as many hours as are equal to the time elapsed between the two observations, allowing 15° 2′ 28″ to every hour, or adding 9″ 85 of time to every hour,† and mark the point on the parallel of the star's declination then under the brass meridian; take the complement of the next altitude in the compasses, and with one foot in this point, describe with the other an arch intersecting the former; the point of intersection will be the zenith of the place, which being brought under the brass meridian, will give the latitude required.‡

* The prob. being performed as directed in the rule, then the distance between the star which is at the horizon and the mer. towards the elevated pole, reckoning on the horizon, the lat. on the mer. or dist. of the pole from the horizon, and the star's co. decl. form a right angled sp. △, one side of which, viz. the co. decl. and the angle at the pole, included between the brass meridian and circle of decl. passing through the star, which is at the hor. being equal to the supplement of the difference of the star's rt. ascensions, are given; hence the third side is given, and may be found by Napier's rule.

† A sidereal day being 23h. 56′ 4″, hence 23h. 56′ 4″ : 1h. :: 360° : 15° 2′ 27″ 9, &c. 2′ 27″ 9 = 9.85 seconds of time.

‡ This prob. may be performed by trigonometry in the same manner as prob. 61, part 2, (see the note to this prob.) thus; A and B being the places

Example 1. On the 31st of April in the afternoon, the altitude of Procyon was observed to be 44° 30′, and one hour after its altitude was 51° 31′, it being southward of the observer; required the latitude?

Ans. Here the complement of the first altitude is 45° 30′ with this extent in the compasses, and one foot in the centre of the given star, describe an arch towards the north; then the given star being brought to the meridian, and the index to 12; turn the globe eastward 1 hour 9″ 83 or 15° 2′ 28″ on the equinoctial, and mark the point under the declination of Procyon on the brass meridian; from this point as a centre, and with the complement of the 2d altitude = 38 30′ in the compasses, describe a second arch intersecting the former; the point of intersection brought to the brass meridian will give the latitude 41° N. nearly.

2. In north latitude on the 1st of April, in the evening, the altitude of Sirius was observed to be 30°, and one hour 15 minutes after his altitude was $19\frac{1}{4}°$; required the latitude of the place of observation?

PROB. 24.

Given one altitude of a star, and the time at which the altitude was taken, to find the latitude.

Rule. WITH the complement of the given altitude in the compasses, taken from the equinoctial (or any other great circle on the globe divided into degrees, &c.) and one foot in the centre of the given star, describe an arch with the other, in a direction contrary to that in which the star appeared when observed; bring the sun's place in the ecliptic to the brass meridian, and set the hour index to 12; then if the time be in the forenoon, turn the globe eastward, but if in the afternoon, westward, as many hours as the time is before or after noon; the degrees then cut by the arch on the brass meridian, will be the latitude required.*

Note. The observed altitude must be previously corrected for the height of the eye and refraction. The same method will also answer for any planet, its declination being given; the observed altitude being first corrected for the height of the eye or dip of the horizon, refraction and parallax. See the note to prob. 19.

of the star at the respective altitudes (see the note to prob. 61, above alluded to) then in the isosceles sp. △ APB, AP or BP the co. decl. of the star, and the ∠ APB, being the elapsed time reduced into degrees (by allowing 15° 2′ 28″ to every hour) are given, hence the ∠ PAB and the side AB is given; and in the sp. △ ABZ, AZ, BZ, the com. of the altitudes and the side AB are given, hence the ∠ BAZ is given, from which the angle PAB being taken, the angle PAZ will be given; and in the triangle AZP, PA, AZ, and the angle at A are given; hence the side ZP, which is the complement of the latitude required, is given.

* This prob. may be calculated in the same manner as prob. 62, part 2. For the time of the star's passage over the meridian may be found by prob. 8, part 2, the difference between which and the time at which the alt. was taken, will be the distance of the star from the mer. or the hour angle, or

Example 1. At 9 o'clock in the evening on the 21st of March, the altitude of Deneb in Leo, was 47°; required the latitude of the place of observation?

Ans. 41° 50′ N. nearly.

2. At 10 o'clock in the evening on the 22d of December, the altitude of Procyon was 27° in north latitude; required the lat.?

3. On the 1st of January, 1810, in north latitude, the correct altitude of Jupiter at 9 o'clock in the evening was 37° 15′, its latitude being 1° 18′ S. and longitude 0s. 15° 48′; required the lat.?

4. On the first of November, 1810, the altitude of Venus at 6h. 30′ in the evening was 10° 35′, being towards the S. W. of the observer, height of the eye being 30 feet above the level of the horizon; the longitude of Venus being 8s. 24° 30′, and latitude 4° 7′ S.; required the latitude of the place, allowing also for parallax and refraction?

5. In north latitude on the 17th of October, 1810, the altitude of the lower limb of the moon, taken by a Hadley's quadrant, at 3 o'clock in the morning, was 61°. Her right ascension being 87° 31′, and declination 18° 14′ N.; required the latitude, allowing for semidiameter, refraction, parallax and dip of the horizon; height of the eye being 20 feet?

PROB. 25.

*The altitudes of two stars having the same azimuth, and that azimuth being given, to find the latitude of the place.**

Rule. PLACE the graduated edge of the quadrant of altitude over both stars, so that each star may be exactly under its given al-

the angle APZ, &c. See the fig. in the note to prob. 62. But as this time is *sidereal time*, it must be converted into degrees, by allowing 15° 2′ 27″ 9. to an hour. (See the note to the foregoing prob.)

Or, The prob. may be calculated as follows; find the sun's rt. asc. from the Nautical Almanac or some good table, and likewise the star's rt. ascen. (see the table at the end of this work) their sum or difference will be the distance between the sun and the star reckoning on the equator (see note to prob. 12, part 3) convert the given time of observation into degrees, allowing 15° to an hour; this will give the sun's dist. from the mer. in degrees, the difference between which and the dist. between the sun and star will give the dist. of the star from the mer. or the angle APZ, &c. as above. This angle being given, and the star's decl. and alt. the calculation is the same as that in the note, prob. 62, as above quoted. Thus in ex. 1, the sun's rt. ascen. (the year being supposed 1812) will be 28″ 4 in time, or 7′ 6″ of a degree, and the star's rt. as. 174° 52′; hence their diff. is nearly 174° 45′, and the dist. of the sun from the mer. being 9 hours or 135°, the dist. of the star is therefore 39° 45′. Now AP the co. decl. = 74° 22′ 40″, AZ the co. alt. = 43°, and the angle APZ = 39° 45′; hence R. : cos. 39° 45′ :: tang. 74° 22′ 40″ : tang. PB 70° 1′, and cos. 74° 22′ 40″ : cos. 70° 1′ :: cos. 43° : cos. BZ 21° 51′, and 70° 1′ − 21° 51′ = 41° 50′ the co. lat. therefore the lat. is 41° 50′ as above.

* In calculating this prob. it will be seen that the altitude and azimuth of either of the stars alone would be data sufficient to solve the prob. For in the triangle PZS or PZs (see the fig. in the note to prob. 13, part 3) ZS or

titude on the quadrant; the quadrant being held in this position, elevate or depress the pole until the division marked 0 on the quadrant, coincides with the given azimuth on the horizon, the elevation of the pole will then be the latitude.

Example 1. The altitude of Aldebaran was observed 45° 45′ when that of Sirius was 30′, their common azimuth, at the same time, being 67½° from the south towards the east or E. S. E. nearly; required the latitude?

Ans. 40° 46′ N. nearly.

2. The altitude of Arcturus was observed to be 40°, and that of Cor Caroli 68°; their common azimuth at the same time being 71° from the south towards the east; required the latitude?

3. The altitude of α Dubhe was 40°, and that of γ in the back of the Great Bear 29½°; their common azimuth at the same time being 30° from the north towards the east; required the latitude?

4. The altitude of Vega or α in Lyra, was observed to be 70°, and that of α in the head of Hercules 39½°; their common azimuth at the same time being 60° from the south towards the west; required the latitude?

PROB. 26.

*Given two known stars having the same azimuth, and that azimuth being given, together with the altitude of one of the stars, to find the latitude of the place.**

Rule. PLACE the graduated edge of the quadrant of altitude over both stars, so that the star whose alt. was taken may be under the same altitude on the quadrant; then proceed as in the foregoing problem.

Note. This problem being similar to the foregoing, the examples there given will answer this, taking one of the altitudes instead of both.

PROB. 27.

Given the altitudes of two known stars observed at different times, and the interval of time between the observations, to find the latitude.

Rule. WITH the complement of the first altitude (taken from the equinoctial) in the compasses, and one foot in the centre of the star whose altitude was first taken, describe an arch; bring the star whose altitude was next taken to the brass meridian, set the

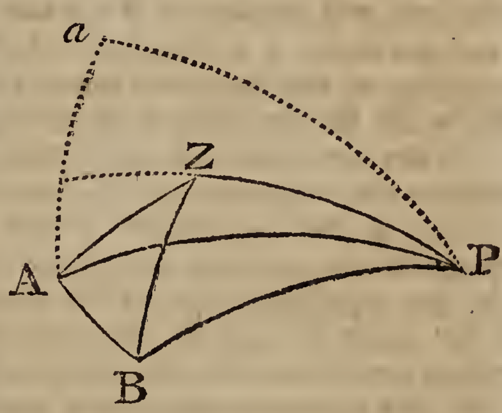
Z*s*, the co. alt. SP or *s*P, the co. decl. of the stars, and the common azim. SZP or *s*ZP are given, therefore the side ZP is given, the compl. of which is the lat. required. Or the prob. may be calculated without the azimuth, thus; in the triangle SP*s*, S*s* the difference of the co. altitudes, and SP, *s*P are all given, hence the angle PSs is given, and therefore its supplement ZSP is given. Again in the triangle ZSP, ZS and SP, and the included angle are given to find ZP, which is therefore given.

* For the solution of this prob. by trigonometry, see the note to the foregoing problem.

index to 12; mark the star's declination on the brass meridian, and also the point cut by the meridian on the equinoctial, or the star's right ascension; turn the globe *eastward* on its axis until the index has passed over as many hours as are equal to the interval of time between the two observations, together with 9″ 83, added for every hour of solar time, or until the point marked on the equinoctial passes over as many degrees as are equal to the same interval, allowing 15° 2′ 28″ for every hour; then mark the point on the globe under the degree of the star's declination on the brass meridian, from this point as a centre, with an extent in the compasses (taken from the equinoctial as before) equal to the complement of the second altitude, describe another arch intersecting the former; the point of intersection will give the zenith, which being brought to the brass meridian, the degree over it will be the latitude required.*

Example 1. In north latitude December 20th, 1806, the true altitude of Menkar in Cetus was 43° 38′, and 1h. 18m. after, the altitude of Rigel was 29° 51′; required the latitude?

Ans. With an extent of 46° 22′ (= 90°— 43° 38′) taken as directed in the rule, and one foot of the compasses in the centre of Menkar, describe an arch with a fine pencil fixed in the other; then Rigel being brought to the meridian, and the index set to 12, or mark the equinoctial as directed; turn the globe eastward 1h. 18′ + 13″, the equation = 1h. 18′ 13″, the interval or sidereal time, or until the point marked on the equinoctional has passed over 19° 33′ 12″ (for 1h. : 15° 2′ 28″ :: 1h. 18′ : 19° 33′ 12″) mark the point on the globe under 8° 26′, the declination of Rigel,



* Let A, B be the two known stars, Z the zenith, and P the pole. Now if the time at which either of the observations was made be given, the altitude of one of the stars will be sufficient to determine the lat. For let the alt. of A and the time at which it was taken, be given; then the distance of the star from the meridian, or the angle APZ will be given (see the note to prob. 24, part 3) and AZ the co. alt. and also AP the co. decl. of the star are given; therefore ZP the co. latitude is given.

If only the interval of time between the observations be given, the prob. may be thus calculated; let A be the place of the star A, when the first observation or its alt. was taken, and *a* its place when the second observation was made, or when the alt. of B was taken; hence the angle AP*a* will be the elapsed time, or the interval between the two observations, which is converted into degrees by allowing 15° 2′ 27″ 9 to every hour. But the angle *a*PB being equal to the difference of the star's right ascensions, is therefore given, and hence the angle APB = *a*PB — *a*PA is given. Now in the triangle APB, the two sides AB, BP = the co. decls. and the included angle are given, therefore the side AB and the angle BAP are given. Again, in the triangle AZB, there are given AZ, BZ the co. alts. and the side AB, hence the angle BAZ is given, and therefore PAZ = BAZ — BAP is given. Lastly, in the triangle PAZ, PA, AZ, and the included angle are given, and therefore PZ is given, the compl. of which is the lat. required.

from this point with an extent of 60° 9′ (= 90° — 29° 51′) describe another arch as before, inserting the former; the point of intersection being brought to the meridian, will give the latitude 49° 32½′ nearly.

2. In north latitude on the 21st of March, 1810, at 9 o'clock at night, the correct altitude of Cor Hydræ, or Alphard, was 41° 15′, its right ascension being 139° 33′ 42″, and declination 7° 50′ 20″ S and one hour after the altitude of Deneb was 57¼°, its right ascension being 174° 50′ 22″, and declination 15° 38′ 5″ N.; required the latitude?

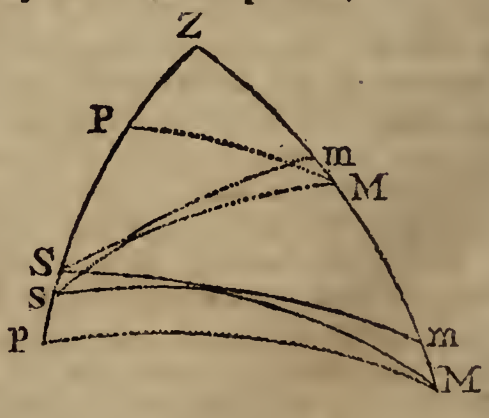
PROB. 28.

To find the distance of the stars or planets from each other in degrees, &c.*

Rule. EXTEND the quadrant of altitude between any two stars or the given places of the planets, so that the division marked 0 may be on one of them, the degrees on the quadrant between that and the other star or planet's place will shew their distance, or the angle which these stars or planets subtends as seen by a spectator on the earth, or rather as seen from the earth's centre.

* In order to find the correct distances of celestial objects, it is necessary to determine their altitudes, which may be accurately found by an *astronomical quadrant* (for the description and use of which, see Vince's Practical Astronomy, which contains the description of the construction and use of all astronomical instruments, 1 vol. 4to.) or by a good Hadley's quadrant, a sextant, or by a repeating circle or circle of reflection. The apparent altitudes being thus found, the true alt. may be found by allowing for the height of the eye and refraction if a fixed star; for the height of the eye, refraction, and parallax, if a planet; and if the sun or moon for their semidiameter, according as the upper or lower limb was taken. A small allowance is also to be made for the aberration of light, as will be shewn in part 4.

From the observed and corrected altitudes, and the observed distance between the two objects, the true distance may be thus computed;



Let Z be the zenith, S the apparent place of the sun or a star, s the true place, M the apparent place of the moon; m its true place; then in the triangle ZSM, there are given SM, the apparent distance, SZ, ZM the complements of the apparent altitudes to find the angle SZM. Let fall from M the perpendicular MP or Mp; then,

$$\text{Tang. } \tfrac{1}{2}\,ZS : \text{tang. } \frac{ZM + MS}{2} :: \text{tang. } \frac{ZM \backsim MS}{2} : \text{tang. } x.$$

If ½ ZS be greater than x, the perpendicular falls within as MP; if less, without as Mp. Then ½ ZS + x = ZP or Zp, and ½ ZS $\backsim x$ = SP or Sp. Moreover if ZS + SM be less than 180°, the perpendicular falls nearest to the lesser side, but if ZS + SM be greater than 180°, the perpendicular falls nearest the greater side; this being premised, then by Napier's rule,

Rad. : tang. ZP or Zp :: co. tang. ZM : co. sine angle at Z.

The same may be performed by a pair of compasses, as is manifest.

Example 1. What is the distance between Betelguex in Orion, and Castor in Gemini ?

Ans. $37\frac{3}{4}^{\circ}$.

2. What is the distance between Procyon and Capella ?

3. On the 2d of January, 1812, at midnight, the moon's right ascension will be 151° 39′ 59″, and her distance from the north pole 79° 17′ ; required her distance from Spica Virginis ?

Ans. 51° 20′.

4. On the 19th of March, 1812, Jupiter will be exactly in the ecliptic, and his longitude will be 2s. 27° 24′ ; required his distance from the sun, and from each of the following stars, viz. Aldebaran, Rigel, Betelgeux, Sirius, Procyon, Castor, Pollux, Capella, the Pleiades ; and also from the moon, whose right ascension at midnight will be 89° 45′ 21″, and declination 18° 34′ N. ?

Now in the triangle sZm, there are given the angle Z, and sZ, mZ, to find sm the true distance ; hence it will be,

Rad. : cos. Z. :: tang. Zm. : tang. Zx the distance between Z and a perp. let fall from m ; then Zs ∽ Zx = sx. And cos. Zx : cos. Zm :: cos sx : cos. sm, the correct distance required. As sx and the angle at Z are of the same or different affection, sm is greater or less than a quadrant.

The learner will also observe, that the mark ∽ denotes the difference of those quantities between which it is placed.

Example. On June 29, 1793, the complement of the sun's apparent alt. or his apparent zenith distance ZS was 70° 56′ 24″, the comp. of the moon's app. alt. or app. zenith distance ZM was 48° 53′ 58″, their app. distance SM was 103° 29′ 27″, and the moon's horizontal parallax was 58′ 35″; their true distance calculated by the above method, will then be 103° 3′ 18″, as required.

The sun's place being given, together with the moon's lat. and long. their distance may be thus found : The diff. of long. of the sun and moon, the moon's lat. and the dist. between the sun and moon (represented on the globe) will form a right angled spherical triangle, the sides of which are given, viz. the diff. of longitude and moon's lat to find the third side, which is the *true* distance between the sun and moon. Whence, making use of Baron Napier's rule, we have this proportion ; Rad. : co. sine diff. long. :: co. sine moon's lat. : co. sine *true* distance required. The rule given in page 147 of the *Nautical Almanac* for 1813, is, in substance, the following ; *log. co. sine diff. long. between the sun and moon* + *log. co. sine moon's lat.* = *log. co. sine true distance*, which is evidently an error, as appears from the above proportion, as it ought to be, *log. co. s. diff. long.* + *log. co. s. moon's lat.* — *rad.* = *log. co. s. true distance.*

The Nautical Almanac above alluded to, is that revised by Mr. John Garnett, New-Jersey, a work which deserves every encouragement, from its extensive utility, and the many important additions made by the Editor. (The error above noticed must evidently have been an oversight, or omission in the printer.) The Nautical Almanac for 1813 is the first distinguished with that considerable advantage of having the accurate *Lunar tables of Mr. Burg*, and the late improved *Solar tables of M. De Lambre*, made use of in its calculation. These tables are corrected and improved by the Rev. Samuel Vince of Cambridge, and published in English. He has adapted them to astronomical time, those of Mr. Burg and De Lambre being adapted to civil time ; so that the year, in the latter, commences at the midnight with which the last day of the former year ends, and in Vince's 12 hours later, &c.

PROB. 29.

Given the true distance of the moon from the sun or a star, and the time at which the observation was made, to find the corresponding time at Greenwich, and the longitude of the place of observation.

Rule. MARK the moon's path on the globe for the noon and midnight preceding and following the time of observation (by probs. 2 and 4. part 3d.) and also the moon's places for every three hours during this interval of time, by taking proportional parts: if it be the distance of the moon from the sun that is given, mark the sun's place in the ecliptic, corresponding to the times in which the moon's respective places were marked in its orbit; then find the true distances of the moon from the sun or star, which are next greater, and next less than the true distance deduced from observation (either with a pair of compasses applied to the sun's places or the centre of the star, and to the corresponding places of the moon at the same time; or taken from the Nautical Almanac) and the difference of these distances (which call D) will give the access of the moon to, or recess from, the sun or star in three hours; then take the difference between the moon's distance at the beginning of that interval, and the distance deduced from observation (which call *d*) and say, D : *d* :: 3h. : to the time the moon is approaching to, or receding from, the sun or star by the quantity *d*; which added to the time at the beginning of the interval, gives the apparent time at Greenwich, corresponding to the given correct distance of the moon from the sun or star; the difference between which and the apparent time at the place of observation, will be the difference of longitude in time, which may be easily reduced into degrees, &c.

Example 1. Suppose that on the 14th of May, 1812, in latitude 40° 42′ 40″ N. at 6h. 3′ 57″ apparent time in the afternoon, the correct distance of the sun and moon's centres was 52° 30′ 40″;

For other methods of finding the distances of celestial objects, see Vince's *Complete System of Astronomy*, or McKay's Treatise on Navigation, &c.

As the learner may be at a loss to determine the parallax of any of the celestial bodies, the following remarks may be necessary.

Observations prove that the diameter of any of the fixed stars is less than 1″ 6, and therefore that they have no sensible parallax. The parallax of the sun resulting from the observations of the transit of Venus in 1761 and 1769, is 8″ 8. The horizontal parallax of the moon is given in pa. 7 of the month in the Nautical Almanac for every noon and midnight, or it may be calculated from probs. 16 and 17 of Mayer's tables. A table of the reduction of latitude and moon's horizontal parallax, for the spheroidical figure of the earth, is also given in page 12 of July in the Nautical Almanac for 1812.

In general the distance of a phenomenon, from the earth : to the semi-diameter of the earth :: cos. apparent altitude of the body : sine of the parallax. See Gregory's Astronomy, b. 2, sect. 7, or Vince's Astronomy, 8vo. ch. 6, where several methods are given. The parallax varies inversely as the distance. De Lambre, in his calculations, makes use of 8″ 6 for the sun's horizontal parallax. See his tables annexed to the 3d edit. of La Land's Astronomy. The distances, &c. of the planets will be given in part 4.

required the apparent time at Greenwich, and the longitude of the place of observation ?

Ans. True distance of the moon from the sun is	52°	30′	40″
Do. by Naut. Alm. on May 14th at 9h.	51	29	25
Do. by do - - at midnight	53	1	17
d =	1	1	15
D =	1	31	52

Hence 1° 31″ 52″ : 1° 1′ 15″ :: 3h. : 2 hours, which added to 9 hours gives 11 hours, therefore 11h. — 6h. 3′ 57″ = 4h. 56m. 3s. which in degrees is 74° 0′ 45″, the longitude of the place of observation, and is west of Greenwich, as the time in Greenwich is later.

2. Suppose that on the 15th of May, 1812, in latitude 39° 57′ N. at 9 o'clock in the afternoon apparent time, the distance of the moon's centre from Regulus was 26° 24′ 27″ ; required the apparent time at Greenwich, and the longitude of the place of observation ?

Ans. True dist. of the moon from Regulus by obs.	26°	24′	27″
True dist. by Naut. Alm. on May 15 at midnight	27	29	1
Do. - - - - - at 15 hours	25	52	56
d =	1	4	34
D =	1	26	5

Then 1° 36′ 5″ : 1° 4′ 34″ :: 3h. : 2h. 0′ 57″ 5, which added to 12 hours, gives 14h. 0′ 57″ 5 ; hence 14h. 0′ 57″ 5— 9h. = 5h. 0′ 57″ 5, the diff. of longitude in time, which in degrees is 75° 14′ 22″, west of Greenwich.

3. On June 29, 1793, in latitude 52° 12′ 35″, the sun's altitude in the morning was by observation 19° 3′ 36″, the moon's altitude was observed to be 41° 6′ 2″, the sun's declination at that time was 23° 14′ 4″, and the moon's horizontal parallax 58′ 35″ ; to find the apparent time at Greenwich, and the longitude of the place of observation ?

Ans. True dist. of the moon from the sun (note to prob. 28)	103°	3′	18″
Do. by Naut. Alm. on June 29, at 3h.	103	4	58
Do. - - - - on June 29, - 6h.	101	26	42
d=	0	1	40
D=	1	38	16

Now 1° 38′ 16″ : 1′ 40″ :: 3h. : 0h. 3′ 3″ which added to 3h. gives 3h. 3m. 3s. the apparent time at Greenwich.

Now to find the apparent time at the place of observation, we have the sun's alt. 19° 3′ 36″, its refraction 2′ 44″, and parallax 8″, hence its altitude was 19° 1′, and therefore its true zenith dist. was 70° 59′ ; also the co. decl. was 66° 45′ 36″ ; hence by the note to prob 48, part 2d, or by the globes, the hour angle is found equal 88° 37′ 44″ in time equal 5h. 54′ 30″ 9, the time before apparent noon, or 18h. 5′ 29″ 1 on June 28th. Hence 29d. 3h. 3m. 3s. the app. time at Greenwich, less 28d. 18h. 5m. 29s. the app. time at the place of obs. gives 8h. 57′ 34″ = the diff. of meridians or diff. long. in time, which in degrees is 134° 22′ 31″, the long. of the place of obs. west of Greenwich.

PROB. 30.

To find what stars lie in or near the moon's path, or what stars the moon can eclipse, or make a near approach to.

Rule. FROM the longitude and latitude of the moon, or her right ascension and declination, taken from the Nautical Almanac, or any good ephemeris, mark the moon's places on the globe, for several days (by problems 2d. and 3d. part 3d.) then by extending the quadrant of altitude or a thread, over these places, you will nearly find the moon's path, and consequently those stars that lie in her way, or that she can make a near approach to.*

Example 1. What stars were in or near the moon's path on the 16th, 17th, 18th, and 19th of May, 1810?

16th.	☽'s right ascension	206°	47′	declination	9°	42′	S.
17th.	- - -	220	43	- -	13	14	S.
18th.	- - -	235	22	- -	16	3	S.
19th.	- - -	250	38	- -	17	53	S.

Ans. Zuben el Chamali, Zuben ha Krabi, β in Libra, &c.

2. On the 4th, 5th, 6th, 7th, and 8th of September, 1812, what stars will be near the moon's way?

4th.	☽'s longitude	4s.	27°	3′	40″	latitude	0°	11′	3″	S.
5th.	- - -	5	9	17	19	-	0	56	22	N.
6th.	- - -	5	21	22	14	-	2	0	22	N.
7th.	- - -	6	3	20	6	-	2	58	22	N.
8th.	- - -	6	15	12	51	-	3	48	9	N.

PROB. 31.

The latitude of a place being given, to find the time of the year at which any known star rises or sets achronically, *that is, when it rises or sets at sun setting.*

Rule. ELEVATE the pole to the latitude of the place, bring the given star to the eastern part of the horizon, and then mark the point of the ecliptic at the western edge, or the point of the ecliptic that sets when the star rises, the day of the month corresponding to this point will give the time when the star rises at sun set, or when it begins to be *visible in the evening.* The globe being

* The situation of the moon's orbit for any particular day may be found thus; find the place of the moon's ascending node in the Nautical Almanac; mark that place and its antipodes (being the descending node) on the globe; take the middle between these two points, and make two marks 5° 8′ 49″ (= the inclination of the lunar orbit to the plane of the ecliptic) on the north and south sides of the ecliptic; so that the northern mark may be between the ascending and descending node, and the southern between the descending and ascending node; a thread extended through these four points, will shew the position of the moon's orbit.

then turned westward on its axis, until the star comes to the western edge of the horizon, observe the degree of the ecliptic cut by the western part of the horizon as before, the day of the month answering to that degree, will shew the time when the star sets with the sun, or when it *ceases to appear in the evening*.

Example 1. At what time does Arcturus rise and set achronically at Ascra* in Bœotia ; the latitude of Ascra, according to Ptolemy, being 37° 45′ N.?

Ans. When Arcturus is in the eastern part of the horizon, the twelfth degree of Aries will be at the western, which answers to the 1st of April,† the time when Arcturus rises achronically ; Arcturus will set achronically on the 30th of November.

* Ascra is a small village in Bœotia at the foot of Mount Helicon, where Hesiod lived, and was probably born, hence called Ascreus. (Virgil Ecl. 6, 70, and Georg. 2, 176.) Strabo says that he came with his father Dio from Cuma, a city of Eolis, opposite to Lesbos, now called Taio Nova. Some are of opinion that he lived before the time of Homer, as his style is more rude and simple ; others that he was cotemporary with Homer (this is the common opinion. Rollin's Anc. Hist. b. 5, art. 9) and others that he lived after Homer. Velleius Paterculus, who lived in the time of the three first Roman emperors, says, in his abridgment of the Roman History, that he lived 120 years after the time of Homer. However, as Carolus Ruæus Soc. Jesu, the learned commentator on Virgil, remarks, 'Lis adhuc pendet.' (See Newton's Chronology.)

† If we allow for the star's refraction, which at the horizon is about 33′, the time will nearly correspond to the 31st of March, and then Arcturus would rise achronically in lat. 37° 45′ N. about 99 days after the winter solstice. Hesiod, in his *Opera et Dies*, lib. 2, verse 185, says,

When from the solstice sixty wintry days,
Their turns have finished, mark, with glitt'ring rays,
From ocean's sacred flood, *Arcturus* rise,
Then first to gild the dusky evening skies.

Hence (supposing Hesiod to be correct) there is, between the time of Hesiod and the present time, a difference of 39 days in the achronical rising of this star ; and as a day answers to about 59′ 8″ of the ecliptic (note to def. 66) 39 days will answer to 38° 26′ 12″, and therefore the winter solstice in the time of Hesiod was in 8° 26′ 12″ of aquarius. Now the precession of the equinoxes being about 50¼″ in a year, we have 50¼″ : 1 year :: 38° 26′ 12″ : 2753 years nearly, since the time of Hesiod ; so that (the places of the stars on the globe being adapted to the year 1800) he must have lived 953 years before Christ by this mode of reckoning. Homer, according to most chronologies, lived 907 years before Christ. Lempriere, in his Classical Dictionary, says that Hesiod lived at the same time. Herodotus however (lib. 2, c. 53) says that Homer wrote 400 years before his time, that is 340 years after the destruction of Troy, which happened 1184 years before Christ, so that, according to Herodotus, Homer lived 844 years before Christ.

The above calculation was made without reflecting that the same allowance should be made for the sun's refraction, which would make the time nearly correspond to the 30th of March, giving the astronomical rising of Arcturus about 98 days after the winter solstice ; differing from the same in Hesiod's time 38 days ; which answers to 37° 27′ 4″, or 7° 27′ 4″ of Aquarius. Hence 50¼″ : 1y. :: 37° 27′ 4″ : 2683 years ; therefore 2683 — 1300= 883 years. This might be rendered more accurate by strict calculation. Keith in his treatise on the Globes, makes the time of Hesiod 990 years before Christ. From the whole we see that there is a strong probability of his be-

2. At what time of the year does Aldebaran rise and set achronically at Athens, in latitude 38° 5′ N. ?

3. When does Sirius rise achronically in New-York, and at what time of the year does it set achronically ?

4. When does Procyon rise at London when the sun is setting, and when does he set at sunset ?

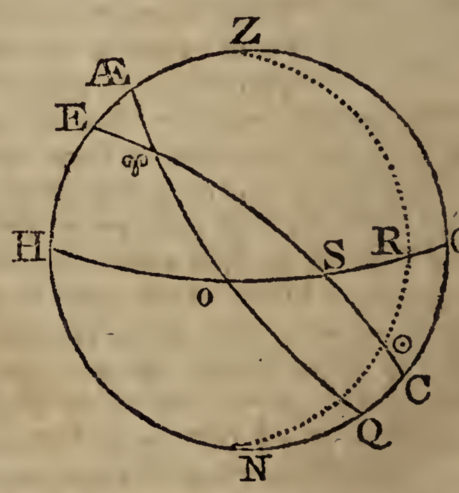
PROB. 32.

The latitude of the place being given, to find the time of the year at which any known star rises or sets cosmically, *that is, rises or sets at sun rising.*

Rule. ELEVATE the pole to the given latitude, bring the given star to the eastern part of the horizon ; then the day of the month corresponding to the degree of the ecliptic cut by the eastern part of the horizon, will give the time when the star rises with the sun ; bring the star to the western part of the horizon, the sign and degree of the ecliptic, then intersected by the eastern part of the horizon as before, will point out on the horizon the time when the star sets cosmically, or at sun rising.

Example 1. At what time of the year do the Pleiades set cosmically at Miletus* in Ionia, in lat. 37° N. according to Ptolemy, and at what time of the year do they rise with the sun there ?

ing cotemporary with Homer. What we have here said may to some seem rather tedious, but the learner will derive much information from it in similar calculations, as the ancients made frequent use of the poetical rising and setting of the stars.



This and the following problem may be solved by trigonometry, as follows ; let HO represent the horizon, HZO the mer. ÆQ the equinoctial, EC the ecliptic, ♈ the point aries, or the intersection of the equinoctial and ecliptic, S the point of the ecliptic which rises with the star, and o the point of the equator ; then in the triangle ♈oS, we have ♈o, the oblique ascen. of the star, the angle at ♈, the obliquity of the ecliptic, and the angle ♈oS, the height of the equator above the horizon (being equal to the co. lat. Note to prob. 19) or its supplement ; hence ♈S is given, and therefore the point S of the ecliptic, which rises with the star, or the star's long. is given, the time corresponding to which, found by the Nautical Almanac, or the globe, will be the time when the star *rises cosmically.* The angle ♈So, is the angle which the ecliptic and horizon make at the rising point. When the sun is in the sign and degree opposite the point S, the star will then rise *achronically.* In a similar the time is found when the star *sets cosmically* or *achronically..*

* *Miletus,* the birth place of *Thales,* was situated in Asia Minor, on the coast of the Egean sea, near the borders of Caria, south of Ephesus, and southeast of the island of Samos, or six miles to the south of the mouth of the river Mœander. This city, as Pliny remarks, was sometimes called Pithyusa, Anactoria and Lelegis; now called Melaxo or Melasso. It was formerly famous for its wool.

Ans. The Pleiades rise with the sun in lat. 37° N. on the 11th of May, and they set at the time of sun rising on the 21st of November.*

Thales laid the first foundation of philosophy in Greece, and founded the Ionian school, where he taught the sphericity of the earth, the obliquity of the ecliptic, and the true causes of the eclipses of the sun and moon. He had exactly foretold the time of the eclipse of the sun, that happened in the reign of Astyages, king of Media, of which mention is made in Rollin's Anc. Hist. He also discovered the solstices and equinoxes, divided the heavens into five zones, and recommended the division of the year into 365 days. When he travelled into Egypt, he discovered an easy and certain method of determining the exact height of the pyramids, by observing the time when the shadow of a perpendicular body was equal in length to the body itself. His life, as well as that of the other wise men, is written by Diogenes Laertius.

* Pliny (in his Natural History, b. 18, c. 25) says, that Thales determined the cosmical setting of the Pleiades to be 25 days after the autumnal equinox. Supposing this observation to be made at Miletus, there will be a difference of 35 days in the cosmical setting of this star; hence 1*d*. : 59′ 8″ of the ecliptic :: 35*d*. : 34° 29′ 40″, which in the time of Thales will make the equinoctial colure pass through 4° 29′ 40′ of scorpio; and 50¼″ : 1*y*. :: 34° 29′ 40″ : 2471 years since the time of Thales. Hence Thales lived 2471 — 1800 = 671 years before Christ, by this mode of reckoning. This time will however be lessened; by allowing for refraction, &c. *Sir I. Newton*, in his chronology, makes it 596 years before Christ; most chronologers make it 600 years. According to *Lempriere* in his Classical Dictionary, he died in the 96th year of his age, about 548 years before the Christian era. The remarkable eclipse predicted by Thales happened in the 545th year before Christ. See Ferguson's Astronomy, page 25.

Some affirm that Thales taught that one intelligent being presides over and governs the universe. Many of the heathen philosophers came to the knowledge of this truth by the light of reason alone; but among the whole there was not one that did not worship the ridiculous Gods of his country with the vulgar, the knowledge of whom in other respects they so much despised, so that we find *Socrates*, the wisest among them, at his dying moments ordering his friend to sacrifice a *cock* to *Esculapius*.

This is another unanswerable argument in favour of that knowledge which the christian possesses of the true God, contrasted with that which a proud, shallow, presumptuous philosophy affords. To him that is destitute of this knowledge, the glimmering ray of human science will yield but little assistance, in dissipating the darkness that surrounds him—Its frigid rules is but a feeble support to resist the violence of human depravity. No light, therefore, but that of the gospel, could dispel the darkness of infidelity in which man was involved; no power but that of religion could resist his lawless passions.

The notions which the heathens had of a providence of immortality, and other truths of a similar nature, were, according to most authors, the effects of a tradition as old as the world, and derived from revelation; but its feeble light was almost extinguished among them, and hence arose their superstition and folly. It is true, the impiety of philosophers have too often been attributed to the sciences which they profess, but with as much reason as the immorality of some christians is attributed to the religion which they pretend to practice. Both evils originate in the corruption of the human heart.

The province of philosophy, rightly understood, is extensive and important. There is no employment more innocent—none more ingenious, and to those who have a taste for science, more amusing—none, in fine,

2. At what time of the year will Procyon rise with the sun, and also at what time will he rise when the sun rises, at Washington city ?

3. At what times of the year will Regulus rise with the sun, and also set when the sun rises at Petersburg in Russia ?

that lead to more important and useful discoveries, than guided by mathematical principles and the result of laborious experiments, patiently to investigate the phenomena and laws of nature, and apply them to those useful purposes for which they are adapted. Conducted by this science, we contemplate with pleasure and advantage those stupendous bodies that exist around us, guided by more stupendous but unerring laws—our faculties and conceptions are thereby expanded; and our mind has something to employ it that bears no small proportion to those noble powers which it possesses. We delight in the beauty and universal harmony which we perceive in these new scenes of wonder which every moment open to our view, from the smallest atom—from the blade of grass which we trample under foot, to the remotest world which we contemplate. Noble and extensive as the human intellect is, between these two extremes its shallow line of reason is soon exhausted. Here man, however, traces the omnipotence, the wisdom and goodness of a being infinitely his own superior ; from his works he feels a desire of being more acquainted with him, and this being, having destined man for himself, nourishes the desire. Thus far philosophy may conduct us, thus far its horizon extends, beyond which there is nothing but shadows and delusion. Religion opens here a brighter prospect, and like the sun which banishes the light of all other luminaries, will have no other light to conduct us but its own. Here futurity developes its extensive prospects, immortality, and not this short span of existence—eternity, not time, arrest our attention. Here omnipotence itself stoops to our assistance—infinite wisdom guides, and informs us of the true source of our condition and misery, and the goodness of this omnipotent being generously affords and points out the remedy. He exhibits the happiness of heaven as a reward for the good, and the torments of hell as punishments for the wicked—He enlightens and gives us a more intimate knowledge of his nature and of our duty towards him—calls himself by the endearing name of Father, and calls us by the loving and exalted title of children ; thus indicating our dignity, and the point in which alone our true nobility consists. These truths are established on divine authority, confirmed by miracles—by human testimony as far as human testimony could go, and by their own internal evidence and necessity.—From the nature of truth they are immutable, and not subject to presumptuous innovation or versatile fancy. They are truths not in the province of philosophy to teach—truths far more important, interesting, and noble, and a mind possessed of all other wisdom and knowledge under the sun, that views them not in this light, is, after all, neither intelligent or wise.

We have already seen (remark after the constellations) how civilized and learned men can act when withdrawn from the salutary restraint of religion ; or when guided by a false philosophy, and under the influence of unrestrained, licentious passions. Their deeds bear awful testimony to the feeble efforts of reason in establishing vain systems of fancied superiority, in opposition to that established by unerring wisdom, whose benignant influence argues its origin, and shews that no reformation is wanting to it, that no substitute can ever supply its place.

PROB. 33.

To find the time of the year when any given star rises or sets helia-cally, *or when a star becomes first visible after emerging from the sun's rays, in the morning before sun rising, or invisible in the evening, on account of its nearness to the sun.*

Rule. ELEVATE the pole to the latitude of the given place; screw the quadrant in the zenith; bring the quadrant to the eastern edge of the horizon, and move it until the star intersects it 12° below the horizon, if the star be of the first magnitude; 13° if the star be of the 2d mag.; 14° if the star be of the 3d, &c.* The point of the ecliptic cut by the quadrant, will shew the day of the month on the horizon, when the star rises heliacally. The given star being brought to the western edge of the horizon, and the quadrant of alt. being moved until intersected by the star as before; the point of the ecliptic then cut by the quadrant will give the day of the month when the star sets heliacally.

Example 1. At what time of the year does sirius, or the dog star, rise heliacally at Alexandria, in Egypt, in lat. 31° 11½′ N.; and when does it set heliacally at the same place?

Ans. On the 4th of August † and 23d of May.

* According to Ptolemy, stars of the first mag. are seen rising and setting when the sun is 12° below the horizon, stars of the 2d mag. when the sun is 13° below the horizon, &c. reckoning 1° for each mag. For the brighter a star is when above the horizon, the less the sun will be depressed before it becomes visible.

† The ancients reckoned the *dog days* from the heliacal rising of sirius, and their continuance to be about 40 days. Hesiod remarks, that the hottest season of the year (or the *dog days*) ended about 50 days after the summer solstice. In the note to prob. 31, it is shewn that the winter solstice, in the time of Hesiod, was in about 8° 26′ 12″ of aquarius, and consequently the summer solstice was in the same degree of leo. Now from the above it appears that sirius rises heliacally when in 12° of leo (corresponding to the 4th of Aug.) and as 59′ 8″ or 1° nearly corresponds to a day, sirius rose heliacally about 4 days after the summer solstice; and if the dog days continued 40 days, they ended about 44 days after the summer solstice. In our almanacs the dog days begin on the 3d of July, which is 12 days after the summer solstice, and end on the 11th of August, which is 51 days after the summer solstice; their continuance is therefore 39 days.

The dog days of the moderns have therefore no reference to sirius or the dog star, for as it varies in its rising and setting according to the latitude of places, it could therefore have no influence, or indicate no change in the temperature of the atmosphere. However as this star rose heliacally at the commencement of the hottest seasons in Egypt, Greece, &c. in the infancy of astronomy; and at a time when astrology referred almost every thing to the influence of the stars, it was natural for those people to imagine that the heat, &c. was the effect of this star's influence, &c. A few years ago the dog days were reckoned in our almanacs from the *cosmical* rising of procyon, viz. on the 30th of July, and continued to the 7th of September; but are now very properly altered, and made to depend on the summer solstice, and not on the variable rising of any particular star whatever.

The solution of the prob. by trigonometry, may be as follows; let S (see the fig. in the note to prob. 31, part 3) be the point of the ecliptic which rises with the star, and let ☉ be the place of the sun in the ecliptic, so that

2. At what time does Aldebaran set heliacally at New-York ?

3. At what time of the year does Arcturus rise heliacally at Washington city ?

4. At what time does Procyon rise and set heliacally at London ?

5. How many years will elapse from 1810, before Sirius will rise heliacally on Christmas day, at Cairo, in Egypt, allowing the precession of the equinoxes to be $50\frac{1}{4}$ seconds ?

PROB. 34.

The latitude of the place and day of the month being given, to find all those stars that rise and set achronically, cosmically, *and* heliacally.*

Rule. RECTIFY the globe for the latitude : then,

1. *For the achronical rising and setting.* Bring the sun's place in the ecliptic to the *western* edge of the horizon ; then all the stars along the eastern edge will rise achronically, and those along the western will set achronically.

2. *For the cosmical rising and setting.* Bring the sun's place to the *eastern* edge of the horizon ; then all the stars along that edge of the horizon will rise cosmically, and those along the western edge will set cosmically.

3. *For the heliacal rising and setting.* Screw the quadrant of alt. in the zenith, turn the globe eastward until the sun's place cuts the quadrant 12° below the horizon ; then all the stars of the 1st magnitude along the eastern edge of the horizon will rise heliacally ; and by continuing the motion of the globe eastward until the sun's place intersects the quadrant in 13°, 14°, 15°, &c. below the horizon, you will find all the stars of the 2d, 3d, 4th, &c. magnitudes, which rise heliacally on that day. By turning the globe

the arc ⊙R of the circle of depression may be 12° or 13°, &c. according as the star is of the 1st, 2d, &c. magnitude ; then in the right angled triangle SR⊙, RS⊙, the angle formed by the ecliptic and horizon (note to prob. 31) and the side R⊙ = 12° or 13°, &c. are given, and therefore the side S⊙ is given, which added to ♈S, gives the arc ♈⊙, and the point ⊙ or the sun's place when the star rises heliacally. The star's heliacal setting may be found in like manner.

* The principal use of this and the foregoing problems, is to illustrate several passages in the ancient writers, as Hesiod, Virgil, Columella, Ovid, Pliny, &c. These different risings and settings of the stars were called *poetical*, because principally used in the writings of the poets. The knowledge of these poetical risings and settings of the stars was much esteemed by the ancients, as it served to adjust the times set apart for their civil and religious duties, and to mark the seasons proper for the several parts of husbandry, the time of the overflowing of the Nile, &c. their knowledge of astronomy being too limited to adjust the length of the year, &c. The knowledge which the moderns have acquired of the motions of the heavenly bodies, renders such observations unnecessary, as an almanac answers every purpose of the husbandman.

This problem being the reverse of the three foregoing, the solution by trigonometry is performed in a similar manner, and is left for the learner's exercise.

westward in a similar manner, and bringing the quadrant to the western part of the horizon, you will find those stars that set heliacally.

Example 1. What stars rise and set achronically, cosmically, and heliacally at New-York on the 4th of December ?

Ans. For the *achronical*, &c. Aldebaran, &c. will rise achronically. Arcturus, &c. will set achronically.

For the *cosmical*, &c. β in Lupus will rise cosmically, &c. Antares will be near the eastern horizon. Algol in Perseus will set cosmically, &c. Betelgeux will be near the western horizon.

For the *heliacal*, &c. Arided in Cygnus will rise heliacally, β in Serpens will set heliacally, &c.

2. What stars rise and set achronically at Petersburg on the 10th of May ?

3. What stars rise and set cosmically at Washington city on the 5th of April ?

4. What stars rise and set heliacally at Philadelphia on the 4th of July ?

5. What stars rise and set achronically, cosmically, and heliacally at London on the 7th of October ?

PROB. 35.

The latitude of the place and the day of the month being given, to find what planets will be above the horizon, after sun setting.

Rule. Rectify the globe for the given latitudes, bring the sun's place to the western part of the horizon, or to 10 or 12 degrees below it ;* then all the planets whose places are in the hemisphere above the horizon, will be visible after sun setting, whose places may be found in an ephemeris for that day and month ; if the motion of the globe be continued westward until the sun's place comes within 10° or 12° of the eastern part of the horizon, all the planets that were above the horizon during this motion, will be visible, and fit for observation on that night.

Example 1. Were any of the planets visible at New-York when the sun had descended 10° below the horizon, on the 1st of January, 1811, their latitudes and longitudes being as follow :

	Latitudes.	Longitudes.		Latitudes.	Longitudes.
☿	2° 6′ S.	9s. 23° 20′	♃	0° 57′ S.	1s. 21° 45′
♀	4 12 N.	9 4 48	♄	1 17 N.	8 20 20
♂	1 29 N.	6 28 32	♅	0 21 N.	7 17 41

☽'s latitude at midnight 1° 0′ 58″ S. long. 0s. 11° 33′ 1″.

Ans. Mercury was near the horizon, Jupiter and the moon were visible.

* The planets are not visible until the sun is a certain number of degrees below the horizon, and these degrees are variable according to the apparent magnitudes and brightness of the planets. Mercury becomes visible when the sun's depression is about 10° ; Venus when the sun is 5° below the horizon ; Mars when the sun is at 11° 30′; Jupiter at 10°; Saturn at 11° ; and Herschel at 17½°.

2. What planets will be above the horizon at New-York on the 1st of December, 1812, and what planets will be visible during that night, or before the sun is within 10° of the eastern part of the horizon ; their latitudes and longitudes being as follow :

	Latitudes.	*Longitudes.*		*Latitudes.*	*Longitudes.*
☿	2° 22′ S.	8s. 26° 59′	♃	0° 32′ N.	4s. 9° 0′
♀	2 9 N.	6 28 21	♄	0 31 N.	9 6 52
♂	1 2 N.	6 24 45	♅	0 14 N.	7 24 40

☽'s lat. at midnight 4° 59′ 27″ N. long. 7s. 11° 24′ 27″.

PROB. 36.

The latitude, day, and hour being given, to find what planets will be visible, or above the horizon, at that hour.

Rule. RECTIFY the globe for the latitude ; bring the sun's place to the meridian, and set the index to 12 ; then turn the globe eastward or westward, according as the time is in the forenoon or afternoon, as many hours as the time is before or after 12 ; fix the globe in this position, and from the latitudes and longitudes of the planets found in the Nautical Almanac, see whether any of them be in the hemisphere which is above the horizon ; such planets will be visible.

Example 1. Were any of the planets whose places are found from example 1st of the foregoing prob. visible at 9 o'clock in the evening, at New-York, on the 1st of January, 1811 ?

Ans. Jupiter and the moon were visible.

2. Will any of the planets whose latitudes and longitudes are given in ex. 2, last prob. be visible at Philadelphia at 4 o'clock in the morning of the 1st of December, 1812 ?

PROB. 37.

Given the latitude of the place and day of the month, to find how long Venus rises before the sun when she is a morning star, and how long she sets after the sun when she is an evening star.*

Rule. RECTIFY the globe for the given latitude ; find the latitude and longitude of Venus in an ephemeris, and mark her place on the globe ; bring the sun's place for the given day to the brass meridian ; then if the place of Venus be to the eastward of the meridian, she is an evening star, or rises after the sun, but if to the westward, she is a morning star, or she rises before the sun.

When Venus is an evening star. Turn the globe westward until the sun's place comes to the western part of the horizon, the index will then shew the time of sun setting ; the motion of the globe being continued westward until Venus comes to the edge of

* Venus is a morning star from inferior to superior conjunction, and an evening star from superior to inferior conjunction.

the horizon, the index will shew when Venus sets; the difference between which and the time the sun sets, will shew how long Venus sets after the sun.

When Venus is a morning star. Find the time of sun rising (prob. 13. p. 2d.) and also the time that Venus rises (prob. 8, part 3d.) the difference between these times will shew how long Venus will rise before the sun.

Note. The same rule will answer to shew when any of the planets rises before the sun and sets after him; and how long.

Example 1. On the 1st of November, 1810, the longitude of Venus was 8s. 24° 30′ lat. 4° 7′ south; will she be a morning or an evening star? If she be a morning star, how long will she rise before the sun at New-York; if an evening star, how long will she shine after sun set?

Ans. Venus was an evening star; the sun set at 5h. 10′ or 10 minutes after 5, and Venus 7h. 20 min. or 20 min. after 7; hence Venus set 2h. 10′ after the sun.

2. On the 19th of November, 1812, Jupiter's longitude will be 4s. 9°, or 9° in Leo, latitude 29′ north; will Jupiter be a morning or an evening star? If a morning star, how long will he rise before the sun; if an evening star, how long will he shine after sun set?

3. On the 13th of October, 1812, the longitude of Venus will be 5s. 3° 40′, and latitude 36′ south; will Venus be then a morning or evening star? If a morning star, how long will she rise before the sun; if an evening star, how long will she shine after sun set?

4. On the 1st of April, 1812, Saturn's longitude will be 9s. 7° 50′, and latitude 52′ N. will he be a morning or an evening star, &c.

PROB. 38.

To find all those places on the earth to which the moon will be nearly vertical on any given day.

Rule. TAKE the moon's latitude and longitude for the given day from an ephemeris, and mark the place corresponding to them, on the globe (by prob. 4.) bring the place to the brass meridian, and observe the degree over it; then all those places having the same or nearly the same latitude, will have the moon vertical when on their respective meridians.

Or, Those places whose latitudes are equal to, and of the same name with her declination for the given time (found in the Nautical Almanac) will have the moon successively vertical on the given day.

Example 1. On the 20th of December, 1810, the moon's longitude at midnight was 6s. 20°, and her latitude 1° 5′ N.; required the places to which she will be nearly vertical that day?

Ans. From the moon's latitude and longitude, her declination is found nearly 7° south ; hence the places are the Sunda Isles, Solomon's Isles, Sana in Peru, Olinda in Brazil, Congo, and Lower Guinea in Africa, &c.

Note 1. When the declination is given, the prob. may be performed by the terrestrial globe alone, being the same as prob. 10, part 2.

Note 2. The places where any of the planets will be vertical, are found in the same manner.

2. On the 1st of November, 1812, the moon's longitude at midnight will be 6s. 14° 54′ 25″, and her latitude 3° 46′ 32″ ; required the places to which she will be nearly vertical when on their meridian that day ?

3. On the 9th of November, 1811, the moon's declination at midnight will be 8° N. ; required where she will be vertical that day ?

4. On the 7th of December, 1811, Jupiter's longitude will be 3s. 3° 20′, and latitude 15′ south ; to what places will he be vertical that day ?

5. On the 21st of October, 1813, the declination of Herschel will be 19° 11′ south ; where will he be vertical on that day ?

6. Required those places where the moon will be vertical when she has her greatest north declination, and also her greatest declination south ?

7. Required those places to which Venus will be vertical when she has her greatest north declination, and required that declination ?

PROB. 39.

To find the time of the moon's southing, or coming to the meridian at any given place, on any given day of the month.

Rule. Elevate the pole to the given latitude ; find the moon's latitude and longitude, or her right ascension and declination from the Nautical Almanac or a good ephemeris, and mark her place on the globe (by prob. 2 and 4) bring the sun's place to the brass meridian, and set the hour index to 12 ; turn the globe westward until the moon's place comes to the meridian, and the hours passed over by the index, will shew the time from noon when the moon will be on the meridian.

OR WITHOUT THE GLOBE.

Find the moon's age by the note to def. 80, which multiply by .81* ($\frac{81}{100}$) and the product will be the hours and decimal parts of an hour, which multiply by 60 for minutes.

OR CORRECTLY, THUS,

Take the difference between the sun and moon's right ascension in 24 hours ; (found by the Nautical Almanac) then say as 24 h.

* The synodic revolution of the moon being according to La Place, 29d. 12h. 44′ 2″ 8, or nearly 29½ days, we have as 29½d. : 24h. :: 1d. : .81h. nearly. Hence the reason of the rule is manifest.

less this difference to 24 h. so is the moon's right ascension at noon diminished by the sun's, to the time of the moon's southing.

Note 1. When the sun's right ascension is greater than the moon's, 24 hours must be added to that of the moon before you subtract.

Example 1. At what time on the 10th of March, 1811, did the moon pass over the meridian of Greenwich,* the moon's right ascension being 172° 10′ 48″, and her declination 2° 55′ N. at noon?

Ans. By the globe the moon came to the meridian at half an hour after 12 at night.

By the note to def. 80. The moon's age is 16 days; hence 16 × .81 = 12.96 hours = 12h. 57m. by this method.

By using the Nautical Almanac—

Sun's rt. as. at noon, 10th March,	= 23h. 19′ 46″ 3
Ditto 11th do.	= 23 23 27 0
Increase of motion in 24 hours	0 3 40 7
Moon's rt. asc. at noon, 10th March,	= 172° 10′ 48″
Ditto 11th do.	= 183 41 27
Increase in 24 hours	11 30 39

and 11° 30′ 39″ = 47′ 2″ 6 (note to prob. 6, part 2) hence 46′ 2″ 6 — 3′ 40″ 7 = 42′ 22″ nearly, the moon's motion in 24 hours exceeds the sun's. Moon's right asc. 172° 10′ 48″ = 11h. 28′ 43″ 2, to which 24h. being added, we have 35h. 28′ 43″ 2, from which the sun's rt. asc. 23h. 19′ 46″ 3, being taken, leaves 12h. 8′ 57″ nearly. Now 24h. — 42′ 22″ = 23h. 17′ 38″ : 24h. :: 12h. 8′ 57″ : 12h, 31m. the true time of the moon's passage over the meridian at night, agreeing with the Nautical Almanac.

2. At what hour on the 5th of February, 1810, did the moon pass over the meridian of Greenwich, the moon's right ascension being 161° 9′, and declination 4° 48′ north?

3. At what hour on the 31st of December, 1811, will the moon pass over the meridian of Greenwich, her declination at noon being 17° 7′ N and right ascension 120° 34′ 6″? (Her right ascension the following day will be 133° 23′ 41″; that of the sun's in time on the 31st 18h. 39′ 7″ 5, and on the 1st of January, 1812, 18h. 43′ 32″ 8.)

4. On the 22d of August, 1812, at what hour will the moon pass the meridian of Greenwich; her right ascension being 329° 37′ 48″, and declination 12° 13′ south?

Note 2. To find the time of the moon's southing or coming to the meridian of any place, different from that of Greenwich. See the note to ex. 8, prob. 18th, part 3d.

* The time of the moon's transit in Greenwich is found calculated in page 7 of the month in the Nautical Almanac.

PROB 40.

The latitude of the place, day of the month, and time of high water at the full and change of the moon being given, to find the time of high water on the given day.

Rule. FIND the moon's southing by the foregoing prob. (or the note to prob. 18th, part 3d.) to which add the time of high water at the full and change of the moon,* and the sum will give the time of high water in the afternoon. If the sum exceed 12 hours, subtract 12 h. 24 min. from it, and the remainder will give the time of high water in the morning ; but if the sum exceed 24 h. take 24 h. 48 min. from it, and the remainder will give the time of high water in the afternoon.

Or, Find the moon's age, and opposite to it in the following table (1st) take out the time in the right hand column corresponding to it, to which add the time of high water at the full and change of the moon, and the sum will shew the time of high water in the afternoon. If the sum exceed 12 h. or 24h. proceed as above.

OR THUS,

Find the moon's horizontal parallax,† and the time of her coming to the meridian, for the given time and place ; from the following table (2d) take out the correction corresponding to this time, and apply it, as the table directs, to the result ; add the time of high water at the full and change of the moon (found as directed above) and the sum will be the time of high water in the afternoon. If the sum exceed 12 or 24 hours, proceed as directed in the first rule.

Example 1. Required the time of high water at London bridge on the 8th of June, 1811. The moon's right ascension at that time being 277° 21′ 2″, and her declination 13° 26′ south ?

Ans. By the globe. The moon came to the meridian at 14 hours, or in the morning at 2h.

Time of high water at the full and change at London 3

Time of high water in the morning 5 hours.

By rule 2d. The moon's age is 18, the time answering to which

		h.	′
in table 1, is		13	54
Time of high water at full and change at London,		3	0
	Sum	16	54
	Subtract	12	24
Time of high water in the morning		4	30

* This is given in the table of the latitude and longitude of places found at the end of this work.

† The moon's horizontal parallax for noon and midnight, and the time of her coming to the meridian of Greenwich, is found in page 7 of the month in the Nautical Almanac, and the latter time may be reduced to any other meridian by the note to prob. 18. The moon's parallax may be reduced to any other meridian by taking proportional parts, and allowing 15° for every hour.

By the Nautical Almanac. The moon will come to the meridian at 14 hours, her horizontal parallax at midnight will be 59′ 35″; hence for 14 hours and parallax 60′, the corresponding correction from tab. 2 to be subtracted, is

	h.	m.
	0h.	13′
From	14	0
Difference	13	47
Time of high water at full and change,	3	0
	16	47
Subtract	12	24
Time of high water in the morning,	4	23

TABLE 1st.

Corr. to be added to the time of high water at full and change.

☽'s age	correct.
0D.	0h 0′
1	0 36
2	1 11
3	1 46
4	2 21
5	3 1
6	3 44
7	4 37
8	5 40
9	6 68
10	8 14
11	9 17
12	10 9
13	10 53
14	11 33
15	12 8
16	12 45
17	13 19
18	13 54
19	14 30
20	15 11
21	15 56
22	16 51
23	18 0
24	19 18
25	20 31
26	21 31
27	22 21
28	23 3
29	23 42
29½	24 0

TABLE 2d.

Correction depending on the angular dist. between the sun and moon, and the dist. of the ☽ from the earth.

Apparent time of ☽'s transit.	*Moon's horizontal parallax.* 54′	55′	56′	57′	58′	59′	60′	61′	*Apparent time of ☽'s transit.*
	add	*add*	*add*	*add*	*add*	*add*	*add*	*add*	
0h	19′	20′	21′	22′	23	25	26′	27′	12h
0½	12	13	13	14	15	16	17	17	12½
1	5	5	5	6	6	6	7	7	13
	sub	*sub*	*sub*	*sub*	*sub*	*sub*	*sub*	*sub*	
1½	2	3	3	3	3	3	3	3	13½
2	10	10	11	11	12	13	13	14	14
2½	17	18	19	20	21	22	23	24	14½
3	23	25	26	27	29	30	32	34	15
3½	29	31	33	34	36	38	40	42	15½
4	34	36	38	40	43	45	47	49	16
4½	38	40	42	45	47	50	52	55	16½
5	40	42	45	47	50	52	55	58	17
5½	39	41	44	46	48	51	54	56	17½
6	35	37	39	41	43	45	47	50	18
6½	25	27	28	30	31	33	35	36	18½
7	11	12	12	13	14	15	15	16	19
	add	*add*	*add*	*add*	*add*	*add*	*add*	*add*	
7½	6	6	6	7	7	7	8	8	19½
8	21	22	23	25	26	27	29	30	20
8½	32	34	36	38	40	42	44	46	20½
9	38	40	42	45	47	50	52	55	21
9½	40	42	45	47	50	52	55	58	21½
10	39	41	43	46	48	51	53	56	22
10½	36	38	40	42	44	47	49	52	22½
11	31	33	35	37	39	41	43	45	23
11½	25	27	28	30	31	33	35	37	23½
12	19	20	21	22	23	25	26	27	24
Apparent time of ☽'s transit.	54′	55′	56′	57′	58′	59′	60′	61′ *Moon's horizontal parallax.*	*Apparent time of ☽'s transit.*

2. Required the time of high water at London on the 10th of May, 1812, the moon's right ascension being 41° 50′ 15″, and declination 11° 16′ north?

3. Required the time of high water at New-York on the 4th of July, 1811, the right ascension of the moon at Greenwich being 256° 39′ 6″, and her declination 17° 49′ south. (The moon passes the meridian of Greenwich on the 4th of July, at 10h. 41′, and on the 5th at 11h. 39′. Her horizontal parallax being nearly 59′.)

4. Required the time of high water at Boston on the 1st of October, 1813, the right ascension of the moon at Greenwich being 264° 26′ 59″, and declination 19° 50′ south; also her passage over the meridian of Greenwich on the 1st of October, being at 5h. 19′, and on the 2d. at 6h. 8′, and her hor. parallax at noon on the 1st of October, at Greenwich being 54′ 17″, and at midnight 44′ 26″?

PROB. 41.

*To describe the apparent path of any given planet or comet, among the fixed stars.**

Rule. FIND the planet's geocentric latitude from an ephemeris or from page 4th of the month in the Naut. Alm. or if a comet,

In table 1st if 1½′ be added for each hour to the daily correction, the sum will be nearly the correction for that time.

In table 2d the distance of the sun from the earth might be also allowed for, but being so small, it is here neglected. See McKay's Treatise on Navigation, tables 5th and 6th. The principles on which these tables are constructed, will be given in part 4th, article tides. See also chap. 5, b. 1, of McKay's Navigation.

* To perform this prob. on a plane or on paper. Draw a straight line to represent the ecliptic, and divide it into any number of convenient equal parts. At the ends of this line draw perpendiculars, and on each of them set off eight or ten of those equal parts northward and southward of the ecliptic; through every one of these parts draw straight lines parallel to the ecliptic, and others perpendicular to these, through the divisions on the ecliptic, these lines will represent the zodiac. Then mark the geocentric lat. and long. (as above) on this zodiac, beginning at the right hand of the ecliptic line, and proceeding towards the left, as the stars appear in a contrary order in the heavens, to what they appear on the surface of the globe; because in the heavens we see the concave part, and are supposed in the centre of the sphere, but on the globe we see the convex, and are supposed to be situated without the sphere of the stars, &c. To describe the principal fixed stars and the constellations near which the planet or comet passes, the sides of the map or the degrees of lat. and also the degrees, &c. on the ecliptic, must be extended so as to take in their latitudes and longitudes. Their lat. and long. must be set off, in a similar manner, from the right to the left. In this manner you will have a complete representation of the heavens with the positions of the several stars, constellations, &c. as they appear to a spectator on the earth. Hence this manner of delineating the stars is useful in learning their places, &c.

The places of the stars may be laid down in like manner, from their right ascensions and declinations, by drawing a portion of the equinoctial in place of the ecliptic, &c.

find its place by observation or from tables constructed for that purpose ; mark those places on the globe for every month, or for several days in each month ; these marks connected will be the path required.

Example. Describe the path of the planet Jupiter for the year 1812, the latitudes and longitudes being as follow :

		Longitudes.			*Latitudes.*		
Jan.	1st.	3s.	0°	1′	0°	11′	S.
—	19th.	2	27	53	0	8	S.
Feb.	1st.	2	26	48	0	6	S.
March	25th.	2	27	58	0	1	N.
Apr.	1st.	2	28	44	0	1	N.
—	13th.	3	0	20	0	3	N.
—	25th.	3	2	13	0	4	N.
May	1st.	3	3	15	0	4	N.
—	13th.	3	5	28	0	6	N.
—	25th.	3	7	52	0	7	N.
June	1st.	3	9	20	0	7	N.
—	13th.	3	11	54	0	8	N.
—	25th.	3	14	34	0	9	N.
July	7th.	3	17	15	0	10	N.
—	19th.	3	19	56	0	12	N.
Aug.	1st.	3	22	50	0	13	N.
—	13th.	3	25	26	0	14	N,
—	25th.	3	27	56	0	16	N.
Sept.	7th.	4	0	28	0	17	N.
—	25th.	4	3	38	0	20	N.
Oct.	7th.	4	5	26	0	22	N.
—	25th.	4	7	32	0	25	N.
Nov.	7th.	4	8	32	0	27	N.
—	25th.	4	9	4	0	31	N.
Dec.	7th.	4	8	50	0	33	N.
—	25th.	4	7	37	0	37	N.

Ans. January 1st. Jupiter will be near η in Gemini. On the 19th near a small star marked h. On March 25th. its motion will be retrograde. On the 13th of April its motion will be again forward, and so continue to the 25th of November, when its motion will again become retrograde. On July 7th. it will be near δ in Gemini. On Oct. 7th. it will be near δ or Asellus Australis in Cancer, &c.

PROB. 42.

*To illustrate the precession of the equinoxes.**

Rule. Elevate the north pole 90° above the horizon, the equinoctial will then coincide with the horizon ; bring the pole of

* The sun returning to the equinox every year, before it returns to the same point in the heavens, shews that the equinoctial points have a retrograde motion from *east* to *west.* The cause of this motion was unknown until *Newton* (prob. 39, b. 3. of his principia) had proved, that it is produced

the ecliptic (or that point on the globe where the circular lines meet) to coincide with that part of the brass meridian which is numbered from the pole towards the equinoctial, and mark the point over it; consider this mark as the pole of the world, and let the equinoctial be considered as the ecliptic, and the ecliptic as the equinoctial; then turn the globe gradually on its axis from *east* to *west*, and the equinoctial points will move the same way, and will describe one revolution round the globe, in the same time that the pole of the world (here represented by the pole of the ecliptic) will describe a circle round the pole of the ecliptic (here represented by the extremity of the earth's axis.) Now as the equinoctial points move *backwards*, or *from east* to *west*, at the rate of $50\frac{1}{4}''$ in a year, or 1° in 71 64 years, this circle will be described in 25791 years. (See the note to def. 74.) In this time the pole of the heavens will also describe a circle, the semidiameter

by the combined actions of the sun and moon on the protuberant matter about the earth's equator; and that this protuberant matter was caused by the revolution of the earth on its axis, which gives the earth the figure of an *oblate spheriod*, flat towards the poles, and elevated towards the equator. (Prob. 19. b. 3. prin. see *Fenn's* System of the Physical World, pa. 61. see also the demonstration of this curious phenomenon in prob. 26. sect. 3. Emerson's Fluxions.) Thus (as La Place remarks) every part of nature is linked together, and its general laws connect phenomena with each other, which, in appearance, have not the most remote analogy.

Hipparchus was the first (as *Ptolemy* informs us ch. 1 b. 7. of his Almagest) who observed this motion of the stars of the zodiac, by comparing his own observations with those which *Aristillus* and *Timocharis* had made in Alexandria about 155 or 160 years before; and *Ptolemy*, by comparing his observations with those of *Hipparchus*, found that all the stars had a similar motion, and as well as *Hipparchus* estimates it at 1° in 100 years. (See his Almagest, ch. 2d and 3d.) In the year 128, before *J. C. Hipparchus* found the longitude of *Spica Virginis* to be 5s. 24°, and in 1750 its long. was found 6s. 20° 21′, the diff. of which is 26° 21′. In the same year he found the longitude of *Cor Leonis* to be 3s. 29° 50′, and in 1750 it was 4s. 26° 21′, the difference of which is 26° 31′. The mean of these two gives 26° 26′ for the increase of long. in 1878 years, or 50″ 40‴ yearly for the precession. *Albategnius*, from the places of *regulus*, observed by *Menelaus* and himself at the distance of 785 years, makes the precession 1° in 66 years. (Chap. 52 of *his book of the knowledge of the stars*.) By comparing the observations of *Albategnius* in the year 878 with those made in 1738, the precession is found to be 51″ 9‴. *Ulugh Beigh* (in the preface to his tables) makes the precession 1° in 70 solar years. *Tycho Brahe* (in his *Progymn*, b. 1.) makes it 1° 25′ in 100 years, or 51″ yearly. (*Tycho's tables*, as also the *Rudolphine*, are given at the end of Nicholas Mercator's Astronomy, latin ed. 1676.) From a comparison of 15 observations of *Tycho* with as many made by *De la Caille*, the precession is found to be 50″ 20‴. *Copernicus*, who considered this motion unequal, makes the mean equal 1° 23′ 40″ 12‴ in 100 years, and *Ricciolus* 1° 23′ 20″ in the same time.—*Street* in his *Astronmia Carolina* makes it 1° 20′. *Bulialdus* in his *Astronomia Philolaica* 1° 24′ 54″, and *Hevelius* 1° 24′ 46″ 50‴, in 100 years. The *first star* of Aries, marked β was at the beginning of the year 1701, according to *Hevelius*, in 29° 0′ 58″ of Aries. *La Land*, from the observations of *De La Caille*, compared with those in *Flamstead's* catalogue, makes the secular precession 1° 23′ 45″, or 50″ 25 in a year.

of which is equal to the obliquity of the ecliptic or 23° 28′, and hence it varies its position a little every year. If from the pole of the world or the above mark on the brass meridian, the complement of the lat. be reckoned upwards (which for New-York, for example, is 49° 17′) and the point where the reckoning ends be marked, this mark will be exactly over the lat. New-York will therefore be under 64° 11′ on the brass meridian, reckoning from the southern point of the horizon or from the equator. And when 12895½ years, being half one entire revolution of the equinoxes, are completed (which may be known by turning the globe half round, or until the pole comes under the opposite degree on the meridian or Aries from the western to the eastern point of the horizon) that point of the heavens which is now 2° 21′ north of the zenith of New-York, will be the *north pole*, as may be seen by the pole's distances from the mark over 64° 11′ on the meridian. This is on supposition that the obliquity of the ecliptic, &c. will not alter this period, &c.

In the same manner it will be found, that in all those places 2° 21′ north of New-York, or in lat. 43° 4′, the north pole will, in the same time be in their zenith, and all those places still farther north, will have the pole in their zenith before this period or before 12895½ years would be expired.

Hence likewise we see that the pole is advancing towards the present equinoctial, &c.

For a further illustration of this prob. see Keil's Astronomy, lect. 8, or La Place's Astronomy, b. 4, ch. 13.

Besides this motion of the equinoctional points, or the pole, there is another called the *nutation*, which depends on similar principles. If the pole of the equator be supposed to move upon the circumference of a small ellipsis, tangent to the celestial sphere, whose centre, which may be regarded as the mean pole of the equator, describes uniformly every year (154″ 63) 50″ 1 of the parallel of the ecliptic on which it is situated; the greater axis of this ellipses, always tangent to the circle of latitude, and in the plane of this great circle, will, according to *La Place*, subtend an angle of about (62″ 2) 20″ 15, and the lesser axis an angle of (46″ 3) 15″. La Place determines the situation of the real pole of the equator upon this ellipsis thus ; let a small circle on the plane of this ellipsis be supposed concentric with it, and its diameter equal to the greater axis of the ellipsis ; let a radius of this circle move uniformly with a retrograde motion, so as to coincide with that half of the greater axis nearest to the ecliptic, every time that the mean ascending node of the lunar orbit coincides with the vernal equinox. From the extremity of this moveable radius, let a perpendicular fall upon the greater axis of the ellipsis ; the point where this perp. cuts the circumference of the ellipsis, will be the place of the real pole of the equator ; this motion of the pole is called its *nutation*. Dr. Maskelyne, by examining the observations of *Bradley*, the first discoverer of this nutation, makes the quantity (62″ 2) 20″ 15 (=58″6) = 18″ 98, which differs but (3″ 6) 1″ 16 from the result found by the tides. This phenomenon being better determined by the tides, induced La Place to take (58″ 2) 18″ 85 as more correct. See the laws of these motions in ch. 13, b. 4, of his Astronomy.

The phenomena of the *precession* and *nutation* throws great light on the figure of the earth, supposed elliptic, as its ellipticity or compression

Remark. The period of the revolution of the equinoctial points, or 25791 years, was called by the ancients a *platonic* year, and they imagined that when this period would be completed, the world was to begin anew, and the same series of things return over again. This idle notion, however, had no other foundation than in their imagination. Whenever we find this sportive faculty permitted to wander unrestrained, its excursions are, in general, more characteristic of extravagant fictions, than the imaginary exploits of a Don Quixote and his squire Sancho; though many of our *prime* philosophers have always shewed an unaccountable disposition to give into extravagancies not warranted by reason or common sense, if they only served to support some favourite opinion or fancied system. In this case truth must always suffer, and become the victim of folly and prejudice. Hence as McLaurin remarks, "False schemes of natural philosophy may lead to atheism, or suggest opinions concerning the deity and the universe, of most dangerous consequence to mankind; and have been frequently employed to support such opinions." (View of Newton's Philosophy. See an account of the Indians or Brahman's division of time, &c. in Bartolomeo's voyage to the East-Indies, ch. 9.)

appears from them not to exeed $\frac{1}{305}$; Bouguer making it $\frac{1}{183}$. Delambre, table 94, makes it $\frac{1}{300}$; most astronomers before him making it $\frac{1}{230}$. And although the *meniscus*, or protuberance at the equator, was supposed solid, &c. in these investigations, the fluidity of the ocean will not change the conclusions, as La Place proves in this remarkable theorem: "*Whatever be the law of the depth of the ocean, and whatever the figure of the spheroid which it covers, the phenomena of the precession and nutation will be the same as if the ocean formed a solid mass with the spheroid.*" The mean obliquity of the ecliptic would be constant, if only the sun and moon acted on the earth, but from the action of the other planets, this is subject to constant variation, and the same cause produces in the equinoxes a direct annual motion of (0″5707) 0″1849. From the actions of the sun and moon alone, the precession, according to La Place, would be (155″20) 50″2848, which diminished by the above quantity gives 50″1 nearly.

The variation in the motion of the equinoxes changes the duration of the *tropical year*, the latter diminishing as the former augments; so that at present the actual length of the year is (12″) 3″888 less than in the time of Hipparchus. But this variation has its limits, and La Place finds that they would be about (500″) 2′ 42″ (a) but that the action of the sun and moon reduces it to (120″) 38″88. (See Emerson's Centripetal Forces, prop. 34, sect. 3.) The length of the tropical year at present is 365d. 5h. 48′ 48″, from which the length of a *sidereal year* may be found by this proportion; (taking the precession of the equinoxes 50″25) 360° — 50″25 : 360° :: 365d. 5h. 48′ 48″ : 365d. 6h. 9′ 11½″ the length of a sidereal year.

The mean length of the day, according to this theory, may be supposed constant, as La Place has shewn in the chapter above quoted, and remarks that this is an important result for astronomy, as it is the measure of time, and of the revolutions of the heavenly bodies.

For more information on this curious subject, consult *Simpson's Miscellaneous Tracts;* D'Alambert's *Recherches sur la Precession des Equinoxes*; Euler, *Mem. de Berlin, tom.* 5, 1749; La Place's *Celestial Mechanics*, &c.

(a) *The translator of La Place makes the above* 500″ = 27′.

ELEMENTS

OF

ASTRONOMY.

PART IV.

OF THE SOLAR SYSTEM.

Having in the preceding part of this work given the learner a comprehensive view of the most useful and interesting parts of practical astronomy, we shall in this fourth part endeavour to give him a general idea of those bodies that exist around us, and of the admirable laws by which they are connected and governed.

The view of the heavens—the innumerable bodies that, from our distant habitation, appear on a serene night, like so many lamps that ornament the firmament—the fixed appearance of some, while others seem destined to no permanent station, must at all times have attracted the attention of mankind, and engaged the most learned in the investigation of their nature.

At first sight their confused, and sometimes insulated appearance, exhibit no traces of order or regularity; but on examination we are astonished to find, amid such magnificent profusion and awful grandeur, such harmony, such order and connexion; and must conclude that a fabric so immense, and at the same time so well proportioned in all its parts—so complicated, and yet directed by laws so simple, evidently points out the wisdom of an architect as far above our limited conceptions, as that power necessary to call the universe into existence.

Who has ever considered without emotion those operations and laws which combine and regulate the distant parts of the world, and so admirably display the greatness, the watchful providence of that intelligent Being who presides over the magnificent scene. What sublime and awful grandeur does this august temple of the Deity exhibit. Thousands of worlds obey his voice and observe his laws. Here the mind is struck with man's little schemes of insignificant and fleeting greatness, when it sees that kingdoms, nations, and the whole earth itself, dwindles into an atom, when compared with the majestic greatness of the heavens. While, on this extensive scale, we contemplate nature in all her perfections, while we behold every part so exactly corresponding to its end, the moral disorder which we witness in man, from an abuse of the freedom and the reason which he possesses, strikes us more forcibly, points out our weakness, humbles our pride, and force us to have recourse to and rely on that beneficent Being who sometimes permits partial evils for greater good. Such extensive views of the immense grandeur and magnificence of creation call the mind from all its little cares and vanities to higher destinies and nobler views, and brings us more and more acquainted with that Being, whom, from his works alone, we learn to venerate and obey.

If then (as De Feller remarks, Philosophical Catechism, vol. 1, pa. 138) " If the thought of God's existence and of our own immortality enlivens all nature ; if without that thought all would be drowned in silence, and in the disconsolate prospect of death and eternal naught, it is chiefly in the sublime regions of the stars that it displays that enlivening power. Bright and powerful luminaries, it is that thought that heightens and dignifies the lustre you shed, it is by that thought that you dispel the horror of the midnight hour, that you adorn the heavens and charm the earth. While you fix my eyes by the radiance and purity of your beams, the liveliness of my faith, the sweetness of my hope, excite in my soul the most delightful emotions !. ..Cheerless philosophy, where you see nothing but sparks, scattered at random over the vast expanse, I see, I hear, the most eloquent, the most indefatigable panegyrists of the Deity." The Atheists' books, says the same author, with all that philosophy can do to render them fashionable, are cold, melancholy performances, they rise only when they borrow the language that confutes their errors. It has been attempted (says a modern writer) to represent the Atheist as a sage, *with whom upon the extinction of faith, reason is become omnipotent* (New Philosophical Thoughts) would it not be better to define him a man over whom reason and faith have lost all their power ? Will it not even be too much for him to be allowed a place in the class of human beings ? Like us, I know, he raises his looks towards heaven ; but like the brute, whose eyes are fixed on the earth, he can discover no connection between it and the Supreme Being.— Heaven has given him that sublime countenance that bespeaks intellect, and perhaps he was made, like man, to possess it to a certain degree, but like the brute, he no where can perceive any traces of it. With the faculty of thinking, he received at his birth, privileges far superior to that of instinct : but is it not the animal senses alone he takes for his guides ? Like man, in short, he enjoys the gift of speech ; but like the brutes, he either never examined nature, or nature never answered him. The sun may go on illuminating the world, and rolling from east to west his refulgent orb : to the brightness of day may succeed the sable majesty of night, ushered in by thousands of radiant stars, hailing with jubilee the greatness of their maker ; the Atheist is deaf to the heavenly concert, though sounding from end to end of the glorious march : millions of living creatures may people our woods and lawns, may soar into the region of the air, may breathe in the ocean's deep gulfs, and perpetuate their various species from age to age : never will they be able to raise his thoughts to the author of life. The constant and regular return of the winter frosts and vernal blooms, of glowing summers and mild maturing autumns, announce to mankind, the God of wisdom and providence ; but order and regularity tell him no more than chaos and confusion — Earth may renew her clothing ; she may dress herself out in her richest attire ; he will gather her fruits, and ascribe them to chance. Insensible in the midst of the great theatre of the universe ; he never will hear that potent voice, that cries out so distinctly : *It is God that made us. (Ipse fecit nos, et non ipsi nos.)* Is he then that being that was destined to contemplate nature ? Is he, while his heart is begirt with ice, and his mind palsied with the apathy of stupidness, is he, I say, qualified to judge of the order, the variety, the riches of all kinds she displays to his view ; and by the beauty, the magnificence, and the aggregate of the work, to raise his thoughts to the power and wisdom of its author ?'

The deeper we penetrate into the works of this great Author, the contemplation of which forms the delight of thinking beings, the more our thoughts are elevated above this little spot on which we are at present confined—the more we despise the trifling pursuits of mortals—the more we pride ourselves in our relation with him, whose goodness has performed so much for us—who gave us such a distinguished place in the scale of beings—endowed us with faculties capable of knowing him, and contemplating such stupendous prodigies of his immense power. Hence we are led to form this pleasing deduction, that minds capable of such deep researches, possessed of qualities so noble and extensive, could not be the ephemeral productions or victims of a day, like the vegetable that perishes, but destined to a nearer approach, to a more extensive knowledge of the great Author of the universe, when time throws off those shackles that retain us in our banishment, and our immortal part is called to a country and state of existence worthy its nature, and the wisdom and goodness of its Creator.

Hence the knowledge of this august and amiable Being is the only science worthy the noble powers and faculties of the soul of man—his service the only service which is truly dignified and honourable—and the enjoyment of him the only possession that can satisfy the boundless desires and the noble and aspiring passions implanted in man ; a strong argument that he is destined, one day, to enjoy that happiness, when this transitory and imperfect view of nature and every thing that occupies and deceives us here, passes away, and our immortal country and state of existence, presents to our view its more sublime and extensive prospects.— These are the pleasing, the important deductions we should draw from viewing the majesty of the heavens—the noble sentiments that should guide and influence our conduct—sentiments which suit well the character of a christian, who knows the indispensible obligations he is under of serving God, and the greatness of the glory which accrues from his service.

Convinced of and impressed with these truths, the contemplation of the works of creation will be no less delightful than useful in conducting us to a knowledge of that infinite Being whom we are so apt to forget. A knowledge which draws the line of distinction between rational beings and the brute creation.

CHAP. I.

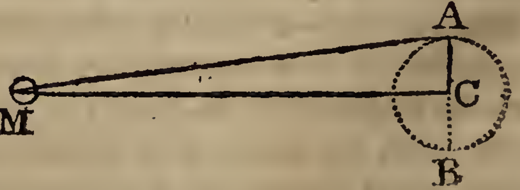
OF THE SUN.

WITH a liberty common to all mankind, the astronomer, like a skilful architect, who, to examine the workmanship of a building, inspects the different apartments, and takes his point of view from different stations; selects his position in various parts of the universe, and there considers the phenomena. Among the innumerable bodies that appear in the heavens, the sun claims the chief place in our system and our first consideration.

To give the young student an idea of the magnitude of this immense body, we find by observation, that if its centre coincided with the centre of the earth, it would fill the whole orbit of the moon, and its surface extend as far again.* The sun is situated near the centre of the orbits of all the planets, and revolves on its axis in 25 days, 14 hours, 8 minutes. This revolution is deter-

* The diameter of the sun, as seen from the earth, appears at a medium under an angle of 32′ or 1920″, and the parallax of the sun or the angle under which the earth's semidiameter appears as seen from the sun, is found from the latest transit of Venus, equal to 8″ 8, and hence the whole diameter would appear under an angle of 17″ 6. Therefore the proportion between the sun's diameter and that of the earth, is as 1920″ to 17″ 6 or $\frac{1920}{17.6} = 109$ nearly; and as the magnitudes of spherical bodies are as the cubes of their diameters (Euclid prop. 18. b. 12. or Emerson's Geom. prop. 18. b. 7.) $109^3 = 1295029$, the number of times the sun is greater than the earth. Again, $109 \times 7911 = 862299$, the number of miles in the sun's diameter, that of the earth being 7911 nearly. (See note to def. 8.)

Now to find the distance of the moon. Let MC represent its mean distance from the centre of the earth C, AC, or CB the earth's semidiameter, and the angle AMC the moon's horizontal parallax, which at the mean radius of the earth, according to La Land, is 57′ 1″. Hence,



As sine 57′ 1″	- -	8.2197069
To Radius	- -	10.
So is AC 3956	- -	3.5972563
To CM 238533	- -	5.3775494

The diameter of the sun being 862299 miles, and the distance of the moon from the earth only 238533 miles, it follows that the sun's diameter is nearly 4 times this distance, and hence the truth of the above assertion.

mined from the motion of certain spots * on its surface, which first appear on the eastern extremity, then advance towards the middle, and at length to the western edge, where they disappear. When they have been absent nearly as long as they were visible, they appear again as at first, finishing their entire circuit in 27 days, 12 hours, 20 minutes. Hence to an observer placed in the

* Black spots of an irregular form are observed at the surface of the sun, whose number, magnitude, and position, are very variable.—They are often very numerous, and of considerable extent; sometimes, though rarely, the sun has appeared pure and without spots for several years together.

It is doubtful by whom these spots were first discovered. *Scheiner*, a German Jesuit and professor of Mathematics in Ingolstadt, *Galileo* in Rome, *David Fabricius*, and *Harriot*, each seemed to have discovered them about the year 1611, and as telescopes were then in use, it is probable that each might make the discovery independent of the other. These spots are supposed to adhere to the sun's body, and hence its rotation has been discovered.

M. Cassini determined the time of rotation from observing the time in which a spot returns to the same situation on the sun's disk, or to the circle of latitude passing through the earth. This time from a great number of observations, he determines to be 27d. 12h. 20′, and the mean motion of the earth in that time being 27° 7′ 8″, we have this proportion; 360° + 27° 7′ 8″ : 360° :: 27d. 12h. 20′ : 25d. 14h. 8′ the time of the sun's revolution on its axis, the motion of the spots being supposed uniform. Their motion is from west to east.

When the earth is in the nodes of the sun's equator, and consequently in its plane, the spots appear to describe straight lines: this happens about the beginning of June and December. As the earth recedes from the nodes, the path of a spot grows more and more elliptical until the earth is 90° from the nodes, which takes place about the beginning of September and March, at which time the ellipsis has its lesser axis the greatest, and is then to the greater axis as the sine of the inclination of the solar equator to radius. Hence the inclination of the solar axis to the plane of the ecliptic is found to be 7½°, or rather this is the angle which the axis of the sun makes with the axis of the ecliptic, or a perpendicular to its plane which passes through the sun's centre. Most of the spots appear always within the compass of a zone, whose breadth, measured on the solar meridian, extends between 29° 42′ and 30° 36′; they have sometimes, however, been seen 29° 36′, and in July 5th, 1780, M. de la Land observed one 40° distant from the solar equator.

There have been various opinions respecting the nature of these spots. *Scheiner* supposed them to be solid bodies revolving round the sun, near its surface. But if they were not on the sun's surface, they would be longer visible than invisible, which is not the case. Moreover, if they revolved about the sun like the planets, their motion would necessarily be in a plane passing through its centre, which seldom happens. Galileo compared them to smoak and clouds, as they varied their figures, increased and sometimes disappeared. Hevelius appears to be of the same opinion in his *Cometographia*, pa. 360. The permanency of most of the spots is however an argument against this hypothesis. *M. de la Hire* supposes that they are solid bodies which swim on the sun's surface, and which are sometimes immersed in the liquid of which he conceives the sun's surface composed.—*La Lande* supposes that the sun is an opake body covered with a liquid fire, and that the spots arise from the opake parts like rocks, which are sometimes raised above the surface by the alternate flux and reflux of the liquid igneous matter of the sun. *Dr. Wilson*, professor of Astronomy at sun, all the planets and fixed stars will appear to revolve from east to west in the space of $25\frac{1}{2}$ of our days nearly, exclusive of the motion which the planets have in their orbits. The northern pole of this revolution will be in that place which an observer, situated on the earth, would refer to the tenth degree of pisces, and in latitude between 83° and 84° north, near the stars δ or π in Draco. The south pole will be in the 10th degree of Virgo, in latitude between 83° and 84° south, near the star α in equuleus pictorius. Besides these apparent motions arising from the motion of the sun on its axis, the planets will be observed to have regular and proper motions in their orbits round the sun, and all from the west towards the east. These motions are performed nearly in the same plane, at least not differing more than 7° or 8°. An observer in the sun will also find, that if he compares any planet with the fixed stars, it will sometimes go slower and sometimes swifter, he will therefore conclude that it is sometimes nearer and sometimes further off. This will also appear more evidently from the change of its apparent diameter. He will likewise observe some perform their revolution in less time than others, and if he has any idea of the laws of gravity,* he will refer this difference of velocity to its proper cause, which is the different distances of these respective bodies. But these distances he cannot so easily observe as if situated on the earth, as on the latter, from its motion, he can select several stations at great distances from each other.

We have observed that the motion of the planets round the sun is not uniform, being subject to very perceptible inequalities, the laws of which form one of the most important objects of astronomy.

Glasgow, opposes La Land, and is of opinion that the spots are excavations, or deep caverns in the luminous matter of the sun, the bottom of which forms the dark spot or umbra formed in the middle. (See the Phil. trans. 1774 and 1783.) La Place in his Astr. vol. 1. b. 1. c. 2. remarks that they are eruptions in the sun's body, of which our volcanoes form but a feeble representation, as they are almost always surrounded by a penumbra, which is enclosed in a cloud of light more brilliant than the rest of the sun, and in the midst of which the spots are seen to form and disappear. Dr. Halley was of opinion that the spots are formed in the atmosphere of the sun. *Dr. Herschel* supposes the sun to be an opake body, surrounded by a very gross atmosphere. He says that if some of the fluids which enter into its composition, should be of a shining brilliancy, while others are merely transparent, any temporary cause which may remove the lucid fluid, will permit us to see the body of the sun through the transparent ones. See the *Phil. trans.* for 1795. Some of these spots have been observed whose diameter exceed 6 or 7 times that of the earth. Dr. Herschel on April 19, 1779, saw a spot which measured 1′ 8″, 06 in diameter, which in length is near 30600 miles. (For 8″, 8 : 1′ 8″, 06 :: 3956 : 30596.) This was visible to the naked eye. For the phenomena of the spots as described by *Scheiner* and *Hevelius*, see Vince's Astronomy, 8vo. pa. 136. Besides the dark spots upon the sun, there are also parts of the sun called *Faculæ*, *Lucili*, &c. which are brighter than the general surface; these abound most in the neighbourhood of spots, or where spots had been recently observed.

* These general laws will be given after the solar system.

Kepler conceived the ingenious idea of comparing the figure of the orbit of the planets with that of an ellipsis,* in one of the foci of which he placed the sun ; and the innumerable observations we have since had, besides the important and numerous discoveries of Newton, leave no doubt concerning the truth of the hypothesis.

Besides this useful discovery, Kepler has made two others no less important, and which all observations and physical reasoning concur to establish, viz. that

The planets, by radii drawn from the sun to their respective centres, describe areas round the sun, proportional to the time of describing them ; and that

The squares of the times in which the planets revolve round the sun, are to each other as the cubes of the greater axis of their orbits.

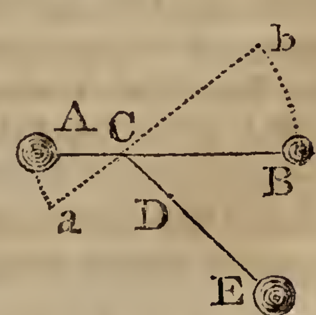
Newton, in his discoveries, has extended the same laws to the secondary planets revolving round their respective primaries.

The sun is not, however, exactly the centre of the planets' motions, but rather the centre of gravity of the whole system;† but

* If the two ends of a thread be tied together, and placed round two pins fastened in a sheet of paper, on a table ; then the thread being uniformly stretched by a black lead pencil, carried round by an even motion of the hand, will trace out an ellipsis, and the points where the pins were fixed are called its foci ; the longer diameter is called the transverse, and the shorter the conjugate. In describing the ellipsis the ends of the string might be fastened to the pins. The nearer the pins are, the more the ellipsis will approach a circle, and the further they are apart, the more *eccentric* will the ellipsis be. When the ends of the thread are fastened to the pins, the length of the thread ought to be equal to the length of the transverse diameter.

It is a known property of the ellipsis, that the sum of two lines drawn from the foci to meet in any point of the cure, is equal to the transverse diameter ; hence other constructions by compasses, &c. will arise. For which and the properties of this figure, see Simpson, Emerson, Hamilton, Milnes, Simson, Vince, or other writers on the conic sections.

† The centre of gravity of a system of bodies is that point round which, if the bodies were suspended, they would remain in equilibrio in any position. (They may be conceived to be suspended by inflexible levers, or in any other manner, from this centre.)



This centre may be thus found ; let the line ACB be supposed an inflexible lever, considered without weight, and let the two bodies A and B be suspended on the ends of the lever ; take the point C in AB so that AC : BC :: B : A, then C will be the centre of gravity between A and B. For if the bodies A and B be made to vibrate about the immovable point C, A and B will describe the arches Aa, Bb, which will be as the velocities of the bodies, and also as the radii AC, CB of the circles ; and hence their velocities are as the radii.—Therefore vel. A : vel. B :: AC : CB, or as B : A (by supposition) whence $A \times$ vel. $A = B \times$ vel. B. But as these products represent the quantity of motion of these bodies (see the laws of motion, next section) which being equal, the bodies will therefore remain in equilibrio round the centre C, which is therefore the centre of gravity required. Again, if A and B be now supposed to act in C, and E another body, the centre of gravity of these three bodies will divide CE in D, so that CD : DE :: E : A + B, &c.

as this centre is generally within the body of the sun, and can never be at the distance of more than the length of a solar diameter from its centre, the sun is therefore generally considered by astronomers as the centre of the solar system. The sun is however agitated by a small motion round the true centre of gravity, owing to the various attractions of the surrounding planets.

As the sun revolves on its axis, his figure, like the rest of the planets, is supposed not to be strictly in the form of a globe, but of an *oblate spheroid* (a figure formed by the revolution of an ellipsis round its shortest axis or conjugate diameter) and therefore flat towards the poles.

An observer placed in the sun can have no vicissitude of day and night, yet the fixed stars and planets will make unequal arches above and below the horizon, as they decline towards either pole, as it happens to the inhabitants of the earth. He will see no shadow or eclipse, but to an eye on the surface of the sun, when the planet is in the horizon, its satellite will sometimes appear in the penumbra of it, or the penumbra of the satellite may appear cast on the disk of the primary planet, which will be known by the colour being something duller. But the whole diameter of a primary planet will appear so small to an observer in the sun, unless assisted by very powerful telescopes or some other substitute, that these appearances can scarce be observed.

At the sun the diameter of Saturn (as laid down by Dr. Gregory in book 6th of his Astronomy) subtends an angle of 18″, that of Jupiter of about 40″, that of Mars only 8″, of Venus an angle of 28″, and of Mercury 20″. *Hugens* makes the diameter of Jupiter near 54″, and that of Saturn without his ring 27″.

The apparent diameters of bodies diminishing as the distance increases, an observer on the earth will therefore form an estimate of the relative change of the sun's distance ; in like manner an observer in the sun will form an idea of the planets' variation in their respective distances ; and as the sun's apparent diameter is greater in the beginning of January than in the beginning of July, it follows that the sun is nearer to our earth in the winter than in the summer. The greatest apparent diameter of the sun, as given in the Nautical Almanacs for 1811 and 1812, on the 1st of January, is 32′ 35″ 6, and the least on the 1st of July is 31′ 31″, the mean between which is 32′ 3″ 3. La Place in his Astronomy (b. 1, c. 1) makes the apparent diameter of the sun, when the velocity of the earth is greatest, equal 6035″ 7,* when the velocity is least, equal

Hence if by this method we find the centre of gravity between the sun and Mercury, between this centre and Venus, between this centre again and the earth, and so on to the remotest planet (their quantities of matter being given) the last centre will be the centre of gravity of the whole system, and the focus of all the planetary orbits. But this will seldom differ much from the sun's centre.

* In La Place's astronomy the quadrant is divided into 100°, each degree into 100′, and each minute into 100″, &c. Hence a degree in the sexagesimal arithmetic adopted in this country is equal $1\frac{1}{9}$° of this cen-

5836″ 3, and its mean diameter 5936″. He says that these quantities should be diminished a few seconds to allow for the effect of *irradiation*, which dilates a little the apparent diameters of luminous bodies.

The mean apparent diameter of the sun being taken about 32′, and if we take the sun's mean distance from the earth to be 95 millions of miles,* its real diameter will be 862399 miles, as before

tesimal division of the quadrant, and therefore $= 11111\frac{1}{9}''$; consequently $\frac{6035'',7 \times 9 \times 60'}{100000} = 32'\ 35''\ 62$, $\frac{5836'',3 \times 9 \times 60'}{100000} = 31'\ 30'',\ 96$, and $\frac{5936 \times 9 \times 60'}{100000} = 32'\ 3'',\ 26$, agreeing with the Nautical Almanac. We shall in the ensuing part of this work, when we have a necessity for quoting La Place, use his own measures, and give their value in the margin, as the reduction of them in the English translation is replete with faults.—The above three numbers are thus given in the margin of the English translation, 32′ 35″, 6, 28′ 49″, and 30.42, 25.

As the apparent diameter of the sun is greatest near the winter solstice, least near the summer solstice, and nearly at a mean in the equinoxes, its apparent diameter may be easily found at any other time sufficiently correct by proportion.

The apparent diameters of the planets are found by a *micrometer* placed in the focus of a telescope ; or the apparent diameter of the sun may be measured in a dark room, by means of the projection of his image through a circular aperture. From these apparent diameters, and the respective distances from the earth, their real diameters may be determined thus :

Let M in the foregoing figure represent the earth, AB the sun's diameter, the angle AMC the apparent semidiameter of the sun = 16′, and MC the distance of the sun from the earth ; to find AB the true diameter, it will be,

Rad.	- - -	10.0000000
To tang. 16′	-	7.6678492
So is 23464.5	-	4.3704112
To 109.2095	-	2.0382604

Hence 109.2095 × 2 = 218.419, which multiplied by 3956, gives 864063.564, the diameter of the sun ; the cube of which divided by the cube of 7911, will give the number of times the sun is greater than the earth.

In the above calculation 23464.5 is taken in place of 23405 (see ch. 2) hence 23464.5 × 3956 = 92825562 miles.

* If in the figure for determining the distance of the moon, &c. (pa. 250) we suppose M to be the sun, AC the earth's semidiameter, and the angle AMC the sun's horizontal parallax 8″ 65, the distance MC is thus found ;

As tang. AMC 8″ 65	- - - - -	5.6219140
To rad. or sine ACM	- - - - -	10.0000000
So is *one* semid. of the earth AC	- -	0.0000000
To 23882.84 semidiameters	- - -	4.3780860

We have here taken the sun's parallax at its mean distance 8″ 65 according to Mr. *Short*, who has taken incredible pains in calculating it from the best observations made on the transits of Venus in 1761, an account of which is given in the philosophical transactions for 1762 and 1763. But from the transit of Venus in 1769, compared with the former, the parallax is found to be 8″ 8.

determined, and its magnitude 1295029 times that of the earth; the diameter of the earth being only 7911 miles, the sun's diameter will be 109 times as great.

Besides the planets and fixed stars, an observer in the sun will discover other bodies of a different nature, called comets, on account of their hairy appearance, as seen from the earth. These are carried in very eccentric orbits round the sun, sometimes approaching near his body, and at other times going off to immense distances from it. These comets will appear to be carried among the fixed stars, some in one direction and some in a contrary, in orbits whose planes are very much inclined to that of the ecliptic, although they regard the sun, or rather the centre of gravity of the solar system, as the focus of their motions.

What we have here said is upon supposition that an observer in the sun is not prevented by its atmosphere (which from late experiments we have reason to conclude is very gross*) from seeing as

Mr Short by taking the semidiameter of the earth 3985 miles, makes the earth's mean distance 95173127 miles. But if the earth's semidiameter be 3956, the earth's mean distance from the sun will be 23882.84 × 3956 = 94480515 miles. Hence in round numbers it is sometimes taken about 95000000 miles, although it is probably less. La Place taking the parallax 8″ 8, makes the sun's mean distance 23405 times the radius of the earth: hence 23405 × 3956 = 92590180 English miles, the sun's mean distance from the earth. There must, however, be some mistake in *La Place's* calculation. (See ch. 2d.)

* Bouguer, by some curious experiments on the intensity of light on different parts of the sun's disk, found that this light was more intense at the centre than near the limb. Two equal and very small portions of the sun's surface seen from the earth, one at the centre of the disk, and the other near its edge, appear to occupy different spaces, which are to each other as radius to the co. sine of the arc of the great circle, which separates these two parts on the sun's surface; this makes the intensity of light increase in this proportion inversely from the centre to the edge of the sun's disk. Bouguer has however found the reverse. In comparing the light of the centre with that of a point distant from the limb by a quarter of the semidiameter, he found the intensities of these two lights in the proportion of 48 to 35. This difference indicates a thick atmosphere round the sun, which weakens its light.

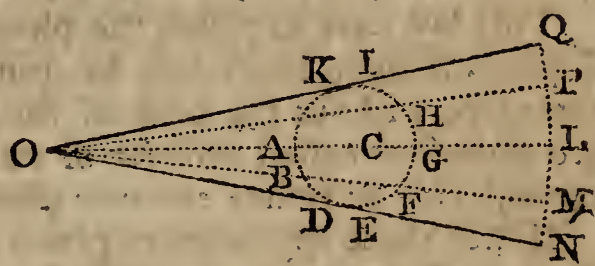
It follows from the preceding results and from the experiments of Bouguer, that the intensities of the light of a star seen from the surface of the sun at the zenith, is reduced 0.24065, and that the sun deprived of its atmosphere would appear $12\frac{1}{7}$ times more luminous. A horizontal stratum of air at the temperature of 0 or zero, the thermometer being divided into 100° from the freezing to the boiling point of water, and under the pressure of a column of Mercury, $0^{me.}$ 76 ought to have 53548 $^{me.}$ (metres) of thickness, to weaken light in the same degree as the sun's atmosphere. This is on supposition that at equal densities the transparency of the sun's atmosphere is the same as that of the air, but of this we are ignorant. Bouguer's experiments deserve also to be repeated in different aspects of the solar disk.

La Place in his astronomy (b. 1, c. 2.) remarks, that the faint light which is visible particularly about the vernal equinox, a little before the rising or after the setting of the sun, and which is called *zodiacal light*, is supposed to be produced from the reflexion of this atmosphere. The far, and as freely as an observer on the earth sees on a clear night, when the moon does not shine. If this be not the case, he may be unable to trace several of the phenomena which we have mentioned; but if he sees them at all, he will observe them as we have described them.

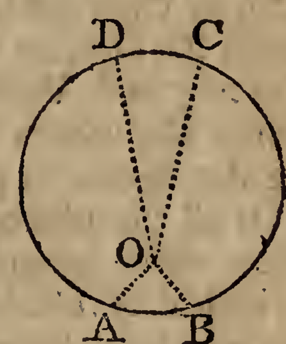
CHAP. II.

OF MERCURY.

We will now transfer our solar observer from that station where the motion of the planets are regular, to the earth, his own habitation, where the phenomena will appear something different. From this point of view the planets will not observe the equal description of areas in equal times as round the sun, but will sometimes appear to move towards the east, at other times towards the west, and sometimes to remain stationary or without any motion.



Let a body revolve in the periphery of the circle A, B, C, D, E, F, &c. and move through equal arches AB, BD, DE, EF, &c. in equal times; and let an eye, in the plane of this circle, view the motion of the body from O. When the body moves from A to B, its apparent motion will be measured by the arch LM, or the angle LOM; while it moves from B to D, its apparent motion is determined by the arc MN, which is less than the former, though described in the same time, when it comes to E, it will still be observed in the same point N; hence during the time that it described the arch DE, it was stationary at N. The motion of the body being continued in its orbit, when it comes to F, it will appear in L, and to have gone backward or retrograde the arc NM, and when it comes to G, it will appear in L, where it appeared before when in A. In like manner when it comes to H, it will appear in P, and at I it will appear at Q, where it will seem *stationary*, while it describes IK. At K it will again go forward as before, and with unequal motions describe the arch QN. This unequal motion is evidently owing to the eye being placed at O, without the orbit ABD, &c. of the body, while the revolving body regards C as the centre round which it regularly revolves. If the eye at



fluid which transmits it to us is extremely rare, since the stars are visible through it; its colour is white, and its apparent figure that of a cone whose base is applied to the sun. The length of the zodiacal light sometimes subtends an angle of 100°, but the atmosphere of the sun does not extend to so great a distance, and cannot therefore be the cause of this light. The true cause is still unknown.

O be in motion during the revolution of the body in the orbit ABD, &c. it will retard or accelerate its motion according as it moves in the same or a contrary direction. This explains the phenomena of Mercury or Venus' motions in their orbits, which are within the orbit of the earth.

If the eye be now placed within the orbit of the body at O, as in the 2d fig. but not in the centre; while the body describes the arch AB, it will seem to move quicker than while it describes its equal CD, which is more distant, because the angle AOB, by which it forms an idea of the motion of the body, is greater than DOC. In this case, however, the body will never appear stationary or retrograde, but will always appear to move forward, though with very unequal motions. But if the point O be in motion, then the phenomena will be different. If the motion of the eye at O be equal to the motion of the body and in the same direction, the body will appear stationary; if the motion be greater, the body will appear to go backward, if less, forward, &c.

This last case explains the phenomena of the motions of Mars, Jupiter, Saturn, Herschel, &c. round the earth, in orbits which are therefore without the orbit of the earth. The two former planets are therefore called *inferior* or rather *interior* planets, and the latter *superior* or rather *exterior* planets.

Hence these appearances prove that the planets do not regard the earth as their centre of revolution, but that they in reality revolve round the sun, as we have before described. Two of the planets, Mercury and Venus, never recede from the sun beyond certain limits, the others are occasionally separated from him by all the angular distances possible.

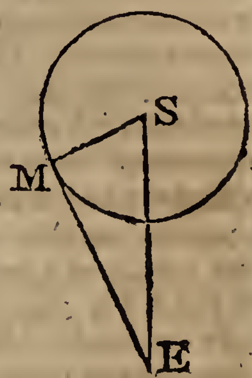
Of all the planets Mercury is nearest to the sun, and the least of those whose magnitudes are accurately known. He performs his periodical revolution round the sun in 87 days, 23h. 15m. 43″6.* His greatest elongation is 28° 20′, and least 17° 36′, the mean of which is 22° 58′, and his distance† from the sun is 35933619.76 miles.

* For the method of finding the planet's periodical revolutions, see the following note.

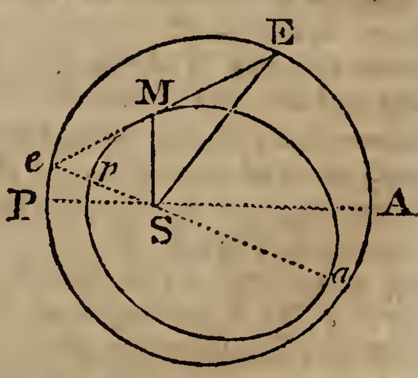
† The distance of Mercury, or any other planet from the sun, may be found by Kepler's rules given in chap. 1; thus, the squares of the periodic times being always as the cube of their mean distances; or which is the same, divide the square of the time in which any planet revolves round the sun by the square of the time of the earth's revolution, the cube root of the quotient will give the *relative distance* of the planet from the sun, which multiplied by the earth's mean distance from the sun, will give the planet's mean distance required.

For Mercury. The earth's periodic revolution is 365d. 5h. 48′ 48″ = 31556928″, the square of which is 995839704797184 (a constant divisor for all the planets) and 23464.5 the distance from the earth to the sun in semidiameters, will be a constant multiplier. (Tang. 8″ 8 (log. t. = 5.6295869) : rad. (log. 10.0000000) :: 1 semidiameter (log. = 0.0000000, : 23464.5 semid. (log. 4.3704131) see notes, ch. 1) 87d. 23h. 15m. 43″ 6 = 7600543″ 6, the square of which is 57768263015500.96, which divided by the former square, gives .058009600076417, the cube root of which is .38711, nearly, the distance of Mercury from the sun, supposing the distance of the earth from the sun to be an *unit* or 1. Hence .38711 × 23464.5 = 9083.34, which mult. by 3956, the radius of the earth, gives 35933693.04 miles, the mean distance of Mercury from the sun. (See table 102 of Delambre, pa. 113, &c. Paris edition.)

The distance of Mercury or any inferior planet from the sun, may also be found by their elongations. If S represent the sun, E the earth, and M Mercury, and EM a tangent to Mercury's orbit; then the angle SEM will be the greatest elongation of the planet from the sun, which angle, if the orbit were circular and the sun in the centre, would be found by saying ES : SM :: Rad. : sine SEM. But as the orbits are elliptic, the angle EMS will not be a right angle, unless the greatest elongation happen when the planet is in one of its apsides. The angle SEM is also subject to variation in proportion to the variation of SE and SM. The greatest angle SEM happens when the planet is in its *aphelion*, and the earth in its *perigee*, and the least, when the planet is in its *perihelion*, and the earth in its *apogee*. M. de la *Lande* finds these elongations equal 28° 20′ and 17° 36′ respectively, the mean of which is 22° 58′. But *Laplace* makes the greatest and least elongations of Mercury = 32° and 18°, or in our measures = 28° 48′ and 16° 12′ respectively, the mean of which is 22° 30′. Now in the triangle SEM taking the angle SEM = 22° 30′, the distance of the earth from the sun SE = 23464.5 of the earth's semidiameters, and SME is a right angle ; hence rad. : SE 23464.5 log. = 4.3704113 :: sine 22° 30′ = 9.5828397 : log. 3.9532510 = 8979.5 nearly, which multiplied by 3956 gives 35522902 miles, the distance of Mercury from the sun by this method ; but an error of a few minutes in the elongation will make a considerable difference ; for taking 22° 58′ instead of 22° 30′, we find 9155.8 semidiameters nearly, = 36220344.8 miles. (See La Land's Astronomy, 3 ed. 1792. art. 1142.)



The distances SE, SM being given, the angle SEM and MSE are also given, the former of which is the greatest elongation of the planet, and the latter the angle of *commutation* or heliocentric distance of the planets (or which is the same, the common mutation or angular distance of any two of the planets among themselves, as seen from the sun.) But this is on the supposition of circular orbits ; however the prob. may be solved nearly in the same manner, the elliptic figure being considered. For the angle SME being given, and the distance SM, and moreover the angles AS*a*, ASM, MSE (nearly equal to the same in circular orbits) the angle ASE will be known, and also SE will be known in magnitude, from which the rest will be given as before. Here the greatest elongation changes according to the different distances of the point M from the aphelion of its orbit; for it is greatest in *a* in its aphelion, less in *p* its perihelion, and a mean in the mean longitude. It is also various, the place M of the inferior planet remaining the same, according as the superior is situated in E or *e*, &c. Laplace remarks that the length of an entire oscillation of Mercury, or return to the same position relatively to the sun, varies from 106 to 130 days, that the mean arc of its retrogradation is about 15° (= 13° 30′) and its mean diameter 23 days ; but that in different retrogradations there is a great difference in these quantities.



By observing two heliocentric places of an inferior planet, its periodic time may be nearly found, though it is more accurate to observe the

The eccentricity of Mercury's orbit* is estimated at one-fifth of its mean distance from the sun Vince makes the eccentricity of his orbit 7955.4 parts of the mean distance of the earth from the sun,

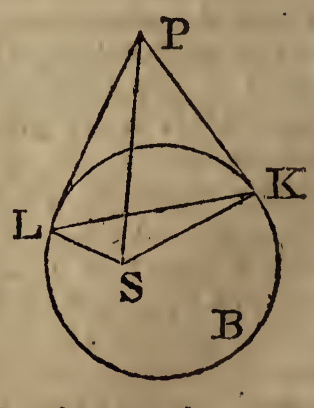
planet when twice successively, in the same node, that is when the planet has no latitude, the time between these observations will be the planets' periodic time, as the motion of the nodes will vary but little during this short period The conjunctions of the inferior planets are also proper for discovering their periods, for then they appear in the same point of the heavens to an observer, either in the sun or on the earth; if the planet be nearer than the sun, it will appear in opposite points. The superior planets when in *opposition* to the sun, also appear in the same point of the heavens or the ecliptic, when seen from the earth, as if observed from the sun, and hence their geocentric and heliocentric places agree. If any of them be then observed, and the time marked, and the same observation be made when it comes to its next opposition, the arch which the planet seen from the sun, has in the elapsed time described, will be thus discovered; then say as that arch : the whole circumference :: the time between the two oppositions : a fourth which will give very nearly the periodic time of the planet. The planet can never be observed in the ecliptic from the earth, which is in the plane of the ecliptic, except when it is also in the ecliptic, and consequently in its node.

The daily mean motion of Mercury, according to Delambre, is 4° 5′ 33″ or 4° 5′ 34″, his mean hourly motion 10′ 14″, in a minute 10″, and in a second 10‴, &c.

The hourly motion of Mercury in miles may be thus found; taking the mean dist. from the sun =35933693.04, this multiplied by 2, gives 71867 386 08 = diameter of Mercury's orbit, which multiplied by 3.1416, gives 225778580.109; hence as 87d. 23h. 15′ 43″ : 225778580.109 :: 1h. : 106940 miles, the hourly motion of Mercury.

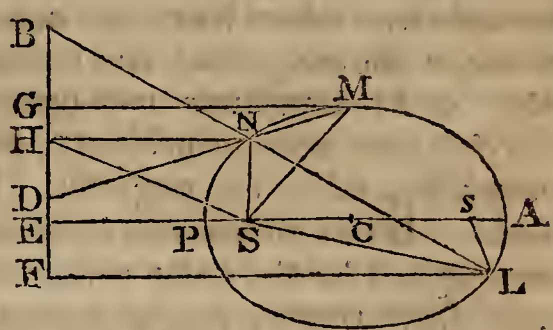
* In order to describe a planet's orbit, or to find its position and eccentricity, the planets' heliocentric place or its place as seen from the sun, and its distance from the sun, must be obtained. Dr. Halley gives the following ingenious method of finding these requisites, with no other data than the periodic time of the planet.

Let KLB be the orbit of the earth, S the sun, P the planet, or rather the point in the plane of the ecliptic on which a perpendicular let fall from the planet meets that plane; when the earth is in K, observe the planet's geocentric longitude (this is calculated in pa. 4 of the month in the Nautical Almanac) and having the theory of the earth, its place in the heavens, or the apparent longitude of the sun is given (found also in pa. 2 of the month in the Nautical Almanac, and its hourly motion, pa. 3.) and hence the angle PKS is given. The planet after completing an entire revolution, returns again to the same point P, at which time the earth being supposed at L, observe the angle PLS the planet's elongation from the sun. Now the times of the observations being given, we have the places K, L of the earth in the ecliptic given, and consequently the angle LSK and the sides LS and SK; wherefore we shall have the angles SKL and SLK, and the side LK. The angles PKS, PLS being likewise known, the remaining angles PKL, PLK will be known. Hence in the triangle PKL, two angles and the side LK being given, the side PL is given; and having the side PL and LS, and the angle PLS, the angle LSP is given, which determines the heliocentric place and its distance from the node according to the ecliptic, and likewise the side SP is given. But as the tangent of the geocentric



supposing this distance 100000. Laplace taking half the greater axis of the earth's orbit or its near distance = 1.000000, makes half the greater axis of Mercury's orbit or his mean distance = 0.387100, and the proportion of his eccentricity to this mean distance for the beginning of the year 1750 = 0.205513, and the *secu-*

latitude, is to the tangent of the heliocentric, so is the curtate distance of the planet from the sun, to its curtate distance from the earth. By observations the planet's geocentric latitude is found, wherefore its heliocentric latitude is given. (These are given in the Nautical Almanac. The method of finding these will be given in chap. 4.) The heliocentric lat. of the planet being thus found, and also its curtate distance from the sun, its true distance can be easily found. (See the note, page 264, &c.) Three heliocentric places of the planet and the corresponding distances from the sun being thus found, we shall find from thence the form of the orbit and the position of the *apsides*, by describing an ellipsis that will pass through three given points.



Let the three given places of the planet be L, M and N, and S the sun's place or the given focus; join LN, and produce it to B, so that SL : SN :: LB : NB; also join MN, and produce it to D, so that SM : SN :: MD : ND; join BD, and from S and L let fall the perpendiculars SE, LF, and divide SE in P, so that LF : LS :: EP : PS, and also make EA : AS in the same ratio. Then AP will be the axis, and the middle point C the centre, A and P the vertices, and C*s* being taken = CS, *s* will be the other focus: whence the ellipsis may be easily described.

From M and N let fall the perpendiculars MG, NH on DB; then by construction SL : SN :: LB : NB, that is by sim lar triangles, as LF : NH, and permutatio SL : LF :: SN : NH. Again by construction SM : SN :: MD : DN, that is by sim. triangles, as MG : NH, hence permut. SM : MG :: SN : NH; also SP : PE :: SL : LF (by constr.) that is as SN : NH, or as SM : MG, and therefore BD is the directrix of the ellipsis, in which are the points N, M, L, and whose focus is S, and vertices A and P. (Emerson's Conic Sections, b. 1, prob 29, or Milnes, part 4, prop. 9.) This prop. is demonstrated nearly in the same manner in Emerson, prop. 85, b. 1, Vince's Ast. c. 13, Keil's Ast. lect. 26, Gregory's Ast. prop 29, b. 3, or Newton's Principia, prop. 21, b. 1. Schol. Newton remarks, that when EP is greater than, equal to, or less than PS, the figure thus described will be either an ellipsis, a parabola or an hyperbola; the point A in the first case falling on the same side of the line BD, as well as the point P; in the second going off to an infinite distance, and in the third falling on the other side of the line BD.

As a *calculation* is preferable to any construction; it may be drawn from the foregoing investigation. Thus, to find NB we have, by division, SL — SN : SN :: LB — NB or LN : NB, but the three first terms are given, because the points N, S and L are given in position, or NS, SL and the angle NSL are given, and hence NL is given, NB is therefore given = $\frac{SN \times LN}{SL - SN}$. Again to find ND we have SM — SN : SN :: MD — ND or MN : ND = $\frac{SN \times MN}{SM - SN}$. (Eucl. 17, 5.) PS and AS are found in the same manner. Moreover in the two triangles NSL, NSM, two sides and the included angles are given (being the distances of the planet from the sun, and the degrees between its observed places in its orbit) hence NL, NM are given, and the angles LNS, SNM, and therefore the angle LNM *lar* increase of this proportion or its increase for 100 years = 0.000003369.* The place of Mercury's aphel. for the beginning of 1750, according to *Vince*, was 8s. 13° 33′ 58″, and its motion in longitude in 100 years 1° 33′ 45″. Mercury's greatest equation, according to the same author, is 23° 40′. (See def 133, p. 2.) According to Laplace, the longitude of the perihelium in 1750 was (81°.7401) 73° 33′ 57″9, and its secular direct motion (1735″5) 9′ 22″ 3. Delambre, tab. 97, makes the place of Mercury in 1810, 9s. 23° 32′, the place of his aphelion 8s. 14° 30′ 14″, the place of its node 1s. 16° 4′ 1″, the motion in longitude of the aphelion in 100 years 1° 33′ 45″, and of the nodes 1° 12′ 10″, both increasing; hence his place at any other time may be found. (These places are set down to mean time in Delambre's tables.) The mean longitude of a planet seen from the sun, is found by adding its mean motions to the *epoch*, or its place for any given year. The longitude of the aphelion taken from the mean longitude of the planet, will give its mean anomaly, and the contrary, &c. In finding the planet's place or his longitude reckoned from the apparent equinox, the *nutation* (see note to prob. 42, part 3) must be applied as given by Delambre, table 11, page 29. His apparent diameter is very variable. It is a minimum or the least when the planet in a morning immerges into the solar rays, or when in the evening it disengages itself from them; it is at its maximum, or the greatest, when it immerges into the sun's light in an evening, or when it again becomes visible in the morning. Its mean apparent diameter, according to Laplace, is (21″ 3) 6″ 9 or nearly 7″, and his apparent

and its vertical opposite angle BND, hence ND and NB being given, the angle BDN is given, therefore in the right angled triangle DHN, the hypot. DN, and the angle at D are given, hence NH is given. Join SH then in the triangle SHN, NH, NS are given, and also SNH (= suppl LNS — BNH or 180° — LNS — BNH) hence SH the angle, NHS, NSH and SHE are given; therefore in the right angled triangle SHE, SE is given, hence we know SA and SP; for HN : NS :: EA : AS and by division HN — NS : NS :: EA — AS or ES : AS $= \frac{NS \times ES}{HN - NS}$. Again, HN : NS :: EP : PS and (Eucl. 18, 5.) HN + NS : NS :: EP + PS or ES : PS $= \frac{NS \times ES}{HN + NS}$. Hence PS and SA being given, their sum or AP is given, and half their difference is the eccentricity SC. Lastly in the triangle S*s*L, we have the sides S*s*, SL (for AP—SL and *s*L = *s*L. Emerson's Conics, prop. 1, b. 1) to find the angle LSA the distance of the aphelion from the observed place L; in the same manner the distance of the aphelion may be found from the observed places M and N.

In the year 1740, on July 17, August 26, and September 6, M. *de la Caille* found three distances of Mercury (the mean distance being 10000) as follows; SL = 10351,5, SN = 11325,5, SM = 9672,166, the angle LSN = 3s. 27° 0′ 36″, NSM = 44° 40′ 4″. From whence its eccentricity (by calculating as directed above) is found = 2099.75, the place of its aphelion 8s. 13° 51′ 14″, and the greatest equation = 24° 3′ 5″. The above scheme must be fitted for these distances by the rules given for constructing the ellipsis. (See other methods in Vince's Complete System of Astronomy, and also his Elements of Astronomy, 8vo.)

* See tables 101, 102, &c. of Delambre.

diameter as seen from the sun is about 17″ 8 nearly. His real diameter is therefore about 3105 miles,* and his magnitude nearly $16\frac{1}{2}$ times less than that of the earth.

It was no doubt difficult at first to recognize the identity of the two stars which were alternately seen in the morning and in the evening, to depart from and return to the sun, but as the one never shewed itself until the other disappeared, it was found to be the same planet which thus oscilated on each side of the sun; its position, apparent diameter, and retrograde motion, confirming this conjecture, and agreeing with the laws of its motion afterwards discovered. In general these laws are very complicated, they do not take place in the plane of the ecliptic, sometimes the planet departs from it, 4° 30′ being its greatest geocentric latitude, or its greatest distance from the ecliptic, as seen from the earth; but this distance as seen from the sun amounts to 7°, being his greatest heliocentric latitude, and equal to the inclination of the plane of its orbit to that of the ecliptic. Its secular variation, according to Laplace, is (55″ 09) 17″ 50. The place of the nodes being the point of intersection of the orbit of the planets and the ecliptic; the place of Mercury's ascending node,† or its longitude at the beginning of 1750, was 1s. 15° 20′ 42″ 8, or taurus 15° 20′ 43″ nearly. Hence the de-

* The mean distance of the earth from the sun being 23464.5 semidiameters, and Mercury's mean distance 9083.3214 semidiameters, the difference is 14381.1786, the distance of Mercury from the earth; and as the magnitudes of bodies vary *inversely* as their distances *nearly*, we have by inverse proportion 23464.5 : 6″ 9 :: 9083.3214 : 17″ 8 nearly, the apparent diameter of Mercury as seen from the sun. Now taking the mean apparent diameter of the sun 32′, and its real diameter = 864065.5, we have 32′ : 6″ 9 :: 864065.5 : 3105.23 miles the diameter of Mercury.

The diameter of Mercury may be also thus found; let S in the annexed figure represent the sun's place, SC Mercury's mean distance = 9083.32 semidiameters of the earth nearly, and the angle CSM, the apparent semidiameter of Mercury as seen from the sun = 8″ 9 nearly; then

Sine SMC = 89° 59′ 54″ 5		10.0000000
Sine CSM =	8″9	5.6247021
So is 9083.32 semid.		3.9582446
To .38278		— 1.5829467

Hence .38278 × 3956 = 3028.5 miles, differing a little from the above.

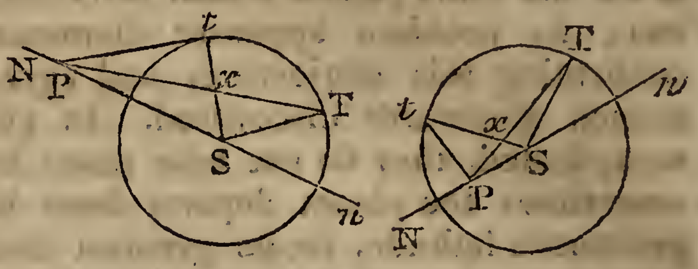
The mean diameter of Mercury is measured when he has his greatest elongation from the sun, his greatest diameter may be thus found by the inverse rule of proportion; 23464.5 : 6″ 9 :: 14381.2 :: 11″ 2 nearly, and his least thus; 23464.5 : 6″ 9 :: 32547.8 (= 23464.5 + 9083.3) : 4″ 9 nearly.

Now if the cube of the diameter of the earth be divided by the cube of the diameter of Mercury, the quotient will give the number of times the earth's magnitude exceeds that of Mercury. For the magnitudes of bodies are as the cubes of their diameters. (Eucl. B. 12 p. 18.) Hence 3105^3 : 7911^3 :: 1 : $\frac{7911^3}{3105^3}$ = 16.5 nearly, the number of times the earth is greater than Mercury.

† The nodes and inclinations of the orbits of the planets, &c. may be thus determined. First to find the position of the line of the nodes.

scending node was in scorpio 15° 20′ 43″. The motion of the nodes is found by comparing their places at two different times, from whence the motion of Mercury's nodes in 100 years is found to be 1° 12′ 10″ according to Vince. Laplace makes the sidereal and secular motion of the node upon the *true* ecliptic diminish (2332.90 seconds) 12 min. 35″ 8.

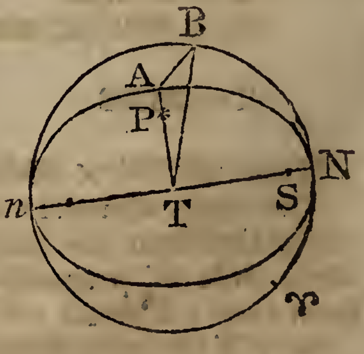
In the annexed figures (adapted to an inferior and superior planet) let S represent the sun, T*t* the earth's orbit, N*n* the line of the nodes of the planet. Let the earth be in T, from which let the planet P be observed, when in the ecliptic, and therefore in its node P; next after one revolution let the planet be observed again in the same node, the earth being in *t*; draw the straight lines ST, PT, S*t*, P*t* Then in the △ (triangle) ST*x*, there are given the ∠ (angle) *t*ST by *the theory of the earth* (see chap. 4) and time between the observations (the motion of the earth in this time is also found in the Nautical Almanac, that of the sun being given) and STP the observed elongation of the planet from the sun, and ST the distance of the earth from the sun; therefore S*x* is found, and also *xt*; S*t* the distance of the earth from the sun at the second observation being given (this distance can always be found from page 3 in the Nautical Almanac, or prob. 5 of Mayer's Tables, the log. there given being adapted to the mean distance 1; their index is increased by 10 when it is negative, which must be allowed for.) Again, in the △ *tx*P, the ∠ *tx*P = T*x*S is given, and the ∠ S*t*P the elongation in the second obs. is given, and likewise *tx*, hence P*t* is given. Lastly, in the △ S*t*P, *t*S, *t*P and the ∠ P*t*S are given, and therefore SP is given, which is the planet's distance from the sun when in its node; the ∠ *t*SP is also given, and therefore the position of the point P is given, the point *t* being the place of the earth as seen from the sun at the second obs. and hence the position of *n* is given, and therefore the position of the line of the nodes N*n* is given.



If the place of the nodes found by the ancients be compared with that found by the moderns, its motion will be given.

If a planet be observed twice in any point of its orbit, as seen from the earth, the place of the planet as seen from the sun, and its distance from the sun, are found in the same manner.

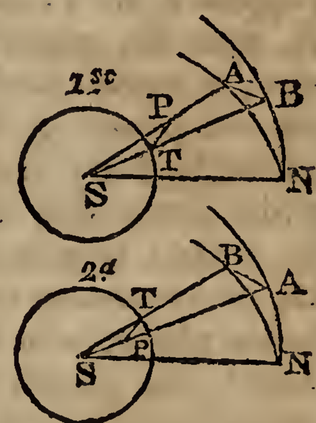
Having the motion of the nodes and the periodic time of the planet, and moreover its place being given for any year, its place for any other year may be easily found. Now the position of the line of the nodes being given, the *inclination of the planet's orbit to the plane of the ecliptic*, can be found thus; let S represent the sun, NB*n* the ecliptic among the fixed stars, NA*n* the planet's orbit, as seen from the sun among them, and NS*n* the line of the nodes. The earth comes twice in the year to this line, and as the *mean time* of its coming to it is given, let the geocentric place A of the planet P be observed at this time (or found in page 4 of the Nautical Almanac, or calculated by Delambre's Astronomical Tables, translated by Vince. See notes to prob. 1 and 3, part 3) and let the latitude thus found be AB, an arch perpendicular to the ecliptic, and longitude ♈B. Now as the longitude of the sun ♈N is known, the difference of these longitudes NB is known. Hence in the



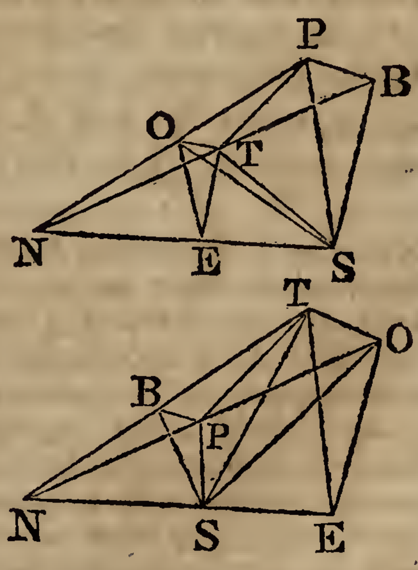
The latitude of Mercury is greater when retrograde and nearest the earth, and less when direct and remotest from it. Moreover if it be in its lower conjunction, or most retrograde and nearest the earth, and at the same time in or near one of its nodes, it will be directly between the observer and the sun. If it be at a considerable distance from a node, it will pass the sun to the northward

right angled spher. △ ANB rt. angled at B, AB, NB are given, and therefore the ∠ ANB the measure of the inclination required will be known.

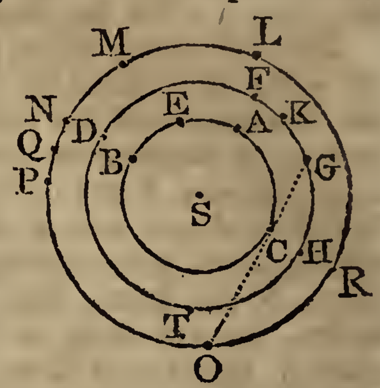
The inclination of the plane of the planet's orbit to the plane of the ecliptic being thus found by observation, the heliocentric place of the planet and his distance from the sun may be found, whenever the planet is in *opposition* to, or *conjunction* with the sun thus; let S represent the sun, T the earth, P the planet in its orbit, NB the ecliptic among the fixed stars, NA the intersection of the planet's orbit with the sphere of the fixed stars, N the node; then SN will be the line of the nodes, the sun being in the plane of the orbit of each planet. Let A and B be the planet and earth's places respectively, as seen from the sun among the fixed stars; and as the planet is either in opposition to the sun, as in fig. 1st. or in conjunction with it, as in fig. 2d. the arch AB will then be the circles of latitude, and therefore perpendicular to the ecliptic. Hence in the spher. rt. angled △ ABN, the ∠ ANB, and BN (found as above) are given, therefore AB and AN are given; but AB is the *heliocentric* lat. and AN is the distance of the planet in its orbit from the node N, as seen from the sun, and therefore the heliocentric place A of the planet P is given. Moreover in the △ PST there are given ST from the theory of the earth, and the ∠ PTS the lat. by observation or *geocentric lat.* or its complement to a semicircle, and also the *heliocentric* lat. PST, therefore PS and PT, the distances of the planet from the sun and earth respectively, are given.



The same may be found for any other aspect of the planet thus; let P be the planet, NS the line of its nodes, and the angle PTB be its apparent or geocentric latitude as seen from the earth in T; let the plane of this latitude be produced until it cuts the plane of the orbit of the planet in PN, and the plane of the ecliptic in the right line BTN. Draw PB perp. to NB, and erect TO prep. to the same, then (prop. 38, Euclid 11.) TO will be perpend. to the plane of the ecliptic, because the plane of the lat. PNB is perp. to the plane of the ecliptic. Let fall TE perp. to NS or NS produced; join OE, which is perpendicular also to NS, and the ∠ OET will be equal to the inclination of the planes of the orbit of the planet and of the ecliptic. In the △ NST there are given ST, the ∠ TSN by the theory of the earth, and the given place of the node, and the ∠ NTS by observation, being the elongation of the planet from the sun, computed in the ecliptic, or its complement to two rt. angles, therefore TN, NS and the ∠ TNS will be given. In the △ TEN rt. angled at E, NT and the ∠ TNE are given, and hence TE is given. In the △ OTE rt. ang. at T, TE and TEO the inclination of the planes of the orbit of the planet and of the ecliptic (found as above) being given, OT is given. In the △ OTN rt. ang. at T, OT and TN are given, hence the ∠ ONT is given. In PNT are given



or southward. But if it be most direct, and at its greatest distance from the earth, and at the same time in or near a node, it will be covered by the sun; if otherwise situated than near its node, it will pass on one side of the sun. When it is nearer the earth and near its node, it is seen in the interval of its disapparition in the evening, and reapparition in the morning, projected like a black spot on the disk of the sun on which it describes a chord.



These motions of the inferior planets may be thus explained; let ABC be the orbit of an inferior planet, and S the sun, the circle LMO the zodiac in the heavens; let the earth be now supposed in T, and the inferior planet in A, near its superior conjunction with the sun; a spectator in T will then evidently see the planet at A in the point L. If the earth had no motion, the inferior planet, while describing the portion AB of its orbit, would appear to have described the portion of the zodiac LM. But in the mean time the earth is in motion, so that when the inferior planet is in B, the earth is in the point of its orbit H, from which the planet in B appears in N. Hence Venus has apparently moved further *eastward* from the earth's motion. But when the planet comes to C, the earth has moved on to G, and then the planet is seen in the point of the zodiac O, GO touching its orbit in C, in which position its apparent motion is nearly equal to the apparent motion of the sun or direct. From this position let the planet move from C to A, and the earth in the same time from G to K, from which the planet will be observed in the zodiac in P, but as it was before observed in O, it will appear to have gone *retrograde* or backwards in the zodiac, through the arch OP, or to have moved *westward* contrary to the order of the signs; and as the planet was *direct* in C, there must be some point of its orbit between C and A where it appeared *stationary*, or without any motion. Let the planet now be in E, and the earth at the

NT, the ∠ TNP and PTN the *geocentric* lat. of the planet or its complement to two right angles, therefore NP is given. In the △ NPB rt. ang. at B, the side NP and ∠ PNB being given, PB and NB will also be given. In BNS, NB and NS and the ∠ BNS are given, hence NSB the heliocentric longitude of the planet computed from its node, and the side SB are given. Then in the △ PBS, rt. ang. at B, PB, BS being known, PS the distance of the planet from the sun, and the angle PSB, which is its *heliocentric latitude*, will be given. Lastly, in the △ PNS all the sides being given, the angle NSP is known, being the heliocentric distance of the planet in its orbit computed from the line of the nodes NS. The mean distance of the earth from the sun may be taken as the measure in finding the planet's distance.

If by this method we find out two other heliocentric places of the planet and the distances from the sun, having likewise the focus of its orbit, which is the sun's centre, an ellipse may be described passing through the given points, as before shewn, which will be the orbit of the planet. The learner will notice that SB is called the *curtate distance* of the planet from the sun.

same time in F, from this point the planet will be seen in Q, and will appear to have moved further backward from P towards the *west*. But when it is seen again in a line that just touches its orbit, its motion will be direct or towards the *east*, between which and the former place the planet will be *stationary* as before. The earth having now come to D, and the planet to C, it will seem to have described the arch QR, and its motion to be quicker *eastward*. Hence when the planet is in its *superior conjunction* with the sun, its motion is always *eastward*, or according to the order of the signs, but when it is in its *inferior conjunction*, its motion is *westward* or contrary to the order of the signs. In the former case it will seem to go forward, in the latter backward or in a contrary direction.*

* Here might be shewn how to find the *position* of a planet when stationary, the time of the station, &c. but as these are subjects of curiosity rather than matters of any real practical utility, the learner is referred to lecture 27 of Keil's Astronomy, or to ch. 15 of Vince's Astronomy, 8vo.

Vince in his Astronomy observes, that the place and time of the *opposition* of a superior or the *conjunction* of an inferior planet, are the most important observations for determining the elements of their orbits, because at that time the observed is the same as the true longitude, or that seen from the sun; whereas if observations be made at any other time, the observed must be reduced to the true longitude, which requires the knowledge of their relative distances, which, at that time, are supposed not to be known.

The conjunctions of the inferior planets may be thus determined; find the diurnal motion of the planet from the sun, and also the diurnal angular motion of the earth; the difference of these motions is the relative diurnal motion, or the quantity by which the planet recedes every day from the earth, as observed by a spectator in the sun. Thus the mean motion of the earth in a day is 360° divided by 365d. 5h. 48m. 48s. or 365.242d.= 59′ 8″3, and that of Mercury 360°÷87d. 23h. 15m. 43″6 or 87.96925d.= 4° 5′ 32″4, the difference of which is nearly 3° 6′ 24″. Hence 3° 6′ 24″ : 360° :: 1 day : 115.88 days, the time wherein Mercury having left the earth will return to her again, or the time between two conjunctions of the same kind. The mean conjunctions of Venus is found in like manner thus; her daily mean motion is 360° ÷ 22d. 16h. 49′ 10″6 or 224.700814d. =1° 36′ 7″6 and 1° 36′ 7″6 — 59′ 3″3 = 36′ 59″3. Hence 36′ 59″3 : 360 :: 1 day : 583.96 days. This will also give the time between any two similar stations as two mean oppositions, &c.

These mean conjunctions, &c. are computed by the planets' mean motion, or on the supposition that they move equably in circular orbits; but as they really move in elliptic orbits, in which their motions are constantly variable, it may happen that the *true conjunctions* may differ a few days from the *mean*; however by having the *mean conjunction* given, the *true conjunction* is thus found; compute by astronomical tables the true places of the earth and the given planet, at the time of mean conjunction, found as above, from which their angular distance as seen from the sun will be given. Now the angular motions of the planets being given for any time, for example for four hours, the difference of these motions will give the access of the planet to, or its recess from, the earth in four hours. Then as this difference of motion : angular distance of the earth and planet :: 4 hours : the time between the mean conjunction and the *true conjunction* required.

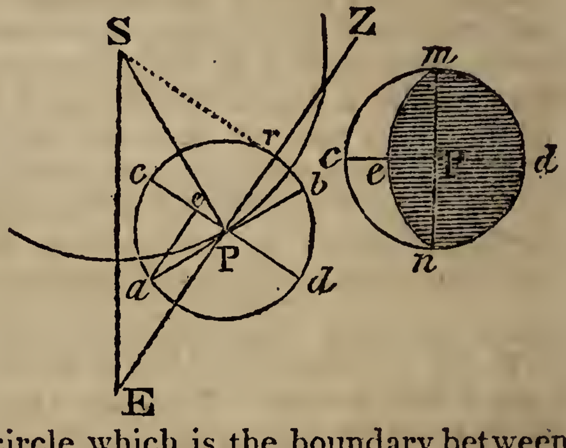
These transits* of Mercury are real *annular* eclipses of the sun, from which we discover that the planet borrows its light from it. When we observe it with a good telescope, it presents phases to us similar to those of the moon, directed in the same manner towards the sun, the variations of which, according to its relative position and the direction of its motion, throw great light on the nature of its orbit.

In his superior conjunction, or at the opposite side of the sun, that side of Mercury which is towards it, is likewise towards the earth, and when visible he appears nearly round. He never appears quite round, as he is either hidden by the sun's body, or the splendour of his rays, and therefore to us invisible ; sometimes he appears in the form of a half moon, and sometimes a little more or less than half his disk is seen. When he is in his inferior conjunction, or between the sun and the earth, the whole of his enlightened side is turned from the earth towards the sun, and hence he appears when seen on the sun's surface, like a dark spot, as has been observed before.†

The above proportion for finding the mean conjunction, &c. may be thus expressed in general; let P = the periodic time of a superior planet, p = that of an inferior, t = the time required; then, proceeding as above, we have $t = \frac{Pp}{P-p}$. And this will hold general for any two similar stations. (See the note, page 24.)

* The method of finding these transits and some other things of a similar nature, will be given in the next chapter.

† To exhibit these phases of the planets at any time; let S be the sun, E the earth, P an inferior planet, *a*P*b* the plane of illumination perpendicular to SP, *c*P*d* the plane of vision perpendicular to EP; draw *ae* perp. to *cd*; then *ca* is the breadth of the visible illuminated part, whose breadth to the eye is *ce*, the versed sine of *c*P*a* or SPZ, SP*c* being the complement of each. Now the circle *cad* will be projected into the right line *cd*, as its plane passes through the eye, but the circle which is the boundary between the illuminated and darkened part of the planet, being seen obliquely, will be an ellipsis (Vince's Con. Sect. p. 36, or Emerson's Project. of the Sphere, sect. 1, prop. 4.) hence if *cmdn* represent the projected hemisphere of the planet, which is next to the earth, *mn*, *cd* two diameters perpendicular to each other ; make *ce* equal the versed sine of SPZ or *c*P*a*, and describe the ellipsis *men*, then *mcnem* will represent the visible enlightened part, as it appears at the earth ; and from the property of the ellipsis (Emerson's Con. Sect. b. 1, prop. 73 and cor. 2) P*m* or P*c* : P*e* :: the semicircle *mcn* : the semi-ellipsis *men*; and by the nature of proportion P*m* : P*c* — P*e* or *ec* :: the area *mcn* : *mcn* — *men* or *mcne*. That is the *semidiameter* : *the versed sine of* *c*P*a* :: *half the whole disc* : *the visible enlightened part*.



Hence the planets *Mercury* and *Venus*, will have the same phases from their inferior to their superior conjunction, as the *moon* has from the new to the full; and the same from the superior to the inferior conjunction as the moon from the full to the new. *Mars* will appear *gibbous* in quadratures, as

As Mercury is always in the neighbourhood of the sun, and that his apparent diameter is so very small, he is seldom seen, and only appears a little after sun-set and again a little before sun rise. The light and heat which this planet receives from the sun, is computed to be about seven times greater than the light and heat which is re-

the angle cPa or its complement will differ considerably from a right angle, and its versed sine therefore from the diameter. In *Jupiter*, *Saturn* and *Herschel*, the angle SPZ never differs so much from 180° as to make these planets appear gibbous, and hence they always appear full orbed.

If P represent the moon; then as EP is small compared with SE, SP, these lines will be very nearly parallel, and the ∠ SPZ nearly equal SEP; hence, *the visible enlightened part of the moon varies nearly as the versed sine of her elongation from the sun.*

The following prob. of finding the position of Venus when brightest, on supposition that her orbit, together with that of the earth, were circular, was proposed by Dr. Halley, and solved by him in the Phil. Trans. numb. 349, and may be equally applied to Mercury.

Draw Sr perp. to EPZ, and take $a =$ SE, $b =$ SP, $x =$ EP, $y =$ Pr; then $b - y =$ the versed sine of SPr (to the rad. SP, as will be evident by describing a circle from the centre P with the distance PS, &c.) which, from the above, varies as the illuminated part; and as the intensity of light varies inversely as the square of its distance (see the following note) the quantity of light received at the earth varies as $\frac{b-y}{x^2} = \frac{b}{x^2} - \frac{y}{x^2}$; but (Euclid. 12 p. 2 b.) $a^2 = b^2 + x^2 + 2xy$; hence $y = \frac{a^2 - b^2 - x^2}{2x}$; this value of y being substituted in the above expression, we get the quantity of light $= \frac{b}{x^2} - \frac{a^2 - b^2 - x^2}{2x^3} = \frac{2bx - a^2 + b^2 + x^2}{2x^3} =$ a maximum, whose fluxion $= \frac{\overline{2b\dot{x} + 2x\dot{x}} + 2x^3 - 6x^2\dot{x} \times \overline{2bx - a^2 + b^2 + x^2}}{4x^6}$

$= \frac{4bx^3\dot{x} + 4x^4\dot{x} - 12bx^3\dot{x} + 6a^2x^2\dot{x} - 6b^2x^2\dot{x} - 6x^4\dot{x}}{4x^6} = \frac{\overline{a^2 - b^2} \times 6x^2\dot{x} - 8bx^3\dot{x} - 2x^4\dot{x}}{4x^6} = 0.$ Hence, clearing it of fractions, and dividing by $2x^2\dot{x}$, we get $\overline{a^2 - b^2} \times 3 - 4bx - x^2 = 0$, by transp. $x^2 + 4bx = 3a^2 - 3b^2$, which solved gives $x = \sqrt{3a^2 + b^2} - 2b$.

If we apply this equation to Mercury, $a = 1$, $b = .3871$ nearly (as calculated pa. 259) and hence $x = 1.00058$; then by trigonometry $a : x + b :: x - b : \frac{x^2 - b^2}{a} = .85131$ nearly, $=$ the difference of the segments of the base SE made by a perpendicular from P; then $\frac{a}{2} + \frac{.85131}{2} =$ the greater segment, and $\frac{a}{2} - .42565 = .07435$. Hence the following proportions;

As $b = .3871 : .07435 ::$ radius : cosine ESP $= 78° 55' 35''$, and $x = 1.00058 : .92565 ::$ rad. : cos. SEP $= 22° 14' 9''$. But the angle ESP at the time of the planet's greatest elongation is about 67°, &c. Hence Mercury is brightest between his greatest elongation and superior conjunction, and at this time his elongation will be 22° 14′ 9″.

ceived by the earth,* the solar disk as seen from Mercury being seven times greater than it appears to us. But the light and heat on this planet are more or less intense in proportion to its distance from the sun. This distance is very variable, the orbit of Mercury being more eccentric than any other planet.

The accelerating gravity of Mercury towards the sun is also seven times greater than on the earth.

It has not as yet been discovered by observation whether Mercury revolves upon its axis, and therefore we are ignorant whether it has the vicissitude of day and night, and still more so of their length. But as all the other primary planets perform this motion on their axis, from analogy it is extremely probable that Mercury is subject to the same law. We are also ignorant whether it has different seasons, because these depend upon the inclination of the axis of its rotation to the plane of the orbit, which it describes about the sun; but this is also unknown.

To an observer in Mercury, all the primary planets that we know would be superior, and appear as Mars, Jupiter, &c. do to us; and it is unknown to us whether the inhabitants of Mercury (if any) see any inferior planet; if not, the argument deduced from the phases of such planets to establish the true system of the world, will be wanting to them: for these phenomena clearly prove, that the planets move round the sun; but although we can observe no planet inferior to Mercury, it does not however follow that there are none, for Mercury itself is seldom seen by us; and a planet that would be much inferior to it, would never be visible on account of its nearness to the sun.

CHAP. III.

OF VENUS.

Venus, the next planet in order, offers the same phenomena as Mercury, with this difference, that its phases are much more sensible, its oscillations or elongations much more extensive, and their period more considerable. Her orbit, including that of Mercury, her periodic time must be greater. According to the latest and best observations, the sidereal revolution of Venus round the sun is 224d. 16h. 49m. 10.5888 sec.† Her greatest elongation, according to La Land, is 47° 48′, and least 44° 57′; her greatest, according to Laplace, is (53°) 47° 42′, and least (50°) 45°; the mean of

* Light or heat, so far as it depends on the sun's rays, decreases in proportion as the square of the distances of the planets from the sun. (Ferguson's Astro. art. 169, Smith's Optics, b. 1. art. 57, or Emerson's Optics, b. 1, prob. 6, and corollaries.) The same may be easily proved of any virtue or fluid substance flowing from or to a centre. See Gregory's Astronomy, b. 1. prop. 48.

† The method of finding the periodic time is given in the notes in chap. 2 prob. 9. See also chap. 7.

these last is 46° 21′. The mean length of its entire oscillation is 584 days.* Here it may be asked why Venus necessarily remains a longer time to the eastward or westward of the sun than the whole time of her entire revolution; but when we consider that the relative motion of Venus is greater than her absolute motion, because while Venus is moving round the sun, the earth is performing its motion round the sun the same way, the question is therefore easily answered. The retrogradations of Venus commence or end when the planet, approaching the sun in the evening or receding from it in the morning, is distant from it according to Laplace 32°, in our measures 28° 48′.† The mean arc of its retrogradation is about (18°) 16° 42′. The distance‡ of Venus from the sun is found from its elongation equal 67165759.2 miles, and from its periodic time 67435662 67 nearly.

The eccentricity of the orbit of Venus, according to Vince or La Land, is 498, the mean distance of the earth from the sun being 100000 of these parts. Hence her eccentricity in miles is 462271.3 nearly §

According to Laplace Venus's mean distance from the sun is 0.723332,|| the earth's being 1 or an unit; proportion of the eccentricity of the semimajor axis for the beginning of the year 1750

* When Venus appears *west* of the sun, she rises before him in the morning, and is called the *morning star*: when she appears *east* of the sun, she shines in the evening after sun set, and is then called the *evening star*: being alternately morning and evening star each 292 days.

† Here we must again notice, that these numbers are given wrong in the English edition of Laplace's Astronomy, 28° 48′ being given 27° 48′, and 16° 42′, 16° 12′.

‡ The distance of Venus from the sun may be found in the same manner as that of Mercury, in chap. 2d. Thus in the triangle SEM (pa. 259) let M now represent Venus, and the angle SEM be taken equal 46° 21′.—Hence rad. : s. 46° 21′ :: 23464.5 : SM 16978.2 the distance of Venus from the sun in semidiameters of the earth, which multiplied by 3956 gives 67165759.2 miles.

The same by the periodic times, &c. 224 d. 16 h. 49 m. 10.6 sec. = 19414150″ 6 the square of which is 376909243519480.36, which divided by 995839704797184 (see the note pa. 259) gives 378473843879 nearly, the cube root of which is .7254. Hence .7264 × 23464.5 × 3956 = 67435662.6748 miles, the mean distance of Venus from the sun by Kepler's rule and extremely near the above, considering the great difference in the principles of calculation, and that an error of a few seconds in the elongation will make a considerable difference. This is a strong proof of the truth of Kepler's laws and of the copernican system. The dist. of Venus from the sun, and her periodic time being given, her hourly motion may be found as in the note pa. 9. for Mercury, thus; 67435662.67 × 2 = 134871325.34 = the mean diam. of her orbit, which multiplied by 3.1416 gives 423711755.688 miles its circumference; then 224 d. 16 h. 49′ 10″ 6 : 1 h :: 423711755.688 : 78569 miles the hourly motion of Venus.

§ For 100000 : 498 :: 92825662 miles the earth's mean distance from the sun : 462271.29876.

|| If .723332 be multiplied by 92825562 miles, the earth's mean distance (see note, pa. 255) the result will be 67143699.4 nearly, the mean distance of Venus according to Laplace.

= 0.006885 ; and the secular diminution of this proportion 0.000062905.

The place of Venus's *aphelion* for the beginning of 1750, according to Vince, was 10s. 7° 46′ 42″, and its motion in longitude for 100 years 1° 21′. Its *greatest equation* is 47′ 20″ — According to Laplace the longitude of the *perihelion* for 1750 was (141° 9759) equal 127° 46′ 41″ 9 or 7° 46′ 41″ 9 in Leo, the sidereal retrograde motion in 100 years (699″ .07) equal 3′ 46 ″4.*

Delambre makes the place of Venus in the beginning of 1800, 4s. 25° 9′ 1″, of her aphelion 10s. 8° 36′ 12″, and of her node 2s. 14° 52′ 8″, and makes the secular variation of the aphelion 1° 21′, and of her node 51′ 40″. Her daily mean motion, according to the same author, is 1° 36′ 8″, her hourly motion is 4′, her motion in one minute 4″, and in one second 4‴, &c.

The apparent diameter of Venus continually varies, which proves that her distance is no less variable. Her distance from the earth being the least at the moment of her transit over the sun's disk, her apparent diameter will then be the greatest, and will decrease until she arrives at her superior conjunction, where her diameter will be the least. The position of the earth in its orbit will also vary it a little. By having the apparent diameter and the planet's distance from the earth at any time, its apparent diameter corresponding to any other distance may be easily found, as it varies nearly in proportion to the distance. The greatest diameter of Venus at a medium, is about 58″.† Her real diameter is therefore 7301.7 miles, and her magnitude is in proportion to that of the earth as 1 : 1.2742.

* Newton in the Scholium to prob. 14, b. 3. of his Principia, says, that by the theory of gravity the aphelions of the planets near the sun, from the action of those more remote, move a little in *consequentia* in respect of the fixed stars, and that in the sesquiplicate proportion of their several distances from the sun ; so that if the aphelion of Mars in the space of 100 years be carried 33′ 20″, in *consequentia* in respect of the fixed stars, the aphelions of the earth, of Venus, and of Mercury. will, in a hundred years, be carried forwards 17′ 40″, 10′ 53″, and 4′ 16″ respectively. See this property demonstrated in Emerson's comment on the Principia, pa. 83.

† Mr. Bliss at Greenwich in 1761, June 6th, from three good observations of Venus on the sun's disk, finds its diameter 58″. Mr. Short in London makes it 58″, and the diameter of the sun 31′ 33″ 24‴. The above observations were made nearly at the same time. Laplace makes the diameter of Venus (177″) 57″ 3 at the moment of her transit. Hence at a medium the diameter is taken equal 58″. Now the mean distance of the earth from the sun being 23464.5 semidiameters, and that of Venus 16978.2 semidiameters ; hence the difference 6486.3 semidiam. is the distance of Venus from the earth, and therefore inversely, 16978.2 : 6486.3 :: 58″ : 22″ 1, the apparent diameter of Venus as seen from the sun.— (The distance of Venus from the earth being here taken to correspond with its greatest diameter.) And again, 23464.5 : 6486.3 :: 58″ : 16″, her apparent diameter at the distance of the sun from the earth, or her mean apparent diameter. Laplace makes this (51″ 54) 16″ 6. Now 31′ 33″ 4 (the sun's ap. diam.) 16″ :: 864065.5 (the sun's real diam. in miles) : 7301.7 miles the diameter of Venus.

Venus does not perform her revolution round the sun exactly in the plane of the ecliptic, but sometimes deviates from it several degrees. At the beginning of 1750, the inclination of the plane of her orbit to that of the ecliptic, was, according to Laplace (3°.7701) 3° 23′ 35″, and the secular variation of this inclination to the *true** ecliptic (13″80) 4″47 increasing. The distance of this planet from the ecliptic, as seen from the earth or its geocentric latitude, will sometimes exceed the inclination of its orbit. In the Nautical Almanac for 1812, Aug. 13, it is made 7° 41′.

The longitude or place of the ascending node of Venus was 74° 26′ 18″, or 14° 26′ 18″ in Gemini, and the descending node was therefore in the opposite sign and degree. The motion of the nodes in 100 years, according to Vince, is 51′ 40″. Laplace makes the sidereal and secular motion of the node on the true ecliptic (—5673″60) 30′38″2 decreasing.†

All the primary planets, except Mercury and Uranus or Herschel, are found to have a rotary motion on their axis, or like the earth a diurnal motion; and from analogy we conclude that these planets observe the same universal law, though at present not within the reach of observation; no telescope possessing sufficient magnifying power to exhibit this phenomenon in these two planets,

Or the diameter may be found by trigonometry, in the same manner as that of Mercury has been found pa. 263, using the angle 11″ in place of 8″ 9, and taking Venus's mean distance 16978.2 semidiameters of the earth; thus,

As sine (90°—11″) 89° 59′ 49″ - -	10.0000000
To sine 11″ - - - - -	5.7269676
So is 16978.2 semid. - - - -	4.2298916
To .90544 semid. - - - -	1.9568592

Hence .90544 × 2 × 3956 = 7163.84 miles the diameter of Venus, by this method, which would more nearly agree with the above if the decimal parts of the seconds, &c. were retained.

Now the cube of the earth's diameter divided by the cube of the diameter of Venus, will give the proportion of their magnitude thus; $\frac{7911^3}{7301.7^3}$ = 1.2742 or log. 7911^3 — log. 7301.7^3 = 0.1052575, the number corresponding to which is 1.2742.

* The true ecliptic is the ecliptic corrected, or when allowance is made for the secular variation. See the note to prob. 49, part 2d. In the same manner the obliquity of the planet's orbit to the plane of the ecliptic, at any time, and its secular variation being given, its obliquity at any other time may be found, as is evident. See the method of finding it by observation, &c. part 4th. ch. 2d. pa. 264, note.

† The secular motion of the nodes being given, their place for any time may be found; and the periodic revolution of the planet round the sun being also given, and its distance from the node, its distance at any other time may from thence be easily found. But the inclination of its orbit to that of the ecliptic being given, its place in the true ecliptic or longitude, and its distance from it or latitude may be easily found, in the same manner as the right ascension and declination of the sun is calculated; by the solution of a right angled spherical triangle.

the one being too near, and the other too far removed from the sun. The cause of this interesting phenomenon is not yet discovered; but from the numberless improvements in natural sciences, it is very probable that in a short time it will develope itself, and probably from its connection with the law of universal gravity, it will throw new light on that intricate subject.

Galileo in 1611, was the first that observed this phenomenon in Venus. In 1666 M. *Cassini* discovered a bright spot upon her straight edge when dichotomised, similar to those on the moon's surface, and found the time of its sidereal motion to be 23h. 16′. In 1726 *Bianchini*, from some observations on Venus, asserted in his *Hesperi et Phosphori nova phenomena*, that the time of her rotation was $24\frac{1}{3}$ days, that her *north pole* answered to the 20th degree of Aquarius, and was elevated 15° or 20° above its orbit, and that her axis continued parallel to itself. M. *Cassini*, the son, makes it about 23h. 20′. *Schroeter*,* from several continued observations of the variation of her horns, and of some luminous points towards the edges of the dark parts, has confirmed Cassini's result, which had been disputed before. He fixes the duration of her rotary motion at 23h. 21′ 7″ 2, and like Cassini has found that the equator of Venus makes a considerable angle with the ecliptic. He has also concluded the existence of high mountains on her surface, from his observations, and the law by which her light varies from her enlightened to her dark side. (Phil. Trans. 1795.) He supposes the planet surrounded with an extensive atmosphere, the refracting power of which differs but little from that of the earth. The cusps or horns appeared sometimes to run 15° 19′ into the dark hemisphere, and hence he computes that the height of the atmosphere, to refract such a quantity of light, must be 15156 Paris feet, or 16146.4 English. But this must depend on the nature and density of the atmosphere, of which we are ignorant. (Phil. Trans. 1792.) Dr. Herschel agrees with M. Schroeter, that Venus has a considerable atmosphere: he has published in the Phil. Trans. for 1793, a long series of observations on this planet, from which he concludes, 1. That the planet revolves on her axis, but that the period and the position of the axis are uncertain; 2. That the planet has a considerable atmosphere; 3. That there are probably hills and inequalities upon her surface, although he has not been able to see much of them, owing, perhaps, to the density of her atmosphere; and 4. That this planet is somewhat larger than the earth, instead of being less, as former astronomers imagined.

* Schroeter, a learned astronomer of Lilienthal, in the Duchy of Bremen. Among others he has published a new work on the height of the mountains of Venus, some of which he makes upwards of 23000 toises, which is more than seven times the height of Chimboraço, in South America. He says that in the moon there are mountains 1000 toises higher than Chimboraço. But Dr. Herschel, considers the height of lunar mountains in general as greatly overrated, and estimates them at no more than half a mile perpendicular height.

M. De la Hire observed with a telescope 16 feet long, mountains in Venus higher than the moon ; but the difficulty of observing those as well as the spots, particularly in northern latitudes where the atmosphere is so dense, renders the result very doubtful.

Venus surpasses in brightness all the other planets and stars, and is sometimes so brilliant as to be seen in the day with the naked eye.* The light and heat which she receives from the sun, are about double to what the earth receives †

The inclination of the axis of Venus to that of her orbit, is, according to most astronomers, about 75°, which is 51° 32′ greater than the inclination of the equator and ecliptic. This is a singular circumstance, and must cause a great variety in the seasons of Venus. Ferguson remarks that the north pole of her axis inclines towards the 20th degree of Aquarius, our earth's to the beginning

* The equation investigated pa. 269, being here applied to Venus, we have $a = 1$, $b = .7254$ (pa. 271) then $x = \sqrt{3a^2 + b^2} - 2b = 1.8778 - 1.4508 = .427$; hence the angle ESP (Venus being supposed at P, see the fig. page 268) = 22° 8′ 26″. For $a : b + x :: b - x : \frac{b^2 - x^2}{a} = .343876$ nearly, = the diff. of the segments of the base a or SE, made by a perp. from P ; then $\frac{a}{2} + \frac{.343876}{2} = .6719$ nearly, = the greater segment ; and $\frac{a}{2} - \frac{.343876}{2} = .3281$ nearly, = the lesser segment. Hence the following proportions ; As $b = .7254 : .6719$ :: rad. : co. sine ESP = 22° 8′ 26″; and $x = .427 : .3281$ :: rad. : co. sine SEP = 39° 47′ 27″ = the elongation of Venus from the sun, when brightest. The angle ESP at the time of the planet's greatest elongation is 43° 40′, according to Vince ; and therefore Venus is brightest between her inferior conjunction and greatest elongation. The angle SPZ is also equal ESP + SEP (Eucl. 1, prop. 32) = 61° 55′ 53″ or 61° 56′ nearly, the versed sine of which is 0.53 nearly, radius being unity ; hence the visible enlightened part : the whole disk :: 0.53 : 2 (note to pa. 268, or art. 195, Vince's Ast.) Venus therefore appears a little more than ¼th illuminated, and answers to the appearance of the moon when 5 days old. Her diameter is here about 39″, and therefore the enlightened part is about 10″ 25. At this time Venus is bright enough to cast a shadow at night. This appearance of Venus takes place about 36 days before and after her inferior conjunction with the sun. For suppose Venus in conjunction with the sun, and when seen from the sun to depart from the earth at the rate of 37′ in 1 day (Vince) we have 37′ : 22° 8′ 26″ :: 1d. : 36 days nearly, the time from conjunction until Venus is brightest. De Lambre makes the daily mean motion of Venus 1° 36′ 8″, and that of the earth = 59′ 8″3 ; hence 1° 36′ 8″ — 59′ 8′ 3 = 36′ 59″ 7, nearly equal 37′, as above.

Vince remarks, that when Venus is brightest, and at the same time is at her greatest north latitude, she can then be seen with the naked eye at any time of the day, when she is above the horizon ; for when her north latitude is the greatest, she rises highest above the horizon, and therefore is more easily seen, the rays of light having to pass through a smaller portion of the atmosphere, in proportion as Venus is elevated. This takes place once in 8 years, Venus and the earth returning to the same parts of their orbits after that interval of time.

† This is found by dividing the square of the earth's distance from the sun, by the square of the distance of Venus from the sun. See the note, pa. 270.

of Cancer. Consequently the northern parts of Venus have *summer* in the signs, where those of our earth have *winter*, and *vice versa*. That the artificial day at each pole of Venus is 112½ of our natural days.* The sun's greatest declination on each side of the equator of Venus amounts to 75°;† hence her tropics are only 15° from her poles, and her polar circles 15° from her equator.—The tropics of Venus are therefore between her polar circles and her poles, contrary to what those of the earth are.

The day in Venus making so considerable a part of her year, the sun will therefore change his decl. so much in one day, that if it be vertical to any place in the tropic, the next day it will be about 26° from it; and in one day he will remove from the equator about 36¼°. So that the sun changes his decl. 14° more at a mean rate in one day on Venus, than in a quarter of a year on the earth.‡

If the inhabitants about the north pole of Venus fix their south or meridian line, through that part of the heavens where the sun has his greatest altitude, or north declination, and call those the east and west points on the horizon, which are 90° from that point where the meridian cuts the horizon; then the following remarkable phenomena will take place. The sun will rise 22½° north of the east, and advancing 112½° (90° + 22¼°) as measured on the horizon, he will cross the meridian at an altitude of 12½°; then making an entire revolution without setting, he will cross it again at an alt. of about 48½°; at his next revolution he will come to his greatest alt. and decl. and cross the meridian at an altitude of 75°; and distant from the zenith of the place 15°. Again, he will descend in the same spiral manner, first crossing the meridian in an angle of 48½°, next in an angle of 12½°, and advancing from thence 112½°, he will set 22½° north of the west; so that after having made 4⅝ revolutions above the horizon, he will descend below it to exhibit similar phenomena at the south pole.

The polar inhabitants of Venus, like those of our earth, have but one day and one night, each of half a year long, or one half of Venus's annual revolution. On Venus, however, the difference

* Or rather half of her annual revolution (see page 270) the sun being visible at her poles during this time.

† Whatever problems we have performed on the terrestrial globe relative to the sun's greatest decl. 23° 28′, &c. may be applied to Venus, on supposition that the greatest declination is 75°.

‡ If any point be taken on the equator of our common globes, and another be taken, in a lesser circle drawn 15° from either pole, at the distance of 90° east or west of the former, and through these points a great circle be drawn, with the quadrant of alt. by which it may be also divided into degrees; this will represent the ecliptic of Venus, and hence the above phenomena may be easily pointed out on the globe.

The great variation in the sun's decl. seems to be providentially ordered, to prevent the great effects of the sun's heat, which on Venus is twice as great as on the earth (pa. 275) as he can shine perpendicularly on the same place but a short time, and on that account the heated places have time to cool.

between the heat of summer and the cold of winter, and also between midday and midnight, is much greater than on the earth, the sun's daily variation and the change of his declination and altitude being much greater on Venus than on the earth. When the sun is in the equinoctial, or over the equator of Venus, one half of his disk appears above the horizon of the north pole, and one half above the horizon of the south pole, his centre being in the horizon of both poles ; and when he descends below the horizon of one, he ascends above the horizon of the other in the same proportion. Hence, in the course of a year, each pole has one spring, one autumn, a summer as long as both, and a winter equal in length to the other three seasons.

At the *polar circles* of Venus, the seasons are nearly the same as at the equator, the distance between both being 15° ; but the winters are not so long, nor the summer so short. The same seasons also happen twice a year.

At Venus's *tropics*, the sun continues about 15 of our weeks without setting, in summer, and as long without rising in winter. While his declination is more than 15°, he does not set to the inhabitants of the adjacent tropic, nor rise to the inhabitants of the other. The seasons are also, at her tropics, nearly the same as at her poles ; the difference being similar to that at the polar circles.

At her *equator* the days and nights are equal, each being about $11\frac{2}{3}$ hours long. The diurnal and nocturnal arches are here however very unequal, particularly when the sun's declination is greatest ; for at this time his meridian alt. is sometimes double his midnight depression, and at other times the reverse. At her equator there are two winters, two summers, two springs, and two autumns every year, owing to the obliquity of the sun's rays when his declination is greatest, being equal to that in the latitude 51° 32′ (75° — 23° 28′) on the earth at the winter solstice. But every winter at the equator is double the length of the summer, the four seasons returning twice in that time, that is in $9\frac{1}{4}$ days.

From the quick change in the sun's declination, the sun's amplitude at rising and setting will differ considerably ; and hence no place has the forenoon and afternoon of the same day equally long, unless at the equator or at the poles.

Where the sun crosses the equator or equinoctial of Venus in any year, he will have 9° decl. from that point on the same day and hour the following year ; and will cross the equator 90° more to the west. This phenomenon will make the *equinoxes* of Venus a quarter of a day, or about 6 of our days later every year, and hence in four annual revolutions the sun will pass vertically in the same places,* &c. &c.

* Many other observations could be made here, but the above are sufficient to enable the learner to pursue the subject at his leisure, and, in a similar manner, to examine the phenomena of the other planets, and exhibit them on the globe, independent of an orrery or any other instrument. The investigation of these curious phenomena, and their representation on the

Venus, when viewed through a telescope, exhibits all the phases of the moon from the crescent to the enlightened hemisphere, though she is seldom observed perfectly round. Previous to the rising of the sun in the morning, when she begins to disengage herself from the sun's rays, she is seen under the form of a crescent, at which time her apparent diameter is at its maximum, being then nearer to us than the sun, and almost in conjunction with him. In proportion as she recedes from the sun, her crescent augments and her apparent diameter diminishes. When she departs from the sun about 45°, she returns towards him again, during which time her enlightened hemisphere is increasing, and her apparent diameter diminishing, until she is again immersed, in the morning, in the solar rays. She is then further from us than the sun; the hemisphere which is turned towards the sun is also towards us, and therefore Venus appears full. Her apparent diameter is then a mininum or the least possible. Here Venus disappears for some time, after which she re-appears in the evening, and produces, in a contrary order, the same phenomena as before. Her crescent diminishes, and her apparent diameter increases as she advances from the sun, and her enlightened hemisphere is turned from the earth. At about 45° distance from the sun, she returns again towards the sun, her crescent diminishing, and apparent diameter increasing, until she again plunges into the sun's rays.

These phenomena evidently prove that Venus's orbit is within that of the earth, and that she revolves round the sun, which is nearly in the centre of her orbit. These results obtained from observations of the phases and apparent diameter of Venus combined with the earth's annual motion round the sun, explain also, in a natural manner, the alternate, direct, and retrograde, motion in longitude of this planet, also her complicated motion in latitude; and the same is true of Mercury. (See page 257.)

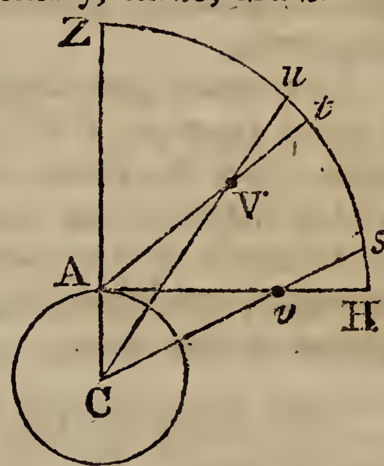
During the interval between Venus's disappearance in the evening and her re-appearance in the morning, she is sometimes seen moving on the disk of the sun, in the form of a dark round spot.

Dr. *Halley* remarks, that when at St. Helena, observing the stars about the south pole, he had an opportunity of observing Mercury passing over the sun's disk, which he observed with the greatest degree of accuracy, by means of a telescope 24 feet long, and found the time of the ingress and egress without being subject to an error of 1″. The lucid line intercepted between the dark limb of the planet and the bright limb of the sun being visible to the naked eye, and the small dent made in the sun's limb by Mercury's entering the disk, appearing to vanish in a moment. From

globes, must afford no small pleasure to those who are well acquainted with what we have delivered in parts 2d and 3d; as these phenomena afford ample scope for inquiry and investigation, and every moment present new scenes of wonder to the mind. These inquiries likewise bring us more acquainted with the variety of those curious and admirable laws displayed in the mechanism of the universe.

this he concluded that the sun's parallax might be accurately determined by such observations, from the difference of the times of the transit over the sun at different places upon the earth's surface,* provided Mercury were but nearer to the earth, and had a greater parallax from the sun. But the difference of these parallaxes, and therefore the difference of times is so small, that the difference of the parallaxes is always less than the solar parallax sought; and hence Mercury was considered unfit for this purpose. Venus was therefore selected, for its parallax being nearly 4 times as great as the solar parallax; and therefore producing a considerable difference between the times at which Venus will be seen to pass over the sun at different parts of the earth, so that the accuracy of the conclusion will be proportionably increas-

* For the method of finding the horizontal parallax of Venus by observation, and from thence by analogy, the parallax and distance of the sun and of all the planets from him. See Ferguson's Astronomy, ch. 23, art. 2.



The following will give the learner a sufficient idea of the nature, &c. of a parallax. Let C be the earth's centre, A the place of a spectator on its surface, V any object, ZH the sphere of the fixed stars, to which the places of all the planets, &c. are referred, Z the zenith, and H the sensible horizon; through the object V conceive the lines AV*t*, CV*u* to be drawn, then *t* is the place of the object as seen from the surface of the earth or its *apparent* place, and *t* its place as seen from the earth's centre C, or its true place, and the arch *tu*, the distance between the apparent and true place, is the *parallax* of the object, and is measured by the angle *t*V*u*. The apparent place of the object observed at the horizon is H, and its true place is *s*, the *horizontal parallax* is therefore H*s* measured by the angle H*vs*, and is greater than any other parallax *tu*. At the zenith Z, the parallax is nothing, for here there is no difference between the true and apparent place. That the angle AVC measures the parallax is thus shewn; the angle ZAV = ZCV + AVC (32 E. 1) AVC is therefore the difference in the zenith distances or places of the body, as seen from the centre and surface of the earth, or the angle under which the diameter of the earth appears, as seen from the object or body V. Now to find this angle we have this proportion, CV : VA :: sine VAC : sine AVC (Simson's Trig. prop. 2) $= \frac{\text{CA} \times \text{sine VAC}}{\text{CV}}$. Now as CA is constant, the earth being supposed a sphere, *the sine of the parallax varies as the sine of the apparent zenith distance directly, and the distance of the body from the centre of the earth inversely*, or as $\frac{\textit{sine}\ \text{VAC}}{\text{CV}}$. Hence appears why the parallax is greatest at the horizon, and nothing at the zenith. If the object be at an indefinitely great distance, it has no parallax; the apparent places of the fixed stars are not therefore altered by it. The parallax depresses an object in a vertical circle, *t* being the apparent and *u* the true place. The parallax varies as the sine of the apparent zenith dist. or $1 : x :: y : xy$, x being the appar. zen. dist. and y the horizontal parallax, radius being 1. To ascertain the parallax at all altitudes, it must therefore be found for some given alt. For different methods by which this is performed, see Keil's Ast. lect. 31 or Vince's Ast. 8vo. ch. 6, pa. 54, &c.

ed, and not liable to any error greater than a small part of a second. (See Motte's abridgment of the Phil. Trans. vol. 1. pa. 243.) The transits of 1761 and 1769 affording every opportunity of putting these observations into practice, astronomers were therefore sent from England, France, &c. to the most proper parts of the earth, to observe both those transits, and the results of their observations give the parallax to a great degree of accuracy.

If the plane of the orbit of Venus coincided with the plane of the ecliptic, she would pass directly between the earth and the sun at each inferior conjunction, and would then appear like a dark spot on the sun for about 7¾ hours, but like the moon's, Venus's orbit only intersects the ecliptic in the nodes, and therefore one half of it is on the north and the other half on the south side of the ecliptic. Hence Venus can never be seen on the sun but at those inferior conjunctions which take place in or near the nodes of her orbit. At all other conjunctions she passes either above or below the sun, and is invisible, her dark side being then turned towards the earth.

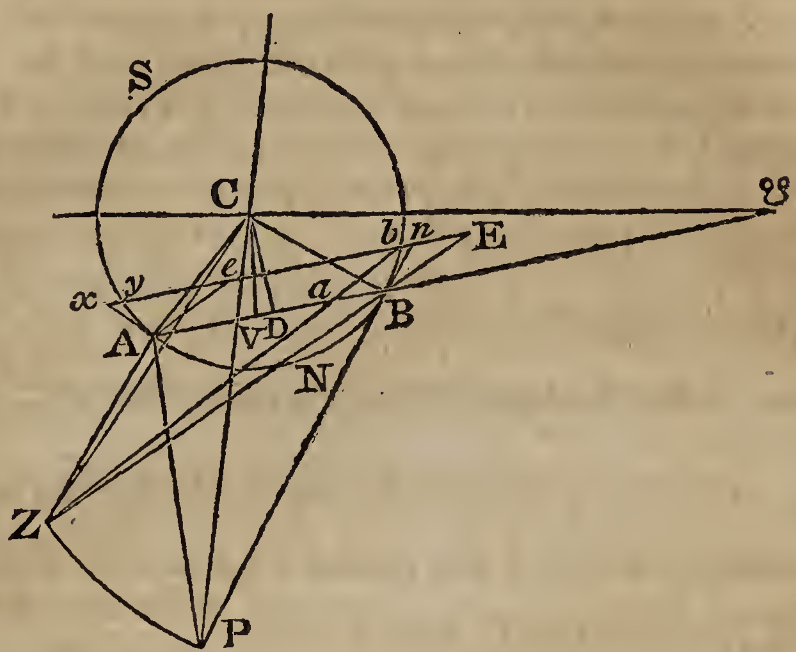
The mean time from conjunction to conjunction of Venus being known (see pa. 275*) and the time of one mean conjunction, the time of all the future mean conjunctions will be given. If those which happen near the node be therefore found,† and the geocentric latitude of the planet be then computed; if it be less than the apparent semidiameter of the sun, there will be a transit of the planet at that time.

* The conjunctions of any number of planets, in circular orbits, may be thus calculated; Let $\frac{Pp}{P-p}$ = the time in which two superior planets would meet from one conjunction to the next (see the note pa. 268.) and let V equal to the periodic time of an inferior planet, taking the planets in order, then by a like process, as in the note pa. 268. $\frac{Pp}{P-p} \times V \div \frac{Pp}{P-p} - V = \frac{PpV}{Pp - V \times \overline{P-p}}$. In like manner it will be found for a fourth, that $\frac{PpV}{Pp - V \times \overline{P-p}} \times Q$ divided by $\frac{PpVQ}{PpV - Q \times \overline{Pp - V \times \overline{P-p}}}$ = the time required, Q being taken in order after V. Whence the general law is manifest. In the same manner oppositions, &c. may be calculated.

This calculation may also be applied to elliptic orbits, provided the formula for the daily revolution be substituted for the daily angular velocity.

† Vince in his Astronomy determines the periods when such conjunctions happen, in the following manner; let P = the periodic time of the earth, p = that of Venus or Mercury; now that a transit may happen again at the same node, the earth must perform a certain number of complete revolutions in the same time that the planet performs a certain number, for then they must come into conjunction again at the same point of the earth's orbit, or nearly in the same position with respect to the node. Let the earth perform x revolutions, while the planet performs y revolutions, then will $Px = py$; hence $x \div y = p \div P$. Now P = 365.256, and for *Mercury* p = 87.968;

The following new method of computing the effect of parallax, in accelerating or retarding the time of the beginning and end of a transit of Venus *or* Mercury *over the sun's disk, was given by the late* Nevil Maskelyne, *astronomer royal in Greenwich, and inserted here for the sake of those who wish to know the principles of this interesting phenomenon, &c. The fig. relates particularly to the transit of* Venus *which happened in* 1769.



Let SN represent the sun, C its centre, P the north pole of the equinoctial, PC a meridian passing through the sun, Z the zenith of the place ADB☋ the relative path of Venus, ☋ being the place of the descending node; A the geocentric place of Venus at her ingress, B at her egress, and D at her nearest approach to the sun's centre, as seen from the centre of the earth, and *b* the apparent place of Venus at

$\frac{p}{P} = \frac{87.968}{365.256} =$ (by resolving it into its continued fractions. (See Trail's Algeb. Appendix No. 1, &c.) $\frac{1}{4}, \frac{6}{25}, \frac{7}{29}, \frac{13}{54}, \frac{33}{137}, \frac{46}{191}$, &c. that is 1, 6, 7, 13, &c. revolutions of the earth are equal to 4, 25, 29, 54, &c. revolutions of Mercury, approaching still nearer to equality the further the series is carried. The first period, or that of 1 year, is not sufficiently exact; the period of 6 years will sometimes bring on a return of the transit at the same node; that of 7 years will bring on the transit's return more frequently; that of 13 still more frequently, &c. There was a transit of Mercury at his descending node in May, 1786; hence all the years at which the transit may be expected to happen at that node, will be found by continually adding 6, 7, 13, 33, 46, &c. to it. In November, 1789, there was a transit at the ascending node; hence by adding the same numbers to this year, the years at which it may be expected to happen, at the same node, will be found. The following years 1799, 1832, 1845, 1878, 1891, &c. are the years for the transit to happen at the descending node; and 1802, 1815, 1822, 1835, 1848, 1861, &c. at the ascending node. For *Venus* $p = 224.7$; hence $\frac{p}{P} = \frac{224.7}{365.256} = \frac{8}{13}, \frac{235}{382}, \frac{713}{1159}$, &c. Hence the periods are 8, 235, 713, &c. years. The transits at the same node will therefore sometimes return in 8 years, but oftener in 235, and still oftener in 713 years. In 1769 there was a transit of Venus in June, at the descending node; the next transits at the same node will be in 2004, 2012, 2247, 2255, 2490, 2498, 2733, 2741 and 2984. In 1639 a transit happened at the ascending node in November; the next transits at the same node will be in 1874, 1882, 2117, 2125, 2360, 2368, 2603, 2611, 2846 and 2854. These transits are found by continually adding the periods, so as to find the years when they may be expected, and then com-the egress to an observer whose zenith is Z; draw baZ, then a is the *true* place of Venus, when the *apparent* place is at b, and bn is the parallax in altitude of Venus from the sun; and the time of contact will be diminished by the time which Venus takes to describe aB. Through b draw $xybn$E parallel to AB, meeting ZB produced in E, and Bn, Ax tangents to the circle, and let CD be perpendicular to AB. Now in the figure abEB, the sides may be considered as rectilinear, on account of their smallness; and from the magnitude of ZB compared with Ba, BE may be considered as parallel to ab, and hence abEB may be considered as a parallelogram; aB may be therefore taken equal bE. Now $b\text{E} = \text{E}n \pm bn$, according as E falls without or within the sun's disk BNS; and En : EB :: sine EBn = cos. CBZ (because CBn is a right angle. 18 Eucl. 3) : sine BnE = sine nBD or nB☋ (29 Eucl. 1) = cos. CBD; (Simson's Trig. prop. 2) hence $\text{E}n \times \cos. \text{CBD} = \text{EB} \times \cos. \text{CBZ}$ (16 Eucl. 6) and therefore $\text{E}n = \frac{\text{EB} \times \cos. \text{CBZ}}{\cos. \text{CBD}}$. Now as $bn \times ny = \text{B}n^2$ (36 Eucl. 3) hence $bn = \frac{\text{B}n^2}{ny} = \frac{\text{B}n^2}{\text{AB}}$ very nearly; but Bn : BE :: sine BEn = sine ZBD (29 Eucl. 1) : sine BnE = cos. CBD; hence $\text{B}n = \frac{\text{BE} \times \text{sine ZBD}}{\cos. \text{CBD}}$, and squaring both sides $\text{B}n^2 = \frac{\text{BE}^2 \times \text{sine ZBD}^2}{\cos. \text{CBD}^2}$; therefore $bn = \frac{\text{BE}^2 \times \text{sine ZBD}^2}{\text{AB} \times \cos. \text{CBD}^2}$. Let p = the horizontal parallax of Venus from the sun; then, as the parallax varies as the sine of the apparent zenith distance, we have Rad. or 1 : sine Zb :: p : BE (note pa. 279) $= p \times$ sine Zb $= p \times$ sine ZB; hence $a\text{B} = b\text{E} = \text{E}n \pm bn$ (and substituting the value of BE) $= \frac{p \times \text{sine ZB} \times \cos. \text{CBZ}}{\cos. \text{CBD}} \pm p^2 \times \text{sine ZB}^2 \times \frac{\text{sine ZBD}^2}{\text{AB} \times \cos. \text{CBD}^2} = p \times \text{sine ZB} \times \cos. \text{CBZ} \times \sec. \text{CBD} \pm p^2 \times \text{sine ZB}^2 \times \frac{\text{sine ZBD}^2 \times \sec. \text{CBD}^2}{\text{AB}}$. (For the secant of an arch is as the cos. reciprocally, or $\sec. = \frac{1}{\cos.}$*) Hence the parallax consists of two parts, one of which varies as p, and the other as

puting the shortest geocentric distance of Venus from the sun's centre at that time, when their conjunction takes place; if this distance be less than the semidiameter of the sun, there will be a transit.

The periods of the planets made use of above, are as given by Vince. Laplace makes the sidereal revolutions of Mercury, Venus, and the earth, 87.969255, 224.700817, and 365.256384 days respectively.

* The radius being a mean proportional between the cos. and sec. (Cor. 2, def. 9, Simson's Trig.) hence Rad.^{2} = cos. × sec. and rad. being 1, we have $\sec. = \frac{1}{\cos.}$. See b. 1, prop. 1, Emerson's Trigonometry.

p^2, the remaining quantities being the same. Let those quantities multiplied by $p = v$, and those multiplied by $p^2 = w$; let $t'' =$ the time in which Venus, by her geocentric relative motion, takes to describe the space p, and let m be the relative horary motion of Venus; then to find this motion we have $m : p ::$ 1 hour or $3600'' : t'' = \frac{p \times 3600''}{m}$. Hence to find the time of describing aB, we have $p : p \times v \underline{+} p^2 \times w :: t : tv \underline{+} tpw$, where by substituting the values of v and w, the time of describing aB, or the effect of parallax in accelerating or retarding the time of contact is given; the upper sign is to be used when CBZ is acute, and the lower sign when obtuse. If CBZ be a right angle very nearly, but obtuse, it may happen that nE may be less than nb, in which case nE is to be taken from nb, according to the rule. The principal part, nE of the effect of parallax, will increase or diminish the planet's distance from the sun's centre, according as the angle ZBC is acute or obtuse; but the small part bn of the parallax will always increase the planet's distance from the centre; hence the sum or difference of the effects, with the sign of the greater is to be taken, with respect to the increase or decrease of the planet's distance from the sun. The second part of the correction in the transits of Venus for 1761 and 1769, did not exceed 9″ or 10″ of time, where the nearest approach of Venus to the sun's centre was about 10′. In the transit of *Mercury*, the first part of the parallax will be sufficient, unless the nearest distance be much greater.

If the *mean* horizontal parallax of the sun be taken = 8″83, then it appears by calculation from the above expression, that the total duration at Wardhus was lengthened by parallax 11′ 16″88, and diminished at Otaheite by 12′ 10″07; the computed difference of the times is therefore 23′ 26″95, but the observed difference was 23′ 10″ (see Vince's Complete System of Astronomy, ch. 25.)

The correct parallax may be therefore accurately found as follows: as the observed difference of the total durations at Wardhus and Otaheite is 23′ 10″, and the computed difference, from the above given parallax, is 23′ 26″ 95, the true parallax of the sun is less than the assumed. Let the true parallax be to the assumed as $1 - r$ to 1; then, from the foregoing expression, the first part of the computed parallax will be lessened in the ratio of $1 - r$: 1, and the second part in the ratio of $\overline{1 - r}^2 : 1$, or of $1 - 2r : 1$ nearly. All the first parts in the above expression, viz. 406″05, 287″05, 341″48, 382″47, in all 1417″05, combine the same way to make the total duration longer at Wardhus than at Otaheite. With respect to the second parts, the effects at Wardhus were $-$ 7″31 and $-$ 8″91; and at Otaheite 1″63 and 4″49, in all $-$ 10″10. Hence $1417''05 \times \overline{1 - r} - 10''10 \times \overline{1 - 2r} = 1390''$, the excess of the total duration at Wardhus above that at Otaheite, or $1417''05 - 10''10 - 1390'' = \overline{1417''05 - 20''20} \times r$, and $r =$

$\frac{16''95}{1396''85} = 0.0121$. Therefore the sun's mean horizontal parallax $= 8''83 \times \overline{1 - 0.0121} = 8''72316$. The mean horizontal parallax of the sun is therefore assumed $= 8\frac{3}{4}''$.

Hence the semidiameter of the earth : its distance from the sun : : sine $8\frac{3}{4}''$: rad. : : 1 : 23575. Now the semidiameter of the earth, according to the late French measures, being 3956 miles; hence the earth's distance from the sun is $23575 \times 3956 = 93262700$ miles. (See note to def. 8.)

The effect of the parallax being determined, the transit affords an easy method of finding the difference of the longitudes of two places where the same observations were made; thus, compute the effect of parallax in time, and reduce the observations at each place to the time, if seen from the centre of the earth, and the difference of time is the difference of longitudes required. For ex. the times at Wardhus, at which the internal contact would take place at the earth's centre, are 9h. 40′ 44″ 6, and 21h. 38′ 25″ 07, the difference of which is 12h. 2′ 19″ 53 = 180° 34′ 53″, the difference of longitude between Wardhus and Otaheite. From the mean of 63 results from the transits of Mercury, Mr. *Short* found the difference of longitude between Greenwich and Paris = 9′ 15″, and from the transit of Venus in 1761 = 9′ 10″. Mayer makes it 9′ 16″, Delambre 9′ 20″, and from trigonometrical calculation 9′ 18″ 8, in time.

The transit of Venus also affords an accurate method of finding the place of the node. For from the observations of Mr. *Rittenhouse* at Norriton or Norristown, within 18 miles N. W. of Philadelphia, the least distance CD was observed to be 10′ 10″; hence drawing CV perpendicular to C☋, cos DCV = 8° 28′ 54″ : rad. : : CD 10′ 10″ : CV 10′ 17″ the geocentric latitude of Venus at the time of conjunction; and 0.72626 (the dist. of Venus from the sun*) : 0.28895 (her distance from the earth) : : CV 10′ 17″ : 4′ 5″ the heliocentric latitude CV of Venus.† (Now considering C☊V a rt. angled triangle, we have tang. V☊C 3° 23′ 35″ (see pa 273) : rad. : : CV 4′ 5″ : C☊ 1° 8′ 52″, which added to 2s. 13° 26′ 34″ the place of the sun, gives 2s. 14° 35′ 26″ for the place of the ascending node of the orbit of Venus. For the beginning of 1800 Delambre makes it 2s. 14° 52′ 8″, and its secular variation 51′ 40″ (see pa. 272.)

Vince determines the time of the ecliptic conjunction as follows; let the difference of longitudes (d) of Venus, and the sun's centre be found for any time (t); and also the apparent geocentric hourly motion (m) of Venus *from* the sun in longitude; then say m : d : : 1 hour : the interval between the time (t) and the conjunction, which interval is to be added to or subtracted from t,

* This distance, &c. differs a little from that given in the notes pa. 271, which see, the above being taken from *Vince*.

† The angle subtended by CV is inversely as the distance from CV.

according as the observation was made before or after the conjunction. In the transit for 1761 at 6h. 31′ 46″, apparent time at *Paris*, M. de la *Land* found d = 2′ 34″4, and m = 3′ 57″4; hence 3′ 57″4 : 2′ 34″4 : : 1h. : 39′ 1″, which subtracted from 6h 31′ 46″, because at that time the conjunction was past, gives 5h 52′ 45″ for the time of conjunction from this observation. The *latitude* at conjunction may be also thus found. The horary motion of Venus in lat. being 35″4; hence 60′ : 39′ 1″ : : 35″4 : 23″, the motion in lat. in 39′ 1″, which subtracted from 10′ 1″2, the observed lat. at 6h. 31′ 46″, gives 9′ 38″2 for the latitude at the time of conjunction.

CHAP. IV.

OF THE EARTH,

AND ITS SATELLITE, THE MOON.

We now come to describe the earth, that part of the System by far the most interesting for us, as it is that which we are destined to inhabit, and of the phenomena of which we are therefore more intimate observers.

Its figure, as composed of land and water, has been already proved to be spherical or nearly so;* but here it becomes necessary to enter a little more into the detail of those arguments on which this important truth is founded.

The first notions that mankind probably formed of the earth were, those which arose from the immediate suggestion of the senses; but by comparing the phenomena together, and examining the nature of the senses themselves, correcting and assisting them; and by a proper application of geometrical and mechanical principles, the scheme of nature soon appeared very different from that which is presented to a vulgar eye. At first sight the surface of the earth appears of an unbounded extent, the clouds, meteors, moon, planets, sun and stars of every degree of magnitude, appear in one azure surface, concave towards the earth, which latter was therefore taken as its centre. It is to *sight*, as M'Laurin remarks,† that we owe our knowledge of the different parts of the system, those objects that are near us falling under the other senses only: but admirable as this sense is, it has its imperfections. Vision depends upon the picture of external objects formed on the *retina*,‡ together with a judgment of the understanding acquired by habit and experience; which is so immediately connected with the sense, that it is impossible, by an act of reflection, to trace it, or when it is erroneous, suddenly to correct it. If vision depended on the picture only, then equal pictures upon the

* See the note to def. 2, and also to prob. 31, part 2.

† View of Newton's Philosophy, 4to. pa. 223.

‡ Retina is the dark coat at the bottom of the eye, on which objects are painted.

retina would suggest ideas of equal magnitudes of the objects; and if the smallest fly was so near that it could cover a distant mountain from it, the fly ought to appear as large as the mountain. But, by habit, we have acquired a faculty of correcting this opinion, or idea of apparent magnitudes or distances; and this, with a quickness of thought almost inconceivable. Hence we see how many fallacies may arise in vision: for as often as we are mistaken in our notion of distance, so often must a corresponding error be produced in our idea of the magnitude of the object.

Thus we would imagine that the moon was of no greater magnitude than about two feet in diameter, if we were not certain, from other sources, that her magnitude is immensely greater. Those objects that are seen in the same direct line, would, if at a considerable distance from the observer, appear equally distant, as both coincide on the *retina;* those that are beyond the reach of distinct vision, would also appear equally distant, as the clouds, the moon, the sun, &c. did we not perceive that the clouds, in passing between us and the moon, concealed it from our view, and that the moon obscures the sun when in conjunction with him. Moreover, a distant body that is in motion, but in a direct line from the eye to the object, will appear at rest, and a body at rest may appear in motion, from our imperceptibly advancing from it, as is the case with passengers in a vessel sailing along the shore, or from land, or of the earth with regard to the sun. Hence as our knowledge of the system must be founded upon the real figures, magnitudes and motions of the bodies of which it is composed, so, rejecting the prejudice produced by our senses, we must, from the apparent phenomena, produce such an account of the real act as may be consonant to reason, and the nature of the objects under consideration.

The simplest proof that we have of the globular figure of the earth, is from her shadow projected on the moon in a lunar eclipse. For this shadow being always bounded by an arc of a circle, it hence follows, that the earth, which projects the shadow, must be of a spherical figure, since no other figure but that of a sphere, when turned in every position with respect to a luminous body, can cast a circular shadow; and that if there were any considerable irregular protuberance on the surface, or any remarkable angle, this would necessarily, at some time, appear by the shadow. We have another proof equally evident in our seeing the further the higher we are elevated. For if the earth was a plane, we could see as far on its surface when no object intervened, as at 1000 miles above its surface; but at one or two miles above the earth's surface we can see much farther, even with the naked eye, than on its surface, and this is true of any part of the earth; moreover the proportion between the distance seen and the height of the spectator above the surface of the earth, will answer to no figure but that of a sphere, or an oblate spheroid, elevated a little towards the equator.* Likewise the calculation of eclipses, of the places of the planets, and in general, all astronomical calculations, are made on the supposition of the earth's spherical figure, and all answer the times, when accurately calculated. When an eclipse of the moon takes place, it is observed by those who live eastward sooner than by those who live westward; and astronomers have found by frequent experience that, for every 15° difference of longitude, an eclipse begins so many hours sooner to those who live eastward, and later to those who live westward; but eclipses would happen at the same time at all places, were the earth a plane, nor could one part of the world be deprived of the light of the sun while another enjoyed the benefit of it. The elevation of the stars, and particularly the N. pole star, in travelling or sailing towards the north, in N. lat. and the depression of those towards the south, clearly prove that the earth is circular from N. to S. and the voyages of the circumnavigators sufficiently prove that the earth is round from east to west. The first who attempted to circumnavigate the globe was Magellan, a Portuguese, who, on the 10th of August, 1519, sailed from Seville, in Spain; in 1522 his ship returned again to St. Lucar, near Seville, on the 7th of September, not having altered its direction, during this time, towards the north or south, except as compelled by the winds or intervening land. Sir Francis Drake was the next, who, in 1577, performed the voyage in 1056 days; afterwards Thomas Cavendish performed it in 777 days, in the year 1586. Lord Anson, Captain Cook, La Perouse, &c.

These arguments clearly prove that the earth is round or nearly so, though common experience shews us that, mathematically or strictly speaking, it is not a sphere, from the mountains, valleys, &c. seen on its surface; but these no more prevent its spherical figure, than grains of dust on an artificial globe, (note to def. 1.) Moreover from the properties of the pendulum, this truth has received further confirmation, (see note to def. 1.) though it appears that the earth is not truly spherical, but rather in the form of a spheroid, and this is also confirmed by the different measures made use of to determine this important point. (See the note to prob. 31, part 2d.)

The earth being thus discovered to be globular, and from the discoveries of the circumnavigators and others, that it had inhabitants on every side of it, it followed, that some must have their heads directed towards that part of the heavens where the feet of the others would point, if the line were continued through the earth, or that their feet must be directly opposite to each other, while each one considered himself upright: but this difficulty

* Although St. Pierre endeavoured to prove that the earth is more elevated towards the poles, yet his reasoning is fallacious (as is evidently proved in the Philadelphia edition of the translation of his works) and contrary to the known laws of the general gravitation of matter and the effects of the rotary motion of the earth.

vanished as soon as the general laws of gravity were discovered, and that it was found that all heavy bodies tended to the centre of the earth, by a force equal to the quantity of matter in them, and therefore that on every part of the surface of the earth bodies were kept in their natural positions, without a possibility of falling off, by means of this law.

No sooner was this law and the globular figure of the earth discovered by *analysis*, than innumerable phenomena and important discoveries were unfolded by the direct method of *synthesis*. Hence followed the whole doctrine of the sphere, the motion of the planets, comets, &c. and those admirable laws set forth in the writings of Newton and his commentators.

This globe being circumscribed and limited as it is, it was natural that some should undertake accurately to discover its dimensions. It is probable that the first attempts were made at a period anterior to those of which history has preserved the record, and that the results have been lost in the physical and political changes which the earth has experienced. The labours of the moderns have, however, been, no doubt, more successful on this head, from the accuracy of their instruments; the following, collected chiefly from Laplace, will exhibit the most accurate result of their observations.

The elevation or depression of the stars, gives the angles which verticals, elevated at the extremity of the arc passed over, form at their point of contact; for this angle is evidently equal to the difference of the meridian altitudes of the same star, less the angle which the arc described would subtend at the centre of the star; and we are certain from observation, that this is insensible. It was then only requisite to measure this arc. It would be a long and tedious operation to apply our measures to so great an extent; it is a much more simple process to connect its extremities by a chain of triangles, to those of a base of 12 or 15,000 feet, and considering the precision with which the angles of these triangles may be determined, its length can be obtained very exactly. It is thus that the arc of the terrestrial meridian which crosses France from Dunkirk to Mountjoy, near Barcelona, has been measured; that part of this arc whose amplitude is equal to the hundredth part of a right angle, and whose central point corresponds to ($51\frac{1}{3}^{\circ}$) $46\frac{1}{5}^{\circ}$, is equal to 100179 metres.

Of all the re-entering, or curve lined, &c. figures, the spherical is the most simple, since it only depends on a single element, the size of its radius; and hence for the facility of calculation, this form was attributed to the earth But the figure of the earth is the result of those laws, which modified by a thousand circumstances, might alter it sensibly from a sphere. Inevitable errors of observation left doubts on this interesting phenomenon, and the Academy of Sciences, in which this great question was anxiously agitated, judged with reason, that the difference of the terrestrial degree, if it really existed, would be principally manifested in the comparison of the degrees at the equator and towards the poles. Academicians from France, and others from different parts of Europe and America, have measured degrees of the meridian in different parts of the world, and their measures incontestibly prove that the earth is not perfectly spherical, from the increase of the degrees from the equator to the poles (see note to prob. 31.) The ellipse being next to the circle, the most simple of the re-entering curves, the earth was considered as a solid formed by the revolution of an ellipsis about its shorter axis; and its compression in the direction of the poles, is a necessary inference from the observed increase of the meridional degrees from the equator to the poles. The radii of these degrees being in the direction of gravity, are, by the law of the equilibrium of fluids, perpendicular to the surface of the sea, with which the earth is in a great measure covered They do not tend as in a sphere, to the centre of the ellipsoid. They have neither the same direction nor the same length as radii, drawn from this centre to the surface, and which, except at the equator, and at the poles, cut it every where obliquely. The point of contact of two adjoining verticals, is the centre of the small terrestrial arc which they comprise between them; if this arc were a straight line, these verticals would be parallel, or would only meet at an infinite distance; but in proportion as they are curved, they meet at a distance so much the shorter. Thus the extremity of the shorter axis being the point where the ellipse approaches more to a straight line, the radius of a degree at the pole, and consequently the degree itself, is of its greatest length; but the contrary takes place at the extremity of the greater axis of the ellipse. At the equator, where the curvature is the greatest, the degree in the direction of the meridian is the shortest. Passing from the second to the first of these extremes, the degrees augment, and if the ellipse is but little flattened, *their increase is very nearly proportional to the square of the sine of the latitude.*

The measure of two degrees in the direction of the meridian, is sufficient to determine the two axis of the generating ellipse, and consequently the figure of the earth, supposing it elliptic. If this be the hypothesis of nature, the same proportion should be found very nearly between the two axes, the degrees of France, of the north, and of the equator, being compared two by two; but this comparison gives differences which it is difficult to attribute to errors of observation alone. The excess of the axis of the equator above that of the pole, taken as unity, is called the compression or ellipticity of the elliptic spheroid; now the degrees of the north, and of France give $\frac{1}{146}$ for the ellipticity of the earth, but the degrees of France and at the equator, give $\frac{1}{334}$, and hence it appears that the earth differs sensibly from an ellipsoid. There is even reason to believe that it is not a solid of revolution, and that its two hemispheres are not equal at each side of the equator. The degree measured by La Caille, at the Cape of Good Hope, in the southern latitude of 33° 18′ 32″4, has been found to be greater than the degree in France, Pennsylvania, and Italy; but it ought to be smaller than all these degrees, if the earth were a regular solid of revolution, formed of two similar hemispheres. Every result leads us to conclude that it is not the case. (See Laplace.)

The figure of the earth being extremely complicated, it is important to multiply the measures of it in every direction, and in as many places as possible. We may always, at every point of its surface, suppose an osculatory ellipse, which sensibly coincides with it, to a small extent round the point of contact.

Terrestrial arches measured in the direction of the meridian and perpendicular to it, compared with observations of latitudes, and of the angles which the direction of the extremities of these arches form with their respective meridians, will give us the nature and position of this ellipsoid, which may not be a solid of revolution, and which varies sensibly at great distances.

The operations which *Delambre* and *Mechain* have executed in France to obtain the length of the metre, determine very nearly the osculatory ellipse of that part of the earth, the latitude being observed at three intermediate points. Two bases of more than 12000 metres have been measured, the one near Melun, the other near Perpignan. And the correctness of the observations is confirmed from this circumstance, that the base at Perpignan deduced from that at Melun, by a chain of triangles which unites them, does not differ $\frac{1}{3}$ of a metre from its measurement, though the distance which separates the two bases is more than 900,000 metres.

The principal results of this important operation, as given by Laplace, are as follows:

OBSERVED LATITUDES.

	Decimal.	Sexagesimal.
Mountjoui	45° 958281	41° 21′ 45″
Carcassone	48 016790	43 12 54
Evaux	51 309414	46 10 42
Pantheon at Paris	54 274614	48 50 49
Dunkirk	56 706944	51 2 10

Arc of the terrestrial meridian comprised between Mountjoui and Carcassone 205621.3 metres, Evaux 534714.5 metres, Pantheon 831536.4 metres, and Dunkirk 1075058.5 metres.

The comparison of these results evidently indicates a diminution in the terrestrial degrees from the pole to the equator; but the law of this diminution seems very irregular. If however the ellipsoid, which satisfies these measures nearer than any other, be required, it is sufficient only to alter the observed latitudes about ($4\frac{1}{2}''$) 0″324.

The compression is then $\frac{1}{150}$, the semiaxis of the pole parallel to that of the earth, is 6344011 metres, and the degree corresponding to the mean parallel, is 99983.7 metres. An error of

($4\frac{1}{2}''$) $0''324$, though very small, is not admissible, considering the great precision of the observations; but this ellipsoid may at least be considered as osculatory to the surface of the earth in France at (51°) 45° 54′ of latitude, and suppose that it coincides with it to an extent of (5° or 6°) 4° 30′ or 5° 24′ round the point of osculation. It gives 100716.9 metres, for the degree perpendicular to the meridian, at (56° 3144) 50° 40′ 58″6 latitude, and by a very exact operation, lately performed in England, it has been found to be 100700.5 metres. This agreement proves that the action of the Pyrenees and other mountains in the south of France, has had very little influence on the latitudes observed at Evaux, Carcassone and Mountjoy, and that the great compression of the osculatory ellipse depends on attractions much more extended, the effect of which is felt in the north as well as the south of France, and even in England, Italy, and Austria: for the degrees, which have been carefully measured, are very nearly the same as on the ellipsoid.

Whatever be the nature of the terrestrial meridians, it is evident, as their degrees diminish from the poles to the equator, that the earth is flattened in the direction of its poles, or that the axis of the poles is less than the diameter of the equator. To explain this, let the earth be supposed a solid of revolution; then it is evident, that the radius of the degree at the north pole, and the series of all the radii from the pole to the equator, which by the supposition continually diminish, form, by their consecutive intersections, a curve which at first touches the axis of the pole, and afterwards separates from it, its convexity being constantly turned towards it, and raises itself towards the pole, until the radius of the meridional degree takes a direction perpendicular to the first; it will then be in the plane of the equator. If this radius of the polar degree be supposed flexible, and that it involves successively the arc of the curve which we have considered, its extremity will describe the terrestrial meridian, and the part of it intercepted between the meridian and curve, will be the corresponding radius of the degree of the meridian. This curve is what geometricians call *the evolute of the meridian.* Let us now consider the intersection of the diameter of the equator with the axis of the pole, as the centre of the earth. The sum of the two tangents to the evolute of the meridian drawn from this centre, the one following the axis of the pole, and the other the diameter of the equator, will be greater than the arc of the evolute which they include between them. Now the radius drawn from the centre of the earth to the north pole, is equal to the radius of the polar degree, *less* the first tangent; the semidiameter of the equator is equal to the radius of the degree of the meridian at the equator, *more* the second tangent. The excess of the semidiameter above the terrestrial radius of the pole, is then equal to the sum of these two tangents, *less* the excess of the radius of the polar degree above the radius of the degree of the meridian at the equator; this last excess is the arc itself of the evolute, which is less than the sum of the extreme tangents. The excess then of the semidiameter of the equator above the radius, drawn from the centre of the earth to the north pole, is positive. It can be proved in the same manner that the excess of the semidiameter of the equator above the radius, drawn from the centre of the earth to the south pole, is positive The whole axis of the poles is therefore less than the diameter of the equator, or, which comes to the same thing, the earth is flattened in the direction of the poles.

Considering every portion of the meridian as a small arc of its osculatory circumference, it is easy to see that the radius drawn from the centre of the earth to that extremity of the arc which is nearest to the pole, is less than the radius drawn from the same centre to the other extremity. From whence it follows, that the terrestrial radii increase from the poles to the equator, if, as all observations indicate, the degrees of the meridian augment from the equator to the poles. The difference of the radii of the degrees of the meridian from the pole to the equator, is equal to the difference of the corresponding terrestrial radii, *more* the excess of twice the evolute above the sum of the two extreme tangents, which excess is evidently positive: thus the degrees of the meridian increase from the equator to the poles in a greater proportion than the diminution of the terrestrial radii. These demonstrations are equally applicable, if the northern and southern hemispheres were not equal and similar, and it is easy to extend them to the supposition of the earth not being a solid of revolution.

It is however remarkable, that the observations made in the northern hemisphere, give the evolute of the meridian from (43° to 73°) 38° 42′ to 65° 42′, very little different from that of an ellipsoid of $\frac{1}{150}$ compression, and of which the mean degree is 99983.7 metres. For this ellipsoid nearly satisfies the measures lately made in France, the degrees measured in Italy and Lapland, and that which has been measured in England perpendicular to the meridian. It also represents the degree of the meridian measured in Austria at (53°) 47° 42′ of latitude, and which Liesganig has found to be 100114.2 metres. Finally, it agrees with the degree of the longitude measured in France at (48° 4′) 43° 33′ 36″ latitude, and of which Cassini and La Caille have fixed the length at 72003.5 metres.

Curves have been constructed at the principal places in France, on the line which has been considered as the meridian of the observatory of Paris, traced in the same manner as this line, with this difference only, that the first side, always tangent to the surface of the earth, instead of being parallel to the plane of the celestial meridian of Paris, is perpendicular to it. It is by the length of these curves, and by the distance from the observatory to the points where they meet the meridian, that the position of these places is determined. This labour, the most useful to *Geography* which has yet been performed, is a model (as Laplace remarks) which every enlightened nation will, no doubt, hasten to imitate.

We shall now proceed to another point no less worthy of attention, that is, *the diurnal motion of the earth on its axis*, a phenomenon which has been so clearly elucidated by the astronomers of the last and present age, that, though contrary to the direct testimony of our senses, the variety of strong and forcible arguments in confirmation of this motion, must effectually dissipate every doubt, and gain the assent of every impartial inquirer.

When we reflect on the diurnal motion to which all the heavenly bodies are subject, we cannot but recognize one general cause which moves and regulates them, or causes them apparently to revolve round the earth. If we consider that these bodies are insulated, with respect to each other, and placed at very different distances from the earth, that the sun and the stars are at much greater distances from it than the moon, and that the variations in the apparent diameters of the planets, indicate great alterations in their distances; and that moreover the comets traverse the heavens freely in all directions, it will be difficult to conceive that it is the same cause which impresses on all bodies a common motion of rotation. But since the heavenly bodies present the same appearance to us, whether the firmament carries them round the earth, considered as immoveable, or whether the earth itself revolves in a contrary direction; it seems much more natural to admit this latter motion, and to regard that of the heavens as only apparent.

The earth is a globe whose diameter is only 7911.2 English miles, as we have shewn in the note to def. 8, part 1; the sun, as we have seen, is incomparably larger; the earth then, which is but a point in comparison of the sun, must turn on its axis in a certain time, or else the sun, stars, &c. revolve round the earth in nearly the same time. Is it not then infinitely more simple to attribute to the globe we inhabit, a motion of rotation on its own axis, than to suppose in masses so immense and so remote as the sun and stars, such an extremely rapid motion as would cause them to revolve in one day round the earth?

But let us suppose that the sun does actually revolve round the earth. Now it is a known principle in the laws of motion (which will be shewn afterwards) that if any body revolve round another as its centre, it is necessary that the central body be always in the plane in which the revolving body moves, whatever curve it describes (Emerson's Astr. p 11.); therefore the diurnal path of the sun, in moving round the earth in a day, must always describe a circle which will divide the earth into two *equal hemispheres.* But this never happens but at the equinoxes, when the sun rises exactly in the east and sets exactly in the west; for in our summer the sun rises to the north of the east, and sets to the north of the west, and when on the meridian, it is nearer to us than the equator, its declination being north; in the winter it rises to the south of the east, and sets to the south of the west, and when on the meridian, is further from us than the equator, and therefore in both cases its diurnal path divides the globe into two *unequal* parts; consequently the sun does not move round the earth.

Moreover we have seen that the pole of the equator seems to move slowly round that of the ecliptic, from whence results the precession of the equinoxes. If the earth be immoveable, the pole of the equator must be likewise immoveable, as it always corresponds to the same point of the terrestrial surface; the ecliptic therefore moves round these poles, and in this motion carries all the heavenly bodies with it. Thus the whole system, or rather the whole *universe*, composed of so many bodies, differing from each other in their magnitudes, motions, and distances, would be again subject to a general motion, which disappears, and is reduced to a simple appearance, if we suppose the terrestrial axis to move round the poles of the ecliptic.

It is no argument against the earth's diurnal motion, that we are not sensible of it; a person on the earth can no more be sensible of its undisturbed motion on its axis, than a person in the cabin of a ship, on smooth water, can be sensible of the ship's motion when it sails along, or turns gently and uniformly round. Carried on with a velocity which is common to every thing that surrounds us,* we are in the case of a spectator placed in a ship that is in motion. He fancies himself at rest, and the shores, the hills, and all the objects placed out of the vessel, appear to him to move. But on comparing the extent of the shore, the planes, and the height of the mountains, with the smallness of his vessel, he recognizes that the apparent motion of these objects, arises from his own real motion. The innumerable stars which occupy the celestial regions, are, relatively to the earth, what the shores and the hills are to the vessel; and the same reasons which convince the navigator of the reality of his own motion, prove to us the motion of the earth.

These arguments are likewise strengthened by *analogy.* We find that the sun, and those planets on which there are visible spots, turn round their axis; and this motion is always from *west to east*, similar to that which the diurnal motion of the heavens indicate in the earth.

There is one effect of the motion of bodies on their axis,† which will enable us to judge with certainty whether this rotation takes place with regard to the earth. By the laws of the gravitation of matter, we comprehend that the centrifugal force which tends to remove every particle of a body from its axis of rotation, should flatten the earth at the poles, and elevate it at the equator; for as the equatorial parts move with the greatest velocity, they

* Laplace's Astr. vol. 1, B. 2, ch. 1.
† Ferguson's Astronomy, Art. 116.

will therefore recede furthest from the axis of motion, and increase the equatorial diameter. That our earth is really of such a figure, we have sufficiently proved in the foregoing articles. This proof receives additional strength from the doctrine of *pendulums*, as is sufficiently proved in Prop. 20. B. 3, Newton's Principia (see note to def. 2.) And as the earth is therefore higher at the equator than at the poles, the fluid parts, or the sea, which naturally seeks its level, would rush towards the polar regions, and leave the equatorial parts dry, if the centrifugal force at the equator did not prevent it.

This centrifugal force, or the tendency that bodies receive from the earth's rotary motion, should likewise diminish the force of gravity, or the weight of bodies at the equator; and hence at the poles the gravity is the greatest, owing to this force being nothing, and moreover that, from the flatness of the earth at the poles, bodies are nearer to the earth's centre, where the force of the earth's attraction is accumulated. We find, from experience, that a pendulum which vibrates seconds near the poles, vibrates slower near the equator, which shews that it is lighter or less attracted there; for as we have remarked before (note to def. 2) the length of pendulums vibrating in the same time, in different parts of the world, are as the force of gravity, or weight of bodies on the earth's surface. Every thing then leads us to conclude, as Laplace remarks, that the earth has really a motion of rotation, and that the diurnal motion of the heavens is merely an illusion produced by it. An allusion similar to that which represents the heavens as a *blue vault*, to which all the stars are fixed, and the earth as a *plane* on which it rests.

But some are apt to imagine* that if the earth turns eastward (as it must from the phenomena) a ball thrown perpendicularly upwards in the air, must fall considerably westward of the place it was projected from; but as the gun or whatever it was projected from, partakes of the earth's motion, it must fall exactly in the same place. A stone dropped from the top of the mainmast of a ship, will fall on the deck, if it meet with no obstacle, as near the foot of the mast when the ship sails as when it has no motion; but persons on shore would observe the ball to describe a curve, if the vessel was sailing, as it partook of two motions, one in the direction of gravity, and the other in the direction of the vessel. (See the laws of motion, after this system.) If an inverted bottle full of liquor be suspended from the ceiling of the cabin, and a small hole be made in the cork to let the liquor drop through on the floor, the drops will fall just as far forward on the floor when the ship sails as when she is at rest.

It is moreover objected from the Scriptures, that at the command of *Joshua* the sun stood still, and that therefore it must have had a previous motion. But those who bring forward this

* Ferguson's Astr. Art. 121. See also Keil's Astr. Lect. 2.

objection, know little of the spirit of the Scriptures, and as little of the idiom of language. The Scriptures were not given us to teach us profound lessons of philosophy or astronomy, but to teach us how to lead a virtuous life; and what is more common, even in the writings of the most accurate philosophers and astronomers, than these expressions, *the sun rises*, *the sun sets*, though they know at the same time that the sun has no such motion at all; and were they to make use of more correct expressions, they would be as unintelligible to the generality of men, as Joshua would, in a similar case, be to the Jewish people*.

* Although the motion of the earth on its axis be established beyond the possibility of doubt, its *cause* has, however, never been investigated; nor can it be deduced from any law consequent of the gravitation of matter. We shall, however, offer a few remarks on it, rather as conjectures than as principles, on which any new theories or systems could be established. The reasoning from strict analogy is conclusive, and the reasoning from analogy in this case, favours a good deal our remarks. It is a general law observed by all the planets, that they perform their revolutions round the sun, always in one and the same direction, that is, in the direction in which it revolves on its axis; that all the primary planets, as far as observations could be made on them, are found to have gross atmospheres surrounding them, and are also observed to have a motion on their axis, all in the same direction as the motion of the earth on its axis; that the secondary planets are found to have little or no atmosphere, and also that they do not perform similar revolutions on their axes, as they are found always to keep the same side turned towards their primary planets. Does it not then follow, that there is some regular cause for these phenomena, which are so constant and so regular? It seems to have no connection with the laws of gravity, and the cause of gravity itself being occult, prevents our forming any just notions of it. The only agent that we can observe is *light* or *heat* (for heat, caloric, &c. whether latent or not, I look upon to proceed from the same principle, though differently modified; this is proved from the writings of many able chymists.) For from the laws and nature of light, the phenomena of the motion of bodies can be accounted for on mechanical principles. It is well known, that as a body, it acts on others, and has a *momentum* proportional to its velocity and quantity of matter; but as its velocity is so very great, and its particles so exceedingly small, this momentum is not easily appreciated: in consequence of this law, it displaces those particles of the atmosphere on which its influence is exerted, and causes a rarefaction; it also *repels* bodies, but as it is bent into a curve, or *refracted* in passing from one medium into another, its force, or momentum, may be thus exerted on the side of a body opposite to that from which it was emitted, and thus cause an *attraction*, or motion of the body towards that from which the *effluvia* of light is emitted, similar to those phenomena produced by the electric fluid, &c. &c. This being premised, let us now consider the phenomenon of the sun's daily apparent motion. We shall find that that part of the atmosphere, over which the sun is perpendicular, is more rarefied than any other, and as the different parts of the earth over which the sun is perpendicular, pass successively under the sun in a direction from *west* to *east*, the whole hemisphere on the east side of the sun, will have its atmosphere more rarefied than the hemisphere west of the sun; and hence on the east side, the rays will act more directly and meet with less resistance, than when acting through the dense atmosphere on the west, and therefore a motion of the earth on its axis must be the consequence of this difference of action. This explication is strengthened from analogy, because all the primary planets have a similar rotation, and are also found to

The diurnal revolution of the earth on its axis being thus established, we shall proceed to that of *its annual motion round the sun.* For we must suppose the sun, accompanied with the planets and satellites, in motion round the earth; or the earth, with the other planets, &c. to revolve round the sun. The appearances of the heavenly bodies, as seen from the earth, are the same in both hypotheses; but the latter is to be preferred, for the following reasons.

have an atmosphere; and the secondary planets, which have no such motion, are found to be destitute of an atmosphere, or if they have any, it is so rare as to be insensible as well as its effects. But the rotation above described could not take place, unless the earth had received a primary impulse in the direction in which it revolves, and as this direction is the same in all the planets, the impulse could not be fortuitous, but must be regulated by some constant and regular law. Now as the sun, which emits the light, is found to have a motion of rotation in this direction, it is in this that the cause of the direction of the planets' rotation is to be looked for. Already an extensive field for speculation is open to our view, and innumerable questions present themselves. The sun has an atmosphere and a similar motion: is this motion produced in a similar manner? What gives this direction to the sun's rotary motion? Whence the sun's light, or why not long since exhausted, being emitted into spaces from which it can never return? These are questions too arduous to be discussed in the compass of a note; but we cannot pass them over in silence. With regard to the first, we must, according to one of the first rules in philosophy, *assign the same causes to the same natural effects, as far as possible.* And hence whatever produces the motion of the earth, a similar cause must produce that of the sun. But whence this cause? Here we are embarrassed, and can produce no satisfactory solution. This we know, that as all bodies gravitate towards each other, in proportion to the quantity of matter which they severally contain, all the bodies in the *universe* would tend to that part where the attraction was the greatest, or to one general centre, unless counterbalanced by a centrifugal force; and the stars must have such a centre, or we must admit of creation *in infinitum.* That the stars have, in reality, some such centre, and also a periodic revolution around it, analogy and observation both concur to prove. Many of the stars are found to have motions, which cannot be accounted for from any other cause; and others are at too great a distance, in the immense expanse, to have their motions sensible. From analogy, we see the distant Herschel, from which the sun appears not much greater than a star, or the still more distant comet, no less than Mercury, regard the sun as the centre of their motion; is it not then rational to conclude, that the sun and stars observe a similar law, and revolve about some common centre? The secondary planets revolve round their primaries as their centres; and both primaries and secondaries regard the sun again as their center, and hence the sun, together with both primaries and secondaries, may regard another body as their centre, &c. It is no objection to say, that such a centre, or such an immense body, has never been observed, for the whole solar system, at the distance of some of the stars, would appear no greater than a point. If then this centre has really an existence, its magnitude must be immensely great beyond conception, or its nature must be different from all those bodies that we have any knowledge of. For in the solar system, the economy of this system requires, that the sun should be much greater than all the other bodies, that they might regard him as their centre; and this we find to be the case. Hence if the stars have a common centre, the magnitude of the body, placed in this centre, must exceed that of all the stars put together; for the aggregate of their attrac-

The masses of the sun, of Jupiter, Saturn, and Herschel, are considerably greater than that of the earth; hence it is much more simple to have the latter revolve round the sun, than that the whole solar system should revolve round the earth. Moreover, by the laws of centripetal forces (given after this system) if two bodies revolve round each other, they perform their revolutions round their common centre of gravity (Newton's Principia, B. 1.

tions on it in one direction, lessened by their aggregate in the opposite, acts as one body, and a greater body or mass of matter can never revolve round a lesser, according to the present laws of nature. And as the sun dispenses light, &c. to the solar system, and regulates the motions of those bodies that exist around it, it is probable that this immense body supplies the whole system of the universe with that light, &c. with which every part of it is replete, and regulates, as a main spring, the motions of the whole machine. What an idea does this give of the universe! But an idea the most simple and consonant to the present laws of nature, and uniting unity and simplicity in the design, with magnificence and awful grandeur in the execution. An idea, which shews the *universe* to be the work of *one* intelligent, sublime Being, who formed and presides over the magnificent structure. And so far is this system from being at variance with the account which *Moses* has given us of the works of creation, that it seems to emanate from the pen of the sacred historian. We read in the 1st ch. of Gen. that " in the beginning God created heaven and earth," where, by the word earth, is meant, according to most interpreters, all that opake matter which enters into the formation of the different bodies of the universe. Next God created *light*, (v. 3) afterwards the *firmament*, (v. 7) or all that space in which bodies are placed; then were those lights made in the firmament of heaven to be for *signs*, and for *seasons*, &c. (v. 14) and to give light upon the *earth*, (v. 15) or those stars which we call fixed, because their motions are not sensible, and which, no doubt, are destined to perform the same functions as our sun. We see, moreover, that the matter of which the sun is composed, is dense and heavy like that of the earth, that it must receive a supply of light from some source to preserve the same uniform splendour, if we except the effects of some spots on its surface (which also shew that parts of it are opake) and that therefore it is only calculated to reflect or emit that light, heat, &c. more copiously: an effect which the planets, from their opacity and contexture, as well as their inferior magnitude, are not calculated to produce. We see, moreover, that those bodies which are nearest the sun, receive most of its light and heat, and that those that are at the greatest distance from him, receive very little more than we do from some of the largest of the stars. Now, reasoning from analogy, if we suppose the body above described to occupy the centre of the universe, and to be the fountain of light and heat to the whole universe, those bodies that are nearest this, must enjoy more of its light and heat; others that are more remote, may be at a loss to know, unless by analogy, whether there be such a body in the universe, as is the case with us; and others may be so remote, as to receive little of its light or heat, and thus remain almost buried in a continual night. If we carry our ideas a little further, we shall find nothing but an immense void, where a ray of light has never penetrated. This is ultimately the view of nature which the present system of philosophy, or the general gravitation of matter, developes—a system, which swallows up, from its immensity, the feeble powers of our reason, and leaves us nothing to build on but conjecture. This is then the utmost stretch of *philosophy*; it may unfold this system, but it can go no further. For ever would it leave us ignorant of our destination, and the great end of our being, did not the author of nature dissipate our doubts, and point them out prop. 57; or prop. 20, sect. 2, Emerson's Centr. Forces) and it is evident, that if the two bodies be of equal magnitude and density, the centre of gravity will be equidistant from each body (see the note p. 253;) but if they be of different magnitudes, the centre of gravity will be proportionally nearer the greater body. If the earth, therefore, remain at rest while the sun revolves round it, its magnitude must be vastly greater than that of the sun; it being

to us by means worthy his infinite wisdom. The magnificence which we behold in creation, is worthy the Great Author; but all this magnificence is one day to vanish—and hence, by his example, he would point out the vanity of all created things, and call our attention to higher destinies and regions. It is not then the magnitude or multiplicity of those orbs that should challenge our estimation; they may excite our astonishment, and produce an awful respect for their Creator; but they are nothing more than inanimate heaps of matter, incapable of knowing that being that called them into existence, and destined one day to perish. Of all the beings then that the universe exhibits to our view, we find only man possessed of that immortal principle destined to survive the wreck of matter, and capable of knowing, serving, and enjoying that great Being that called it into existence. One immortal soul is therefore more precious in the sight of its Creator, than all those vast orbs that roll their immense masses through the expanse of heaven; and hence we need not wonder that he has done so much for its preservation. To examine the question, whether those bodies are inhabited as well as our earth, would be a futile as well as useless inquiry, as it is evident we can never, in our present state of existence, know any thing of the matter. The Creator (as De Feller remarks) undoubtedly could, for his own glory, and to display the treasures of his wisdom and power, do great and beautiful things without any reference to man, or to any rational creature. This is the opinion of many learned writers, and particularly of St. Augustine, St. Thomas, Petavius, Leibnitz, &c. and the sacred writings declare, that *Universa propter semet ipsum operatus est Dominus*. Prov. 164. "God," says *Hugens* (Plurality of Worlds, ch. 8) "is himself the spectator of the works he has created; and who can doubt, but that he who made the eye can see very well, and delights in doing so? Inquire no further. Is it not for that that he created man, and all that is contained in the universe?" Pythagoras's music of the celestial spheres, is an allegorical expression of the pleasure which intellectual beings take in viewing them. Young, in his Night Thoughts (Night 9th) considers the stars as grand refulgent thrones, on which the ministers of the Eternal sit in majestic state, executing throughout the universe the decrees of his love or his vengeance. But to spare the mind of man from amusing itself with vain systems and philosophic dreams, *religion* saves for it this useless waste of time, and calls it to more important studies. It points out all that is necessary as regards our own destination. To reject this knowledge, because it does not point out the destination of beings which it would be useless for us to know, and of whose existence we are perfectly ignorant, would be folly in the extreme. To say that the Christian religion is unphilosophical, is equally frivolous. It advances no one principle contrary to the known established laws of nature; and while absurd systems of philosophy and false schemes of nature inundated the world, a few expressions in the sacred writings contained more sound sense, and genuine philosophy, than all those reveries. Let any of these philosophers solve those important questions found in the book of Job, or shew that they import any thing contrary to the true system of nature. "*Hast thou considered the breadth of the earth? Tell me if thou knowest all things? Where is the way where light dwelleth, and where is the place of darkness?* (ch. 38, v. 18, 19). It cannot be here the contrary to the laws of nature for a heavy body to revolve round a lighter one, as its centre of motion; for the lighter one must be at a greater distance from the common centre of gravity, and must have a greater velocity to counterbalance the attraction of the other. Now the sun is found, from observation, not only to exceed the earth in magnitude, but so far to exceed the magnitudes of all the planets in the solar system, that the common centre of gravity of the whole is almost constantly within its body, so that its motion round the common centre of gravity of the whole system, is scarcely perceptible to the nicest observers. The earth, therefore, and all the planets, must revolve round the sun.

The analogy of the earth with the other planets (as Laplace remarks, Astr. B. 2, ch. 3) confirms the hypothesis of its annual revolution. Like Jupiter it revolves on its axis, and is accompanied by a satellite. An observer on the surface of Jupiter would conclude that the solar system was in motion round him, and the magnitude of that planet would render this illusion less improbable than for the earth. Is it not, therefore, reasonable to suppose, that the revolution of the solar system round us, is likewise only an illusion? Let us examine the phenomena of the earth and the

darkness produced by the absence of the sun, or the light caused by his presence, that is meant, for then the question would be too trifling, being proposed by God himself. It is in this book that we find this remarkable sentence, "He (God) stretched out the north over the empty space, *and hangeth the earth upon nothing*." (ch. 26, v. 7.) What more philosophical than the latter part of this sentence? We also find in it, speaking of the wicked, this no less remarkable sentence: "*He shall drive him out of light into darkness, and shall remove him out of the world*." (ch. 18, v. 18.) We find the same idea, expressed on a similar occasion, in St. Matthew. "*And the unprofitable servant cast ye out into the exterior darkness. There shall be weeping and gnashing of teeth*." (ch. 25, v. 30. see also ch. 8, v. 12. ch. 22, v. 13.) Until philosophers point out that this *exterior darkness*, &c. has got no existence—until they bring us intimately acquainted with the extremes of nature, their arguments against the sacred writings, drawn from their knowledge of a little corner or point of the universe, must not only be inconclusive, but ridiculous and vain. It is with a view of shewing the folly of shallow philosophers, who pass the bounds of their knowledge to attack incontestible truths, that the above remarks have been made—to shew how little we know as yet of the system of the universe, and the design of the Author of this sublime structure, in its formation—and to induce those who are in possession of the most sublime philosophy that man can learn, I mean the *Christian religion*, to appreciate that sacred treasure, and despise those vain systems that have no other support but the imaginations of their authors.

The remarks that have been made on the system of the universe are not novel. They are deductions strictly drawn from Newton's Philosophy. And either this Philosophy, now universally received, must be false, or the general conclusions cannot be denied. The remarks are therefore principally calculated for those who are versed in the principles of this Philosophy, as a superficial view of the system of nature, may produce notions unfavourable to religion and to sound philosophy. What is offered is, however, offered with that diffidence and distrust, which every conjecture or hypothesis ought to inspire, that is not the result of accurate observation, or strict calculation.

planets from the sun's surface. All these bodies will appear to move from *west* to *east;* this identity, therefore, indicates a motion of the earth; but that which proves it, evidently is the law which exists between the times of the revolutions of the planets and their distances from the sun. They perform their motions round it slower in proportion as their distances are greater, and in such a manner, that *the squares of the periodic times are proportional to the cubes of their mean distances* (Principia, Phenomenon 5, B. 3.) From this remarkable law, the length of a revolution of the earth, supposing it in motion round the sun, to correspond with the earth's distance, should be exactly a sidereal year, as is really the case; this, therefore, is an incontestible proof that the earth moves like the other planets, and is subject to the same laws. Another argument, still more incontestible, is the following, that the force of gravity, which balances the centrifugal force in the other planets, and retains them in their respective orbits, should likewise act on the earth; and that the earth must, therefore, oppose to this action the same centrifugal force. Hence the consideration of the celestial motions, as observed from the sun, leaves no doubt of the real motion of the earth.

An observer on the surface of the earth, has another evident proof in the phenomenon of the *aberration* (Laplace, B. 2, ch 3) which is a necessary consequence of it. *Roemer*, about the end of the 17th century, observed that the eclipses of the satellites of Jupiter happened sooner about the oppositions of this planet, and later towards the conjunctions; this led him to conjecture, that light was not transmitted instantaneously from those bodies to the earth, but took a perceptible interval of time to traverse the diameter of the sun's orbit. Now, Jupiter being nearer to us in his oppositions than conjunctions, by a distance equal to the sun's orbit, the eclipses ought therefore to happen to us sooner in the first case than in the latter, by the time which the light takes to traverse the sun's (or rather the earth's) orbit; and the retardation of these eclipses so exactly correspond to this law, that it is impossible to refuse assent to it. It is therefore found, that light takes about $8' \, 7''5$ in passing from the sun to the earth, at its mean distance.

A star near the constellation *Draco*, that passed near the zenith, was observed by Messrs. *Molyneux*, *Bradley*, and *Graham*, with an instrument contrived by the latter, with a view of discovering its parallax. They soon discovered that the star did not always appear in the same place in the instrument, but that its distance from the zenith varied, and that the difference of its apparent places amounted to $21''$ or $22''$. This star was γ draconis, near the pole of the ecliptic. They made similar observations on other stars, and found a like apparent motion in them, proportional to the latitude of the star. This motion was by no means such as could result as the effect of a parallax; and it was some time before they could discover any method of accounting for this new and strange phenomenon ; but Dr. *Bradley*, at length, resolved all its variety in a satisfactory manner, by the motion of light and the annual motion of the earth compounded together. For as the earth describes 59′ 8″ of her orbit in a day = 3548″, and that light comes from the sun to us in 8′ 7″5, we have this proportion, 24 hours or 86400″ : 8′ 7″5 or 487″5 : : 3548″ : 20″ very near, the aberration of light or the change in the star's place ; and this is what Dr. *Bradley* has made it. And hence it affords as sensible a demonstration of the motion of the earth round the sun, as the increase of degrees and the force of gravity in passing from the equator to the poles, afford of the revolution of the earth on its axis. We shall give the principles of the aberration of light more at large when we come to treat of the fixed stars.

It is objected against the annual motion of the earth, in its orbit round the sun, that if it really had such a motion, the annual parallax of the stars, or the angle under which the diameter of the earth's orbit would appear, as seen from a fixed star, should make a considerable difference in the position of the star observed at two different times ; but it is never found to make the least difference, though observed with our nicest instruments. To understand this more clearly, as the axis of the earth keeps always parallel to itself, it would follow, that if it pointed to any star at one time of the year, in six months after it ought to point to another, distant from the former by the angle under which the whole diameter of the earth's orbit appears from the star ; but it is found not to deviate a single second from its former position. Now this objection, the most forcible that has been brought against the earth's motion, vanishes when we come to consider the immense distance of the fixed stars, of which we may form an idea thus : if we should suppose the distance between us and a fixed star to be divided into 1000 equal parts, and that a spectator, after having passed over 999 of those parts, should view it from the last division, or at $\frac{1}{1000}$ part of the whole distance, it would not appear larger than to the naked eye ; because a telescope that magnifies 1000 times, though it will render it brighter, will not sensibly magnify its diameter. Herschel's 20 feet telescope magnified 460 times, and his 40 feet telescope magnified some thousand times ; and Herschel confirmed the above assertion. Hence the immense distance of the fixed stars, indicated by these observations, so far from being an objection against the earth's annual motion, rather confirms it.

The earth's annual motion round the sun being thus established, we shall now shew how the quantity of this motion is estimated. We know, as the earth regards the sun as the centre of its motion, that in whatever part of the heavens the earth actually is at any time, the sun must be directly in the opposite point at the same time ; and that therefore, if the sun's place be observed in the heavens, the opposite point is the place of the earth ; now as the earth advances round the sun, the sun will seem to perform a similar motion in the heavens, and hence, if we compute this apparent motion of the sun among the fixed stars, it will give the earth's real motion round the sun.

From comparing the sun's right ascension every day, with the right ascensions of the fixed stars lying to the east and west, the sun is found constantly to recede from those on the west, and approach those on the east; its apparent annual motion is therefore found to be from west to east, and as the earth performs a similar motion in an opposite part of the heavens, the earth's real motion must also be from west to east. The interval of time from the sun's leaving any fixed star, until its return to the same star again, is called a *sidereal year*, being the time in which the sun completes its apparent revolution among the fixed stars, or in the ecliptic. But the sun, after it leaves either of the equinoctial points, returns to it again sooner than it returns to the same fixed star; and this interval is called a *solar* or *tropical year*, because the time of the sun's leaving one equinox until its return to it again, is equal to the time from its leaving one tropic until its return again. This solar or tropical year, is that on which the return of the seasons depends.

To find the length of a sidereal year. On any day when the sun passes the meridian, take the difference between its right ascension, when on the meridian, and that of a fixed star; call this difference (a) and when the sun returns to the same part of the heavens the next year, compare its right ascension with that of the same star, for two days, that is, when their difference (d) of right ascension is less, and also when greater (e) than the difference (a) before observed; then $e - d$ is the increase of the sun's right ascension in the time t; and as the increase of right ascension may be considered uniform for a small time, we have $e - d : a - d :: t : T$ the time in which the sun's right ascension is increased from the sun's place, when the difference d of his right ascension, and that of the star, was less than a. This time T being therefore added to the time of the observed right ascension of the sun, when the quantity d was found, will give the time when the sun is at the same distance a from the star, as when observed the former year; and the interval of these times is therefore a sidereal year. About March 25, June 20, September 17, and December 20, is the best time for these observations, as the sun's motion in right ascension is then uniform.

If, instead of repeating the second observations the following year, there be an interval of several years, and if the observed interval of time, when the difference of the right ascensions of the sun and star was found to be equal, be divided by the number of years, the length of the sidereal year will be given more exactly.

On April 1st, 1669, at 0h. 3′ 47″ mean solar time, M. *Picard* observed the difference of longitude between the sun and *Procyon* to be 3s. 8° 59′ 36″, which is the most ancient observation of this kind, the accuracy of which can be depended on. (See Hist. Celeste, par M. *le Monnier*, p. 37.) On April 2d, 1745, M. *de la Caille* found, by taking their difference of longitude on the 2d and 3d, that at 11h. 10′ 45″, mean solar time, the difference of their longitudes was the same as at the first observation. Now as the sun's revolution is nearly 365 days, it is manifest that it made 76 complete revolutions, in respect to the same fixed star, in the interval between the two observations, or in 76 years, 1d. 11h. 6′ 58″. In these 76 years, there were 58 of 365 days, and 18 bissextiles of 366 days each; hence in the interval, there were 27759 d. 11h. 6′ 58″, which being divided by 76, the quotient is 365 d. 6h. 8′ 47″, the length of a sidereal year. According to Laplace, the length of a sidereal year is 365.256384 days, or 365 d. 6h. 9′ 11″5776, which is the most accurate, being computed from the best observations.

To find the length of the Solar or Tropical Year. Take the meridian altitude *a* of the sun, on any day when it is nearest to the equinox; then the following year, let its meridian altitude be taken on two days, as follow: one when its altitude *m* is less than *a*, and next when its altitude *n* is greater than *a*; then *n* — *m* is the increase of the sun's declination in 24 hours. Also when the declination has increased by the quantity *a* — *m*, from the time when the meridian alt. *m* was observed, the declination will then become *a*; and as the increase of the declination may be considered as uniform for one day, we have this proportion, *n* — *m* : *a* — *m* : : 24h. : the interval from the time the sun was on the meridian on the first of the two days, until the sun has the same declination *a*, as at the observation the foregoing year; hence this time being added to the time when the sun's altitude *m* was observed, will give the time when the sun's place in the ecliptic had the same situation, in respect to the equinoctial points, which it had at the time of observation the preceding year; the interval of these times is the length of a tropical year.

If, as in the method for determining the sidereal year, there be an interval of several years between the observations, and that the interval between the times when the declination was found to be the same, be divided by the number of years, the length of the tropical year will be obtained more exactly.

On the 20th of March, 1672, M. *Cassini*, the father, observed the meridian alt. of the sun's upper limb to be 41° 43′, at the Royal Observatory at Paris; and on March 20, 1716, M. *Cassini*, his son, observed the mer. alt. of the upper limb to be 41° 27′ 10″, and on the 21st, to be 41° 51′; the difference of these two latter altitudes was then 23′ 50″, and of the two former 15′ 50″; hence 23′ 50″ : 15′ 50″ : : 24h. : 15h. 56′ 39″; therefore, on March 20, 1716, the sun's declination, at 15h. 56′ 39″, was the same as on March 20, 1672. Now the interval between these two observations was 44 years, of which 36 consisted of 365 days, and 10 of 366 each, that is in all, 16070 days; hence the whole interval between the equal declinations was 16070 d. 15h. 56′ 39″, which divided by 44, gives 365 d. 5h. 49′ 0″ 53‴, the length of a tropical year, from these observations. The length of a tropical year, or the return of the sun to the same equinox, from the best observations, as given by Laplace in his Astronomy, is 365.242222 or 365 d. 5h. 48′ 47″98. Hence the sidereal year exceeds the tropical by 0.014162 of a day. The equinoxes have therefore a retrograde motion in the ecliptic, or in a direction contrary to that of the sun, by which they describe every year an arc equal to the mean motion of the sun in the interval of 0.014162 of a day; hence 1 d. : 0.014162 d. : : 59′ 8″3 : 50″151, the *precession of the equinoxes.*

The precession of the equinoxes being given, and also the length of a tropical year, the length of a sidereal may be easily found, as shewn in the note to prob 42, part 3d.

There is another year, called by astronomers the *anomalistic year*, and is the time from the sun's leaving his apogee until his return to it again. Now the progressive motion of the apogee in a year, according to *Vince*, is 11″75, and hence the anomalistic must exceed the sidereal year by the time the sun takes in moving over 11″75 of longitude at its apogee; but the sun's motion in longitude, when in its apogee, is 58′ 13″ in 24 hours; hence 58′ 13″ : 11″75 : : 24h. : 4′ 50″6384, which added to 365 d. 6h. 9′ 11″5776, gives 365 d. 6h. 14′ 2″216, the length of the anomalistic year. The motion of the apogee here given, is that determined by M. *de la Lande*, from the observations of M. *de la Hire*, and those of Dr. *Maskelyne*, agreeing also with Cassini's determination. M. Laplace makes the sidereal and secular motion of the earth's *perihelion* (3671″63) 19′ 49″ 60812, which gives 11″896 yearly. *Delambre* makes the mot. of the apog. in a year 62″ which includes the precession, &c. *Mayer* makes it 66″.

The longitude of the earth's perihelion at the beginning of 1750, according to *Laplace*, was (309° 579′) 278° 37′ 15″96. The mean longitude of the earth, reckoning from the mean vernal equinox at the epoch of the 31st December, 1749, at noon, mean time at Paris, was (311° 1218) 280° 0′ 34″632.

In accounting for the cause of the planets' revolutions round the sun, philosophers had recourse to various hypotheses. The ancients invented their solid orbs, and *Descartes* vortices; but both were imaginary fictions, void of proof. *Newton* was the first that built his explanations on actual experiment and observation, and fully investigated the laws of motion resulting from the gravitation of matter. He seemed possessed of all that could qualify him for this arduous task; and the innumerable mathematical theorems and inventions which he discovered in his inquiries will, probably for ever, remain the greatest monument of human ingenuity.

The substance of Newton's discoveries, relative to the cause of the planets' motions, we shall give here, principally collected from *Cote's* preface to *Motte's* translation of the Principia. That we may begin our reasoning from what is most simple and nearest to us, let us first consider what is the nature of gravity with us on the earth. All agree, that every circumterrestrial body gravitates towards the earth; that no bodies really light are to be found, as experience shews; that those bodies which are relatively light, are not really so, but apparent only, and arising from the preponderating gravity of the contiguous bodies, or the fluids in which they are immersed; that all bodies gravitate towards the earth, and the earth in like manner towards bodies;* that the action of gravity is mutual and equal on both sides; that the weights of bodies, at equal distances from the centre of the earth, are as the quantities of matter in the bodies;† this is proved from the equal acceleration of all bodies that fall from a state of rest by the force of their weights, the resistance of the air being taken away; and this is yet more accurately proved from the doctrine of pendulums; that the attractive forces of bodies, at equal distances, are as the quantities of matter in the bodies;‡ and that therefore the attractive force of the entire bodies arises from, and is compounded of, the attractive forces of the parts,§ so that terrestrial bodies must attract each other mutually, with absolute forces, that are as the matter attracting. This being the nature of gravity on the earth, the following will shew what its nature is in the heavens.

Every body perseveres in its state, either of rest or of uniformly moving in a right line, unless it is compelled to change that state by other forces impressed;|| and hence it follows, that bodies that move in curve lines, and therefore continually deflect from the right lines that are tangents to their orbits, are, by some continued force, retained in those curve lined paths. Now, as the planets move in curve lined orbits, there must be some force operating, by whose repeated actions they are perpetually made to deflect from the tangents.

It is a mathematical principle, the demonstration of which we shall give when we come to treat of the doctrine of centripetal forces, that all bodies that move in a curve line described in a plane, and which by a radius drawn to any point, whether quiescent, or any how moved, describe areas about that point proportional to the times, are urged by forces directed towards that point.¶ Since then all astronomers agree, that the primary planets describe about the sun, and the secondary planets about their respective primaries, areas proportional to the times, it follows, that the forces by which they are deflected from the rectilinear tangents, and made to revolve in curve lined orbits, are directed towards the bodies that are situated in the centres of the orbits.

* Principa Scholium after the laws.
† Principia, B. 3, prop. 6.
‡ Principia, B. 1, prop. 69, cor. 3, and prop. 7, B. 3.
§ Principia, cor. 1, prop. 7, B. 3.
|| Principia, Law 1, B. 1.
¶ Principia, B. 1, prop. 2.

This force may therefore not improperly be termed *centripetal* in respect of the revolving body, and in respect of the central body *attractive*, whatever cause it may be imagined to arise from.

The following is also mathematically true; that is, if several bodies revolve with an equable motion in concentric circles, and the squares of the periodic times be as the cubes of the distances from the common centre; the centrifugal forces will be reciprocally as the squares of the distances.* Or if bodies revolve in orbits that are nearly circles, and the apsides of the orbits rest, the centripetal forces of the revolving bodies will be reciprocally as the squares of the distances; and both these cases hold in all the planets, as observations fully testify.

From what has been hitherto said, it is evident that the planets are retained in their orbits by some force perpetually acting upon them; that that force is always directed towards the centre of their orbits; that its efficacy is augmented in proportion as the centre is approached, and diminished as its distance increases from the centre; and that it is augmented in the same proportion as the square of the distance is diminished, and diminished in the same proportion as the square of the distance is augmented. Now, by making a comparison of the centripetal forces of the planets and the force of gravity, we shall find that they are, in effect, of the same kind; for to have them of the same kind, it is only necessary that both observe the same laws, &c.

Let us therefore first consider the centripetal force of the moon, being nearest to us. The rectilinear spaces which bodies, let fall from rest, describe in a given time at the very beginning of the motion, when the bodies are urged by any forces whatsoever, are proportional to the forces (as will be shewn after the solar system.†) Hence the centripetal force of the moon, revolving in its orbit, is to the force of gravity at the earth's surface, as the space which, in a very small particle of time, the moon, deprived of all its centrifugal force, and descending by its centripetal force towards the earth, would describe, to the space which a heavy body would describe, when falling by the force of its gravity near the earth, in the same given particle of time. The first of these spaces is equal to the versed sine of the arch described by the moon in the same time,‡ because that versed sine measures the translation of the moon from the tangent, produced by the centripetal force, and may therefore be computed, the periodic time of the moon, and its distance from the centre of the earth being given.§ The last space is found by experiments of pendulums,

* Principia, B. 1, prop. 4, cor. 6, and B. 3, prop. 2, also cor. 1, prop. 45, B. 1.

† Principia, B. 1, Lemma 10, cor. 3.

‡ Principia, B. 1, sect. 2, prop. 1, cor. 4.

§ The mean distance of the earth from the moon being taken = 238533 miles (see the note p. 250;) hence, the moon's orbit being nearly circular, its diameter is 477066 miles, and its circumference 477066 ×

as shewn by Mr. *Hugens*.* Therefore by making a calculation, we shall find that the first space is to the latter, or the centripetal force of the moon revolving in her orbit, to the force of gravity at the surface of the earth, as the square of the semidiameter of the earth, to the square of the semidiameter of the orbit. But by what has been shewn before, the very same ratio holds between the centripetal force of the moon, revolving in its orbit, and the centripetal force of the moon near the earth's surface. Therefore,

3.1416 = 1498750.5456. Now the moon's sidereal revolution is 27 d. 7h. 43′ 11″ = 2360591″; hence this proportion 2360591″ : 1′ or 60″ : : 1498750. 5456 miles : 38.0942 miles the moon will describe in 1′, which is nearly equal to the tangent of 1′ to her orbit. But as the secant of the same arc of 1′ less the radius, gives her distance fallen towards the earth in her orbit by the power of gravity in 1′, we have rad.^{2} or 238533^2 × tang.^{2} or 38.0942^2 = secant^2 = 56837993540.16807364, the square root of which is 238533.0030418 miles nearly, from which the semidiameter of the orbit being subtracted, and the remainder .0030418 being multiplied by 5280 (the feet in a mile) gives 16.060704 feet, the moon's descent in 1′ in her orbit towards the earth, by the force of gravity. This may be also calculated from Cor. 9, prop. 4, B. 1, *Principia.* If we now divide the moon's distance from the earth 238533 miles, by 3956 miles, the earth's semidiameter, there will result 60.29 nearly, the moon's distance in semidiameters of the earth. Now as the gravity of the moon increases in proportion as the square of her distance from the centre of the earth decreases (Emerson's Tracts, prop. 13, p. 23;) hence her gravity at the surface of the earth would be 60.29 × 60.29 = 3634.8841 times greater, and therefore at the earth's surface the moon would fall towards the earth, or deflect from its tangent 3634.8841 × 16.06 = 58376.238646 feet in 1′; hence as the spaces described by falling bodies are as the squares of the times of falling (Emerson's Tracts, prop. 13, p. 23) the same power would carry the moon 60 × 60 = 3600 times less space in 1″ than in 1′, and therefore 58376.238646 ÷ 3600 = 16.215 feet, the space the moon would fall in 1″ at the earth's surface. This might also be computed from the equal description of areas, whatever be the distance of the body. Now to compare this with the gravity of terrestrial bodies, found by the pendulum: by a very accurate experiment, *Borda* has found that the length of pendulums vibrating seconds, at the Observatory at Paris, and reduced to a vacuum, is 0.741887 metres, or 29.208833077 English or American inches. The seconds here used by *Borda* must be those adopted in the French measures; and as they divided the *day* into 10 hours, the hour into 100′, and the minute into 100″; hence the number in the day, French measure, is 100000″, but in our division of the day it is 86400″, consequently 86400 : 100000 or 108 : 125 : : 1″ (English or American) : 1″1574 Paris nearly. Now as the lengths of pendulums, describing similar arches, are as the squares of the times of vibration (Emerson's Tracts, prop. 25) we have this proportion; 1″: 1″1574^2 or 1′ 33957476 : : 29.208833077 inches : 39.127315559 inches nearly, the length of a pendulum vibrating seconds, in our measures, in the latitude of Paris, half of which is 19.5636577795 inches. And as the square of the diameter of a circle : the square of its circumference : : half the length of a pendulum : the space described by a falling body in the time of one vibration (Emerson's Tracts, prop. 24, cor. 5) hence this proportion; 1^2 : 3.1416^2 or 9.86965056 : : 19.5636, &c. : 193.086465959, &c. inches = 16.09 feet nearly, the space

* Newton's Principia, B. 3, prop. 4.

the centripetal force near the surface of the earth, is equal to the force of gravity; and hence these two forces are identically the same. For if they were different, these forces united would cause bodies to descend to the earth with twice the velocity produced by the force of gravity alone. Hence it is evident, that the force which retains the moon in its orbit, is the force of terrestrial gravity extending to it. And it is reasonable to suppose that this virtue should extend to vast distances, as we find no sensible diminution of it on the tops of the highest mountains. Now as the moon gravitates towards the earth, so the earth, on the other hand, gravitates towards the moon, which is also confirmed from the phenomena of the tides and the precession of the equinoxes; which arise from the actions of the sun and moon on the earth. Hence also we discover by what law the force of gravity decreases at great distances from the earth: for as gravity does not differ from the moon's centripetal force, and that this is reciprocally proportional to the squares of the distances; it follows, that it is in that same ratio the force of gravity decreases. We do not consider here the small deviations arising from the actions of the sun and planets.

In like manner the same reasoning may be applied to the primary planets. The revolutions of the primary planets round the sun, and of the secondary planets round their respective primaries, are phenomena of the same kind with the revolution of the moon round the earth; and as it has been found that the centripetal forces of the primary planets are directed towards the centre of the sun, and those of the secondaries to their respective primaries,

that bodies fall in a second on the earth's surface, differing but little from that calculated from the moon's motion. Owing to the earth's centrifugal force, the gravity of bodies is diminished at the earth's surface, in advancing from the poles to the equator, nearly as the versed sine of double the lat. (Newton's Principia, prop. 20, B. 3) and the moon's gravity is also diminished by the sun's action. Laplace gives this diminution $\frac{1}{358}$ part. B. 4, ch. 1. Newton makes this something different. Prop. 3, B. 3, Principia. When these quantities are allowed, the forces will come out very nearly equal; and hence the force of gravity on the earth's surface, is the same force which retains the moon in her orbit.

☞ We must remark here, that the division of the day above mentioned, is taken from p. 162, of *Laplace's* Astronomy, vol. 1, as translated by *J. Pond;* and that this must be the division used by *Borda,* as no other would correspond to the length of this pendulum. This circumstance I have discovered in making the above calculations; and hence, wherever Laplace makes use of *seconds of time,* it must be the above division that he adopts, though he no where mentions it. This is very necessary to be known by those who make use of Laplace's works, as his translator, *J. Pond,* takes no notice of it. He makes the seconds of time the same as the seconds of a degree, according to the division of the quadrant adopted in France, which, besides the other errors in reducing the French measures, is a source of error, in the translation, through the whole work. See p. 173, vol. 1. of the translation, &c.

in the same manner as that of the moon is directed towards the earth; and that moreover all these forces are reciprocally proportional to the squares of the distances from the respective centres, as we have shewn to be the case with the moon; we must therefore conclude, that the nature of all these forces is the same. Therefore, as the moon gravitates towards the earth, and the earth again towards the moon, so also the secondary planets gravitate towards their primaries, and the primary planets again towards their secondaries; and in like manner, the primary planets towards the sun, and the sun again towards the primary planets. Hence the action of gravity is mutual between the sun and all the planets; for the secondary planets, while they accompany the primary, revolve at the same time with the primary round the sun. And Newton further confirms this *general gravitation of matter* from the inequalities of the moon, &c. the theory of which is clearly explained in the 3d Book of his Principia Hence also we conclude, from analogy, that the gravitation of matter is *universal*, and that therefore the whole *solar system* gravitates towards the fixed stars, and the fixed stars towards the solar system. The motions of the comets evidently shew, that the action of the sun, or its attractive virtue, is propagated on all sides to prodigious distances; for from the discoveries of the penetrating Newton, it is now evident, that the comets describe conic sections round the sun, having their foci in the sun's centre, and by radii drawn to the sun, describe areas proportional to the times; and also that the forces by which they are retained in their orbits, respect the sun, and are reciprocally proportional to the squares of the distances from his centre *

The foregoing conclusions are grounded on this axiom or rule, laid down by Newton in the beginning of the 3d Book of his Principles, and now received by all philosophers, viz. that "*to the same natural effects we must, as far as possible, assign the same natural causes.*" For no one can doubt, if gravity be the cause of the descent of a stone in *Europe*, that it is also the cause of the like descent in *America*. If in *Europe* the attraction of the earth be propagated to all kinds of bodies, and to great distances, can any one doubt that the same happens in *America* or in *China*, &c.? If this rule were not admitted, then nothing could be affirmed of the properties of bodies in general. The nature of particular things being known from observations and experiments, from these, as from certain data, we judge of the nature of such bodies in general. And hence, as we find that all bodies, whether on the earth or in the heavens, are heavy, as far as we can make any experiments or observations on them, we must therefore allow, that gravity is found in all bodies universally. In like manner all bodies, that come under our observations, are extended, movea-

* Principia, B. 3, prop. 40, and corollaries.

ble, and impenetrable; and from thence we conclude, that all bodies, even those we have made no observations on, are extended, moveable, and impenetrable. In like manner all bodies, that we have made any observations on, are found to be heavy; hence we conclude, that weight is a universal property of all bodies in general. Hence the gravity or weight of the fixed stars can no more be denied, though not as yet precisely observed, than their extension, mobility, or impenetrability, which no one will deny, though these qualities are no less out of the reach of observation.

It is thus that, from strict analogy, we can apply the knowledge which we obtain of bodies from experiments and observations on the earth, to those that we can have no such access to; and as Newton remarks (Rule 4th, B. 3, Prin.) "In experimental philosophy, we are to look upon propositions, collected by general induction from phenomena, as accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined, till such time as other phenomena occur, by which they may be either made more accurate, or liable to exceptions."

As the earth performs its motion round the sun in an orbit which is not circular but elliptical, having the sun in one of the foci, it follows, that the earth must at some times approach nearer to the sun than at others, and will therefore take more time in describing that half of its elliptic orbit, in whose focus the sun is, than the other, in consequence of that general law, first observed by *Kepler*, that is its describing equal areas round the sun in equal times. It is in our winter that the earth is in that part of its orbit where its velocity is greatest; and hence astronomers observe, that the earth is more rapid in the winter half of its orbit, than in the summer by about 7 or 8 days. It follows also, that in the winter we are nearer the sun than in summer, although in winter the season is colder and more inclement; but this phenomenon is easily explained from the sun's rays falling more perpendicularly on us in summer than in winter,* from their acting on the same place a longer time,† the days being longer in summer than in winter, and from their passing through a more dense and extensive part of the atmosphere. That the sun is actually nearer to us in winter than in summer, is also proved from the increase of his apparent diameter in this season,‡ as observed by all astronomers. The unequal motion of the earth in its orbit, will be more fully explained afterwards.

* Thus the heat in the torrid zone, is not caused from that part of the earth being nearer the sun, but from the sun's rays being darted perpendicularly on it, and through a comparatively small portion of the atmosphere.

† In the northern regions, the accumulation of the sun's heat is so great during their short summer, that it is sufficient for vegetation, &c. which takes place in these inclement regions, much quicker than in more temperate latitudes.

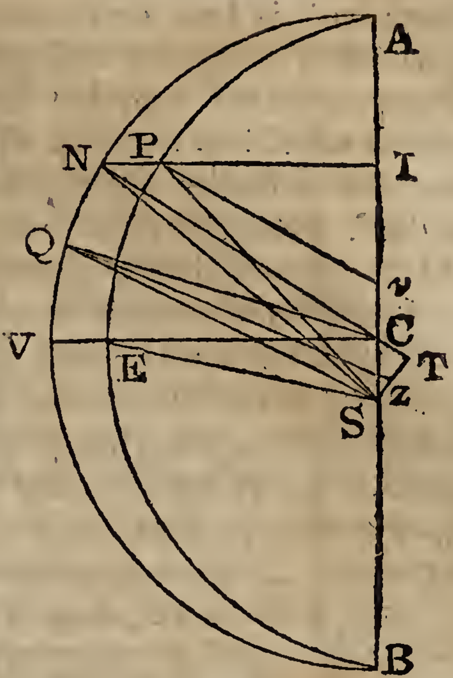
‡ See the table, p. 155.

The earth's axis makes an angle of 23° 28′ with a perpendicular to the plane of the ecliptic, or its orbit, and keeps always the same oblique direction during its annual course; the north pole is therefore turned towards the sun during one part of the earth's revolution, and the south pole is turned towards it, in like manner, during the other; and this is the cause of the different seasons, as *spring*, *summer*, *autumn*, and *winter*. If a small ball of wood, or any other substance, be procured, having the ecliptic, the equator, the tropics, polar circles, and a few meridians, delineated on it, and also a small wire passing through the poles of the equator; if this ball be carried round a lighted candle placed on a table, either in a circle, or in the curve of an ellipsis, having the candle placed in one of the foci, and the axis of the earth, during the motion, be always kept parallel to itself, the enlightened part of the earth will exhibit the different seasons in a pleasing and satisfactory manner. If the 12 signs be delineated on the ellipsis or circle, a line drawn from the ball through the candle will point out, on the opposite side of the curve, the sun's place, corresponding to that of the earth, pointed out by the ball.*

We shall now give the *theory of the earth's motion*, or rather the theory of the planets' motions in general, in elliptic orbits, about their common focus. This has been given by various authors, as Sir Isaac Newton, in his Principia, Dr. D. Gregory, Keil, and others, in their respective treatises on astronomy; but the most concise is that given by Vince in his Astr. from which we shall collect most of what we shall give on this subject.

As the orbits, which are described by the primary planets revolving round the sun, are ellipses, having the sun in one of the foci, and as each planet describes about the sun equal areas in equal times, it is from these principles that we shall deduce such consequences, as will be found necessary in our inquiries respecting their motions. From the variation in the planets' distances from the sun, and their describing equal areas in equal times, it is evident that they must move with unequal angular velocities round the sun. The principal proposition, therefore, on which the planets' theory depends, is the following. *The periodic time of a planet, the time of its motion from its aphelion, and the eccentricity of its orbit being given; to find its angular distance from its aphelion, or its* true anomaly, *and its distance from the sun.* This problem was first proposed by *Kepler*, and has therefore obtained the name of *Kepler's Problem.* (See Keil's Astr. Lect. 23 and 24.) Kepler knew no direct method of solving the problem, and therefore performed it by long and tedious trials.

* See part 2d, where this subject is fully elucidated on the globes.



Let AEB be the ellipse described by the planet round the sun at S in one of its foci, AB the greater axis, EC half the lesser axis, A the aphelion, B the perihelion, P the planets' place, AVB a circle, C its centre. Let NPI be drawn perp. to AB, join PS, NS and NC, on which produced let fall the perp. ST. Now let a body be supposed to move uniformly in a circle from A to Q with the *mean* angular velocity of the body in the ellipsis, whilst the body moves in the ellipse from A to P; then the angle ACQ is the *mean*, and the angle ASP the *true* anomaly; the difference of these two angles is called the *equation of the planets' centre*, or *prostapheresis*. Let p = the periodic time in the ellipse or circle (the periodic time being equal in both by supposition) and t = the time of describing AP or AQ; then, as the bodies in the ellipse and circle describe equal areas in equal times about S and C respectively, we have

area AQC : area of the circle : : $t : p$, and
area of the ellipse : area ASP : : $p : t$; also
area of circ. : area of ellip. : : area ASN : ASP;*

hence area AQC : area ASP : : area ASN : area ASP; therefore, area AQC = area ASN; take away the area SNC which is common to both, and the area QCN = SNC; but QCN = $\frac{1}{2}$QN × CN; therefore ST = QN Now if t be given, the arc AQ will be given; for as the body in the circle moves uniformly, it will be $p : t$: : 360° : AQ. In this manner, the mean anomaly for any given time may be found, the time when the planet was in the aphelion being given; and therefore, if ST or NQ be found, the ∠ NCA, which is called the *eccentric anomaly*, will be given, from whence, by one proportion, as we shall presently shew, the ∠ ASP the *true* anomaly will be given. The prob. is therefore reduced to this; to find a triangle CST such, that the ∠ C + the degrees of an arc = ST may be equal to the given ∠ ACD. M. de la *Caille*, in his Astronomy, gives an expeditious method of performing this by trial, as follows: Find the arc of the circumference of the circle AQB that is equal to CA, by saying as 3.1416 : 1 : : 180° : 57° 17′ 44″8 = the number of degrees in an arc = CA; hence CA : CS : : 57° 17′ 44″8 : the degrees of an arc = CS. Now assume the ∠ SCT, multiply its sine into the degrees in CS, and to the product add the ∠ SCT, and if the sum be equal to the given angle ACQ, the supposition was right; if not, add or subtract the difference to or from the first supposition, according as

* Vince's Conic Sect. 2d ed. prop. 7 of the ellip. cor. 3 and 4.

the result is less or greater than ACQ; this operation being repeated, a few trials will give the accurate value of SCT. The degrees in ST may be most readily obtained by adding the logarithm of CS to the log. of the sine of the angle SCT, and lessening the index by 10, the remainder will be the log. of the degrees in ST. Having thus found the value of the arc AN, or the angle ACN, we shall now show how to find the angle ASP.

Let v be the other focus, and let AC = 1; then $SP^2 - Pv^2 = vS^2 + 2\,vS \times vI$ (Eucl. 12 prop. 2 B.) $= (vS + 2\,vI) \times vS = (2\,Cv + 2\,vI) \times 2\,SC = 2\,CI \times 2\,SC$; hence $SP + Pv : 2\,CI :: 2\,SC : SP - Pv$ (because the difference of the squares of two quantities, is equal to the rectangle of their sum and difference) or $2 : 2\,CI :: 2\,SC : SP - \overline{2 - SP}$,* or dividing by 2, $1 : CI :: SC : SP - 1$; hence $SP = 1 + CS \times CI = 1 + CS \times \cos.$ $\angle$ ACN. But $\frac{1 - \cos. ASP}{1 + \cos. ASP} = \text{tang.}\ \tfrac{1}{2}\ ASP^2$ (see Vince's Trig. art. 94) also SP, or $1 + CS \times \cos. ACN : \text{rad.} = 1 :: SI$, or $CS + CI$ or $CS + \cos. ACN : \cos. ASP$ (Vince's Trig. art. 125) $= \frac{CS + \cos. ACN}{1 + CS \times \cos. ACN}$. Hence tang. $\tfrac{1}{2}\ ASP^2 \left(= \frac{1 - \cos. ASP}{1 + \cos. ASP}\right) =$
$$\frac{1 + CS \times \cos. ACN - CS - \cos. ACN}{1 + CS \times \cos. ACN + CS + \cos. ACN} = \frac{1 - CS + \cos. ACN \times \overline{CS - 1}}{1 + CS + \cos. ACN + \overline{CS + 1}}$$
$$= \frac{SB - \cos. ACN \times SB}{SA + \cos. ACN \times SA} = \frac{1 - \cos. ACN}{1 + \cos. ACN} \times \frac{SB}{SA} = \text{tang.}\ \tfrac{1}{2}\ ACN^2 \times \frac{SB}{SA}$$
(Vince's Trig. art. 95.) hence $SA^{\frac{1}{2}} : SB^{\frac{1}{2}} ::$ tang. $\tfrac{1}{2}$ ACN : tang. $\tfrac{1}{2}$ ASP, therefore we get ASP the *true* anomaly required.

Example. Required the true place of *Mercury*, on August 26, 1740, at noon, the equation of the centre, and his distance from the sun.

According to la *Caille*, Mercury was in its aphelion on Aug. 9, at 6h. 37′. Hence on Aug. 26, it had passed its aphelion 16 d. 17h. 23′; therefore 87 d. 23h. 15′ 32″ (his periodic rev.) : 16 d. 17h. 23′ :: 360° : 68° 26′ 28″ the arc AQ or *mean* anomaly. Now, according to la *Caille*, CA : CS :: 1011276 : 211165 :: 57° 17′ 44″8 : 11° 57′ 50″ = 43070 seconds, the value of CS reduced to the arc of a circle, the log. of which is 4.6341749. Also 68° 26′ 28″ = 246388″; and assuming the $\angle$ SCT = 60° = 216000″, the operation to find the $\angle$ ACN will be as follows:

* The reason of taking 2 for SP + Pv is, because from the property of the ellipse (Emerson's Con. Sect. prop. 1, B. 1) the sum of the lines SP, Pv, drawn to any point P of the curve, is always equal the transverse axis AB = 2 AC; but as AC in this case = 1, AB, or SP + Pv = 2.

4.6341749		
9.9375306 log. of - -	216000	$= a$
4.5717055 - - - - - -	37300	
	253300	
	246388	
	6912	$= b$
4.6341749		
9.9287987 - - - - - -	209088	$= a - b = 58° 4' 48'' = c$
4.5629736 - - - - - -	36557	
	245645	
	246388	
	743	$= d$
4.6341749		
9.9297694 - - - - - -	209831	$= c + d = 58° 17' 11'' = e$
4.5639443 - - - - - -	36639	
	246470	
	246388	
	82	$= f$
4.6341749		
9.9296626 - - - - - -	209749	
4.5638375 - - - - - -	36630	
	246379	
	246388	
	9	$= h$

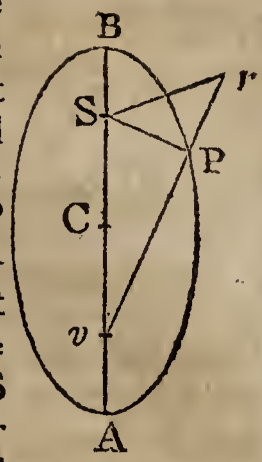
Hence, as the difference between the value deduced from the assumption and the true value, is now diminished about nine times every operation, the next difference would be $1''$; hence $h + g - 1'' = 58° 15' 57''$ the true value of the angle ACN the *eccentric* anomaly. Hence, from the proportion laid down above, the *true* anomaly is found by logarithms, thus:

Log. tang. 29° 7′ 58½″ - - -	9.7461246
½ log. SB = 800111 - - - -	2.9515751
	12.6976997
½ log. SA = 1222441 - - -	3.0436141
Log. tang. 24° 16′ 15″ - - -	9.6540856

Hence the *true anomaly* is 48° 32′ 30′. Now the aphelion A was in 8s. 13° 54′ 30″, therefore Mercury's true place was 10s. 2° 27′; hence, from what we have shewn above, 68° 26′ 28″ — 48° 32′ 30″ = 19° 53′ 58″, the *equation of the centre*. Also SP = 1 + CS × cos ∠ ACN = 1.10983, the distance of Mercury from the sun, the radius of the circle, or the planets' mean distance being unity. Vince remarks, that the above method of computing the *eccentric* anomaly, appears to be the most simple and easy of application of all others. and capable of any degree of accuracy. From the same method, we are able to compute, at any time, the place of a planet in its orbit, and its distance from the sun.

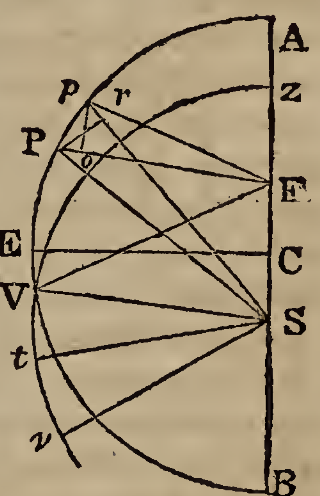
As the bodies Q and P were supposed to depart from A at the same time, and will coincide again at B, AQB, APB being each described in half the time of a revolution; and as the planet moves with its least angular velocity at A; for as AC is its greatest dist. from the sun, the arc which it describes must be proportionally smaller, to have the areas described in the same time equal; therefore from A to B, or in the *first* six signs of anomaly, the angle ACQ will be greater than ASP, or the mean will be greater than the *true* anomaly; but from B to A, in describing the other half of its orbit, or in the *last* six signs, as the planet at B moves with its greatest angular velocity, being nearest the sun, the *true* will be greater than the *mean* anomaly When the equation is greatest in going from A to B, the *mean* place is before the *true* place by the equation, and in the remaining half of the orbit the *true* place is before the *mean* place by the equation; hence from the time the equation is greatest, until it becomes greatest again, the difference between the true and mean motions is twice the equation. From apogee to perigee, the true and mean motions are the same.

There is another method ascribed to *Seth Ward*, Professor of Astronomy, at Oxford, which, though less accurate than the method given above, yet, as in many cases it serves as a useful approximation, and renders the calculation more simple and easy, we shall here briefly explain it.



Ward assumed the angular velocity about the focus v to be uniform (the sun being supposed in the other focus S) and therefore made it represent the *mean anomaly*. Let vP be produced to r, and make Pr = PS; then in the triangle Svr, rv + Sv : rv — Sv : : tang. $\frac{1}{2}$ of the angles vSr + vrP : tang. half their difference vSr — vrS (Emerson's Trig prop. 6, B. 2, or Simson's prop. 3) but $\frac{1}{2}$ (rv + vS) = $\frac{1}{2}$ AB + $\frac{1}{2}$ vS = AS; (for rv = SP + Pv = AB) and $\frac{1}{2}$ (rv — vS) = $\frac{1}{2}$ AB — $\frac{1}{2}$ vS = SB; likewise tang. $\frac{1}{2}$ the sum of the angles vSr + vrS = tang. $\frac{1}{2}$ ∠ AvP (Eucl. p. 32, B. 1) and tang. $\frac{1}{2}$ diff. of the angles vSr — vrS = (Pr being = PS) tang. $\frac{1}{2}$ diff of the angles vSr — PSr (Eucl. 5, B. 1) = tang. $\frac{1}{2}$ ∠ ASP; hence *the aphelion distance : perihelion dist. : : tang. of* $\frac{1}{2}$ *the mean anomaly : tang.* $\frac{1}{2}$ *the true anomaly*. This is called the *simple elliptic hypothesis*. In the *earth's* orbit, which is nearly circular, the error is never greater than 17″; the error is greater in the orbits of *Mars* and *Mercury*, and hence *Bulialdus* corrects this theory to adapt it to these planets (see Keil's Astr Lect. 24.) In the orbit of the moon, the error may amount to 1′ 35″. By *Ward's* hypothesis, the computed place is *more backward* than the true, for 90° from the aphelion and perihelion, and for the other part it is *more forward*.

That *Ward's* hypothesis of the uniformity of the angular velocity about the focus v is not true, may be shewn as follows:



From the centre S at the distance SV = $\overline{AC \times CE}^{\frac{1}{2}}$ (or so that SV may be a mean proportional between the semi-transverse AC, and semi-conjugate CE of the ellipse AEB) describe the circle zV; then the area of this circle will be equal the area of the ellipse (see Vince'e Con. Sect. Ellip. pr. 7, cor. 5) let a body be supposed to move with a uniform motion through the periphery of the circle, in the same time that the planet performs one revolution in the ellipse; and let the body and the planet commence their motion at the same time, the planet from A and the body from z, so that the planet may describe AP in the same time that the body describes zv; then the angle zSv is the *mean*, and ASP the *true* anomaly. Take p indefinitely near P and join SP, and draw Pr, po, perpendicular to Sp, FP respectively; then Pr = po; but the angle PFp varies as $\frac{po}{PF}$* = $\frac{pr}{PF}$; but the area PSp is given in a given time; therefore

* The angle PFp increases in proportion as po increases, all other circumstances remaining the same; and it diminishes in proportion as PF increases, po remaining constant, or is inversely as PF; hence when both vary, the angle PFp varies as $\frac{po}{PF}$.

Pr varies as $\frac{1}{PS}$; hence the angle PFp, described in a given time, varies as $\frac{1}{PF \times PS}$, which is not a constant quantity. Now the angle PFp : ∠ PSp : : PS : PF : : $\frac{1}{PF \times PS}$: $\frac{1}{PS^2}$: and as equal areas are described in equal times in the circle and ellipse about S, the angular velocity in the circle, about S, is equal $\frac{1}{SV^2}$.* Therefore the angular velocity about S is greater or less than the mean angular velocity, according as PF × PS is less or greater than SV^2, or than AC × CE. Also, the angular velocity about S, is the same in similar points of the ellipse in respect to the centre, or at equal distances from the centre. From the above investigation, the *greatest equation of the centre* may be found, the dimensions of the orbit being given. For while the angular velocity of the body in the circle, is greater than that of the planet in the ellipse, about S, the equation will increase, the planet and body commencing their motion from A and *z* together; when the angular velocities are equal, the equation is then greatest; and this takes place when $\frac{1}{SP^2} = \frac{1}{SV^2} = \frac{1}{AC \times CE}$, or when AC × CE $= SP^2$; hence SP is given. Let this value of SP be represented by SV, then as SV is known, FV (= 2 AC — SV) will be given; and as SF is given, we can find the angle FSV, the true anomaly. Hence (see the fig. p. 313) by what we have shewn in determining the angle ASP (p. 314) $SB^{\frac{1}{2}} : SA^{\frac{1}{2}}$: : tang. ½ *true* anomaly : tang. ½ *eccentric* anom. ACN, or tang. ½ SCT; and as SC is given, ST or its equal NQ is likewise given; now to convert this into degrees, we have this proportion, rad. or 1 : NQ : : 57° 17′ 44″8 : the degrees in NQ, which added to or subtracted from the angle ACN, gives ACQ the *mean* anomaly; the difference between which and the *true* anomaly, is the greatest equation. The equation at any other time may in like manner be found, SP being given.

The greatest equation being given, the *eccentricity*, and therefore the dimensions of the orbit may be found. For, as is plain from the last article, the equation is greatest when the distance is a mean between the semi-transverse and semi-conjugate of the elliptic orbit, and therefore in orbits nearly circular, the body must be nearly at the extremity of the conjugate or minor axis, and hence the angle NCA or SCT will be nearly a right angle, ST will be therefore nearly equal to SC; and also the angle NSA nearly equal PSA. Now the angle NCA — NSA (or PSA) = SNC, and QCA — NCA = QCN; these being added, we have QCA — PSA = QCN + SNC = 2 QCN nearly (NC being nearly parallel to QS) that is, the difference between the *true* and *mean* anomaly, or the *equation of the centre*, is nearly double the arc QN, or double ST, or very nearly twice SC. Hence 57° 17′ 48″8 : half the greatest equation : : rad. 1 : SC the eccentricity. If the

* This and similar properties will be demonstrated in the laws of motion.

orbit be considerably eccentric, compute the greatest equation to this eccentricity; then as the equation varies nearly as SC, we have this proportion; as the computed equation : the eccentricity found : : given greatest equation : true eccentricity.

Thus, if with la *Caille*, we suppose that Mercury's greatest equation is 24° 3′ 5″ (see p. 262) then 57° 17′ 44″8 : 12° 1′ 32″5 : : 1 : .209888 the eccentricity very nearly. Now the greatest equation, computed from this eccentricity, is 23° 54′ 28″5; hence 23° 54′ 28″5 : 24° 3′ 5″ : : .209888 : .211165 the true eccentricity. *Delambre* makes the eccentricity of Mercury 79855.4, his mean dist. being 38710. By taking the mean distance of the earth from the sun 100000, *Vince* makes the *eccentricities* and *greatest equations* of the planets as follow; *Mercury* eccen. 7955 4, great. equat. 23° 40′; *Venus* eccen. 498, gr. eq. 47′ 20″; the *Earth* eccen. 1681.395, gr. eq. 1° 55′ 36″5; *Mars* eccen. 14183.7, gr. eq. 10° 40′ 40″; *Jupiter* eccen. 25013.3, gr. eq. 5° 30′ 38″3; *Saturn* 53640.42, gr. eq. 6° 26′ 42″; and *Herschel* eccen. 90804, gr. eq. 5° 27′ 16″. M. *Delambre*, in his tables annexed to *La Lande's* Astr. 3d ed. makes the greatest equations of the planets for the respective years, as follow; Mercury 23° 39′ 39″ tab. 101; Venus 47′ 20″, tab. 108, year 1780; Earth 1° 55′ 2″4, tab. 5, year 1780; Mars 10° 40′ 39″, tab. 115, year 1770; Jupiter 5° 30′ 37″7, tab. 124, 1750; Saturn 6° 26′ 41″7, tab. 147, 1750; Herschel 5° 21′ 2″7, tab. 165, year 1780. In these tables, Delambre gives the equation and its secular variation, for every degree of the planets' mean anomaly. *Laplace*, taking the mean distance of the earth from the sun = 1, makes the proportion of the eccentricities of the semi-major axes, for the beginning of the year 1750, as follows; Mercury 0.205513, Venus 0.006885, the Earth 0.016814, Mars 0.093808, Jupiter 0.048877, Saturn 0.056223, Uranus or Herschel 0.046683. He gives the secular variation of this proportion as follows; the sign — indicates a diminution. Mercury 0.000003369, Venus — 0.000062905, the Earth — 0.000045572, Mars 0.000090685, Jupiter 0.000134245, Saturn — 0.000261553, Herschel — 0.000026228.

The eccentricity and true anomaly being given, the mean anomaly may be readily found by a direct solution, as follows; the eccentricity being given, the ratio of the transverse and conjugate, or the major and minor axes, which is the ratio of NI : PI (Emerson's Conic Sect. prop. 19, B. 1) is given; for as AC, CS are given (see the fig. p. 313) we have $GC = \overline{(SG^2 - SC^2)}^{\frac{1}{2}} = (\overline{AC + SC} \times \overline{AC - SC})^{\frac{1}{2}}$. Hence the angle ASP being given, we have PI : NI : : tang. ASP : tang. ASN; therefore in the triangle NCS, NC, CS, and the angle CSN are given, and therefore the angle SCN is given by Trig. the supplement of which is the angle ACN or SCT; hence in the rt. angled triangle STC, SC and the angle SCT are given, therefore ST, which is equal NQ, is given; this arc, being the measure of the equation, may be found by this proportion; rad. : ST : : 57° 17′ 44″8 : the degrees in NQ, which added to ACN, gives ACQ, the mean anomaly.

The mean hourly motion of a planet being given, the hourly motion in its orbit may be found in the following manner:

The planets' hourly motion in its orbit, is found immediately from what we have shewn above in the correction of *Ward's* Theory; for it appears from thence, that the angles PS*p*, VS*t* described by the planet at P in the ellipse, and the body V in the circle in the same time, are as $SV^2 : SP^2$, or as $AC \times CE : SP^2$ (see the fig. p. 317) hence $PSp = VSt \times AC \times CE \div SP^2$, which is the hourly motion of a planet in its *orbit*, the angle VS*t* being the *mean* motion of the planet in an hour. For greater accuracy SP must be taken at the middle of the hour. Tables of the planets' hourly motions in their orbits may be thus easily computed.

OF THE MOON.

THE moon being the nearest celestial body to the earth, and, next to the sun, the most remarkable and interesting in our system; interesting not only from its resplendent appearance, but also from its various phases, which afford us a measure of time so remarkable, that it has been primitively in use among all people. Ancient history testifies, that the Hebrews, the Greeks, the Romans, and in general all the ancients, used to assemble at the time of new or full moon, to testify their gratitude for its manifold uses. It is no wonder, therefore, that the ancient astronomers were always attentive to discover its motions; and their observations, handed down to succeeding astronomers, enable them to settle her mean motion more accurately than could be done by modern observations alone. It was from the observations of some ancient eclipses, that Dr. *Halley* discovered an acceleration in her mean motion.

The proper motion of the moon in her orbit, is, like the sun or rather the earth, from *west* to *east*; and her place being compared with the fixed stars in one revolution, she is found to describe an orbit inclined to the ecliptic; her motion also appears not to be uniform; and the position of her orbit, and the line of its *apsides*, are observed to be subject to a continual change. These and other phenomena we shall explain in the following remarks.

The *mean motion of the moon* is found thus: observe her place at two different times, then the mean motion during this interval is given, on supposition that the moon had the same situation with regard to her apsides during each observation; if not, it will be sufficiently exact, if the interval of the times be very great. Hence the moon's places, at a small interval of time from each other, being compared, we get the mean time of a revolution nearly; and then at a greater interval, the mean time of her revolution is obtained more correctly. The moon's place may be determined directly from observation, or deduced from an eclipse.

M. *Cassini*, in his Astronomy, p. 294 (as *Vince* remarks) observes that on Sept. 9, 1718, the moon was eclipsed, the middle of which eclipse happened at 8h 4′, when the sun's true place was 5s. 16° 40′. Having compared this with another eclipse, the middle of which was observed at 8h. 32′, on August 29, 1719, when the sun's place was 5s. 5° 47′, the interval gives 354 d. 28′, in which the moon made 12 revolutions and 349° 7′ over; hence 354 d. 28′ being divided by 12, rev. + 349° 7′ part of a revolution, or 354.0125 days divided by 12.96947685 = 27 d. 7h. 6′ 7 for the time of one revolution. From two eclipses, in 1699 and 1717, the time was found to be 27 d. 7h. 43′ 6″.

The moon was observed to be eclipsed at Paris, on Sept. 20, 1717, the middle of which eclipse was observed at 6h. 2′. And *Ptolemy* remarks, that a total eclipse of the moon was observed at Babylon, on March 19th, 1720 years before Christ, the middle of which was at 9h. 30′ at that place, or 6h. 48′ at Paris. The interval of these times was 2437 years, 147 days less 46′, of which 609 were bissextiles; this being divided by 27 d. 7h. 43′ 6″, gives a little more than 32585.5 revolutions. Now the difference of the sun's places, and therefore of the moon's,* as observed at both observations, was 6s. 6° 12′; therefore the moon had made 32585 revolutions, 6s. 6′ 12′ in the interval of 2437 y. 174 d. — 46′, which gives 27 d. 7h. 43′ 5″ for the mean time of one revolution. This determination is very exact, as the moon was at each time, very nearly, at the same distance from her apside. Her *mean diurnal motion* is therefore 13° 10′ 35″, and her mean *hourly* motion 32′ 56″ 27‴5. M. de la *Land* makes her mean *diurnal* motion 13° 10′ 35″ 02784394. *Delambre* in his tables (tab. 28) has it 13° 10′ 35″. This is the mean time of a revolution with respect to the equinoxes.

The annual precession of the equinoxes being 50¼″, or nearly 4″ in a month; hence the moon's mean revolution must be greater with respect to the fixed stars, than with respect to the equinox, by the time in which she describes 4″ with her mean motion, which is about 7″. Hence the time of a *sidereal* revolution of the moon is 27 d. 7h. 43′ 12″. *Laplace* makes the length of her sidereal revolution at the commencement of 1750 = 27 d. 32166118036, or 27 d. 7h. 43′ 11″5 nearly.

The *acceleration* of the moon, before taken notice of, though but little sensible, since the most ancient recorded eclipse will be developed in progress of time, as *Laplace* remarks, though an immense number of ages would be necessary to determine it by observations. The discovery of its cause has, however, antici-

* The place of the moon at the eclipse is here taken the same as that of the sun, which is not accurate unless when the eclipse is central; for this long interval it is, however, sufficiently accurate.

pated this immense length of time, and shewn that this acceleration is periodical.

Laplace, whose penetration has enabled him to discover most of the *secular* variations in our system, from his profound investigations, and strict application of the laws of gravity, has elucidated this as well as many other intricate subjects, in so satisfactory a manner, that much of our observations on the moon will therefore be collected from him, as time would not permit, at present, our entering deeply into their investigation.

The moon moves in an elliptic orbit, in one of whose foci the earth is situated. Her radius vector, or a line drawn from her to the earth, describes about this point equal areas in equal times. The *eccentricity* of her orbit is 0.0550368, her mean distance from the earth being taken as unity; which gives for the *greatest equation* of her centre (7°0099) 6° 18′ 32″076. The lunar *perigee* has a direct motion, that is, in the same direction as the motion of the sun, and the length of its sidereal revolution is 3232.46643 days or 8 y. 312 d. 11 h. 11′ 39″552.

If the place of the moon be observed as often as possible during a whole revolution, and the true and mean motions be compared, the difference will be double the equation. If there should happen to be found two observations, where the difference of the true and mean motions is nothing, the moon must then have been in her apogee in the one, and in her perigee in the other, as is evident from the theory of the planets' motions, given in chap. 4. *Mayer* makes the greatest *eccentricity* 0.05503568, and the *greatest equation* corresponding 6° 18′ 31″6. In his last Tables, published by Mr. *Mason*, under the direction of Dr. *Maskelyne*, he makes it 6° 18′ 32″. *Delambre*, in his Tables (tab. 50) makes it 6° 18′ 31″6.

The place of the *apogee* may be thus determined, from M. *Cassini's* observations; the greatest equation = 5° 1′ 44″5; hence 57° 17′ 48″8 : 2° 30′ 52″25 : : AC = 100000 : CS = 4388 (see p. 319) = the moon's eccentricity at that time. This eccentricity is, however, subject to a variation, being the greatest when the apsides are in the syzygies, and least when in the quadratures. Now let v be the focus in which the earth is situated (see the small fig. p. 317) then taking BSP for the mean anomaly, BvP being the true anomaly, their difference SPv (Eucl. 32, B. 1) is the *equation of the orbit*, which equation is here 37′ 50″5; and as PS = Pr, the angle vrS = 18′ 55″25; hence (Trigonom.) vS = 8776 : vr = 200000 : : sine vrS = 18′ 55″25 : sine vSr or its supplement BSr, = 7° 12′ 20″, from which let vrS = 18′ 55″25 be taken, and we have BvP = 6° 53′ 25″ = the distance of the moon from its apogee; to this let the true place of the moon = 2s. 19° 40′ be added, the sum gives 2s. 26° 33′ 25″ for the place of the apogee on December 10, 1685, at 10h. 38′ 10″ mean time at Paris. Hence this may be considered as an *epoch* of the place of the apogee.

The *mean motion of the apogee* may be thus determined; let its place be found at different times, and let the difference of these places be compared with the interval of the time between. In performing this, the observations must be first taken at a small distance from each other, as we might be deceived in a whole revolution; then those observations at a greater distance may be compared. Thus the mean annual motion of the apogee is found, according to *Mayer*, = 40° 39′ 50″ or 1s. 10° 39′ 50″, its monthly motion = 6′ 41″, its hourly mot. = 17″, &c. *Delambre* makes its annual motion = 1s. 10° 39′ 50″5,* in a month 6′ 41″, &c.

To determine the place of the moon's nodes. The moon's place is directly opposite to the sun in a *central* eclipse of the moon, and hence the moon must then be in her node; if the true place of the sun be then found by calculation, or rather by observation, the opposite sign and degree, &c. in the ecliptic, will be the true place of the moon, and consequently the place of her node.

M. Cassini, in his Astr. p. 281, says, that on April 16, 1707, a central eclipse was observed at Paris, the middle of which took place at 3h. 48′ apparent time. Now the sun's true place calculated for that time, was 0s. 26° 19′ 17″; hence the place of the moon's node was 6s. 26° 19′ 17″. The moon, at that time, passed from north to south lat. and therefore this was the descending node. The place of the node is always ready calculated in the astronomical tables, with its mean motion.

The place of the nodes may be also determined as in p. 264. *The mean motion of the nodes* may be determined, by finding the place of the nodes at different times, from which its motion, in the interval, will be given; the greater the interval the more accurate will the motion be discovered. Mayer and Delambre, both make the mean annual motion of the nodes 19° 19′ 43″. The motion of the nodes is *westward*, contrary to the order of the signs. The length of their sidereal revolution, according to *Laplace*, is 6793.3009 days, or 18 y. 223 d. 7h. 13′ 17″76. Their motion is subject to several inequalities, of which the greatest, according to *Laplace*, is proportional to the sine of double the angular distance of the sun from the ascending node of the lunar orbit, and at its maximum amounts to (1° 8105) 1° 40′ 46″02.

The moon's *nodes* being those points where the lunar orbit cuts the orbit of the earth, or the ecliptic; and the angle formed by the planes of these orbits, the same as the *inclination of the orbit of the moon to the ecliptic*, we shall now shew how to find this inclination. When the moon is 90° distant from her nodes, it is evident that she has then her greatest latitude, and that this latitude will measure the inclination of her orbit, in the same manner as the sun's greatest declination measures the inclination of the

* As Delambre (tab. 27) only gives the moon's mean motion and mean anomaly for entire years, it is necessary to remark, for the learner's sake, that if the mean anomaly be taken from the mean motion, the remainder will give the motion of the apogee.

equator and ecliptic. Hence if, when the moon is 90° from her nodes, her right ascension and declination be observed, and from thence her latitude be computed (see the note to prob. 3, part 3d) this will be the inclination of her orbit for that time. If similar observations be made for every distance of the sun from the earth, and for every position of the sun in respect of the moon's nodes, the inclination at those times will be observed. It appears, from these observations, that the inclination of the moon's orbit to the ecliptic is variable, and that the *least inclination* is about 5°, which takes place when the nodes are in the quadratures; and the *greatest* about 5° 18′, which is found to happen when the nodes are in the syzygies. The inclination is also found to depend on the sun's distance from the earth *Laplace* makes the inclination (5°.7188) 5° 8′ 48″9. He makes the greatest inequality in its variation (0° 1631) 8′ 48″444, and remarks, that it is proportional to the cosine of the same angles on which the inequality of the motion of the nodes depends.

The moon would always describe the same ellipse, in her revolution round the earth, if this revolution were not disturbed by the action of the sun; the principal axis of her orbit would remain invariable, her periodic times would be the same, and the inclination of her orbit to the ecliptic, as well as the place of her nodes, would remain fixed; but from the sun's action, her motions become subject to so many irregularities, that to establish her theory, and calculate her place truly, is one of the greatest difficulties in physical or practical astronomy. These irregularities are, however, evidently connected with the sun's position

The moon's motion being examined for one month, it will be found that it is subject to an irregularity which sometimes amounts to 5° or 6°, but that every 14 days this irregularity disappears.* If these observations be continued for different months, it will also appear that the points where the inequalities were the greatest, were not stationary, but advanced forwards about 3° in a month, so that, in respect to the apogee, the moon's motion was about $\frac{1}{120}$ less than her absolute motion; and hence the apogee's progressive motion has been discovered. This *first* inequality, or *equation of the orbit*, was determined by *Ptolemy*, from three lunar eclipses observed at Babylon, in the years 719 and 720 before J. C. by the Chaldeans; he found it amounted to 5° 1′ when greatest. But he soon found that this would not account for all the irregularities of the moon, as her distance from the sun, observed both by *Hipparchus* and himself, sometimes agreed with this inequality and sometimes did not. He found that this *first* inequality would give the moon's place sufficiently correct, when the apsides of her orbit were in the quadratures; but that when the apsides were in the syzygies, he discovered that there was a further inequality of $2\frac{2}{3}$°, which, in this case, made the whole inequality amount to about

* Vince.

7⅔°. This *second* inequality is called the *evection*, and arises from the variation of the eccentricity of the moon's orbit. Hence Ptolemy found that the moon's inequality varied from 5° to 7⅔°, and at a mean was therefore 6° 20′. *Mayer* makes it 6° 18′ 31″6. It is therefore extraordinary, how Ptolemy, in a point of so delicate a nature, should have determined this to so great a degree of accuracy.

Laplace makes the evection (1° 4902) 1° 20′ 28″248, and remarks, that it is proportional to the sine of double the mean angular distance of the moon from the sun, *minus* the mean angular distance of the moon from the perigee of its orbit. In the oppositions and conjunctions of the moon with the sun, it is confounded with the equation of the centre, which it constantly diminishes, and hence the ancient astronomers, who only determined the elements of the lunar theory by means of eclipses, with a view of predicting these phenomena, always found the equation of the centre less than the truth, by the whole quantity of the evection.

There is another inequality observed in the moon, which is called the *variation*; this inequality disappears in eclipses, or in the conjunctions and oppositions, and could not have been discovered from the observation of those phenomena. It also disappears in those points where the sun and moon are distant from each other 90°. It is at its *maximum*, and amounts to 35′ 40″992, when their mutual distance is 45°; from whence it is inferred, that it is proportional to the sine of double the mean distance from the sun.

The last inequality which we have to observe, is that known by the name of the *annual equation*, caused by the moon's motion being accelerated when that of the sun is retarded, and the contrary. The law of this inequality is exactly the same as that of the equation of the centre of the sun,* but with a contrary sign; at its maximum it is (0° 2064) 11′ 8″736. This inequality in eclipses becomes confounded with the equation of the centre of the sun, and in calculating the instant of these phenomena, it is indifferent whether these two equations be considered separately, or the annual equation of the lunar theory be suppressed to augment the equation of the sun's centre. This is one principal reason why the ancient astronomers gave too great a value to this last equation, and assigned too small a value to the equation of the sun's centre affected by the evection.

The following exhibits, at one view, the *revolutions of the moon*, of its *apogee* and *nodes*, as determined by M. de la *Lande*.

	d.	h.	′	″
Tropical revolution - - - - - - -	27	7	43	4,6795
Sidereal revolution - - - - - - -	27	7	43	11,5259
Synodic revolution - - - - - - -	29	12	44	2,8283

* The equation of the sun's centre at its maximum, according to Laplace, was in 1750, equal (2° 1409) 1° 55′ 36″516. The method of finding this is given in p. 313.

		d.	h.	′	″
Anomalistic revolution - - - - - -		27	13	18	33,9499
Revolution in respect to the node - -		27	5	5	35,603
Tropical revolution of the apogee	8 y.	311	8	34	57,6177
Sidereal revolution of the apogee	8	312	11	11	39,4089
Tropical revolution of the node	18	228	4	52	52,0296
Sidereal revolution of the node	18	223	7	13	17,744
Diurnal motion of the moon in respect to the equinox	13° 10′ 35″,02784394				
Diurnal motion of the apogee - -		0	6	41,	069815195
Diurnal motion of the node - -		0	3	10,	638603696

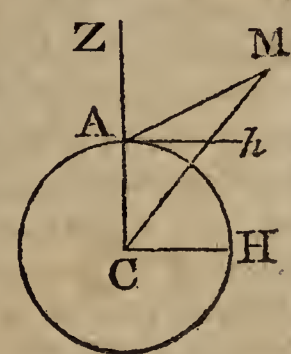
Nevil Maskelyne finds, from the new Tables of M. *Burg* and *Delambre*, that the mean longitude of the moon, at the middle of the year 1813, including the secular equation and new equation of 180 years, will be 9s. 0° 25′ 48″2; mean anomaly with secular equation 7s. 25° 59′, and the supplement of the node with the secular equation 7s. 8° 13′ 15″6.

The apparent diameter of the moon varies in a manner analogous to her motions. This diameter may be measured at the time of full moon, by a *micrometer* placed in the focus of a telescope, or it may be measured by the time of its passing over the vertical wire of a *transit telescope*; but this must be within one or two hours of the time of full moon, before the visible disk is sensibly changed from a circle. The diameter may be thus found from the time of its passing over the meridian; let d'' = the moon's horizontal diameter, c = sec. of her declination, and m = the length of a lunar day, or the time from the moon's passage over the meridian on the day of observation, to the time of her passage over the meridian, on the next day. Then (art. 8 of the last note to prob. 19, part 3, p 216) cd'' = the moon's diameter in right ascension; therefore $360° : cd'' :: m :$ the time (t) of passing the meridian; hence $d'' = 360° \times \frac{t}{cm}$. If the time be observed when the limb of the moon comes to the meridian, the time when the centre comes to it can be found, by adding to or subtracting from, the time when the first or second limb comes to the meridian, half the time of the moon's passage over the meridian.

Albategnius made the diameter of the moon vary from 29′ 30″ to 35′ 20″, and hence found the mean = 32′ 25″. *Copernicus* found the diameter to vary from 27′ 34″ to 35′ 38, and therefore the mean to be 31′ 36″. *Kepler* made the mean diameter 31′ 22″. M. de la *Hire* made it 31′ 30″. M. *Cassini* made the diameter from 29′ 30″ to 33′ 38″. La *Lande*, from his own observations, found the mean diameter = 31′ 26″, and the extremes from 29′ 22″, when the moon is in apogee and conjunction, to 33′ 31″ when in perigee and opposition. The mean diameter here taken is the arithmetic mean between the greatest and least; the diameter at the mean distance being 31′ 7″. *Delambre*, in his tables (table 91) gives the extreme horizontal diameters 29′ 30″ and 33′

30″ respectively; and also its augmentation in every degree of altitude corresponding to its respective horizontal diameters, and corrects these altitudes of refraction by tab. 93 (see articles 1510, 2247, and 2248, La *Lande's* Astr. 3d ed.). *Laplace* makes the apparent diameter at the moon's greatest dist. = (5438″) 29′ 21″, 912, and at her least dist. = (6207″) 33′ 31″068. Taking the apparent diameter at her mean distance = 31′ 7″, her real diameter is found to be 2159 07 miles,* and her magnitude is about $\frac{1}{49}$ of the magnitude of the earth.

When the moon appears in the horizon, she is then an entire semidiameter of the earth more distant from a spectator on the earth's surface, than when she appears in the zenith; hence it follows, that her apparent diameter must augment in proportion as her altitude increases from the horizon. Let C be the centre of the earth, A the place of a spectator on its surface, Z his zenith, M the moon; then, as the sides of triangles are as the sines of their opposite angles, we have sine CAM or its suppl. ZAM : sine ZCM : : CM : AM = CM × sine ZCM ÷ sine ZAM; but the apparent diameter varies inversely as its distance; hence the apparent diameter will vary as sine ZAM divided by sine ZCM, the moon's distance from the centre of the earth being supposed constant. Now in



* In the fig. p. 250, let M represent the place of the earth, AB the moon, the angle AMC half its semidiameter $= \frac{31' \, 7''}{2} = 15' \, 33''5$; hence the angle MAC = 90° — 15′ 33″5 = 89° 44′ 26″5. MC is the moon's distance from the earth = 238533 miles (see p. 250.) Now conceive a straight line to be drawn from M to B, then in the triangle AMB it will be,

As sine ∠ A = 89° 44′ 26″5 - - -	9.9999956
To sine AMB 31′ 7″ - - - - - - -	7.9567133
So is the dist. MB = 238533 - - -	5.3775494
	13.3342627
To AB the moon's diam. 2159 07 - -	3.3342671

The diameter might also have been thus calculated; rad. : sine 15′ 33″5 : : MA : AC the moon's semidiameter. Or it might be calculated in the same manner as the diameter of Mercury has been calculated in the first part of the note, p. 263.

Now the cube of the earth's diameter $= 7911^3$ divided by the cube of the moon's diameter $= 2159.07^3$ will give the proportion of their magnitudes: thus, log. 7911^3 — log. 2159.07^3 = 11.6946942 — 10.0028013 = 1.6918929; the number corresponding to this log. is 49.19, which shews that the magnitude of the earth is something more than 49 times that of the moon.

Keith, in his Treatise on the Globes, has calculated the diameter of the moon in a similar manner, but makes the angle at A, from data the same as the above, with respect to the angles, = 89° 59′ 44″26½‴, and hence all his conclusions, resulting from these erroneous premises, must be false.

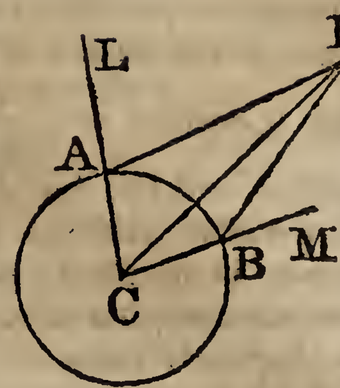
the horizon, taking sine ZAM ÷ sine ZCM as equal to unity or one, we have this proportion ; 1 : sine ZAM ÷ sine ZCM, or s. ZCM : s. ZAM,* or cos. true alt. MCH (a) : cos. apparent alt. MAh (b) : : the horizontal diameter to the diameter at the apparent alt. b. Hence the horiz. diam. : its increase : : cos. a : cos. $b - a = 2$ sine $\overline{\frac{1}{2}a + \frac{1}{2}b} \times$ sine $\overline{\frac{1}{2}a - \frac{1}{2}b}$ rad. being 1 (Emerson's Trig. B. 1, prop. 3, cor. 4, or Vince's Trig. art. 111.) Hence (16 Eucl. 6) the increase of the semidiameter = hor. semid. $\times \frac{\text{sine}\,(\frac{1}{2}a + \frac{1}{2}b) \times \text{sine}\,(\frac{1}{2}a - \frac{1}{2}b)}{\text{cos.}\,a}$. From this expression, a table of the increase of the semidiameter for any horizontal diameter, may be easily constructed ; and for any other horizontal semidiameter, the increase will vary in the same proportion.

The *moon's parallax* is the next subject that requires our consideration. Various methods have been given by authors, but the following are the principal.

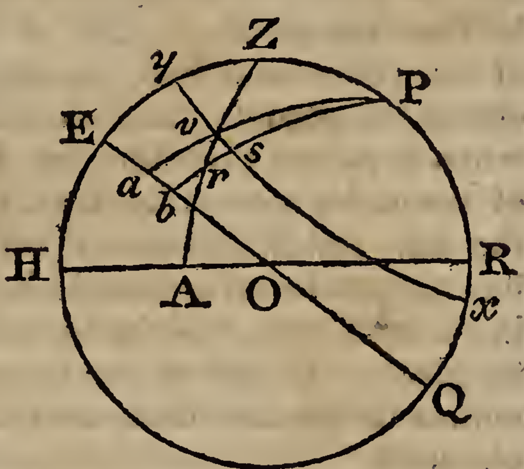
First method. Let the meridian altitudes of the moon be taken when she is at the greatest north and south latitudes, and let these altitudes be corrected for refraction ; then if there were no parallax, the difference of these corrected altitudes would be equal to the sum of the two latitudes of the moon ; hence the difference between the sum of the two latitudes, and the difference of the altitudes, will be the difference between the parallaxes at the two altitudes. Now from thence to determine the parallax itself, let S be the sine of the greatest, and s of the least, apparent zenith distance, and P, p, the sines of the corresponding parallaxes ; then, as the parallax varies as the sine of the zenith distance, when the distance is given (see the note, p. 279) we have S : s : : P : p ; hence $S - s : s : : P - p : p = \frac{S \times (P - p)}{S - s}$ (17 Eucl. 5) the parallax at the greatest altitude. As the above calculation is on supposition that the moon is at the same distance in both observations, which will generally not be the case ; one of the observations must be reduced to what it would have been had the distance been the same as the other, the parallax being inversely as the distance (note, p 279) If the moon pass through the zenith of one of the observers, the difference between the sum of the two latitudes and the zenith dist. at the other observation, will be the parallax at that altitude.

* For 1 $\times$ sine ZAM = $\frac{\text{s. ZAM}}{\text{s. ZCM}} \times$ sine ZCM therefore, &c. (16 Eucl. 6)

Second method, for any planet. Let the planet P be observed from two places A, B, in the same meridian; then the angle APB is the sum of the two parallaxes at both places. The parallax APC or sine APC = *hor. par.* × sine PAL (p. 279) and parallax BPC or s. BPC = *hor. par.* × sine PBM; hence *hor. par.* × (s. PAL + s. PBM) = APB; therefore *hor. par.* = APB divided by the sum of these two sines. If the meridians of the places differ, the variation of the planets' declination, in the interval of the passages over the meridians of the two observations, must be known.*



Third method, answering for any planet. Let EQ be the equator, P the pole, Z the zenith, *v* the true place of the planet, and *r* the apparent place as depressed by the parallax in the vertical circle ZA; let the circles of declination P*va*, P*rb*, be drawn; then *ab* is the parallax in right ascension, and *rs* in declination. Now *vr* : *vs* : : rad. 1 : sine *vrs*, or Z*v*P (see Vince's Trig. art. 125) and *vs* : *ab* : : cos. *va* : rad. 1 (see the note to prob. 35, part. 2) hence *vr* : *ab* : : cos. *va* : sine Z*v*P; therefore *ab* = *vr* × sine Z*v*P ÷ cos. *va*; but *vr* = *hor. par.* × s. *v*Z (note, p. 279) and the sides of spher. triangles being as the sines of the angles opposite to them, s. *v*Z : s. ZP : : s. ZP*v* : Z*v*P = s. ZP × s. ZP*v* ÷ s. *v*Z. Hence, by substitution *ab* = *hor. par.* × s. ZP × s. ZP*v*. ÷ cos. *va*. Therefore, for the same star, the parallax in right ascension varies as the sine of the hour angle, where the *hor. par.* is given. The *hor. parallax* is also = *ab* × cos. *va* ÷ s ZP × s. ZP*v*.



The apparent place *b* on the equator, is to be east of *a*, the true place, for the eastern hemisphere, or that hemisphere east of the meridian, and therefore the right ascension is increased by the parallax; but in the western hemisphere, *b* lies to the west of *a*, and therefore the right ascension is diminished. Hence, if the right ascension be taken before and after the meridian, the whole

* Ex. On Oct. 5, 1751, M. de la *Caille* observed Mars to be 1′ 25″8 below the parallel λ in Aquarius, at the Cape of Good Hope, and to be 25° distant from the zenith. On the same day, at Stockholm, Mars was observed to be 1° 57′7 below the parallel of λ. and his zenith distance to be 68° 14′. Here then the angle APB = 31″9, and the sines of the zenith distances being 0.4226 and 0.9287, the horizontal parallax was 23″6. If the ratio of the distance of the earth from Mars and the sun respectively, be given, the *sun's hor. parallax* will therefore be given, the parallaxes of the planets being inversely as their distances. (Note, p. 279.)

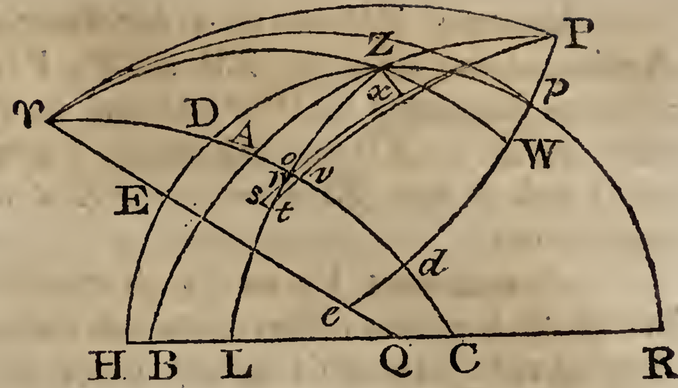
change of parallax in right ascension, between the two observations, is the sum s of the two parts before and after the meridian, and therefore $= \frac{vr}{\cos.\ va} \times S$ the sum of the sines of the two hour angles, and the *hor par.* $= s \times \cos.\ va \div \sin.\ ZP \times S$. There is no parallax in rt. as. on the mer for then the value of ab, shewn above, is nothing, as the angle ZvP vanishes. As the spher. △ vrs is rt. angled at s, rs, the parallax in decl. may be easily found by *Napier's rules.*

In the application of the above investigations, the rt. as. of the planet when it passes the mer. compared with that of a fixed star, must be observed, as then there is no parallax in rt. as. Let the diff. of their rt. ascensions be again observed, 6 hours after, and let the change of the diff. d between the apparent rt ascensions of the planet and star, during that time, be observed. Again, to obtain the planets' true motion in rt. as. let its rt. as. be observed, when it passes the mer. for 3 or 4 days; then, if in this interval of time, its motion in rt. as. between taking the rt. ascensions of the star and planet on and off the mer. be equal to d, the planet has no par. in rt. as. but if it be not $= d$, the diff. is the parallax in rt. as.; from which, by what is shewn above, the *hor. par.* will be given. If one of the observations be made before the planet comes to the meridian, and the other after, a greater diff. will be obtained.*

As the right ascen. and decl. is thus affected by the parallax, it is evident that the lat. and long. of the moon and planets must, in like manner, be affected by it; and as the determination of this, in respect to the moon is, in many cases, particularly in solar eclipses, of great importance, we shall here shew how to compute it, on supposition that the lat. of the place, the time, and therefore the sun's rt. as. the moon's true lat. and long. with her hor. parallax, are given.†

* Ex. *Mars* was very near a star of the 5th mag. in the eastern shoulder of Aquarius, on Aug. 15, 1719, at 9h. 18′ in the evening, and in 10′ 17″ he followed the star; on the 16th, at 4h. 21′ he followed the star in 10′ 1″; hence in that interval, the appar. rt. as. of Mars had increased 16″ in time. But from observations made in the mer. for several days after, Mars, from its proper motion in that time, approached the star only 14″; therefore the effect of parallax, in the interval of the observations, was 2″ in time, or 30″ in motion. Now the decl. of Mars was 15°, the co. lat. 41° 10′, and the two hour angles 49° 15′, and 56° 39′; hence the *hor. par.* $= 30'' \times \cos.\ 15° \div \sin.\ 41°\ 10' \times (\sin.\ 49°\ 15' + \sin.\ 56°\ 39') = 27\frac{1}{2}''$. But the dist. of the earth from Mars, was to its dist. from the sun, at that time, as 37 : 100, whence the sun's parallax comes out $= 10''17$, but this is too great by nearly $1\frac{1}{2}''$.

† The following solution is principally taken from *Vince.*



Let HZR be the meridian, ♈EQ the equator, *p* its pole; ♈DC the ecliptic, P its pole, ♈ the beginning of Aries, HQR the horizon, Z the zenith, ZL a vertical circle, or secondary to the horizon passing through the true place *r*, and apparent place *t* of the moon; draw P*t*, P*r*, which produce to *s*, and draw the small circle *ts*, parallel to *ov*; then *rs* is the *parallax in lat.* and *ov* the *parallax in longitude.** Draw the great circles ♈P, PZAB, P*pde*, and ZW perp to P*e*; then as ♈P = 90°, and also ♈*p* = 90°, ♈ is the pole of P*de* (see def. 6, or Simson's Spher. Trig. annexed to his Euclid, cor. to prop. 3) and hence *d*♈ = 90°; therefore *d* is one of the solstitial points *Cancer* or *Capricorn*; draw Z*x* perp. to P*r*, and join Z♈, *p*♈. Now ♈E or the ∠ ♈*p*E or Z*p*♈, is the rt. as. of the midheaven, which is known (see the note to prob. 16, part 3) PZ = AB (being each the comp. of AZ) the alt. of the highest point A of the ecliptic above the horizon, or nonagesimal degree, and ♈A, or the angle ♈PA is its longitude; also Z*p* = co. lat. of the place, and the ∠ Z*p*W is the diff between the rt. as. of midheaven ♈*p*E and ♈*e*. Now in the rt. angle △ Z*p*W, rad. × cos ∠ *p* = tang *p*W × cot. *p*Z (Napier's rule) hence (16 Eucl. 6) cot. *p*Z : rad. : : cos. *p* : tang. *p*W; or by logarithms,

log. tang. *p*W = 10, + *log. cos.* *p* — *log. cot.* *p*Z;

therefore PW = *p*W ± *p*P, where the *upper* sign takes place when the sign of the midheaven is *less* than 180°, and the *lower* sign when *greater*. Also in the triangles WZ*p*, WZP, we have sin. W*p* : sin. WP : : tang. WPZ : tang. W*p*Z (Vince's Trig. art. 231, or Simson's prop. 26) : : cot. W*p*Z : cot. WPZ, or tang. AP♈ (the tangents being reciprocally as the cotangents, Emerson's Trig. prop. 1, cor. 4, or Vince, art. 82) therefore,

log. tang. AP♈ = *ar. co. log. sin.* W*p*† + *log. sin.* WP + *log. cot.* W*p*Z — 10

or, *log. tang.* AP♈ = *log. sin.* WP + *log. cot.* W*p*Z — *log. sin.* W*p*;

and as ♈*o*, or ♈P*o*, the true long. of the moon is given, AP*o*, or ZP*x* is therefore given. Also in the triangle WPZ; cos. WPZ, or sin. AP♈ : rad. : : tang. WP : tang. ZP (Simson's Spher. prop. 20, or Vince's, art. 219) therefore,

* See Keil's Astronomy, Lect. 21, or Gregory's Astr. B. 2, sect. 8, where this subject is also fully investigated.

† The *arithmetical complement* of any logarithm is what it wants of 10, or 20, and is used to avoid subtraction; thus the ar. com. of 2.6963564 is 7.3036436. Hence in the above proportions the ar. com. log. s. W*p* being added and 10 subtracted, is the same as subtracting log. sin. W*p*, as is evident. However, there seems to be more perplexity, particularly for beginners, in using the ar. co. than the simple log.

log. tang. ZP = 10, + *log. tang.* WP — *log. sin.* AP♈.

Again, in the triangle ZPr, ZP, Pr, and the angle P are given, whence the angle ZrP or srt may be thus found; in the rt. angled triangle ZPx, ZP and the angle P are given; hence (by Napier's rule) rad. × cos. ZPx = cot. PZ × tang. Px; which resolved by logs. gives

log. tang. Px = 10, + *cos.* ZPx — *log. cot.* PZ;

hence rx is given; therefore sin. rx : sin. Px : : tang. ZPx : tang. Zrx or trs (Simson's Spher. prop. 26) which in logarithms is,

log. tang. Zrx = *ar. co. log. sin.* rx + *log. sin.* Px + *log. tang.* ZPx — 10;

also in the rt. angled triangle Zrx, we have (by Napier's rule) rad. × cos. Zrx = cot. Zr × tang. rx; therefore,

log. cot. Zr = 10, + *log. cos.* Zrx — *log. tang.* rx.

with this *true* zenith dist. Zr, let the parallax be found (note, p. 279) as if it were the *apparent* zenith distance, and the true parallax will be given *nearly*; let this par. be therefore added to the *true* zenith dist and the *apparent* zenith dist. will be given *nearly*, to which let the parallax be again computed (p. 279) and the true parallax rt will be obtained extremely near; then in the rt. angled triangle rst, which may be considered as plane, we have rad. : cos. r : : rt : rs, the *parallax in latitude* (Simson's Trig. prop. 1) hence, *log.* rs = *log.* rt + *log. cos.* r — 10 = *log. par. lat.* Also rad. : sin. r : : rt : ts; therefore *log.* ts = *log.* rt + *log. sin.* r — 10; hence cos. tv : rad. : : ts : ov, the *paral. in longitude* (see the note to prob. 35, part 2.)

Ex. On January 1, 1771, at 9h. apparent time, in lat. 53° N. the moon's true longitude was 3s. 18° 27′ 35″ and lat. 4° 5′ 30″ S. and her horizontal parallax 61′ 9″; to find her parallax in lat. and long.

The sun's rt. as. by the Tables, was 282° 22′ 2″, and his dist. from the mer. = 9h. × 15° = 135°; also the rt. as. of the mid-heaven was 57° 22′ 2″;* hence the whole operation for the solution of the triangles will be as follows.

In the triangle ZpW.

ZpW =	32° 37′ 58″	- - - 10, + cos.	19.9253864
Zp =	37 0 0	- - - - - - cot.	10.1228856
pW =	32 23 57	- - - - - - cot.	9.8025008

pW + pP = 23° 28′ = PW = 55° 51′ 57″.

† 360° — 282° 22′ 2″ = 77° 37′ 58″: hence 135° — 77° 37′ 58″ = 57° 22′ 2″. ZpW = 90° — 57° 22′ 2″ = 32° 37′ 58″.

In the triangles WpZ, WPZ.

pW	= 32° 23′ 57″	ar. co. s.	0.2709855
PW	= 55 51 57	sin.	9.9178865
ZpW	= 32 27 58	cot.	10.1935941
AP♈	= 67 29 8	tan.	10.3824661

oP♈ = ☽'s *long.* 108° 27′ 35″ — AP♈ = 40° 58′ 27″.

In the triangle WPZ.

WP	= 55° 51′ 57″	10, + tan.	20.1688210
AP♈	= 67 29 8	sin.	9.9655700
ZP	= 57 56 36	tan.	10.2032510

In the triangle WPZ.

ZPx	= 40° 58′ 27″	10, + cos.	19.8779500
ZP	= 57 56 36	cot.	9.7967445
Px	= 50 19 33	tan.	10.0812055

Pr = 90° + 4° 5′ 30″ = 94° 5′ 30″, hence 94° 5′ 30″ — Px = 43° 45′ 57″ = rx.

In the triangles ZPx, Zrx.

rx	= 43° 45′ 57″	ar. cos. s.	0.1600743
Px	= 50 19 33	sin.	9.8863144
ZPx	= 40 58 27	tan.	9.9387676
Zrx	= 44 1 16	tan.	9.9851563

In the triangle Zrx.

Zrx	= 44° 1′ 16″	10, + cos.	19.8567795
rx	= 43 45 57	tan.	9.9812846
Zr	= 53 6 10	cot.	9.8754949

*Zr	= 53° 6′ 10″	sin.	9.9029362
Hor. par.	61′ 9″ = 3669″	log.	3.5645477
rt uncorrected	= 2934″ = 48′ 34″	log.	3.4674839

App. zen. dist. Zt†	= 53° 55′ 4″ nearly	sin.	9.9075042
Hor. par.	3669″	log.	3.5645477

* See the latter part of the note, p. 279.

† Zt = Zr 53° 6′ 10″ + rt 48′ 54″ = 53° 55′ 4″.

Tri. *trs.*

Par. *rt* corrected = 2965″ = 49′ 25″	log.	3 4720519
trs or *Zrx* = 44° 1′ 16″ - - - - -	cos.	9 8567795
rs par. in *latitude* = 2132″ = 35′ 32″	log.	3.3288314

Tri. *trs.*

rt corrected = 2965″ - - - - - - -	log.	3.4720519
trs = 44° 1′ 16″ - - - - - - - -	sin.	9 8419369
ts = 2061″ = 34′ 21″ - - - - - -	log.	3.3139888

The true latitude *ro* being = 4° 5′ 30″ S. hence,

Appar. lat. *tv* = *ro* + *rs* = 4° 41′ 2″	cos.	9.9985472
ts = 2061″ - - - - - - - - - 10, +	log.	13,3139888
ov par. in *longitude* = 2067″ = 34′ 27″	log.	3.3154416

Note 1. The value of *tv* is *ro* ∓ *rs*, according as the moon has N. or S. lat.

Note 2. The order of the signs being from *west* to *east*, from A towards C is *eastward*, and from A towards ♈ is *westward;* now as the parallax depresses the hody from *r* to *t*, it increases the longitude from *o* to *v* ; but if the point *o* had been on the other side of A, *ov* would be the contrary way ; hence when the body is to the *east* of the nonagesimal degree, the parallax *increases* the longitude ; and when to the *west*, it *diminishes* the longitude.

Ex. 2. On June 29, 1813, at 7h 3′ 57″ apparent time, in the evening, at New-York, lat. 40° 42′ 40″, the moon's true longitude will be 4s. 1° 11′ 36″, and latitude 52′ 10″ S, and her horizontal parallax 59′ 16″ ; required her parallax in lat. and longitude ?

The sun's rt. as. will be 6h. 33′ 40″3 in time = 98° 25′ 4″5, by the Naut. Alm and his distance from the mer. will be 7h. 3′ 57″ = 105° 59′ 15″ ; also the rt. as. ♈E of the *medium cœli* will be 204° 24′ 19″5 *

From *Mayer's* tables the moon's greatest parallax (or when she is in her perigee and in opposition) is 61′ 32″ ; her least parallax (or when in her apogee and conjunction) is 53′ 52″ in the lat. of Paris. The arithmetical mean of these is 57′ 42″ ; but this is not the parallax at the mean dist. as the par. varies inversely as the dist. the par. at the mean dist. is therefore 57′ 24″, an harmonic mean between the two.† *Laplace* makes the moon's par. at her dist. from the earth, which is an arith. mean between the two extremes = (10676″) 57′ 39″024, so that at the same dist. at which the moon appears to us to subtend an angle of (5823″) 31′ 26″652,

* To find the rt. as. of midheaven ; the sun's rt. as. 98° 25′ 4″5 + his dist. from the mer. 105° 59′ 15″ = 204° 24′ 19″5.

† *Harmonic ratio,* is when a quantity is divided into three parts, so that the whole is to one part, as the second part to the third. When the second and third are equal, it is called *harmonic proportion continued.* Emerson's Doctrine of Prop. sect. 2, def. 14.

the earth would appear under an angle of (21322″) 1° 55′ 18″048. M. de *Lambre* re-calculated the parallax from the same observations from which *Mayer* calculated it, and found that it did not exactly agree with *Mayer's*. He made the equatorial parallax 57′ 11″4. M. de la *Landé* makes it 57′ 5″ at the equator, 56′ 53″2 at the pole, and 57′ 1″ for the mean radius of the earth, supposing the diff. of the equatorial and polar diameters to be $\frac{1}{300}$ of the whole. From the formula of *Mayer* (at the end of his tables) the equatorial parallax is 57′ 11″4.*

During the course of a lunation, or synodic revolution, the moon constantly exhibits very singular phenomena, which we call her *phases*. At the moment that the moon passes between the earth and the sun, in her revolution round the earth, which she regards as her centre, the enlightened half of her is then entirely turned towards the sun, and the other dark half is towards the earth; in this case, the moon will therefore be invisible to us, and this position of the sun and moon is termed the *conjunction* or *new moon*. The moon remains invisible during 3 or 4 days; because for a day or two both before and after conjunction, her *crescent* is so small, and her light so obscured by the sun's rays, that she escapes the nicest observation. After disengaging herself in the evening from the rays of the sun, she re-appears towards the *east* with a slender crescent, convex towards the sun, which increases with her distance; in about $7\frac{1}{3}$ days after the conjunction it becomes a semicircle, at which time she will come to the meridian about 6 o'clock in the evening, when the moon is 90° distant from the sun; moving still eastward, she becomes an entire circle of light in about $14\frac{1}{2}$ days, when she is in *opposition* with the sun, at which time she will come to the meridian at midnight; hence in this position she appears full, and is therefore called *full moon*. When she afterwards approaches the sun, this luminous circle is changed into a crescent, which diminishes according to the same degrees it had increased before, until, in the morning it becomes immersed in the solar rays. The lunar crescent being always turned towards the sun, evidently indicates that it is from the sun the moon receives her light. The law of the variation of her phases we have given in p. 269, and the method of delineating the *phases* may be easily collected from what is given in p. 268. These phases are renewed at every conjunction, and their return depends on the excess of the moon's synodical motion above that of the sun, which excess is called the *synodical motion of the moon*. The length of the synodic revolution of the moon, or the period of her

* M. *John Machin*, Astron. Prof. Gresh. Col. has, at the end of *Motte's* translation of the Principia, given the laws of the moon's motions according to gravity; that is, her *variations*, her *inequalities* during a revolution, &c. the *motion of the nodes*, the *inclination of the plane of her orbit to that of the ecliptic*, the *variation of the areas described about the sun*, the *motion of her apogee*, the *variation of the eccentricity of her orbit*, the *equation of the apogee*, *equation of the centre*, and other things of a similar nature.

mean conjunction, according to *Laplace*, is 29.530588 days (see pages 325, 326) it is to the tropical year nearly as 19 : 235, that is, 19 solar years for about 235 lunar months.

It is in those points of the moon's orbit called the *syzygies*, that she is in conjunction or opposition with the sun ; in the first point she is *new*, in the second *full*. In the *quadratures*, when she is distant from the sun 90° or 270°, reckoning in the direction of her proper motion, or in her first and second quarters, we see half of her enlightened hemisphere, strictly speaking, we see a little more, for when the exact half is presented to us, the angular distance of the moon from the sun is a little less than 90°. At this instant, as *Laplace* remarks, the enlightened being separated from the obscure part of the moon by a straight line, the radius drawn from the observer to the centre of the moon, is perpendicular to that which joins the centres of the moon and sun : so that in the triangle formed by the straight lines which joins those centres and the eye of the observer, the angle at the moon is a right one ; hence the distance of the earth from the sun, may be determined in parts of that of the moon from the earth. This method is, however, very inaccurate, from the difficulty of fixing with precision the instant when half of the lunar disk is enlightened ; however, it is to this method we owe the first just notions that were formed of the immense magnitude of the sun, and of his distance from the earth. An observer will moreover observe, that from *new* to *full* moon the phases are *horned*, *half moon*, and *gibbous*, and as the enlightened or convex side of the moon is always turned to the sun, the crescent, or irregular side will appear towards the east, or, if the spectator be in *north* lat. towards the left. From the full to the change, the phases appear in this order, *gibbous*, *half moon*, and *horned ;* in these positions, the convex or enlightened side will appear towards the east, and the horns or crescent towards the west, or to the right hand. The earth exhibits to the moon similar phases ; when she is *new* to us, the earth is *full* to her, and when she is in her first quarter to us, the earth is in her third quarter to her, &c. In consequence of this, one half of the moon will have no darkness at all, the earth affording her a much greater light in the sun's absence than she does to us ;* while the other half has about 14½ days darkness and 14½ days light alternately. As the moon's axis is almost perpendicular to the ecliptic, she has scarce any difference of seasons.

In north lat. all the full moons, in the winter, happen when the moon is on the north side of the equinoctial ; as they always happen when the moon is directly opposite the sun. While the moon passes from Aries to Libra, she will be visible at the north pole, and from Libra to Aries she will be invisible there ; hence, at the north pole there is alternately a fortnight's moonlight and a fort-

* As the surface of the earth is about 13 times greater than that of the moon, it affords 13 times more light to the moon than the moon does to the earth.

night's darkness. The same phenomena will take place at the south pole, in our summer, during the sun's absence.

The explanation of the moon's phases naturally leads us to that of *eclipses;* but as this subject merits a separate chapter, we shall give it in the following part of the work. The influence of the moon on the waters of the ocean shall also be explained, when the laws of gravity, &c. on which it depends, are first investigated.

Before the first and after the last quarter, but principally about the time of new moon, we can sometimes distinguish that portion of the lunar disk which is not enlightened by the sun; this feeble light is called *lumière cendrée*, and is caused by the light reflected from the illuminated hemisphere of the earth on the moon's disk. This is evident from its being more perceptible at the new moon, when the greatest part of the earth's enlightened hemisphere is turned towards the moon. According to Dr. *Smith*, the proportion of moonlight to daylight, at the full moon, is 90000 to 1. *Emerson*, in his Optics (B. 1, prop. 20) makes it as 96000 to 1.* But *Bouguer* has found, by experiment, that it is as 300000 to 1.† This is the reason why the light of the moon, collected in the focus of the largest mirrors, produces no sensible effect on the thermometer.

The moon's disk is greatly diversified with spots or inequalities, which have been accurately described. Through a telescope, those spots have the appearance of hills, valleys, &c. They, however, shew us that the moon always presents to us very nearly the same hemisphere, and that she revolves upon her axis in a period equal to her revolution round the earth.‡ From the best observa-

* *Emerson* shews (Optics, B. 1, prop. 20, cor. 1) that moonlight is to daylight as half the square of the moon's radius, to the square of the moon's distance, when she is full. And in the quadratures as ¼ the square of the moon's radius to the square of the moon's distance; and shews that from the same principle, the light of any other body, compared with daylight, may be found.

† In the *English edition* of *Laplace*, this is given as 300 : 1.

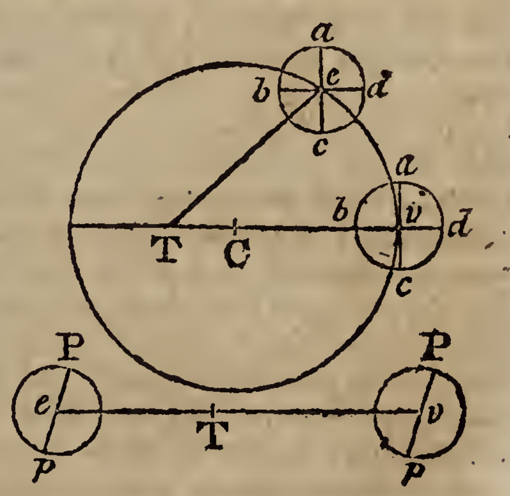
‡ Each of the moon's spots have been distinguished by a proper name, principally from the most noted astronomers, philosophers, and mathematicians, or from their respective appearances. Thus *Sinus roris, Mare frigorum, Oceanus procellarum, Terra siccitatis, Palus nimborum, Copernicus, Keplerius, Grimaldi, Galileo, Herschel's volcano,* &c. Many astronomers have given maps of the face of the moon; but the most celebrated are those of *Hevelius* in his *Selenographia,* in which he has represented the different phases of the moon during an entire revolution. *Florentius, Langrenus, Grimaldus,* and *Ricciolus,* have each distinguished himself in describing the lunar spots, &c. *Langrenus* and *Ricciolus* denoted the spots by the names of the principal philosophers, mathematicians, &c giving the names of the most celebrated characters to the largest spots. *Hevelius* marked them with the geographical names of places on the earth. The former distinction is, however, generally followed, though *Mayer* prefers *Hevelius's* figures: see Keil's Astr. lect. 10. The best and most complete representation of the moon's disk, is that drawn on Mr. *Russel's lunar globe,* published a few years ago. This globe not only shews the *libration* of the moon in the most perfect manner, but is also a complete picture of the mountains, pits, shades, &c. on her surface.

tions, these spots are found to be produced by the mountains and valleys on the moon's surface. This is evident from the irregularity of that part of her surface which is turned from the sun; for if her surface was perfectly level or smooth, the illuminated part of her disk, at the quadratures, would be separated from the dark by a straight line; at all other times, this line would appear of an elliptic form, convex towards the enlightened part of the moon, in the 1st and 4th quarters, and concave in the 2d and 3d; but these lines, so far from appearing regular and well defined, particularly when the moon is viewed through a telescope, that they always appear notched and broken in innumerable places. In all situations of the moon, the elevated parts always cast a triangular shadow with its vertex turned from the sun; on the contrary, the cavities are always dark on the side next the sun, and illuminated on the opposite side; moreover, when the sun becomes vertical to some of these parts, there is no shadow perceptible; hence these are mountains, and those that are dark on the side next the sun, are cavities; for these appearances are exactly conformable to what we observe of hills and valleys on the earth. It is not, therefore, singular that the edge of the moon, which is always turned towards the sun, is regular and well defined, and that no indented parts are seen on her surface at the time of full moon; for the shining spots on her surface would not be perceptible, did not the shade or dark part separate them from the illuminated part of the disk; but in the above circumstances, there is more of the dark part turned towards a spectator on the earth, all being equally and more strongly enlightened. The dark parts by some have been thought to be seas; but the irregularity of the line between the enlightened and dark parts, shews that there can be no very large tracts of water, as such a regular surface would necessarily produce the line perfectly free from any irregularity. On the dark part of the moon's disk, near the confines of the lucid part, some bright spots are perceptible; these shining spots are supposed to be the summits of high mountains, which are enlightened by the sun's rays, while the adjacent valleys, near the enlightened part, are entirely dark. On this supposition, astronomers have determined the height of some of these mountains; the method of performing which we shall presently shew.

Continued observations on the lunar disk, have discovered some small changes in these appearances, so that the same side of the moon is not always exactly turned towards the earth, the spots that lie near the edge or limb, successively appearing and disappearing by periodical oscillations, which have been distinguished by the name of the *libration of the moon.*

This phenomenon arises from *four* principal causes. 1. From the observer not being placed at the centre of the earth, but at its surface. *Galileo*, who first observed with a telescope the moon's spots, discovered this circumstance; he observed a small daily variation, arising from the motion of the spectator about the earth's centre, which caused a little of the moon's western limb to disappear, from her rising to her setting, and brought into view a small portion of the eastern limb. For the visual ray, drawn from the eye of an observer to the moon's centre, determines the middle of the visible hemisphere, and it is evident, that from the effect of the lunar parallax, this radius cuts the surface of the moon at different points, according to her alt above the horizon. 2. *Galileo* likewise observed that the north and south poles of the moon, and the part of the surface that are near them, alternately appeared and disappeared; this is called the *libration in latitude*, and is caused from the axis of the moon not being perpendicular to the plane of her orbit, it making an angle of about 1°.43′ with a perp. to the plane of the ecliptic. In supposing this axis to maintain its parallelism during the moon's revolution round the earth, it inclines more or less to the radius vector of the moon, as observed from the earth; and the angle which is formed by these two lines, is therefore acute during one half of the revolution, and obtuse during the other half. 3. The third cause, is the unequal angular motion of the moon about the earth, and her uniform motion about her axis, which makes a little of the eastern and western parts alternately appear and disappear, the period of which is a month; this is called the *libration in longitude*.* 4. The fourth cause of the libration arises from the attraction of the earth upon the moon, in consequence of its spheroidical figure.

* The libration in longitude would not take place, if the moon's angular motion about the earth were equal to her angular motion about her axis. For if T be the earth, *abcd* the moon at *v* and *e*; let *avc* be perp. to T*v*, then *abc* is that hemisphere of the moon at *v* which is next the earth. Now when the moon comes to *e*, if she had no motion on her axis, *bed* would be parallel to *bvd*, and the same face would not be turned towards the earth. But if *b* was brought to coincide with the line T*e*, by the moon's rev. on her axis in the direction *abc*, the same face would remain turned towards the earth; in this case the moon would have revolved, on her axis, the angle *be*T, which is equal to the alternate angle *e*T*v*, the angle which the moon has described about the earth.



The same face of the moon is always turned towards the earth, when in the same point of her orbit, and hence, from what we have now shewn, the time of her rev. in her orbit, is equal to the time of her rev. on her axis. But as her angular motion *e*T*v* about the earth is unequal, while that on her axis is equal, these two angles cannot continue equal, and hence, from the above, the same face cannot continue towards the earth, but in the intermediate points, must vary sometimes a little more to the east, and sometimes to the west. The *greatest* libration in *longitude* is therefore nearly equal to the equation of the orbit, or at its maximum about $7\frac{1}{2}°$; this would be accurately so, if the moon's axis were

It is an extraordinary circumstance, as *Vince* remarks, that the time of the moon's revolution on her axis, should be equal to that in her orbit; and still more extraordinary, that all the secondary planets should observe the same law Sir *Isaac Newton* has computed (Prin. B. 3, prop. 37) from the altitude of our tides, that the alt. of the moon's tides must be 93 feet, and that therefore the figure of the moon is a spheroid, whose greatest diameter produced, would pass through the centre of the earth, and exceed the diameter perp thereto by 186 feet. Hence it is, says he, that the same face of the moon always respects the earth; nor can the body of the moon possibly rest in any other position, but would always return by a libratory motion to this situation, from the earth's attraction. But it has been shewn (p. 338) that there can be no large tracts of water on the moon's surface, and hence *Newton's* supposition cannot account for this phenomenon. The supposition of Dr. *Mairan* is, that the hemisphere of the moon next the earth is more dense than the opposite one, in which case the same face would be kept towards the earth, from the earth's attraction. We have pointed out a more probable cause (note, p. 296, &c.) from the moon's having little or no atmosphere.

Whether the moon has an *atmosphere* or not, is a question, however, that has long been agitated by various astronomers. *Schroeter*, of Lilienthal, in the duchy of Bremen, endeavours to establish the existence of an atmosphere from the following considerations. 1. He observed the moon when $2\frac{1}{2}$ days old, in the

perp to her orbit; for the equat. of the orbit, or the diff between the true and mean motion, is equal to the diff. of her mot. about her axis, and her true motion, which is the libration. As there is no equation of the orbit in apogee and perigee, the same face will then be turned towards the earth. Let T, in the above small fig represent the earth, M the moon, P*p* its axis, not perp. to the plane of the orbit *ev*, then at *e* the pole P will be visible to the earth, and at *v* the pole *p* will be visible; hence as the moon revolves about the earth, the poles will alternately appear and disappear, which explains the libration in *latitude*. Our seasons are caused in a similar manner from the obliquity of the ecliptic. From what we have here shewn, it is evident, that one half of the moon is never visible at the earth; and that the time of its rotation about its axis being one month, the length of the lunar days and nights will be each nearly a fortnight, being subject but to a small variation, as the moon's axis is nearly perp. to the ecliptic.

Hevelius observes, that the libration in *lat.* was the greatest when the moon was at her greatest north lat. the spots which are near the northern limb being then nearest to it; and that the spots receded from that limb, as the moon advanced from thence, until she came to her greatest lat. S, where the spots near the southern limb were then nearest to it. He found this variation to be about 1′ 45″, the moon's diam. being 30′. It therefore follows, that when the moon has her greatest lat. a plane passing through the earth and moon, perp. to the plane of the moon's orbit, will pass through the moon's axis; the moon's equator must therefore intersect the ecliptic in a line parallel to the line of the nodes of the moon's orbit, so that in the heavens, the nodes of the moon's orbit coincide with those of her equator (see Vince's Astr.)

evening soon after sun-set, before the dark part was visible, and continued to observe her until it became visible. The cusps or horns, appeared tapering in a very sharp, faint prolongation, each exhibiting its further extremity faintly illuminated by the sun's rays, before any part of the dark hemisphere was visible; afterwards the whole dark limb appeared illuminated. The prolongation of the cusps beyond the semicircle, *Schroeter* thinks, must arise from the refraction of the sun's rays by the moon's atmosphere. He also computes the height of the atmosphere, and finds it = 1356 Paris feet, when it is capable of refracting light enough into the dark hemisphere to produce a twilight, more luminous than the light reflected from the earth, when the moon is about 32° from the new; and that the greatest height, capable of refracting the solar rays, is 5376 feet. 2. At an occultation of Jupiter's satellites, the 3d disappeared, after having been about 1″ or 2″ of time indistinct; the 4th became indiscernible near the limb; this was not observed of the other two. *Phil. trans.* 1792 There is another argument brought forward to prove the existence of a lunar atmosphere, taken from the appearance of a luminous ring round the moon, in the time of solar eclipses; this has been particularly observed in the total *eclipse* of the sun in 1706, and in another in 717, when, during the time of total darkness, certain streaks of light were seen to dart from different places of the moon, during the time of total darkness. These were imagined to be flashes of lightning, and hence the existence of clouds and vapours, and an atmosphere, has been inferred. These flashes are also, by some, supposed to be connected with such appearances, as Dr. *Herschel* has concluded,* to be volcanoes, which have also been considered as a proof of the lunar atmosphere.

On the other side it is urged, that as the moon constantly ap-

* On April 19, 1787, Dr. *Herschel* discovered three volcanoes in the dark part of the moon; two of which appeared to be almost extinct, but the third shewed an actual eruption of fire, or luminous matter, resembling a small piece of burning charcoal, covered by a very thin coat of white ashes; it appeared about as bright as such a coal would be seen to glow in faint daylight. The adjacent parts of the *volcanic mountain* seemed faintly illuminated by the eruption. *Ulloa*, in an eclipse of the sun, discovered a similar eruption, several years ago; it appeared like a star near the moon's edge. Another eruption appeared on May 4, 1783. *Phil trans.* 1787. On March 7, 1794, a few minutes before 8 o'clock in the evening, a bright spot was observed, with the naked eye, on the dark side of the moon, by Mr. *Wilkins*, an eminent architect of Norwich; he conjectured that he saw it about five minutes. London, *Phil. trans.* 1794. On April 13, 1793, and on Feb. 5, 1794, the celebrated Mr. *Piazzi*, of Palermo, observed a bright spot on the dark part of the moon, near *Aristarchus.* Several other astronomers have observed the same phenomenon. *Laplace* remarks (Astr. ch. 4, B. 1) that the crown of pale light which has been perceived round the lunar disk, is probably the solar atmosphere, for that its extent cannot accord with that of the moon, as we are assured by eclipses of the sun and stars, that the lunar atmosphere is nearly insensible.

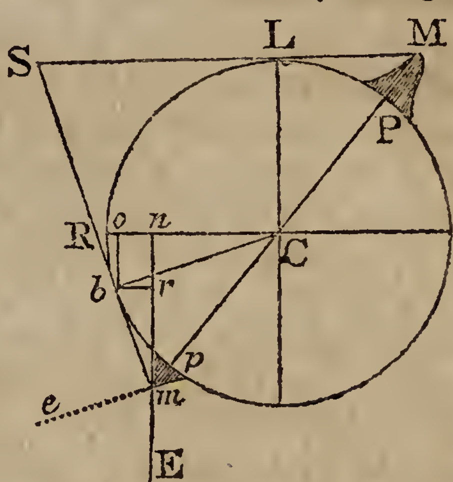
pears with the same brightness, when there are no clouds in our atmosphere, she cannot be surrounded with an atmosphere, at least like ours, which is so variable in its density, and so frequently obscured by clouds and vapours. And *Vince* remarks, that if there were much water on her surface, or an atmosphere, as conjectured by some astronomers, the clouds and vapours might easily be discovered by the telescopes we have now in use; but no such phenomena have ever been observed. *Laplace* says, that the atmosphere which we may suppose to surround the moon, inflects the luminous rays towards her centre, and if (as should be the case) the atmospherical strata are rarer in proportion as they are removed from the surface, these rays, in penetrating into them, will be inflected more and more, and will describe a curve concave towards her centre. An observer in the moon will not cease to see a star until it is depressed below the horizon, an angle called the *horizontal refraction.* The rays emanating from this star, seen at the horizon, after having first touched the moon's surface, will continue to describe a curve similar to that by which they arrived; thus an observer placed behind the moon, relatively to the star, will see it in consequence of the inflection of the lunar atmosphere. The diam. of the moon is not sensibly augmented by the refraction of its atmosphere; and hence a star, eclipsed by the moon, would appear eclipsed later than if this atmosphere did not exist; and would, for the same reason, sooner cease to be eclipsed. Thus the effect of a lunar atmosphere would be principally perceived in the eclipses of the sun and stars by the moon. Very exact and numerous observations have scarcely indicated a suspicion of this influence; and, according to *Laplace*, the horizontal refraction, at the surface of the moon, does not exceed (5″) ″62. At the surface of the earth, this refraction is at least 1000 times greater.* The lunar atmosphere, therefore, if any exist, must be extremely rare, and even superior to that produced in the best air-pumps.

Some remark, that if we reason from analogy, the advocates for an atmosphere have the advantage over those who contend that there is none; but the reverse is, in reality, the case. For it is not analogy to compare the phenomena of a secondary planet with those of a primary: the phenomena of the moon, being compared with those on our earth. Whereas, reasoning from strict analogy, we should compare her phenomena with those similar phenomena

* The horizontal refraction on the earth is about 33′ = 980″ (see p. 155.) *Newton* has shewn (cor. 5, prop. 37, B. 3) that the accelerative gravity, or weight of bodies, on the surface of the moon, is about *three* times less than on the surface of the earth, and as the expansion of the air is reciprocally as the weight that compresses it; hence the moon's supposed atmosphere, being pressed or attracted towards the moon's centre, by a force only one third of that which attracts our air towards the earth's centre, it follows, that the lunar atmosphere is only one third as dense as that of the earth, and is therefore, from the laws of refraction, too rare to produce any sensible refraction.

in the secondary planets. Now the secondary planets, from the observations of the most skilful astronomers, are found, like the moon, to have little or no atmosphere; and hence the probability, from analogy, is in favour of those who contend that there is no atmosphere.*

We have shewn that the bright spots which are sometimes visible on the moon's disk, are the tops of high mountains, the shadow of these, projected on the planes, varying with the sun's position; upon or near the edge of the enlightened part of the disk, we see these mountains forming an indented border extending beyond the line, which separates the illuminated and dark part; by a quantity from which, being measured, their altitude may be determined. The method used by *Hevelius*, *Riccioli*, and others, to determine this alt. is the following. Let SLM be a ray of light from the sun, passing the moon at L, and touching the top of the mountain at M; then the space between L and M will appear dark. Now by means of a micrometer, the ratio of LM to the moon's diameter, or its half LC, may be determined; hence LC being given, LM is therefore given; then (47 Eucl. 1) $CM = \sqrt{CL^2 + LM^2}$ is given; but $CP = CL$, hence $PM = CM - CP =$ the height of the mountain, is given.



Keil, in his Astr. lect. 10, remarks, that *Riccioli* observed the illuminated part of the mountain St Catharine, on the 4th day after new moon, to be distant from the confines of the lucid surface about $\frac{1}{16}$ part of the moon's diameter, or $\frac{1}{8}$ part of her semidiameter LC; hence LC being $= 8$, and $LM = 1$, we have $CM = \sqrt{64 + 1^2} = \sqrt{65} = 8.062$, and hence $PM = .062$; therefore PC or LC : PM :: 8 : .062. Now taking the moon's semidiameter $= 1079$ miles (p. 66 or p. 344) the height of this mountain will be $= \frac{1079 \times .062}{8} = 8.1$ miles nearly. *Galileo* makes $LM = \frac{1}{16}$ of LC, from which the height of the mountain will be 5.07 miles; and *Hevelius* makes $LM = \frac{1}{13}$ of LC, from which the mountain's height $= 3.15$ miles.

The foregoing method, as Dr. *Herschel* observes (Phil. trans. 1781) is only applicable when the moon is in her quadratures; he has therefore given the following general method. Let E represent the earth (see the last. fig.) draw E*mn* and *bo* perp. to the moon's rad. RC, and *br* paral. to *on*, also *me* perp. to S*m*; then, to an observer at E, the line *mb* will not measure its full length,

* We have been rather diffuse on this article; but the importance of the subject deserves to be strictly examined, as it may lead to important results relative to the planets' motions on their axes, &c. see the note, p. 296.

being projected into the line br or on, which will therefore be the lines observed by the micrometer; but when the earth is in quadratures at e, the line bm will measure its full length. From the observed quantity bm or on, when the moon is not in her quadratures, to find bm we have the following proportion: the triangles brm, bCo, are similar; hence $bo : bC :: br : bm = \frac{bC \times br}{bo}$. But bC is the radius of the moon, and br or on the observed dist. of the moon's projection; also bo is the sine of the angle $RCb = obS$ = the distance, or elongation of the moon from the sun, very nearly, which may be easily found by calculation, or from the Nautical Almanac; hence $bm = br$ divided by the sine of the elongation, radius being unity, from which mp, the height of the mountain, is found as before.

In June, 1780, at 7 o'clock, Dr. *Herschel* measured bm or br, and found it = 40″625, for a mountain in the south-east quadrant; the moon's elongation was 125° 8′, the sine of which = .8104; hence 40″625 ÷ .8104 = 50″13, the angle under which bm would appear, if seen directly. Now Cb, the moon's semidiameter, was 16′ 2″6, and if, with *Vince*, we take its length = 1090 miles,* we have 16′ 2″6 : 50″13 : : 1590 : bm = 56.78 miles; hence mp = 1.47 miles, the height of the mountain.

Dr. *Herschel* has determined the height of several other mountains, and thinks that the height of the lunar mountains is in general greatly overrated, and that, a few excepted, they do not exceed half a mile in their perpendicular elevation. He observes, that it should be examined whether the mountain stands upon level ground, that the measurement may be exact: as a low tract of ground, between the mountain and the sun, will make its alt. greater, and elevated places will make it lower than its true height.

* The moon's semidiameter may be thus calculated. In the fig. p. 250, let M represent the earth, AB the moon, AC its semidiameter; then MC her dist. from the earth being 238533 miles (p. 250) and her appar. diam. at her mean dist. from the earth = 31′ 7″ (p 326) or semidiam. or the angle AMC = 15′ 33″5, we have this proportion:

As Cos. 15′ 33″5 - - -	9.9999956
To Sine 15′ 33″5 - - -	7.6554652
So is 238533 - - - -	5.3775494
	13.0330146
To AC 1079 miles - -	3.0330190

Having the diameter of the moon given = 1079 × 2 = 2158 miles, its magnitude from this $= \frac{7911^3}{2158^3}$ = log. 7911^3 — log. 2158^3 = 1.6925400 the number corresponding to which, is 49.26 nearly, the number of times the earth is greater than the moon. See this also calculated. p. 327.

† For more information on the moon's phenomena, the reader is referred to *Newton's* Principia, B. 3, *Laplace's* Celestial Mechanics, *Mayer's* Lunar Theory, or the late Tables of M. *Burg* as published by *Vince*.

Schroeter, on the contrary, asserts, that there are mountains in the moon much higher than any on the earth, and instances one above 1000 toises higher than Chimbaraço.* But, as we have seen above, a small error in taking the angle with the micrometer, will produce a great error in the height of a mountain.

Before we finish this chap. it may not be improper to say a few words on the *comparative astronomy* of the moon. In one of her hemispheres the inhabitants in the moon (if any) constantly see the earth, but in the other never, except, from her libration, those who are situated near the limits of her disk. To those the earth sometimes rises a little above the horizon, and sometimes apparently moving backward, subsides below it. In the hemisphere, from which the earth is visible, it seems as it were fixed to the same point of the heavens, except a small oscilatory motion from the moon's libration, whilst in the space of a natural day, the sun and stars move towards it from the east, and then advance from it towards the west, the earth's atmosphere concealing the stars, some time, from a lunar observer. To such of the lunarians as live near the middle of the disk, the earth appears continually vertical ; but to some it appears to decline towards the north, and to others towards the south ; to some towards the east, and to others towards the west ; the declination being in proportion to their dist. from the middle of the disk. To the inhabitants who reside near those limits which divide the visible from the invisible hemisphere of the moon, one half of the earth will appear always in their horizon like a stupendous flaming mountain, its orb appearing to them nearly 14 times larger than the moon's disk appears to us ;† but those that dwell in a circle of the moon, passing from the poles through the middle, will have the earth always in their meridian, &c. &c ‡ In her compound motion round the earth and sun, her path is every where concave towards the sun, as McLaurin has shewn in his account of Newton's discoveries B. 4. ch. 5. The force, as he remarks, that bends the course of a satellite into a curve, when the motion is referred to an immoveable plane, is, at the conjunction, the difference of its gravity towards the sun, and of its gravity towards its primary ; when the former prevails over the latter, the force that bends the course of the satellite tends towards the sun ; hence the concavity of the path is towards the sun, and this is the case of the moon, as he proves, in the ch. above

* This author has also lately published a new work, on the height of the mountains of Venus, some of which he makes 23000 toises in perp. height, an alt. seven times more than that of Chimboraco.

† *Laplace* remarks, that at the same distance at which the moon appears under an angle of 5823″ (in his measures) the earth would subtend an angle of 21352″; hence the ratio of their diameters are nearly as 3 to 11 ; and as the areas of circles are as the squares of their diameters (2 Eucl. 12) we have 11^2 divided by $3^2 = 13\frac{4}{9}$ nearly.

‡ For more information on this curious subject, consult Gregory's Astr. B. 6, prop. 9.

quoted.* McLaurin further remarks, that when the gravity towards the primary exceeds the gravity towards the sun, at the conjunction, then the force that bends the course of the satellite tends towards the primary, and therefore towards the opposition of the sun; therefore the path is there convex towards the sun: and this is the case with *Jupiter's satellites*. When these two forces are equal, the path has, at the conjunction, what is called by mathematicians a point of *rectitude*;† in which case, however, the path is concave towards the sun throughout.

CHAP. V.

OF MARS.

Mars is the next planet in order after the earth; he always appears of a dusky red colour, and though sometimes apparently as large as Venus, yet he never appears so bright. From his red and dull appearance it is very probable that he is encompassed with a gross, cloudy atmosphere.‡

We have shewn that the orbits of Mercury and Venus are within the orbit of the earth; that these planets seem to accompany the sun, like satellites, their mean motion round the earth being the same as that of the sun; that they never recede from the sun, so as to be seen in opposition, or even a quadrant or 90° from him, and are only visible a few hours in the morning before the sun rises, and a few hours in the evening after he sets; and that they are sometimes seen to pass over the sun's disk in the form of a dark round spot, which phenomena evidently prove that they are situated within the earth's orbit. But Mars, being the first planet situated without the orbit of the earth, exhibits to the spectator different appearances. He is sometimes seen in conjunction with the sun, but he is never seen to transit or pass over his disk. He recedes from the sun to all possible angular distances, is sometimes in opposition, comes to the meridian at midnight, or rises when the sun sets, and sets when the sun rises; at this time he

* See also *Rowe's* Fluxions, 2d ed. pa. 225, A Treatise on Ast. by *O. Gregory*, art. 458, or Ferguson's Ast. art. 266. See also Dr. D. Gregory's Ast. b. 4, where the theory of the secondary planets are fully established; or B. 3, Newton's Prin.

† See Simpson's Fluxions, vol. 1. sect. 8.

‡ Emerson, in his Optics (B. 1. prop. 12. cor. 4) has shewn, that the red rays are the strongest, are the least refracted, or turned out of their way, and penetrate furthest into a resisting medium. And that the rest of the colours grow weaker in order, the violet being the weakest. This is also proved from the observations of those who dive into the sea; for the deeper they go the redder the objects appear, the other rays being reflected back. And we see that the sun and moon always appear ruddy in the horizon, where their light has to pass through a greater portion of the atmosphere, than when they are in the zenith.

appears brightest being nearest the earth. The disk of Mars, when viewed through a telescope, changes its form and becomes sensibly oval, according to his relative position from the sun; sometimes appearing *round*, at other times *gibbous*, but never *horned*. These phenomena evidently shew, that Mars moves in an orbit more distant from the sun than that of the earth (see p. 257) and that it is from the sun he receives his light. When viewed from the earth, he appears to move sometimes from *west* to *east*, at other times from *east* to *west*, and sometimes he appears *stationary*; which shews that he does not regard the earth as his centre of motion, not observing the equal description of areas; (see pages 258, 306 and 307) but when viewed from the sun Mars observes this law, and always performs his motion from *west* to *east*, which proves that he regards the sun as the centre of his motions.

The mean length of the *sidereal* revolution of Mars is 686,979579 days, or 1 y. 321 d. 23 h. 30′ 35″ 6. His *synodic* revolution is 1 y. 321 d. 22 h. 17′ 56″.* His motion is very unequal. In the morning when he begins to be visible, it is then *direct* and its velocity the greatest; it becomes gradually slower until the planet's elongation from the sun is about 136° 48′, where he becomes *stationary*; after which his motion becomes *retrograde*, increasing in velocity until the planet is in opposition with the sun. At this time his velocity is a *maximum*, after which it diminishes and again becomes nothing, when Mars is distant from the sun 136° 48′ as before. His motion then becomes again direct, after having been retrograde during 73 days; in which interval, the arc of retrogradation, described by the planet is about 16° 12′, immerging at length, in the evening into the sun's rays. Mars renews these singular phenomena at every opposition, with considerable variations, however, in the extent and duration of his retrogradations.

The rotation of Mars on its axis is found, from some spots on his surface, to be from *west* to *east*. M. *Cassini*, in 1666, discovered some well defined spots, from which he determined the time of the rotation to be 24 h. 40′. M. *Miraldi* determined the time of rotation to 24 h. 39′. Dr. *Herschel* makes the time of rotation = 24 h. 39′ 21″ 67, without the probability of a greater error than 2″ 34; he remarks that the spots are *permanent*, and

* The mean motions of the planets being given from the ast. tables, the length of their synodic, or *tropical year*, can be easily found, by saying, as the mean motion for a year to 360°, so is 365 days to the synodic revolution. Thus for Mars, the mean motion in a year, according to *Delambre* (tab. 112) is 6s. 11° 17′ 10″; hence 191° 17′ 10″ : 360° :: 365d. : 686d. 22h. 17′ 56″ nearly. Independent of the tables the length of a *tropical* year may be found, by saying, as 360° to 360° *less* the precession of the equinoxes, during the time, so is the length of the planet's *sidereal* revolution, to the length of its *synodic* or *tropical* revolution. And on the contrary the *tropical* rev. being given, the sidereal is found as shewn before, pa. 246. See also pa. 303, 304, &c.

that this planet has a considerable *atmosphere*. *Laplace* makes this motion on its axis 1.02723 days = 1 d. 0 h. 39′ 12″ 672, and on an axis inclined to the ecliptic in an angle of (66° 33′) 59° 41′ 49″ 2. Dr *Herschel (phil. trans.* 1784*)* makes the axis of Mars inclined to the ecliptic in an angle of 59° 42′, and to his orbit in an angle of 61° 18′; and the north pole to be directed to 17° 47′ of Pisces upon the ecliptic, and 19° 28′ on his orbit. He makes the ratio of his diameters as 16 : 15; but Dr. *Maskelyne*, who carefully observed Mars at the time of opposition, could perceive no difference in his diameters.

According to *Vince*, the relative *mean* distance of Mars from the sun is 152369, that of the earth being 100000. This distance may be found, by *Kepler's* rule, from the periodic time (see p 253*) or by the note p. 260 *Kepler* has also given the following method (in his works, *de motibus stellæ Martis.*) Let S (fig. p. 260) represent the sun, P Mars, L, K, two places of the earth, when Mars is in the point P of his orbit. When the earth was at L, Kepler observed the difference between the longitude of the sun and that of Mars, or the angle PLS; in the same manner he observed the angle PKS Now the places L, K, of the earth in its orbit being known (which for any given time may be found from Astr. tables, or the Naut Alm pa. 11, being the point opposite the sun's place) the distances LS, KS, and the angle LSK, will be given; hence in the triangle LSK, LS, SK, and the angle LSK, are given, to find LK. and the angles SLK, SKL; hence the angles PLK, PKL, and the side LK, are given, to find PL; and lastly, in the triangle PLS, PL, LS, and the angle PLS are given, to find PS the distance of Mars (or of any other planet) from the sun.†

* The learner will take notice, that the mean dist. of a planet from the sun, is equal to half the transverse, or greatest diameter of its orbit. For the mean dist. is expressed by a line drawn from the focus of the orbit, to the extremity of the conjugate axis, and this line is always equal to half the transverse. (Emerson's Conic Sect. b. 1, prop. 2.)

† *Kepler* also determined the angle PSL, or the diff. of the heliocentric long. of Mars and the earth. From the above method, from his observations on Mars in the aphelion and perihelion (as he had before determined the position of the line of the apsides) he determined the dist. of Mars from the sun in his aphelion to be 166780, and in the perihelion 138500, the earth's mean dist. from the sun being 100000; the mean dist. of Mars was therefore 152640, and the eccentricity of his orbit 14140. He also determined three distances of Mars as follow; 147750, 163100 and 166255; he calculated these same distances, on supposition that the orbit was circular, and found them equal 148539, 163883 and 166605; and therefore the errors of the circular hypothesis were 789, 783 and 350 respectively. From these observations, *Kepler*, relying on *Tycho Brahes'* observations, first concluded that the orbit of Mars must be *oval*. After this discovery *Kepler* discovered the relation between the mean distances and the periodic times of the planets, and thus laid the foundation of the *Principia* of *Newton*, and consequently the foundation of all *Physical Astronomy*. The angle under which the semidiameter of the earth's orbit, or the *parallax of its annual orbit*, as seen from Mars, being equal to the angle LPS, is, at a medium equal 41°, the greatest being 47° 24′.

Laplace makes half the greater axis of the orbit of Mars, or his *mean distance* equal 1.523693 the earth's being 1. He also makes the proportion of the eccentricity of the greater axis 0.093808, and the secular augmentation of this proportion 0.000090685. *Vince* makes this eccentricity 14183.7, the earth's mean dist. from the sun being taken 100000; and also makes the greatest equation equal 10° 40′ 40″. *Delambre* (table 116) has given the log. of the mean dist. of Mars from the sun, for every degree of his mean anomaly, allowing the secular corrections, &c. He makes his greatest equation (tab. 115) 10° 40′ 39″.

Laplace remarks that the variations in the *apparent diameter* of Mars are very great; he makes it in its mean state about (30″) 9″ 72, and when greatest (90″) 29″ 16 when the planet approaches his opposition. M. *Mollet* in his *Etude de Ciel* (p. 223) makes his mean diameter 11⅓″. Some make the greatest appar. diam. 25″. *Laplace* further remarks, that when the app. diam. is greatest, the *parallax* of Mars becomes sensible, being then nearly double that of the sun, or equal to 17″ 6. *M la Caille* makes his horizontal parallax at the time of opposition 23″ 6.—(see p 329.) From other observations he makes the hor. par. = 27½″. (3d. method of determining the paral. pa. 330.) The same law which exists between the parallax of the sun and Venus, exists also between that of the sun and Mars, as we have shewn pa. 279, and this last parallax had given a near approximation to the sun's paral. before the transit of Venus had more accurately determined it (See pa. 330.)

From the periodic time, the *mean distance** of Mars is 142088087.7 miles; his *real diameter* 4887 miles,† and his *mag-*

* The planet's synodic rev. = 686d. 22h. 17′ 56″ (pa. 347) = 59350676″, the square of which is 3522502741656976, which being divided by the sq. of the earth's per. rev. = 995839704797184 (pa. 258) gives 3.537208573 nearly, the cube root of which is 1.5307 nearly, the dist. of Mars from the sun, that of the earth being an *unit* or 1; hence 1.5307 × 23464.5 (see pa. 258) = 35917.11015 dist. of Mars in semidiam. of the earth; therefore 35917.11015 × 3956 = 142088087.7 miles the mean dist. of Mars from the sun.

† Now taking the hor. par. at the time of opposition 23″6 as given by *La Caille.* Then in the fig. pa. 250, let M represent Mars, and AB the earth, and we have,

As sine AMC 23″6 - - -	6.0583927
To radius or sine 90° - -	10.0000000
So is AC = 1 semidiam. -	0.0000000
To MA = 8741.93 semidiam.	3.9416073

Hence Mar's dist. from the earth at his oppos. is 8741.93 semid. of the earth, and taking his appar. diam. at oppos. = 29″16, it will be 23464.5 : 8741.93 :: 29″16 : 10″86, the app. diam. of Mars as seen from the earth at a dist. equal that of the sun; therefore 32′ : 10″86 :: 864065.5 (the sun's diam. pa. 255) : 4887 miles, the diameter of Mars.

nitude is something more than $\frac{1}{4}$ of that of the earth.* His velocity in his orbit round the sun is about 54152 miles an *hour*;† and his *light* and *heat* is in proportion to that on the earth as 0.43 to 1 nearly.‡ The distances of the *exterior* planets may also be found from the parallax of the earth's annual orb, as shewn before.§

The *mean longitude* of Mars at the commencement of 1750, reckoning from the mean vernal equinox at the epoch of the 31st of December, 1749, at noon, mean time at Paris, was, according to *Laplace* (24° 4219) 21° 58″ 46″ 956. Longitude of the *perihelion* at the beginning of 1750 (368° 3005) 321° 28′ 13″ 62. The sidereal and *secular* progressive motion of the perihelion (4834″ 57) 26′ 6″ 4. The *inclination* of his *orbit* to the plane of the ecliptic (2° 0556) 1° 51′; and its *secular* retrograde variation (4″ 45) 1″ 44. Longitude of the *ascending node* upon the ecliptic (52° 9377) 47° 38′ 38″ 148; and its *sidereal* and *secular retrograde* motion upon the *true* ecliptic (7027″ 41) 37′ 56″ 88 *Vince* makes the place of the aphelion for 1750, 5s. 1° 28′ 14″, and its secular motion in longitude 1° 51′ 40″. In 1760 (Bissextile) *Delambre* makes the mean place of the aphelion 5s. 1° 39′ 34″, and of the node 1s. 17° 43′ 18″. In 1810 (com. year) he makes the place of the aphel. in 5s. 2° 35′ 24″, and of the node 1s. 18° 6′ 38″. According to him the *annual* mean motion of Mars is 6s 11° 17′ 10″, of the aphel. 1′ 7″, and of the node 28″; the *daily* mean motion is 31′ 27″.

The learner should take notice, that all the epochs in the Astronomical tables are reckoned from noon on December 31, for common years, and from January 1st for the bissextiles.

* To determine his magnitude, we have 7911^3 divided by 4887^3 = log. $7911^3 - 4887^3$ = .6275673, the number corresponding to which is 4.234, which shews that the earth is more than four times as large as Mars. It is probably greater in proportion, as we have taken Mars' greatest diam. as given by *Laplace*; if 25″ were taken the proportion would be greater.

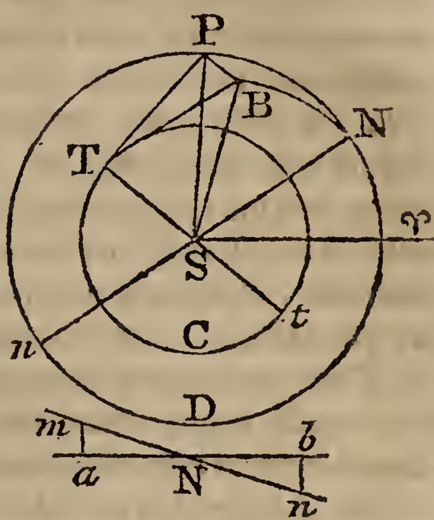
† The mean dist. 142088087.7 × 2 × 3.1416 = 892767872 miles nearly, the circumference of the orbit of Mars; and therefore 59350676″ : 1h. or 3600″ :: 892767872m. : 54152 miles.

‡ The effects of light and heat being reciprocally proportional to the squares of the distances from the centre from which they are propagated; hence the sq. of the earth's distance from the sun divided by the sq. of the dist. of Mars, the quot. will be the comparative heat, &c.

§ Thus in the fig. pa. 260, in the right angled triangle SLP, taking the angle LPS at a medium = 41° (see pa. 348) and SL, the earth's mean dist. from the sun = 23464.5 semidiam. SP, the dist. of Mars from the earth is found = 35766 semid. of the earth.

In *Mercury* and *Venus* as the *sidereal* rev. had been taken in place of the *tropical*, to correspond with the *tropical* rev. of the earth, in finding their dist. from the sun (pa. 258 and 271) this will therefore alter a little the distances thus determined. When the sidereal rev. of the planets is used, the *sid.* rev. of the earth should also be used, which is nearly 365d. 6h. 9′ 11″ = 31558151″, the sq. of which is 995916894538801, a *constant divisor*, for the *sidereal* rev. of the planets. The examples of different authors, who could scarce be suspected of the above mistake, was the cause of this inadvertent error, which the reader, knowing the principle, may correct, at his leisure.

Now the places of the aphelion and nodes being given, and the planet's *true anomaly* (found from the *theory of the planet's motions*, given in ch. 4. see pa. 312) its distance from the *node*, which is called the *argument of latitude*, is given ; from which we can find the central distance of the planet, and its *curtate distance* from the sun, or the dist. from the sun to that point where a perp. let fall from the planet meets the ecliptic. From these data the geocentric place of the planet may be easily found. Let TCt be the earth's orbit, and T the earth's place ; DNP the orbit of a planet, and P the planet's place ; S the sun ; nSN the line of the nodes.—From P let fall on the plane of the ecliptic the perp. PB, join SB and produce it until it meets the planet's place reduced to the ecliptic, found by the arch PN, (the planet's dist. from the node) and the inclination of the orbit to the plane of the ecliptic, which are given (for *Mars* see page 350) but the place of the *earth* seen from the sun, or the point opposite the sun's place, is given ; and hence the angle between them, or the angle TSB, which is called the *angle of commutation*, is given. Then in the triangle STB, ST the earth's dist. from the sun, and SB the *curtate dist.* of the planet, are given (see pa. 265) the angle TSB is given, being the *elongation* of the planet from the sun, or the arc of the ecliptic intercepted between the sun's place, and that of the planet reduced to the ecliptic ; as also TB the *curtate dist.* from the *earth*. And as the sun's place is given, the place of the planet, as seen from the earth, is likewise given. Again in the triangles PSB, PTB, right angled at B, *tang.* PSB : *tang.* PTB :: TB : SB :: *sine of the commut.* TSB : *sine of the elong.* STB. Hence as *sine of the commutation* : *sine of the elong* :: *tang. heliocentric lat.* : *tang. geocentric lat.* In this manner the *geocentric* place, or the place seen from the earth, and the lat. of a planet is found for any time. (See Dr. *Gregory*'s Astr. b. 3. where several curious and important points relative to the theory of the primary planets, are fully investigated.)



The following observations principally collected from *Delambre* and la *Lande*, will be of use to the learner.

The *mean longitude* of a planet, seen from the sun, as also that of the *sun* and *moon*, is found by adding the *epoch* to the *mean motions*.*

* Thus if it be required to find the *moon's mean longitude, mean anomaly,* and *place of the node,* on the 19th of May, 1819, at 5 o'clock in the afternoon, at New-York. If we make use of *Delambre's* tables, the time must be reduced to the meridian of Paris, and as the diff. of long. is 76° 20′ 42″, the

The longitude of the *aphelion* taken from the planet's mean longitude, will give the mean anomaly If the sun's longitude, increased by 20″, on account of the aberration (pa. 302) be taken from the reduced heliocentric long. if the remainder exceeds 6 signs, subtract it from 12 signs : half this *commutation*, or suppl. of 12 signs, is called the *semi-commutation*. Also if the difference between the log. of the sun's dist. from the earth, and that of a planet's dist. from the sun being found, and so be added to the caracteristic of the remainder, let this diff be found in the log. tangents, and from the corresponding angle let 45° be subtracted (*la Lande's* Astr. Art. 3850) the log. tang. of the remainder added to the log. tang. of the semi-commutation, will give the log. tang. of an angle, which *added* to the semi-commutation for the *superior* planets, or *subtracted* from the semi-commutation for the *inferior*, the sum or remainder will give the planet's *elongation* respectively (*La Lande*, Art. 1142.)

The *geocentric longitude* is found by adding the elongation to the sun's long. when the commutation is less than 6 signs, or subtracting it when the commut. is greater.

La Lande (art. 1146) gives the following rule for finding the planet's dist. from the earth ; as sine elong. : sine commut. :: planet's dist. from the sun reduced to the ecliptic : the dist. from the earth on the plane of the ecliptic : this distance divided by the cosine of the geocentric lat. gives the direct distance from the earth to the planet. Also the *diameter* of the planet for the mean dist. of the sun, being divided by the dist. of the planet from the earth, will give its actual and apparent diam. seen from the earth. (*La Lande*, art. 1391 and 1384.) And the sun's parallax being divided by the same dist. of the planet from the earth, gives the hor. parallax of the planet. (*La Lande*, Art. 1631.)

Vince shews how to reduce the places of the planet's seen from the earth to their places seen from the sun, as follows.

Let T be the place of the earth (see the fig. pa. 351) P the planet, S the sun, ♈ the point aries ; let PB be drawn perp. to the ecliptic, and TS be produced to *t*. Let the longitude of the sun seen at *t*, at the time of observation be computed (note to prob. 1 and 3, part III) and the opposite point in the ecliptic is the long. of the earth at T, or the angle ♈ST ; compute also the long. of the planet, or the angle ♈SB (pa. 195) and the difference of these two angles is the angle of *commutation* TSB. The place of the planet in the eclip-

difference of time is 5h. 5′ 22″8 ; hence the corresponding time in Paris is 10h. 5′ 22″, which is the time for which we must calculate.

	Moon's long.	*Mean anom.*	*Sup. of the node.*
Epoch for 1819 (tab. 26)	10s. 26° 39′ 31″3	7s. 18° 8′ 37″4	11s. 4° 11′ 23″0
May 19 (tab. 28)	1 1 31 9 0	16 2 0 2	7 21 38 8
Motion for 10h. (t. 29)	5 29 24 6	5 26 37 5	1 19 4
Motion for 5′	2 44 7	2 43 3	0 7
Motion for 22″8	12 5	12 4	
Mean long. reg.	0 3 43 2 1	8 9 40 10 8	11 11 34 21 9

tic being observed, and the sun's place being known, we have the angle BTS, the elongation in respect of the longitude; hence the angle SBT, which is the measure of the diff. of the planet's places, as seen from the earth and the sun, is given; therefore the *geocentric* place of the planet being known, its *heliocentric* will be known. Moreover *tang.* PTB : *rad.* :: BP : TB (Simson's Trig. prob 1.) and *rad.* : *tang.* PSB :: BS : BP; hence *tang.* PTB : *tang.* PSB :: BS : TB :: *sine* STB : *sine* TSB; that is, *sine of the elong. in long. : sine diff. long. of the earth and planet :: tang. geocentric lat. : tang. heliocentric lat.* When the lat. is small SB : TB :: PS : PT very nearly, which, in opposition, is very nearly as PS : PS — ST. Or the values of PS and ST can be computed with more accuracy (see the method in pa. 314) than we can compute the angles STB, TSB. The *curtate* dist. ST of the planet from the sun, is found by this proportion *rad.* : *cos.* PST :: PS : SB.

The place of the planet's *node* may be thus determined; find its heliocentric lat. immediately before and after it has passed the node, and let n, m, be the places in the orbit, and b, a, the corresponding places reduced to the ecliptic; then the triangles nbN maN, which may be considered as rectilinear, being similar, we have nb : ma :: Nb : Na; hence $nb + ma$: nb :: Nb + Na (ab) : Nb (5 Eucl. 18) or alternately, $nb + ma$: ab :: nb : Nb, that is, *the sum of the two latitudes : diff. of the longitudes :: either lat. : dist. of the node from the long. corresponding to that lat.* Or it will be very nearly as accurate to take both latitudes from the earth, when the observations are made in opposition. If the dist. of the observations exceed 1°, this rule will not be sufficiently accurate, in which case we can compute by spherical trig. thus, in the rt. angled spher. triangle aNm, we have, by *Napier's* rule, rad. × sine aN = tang. ma. × cot. ∠ N; but rad. being = 1, and sine aN = sine $\overline{ab - bn}$; hence, sine $\dfrac{\overline{ab - \text{N}b}}{\text{tang. } ma} = \text{cot. N}$; and *(Napier)* rad. × sine Nb = tang. nb × cot. N; hence cot. $\text{N} = \dfrac{\text{sine N}b}{\text{tang. } nb}$; but sine $\overline{ab - \text{N}b}$ = sine ab × cos. Nb — sine Nb × cos. ab (Vince's Trig. art. 101, or Emerson's b. 1. prob. 6.) hence, $\dfrac{\text{sine } ab \times \text{cos. N}b - \text{sine N}b \times \text{cos } ab}{\text{tang. } ma} = \dfrac{\text{sine N}b}{\text{tang. } nb}$;

therefore $\dfrac{\text{s. } ab \times \text{cos. N}b}{\text{tan. } ma} = \dfrac{\text{s. N}b}{\text{t. } nb} + \dfrac{\text{s. N}b \times \text{cos. } ab}{\text{tan. } ma} =$

$\dfrac{\text{s. N}b \times \text{tan. } ma + \text{cos. } ab \times \text{tan. } nb}{\text{tan. } nb. \times \text{tan. } ma}$; hence $\dfrac{\text{s. } ab \times \text{tan. } nb}{\text{tan. } ma + \text{cos. } ab \times \text{tan. } nb}$

$= \dfrac{\text{s N}b}{\text{cos N}b} =$ (Emerson's Trig. b. 1. prob. 1. cor. 4.) tang. Nb.*

* Mr. *Bugge* having observed the rt. as. and decl. of *Saturn*, from thence found the following heliocentric longitudes and latitudes (see prob. 3, part 3, and the above rule for the heliocentric lat. &c.)

The *inclination of the orbit* may be thus determined; *bn* the lat. of the planet, and *b*N its dist. from the node on the ecliptic being given, we have *sine b*N : *tan. nb* :: *rad.* : *tan.* ∠ N; or by *Napier's* theorem *tang. bn* : *rad.* :: s. *b*N : *cot.* N the incl. required. The observations must not however be taken near the node, in determining the incl. as a very little error in the lat. will make a considerable error in the inclination.*

Or the *incl.* may be thus found Find the angle PSB, as shewn above, then the place of the planet and that of its nodes being given, BN is given; hence (Simson's Spher. Trig. prob. 17.) sine BN : tang. PB :: rad. : tang. PNB the incl. of the orbit. Or by *Napier's* rule, rad. × s. BN = tan. PB × cot. PNB; hence tan. PB : rad. :: s. BN : cot PNB.

On March 27, 1694, at 7h. 4′ 40″, at Greenwich, Mr. *Flamstead* determined the right ascension of Mars to be 115° 48′ 55″ and his decl. 24° 10′ 50″ N. hence the geocentric long found as directed above, was *Cancer* 23° 26′ 12″, and lat. 2° 46′ 38″.—Let P represent the place of Mars, B his place reduced to the ecliptic, and S, T, the places of the sun and earth respectively; then the true place of Mars, by calculation, as seen from the sun, was *Leo* 28° 44′ 14″, and the sun's place *Aries* 7° 34′ 25″; hence the diff. of these places or the angle BTS = 105° 51′ 47″; and the earth's place being *Libra* 7° 34′ 25″, if the place of Mars be taken from it, the remainder is the angle TSB = 38° 50′ 11″; hence from the above, sine 105° 51′ 47″ : sine 38° 50′ 11″ :: tang. PTB = 2° 46′ 38″ : tang. PSB = 1° 48′ 36″. Now the place of the node was *Taurus* 17° 15′, which subtracted from *Leo*

1784.	*Appar. time.*			*Helioc. long.*				*Helioc. lat.*			
July 12	at 12h.	3′	1″	9s.	20°	37′	29″	0°	3′	13″	N.
20	11	29	9	9	20	51	53	0	2	41	
Aug. 1	10	38	25	9	21	13	17	0	1	34	
8	10	9	0	9	21	26	2	0	0	56	
21	9	14	59	9	21	49	27	0	0	2	
27	8	50	19	9	22	0	12	0	0	27	S.
31	8	33	47	9	22	7	32	0	0	50	
Sep. 5	8	13	45	9	22	16	28	0	1	21	
15	7	33	45	9	22	34	32	0	1	59	
Dec. 8	6	4	23	9	23	16	15	0	3	35	

From the obs. on August 21 and 27, the triangles being considered as plane, N*b* = 44″ 5, from the observations on the 21 and 31, N*b* = 42″ 5; and from those on Aug. 21, and Sept. 5, N*b* = 40″; the mean of these gives N*b* = 42″. Mr. *Bugge* makes N*b* = 41″, either from taking the mean of more obs. or computing from spher. hence the heliocentric place of the descending node was 9s. 21° 50′ 8″ 5. On Aug. at 9h. 12′ 26″ true time, *Saturn's* hel. long. was 9s. 21° 49′ 27″, and on 27, at 8h. 49′ 23″ true time, it was 9s. 22° 0′ 12″; hence in 5d. 23h. 36′ 57″ *Saturn* moved 10′ 45″ in long. therefore 10′ 45″ : 41″ :: 5d. 23h. 36′ 57″ : 9h. 7′ 44″ the time of describing 41″ in long. which being added to Aug. 21, 9h. 12′ 26″, gives Aug. 21, 18h. 20′ 10″, the time when *Saturn* was in its node.

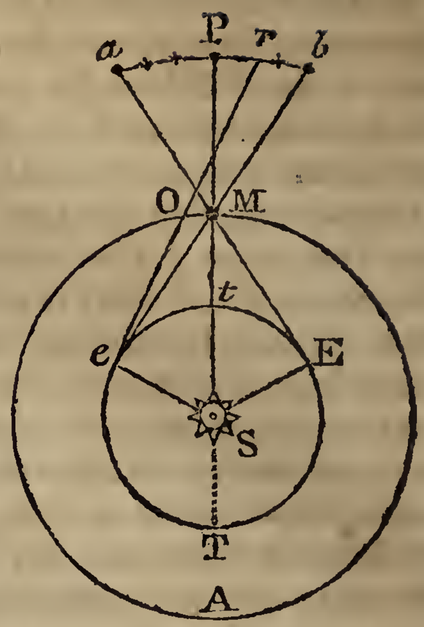
* The obs. on July 20 (see the last note) gives the angle 2° 38′ 15″; that on Oct. 8, gives it = 2° 22′ 13″; the mean of these is 2° 30′ 14″ the inclination of *Saturn's* orbit to the ecliptic from these observations.

28° 44′ 14″, gives 101° 29′ 14″ = BN the dist. of Mars from his node; hence sine BN 101° 29′ 14″ : tang. PB 1° 48′ 36″ :: rad. : tan. PNB = 1° 50′ 50″ the inclination of his orbit. Mr. *Bugge* makes the incl. 1° 50′ 56″ 56 for March, 1788. M. de la *Lande* makes it 1° 51′ for 1780.

If we conceive lines to be drawn from *t* to P and B. it is evident that the angle PTB will be much greater than P*t*B; and that also the angle PTB is greater than PSB, while T is nearer to P than S, and less when further from P, as at *t*; hence when the earth is in T, the *geocentric* lat. of *Mars* is greater than his *heliocentric*, but when the earth is in *t*, the *heliocentric* is greater than the *geocentric*; hence the visible lat. of Mars vary according to his various positions; so that, other circumstances remaining the same, his latitude is greater the nearer he approaches his *opposition* with the sun, and becomes less as he approaches his *conjunction*. The same reasoning may be applied to the other *superior* planets *Jupiter*, *Saturn*, *Herschel*, and also the newly discovered planets *Ceres*, *Pallas*, *Juno* and *Vesta*.

When the planets are in *opposition* to the *sun*, they rise when the sun sets, and set when he rises; after they depart from the opposition, they appear to the *eastward* of the sun, and after sun set they are visible in the *evening* until their *conjunction* with the sun, when they rise and set with him. As they recede from the sun, after their conjunction, they are *visible* only in the *morning* before sun rise, for they set in the evening before the sun. When they come to their opposition again, these phenomena will appear in order as before, &c. In their *oppositions* their appar. *diameters* appear much larger than in *conjunctions*, being nearest the *earth* in one position, and furthest in the other; the diff in their distances, in these two positions, being equal to the diameter of the earth's orbit; and, as this bears a considerable proportion to the dist. of Mars, being about *five* times nearer the earth in *opposition* than in *conjunction*, his app. diam. will be 5 times greater in the one than in the other, and hence, as his visible disk and lustre increases as the squares of the app. diam. he will in oppos. be twenty times larger and brighter than in conjunction.

All the *superior* planets observed from the sun, will appear to move regularly the same way, though with unequal angular motions, arising from their different distances, but yet so as to observe the *general law* of describing equal areas in equal times, round the sun. But when observed from the earth, their appearances are very different: they sometimes move forward, or *direct*, that is from *west* to *east*, at other times *retrograde*, or from *east* to *west*, and at other times they appear immoveable or *stationary*.—



(See pa. 258.) Let TE*e* be the orbit of the earth, AMO the orbit of a superior planet, as *Mars*, and S the sun's place. Now let the earth be at T when Mars is at M, then Mars and the sun will be in *conjunction*; but if the earth be at *t* when Mars is at M, they will be in *opposition*. If the earth remained fixed at T, the motion of Mars would appear always *direct*; but the earth moving in its orbit quicker than Mars in his, the latter will therefore be overtaken by the earth, and afterwards the earth will advance before him, so that he will seem to move backwards. If we now suppose the earth at E when Mars is at M, he will be seen in the heavens among the fixed stars at *a*, and for some time before and after he has been at E, he will appear nearly in the same point *a*, that is, he will be *stationary*. While the earth moves in its orbit through the space E*te*, if Mars remained without any motion at M, he would appear to move in a *retrograde* direction, through the arc *a*P*rb*, among the fixed stars, and would again appear *stationary* at *b*; but if while the earth moves from E to *e*, Mars moves from M to O, his *retrograde* motion will be nearly represented by the arc *a*P*r*. Hence Mars will have two stationary and one retrograde appearance.

What we have here shewn with respect to Mars, is also true of *Jupiter*, *Saturn*, and all the superior planets. But Saturn's regressions are more frequent than Jupiter's, and Jupiter's than those of Mars, because the earth overtakes them, or passes between them and the sun, oftener than between the sun and Mars; but the retrogressions of Mars are greater than those of Jupiter, and Jupiter's greater than Saturn's, &c. For more information on this subject see Keil's Astronomy, lect. 27.

From what we have given above it will be seen that *Mars' year* is nearly equal to two of our years; but that the natural day is only a little greater than ours. The axis of Mars' diurnal revolution being nearly perp. to the plane of its orbit, it follows, that the artificial day, exclusive of twilight, is almost constantly equal to the night, on every part of this planet's surface; and that therefore there is little diff. between his *summer* and *winter*. However, from the different inclinations of the sun's rays to the horizon, as on the earth at the equinoxes, the heat constantly diminishes from his equator to the poles; and hence Dr. *Gregory* supposes the *Fasciæ*, or belts of Mars, to be produced from this phenomenon. For as the same degree of heat always continues in the same climate, it is probable that the spots in Mars, like the clouds and snow on the earth, being produced by heat and cold, are extended, according to the climate, and make *Fasciæ* or belts parallel to the circles of Mars' diurnal motion. Similar phenomena existing in Jupiter, who, like Mars, has a perpetual equinox, strengthens the conjecture. (See *Gregory's* Astr. b. 6. prob. 4.)

In our earth and moon, an observer in Mars will have a phenomenon, which is not seen by us, that of an *inferior* planet with a *satellite;* though they will never appear to him to be one quarter of a degree from each other. To him our earth will appear about as large as Venus does to us, and its *elongation* from the sun, will never appear to him to be more than about 48°. Like Mercury and Venus, it will sometimes be seen by him to pass over the sun's disk. Venus will be as seldom seen by him as we see Mercury, and Mercury will never be visible to him, unless assisted with good telescopes or other substitutes. We have remarked before (pa. 277) that, as we are now better acquainted with the planet's motions and phenomena, than in any preceding period, these phenomena represented on the globe, or with an *orrery*, would be very interesting. And from what is shewn, in parts 2 and 3, the learner will be able to pursue this entertaining subject at his leisure.

CHAP. VI.

OF THE NEW PLANETS,

CERES, PALLAS, JUNO AND VESTA.

Herschel, in 1780, having enriched our system with a new planet, it seemed at that time, that in this regard, all was then discovered, and that the number of the planets was fixed to *seven;* but recent discoveries have shewn that they are not thus limited, and that we are, as yet, far from being acquainted with their number. We have spoken of the new planets discovered, since this time, in pa. 47, note to def. 119, but here it becomes necessary to speak more particularly of them.

The celebrated M. *Piazzi*, astronomer royal at Palermo in Sicily, on the 1st of January, 1801, augmented the number of the planets already known in our system, by adding another, to which he gave the name of *Ceres*,* called *Ceres Fernandea*, in honour of Ferdinand IV. king of the Two Sicilies. Her orbit he found to be situated between the orbits of Mars and Jupiter, at the distance, according to some, of about 94 millions of leagues, or according to others of $2\frac{4}{5}$ times that of the earth, from the sun. Her periodic

* *Ceres* and the names of the other newly discovered planets, *Pallas, Juno,* and *Vesta,* were given these planets in allusion to the heathen names given the other primary planets, to preserve an uniformity and similarity, with the names in the ancient system. The planet *Herschel* or *Uranus* being exterior or superior to *Jupiter,* has received a title of greater antiquity, but these being interior or inferior, have received titles which indicate a more recent date. For their mythological explication see *Lempriere's* Classical Dictionary, or *Littleton's* Latin Dictionary.

revolution is, according to some, 4 years 7 months, and according to others, 4 years 8 months; but the elements of her theory is as yet very imperfectly known. She is found not to be confined within the ancient limits of the zodiac; she is invisible to the naked eye, being of the 8th magnitude, and cannot therefore be seen without a good telescope. Her diameter, according to Dr. *Herschel*, is about 162 miles.

Dr. *Olbers*, of Bremen, discovered a 9th planet on the 28th of March, 1802, to which he gave the name of *Pallas*. It is of the 7th magnitude, and was then situated near the northern wing of the constellation Virgo. Her orbit is about equally placed between Mars and Jupiter, and stated to be at nearly the same distance from the sun as *Ceres*. The theory of the phenomena of this planet is still less known than that of *Ceres*, and hence the account of her dist. period, magnitude, &c. must be very imperfect. Her periodic revolution is reckoned to be about 1 month more than the planet of *Piazzi*, and her diameter about 95 miles.

These discoveries surprized all the astronomers, in pointing out to them new planets which till then had escaped their researches, and attended with phenomena, which they had never before observed. Two planets placed at nearly equal distances from the central body (the sun) is a phenomenon entirely new, and which may give place to very extraordinary results, altogether unforeseen or unexpected. It is true that these planets move in different *planes*, and that the *eccentricity* of their orbits is not the same; but after all it may possibly take place, that their approach may be too near; and, if so, it will then be curious to observe the effects which would result from this too near proximity.

Some German astronomers, having considered the relative distances of the planets from the sun, concluded that there was wanting another planet between Mars and Jupiter, and hence they endeavoured to find it out; but their wishes are more than gratified in the discovery, not only of *one* or *two*, but even of *four*. However some, before the discovery of the two latter, to account for the phenomenon of the two planets being equally dist. from the sun, asserted that they were but one planet divided into pieces, &c.

Mr. *Harding* of Lilienthal in the duchy of Bremen, on September 1, 1804, discovered a 10th planet which he called *Juno*. This new planet is also found to be at nearly the same distance from the sun as the former two, and it is not yet decided with certainty, which of the three is the nearest or the most remote from the sun. As she appears like a star of the 8th magnitude, she is not therefore visible to the naked eye.

On the 29th March, 1807, Dr. *Olbers* discovered another new planet, which he called *Vesta*, now the 11th in our System, in the order of discovery: it is very remarkable that this planet is found to be apparently at the same distance from the sun as the three already mentioned. At the time that it was discovered, that is on the 29th March, 8h. 21′, its rt. ascen. was 184° 8′, and declination

11° 47′. In size this planet appears like a star of the 5th magnitude. If the phenomena of *two* planets, nearly at the same dist. from the sun appeared strange to astronomers, that of *four* must appear still more extraordinary; however it is more than probable that since the creation of the world, they have like the comets performed their motions in their respective orbits, without clashing with each other, or producing any of those strange phenomena, resulting from their too near proximity; and it is equally probable, that the wisdom of the Creator has regulated their motions in such a manner, as to prevent any of these accidents taking place, while the Solar System exists.

CHAP. VII.

OF JUPITER,

AND HIS FOUR SATELLITES.

Jupiter is the next planet, in order, in our System, and also the largest of all the planets, so that notwithstanding his great distance from the sun and earth, he appears to the naked eye almost as large as Venus, though not so bright. Jupiter when in opposition to the sun, appears larger and more luminous than at other times, being then much nearer to the earth, than a little before or after his conjunction.* Jupiter will be a *morning star* when his longitude is less than that of the sun, and will therefore appear in the *east* before the sun rises; but when his longitude is greater than the sun's, he will be an *evening star*, and will be in the *west* after sun set. Jupiter's periodic or *sidereal* revolution from *west* to *east*, is, according to Laplace, 4332.602208 days or 11y. 31d. 14h. 27′ 10″77. *Vince* makes his periodic revolution 11y. 315d. 14h. 27′ 10″8.†

Jupiter before his *opposition*, is subject to the same apparent inequalities as Mars, as we have before remarked, and when he is about 115° 12′ distant from it, his motion becomes *retrograde*, his velocity increases until the opposition, and after which it dimin-

* Jupiter, when in conjunction with the sun, rises, sets, and comes to the meridian with the sun; but when in opposition, he rises when the sun sets, sets when the sun rises, and comes to the meridian at midnight.

† Here there are two days difference between *Vince* and the other astronomers (*except those who copy him*) this might be accounted for allowing only 365 days for the year, in reducing the days from *Laplace*; whereas there are, at least, two leap years; but *Vince*, where he makes the same calculation for *Saturn's sidereal* revolution, takes 365 days for the year without any allowance for bissextile. Calculating from *Jupiter's* mean motion according to *Delambre* (tab. 119) which is at the rate of 30° 20′ 31″7 in a year, we have 30° 20′ 31″7 : 360° :: 365d. : 4330d. 14h. 39′ 49″2; but this is the mean *tropical* revolution which is considerably less than the *sidereal*; and hence *Vince* must have here fallen into an error. Keith in his Treatise on the Globes, makes the same mistake.

ishes, until the planet in his approach towards the sun, is again only 115° 12′ distant from him. The duration of this retrograde motion is 121 days, according to *Laplace*, and the arc of retrogradation 9° 54′; there are, however, perceptible differences in the duration and extent of the regressions of Jupiter.

The semimajor axis of Jupiter's orbit, or his *mean distance* from the sun, is, according to Laplace 5.202592, the earth's being 1, the proportion of the *eccentricity* of this axis for the beginning of 1750 is 0.048877, and the secular increase of this proportion 0.0001434 5. *Vince* makes the relative mean dist. of Jupiter 520279, and the eccentricity of his orbit 25013,3, the mean dist. of the earth from the sun being 100000. The *mean longitude* of Jupiter, reckoned from the mean vernal equinox at the epoch of the 31st of December, 1749, at noon, mean time at Paris, is reckoned according to *Laplace* (4°1201) 3° 42′ 29″124; longitude of the *perihelion* at the beginning of 1750 (11°5012) 10° 21′ 3″888; its sidereal and *direct secular* motion (2030″25) 10′ 57″801. *Vince* makes the place of the *aphelion* for the beginning of 1750, 6s. 10° 21′ 4″, and its *secular* motion in longitude 1° 34′ 33″. *Delambre* in his tables, makes the mean long. of Jupiter for the beginning of 1811, mean time at Paris, 1s. 25° 44′ 33″7, of his *aphelion* 6s. 11° 8′ 45″, and of his node 3s. 8° 30′ 40″. He makes Jupiter's mean annual motion 1s 0° 20′ 31″7, that of his aphelion 57″, and of his node 36″; and the *secular* motion of the aphelion 1° 34′ 33″, and of the nodes 59′ 30″ The mean motion of Jupiter for a *month* is therefore 2° 34′ 37″2, of the aphel. 5″, and of the nodes 3″ For a *day* the mean mot. is 4′ 59″ 3, for an *hour* 12″5, and for a *minute* 0″2. The *inclination* of Jupiter's *orbit* to the plane of the ecliptic, at the beginning of 1750, according to *Laplace*, was (1° 4636) 1° 19′ 2″064, and its *secular* variation to the *true* ecliptic (—67″40) 21″8376 decreasing. *Vince* makes his inclination 1° 18′ 56″. According to *Laplace*, the longitude of the ascending node, on the ecliptic, at the beginning of 1750, was (108° 8062) 97° 55′ 32″088, and its *sidereal* and *secular* mot. on the *true* ecliptic (—4509″5) 24′ 21″078 decreasing. *Vince* makes the long. of the nodes for the beg. of 1750, 3s 7° 55′ 32″, and its *secular* motion 59′ 30″. The *greatest equation* for Jupiter, according to *Vince*, is 5° 30′ 38″3, according to *Delambre* 5° 30′ 37″7. (See tab. 124 of Delambre, where its secular var. is also given for every degree of the mean anomaly.) *Laplace* remarks that the path of Jupiter occasionally deviates from the ecliptic (3° or 4°) 2° 42′ or 3° 36′.

Jupiter is observed to have several obscure *belts* or stripes on his surface, which are parallel among themselves and also to his equator, and therefore nearly parallel to the ecliptic; there are likewise other spots, the motion of which has demonstrated the rotation of this planet from *west* to *east* upon an axis nearly perp. to the plane of the ecliptic, and in a period, according to *Laplace* of 0.41377 days, or 9h. 55′ 49″7. *M. Cassini*, from a remarkable spot which he observed in 1665, found the time of rotation to be 9h 56′. In Oct. 1691, he observed two bright spots almost as broad as the belts ; at the end of the month he saw two more, from which he found the rotation of the planet to be performed in 9h. 51′ ; he also found that some spots near the equator revolved in 9h. 50′, and in general he found that the nearer the spots were to the equator the quicker they revolved ; he also observed, that several spots which at first were round, grew long by degrees, in a direction parallel to the belts ; and divided themselves into two or three spots. These variations in some of the spots, and the sensible difference in the period of rotation of others, induce the opinion that they are not attached to Jupiter ; they appear to be clouds which are transported by the winds with various velocities, in an atmosphere extremely agitated.* M. *Miraldi* from several observations of the spot observed by *Cassini* in 1665, found the time of rotation to be 9h. 56′, and concluded that the spots had a dependence upon the contiguous belt, the spot never appearing without the belt, though the belt appeared without the spot. It continued to appear and disappear until 1694, and then disappeared until 1708 ; he therefore concluded that the spot was some effusion from the belt upon a fixed place of Jupiter's body, as it always happened in the same place. Dr. *Herschel* found the rotation of different spots to vary, and that the time of the rotation of the same spot diminished. The spot in 1788 revolved as follows ; from Feb. 25, to March 2, it revolved in 9h. 55′ 20″ ; from March 2 to 14, in 9h. 54′ 58″ ; from April 7 to 12, in 9h. 51′ 35″. He observes that this is agreeable to the theory of equinoctial winds, as the spot may require some time before it can acquire the velocity of the wind ; and he further remarks, that if Jupiter's spots were observed in different parts of its revolution to be accelerated and retarded, it would amount almost to a demonstration of its monsoons, and their periodical changes. M *Schroeter* makes the time of rotation 9h. 55′ 36″ 6 ; he found the same variations as Herschel.

According to *Laplace* the *apparent diameter* of Jupiter, in his opposition, or when it is greatest, is (149″) 48″ 276, and his mean diameter, in the direction of his equator, is (120″) 38″ 88. From the great magnitude of Jupiter, and his quick revolution on his axis, it is found that he is flatter towards the poles than at the

* The belts of this planet are also subject to very great variations, both as to number and figure ; eight have been sometimes seen at once, and at other times only one ; they are sometimes found to continue three months without any change, and sometimes a new belt has been formed in one or two hours. Large spots have been also seen in these belts, and when a belt disappears, the contiguous spots disappear likewise. Hence, from their being subject to such sudden changes, it is very probable that they do not adhere to the body of Jupiter, but are produced and exist in his atmosphere. If this be the case, those that are produced in one or two hours, must require an agitation or velocity in the air, much greater than we experience in the greatest hurricanes.

equator, *Laplace* makes the polar diameter : equatorial diam. :: 13 : 14. Mr. *Pound* makes this proportion as 12 : 13. Mr. *Short* as 13 : 14. Dr. *Bradley* as 12.5 : 13.5. Sir I. *Newton* makes the ratio from theory as $9\frac{1}{3}$: $10\frac{1}{3}$ (prop. 18. b. 3. prin.) and on supposition that Jupiter is more dense towards the equator, he makes the ratio from 12 : 13 or from 13 to 14. The diameters as observed by Mr. *Pound* vary from 11 : 12 to $13\frac{1}{2}$: $14\frac{2}{3}$ (see the principia prob. 19. b. 3.) *Newton* makes the greatest diam. 37″ and least 33″ 25‴; but after allowing 3″ for refraction, he makes them 40‴ and 36″ 25‴ respectively.

Jupiter's *mean dist.* from the sun, as calculated from his periodic time, is 483342701.3 miles,* his *hourly* velocity in his orbit is 29206 miles,† taking his mean diameter 38″ 8 or 39″ nearly, his *real diameter* will be 73687 miles,‡ and his *magnitude* 807 times that of the earth.§ The *light* and *heat* which he receives from the sun is about $\frac{1}{27}$ of that which the earth receives.‖

It is a remarkable result which the nicety of modern observations has determined, and which may be collected from what is said above, that, while the eccentricities, inclinations to a fixed

* The sidereal revolution of Jupiter being 11y. 317d. 14h. 27′ 11″ nearly = 374336831″, the square of which is 140128064043122561, this divided by 995916894538801, the square of the earth's sidereal revolution (see pa. 350) gives 140.702567, the cube root of which is nearly 5.207, the relative dist. of Jupiter from the sun, that of the earth being 1; hence 23464.5 × 5.207 = 122179.6515, Jupiter's dist. from the sun in semidiameters of the earth, which multiplied by 3956, gives 483342701.334 miles, the mean dist. of Jupiter from the sun.

† 483342701.3 × 2 × 3.1416 = 3036938860.8 miles, the circumference of Jupiter's orbit; hence 4332d. 14h. 27′ 11″ : 1h. :: 3036938860.8 : 29206 miles, the hourly velocity of Jupiter.

‡ 483342701 — 92825562 (pa. 255 and 258) = 390517139 miles, the dist. of the earth from Jupiter. Now by the rule of three, inversely, 390517139 : 39″ :: 92825562 : 164″073, the apparent diam. of Jupiter at a dist from the earth equal to that of the sun; hence 32′ : 862299 :: 164″073 : 73687.4 miles, the diameter of Jupiter. The diam. may be also determined by trig. in the same manner as that of Mercury, pa. 263, or thus, in the fig. pa. 250, let M represent the earth, AB Jupiter; then the angle AMC = 19″5, and MC = 122179.6 — 23464.5 = 98715 nearly, Jupiter's dist. from the earth in semidiameters of the earth; then cos. 19″5 (log. = 10) : sine 19″4 (log. = 5.9754667) :: 98715 (log. = 4.9943831) : 9.3293 (log. = 0.9698498) the semidiameter of Jupiter in semidiam. of the earth; hence 9.3293 × 2 = 18.6586, Jupiter's diam. which multiplied by 3956, gives 73813.4, Jupiter's diameter in miles.

§ The cube of the diameter of Jupiter, divided by the cube of the earth's diameter $= \frac{73687^3}{7911^3} =$ (log. 2.9074785) 806.7. Or taking 73813 we get 812.3 nearly. A more accurate method for determining the mag. and dist. of Jupiter will be given in treating of his satellites

‖ The relative mean dist. of Jupiter from the sun is 5.207, that of the earth being 1; hence, the effects of light and heat being reciprocally as the distances from the centre whence they are propagated, we have $\frac{5.207^2}{1^2} = 5.207^2 = 27.1$ nearly.

plane, the position of the nodes and perihelia of the planetary orbits, are subject to small variations, which, as *Laplace* remarks, appear, up to the present time, to have increased proportional to the times, their *greater axis, half of which is their mean distances from the sun, appear to be always the same.* These variations only becoming sensible through the lapse of ages, have been therefore called *secular inequalities*. We shall speak more fully of these after the laws of gravity, &c. Small inequalities have been likewise remarked which disturb the periods of the planets, as we have shewn with respect to the sun (pa. 246) but these inequalities are principally sensible in *Jupiter* and *Saturn*, from their mutual actions and respective situations; these particular causes are found to alter at length the elements of their orbits, as we shall afterwards shew.

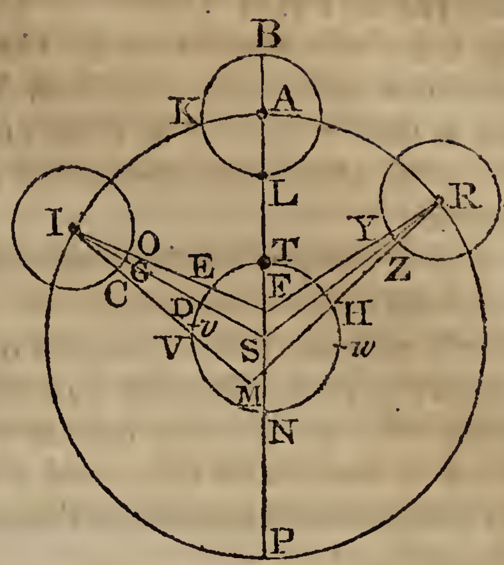
For the *comparative astronomy* as regards Jupiter, or the phenomena that would appear to an eye in Jupiter, see Dr. *Gregory's* Astronomy, b. 6. prob. 5.

OF THE SATELLITES OF JUPITER.

Gallileo, the inventor of the telescope, was the first who discovered that there are *four* small stars or satellites invisible to the naked eye, which constantly accompany Jupiter, and perform their revolutions round him. He called them *Medicea sidera* or *Medicean stars*, in honour of the family of the *Medici*, his patrons. This discovery, which he made on January 8, 1610, was a very important one in its consequences; the eclipses of these satellites furnishing one of the best methods of determining the longitudes of places on the earth. From these eclipses *Roemer* was led to the discovery of the progressive motion of light, from which Dr. *Bradley* was enabled to account for the apparent motion of the fixed stars, called the *aberration.*

The relative situations of these satellites to each other vary every moment; they are observed to oscilate on each side of Jupiter, and it is from the extent of these elongations, that they are classed; that being called the *first* satellite whose oscilation is the least, and so on in order. They are frequently concealed from our view or eclipsed by the shadow of Jupiter, while performing their revolutions from *west* to *east;* and when they move from *east* to *west*, they are observed to pass over his disk, and project a shadow which then describes a chord of this disk. From these phenomena we discover that Jupiter and his satellites are *opake* bodies enlightened by the sun; and also that they revolve round Jupiter in the same direction that Jupiter revolves round the sun. The three first satellites are always eclipsed when they are in opposition to the sun; they are often found to disappear when at some distance from the planet's disk, and the duration of their eclipses is different at different times; the third and fourth sometimes re-appear on the same side of the disk, and the fourth sometimes passes through its opposition without being at all eclipsed. These disappearances are perfectly similar to eclipses of the moon, and are evidently produced by the conical shadow of Jupiter, which, relatively to the sun, is projected behind him; for the satellites are found to disappear in opposition, or on that side of Jupiter where the shadow is projected; they are eclipsed nearest to the disk when the planet is nearest his opposition; and the duration of their eclipses exactly corresponds to the time they should employ in traversing the shadow of Jupiter in his various positions. From the same phenomena it appears, that the planes of their orbits do not coincide with that of Jupiter's orbit; as in that case they would always pass through the centre of Jupiter's shadow, and therefore, at every opposition of the sun, an eclipse would take place, and of the same, or very nearly the same duration. By comparing eclipses at long intervals, observed near the planets' opposition, we have the most accurate methods of determining their motions. It is thus discovered that the orbits of the satellites of Jupiter are nearly circular and uniform; for this hypothesis very nearly corresponds with those eclipses which take place while the planet is in the same position with respect to the sun. When the planes of the orbits pass through the eye, the satellites will appear to describe straight lines, passing through the centre of Jupiter's disk; when this is not the case, they will appear to describe ellipses of which Jupiter is the centre.

The following is, in substance, the method given by *Vince* and other astronomers for determining the *periodic times and distances of Jupiter's satellites*. The mean time of their *synodic* revolutions, or of their revolutions relatively to the sun, is thus found; let the passage of a satellite over the body of Jupiter be observed, when Jupiter is in opposition, and let the time be marked when it appears to be exactly in conjunction with the centre of Jupiter, this will also be the time of conjunction with the sun. Let the same observation be repeated after a considerable interval of time, Jupiter being in opposition, and divide this interval by the number of conjunctions with the sun during this time, and the result will be the satellites *synodic* revolution. This is the revolution on which the eclipses depend, and is therefore the most important to be considered; but on account of the equation of Jupiter's orbit, this will not give the *mean* time of a synodic revolution, unless Jupiter were at the same point of his orbit at both observations; when this is not the case, we must proceed in the following manner.



Let AIPR be the orbit of Jupiter, S the sun in one focus, about which the motion may be considered as uniform, the eccentricity of the orbit, or SF being small. (*Ward's* Theory, pa. 317) Let Jupiter be in A his aphelion, in opposition to the earth at T; and L a satellite in conjunction; let I be the place of Jupiter at his next opposition with the earth at D, and the satellite in conjunction at G; then if the satellite had been in O, it would have been in conjunction with F, or in mean conjunction; therefore before it comes to its mean conjunction, it must describe the angle FIS; this angle is the equation of the orbit, according to *Ward's* or the *simple elliptic hypothesis*, which, as the eccentricity is small, is here used. The angle FIS therefore measures the difference between the mean *synodic* revolutions with respect to F and the synodic revolution with respect to the sun S. Let n = the number of the satellites revolutions as respects the sun; then $n \times 360° - SIF$ = the revolutions as respects F; hence $n \times 360° - SIF : 360° ::$ the time between the two oppositions : to the time of a *mean* synodic revolution about the sun.

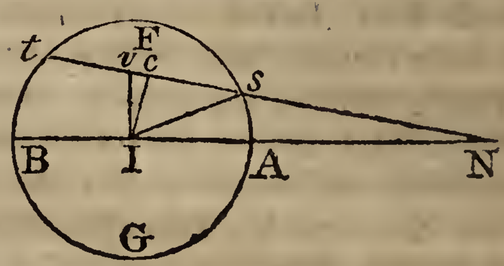
The difference between the times of any two successive revolutions, with regard to S and F respectively, is as the variation of the equation of the orbit, or of the angle FIS; for the satellite being at O at the mean conj. and at G when in conj. with the sun, it is evident that if FIS continued the same, the time of a rev. in respect to S, would equal the time in respect to F, or that of the mean synodic revol. When Jup. is at A, this equation vanishes, and the conjunctions at F and S happen at the same time. When Jup. comes to I, the mean conj. at O takes place after the true conj. at G by the time of describing FIS. This astronomers call the *first* inequality; and this inequality of the times of the intervals of the true conjunctions, affect the times of the eclipses of the satellites.

As it however seldom happens, that a conjunction of the satellite takes place when Jupiter is in opposition, the following method must be used to find the time of a mean revolution, when he is not in opposition. Let the earth be in H when the satellite is in Z, in conjunction with Jup. at R, and again in V when the satellite is in C, in conj. with Jup at I; let RH, IV, be produced until they meet in M; then Jupiter's motion round the earth, in this interval, is the same as if the earth was fixed at M. Now the diff between the true and mean mot. of Jup. is RFI — RMI = FIM + FRM (32 Eucl 1) which shews the excess of the number of mean revolutions in respect to F, above the same number of apparent revolutions in respect to the earth; hence $n \times 360°$

— FIM — FRM : 360° :: the time between the observations : the time of a mean synodic revolution of the satellite If C and Z be at the other side of O and Y, the angles FIM, FRM must be added to $n \times 360°$; if C be on one side and Z at the other, one must be added and the other subtracted according to the circumstances.

From the great brightness of Jupiter, it is difficult to determine accurately the time when the satellite is in conjunction with Jupiter's centre, in its passage over his disk, and hence it is determined by observing the satellites entrance upon the disk, and its going off; but as this cannot be so accurately determined as the times of *immersion* into, and *emersion* from the shadow of Jupiter, the eclipses will therefore determine the time of conjunction more accurately.

Let I be the centre of Jupiter's shadow FG ; N*st* the orbit of a satellite, and N its node upon the orbit of Jupiter, let I*v* be drawn perp. to IN, and IC to N*t* ; and when the satellite comes to *v*, it is in conj.* with the sun. The time of this conjunction is found thus ; the immersion at *s* and emersion at *t* of the 2d, 3d and 4th satellites, may be sometimes observed, the middle of the time between which will give the middle of the eclipse at *c* ; and hence NI and the ∠ N being given, *cv* may be found, and therefore the time of conjunction at *v*. If both the immersion and emersion cannot be observed, let the time of either be observed, and after a long interval, let the time of the same be again observed, when an eclipse happens in the same situation with respect to the node, as nearly as possible ; from the interval of these times, the time of a revolution will be obtained.



M. *Cassini*, by these methods, found the times of the four satellites mean *synodic* revolutions to be as follow, viz. *First* 1d. 18h. 28′ 36″, *Second* 3d. 13h. 17′ 54″, *Third* 7d. 3h. 59′ 36″, and *Fourth* 16d. 18h. 5′ 7″.

Hence it follows that 247 revolutions of the first satellite are performed in 437d. 3h. 44′ ; 123 revolutions of the 2d in 437d. 3h. 41′ ; 61 revolutions of the 3d in 437d. 3h. 35′ ; and 26 revolutions of the 4th in 435d. 14h. 13′. It appears therefore that after an interval of 437 days, the three first satellites return to their relative situations within 9′.

The synodic revolutions of the satellites and the mean motion of Jupiter being given, their *sidereal*, or *periodic* revolutions may be easily found thus ; let $x°$ be the mean angle described by Jupiter during a synodic revolution of the satellite, then in the return of the satel. to the mean conj. it will have to describe this an-

* A satellite is said to be in conjunction, both when it is between the sun and Jupiter, and when it is opposite to the sun ; the latter is called superior, the former inferior conjunction.

gle to complete its periodic revolution ; hence we have this proportion $360° + x°$: 360° :: time of a synodic revolution : time of a periodic revolution. The periodic revolutions of the satellites are therefore as follow ; the sidereal revolution of the *First* is 1d. 18h. 27′ 33″ ; of the *Second* 3d. 13h. 13′ 42″, of the *Third* 7d. 3h. 42′ 43″ ; and of the *Fourth* 16d. 16h. 32′ 8″. *Newton* (Prin. b. 3. phen. 1.) makes them nearly the same. *Laplace* gives their sidereal revolutions as follow ; the *First* 1.769137787069931 days = 1d. 18h. 27′ 38″ 5 ; the *Second* 3.551181016734509 days = 3d. 13h. 13′ 42″ ; the *Third* 7.154552807541524 days = 7d. 3h 42′ 33″ 3, and the *Fourth* 16.689019396008634 days = 16d. 16h. 32′ 11″ 27.

At the beginning of 1700 the mean longitudes of the satellites were as follow : *First* 77° 15′ 51″ 084, of the *Second* 75° 13′ 37″ 948, of the *Third* 164° 12′ 16″ 38, and the *Fourth* 227° 50′ 20″ 58.

The distances of the satellites being compared with the duration of their revolution, the same proportion has been observed as in the primary planets, and hence the dist. of one being obtained, the distances of the others may be therefore found, *the squares of the periodic times being as the cubes of their mean distances from Jupiter.* The greatest heliocentric elongation of the 4th satel. from Jupiter's centre, was taken by M. *Pound* with a micrometer in a telescope 15 feet long, and at the mean distance of Jupiter from the earth, was found about 8′ 16″, that of the 2d. taken with a micrometer in a telescope 123 feet, was found = 4′ 42″, and from the periodic times the others were found 2′ 56″ 47‴ and 1′ 51″ 6‴ respectively, and from *Newton's* determination Jupiter's diam. at its mean dist. being taken $37\frac{1}{4}$″, the distances of the satellites are found to be 5.965, 9.494, 15.141 and 26.63 semidiameters of Jupiter, respectively. (See b. 3, prin. phen. 1.) *Newton* also remarks that with the 123 feet telescope, Jupiter's diameter reduced to the earth's mean distance proved always less than 40″, never less than 38″, and generally 39″. *Laplace*, taking the diameter of Jupiter's equator (120″ 37) 39″ nearly, finds the mean distances of the satellites from his centre, the mean distance of Jupiter from the being taken 1, as follow ; 1st. 5.6973, 2d. 9.065898, 3d. 14.461628, and 4th 25.436. The distances may be also thus found. When a satellite passes over the middle of Jupiter's disk, let the whole time of its passage be observed, then, *the time of a revolution : time of passage over the disk* :: 360° : *the arc of its orbit corresponding to the time of its passage over the disk ; hence, sine of half this arch : rad. :: Jupiter's semidiameter : the distance of the satellite.* See the distances thus determined by M. *Cassini*, also by *Borelli* and *Townley* given by *Newton*. (Pa. 207 Motte's trans.)

By knowing the greatest elongations of the satellites in minutes and seconds, their distances from the centre of Jupiter, compared with Jupiter's mean distance from the earth is found by saying, *sine of the satellite's greatest elongation : radius : distance of the satellite from Jupiter : the mean distance of Jupiter from the earth.*

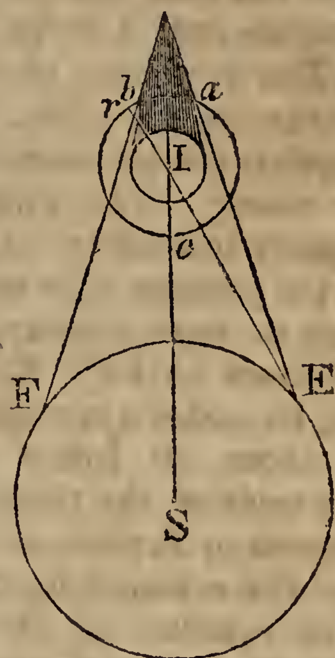
The distances of Jupiter and the sun from the earth may be also compared with each other, by knowing the position of the satellites as seen from Jupiter's centre, which is easily determined from their periodic times, on supposition that they revolve in circular orbits round him. Let the total duration of an eclipse of the third satellite, for example, be observed ; and at the middle of the eclipse the satellite is nearly in opposition to the sun, as seen from Jupiter's centre. Now its position in the heavens, as observed from Jupiter's centre, is the same as this centre seen from the sun ; and from the periodic time of the sun, or from direct observation, the position of the earth as seen from the sun's centre is given ; hence if a triangle be conceived to be formed by the right lines which join the centres of the sun, the earth, and Jupiter, the rectilinear distance from Jupiter to the earth, and also to the sun, in parts of the distance of the sun and earth, will be given, at the instant of the middle of the eclipse. By this method it is found that the distance of Jupiter, when his apparent diameter is 38″ 88, is at least *five* times that of the sun from us. At this distance the diameter of the earth would not appear under an angle of 3″ 56, and hence the magnitude of Jupiter is at least 1000 times greater than the earth's, from these data.

The apparent diameters of the satellites being insensible, their magnitudes cannot, therefore, be exactly measured. The attempt to appreciate it has been made by observing the time they take to penetrate the shadow of the planet ; but the various powers of telescopes and other circumstances, have produced a great discordance in these observations.

Cassini suspected that the satellites had a *rotary motion on their axis*, as in their passage over Jupiter's disk, they were sometimes visible, at other times not : he therefore conjectured that they had spots on one side and not on the other, and that they were rendered visible in their passage when the spots were next the earth At different times they likewise appear of different magnitudes and of different brightness. The 4th appears generally the smallest, but sometimes it appears the largest ; and the diameter of its shadow on Jupiter, appears sometimes greater than the satellite The 3d also appears to vary its magnitude, and the same is observed to take place with regard to the other two. From similar circumstances also Mr. *Pound* concluded that they revolved on their axis. The comparative brightness of the satellites ought to afford information concerning their rotary motion. Dr. *Herschel* has observed, that they surpass each other alternately in brilliance, a circumstance that enables us to judge of their respective light. The relation of the maximum and minimum of their light with their mutual positions, has persuaded him that they revolve upon themselves like our moon, in the period equal to the duration of their respective revolutions round Jupiter. (*Phil. trans.* 1797.)

Maraldi had already found the same result for the 4th satellite, from the returns of the same spot observed on its disk, in its passage over the planet.

The eclipses of Jupiter's satellites deserve to be particularly considered, not only for themselves, but also as they serve to settle the longitude of places, and to explain one of the most interesting results in modern philosophy; that is the motion or propagation of light in a determined time, as we have before noticed.



Let S be the sun, EF the orbit of the earth, I Jupiter, *abc* the orbit of one of his satellites. When the earth is at E before Jupiter's opposition, the spectator will observe the *immersion* at *a*; but if the satellite be the 1st, from its nearness to Jupiter, the *emersion* is never visible, the satellite being then always behind the body of Jupiter; the other three satellites may have both their immersions and emersions visible; this will, however, depend on the earth's position. When the earth is advanced to F after opposition, the emersion of the 1st is then seen, but the immersion can never be seen in this position; but both the emersion and immersion of the other three *may* be visible at this time. Let EI*r* be drawn; then denoting the centre of the shadow, or the middle point between *a* and *b* by *s*, *sr*, the distance of the centre of the shadow from the centre of Jupiter, referred to the orbit of the satellite, is measured at Jupiter by *sr*, or by the angle *s*I*r*, or EIS. The satellite may be hidden behind Jupiter at *r* without being eclipsed, which is called an *occultation*. When the earth is at E, the conjunction of the satellite happens *later* at the earth than at the sun; but *sooner* when the earth is at F.

The diameter of Jupiter's shadow, at the distance of any of the satellites, is best found by observing the time of an eclipse, when it happens at the node, at which time the satellite passes through the centre of the shadow; for the time of a synodical revolution : the time the satellite is passing through the centre of the shadow :: 360° : the diameter of the shadow in degrees. But when in the nodes, the immersion and emersion of the first and second satellite cannot be seen. Hence astronomers compare the immersions some days *before* the opposition of Jupiter with the emersions some days *after*, and then the number of synodical revolutions being known, the time of the transit through the shadow, and thence the corresponding degrees are found. But from the eccentricity of some of the orbits, the times of the central transit must vary. Example. The second satellite is sometimes found to pass through the centre of the shadow in 2h. 50′, and sometimes to be 2h. 54′ in passing; this indicates an eccentricity.

The duration of the eclipses being very unequal, shews that the orbits are inclined to the orbit of Jupiter, sometimes the fourth satellite passes the opposition without being eclipsed, as noticed before. The duration of the eclipses depends on the situation of the nodes with respect to the sun, as in a lunar eclipse ; when the line of the nodes passes through the sun, the satellite will pass through the centre of the shadow ; but as Jupiter revolves round the sun, the line of the nodes will be carried out of conjunction with the sun, and the times of the eclipse will be shortened as the satellite only describes a chord of the section of the shadow.

The *ellipticity* of the orbit of the first satellite, as *Laplace* remarks, is insensible, its plane nearly coincides with the equator of Jupiter, the inclination of which, to the planet's orbit is (4° 4444) 4° nearly. The ellipticity of the orbit of the second satellite is equally insensible. Its inclination to Jupiter's orbit varies, as well as the position of its nodes. These variations according to *Laplace*, may be represented nearly by supposing the orbit of satellite inclined about (5182″) 27′ 58″ 968 to the equator of Jupiter, and giving its nodes a retrograde motion on this plane, the period of which is about 30 Julian years. A slight ellipticity is observable in the orbit of the third satellite. The extremity of its greater axis nearest to Jupiter, and which is called *perijove*, has a direct motion, and the eccentricity of its orbit is subject to perceptible alterations. The *equation of the centre*, towards the end of the last century, was at its maximum nearly = (2661″) 14′ 22″ 184, then diminishing, about 1775, it was at its minimum = (759″) 4′ 5″ 916. The variations in the inclination of its orbit, and the position of its nodes, may be nearly represented by supposing the orbit inclined about (2244″) 12′ 7″ 056 to the equator of Jupiter, and the period of the retrograde motion of its nodes = 137 years. The orbit of the 4th satellite has a very sensible ellipticity, its perigee has a direct motion of about (7852″) 42′ 24″ 048. Its inclination to the orbit of Jupiter is (272″) 1′ 28″ 128, in consequence of which this satellite passes behind Jupiter relatively to the sun without being eclipsed. The cause of these irregularities, &c. is explained in ch. 6. b. 4, *Laplace's* Astronomy.

The mean motion of the satellites are such, that the motion of the first satellite plus that of the third, is nearly equal three times that of the second. Hence the same proportion evidently exists between their mean synodical motions.

The mean longitudes, whether synodical or sidereal, of the three first satellites seen from Jupiter's centre is such, that the motion of the first satellite, minus three times that of the second plus, twice that of the third is nearly, or exactly, equal the semicircumference.

The periods and the laws of the principal inequalities in these satellites are the same. The inequality of the first advances or retards its eclipses (233″) 1′ 15″ 492 in time, at its maximum. This is found to disappear when the two first satellites seen from the centre of the planet are in opposition to the sun at the same time ; it afterwards increases, and is the greatest possible when the first satellite at its opposition is 45° more advanced than the second. It is again nothing when the satellite is 90° more advanced ; after this it takes a contrary sign and retards the eclipses, it augments to 135° distance between the satellites where it is at its negative max. It then diminishes and disappears, when 180° distance. In the second half of its circumference it follows the same laws as in the first. Hence in the first satellite there is found an inequality of (5258″) 28′ 23″ 592 at its max. and proportional to the sine of twice the excess of the mean longitude of the first satellite above that of the second, an excess = the difference of the mean synodic longitudes of the two satellites. The period of this inequality is not four days, and yet it transforms itself into a period of 437d. 18h. in the eclipses of the first satellite

Let the two first satellites be supposed to set out together from their mean oppositions to the sun. At every circumference which the first satellite describes, in consequence of its mean synodic motion, it will be in the mean opposition. If we now imagine a fictitious star, whose angular motion may be equal to the excess of the mean synodic motion of the first satellite above twice that of the second, then twice the difference of the mean synodic motions of the two satellites will be, in the eclipses of the first, equal to the multiple of its circumference. *plus* the motion of the fictitious star. The sine of this last motion will be then proportional to the inequality of the first satellite in its eclipses, and may represent it. Its period is equal to the duration of the motion of the fictitious star, which from the mean synodic motions is 437.75 days.

The inequality of the second satellite follows a law similar to that of the first, with this difference, that it has always a contrary sign. It accelerates or retards the eclipses (1059″) 5′ 43″ 116 in time,* at its maximum. It disappears when the two first satellites are at the same time in opposition to the sun. It then retards more and more the eclipses of the second satellite, until the two satellites are 90° distance from each other, at the instant of these phenomena. This retardation diminishes and becomes nothing when the satellites are distant 180° ; beyond which term the eclipses advance in the same manner as they were before retarded.—From these observations it has been concluded, that there is an inequality of (11923″) 1° 4′ 23″ 052 at its maximum in the motion of the second satellite proportional (but affected with a contrary sign) to the sine of the excess of the mean longitude of the first satellite above that of the second, which is equal the difference of the mean synodic motions of the two satellites. If we conceive

* These quantities being principally taken from *Laplace,* his measures are therefore retained, and the corresponding measures used in this country given ; but in his seconds of time, he probably retains the late French division of the calendar (see the note page 308) if so, 1059″=15′ 34″976.

the two satellites to set out together as before, the second satellite will be in its mean opposition at each circumference, it will describe in consequence of its mean synodical revolution. If as before, a star be taken whose angular motion may be equal the excess of the mean synodical motion of the first satellite above twice that of the second, then the difference of the synodical motion of the two satellites will be, in the eclipses of the second, equal to a multiple of its circumference, plus the motion of the fictitious star The inequality of the second satellite in its eclipses, will then be proportionable to the sine of the motion of this imaginary star. Hence the period and law of this irregularity is the same as that of the first satellite

The influence of the first on the second satellite is very probable ; but if the third satellite produced in the motion of the second an inequality similar to that which the second seems to produce in the motion of the first, that is proportional to the sine of double the difference of the mean longitude of the second and third satellites, this new inequality would confound itself with that of the first satellite; for, from the relation which the longitudes of the three first satellites have to each other, as before remarked, the difference of the mean longitudes of the two first, is equal to half the circumference, plus twice the difference of the longitude of the second and third satellites ; so that the sine of the first difference is the same as the sine of twice the second difference with a contrary sign. The inequality produced by the third satellite in the motion of the second, would therefore have the same sign, and follow the same law, as the inequality observed in this motion ; hence it is probable that this inequality is the result of two inequalities depending on the first and third satellites. If by the succession of ages, says *Laplace*, the relation between the mean longitudes of these three satellites should cease to exist, these two inequalities, at present confounded, would separate, and their respective values might be known. But according to observation this relation should subsist for a long period *Laplace* shews (in the fourth book of his astronomy) that it is rigorously exact.

Finally, the inequality relating to the third satellite in its eclipses, compared with the respective positions of the second and third satellites, offers the same proportion as the inequality of the two first satellites. Hence in the motion of the third satellite there exists an inequality proportional to the sine of the excess of the mean longitude of the second satellite above that of the third, which at its maximum is (827″) 4′ 27″948.

If we suppose a star, whose angular motion may be equal to the excess of the mean synodical motion of the second satellite, above twice the mean synodical motion of the third, the inequality of the third satellite will be, in its eclipses, proportional to the sine of the motion of this fictitious star. Now in consequence of the proportion which exists between the mean longitudes, the sine of this motion is, exclusive of the sign, the same with the motion of the first fictitious star which we have considered. The inequality of the third satellite in its eclipses, has the same periods, and follows the same laws, as those of the two first satellites.

These are the principal inequalities in the motions of the three first satellites of Jupiter. Dr. *Bradley* first remarked them, but *Wargentin*, whose tables of these satellites are used in calculating the Nautical Almanac, has since investigated them with the greatest accuracy.

From these tables of *Wargentin* the *configurations*, or the *proportional distances*, of the satellites of Jupiter and their eclipses are calculated in the Nautical Almanac.

The eclipses of Jupiter's satellites, besides their being useful at sea (see pa. 53) are, as Dr. *Maskelyne* remarks, observed by astronomers on land, as well in order to provide materials for improving the theories and tables of their motions, as for the sake of comparison with the corresponding observations, which may be made by persons in different parts of the globe, whereby the longitude of such places will be accurately ascertained. He also remarks, that it is to be lamented that persons who visit distant countries, are not more diligent to multiply observations of this kind ; for want of which, the observations made by astronomers in established observatories lose half their use, and the improvement of geography is retarded.* The eclipses set down in the *Ephemeris*, pa. 3 of the month, will serve to shew the times when these observations should be attended to ; having first from the difference of longitude in time, if he be under any meridian different from *Greenwich*, found the apparent time at which the eclipse will happen at his meridian nearly, that at *Greenwich* being given. (See prob. 6, page 58.) He must also have his watch or clock, previously, well regulated, by some of the methods given in parts first or second, to know the *mean* time exactly at which the observation is to be made Equal altitudes of the sun or stars is, perhaps, the best method.†

* As so useful a service can be rendered to the public, and to science in general, from multiplied observations of this and a similar nature, it is truly to be regretted that *public observatories* are not erected in some of our principal cities in the United States, and that there are to be found only one solitary observatory near *Boston*, which is of any credit to the country. It is, however, to be hoped, that a point of such extensive utility and importance to a country, will not be much longer overlooked by an enlightened public ; and that *New-York*, which is so well situated for, and calculated to support a public undertaking of this nature, will not be backward in setting the example to her sister states.

† In prob. 22, pa. 83, we have shewn how a watch or clock may be regulated, and how its rate of going may be ascertained ; but as this point is extremely useful in determining the longitude, &c. we shall here insert the following observations collected from *Vince's* astronomy, for finding the *rate of going*, and from thence the *mean* rate, &c. Suppose, for instance, that the rate of a watch for thirty days be examined, and that on some of those days it gains and on others loses ; if the quantities which it has gained be added, and found for example to amount to 17″, and the quantities

Dr. *Maskelyne* further remarks, that, the observer, being in a place whose longitude is well known, should be settled at his telescope three minutes before the expected time of an immersion or emersion of the three first satellites; and ten minutes before that of the fourth. If the longitude of the place be very uncertain, he must begin to look out for the eclipse proportionally sooner. Thus, if the longitude be uncertain to 3°, answering to 12′ of time, he ought to fix himself at his telescope 12′ sooner than is mentioned above, &c. However, when he has observed one eclipse of any satellite, and thereby found the error of the tables, he may allow the same correction to the calculations of the *Ephemeris* for several months, which will advertise him nearly of the time of expecting the eclipses of the same satellite, and dispense with his attending so long. (See pa. 178.)

The immersions or emersions of any satellite being carefully observed in any place according to *mean* time (the eclipses of the satellites being now computed to *mean* time, in the Nautical Almanac, instead of *apparent* time as formerly) the longitude from *Greenwich* is found immediately, by taking the difference of the observed time from the corresponding time shewn in the *Ephemeris*, which must be turned into degrees, &c. and will be east or west from *Greenwich*, as the time observed is more or less than that of the *Ephemeris*.

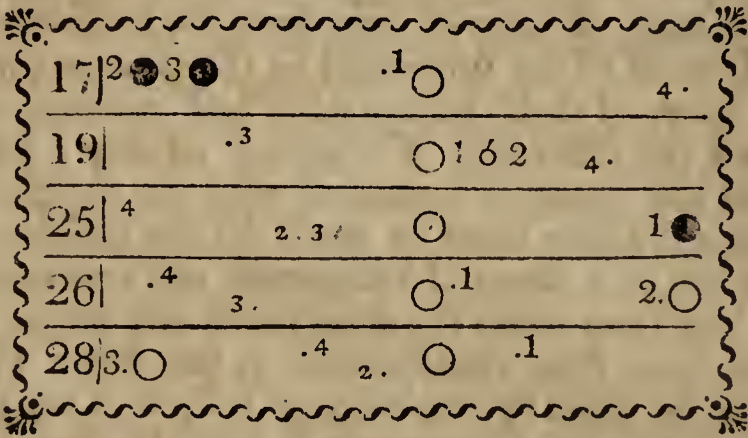
Dr. *Maskelyne* also observes that a corresponding observation of an eclipse of a satellite of Jupiter, made under a well known meridian, is to be preferred to the calculations of the *Ephemeris* for

which it has lost to 13″, then the difference 4″ is the *mean* rate of gaining for 30 days; hence 0″ 133 is the *mean daily rate* of gaining. Or the daily rate may be thus obtained; take the *difference* between what the watch was too fast or too slow on the first and last days of observation, if it be too fast or too slow on each day; but their *sum* if too fast on one day, and too slow on the other; and divide by the number of days between the observations. (See *Wales's method of finding the longitude at sea.*) To find the time at the place of trial at any future period by this watch, the time gained or lost by the watch must be obtained; then 0″ 133 × the number of days from the end of the trial, being the quantity gained according to the above mean rate of gaining, and the true time affected by the error at the end of the trial, is supposed to be obtained. This will, however, only diminish the probable error of the watch; as the temperatures of the air, and the imperfection of the workmanship, will cause some change in the *rate* of the watches going. Hence when the watch is used to determine the *longitude* at sea, the observer when he goes ashore, if time permits, should compare his watch, for several days, with the *mean* time deduced from the sun's or a star's altitude, to determine its rate of going more correctly. And whenever he comes to a place whose longitude is known, the watch may be corrected and set to Greenwich time. Without these or similar means of adjusting the watch, its error in long voyages may be very considerable, and therefore the longitude deduced from it proportionably erroneous; but in short voyages, and in carrying on the longitude from one known place to another, or in keeping the longitude from that which is deduced from a lunar observation until another is obtained, the watch is undoubtedly very useful; and hence in navigation and geography it may be rendered of great service.

comparing with an observation made in a meridian whose longitude is required; but if no corresponding observation can be obtained, as is frequently the case, it will be best to find what corrections the calculations of the *Ephemeris* required, by the nearest observations to the given time that can be obtained; which corrections applied to the calculation of the given eclipse in the *Ephemeris*, renders it almost equivalent to an actual observation. In the actual making the observations, the observers should be furnished with the same kind of telescopes, should make allowance for the different states of the air, and remove themselves from all warmth and light, for a little time, before making the observation, that the eye may be reduced to a proper state; and this precaution is also to be attended to, when the difference between the apparent and true times of immersion and emersion is to be settled. If two telescopes show the disappearance or appearance of the satellite, at the same distance of time from the immersion or emersion, the difference of the times will be the same as the difference of the true times of their immersions and emersions, and will therefore shew the difference of longitude exactly; but if the observed time at one place be compared with the computed time at another, then allowance must be made for the difference between the apparent and true times of immersion and emersion, in order to obtain the true time when the observation was made, to compare with the true time from computation at the other place. This difference may be found by observing an eclipse at any place whose longitude is known, and comparing it with the time by computation. At an immersion when the satellite enters the shadow, it grows fainter and fainter, until at last the quantity of light is so small, that it becomes invisible, even before it is immersed in the shadow; hence the instant that it becomes invisible, will depend on the quantity of light which the telescope receives, and its magnifying power. Therefore the instant of the disappearance of a satellite will be later, the better the telescope is, and its emersion will appear sooner.

The apparent position of Jupiter's satellites with respect to each other, and to Jupiter, or their *configurations*, are given in pa. 12 of the month in the Nautical Almanac, at such an hour of the evening or night, as they are most likely to be observed, and serve to distinguish the satellites from one another. Jupiter is distinguished by the mark O, and the satellites by points with figures annexed, the 1 signifying the 1st satellite, 2 the 2d, &c. When the satellite is approaching towards Jupiter, the figure is put between Jupiter and the point, but when receding from him, the figure is put on the other side of the point. The satellites are in the superior parts of their orbits, that is above the orbit of Jupiter or furthest from the earth, when they are marked to the right hand, or west of Jupiter approaching him; or to the left hand or east of Jupiter receding from him; but are in the inferior parts of their orbits or nearest to the earth when they are marked to the right hand or west of Jupiter receding from him, or to the left or east of Jupiter approaching him. The satellites that are above the orb of Jupiter has north latitude, those below south latitude — The cypher 0, sometimes annexed to the figure of the satellite towards the margin, signifies that it is invisible on the face of Jupiter, and the black mark ●, signifies that it is invisible, being eclipsed in Jupiter's shadow, or behind Jupiter, eclipsed by his body.

The following exhibits the configurations of the satellites of Jupiter at 4 o'clock in the morning, *Greenwich time* (or 3′ 56″ after eleven o'clock at night, in New-York) on the following days in September, 1813, viz. 17, 19, 25, 26, and 28, as given pa. 12 of the month in the Nautical Almanac, an explanation of which will render that page intelligible to young students, for any other year and month.



On the 17th day of the month, given above, the second and third satellites distant from Jupiter as represented, will be eclipsed at 4 o'clock in the morning, at Greenwich, or at 3′ 56″ after 11 at night, in New-York ; the first satellite is to the left hand of Jupiter, and in north latitude, the fourth satellite to the right and in south latitude.

On the 19th the third satellite at 4 o'clock, at Greenwich, will be to the left of Jupiter in north latitude, or higher than Jupiter ; the first and second will be in conjunction, or appear as one, on the right hand of Jupiter, and the fourth will be in south latitude further from Jupiter.

On the 25th the fourth satellite will appear to the left of Jupiter above his orbit, or in north latitude, and approaching towards Jupiter, the second and third will be also to the left, in south latitude, approaching towards Jupiter, the first will be eclipsed to the right hand of Jupiter.

On the 26th the fourth will be above Jupiter's orbit, the third below it, both to the left ; the first will be near the body of Jupiter, in south latitude and receding from him ; the second will appear like a bright spot on the disk of Jupiter.

On the 28th the third is invisible on the disk of Jupiter, the fourth in north, and the second in south latitude, both to the left ; the first will appear to the right of Jupiter above his orbit, and receding from Jupiter.

CHAP. VIII.

OF SATURN,

HIS SATELLITES AND RING.

SATURN is the next planet in order, in the solar system, after Jupiter ; he shines with a pale, feeble light, being the most remote from the sun of any of the planets that are visible without a telescope. When viewed through a good telescope, the singularity of his appearance engages the attention of the young astronomer ; being surrounded by a *ring*, the only phenomenon of the kind observed in the Solar System.

As the periods of the superior planets depends on their *oppositions* to the sun, we shall here shew how to find these oppositions, and from thence the periodic times, this article being omitted in the preceding chapters.

To determine the time of opposition ; Let the time when the planet is nearly in that situation be observed, the time at which it passes the meridian (note to prob. 8) and also its right ascension (prob. 1. pa. 192) let its meridian altitude be also found, and likewise the meridian altitude of the sun ; and let the observations be repeated for several days. From the observed meridian altitudes let the declinations be found (see the note pa. 140) and from the right ascensions and declinations compute the latitudes and longitndes of the planet (note to prob. 3, pa. 195) and the longitudes of the sun. Then let a day be taken when the difference of their longitudes is 180°, and reduce the sun's longitude on that day, found from observation when it passes the meridian, to the longitude found at the time (t) the planet passed, by finding from observation or computation at what rate the longitude then increases. Now as the planet is retrograde in opposition, the difference between the longitudes of the planets and sun increases by the sum of their motions. Hence the following rule : As sum (S) of their daily motions in longitude : the difference (D) between 180° and the difference of their longitudes reduced to the same time (t)* (subtracting the sun's longitude from the planet's to obtain the difference reckoned from the sun according to the order of the signs) :: 24h. : the internal between that time (t) and the time of opposition. This interval added to, or subtracted from that time (t) according as the difference of their longitudes was greater or less than 180°, gives the time of opposition. If this be repeated for several days, and the mean of the whole taken, the time will be obtained more accurately. If the time of opposition, found from observation, be compared with the time by computation from

* The diff. shewing how far the planet is from oppos. The proportion is evident from this principle, that the sun approaches the star by spaces proportionable to the times ; hence the spaces S and D must be as the time 24h. and the time (t) to opposition.

the tables, the difference will be the error of the tables, which may serve as means of correcting them.

Example. M. de la *Lande*, on October 24, 1763, observed the difference between the right ascension of β *Aries*, and *Saturn*, which passed the meridian 12h. 17′ 17″ apparent time, to be 8° 5′ 7″, the star passing first. Now the apparent right ascension of the star at that time was 25° 24′ 33″ 6 ; hence the apparent right ascension of Saturn was 1s. 3° 29′ 40″ 6 at 12h. 17′ 17″ apparent time, or 12h. 1′ 37″ mean time. On the same day he found from observation of the meridian altitude of Saturn, that his declination was 10° 35′ 20″ N. His longitude is therefore found, from his right ascension and declination = 1s. 4° 50′ 56″, and latitude 2° 43′ 25″ S. At the same time the sun's longitude was found by calculation to be 7s. 1° 19′ 22″, which taken from Saturn's longitude gives 6s. 3° 31′ 34″ ; hence Saturn was 3° 31′ 34″ beyond opposition, but being retrograde, will afterwards come into opposition. From the observations made on several days at that time, Saturn's longitude was found to decrease 4′ 50″ in 24h. and by computation the sun's longitude increased 59′ 19″ in the same time, the sum of which is 64′ 49″ ; hence 64′ 49″ : 3° 31′ 34″ :: 24h. : 78h. 20′ 20″, which being added to October 24, 12h. 1′ 37″, gives 27d. 18h. 21′ 57″ for the time of opposition. The longitude of Saturn at the time of opposition may therefore be found by saying 24h. : 78h. 20′ 20″ :: 4′ 50″ : 15′ 47″, the retrograde motion of Saturn in 78h. 20′ 20″, which taken from 1s. 4° 50′ 56″, leaves 1s. 4° 35′ 9″, the longitude of Saturn at the time of opposition. In like manner the sun's longitude may be found at the same time, in order to prove the opposition ; for 24h. : 78h. 20′ 20″ :: 59′ 59″ : 3° 15′ 47″, which added to 7s. 1° 19′ 22″, the sun's longitude at the time of observation gives 7s. 4° 35′ 9″ for the sun's longitude at the time of opposition, which is exactly opposite that of Saturn. Hence the latitude of Saturn may be found at the same time, by observing in like manner the daily variation, or by computation from the tables, the elements of its motions being known, and the tables constructed : whence it appears, that in the interval between the times of observation and opposition, the latitude had increased 6″, and was therefore 2° 43′ 31″. Thus, *the times of opposition of all the superior planets are found.*

Vince remarks, that from the conjunctions* and oppositions of the planets, their *mean* motions could be readily determined, if the place of the aphelia, and eccentricities of their orbits were previously known ;† for then the equation of the orbit could be found (pa. 313) and the *true* reduced to the *mean* place ; hence the *mean* places being determined at two different times, the mean motion corresponding to the interval between these times will be

* See the method of determining the conjunctions of the inferior planets, pa. 267. See also pa. 260 and pa. 280.

† The method of finding these is given pa. 261, &c.

given. The place of the aphelion is however best determined from the mean motion. To determine, therefore, the mean motion, independent of the place of the aphelion, such oppositions or conjunctions must be sought, as take place nearly in the same points of the heavens; for the planet being then very nearly in the same point of its orbit, the equation will be very nearly the same at each observation, and therefore the comparison between the true places will be nearly a comparison of their mean places. The equation must be considered, if it differ much in the two observations. Now by comparing the modern observations, the time of a revolution will be nearly obtained; and then, by comparing the modern with the ancient observations, the mean motion may be accurately determined; for any error, being divided among a great number of revolutions, will become very small with respect to one revolution. The following example (*M. Cassini Elem. d'Astron.* pa. 362 or *Vince*) will best illustrate this article.

On September 16, 1701, *Saturn* was in opposition at 2h. when the sun's place was *Virgo* 23° 21′ 16″, and Saturn therefore in *Pisces* 23° 21′ 16″, with 2° 27′ 45″ S. latitude On September 10, 1730, the opposition was at 12h. 27′, and Saturn in *Pisces* 17° 53′ 57″, with 2° 19′ 6″ S. latitude. On September 23, 1731, the opposition was at 15h. 51′, in *Aries* 31′ 50″, with 2° 36′ 55″ S. latitude. Now the interval of the two first observations was 29 years (of which 7 were bissextiles) wanting 5d. 13h 33′; and the interval of the two last was 1y. 13d. 3h. 24′. The difference of the places of Saturn was also, in the two first observations, 5° 27′ 19″, and in the two last 12° 36′ 53″. Hence, from these observations, Saturn moved over 12° 36′ 53″ in one year; therefore 12° 36′ 53″ : 5° 27′ 19″ :: 1y. 13d. 3h. 24′ :: 163d. 12h. 41′, the time of moving over 5° 27′ 19″ very nearly, Saturn being nearly in the same part of its orbit, and will therefore move nearly with the same velocity; this therefore, *added* to the interval between the two first observations (as Saturn, at the 2d observation, wanted 5° 27′ 19″ from being up to his place at the 1st) gives 29 years, 164d. 23h. 8′ for the time of one revolution. Hence 29y. 164d. 23h. 8′ : 365d :: 360° : 12° 13′ 23′ 50‴, the *mean* annual motion of Saturn in a common year of 365 days, that is, on supposition that it moves uniformly. This being divided by 365, gives 2′ 0″ 28‴ for Saturn's mean daily motion.*

* The mean motion thus determined will be sufficiently accurate to determine the number of revolutions which the planet must have made, when we compare the modern with the ancient observations, in order to determine the mean motion more accurately. *Delambre* in his tables (tab. 142) makes Saturn's mean mot. in a Julian or common year 12° 13′ 36″ 8; hence 12° 13′ 36″ 8 : 360° :: 1y. : 161y. 19h. 20′ 18″ 07 the time of Saturn's revolution. He makes Saturn's mot. for a *day* 2′ 0″ 6, for an *hour* 5″, and for a *minute* 0″ 1. These tables of *Delambre* are calculated from the theory of *Laplace*, and examined from a multitude of observations. See the demonstrations of the principles in the memoires of the Academy for 1785 and 1786.

The most ancient observation which we have of the opposition of Saturn, was on March 2, in the year 228, before Jesus Christ, at 10 o'clock in the afternoon, in the meridian of Paris, Saturn being then in *Virgo* 8° 23′, with 2° 50′ N. latitude. February 26, 1714, at 8h 15′, Saturn was found in opposition in *Virgo* 7° 56′ 46″, with 2° 3′ N. latitude. From this time 11 days must be subtracted (if the observation were made after the year 1800, 12 days should be subtracted, &c. see page 16) to reduce it to the same stile as at the first observation, and therefore this opposition happened on February 15, at 8h. 15′; the difference between these two places was, therefore, only 26′ 14″. Also, the opposition in 1715, was on March 11, at 16h. 55′, Saturn being then in *Virgo* 21° 3′ 14″, with 2° 25′ N latitude. Now between the two first observations, there were 1942 years (of which 485 were bissextiles) wanting 14d 16h. 45′, that is, 1943 common years, and 105d. 7h 15′ over. The interval between the times of the two last oppositions was 378d 8h. 40′, during which time Saturn had moved over 13° 6′ 28″; hence 13° 6′ 28″ : 26′ 14″ :: 378d. 8h. 40′ : 13d. 14h. which added to the time of oppos. in 1714, gives the time when Saturn had the same longitude as at the oppos. in the year 228 before J. C. This being therefore added to 1943 com. years, 105d. 7h. 15′, gives 1943y. 118d. 21h. 15′, in which interval Saturn must have made a certain complete number of revolutions. Now having found above from the modern observations, that the time of one revolution must be nearly 29 com. y. 164d. 23h. 8′, it follows that the number of rev in the above interval was 66; this interval being therefore divided by 66, gives 29y 162d. 4h. 27′ for the time of one revolution. From comparing the oppositions in the years 1714 and 1715, the true mot. of Saturn appears to be very nearly equal his mean mot. which shews that the oppositions were observed very near the mean distance, and that therefore, the mot. of the aphel. could not have caused any considerable error in the determination of the mean mot. Hence the mean annual mot. is 12° 13′ 35″ 14‴, and the mean daily mot. 2′ 0″ 35‴. Dr *Halley* makes the annual mot. to be 12° 13′ 21″. *M de Laplace* and *Delambre* make it 12° 13′ 36″8. As the revolution here determined is, that in respect to the *long.* of the planet, it must be a *tropical* revolution; hence to find the *sidereal* rev. we have this proportion, 2′ 0″ 35‴ : 24′ 42″ 20‴ (the precession in the time of a tropical revol. see pa 246 and 305) :: 1 day : 12d. 7h. 1′ 57″, which added to 29y 162d. 4h 27′, gives 29y. 174d 11h. 28′ 57″, the length of a sidereal year of Saturn. From more correct observations *Vince* makes it 29y. 174d. 1h. 51′ 11″2. In this manner the *periodic times* of all the *superior* planets are found.

Laplace makes the *sidereal rev.* the same as *Vince* 10759.077213 days, or 29y. 174d 1h. 51′ 11″2. The semimajor axis of his orbit, or his *mean distance* 9.540724, the earth's being 1; the proportion of the *eccentricity* of half the greater axis for the beginning of 1750, 0.056223 ; the *secular variation* of this proportion 0.000261553 diminishing ; the *mean longitude* of Saturn at the beginning of 1750, reckoning from the mean vernal equinox at the epoch of the 31st of December, 1749, at noon, mean time at Paris (257°0438) 231° 20′ 21″912 ; long. of the *perihelion* at the beginning of 1750 (97°9466) 88° 9′ 6″984 ; the sidereal and *secular* direct motion of the perihelion (4967″64) 26′ 49″51536 ; the *inclination of the orbit* to the ecliptic at the beginning of 1750 (2°7762) 2° 29′ 54″888 ; the *secular var.* of the inclination to the true ecliptic (— 47″87) 15″5098 decreasing ; longitude of the ascending *node* upon the ecliptic at the beginning of 1750 (123°9327) 111° 32′ 21″948 ; the sidereal and *secular* mot. of the node on the *true* ecliptic (— 5781″54) 31′ 3″21896, retrograde.

According to *Vince*, the relative *mean distance* of Saturn from the sun is 954072, that of the earth being 100000 ; the place of his *aphelion* for the beginning of 1750, was 8s. 28° 9′ 7″ ; its *secular* motion 1° 50′ 7″ ; the *eccentricity* of his orbit 53640.42 ; the *greatest equation* 6° 26′ 42″ ; the *longitude* of his *node* for the beginning of 1750, 3s. 21° 32′ 22″ ; its *secular* motion in respect to the equinox, 55′ 30″ ; and the *inclination* of his *orbit* to the plane of the ecliptic 2° 29′ 50″. *Delambre* for the beginning of 1800, makes the mean place of *Saturn* 4s. 3° 5′ 9″ 9, of the *aphelion* 8s. 29° 4′ 10″, and of the *node* 3s. 21° 56′ 40″.—For the beginning of 1812, he makes his place 8s. 29° 54′ 33″ 5, that of his *aphelion* 8s. 29° 17′ 23″, and of his *nodes* 3s. 22° 2′ 58″. The *annual* mean motion of the aphelion, according to *Delambre*, is 1′ 6″, and of the node 32″. The *greatest equation* of Saturn in his orbit, for 1750, according to the same author, is 6s. 26° 41′ 7, and its secular variation 110″ 24.

The periodic motion of Saturn in his orbit is from *west* to *east*, and nearly in the plane of the ecliptic ; it is subject to inequalities similar to those of Jupiter and Mars. Saturn commences and finishes his *retrograde motion* when the planet, before and after his opposition, is about 108° 54′ distant from the sun. The duration of this retrogradation is nearly 131 days, and the arc of retrogradation about 6° 18′. At the moment of his opposition, his *diameter* is a maximum, and its *mean* magnitude, according to *Laplace*, is (54″ 4) 17″ 6256, or 17″ 6 nearly.

From Saturn's periodic revolution, his mean distance from the sun is found to be 895351645.2 miles ;* and his progressive motion in his orbit is 21786.5 miles an *hour*.† His *real diameter* is

* Saturn's per. rev. = 10759d. 1h. 51′ 11″ nearly, = 929584271″, the sq. of which is 864126916890601441, this being divided by 995916894538801, the sq. of the seconds in a sidereal year (see pa. 350) gives 867.669705, the cube root of which is 9.5378 nearly, the relative dist. of Saturn from the sun. Hence 9.5378 × 23464.5 = 223799.7081 dist. of Saturn in semidiam. of the earth, which multiplied by 3956, gives 895351645.2436 miles, the dist. of Saturn from the sun.

† Saturn's dist. from the sun being multiplied by 2, and then by 3.1416, gives 5625673457.4 nearly the circumference of his orbit ; hence 10759d.

67624 miles, * and his *magnitude* 624.6 times that of the earth.† The *light* and *heat* which he receives from the sun about $\frac{1}{90}$ of the light and heat which the earth receives ‡

Cassini and *Fato* in 1683, suspected that Saturn revolved on his axis, from having one day observed a bright streak which disappeared the next, when another came into view near the edge of his disk; these streaks are called *Belts*. *Cassini* considered these belts as clouds floating in the air; and having a curvature similar to the exterior circumference of the ring, he concluded that they ought to be nearly at the same distance from the planet, and that therefore the *atmosphere* of Saturn extended to the ring Dr. Herschel found that the arrangement of the belts always followed the direction of the ring, so that when the ring opened, the belts shewed an incurvature answering to it. And during his observations on June 19, 20 and 21, 1780, he saw the same spot in three different situations. He in consequence conjectured that Saturn revolved about an axis perpendicular to the plane of his ring. This conjecture receives a greater degree of probability from the planet being an oblate spheriod, the equatorial diam. or the diam. in the direction of the ring, being to the diam. perp to it, or the polar diam. in the proportion of about 11 : 10 according to Dr. Herschel; the measures being taken with a wire micrometer prefixed to his 20 feet reflector. The attraction of the ring, however, contributes to produce this effect. He afterwards verified the truth of his conjecture, having determined from direct observation, that Saturn revolves on his axis from *west* to *east* in 10h 16′ 0″ 4. *Phil. trans.* 1794. He has also observed *five* belts nearly parallel to Saturn's equator.

For the phenomena that would appear to an observer situated in Saturn, see Dr. Gregory's Astr. b. 6, prob 6. From what has been delivered these may be easily conceived, and most of them represented on the globes.

1h. 51′ 11″ : 1h. or 3600″ :: 5625673457.4 : 21786.5 miles the hourly velocity of Saturn in his orbit.

* Saturn's dist. from the sun, at opposition, is 223799.7 of the earth's semidiameters, from which 23464.5, the earth's dist. in semid. being taken, leaves 200335.2 semidiameters of the earth, Saturn's dist. from the earth.—Now taking his appar. diam. at oppos. 17″ 6, we have inversely 200335.2 : 17″ 6 :: 23464.5 : 150″ 265, the appar. diam. of Saturn at a dist. from the earth equal to that of the sun. Hence 32′ : 150″ 265 :: 864065.5 (sun's diam.) : 67624 miles the diameter of Saturn. The diam. may be also found in the same manner as Jupiter's, pa. 362.

† For $\frac{67624^3}{7911^3}$ = (log. 2.7956085) 624.6.

‡ Saturn being about 9½ times further from the sun than the earth, his heat and light (being as the square of the dist.) will therefore be 90¼ times less than the earth's.

OF SATURN'S RING.

SATURN, when viewed through a good telescope, makes a more remarkable appearance than any of the other planets. Galileo, in 1610, first discovered his extraordinary shape; the planet appearing to him like a large globe between two small ones. In 1612 he was surprized to find only the middle globe; but afterwards he discovered again the globes on each side, and found that their magnitude and form were extremely variable; sometimes they appeared round, at other times oblong, sometimes semicircular, then with horns towards the globe in the middle, and by degrees growing so long and wide as to encompass it, as it were, with an oval ring. *Huygens* in 1656, from the improvements he had made in grinding glasses, was able to announce the curious discovery, that these strange phenomena are produced by a large thin *ring*, which surrounds the globe of Saturn, and which is every where separated from it. He made the space between the globe and the ring something greater than the breadth of the ring, and the greater diameter of the ring (which generally appears elliptic except when the eye of the spectator is in its plane, when it appears like a straight line) to that of the globe as 9 : 4. Mr. *Pound* made this prop. as 7 : 3. The best description of this singular phenomenon is that given by Dr. *Herschel* in the *Phil. trans.* for 1790. The following is the substance of his account.

The black disk, or belt, or Saturn's ring, is not in the middle of its breadth; nor is the ring subdivided into many such lines, as some astronomers represent; but there is one single dark, considerable broad line, belt, or zone, which he has constantly found on the north side of the ring. As this dark belt is subject to no change whatever, it is probably owing to some permanent construction of the ring's surface. This construction cannot be owing to the shadow of a chain of mountains, since it is visible all round the ring; for at the ends of the ring there could be no shade; and the same arguments will hold against any supposed caverns. It is moreover pretty evident, that this dark zone is contained between two concentric circles, as all the phenomena correspond to the projection of such a zone.

The matter of the ring is undoubtedly no less solid than the planet itself; and it is observed to cast a strong shadow on the planet. The light of the ring is also generally brighter than that of the planet; for the ring appears sufficiently bright when the telescope affords scarcely light enough for Saturn. Dr. *Herschel* next observes the extreme thinness of the ring: he frequently saw the 1st, 2d, 3d, and 4th satellites pass before and behind it, in such a manner as to serve as excellent micrometers to measure its thickness. For an account of these phenomena, consult the *phil. trans.* for 1790 and 1792; or *Vince's* astr. many particulars will also be found in Dr. *Gregory's* astr.

From a series of observations upon luminous points of the ring, he has discovered that it has a rotation about its axis, the time of which is 10h. 32′ 15″4. The ring is invisible, with the telescopes in common use among astronomers, when its plane passes through the sun, or the earth, or between them; in the first case the sun shines only upon its edge, which is too thin to reflect sufficient light to render it visible; in the second case, the edge only being opposed to us, it is not visible, for the same reason; in the third case, the dark side of the ring is exposed to us, and therefore the edge, being the only luminous part which is towards the earth, is invisible on the same account. Dr. *Herschel*, with his large telescopes has been, however, able to see it in every situation. He thinks that the edge of the ring is not flat but spherical, or spheriodical. M. de la *Lande* thinks that the ring is just visible with the best telescopes in common use, when the sun is elevated 3′ above its plane, or three days before its plane passes through the sun; and when the earth is elevated 2′ 20″ above the plane, or one day from the earth's passing it The difference of the telescopes and the state of the atmosphere, will make 10 or 12 days difference in the time of its becoming invisible.

Dr. *Herschel*, from his observations on the ring, thinks that he has sufficient reason to conclude, that Saturn has two concentric rings, situated in one plane, which is probably not much inclined to the equator of the planet The dimensions of the rings and the space between them, are in the following proportion, as nearly as they could be ascertained.

	Parts.		*Miles.*
Inner diameter of the smaller ring, - -	5900	or	146345
Outside diameter of do. - - -	7510		184393
Inner diam. of the larger ring, - - -	7740		190248
Outside diam of do. - - -	8300		204883
Breadth of the inner ring, - - - - - -	805		20000
Breadth of the outer ring, - - - - - -	280		7200
Breadth of the dark zone, or vacant space between the rings, - - - - - - -	155		2839

Dr. *Herschel*, from the mean of a great many measures of the diameter of the larger ring, makes it 46″ 677 at the mean dist. of Saturn. Hence his diam. : the earth's diam :: 25.8914 (according to *Vince*) : 1. From the above proportion, therefore, the diameter of the ring must be 204883 miles;* and the dist. of the two rings 2839 miles.

From the oblique position of the ring, though circular, it appears elliptical, and it appears most open when Saturn is 90° from the nodes of the ring upon the orbit of Saturn; or when Saturn's long. is about 2s. 17°, and 8s. 17°. In this situation the lesser

* By taking the appar. diameter of Saturn = 17″ 6, we have 17″ 6 : 46″ 677 :: 67624 miles the diam. of Saturn (see pa. 382) : 179345 miles the diameter of the ring. If 17″ 6 the appar. diam. of Saturn at his oppos. were reduced to Saturn's mean dist. from the earth, the diameter of the ring would come out greater, as the appar. diameter of Saturn would be diminished.

axis is very nearly equal to half the greater, when the observations are reduced to the sun; and therefore the plane of the ring makes an angle of about 30° with Saturn's orbit.

In the Mem. de l'Acad. at Paris, 1787, M. de *Laplace* supposes that the ring may have many divisions; but Dr. *Herschel* remarks that no observations will justify this supposition. *Laplace* makes the inclination of the ring to the plane of the ecliptic (54° 8) 49° 19′ 12″. He remarks that the plane of the ring meeting the solar orbit at every semi revolution of Saturn, the phenomena of its disappearance and reappearance return every fifteen years, but frequently with very different circumstances, two disappearances, and two reappearances may occur in the same year, but never more. The incl. of the ring to the ecliptic is measured by the largest opening which the eclipse presents to us. As the earth is in the plane of the ring when it disappears or reappears, the position of its node may be determined by the appar. situation of Saturn. *Laplace* further remarks, that all the disappearances and appearances from which the same sidereal positions of the nodes of the ring result, take place when its plane meets the earth.—The others when the same plane meets the sun. It may therefore be known by the situation of Saturn when the ring disappears or reappears, whether this phenomenon is produced by the sun or the earth. When the plane passes through the sun, the position of its nodes gives that of Saturn, as seen from the sun's centre, and the rectilinear dist. of Saturn from the earth may be determined as that dist. of Jupiter is by the eclipses of his satellites.—It is thus found that Saturn is about 9½ times further from us than the sun, when his appar. diam. is 17″ 6. For more information on the phenomena of the ring, and the manner of determining them, consult Dr. *Gregory's* Astr. sect. 13, b. 4, and ch. 6. b. 6. See also *Newton's* prin. phen. 2. b. 3.

The phenomena of the ring to an eye placed in Saturn make an interesting and curious part of the comparative Astr. of Saturn, for which the reader is referred to *Gregory's* Astr. b. 6. ch. 6. A learner who understands what is here delivered, will easily conceive these phenomena, and the globe will very much assist in exhibiting them; thus, if the equator of the artificial globe be made to coincide with the horizon, and the globe be turned on its axis from west to east, its mot. will represent that of Saturn on his axis, and the wooden horizon will represent the ring, especially if it be supposed a little more dist. from the globe. This ring will cause a great variety in the days and nights in Saturn, which, from its rapid mot. on his axis, are shorter than ours. There is also a much greater diff. between summer and winter on Saturn's globe than on the earth, as well on account of the duration on each, and the sun's great decl. from the equator, as on account of the meridian darkness in winter, from the interposition of the ring which hides the sun.

OF THE SATELLITES OF SATURN.

SATURN has besides his ring, seven little secondary planets or satellites, which perform their motions round him from *west* to *east*, in orbits nearly circular. One of them, which till lately was reckoned the 4th in order, was discovered by *Huygens* in 1655, with a telescope 100 feet long; he published tables of its mean motion in 1659, which were afterwards corrected by Dr. *Halley* in 1682. M. *Cassini*, with telescopes 100 and 136 feet long, discovered the 5th in 1671, the 3d in 1672, and the 1st and 2d in 1684; he afterwards published tables of their motions, and called them *Sidera Lodoicea*, in honour of *Louis le Grand*, in whose reign and observatory they were discovered. These tables were afterwards reformed and corrected by Dr. Halley from Mr. *Pound's* observations. Dr. Halley observes that the four innermost satellites describe orbits very nearly in the plane of the ring, which he says is, as to sense, parallel to the equator; and that the orbit of the 5th is a little inclined to them. The periodic times of the five satellites, and their dist. in semid. of the ring, as determined by Mr. *Pound*, with a micrometer fitted to the 123 feet telescope given by *Huygens* to the *R. Society*, are as follow; *first*, 1d. 21h. 18′ 27″ dist. 2.097; *second*, 2d. 17h. 41′ 22″ dist. 2.686: *third*, 4d. 12h. 25′ 12″ dist. 3.752; *fourth*, 15d. 22h. 41′ 12″ dist. 8.698; and *fifth*, 79d. 7h. 49′ dist. 25.348. The distances in semid. of Saturn as given by *Pound*, are 4.893, 6.286, 8.754, 20.295, and 59.154 respectively. The above distances were deduced from that of the 4th. which was measured, from the proportion between the squares of the periodic times and the cubes of their distances, and found to agree with observation. *Cassini*, from his own observations, makes the periods the same except the 5th, which he makes 79d. 7h. 48′. He makes their dist. in semid. of the ring as follow; $1\frac{14}{15}$ (or $1\frac{19}{20}$) $2\frac{1}{2}$, $3\frac{1}{2}$, 8, 8, and 23 (or 24 *Newton's* prin. b. 3. phen 2.) their dist. from the periodic times in sem. of the ring 1.93, 2.47, 3.45, 8, 23.35; and their dist. at the mean dist. of Saturn 43″ 5, 56″, 1′ 18″, 3′, and 8′ 42″ 5 respectively. *Herschel* makes the dist. of the 5th 8′ 31″ 97, which is probably more exact. Mr. *Pound* found the dist. of the 4th satel. 3′ 7″, when it was very near its greatest eastern digression; hence at the mean dist. of the earth from Saturn, that distance becomes 2′ 58″ 21. Sir *Isaac Newton* (b. 3. prop. 8, cor. 1.) makes it 3′ 4″.

The periodic times for Saturn's satellites are found in the same manner as for those of Jupiter (pa. 365.) To determine these, *Cassini* chose the time when the semiminor axis of the ellipsis which they describe, were the greatest, as Saturn was then 90° from their node, because the place of the satellite in its orbit is then the same as upon the orbit of Saturn; whereas in every other case, it would be necessary to apply the reduction to obtain its place in its orbit. As Saturn and his satellites cannot be seen at the same time, without difficulty, in the field of view of a telescope, their distances have sometimes been measured by observing the time of the passage of Saturn over a wire adjusted as an hour circle in the field of the telescope, and the interval between the times when Saturn and the satellite passed.

By comparing the places of the satellites with the ring in different points of their orbits, and the greatest minor axes of the eclipses which they appear to describe, compared with the major axes, the first four are found to have the planes of their orbits very nearly in the plane of the ring, and are, therefore, *inclined* to the orbit of Saturn about 30°; but according to *Cassini* the son, the orbit of the 5th makes an angle with the ring of about 15°. *Cassini* places the *node of the ring*, and consequently the nodes of the first four satellites, from what we have just now remarked, in 5s. 22° upon the orbit of Saturn, and 5s. 21° upon the ecliptic.— *Huygens* found it equal 5s. 20° 30′. M. *Maraldi*, in 1716, determined the long. of the node of the ring upon the orbit of Saturn to be 5s. 19° 48′ 30″, and upon the ecliptic 5s. 16° 20′. The node of the 5th satel. is placed by *Cassini* in 5s. 5° upon the orbit of Saturn. M. de la *Lande* makes it 5s. 0° 27′ From the observation of M. *Bernard* at Marseilles, in 1787, it appears that the node of this satellite is retrograde. Dr. *Halley* discovered that the orbit of the 4th satellite was eccentric; for, from its mean motion, he found that its observed place was at one time 3° more forward than by his calculations, and at other observations 2° 30′ backward This indicated an eccentricity; and he placed the line of the apsides in 10s. 22°. *Phil. trans.* No. 145. Or *Vince's* astr. from which the principal part of our acct. of the satellites is extracted.

The revolutions and mean motions of the satellites are given by *La Lande* as follow. In this table the satellites are numbered from Saturn, as they were before the discovery of the other *two* by Dr. *Herschel*, whose orbits are situated nearer to Saturn than any of the other five.

Sat	*Diur. mot.*				*Mot. in 365d.*				*Period Revol.*				*Synod. Revol.*			
I	6s.	10°	41′	53″	4s.	4°	44′	42″	1d.	21h.	18′	26″222	1d.	21h.	18′	54″778
II	4	11	32	6	4	10	15	19	2	17	44	51,177	2	17	45	51,013
III	2	19	41	25	9	16	57	5	4	12	25	11,100	4	12	27	55,239
IV	0	22	34	38	10	20	39	37	15	22	41	16,022	15	23	15	23,153
V	0	4	32	17	7	6	23	37	79	7	53	42,772	79	22	3	12,883

Newton in his prin. b. 3. prob. 17, remarks, that the outermost satellite of Saturn seems to revolve about its axis with a motion like that of the moon, having the same face continually turned towards Saturn. For in its revolution round Saturn, as often as it comes to the eastern part of its orbit, it is scarcely visible, and generally quite disappears; which he says is probably occasioned by some spots on that part of its body which is then turned towards the earth. *Newton* remarks the same of Jupiter's satellites. Dr. *Herschel* has confirmed this conjecture, by tracing regularly the periodical change of light through more than ten revolutions, which he found, in all appearances, to be cotemporary with the return of the satellite to the same situation in its orbit. M. *Bernard*, at Marseilles, from his observations in 1787, has further confirmed this result. Hence this equality in the period of rotation and revolutions appears to be a general law of the motion of the satellites, and a remarkable instance of analogy in this part of the Solar System.

In the *phil. trans.* for 1789 and 1790, Dr. *Herschel* gives an account of the discovery of two other satellites, with some of the elements of their motions, and tables for calculating them.

The distances of these satellites from the centre of Saturn are 36″ 7889, and 28″ 6689; and their periodic times are 1d. 8h. 53′ 8″ 9, and 22h. 37′ 22″ 9. The planes of the orbits of these satellites lie so near the plane of the ring, that their difference cannot be perceived.

According to *Laplace*, if the semidiameter of Saturn seen at his mean distance from the sun be taken as unity, the *distances* of the satellites from its centre will be as follow: *First*, 3.080, *Second*, 3.952, *Third*, 4.893, *Fourth*, 6.268, *Fifth*, 8.754, *Sixth*, 20,295, and *Seventh*, 59 154; and the durations of their *sidereal revolutions*, 0.94271 days = 22h. 37′ 30″ 144; 1·37024 days = 1d. 8h. 53′ 8″ 736; 1.8878 days = 1d. 21h. 18′ 25″ 92; 2.73948 days = 2d. 17h. 44′ 51″ 072; 4.51749 days = 4d. 12h. 25′ 11″ 136; 15.9453 days = 15d. 22h. 41′ 13″ 92; and 79.3296 days = 79d. 7h. 54′ 37″ 44 respectively, the satellites being taken here in order, as their respective orbits are situated from Saturn. These mean distances of the satellites, as *Laplace* remarks, being compared with the durations of their revolutions, the beautiful proportion of *Kepler*, relative to the planets, and which we have seen to exist in the satellites of Jupiter, is here again found to take place.

CHAP. IX.

OF URANUS OR HERSCHEL,*

AND HIS SATELLITES.

This is the remotest of the planets belonging to the Solar System, that has hitherto been discovered. From its minuteness it had escaped the observation of ancient astronomers. *Flamstead* at the end of the last century, and *Mayer* and Le *Monnier* in this, had observed it as a small star; and according to F. de *Zach's* account of this planet in the memoirs of the Brussels academy,

* This planet is called by the English the *Georgium Sidus,* in honour of the present King George III. In the Naut. Alm. it is called the *Georgian.* By foreigners it is generally called *Herschel,* in honour of the discoverer. The royal academy of Prussia and some others call this planet *Ouranus* or *Uranus*. *Laplace* calls it by the same name, but *Delambre* in his tables

1785, there was then in the library of the prince of Orange, four observations of this planet considered as a star, in a catalogue of observations written by *Tycho Brahe*. It was not, however, until 1781, that Dr. *Herschel* discovered its motion, and soon after, by observing carefully, he was able to ascertain that it was a true planet.

Its *apparent diameter* is so small that it can seldom be seen by the naked eye. When viewed through a telescope of small magnifying power, it appears like a star of the 6th or 7th magnitude. *Laplace* makes its apparent diam. about (12″) 3″888. In a clear night, in the absence of the moon, it may be perceived, by a good eye, without a telescope; at the beginning of 1812, its place, as given in the Naut. Alm. will be long. 7s. 22° 2′, lat. 18′ N. and decl. 18° S. At the beginning of 1813, it will be in long. 7s. 26° 23′, lat. 14′ N. and decl. 19° 8′ S. And on the 31st Dec. 1813, it will be in long 8s. 0° 36′, lat. 11′ N. and decl. 20° 7′ S. Like Mars, Jupiter and Saturn, its motion is from *west* to *east* round the earth. According to *Vince*, its periodic revolution is performed in 83 years, 150d. 18h. The place of its *aphelion* for the beginning of 1750, is 11s. 16° 19′ 30″, and its *annual progressive motion*, according to M. de la *Grange* 3″ 17, from the action of Jupiter and Saturn; and therefore its motion in long. = 50″ 25 (the precession of the equin.) + 3″ 17 = 53″ 42. The longitude of the nodes in the beginning of 1750 was 2s. 12° 47′. The annual motion of its *nodes*, according to La *Grange*, is 12″ 5 from theory; according to La *Lande*, who takes a different density for Venus, it is 20″ 40‴, which he uses in his tables. The *inclination* of its orbit to the ecliptic is 46′ 20″. Its *distance* from the sun is 1918352, that of the earth being 100000. The *eccentricity* of its orbit 90804. Its *greatest equation* is 5° 27′ 16″.

Laplace makes the *sidereal revolution* of *Uranus* 30689 days, or 84 years 29 days. His mean *distance* or half the greater axis of his orbit, 19.18362, that of the earth being 1. The proportion of the *eccentricity* of half the greater axis of his orbit, for the beginning of 1750, 0.046683. The *secular* variation of this proportion — 0.000026228, the sign — indicating a diminution. The *mean long.* at the beginning of 1750, reckoning from the mean vernal equinox, at the epoch of the 31st Dec. 1749, at noon, mean time at Paris was (352° 962) 318° 33′ 53″ 64. The longitude of the *perihelion* at the beginning of 1750, was (185° 1262) 166° 36′ 48″ 888. The sidereal and *secular* progressive motion of the perihelion (759″ 85) 4′ 6″ 1914. The *inclination* of the orbit to the ecliptic at the beginning of 1750 (0° 8599) 46′ 26″076. The *secular* progressive var. of the *incl.* to the true ecliptic (9° 38) 3″ 039. Longitude of the ascending *node* upon the

calls it *Herschel*. It is called *Uranus* in allusion to the names of the heathen deities by which the other planets are distinguished, as before remarked; thus *Uranus* was the father of Saturn, *Saturn* the father of Jupiter, *Jupiter* the father of Mars, &c. *Herschel* discovered this planet at *Bath* in England, on the 13th of March, 1781.

ecliptic at the beginning of 1750 (80° 7015) 72° 37′ 52″ 86.—And the sidereal and secular retrograde motion of the node upon the true ecliptic (10608″) 57′ 16″ 992.

Delambre in his tables (tab. 160) gives his *mean place* for the beginning of 1812, 7s. 15° 4′ 9″5, of his *aphelion* 11s. 17° 31′ 23″, and of his node 2s. 12° 54′ 6″. His mean mot. for 365 days 4° 17′ 44″2, of his aphel. 53″, and of his node 16″, his mean motion for an *hour* is 1″8. His greatest equat. for 1780, 5° 21′ 2″ 7.

Laplace remarks that his motion, which is nearly in the plane of the ecliptic, begins to be retrograde when, previous to the opposition, the planet is (115°) 103° 30′ distant from the sun. The motion ceases to be retrograde when, after the opposition, the planet in his approach to the sun is only 103° 30′ distant from it. The duration of his retrogradation is about 151 days, and his arc of retrogradation (4°) 3° 36′. He further remarks, that if the dist. of Uranus were to be estimated by the slowness of his motion, it should be on the confines of the planetary system.

From the periodic time of this planet, given above, his distance from the sun, &c. may be found as for the other planets. The ratio of his diam. to that of the earth's is given as 4.32 : 1, hence its magnitude is more than 80 times that of the earth. His hourly vel. in his orbit; the light and heat on his surface, &c. may also be found as for the other planets.

OF THE SATELLITES OF HERSCHEL.

Dr. Herschel has discovered six satellites moving round this planet, in orbits almost circular and nearly perpendicular to the plane of the ecliptic. The first two he discovered on Jan. 11, 1787, of which he gives an account in the *Phil. Trans.* for 1787.

In the *Phil. Trans.* for 1788, he gives a further account of this discovery, together with their periodic times, distances, and positions of their orbits, as far as he was then able to ascertain them.

The most convenient method, as *Vince* remarks, of determining the periodic time of a satellite, being, either from its eclipses, or from taking its positions in several successive oppositions of the planets; but as no eclipses happened since the discovery of the satellites, and that it would be too tedious to put in practice the latter method, *Herschel*, therefore, took their situations whenever he could ascertain them with some degree of precision, and then reduced them, by computation, to such situations as were necessary for his purpose. In computing their periods round their primary, he has taken the synodic revolutions, as the positions of their orbits, at the times when their situations were taken, were not sufficiently known, to get very *accurate* sidereal revolutions. The mean of several revolutions gave the synodic rev. of the first satellite 8d. 17h. 1′ 19″3, and of the second 13d. 11h. 5′ 1″5. The epochs from which their situations may, at any time, be computed, are, for the *first*, Oct. 19, 1787, at 19h 11′ 28″; and for the *second*, at 17h 22′ 40″; at which times they were 76° 43′ north, following the planet.

Dr. Herschel has also determined their distances from the planet; one of which he obtained by observation; and the other from the periodic times. While making his observations to discover the dist. of the second, its orbit seemed elliptical. He found its greatest elongation to be 46″46, and its elong. at the mean dist. of the primary from the earth 44″23, which will be the true dist. very nearly, on supposition that the satellites move in circular orbits; hence by *Kepler's* rule, the dist. of the second sat. comes out 33″09. In this calculation the synodic rev. were used for the sidereal, which will make but little error.

In determining the inclinations of the orbits and places of their nodes, Herschel could not determine which part of the orbit was inclined *to*, and which *from* the earth; he therefore computed them on both suppositions, and found that the orbit of the 2d sat. is inclined to the ecliptic 99° 43′ 53″ 3, or 81° 6′ 4″ 4; its ascending node upon the ecliptic is in 5s. 18°, or 8s. 6°, and when the planet comes to the ascending node of this satellite, which happened about the year 1799, and will again take place about the year 1818, at which time there will be an eclipse of this and the 1st satellite, when they will appear to ascend through the shadow of the planet, in a direction almost perp. to the ecliptic. M. *Delambre* makes the ascending node in 5s. 21°, or 8s. 9° from Dr. Herschel's observations. The situation of the orbit of the first satellite does not materially differ from that of the second. The light of the satellites is extremely faint; the 2d is the brightest, but the difference is small. Here, as in Jupiter's satellites, these two are called 1st and 2d satellites, and are so in the order of discovery, but from the four other satellites which *Herschel* has discovered to revolve round this planet (*Phil. trans.* 1798) this order is changed, and the 1st is now the 2d, and the 2d the 4th.

Most astronomers give their *distances* from the planet, and their periods as follow.

	I.	II.	III.	IV.	V.	VI.
Dist.	25″5	33′	38″57	44″2	88″4	176″8
Rev.	5d. 21h. 25′	8d. 17h. 1′19	10d. 23h. 4′	13d. 11h. 5′	38d. 1h. 49′	107d. 16h. 40′

Laplace says, that if we take for unity the semidiameter of Uranus, supposed (6″) 1″944 seen at the mean dist. of the planet from the sun, the distances of his satellites will be 13.120, 17.022, 19.845, 22.752, 45.507, 91.008; and the durations of their sidereal revolutions 5.8926 days, 8.7068d. 10.9611d. 13.4559d. 38.075d. and 107.6944 days respectively. These durations, as Laplace remarks, with the exception of the 2d and 4th, have been concluded from the greatest observed elongations, and from Kepler's rule, as regards the primary planets, (see pa. 253) a rule which observation has confirmed with regard to the 2d and 4th satellites of Herschel, so that it should be considered as a general law of the motion of a system of bodies round a common focus.

It is a singular circumstance, that the orbits of those satellites are found to be nearly perp. to the ecliptic, and still more singular, that they perform their revolutions round Herschel in a retrograde order, that is contrary to the order of the signs. The first is probably the cause of the latter; and if properly examined, might therefore throw much light on the general cause of the regular law observed in all the planets, in following the direction of the sun's motion on his axis; and also of all the satellites, except those of Herschel, in performing their motions in the direction of the *diurnal* revolution of their primaries. If the action of the sun in moving on its axis, carry the planets, or that of the planets the satellites, it is plain that the more oblique their orbits are to the equator of the body, the less will the effect of the body be upon that which regards it as its centre.

CHAP. X.

OF THE NATURE AND MOTION OF COMETS.

Besides the primary planets and their satellites already described, there are, belonging to our system, other bodies called *Comets*, from their hairy appearance; these appear suddenly in the planetary regions, and again disappear; they are *supposed* to move round the sun in elliptic orbits, like the planets, but very eccentric, so that the Comet is visible but in a small part of it. They are distinguished from other stars from their being attended with a long train of light, always opposite the sun, and which is of a fainter lustre the further it is from the body. Hence comets are commonly divided into *bearded*, *tailed*, and *hairy*; this division, however, relates not to different comets, but rather to the several appearances of the same comet. Thus, when the comet is westward of the sun, and moves from it, it is said to be *bearded*, because the light precedes it in the manner of a beard; when the comet is west of the sun, and therefore sets after him, it is said to be tailed, because the light or train follows it; lastly, when the comet is in opposition to the sun, the train is hidden behind the body of the comet, except a small portion that surrounds it like a border of hair, or *coma*, whence called *hairy*, and whence the comet derives its name.

Like the other stars, the comets participate in the diurnal motions of the heavens, and thus combined with the smallness of their parallax, proves that they are not meteors generated in the atmosphere, but that they are much higher than the moon, and in the regions of the planets (*Newton* prin. b. 3. Lem. 4.) Though the opinion prevailed among many of the ancient philosophers that they were meteors, &c. yet the most ancient and learned of them supposed comets to be eternal or constant bodies of the world, which like planets perform their revolutions in stated times. See Newton pr. b. 3, or Dr. Gregory's Astr. b. 5. sect. 1. where their opinions, &c. are given. (The *phase* observed in the comet of 1744, of which only half the disk was enlightened, proves that they are *opake* bodies, which receive their light from the sun.)

Among the moderns *Tycho Brahe* was the first who, after diligently observing the comet of 1577, and finding that it had no sensible diurnal parallax,* assigned it its true place in the planetary regions. Few comets have approached the earth so near as to have a diurnal parallax, they however afford sufficient indications of an annual parallax. This shews that they are not so distant as the fixed stars.†

There have been various theories concerning the nature of comets, which it would be too tedious here to detail ‡ (they may be

* *Kiel* in his astr. lect. 17, gives the following simple method of discovering whether the comet has any sensible *parallax.* A comet before it disappears moves so slowly, that it seems to be almost without any motion, and it may be twice observed in this manner, before and after the perihelion ; these places of the comet being selected, then, when it is very high above the *horizon*, take any two stars between which the comet lies in a right line parallel to the horizon, which may be easily found by extending the thread before the stars ; next when the comet approaches the horizon, let the thread be extended again as before, and if the comet is found to be in the same straight line with the stars as before, it is a proof that it has no sensible *parallax*, and must be at an immense distance from us. No error can arise here from refraction, as it equally affects both the stars and comet. *Kiel* also gives the following method ; let the comet be observed when it is near the *eastern* part of the *horizon*, and in a right line with two stars that are both in the same circle, which is perp. to the horizon ; and afterwards when the stars rise higher, and are not in the same vertical circle as before, if the comet still appear to be in the same right line with them, it can have no sensible parallax ; and hence its course must be very high in the heavens. If it should be found more depressed than to appear in the right line that joins the stars, it must necessarily have a parallax. And if during the observations, the comet has a proper motion of his own, this motion must be allowed for, in proportion to the time between the observations. The parallax here spoken of is the *diurnal.* The want of this *parallax* afforded an argument of placing the comets higher than the *moon ;* but their being subject to an *annual* paral. proves their descent into the planetary regions. The reason of these methods may be easily understood from considering the earth's motion, and the nature of a parallax.

† *Hevelius* observes, that these motions of the comets are inexplicable, but on the supposition of the earth's motion round the sun ; which therefore affords another proof of the truth of this hypothesis. See *Newton's* prin. pa. 380, Motte's translation.

‡ The principal is that of Sir *Isaac Newton.* He says (*prin.* b. 3, prop. 41.) that the comets are solid, compact, fixed, and durable bodies, like the bodies of the planets ; in a word, they are a kind of planets which move in very oblique orbits every way with the greatest freedom ; persevering in their motions even contrary to the planets direction ; and that their tail is a very thin slender vapour, emitted by the head or nucleus of

found in the *principia*, in *Gregory's* ast. b. 5, or in *Vince's* astr.) but the truth or falsehood of any one of these theories may be tried from the following phenomena of comets, collected from *Ree's Cyclopedia* (Philadelphia ed.)

First. Those comets which move according to the order of the signs, do all, a little before they disappear, either advance a little slower than usual, or else go retrograde, if the earth be between them and the sun ; and more swiftly if the earth be situated in a contrary part. On the contrary, those which proceed contrary to the order of the signs, advance more swiftly than usual, if the earth be between them and the sun ; and more slowly, or go retrograde, when the earth is in a contrary part.*

2. As long as their velocity is increased, they move nearly in great circles : † but towards the end of their course, they deviate from their circles ; and when the earth advances in one direction, they advance the contrary way.

3. They move round the sun in ellipses, having one of their foci in the centre of the sun : and by radii drawn to the sun describe areas proportional to the times.‡

the comet heated by the sun. The truth of these principles appear from their being perfectly conformable to the above phenomena. That the comets are solid, appears from the heat they are capable of sustaining, as appears from that of 1680, whose heat, according to *Newton*, was 2000 times greater than that of red hot iron.

* This is evident, as the course of the comets is among the planets, and must therefore follow the same laws.

† *Keil* remarks (lect. 17. astr.) that if the distance of the comet be observed every day from two *fixed stars*, whose longitudes and latitudes are known, and the places computed from these distances be marked on the surface of a celestial globe, the course of the comet will thus be found to be a portion of a great circle, allowance being made for the earth's motion. From this it is manifest, that the comet moves in a plane passing through the eye of the spectator, or more exactly through the *sun* ; for all visible motion that is made in such a plane, however inclined to the ecliptic, will always appear to be in the periphery of a great circle. The comet's deviation from a course in a great circle, or the variation in the comet's orbit with the ecliptic, is only *apparent*, and does not arise from the real motion of the comet, but from that of the earth, as was shewn in the *planets*, whose distances and incl. to the ecliptic vary according to their different positions, as seen from the earth ; while they are regular, as seen from the sun. *Newton* says that this arises from their *parallax*. See his prin. b. 3. Lemma 4.

‡ They are supposed, for the ease in calculation, to move in parabolic *orbits*, which, in that part of it near the sun, is sufficiently correct for the elements of their elliptic orbits, as they are very eccentric. *Newton* remarks, from considering the curvity of their orbits, that when they disappear, they are much beyond the orbit of Jupiter, and that in their perihelion they frequently descend below the orbits of Mars and the inferior planets. (*Prin.* b. 3. Lemma 4.) He has fully demonstrated, that every body placed in our planetary system, should be attracted by the sun, with a force reciprocally proportional to the squares of the distances, which, in conjunction with the projectile force, would cause the body to move in a conic section about the sun placed in the focus, and describe areas proportional to the times. He also shews, that if the same comet ever return to our sys-

4. The light of their bodies, or nuclei, increases in their recess from the earth towards the sun ; and on the contrary, decreases in their recess from the sun.*

5. Their tails appear the largest and brightest immediately after their transit through the region of the sun, or after their perihelion ; because then they are most heated, and must therefore emit a greater quantity of vapours †

6. The tails always decline from a just opposition to the sun towards those parts which the bodies or nuclei pass over, in their progress through their orbits ; because all smoke or vapours emitted from a body in motion, tends upwards in an oblique direction, and receding from that part towards which the smoking body proceeds.

7. This declination *cæteris paribus*, is the smallest, when the heads, or nuclei, approach nearest the sun ; and is still less near the nucleus of the comet, than towards the extremity of the tail. Because the vapour ascends with more velocity near the head of the comet, than in the higher extremity of the tail ; and also when the comet is at a less distance from the sun than when at a greater. See Dr. Gregory's astr. b. 5, prop. 4. cor. 1, &c. In this prop. and corollaries, many interesting remarks concerning the tails of comets are given. See also the principia, b. 3, Lemma 4, and prop. 41.

8. The tails are somewhat brighter and more distinctly defined in their convex than in their concave part, because the vapour in the concave part, which goes first, being somewhat nearer and denser, reflects the light more copiously.

9. The tails always appear broader at their upper extreme than near the centre of the comet, because the vapour in a free space is perpetually rarified and dilated, as is also the case with any virtue passing from a centre.

10. The tails are always transparent, and the smallest stars appear through them, because it consists of thin vapour, &c.

Hence the hypothesis of Newton, given in the note to pa. 393, exactly agrees with the phenomena. Newton, at the end of the third book of his Principia, fully illustrates this hypothesis, and gives many other interesting particulars concerning the nature of comets. Dr. Gregory enters more fully into the investigation of these particulars. See b. 5 of his Astronomy.

The nuclei, or the *heads*, or rather the *bodies* of comets, when viewed through a telescope, appear differently, or with different

tem, it must describe an ellipsis, though very eccentric. See Dr. *Halley's Synopsis of the astronomy of comets*, at the end of Dr. *Gregory's* astronomy.

* As they are in the regions of the planets, their access towards the sun bears a considerable proportion to their whole distance. See *Newton's* observations on the comet of 1680.

† As the heads of the comets are illuminated by the sun, this light being reflected towards the earth, renders them visible, and shews that they are not in the region of the fixed stars, for any of the planets which are only illuminated by the light of the fixed stars, are not visible on the earth.

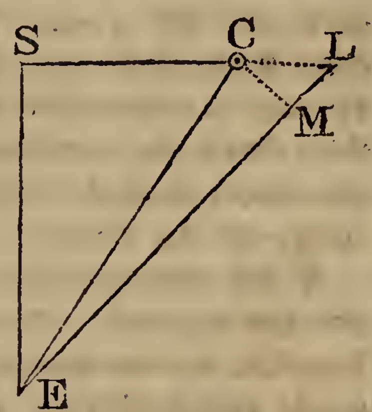
phases, from those of the fixed stars or planets. They are subject to apparent changes, which Newton considered as performed in their atmosphere ; and this opinion was confirmed by observations of the comet in 1744. Hist. Acad. Sciences, 1744.

Tycho, *Hevelius*, and some others, give various estimates of the *magnitude* of comets, but their estimates are not sufficiently accurate to be depended on ; for it appears that they did not distinguish between the nucleus and the surrounding atmosphere. Tycho computes that the true diameter of the comet in 1577, was in proportion to the earth's diameter as 3 to 14. *Hevelius* found the diam of the nucleus of the comet of 1661, and also that of 1665, at its commencement, to be less than a 10th part of the diam. of the earth ; and that of 1652, from its parallax, and appar. mag. of its head, he computes on the 10th Dec. to be to the diam. of the earth as 52 to 100. He found the true diam. of the comet of 1664 to be six times that of the earth, at another time not much more than 2½ diameters The diameter of the atmosphere is sometimes 10 or 15 times greater than that of the nucleus. *Flamstead* observed the comet of 1682, with a telescope of 16 feet, and found with a micrometer the least diam. of its head = 2′, but the nucleus scarcely a tenth part, or about 11″ or 12″. From comparing the appar. dist. and mag. of comets, some have been found larger than the moon, and even equal to some of the primary planets. The diam. of that of 1744, when at the distance of the sun from us, measured about 1′, and therefore its diam must be about three times the diam of the earth : at another time the diam. of the nucleus was nearly equal that of Jupiter.

Hence Newton shews that more comets are seen in the hemisphere towards the sun, than in that which is opposite to it. For as comets shine by the reflected light of the sun, they will not become visible to us until their light, so reflected, is strong enough to affect our eyes : and moreover, as comets are principally rendered conspicuous from their tails, which they do not emit until heated by the sun, it is evident that to have the comet or its tail visible, it must come within a defined distance of the sun And, as *Newton* remarks (Cor. 2, lem. 4, b. 3) comets descending into our parts, neither emit tails, nor are so well illuminated by the sun, as to discover themselves to our naked eyes, until they are come nearer to us than Jupiter. But the far greater part of that spherical space which is described about the sun, with so small an interval, lies on that side of the earth which regards the sun ; and the comets in that greater part are more strongly illuminated, as being generally nearer the sun. This is more fully elucidated in prop. 5, b. 5, *Gregory's* Astr. where it is further proved (Scholium) that by how much nearer a comet must be to the sun before it becomes visible, by so much does the number of the comets, seen in the hemisphere towards the sun, exceed the number of those which appear in the opposite hemisphere. If, as *Newton* remarks, they were visible in the regions far above Saturn, they would appear more frequently in the parts opposite the sun.

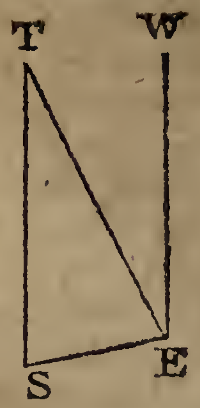
Comets are always surrounded with a very gross dense atmosphere, and from the side opposite the sun project a tail, which increases as the comet approaches its perihelion, immediately after which it is longest and most luminous. That the tail depends on the sun, is evident from these phenomena. The tail is so rare, that the smallest stars are seen through it, and hence the opinion of the ancient philosophers was, that the tail is a very thin, fiery vapour arising from the comet. *Apian*, *Cardan*, *Tycho*, *Snell* and others among the moderns, were of a different opinion, and imagined that the sun's rays were propagated through the transparent head of the comet, and refracted as in a lens. But this is contrary to the laws of Dioptrics, nor does the figure of the tail answer to it; and moreover there must be some reflecting substance like dust in a room, &c to render the rays visible to an eye placed sideways from it. *Kepler* supposed that the tail was produced by the gross parts of the comet being carried away by the sun's rays. *Hevelius* thought that the thinnest parts of the comet's atmosphere were rarified, and driven towards the parts turned from the sun. De. *Cartes* considered the tail as produced by the refraction of light, from the nucleus of the comet to the eye of the spectator. If this were the case, the planets and principal fixed stars ought likewise to have tails; nor would the tails, as Dr. Gregory remarks, be free from the colour of the rainbow, which always accompany refracted light. (See other opinions, &c. in Gregory's Ast. prop. 4, b. 5.) Dr. Gregory remarks, that the most obvious opinion to any one that looks at a comet, is, that the tail has its origin from the head. He shews this to be the opinion of the principal of the ancient philosophers, and also that of *Newton*, who says, *that the tail is nothing else but a very thin vapour, which the head or nucleus of the comet emits by its heat.* He shews that the atmosphere of comets will furnish vapours sufficient to form their tails, from this principle, that if the air should expand itself according to this law, which is confirmed by experience, viz. that the spaces in which it is compressed are reciprocally proportional to the weights compressing it; a globe of air of an inch diameter, if it should become as rare as it would be at the height of a semidiameter of the earth, would fill all the planetary regions as far as the sphere of Saturn, and far beyond. (See this demonstrated in prop 3, b. 5, Gregory's Ast.) Hence he supposes that when the comet is descending to its perihelion, the vapours behind the comet in respect to the sun, being rarified by the sun's heat, ascend and carry with them the particles of which the tail is composed, as air rarified by heat carries up the particles of smoke in a chimney. Since then the coma or atmosphere of a comet is ten times higher than the surface of the nucleus, reckoning from its centre; the tail ascending much higher, must necessarily be immensely rare; and hence the stars appear so visibly through it. The ascent of the vapours will be promoted by their circular motion round the sun. When the tail is thus formed, like the nucleus, it gravitates towards the sun, and by the projectile force received from the comet, it describes an ellipse about the sun, and accompanies the comet, &c. (See the *Prin.* or Gregory's Astr.) The vapours of the comets being thus rarified and dilated, may be scattered through the heavens, and mix with the atmosphere of the planets. *Mairan* supposes that the tails are formed out of the luminous matter that composes the sun's atmosphere, which is supposed to extend as far as the orbit of the earth, and to furnish matter for those northern lights, called the *Aurora Borealis.* He calls the tail of a comet the *Aurora Borealis* of the comet. This hypothesis *La Lande* combines with that of *Newton.* He thinks that part of the vapour which form them, arises out of the atmosphere rarified by heat, and is pushed forward by the force of the light streaming from the sun ; and also that a comet passing through the sun's atmosphere is drenched therein, and carries away some of it. *Euler* (Mem. de l'Acad. de Berlin, tom. 2, pa. 117, seq) thinks that there is great affinity between these tails, the zodiacal light, and the aurora borealis ; and that the common cause of them all is the action of the sun's light on the atmosphere of the comets, of the sun, and of the earth. It may, from thence happen, that the vel. of the comet in its perihelion may be so great, that the force of the sun's rays may produce a new tail before the old one, varied from the comet's mot. in its orbit and about an axis, can follow ; in which case the comet might have two or more tails. The possibility of this is confirmed by the comet of 1744, which was observed to have several tails while it was in perihelion. Dr. Hamilton, in his *Philosophical Essays*, urges several objections against the Newtonian hypothesis. He observes that we have no proof of the existence of a solar atmosphere ; and if we had, that when the comet is moving in its perihelion, in a direction at right angles to the direction of its tail, the vapours which then arise, partaking of the great velocity of the comet, and being also specifically lighter than the medium in which they move, must suffer a much greater resistance than the dense body of the comet, and therefore ought to be left behind, and would not appear opposite the sun ; and afterwards they ought to appear towards the sun. Besides, if the splendour of the tails be owing to the reflection and refraction of the sun's rays, it ought to diminish the lustre of the stars seen through it, which would have their light reflected and refracted in like manner, and consequently their brightness diminished. He concludes that the tail of a comet is composed of a matter which has not the power of refractity or reflecting the rays of light ; but that it is a lucid or self shining substance ; and from its similarity to the *Aurora Borealis*, produced by the same cause, and a proper electrical phenomenon. He conjectures that the use of the comets are destined to supply the sun with fresh fuel, in place of what he loses from the emission of light. This he conjectured from the proximity of the comet of 1680 to the sun, and the resistance it must receive from the sun's atmosphere. *Hevelius* informs us that he observed the comet of 1665 to cast a shadow upon the tail, a dark line appearing in its middle. Cassini observed the same phenomenon in the comet of 1680 ; and the same appearance was taken notice of by a curious observer in the comet of 1744.

The *lengths of the tails* of comets are various, and depend on a variety of circumstances. *Longomontanus* mentions a comet that in 1688, Dec. 10th. had a tail which appeared under an angle of 140° ; that of 1680 on the month of Dec. when it was scarcely equal in light to the stars of the second mag. emitted a remarkable tail, extending 50°, 60°, or 70°, and more ; the comet of 1744 had a tail extending 16° from its body, and which, allowing the sun's parallax to be 10″, must have been about 23 millions of miles in length. The diameter of its body was equal to that of Jupiter. The tail of the comet of 1759, according to M. *Pingre*, subtended an angle of 90° ; but the light was very faint. The length of a comet's tail may be thus found ; let S represent the sun, E the earth, C the comet, CL the tail when directed from the sun ; then the place of the comet being given, we have the angle ECL, the side EC, and the angle CEL, the angle under which the tail appears ; hence CL the length of the tail is given. If the tail deviate by an angle LCM found from observation, we shall then know the angle ECM, with CE and the angle CEM, to find CM.



The analogy between the periodic times of the planets and their *distances* from the sun, takes place also in the comets ; hence the comet's mean distance may be found by comparing its period with the time of the earth's revolution round the sun : thus the period of the comet which appeared in 1531, 1607, 1682, and 1759, being about 76 years, its mean dist. is found by saying as 1^2 (or 1 year) : 76^2 (= 5776) :: 100^3 (or 1000000) : 5776000000 the cube of the comet's mean dist. the cube root of which is 1794, the mean dist. in such parts as that of the earth contains 100. If the per. dist. of this comet 58 be taken from 3588, double the mean dist. the aphel. dist. will be given = 3530, which is a little more than 35 times the earth's mean dist. from the sun. In like manner the aphel. dist. of the comet of 1680 is found to be 135 times the earth's mean dist. from the sun, its period being supposed 575 years ; so that in the aphel. it is more than 14 times more distant from the sun than Saturn.

If the tail of the comet be supposed directed from the sun, the limit of the comet's *distance* may be easily ascertained from it.—

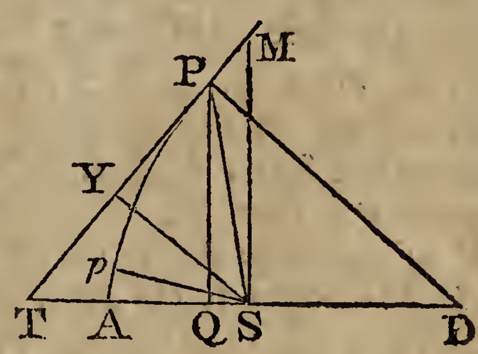


Let S be the sun, E the earth, ET the line in which the head of the comet appears, EW the line in which the extremity of the tail is observed, and draw ST parallel to EW, then the comet is within the dist. ET; for if the comet were at T, the tail would be directed in a line parallel to EW, and therefore it could not appear in that line. The angle TEW being given from observation, and therefore ETS its equal, together with TES the angular distance of the comet from the sun, and ES (from the theory of the earth, part 4. ch 4. or the Nautical Alm.) hence ST the limit is given by saying sine ETS : sine TES :: ES : ST. Or it may be found as in the following example. On Dec. 21, 1680, the comet's elongation from the sun was 32° 24′, and length of the tail 70°; hence ST : SE :: sine 32° 24′ : sine 70° :: 4 : 7 nearly; therefore the comet's dist. from the sun was $\frac{4}{7}$ of the earth's dist. from the same.—Hence *Newton* found that all comets, while visible, are not further from the sun than 3 times the earth's dist. from the sun. But this computation depends on the goodness of the telescope, and the mag. of the comet.

With regard to the *motion* and *periods* of comets, astronomers are not agreed. *Newton*, *Flamstead*, *Halley*, *Gregory*, and all the English astronomers, and also *Cassini* and others of the French, seem satisfied that they return. But de la *Hire* and others supposed not. It would be too tedious to give their respective reasonings on this subject. Dr. *Halley* was the first who predicted the return of a comet, and found that it was one and the same comet which appeared in 1682, 1607, 1513, 1456 and 1305.—Dr. Halley in his *Synopsis of the Astronomy of Comets* (see Dr. Gregory's astr.) shews that comets describe ellipses, and not parabolas or hyperbolas, and thence ventures to foretell the return of the comet of 1682, about the end of 1758 or beginning of 1759; it appeared Dec. 14, 1758. He also shewed that it was the same as the comet of 1607, 1513, 1456, and 1305. In this computation he allowed for the action of *Jupiter* on this comet, which he found would increase its periodic time about a year. M. *Clairaut* computed the effects both of *Saturn* and *Jupiter*, and found that Saturn would retard its return, in the last period, 100 days, and Jupiter 511 days. He, therefore, determined the time when the comet would come to its *perihelion* to be in April 15, 1759, observing that he might err a month from neglecting small quantities in the computation. It passed the perihelion on March 13, within 33 days of the time computed. If we suppose that the time meant by Dr. *Halley* to be that of the comet's passing the perihelion, and not of its first appearing; the addition of the 100 days, from the action of Saturn, which he did not consider, will bring it very near the time in which it did pass the perihelion, and will also shew that Jupiter's effect on its motion, as computed by him, was very accurate. He also observed that the action of Jupiter, in the descent of the comet towards its perihelion in 1682, would tend to increase the inclination of its orbit, and accordingly the incl. in 1682 was found 22′ greater than in 1607.

From the observations of M. Messier upon a comet in 1770, M. *Edric Prosperin*, member of the Royal Acad. of Stockholm and Upsal, shewed that a parabolic orbit would not answer to its motions, and therefore recommended to astronomers to seek for the elliptic orbit. This laborious task was undertaken by M. *Lexell*, who has shewn, that an ellipsis in which the periodic time is about 5 years and 7 months, agrees very well with the observations.—(*Phil. trans*. 1779.)

The ellipsis which the comets describe being very eccentric, astronomers, for the ease in calculation, suppose them to revolve in parabolic orbits, for those parts of their orbits which are within the reach of calculation; on this supposition they can very accurately find the place of the perihelion of a comet, its dist. from the sun, the inclination of the plane of its orbit to the ecliptic, and the place of its node; which are the elements of the comet's orbit. Before we can, however, determine the orbit of a comet, from observation, it will be necessary to premise such particulars relative to the motion of a body in a parabola, as may be requisite for such an investigation.

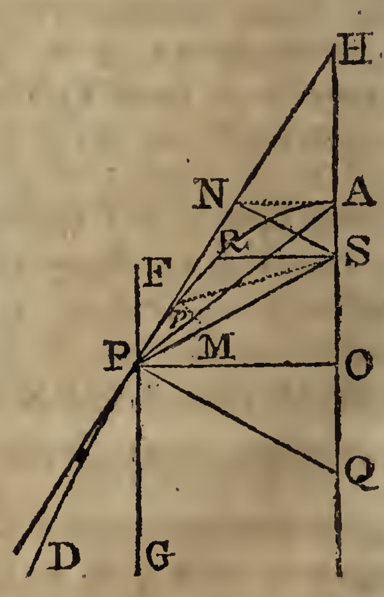


Let APM be a parabola, S its focus, A the vertex, P the place of the body; draw PQ perp. to AS, PD perp. to the tang. PT, and SM perp. to AD. Now by the property of the parabola $QD = \frac{1}{2}$ the latus rectum (Emerson's Conic Sect. prob. 11, b. 3) hence if AS be taken $= 1$, then $QD = 2$; also the angle $PSA = 2PDA$; therefore if QD be made rad. PQ will be tang. of PDA or $\frac{1}{2}$ PSA; hence to the rad. AS, PQ will be twice the tang. of $\frac{1}{2}$ PSA; so that if t be taken = the tang. of (z) half the true anomaly PSA, to the radius $AS = 1$, $2t = PQ$. Also $AQ \times 4AS = PQ^2$ (Emerson's con. sect. b. 3. prop. 3, cor. 3) hence $AQ = t^2$; also the area $AQP = \frac{2}{3} AQ \times QP$ (Em. con. b. 3, pr. 55) $= \frac{2}{3} t^2 \times 2t = \frac{4}{3} t^3$; and as $QS = 1 - t^2$, the area $QPS = \frac{QS \times PQ}{2} = t - t^3$; hence the area $ASP = \frac{1}{3}t^3 + t$; and the area $ASM = \frac{4}{3}$ (AS being $= \frac{1}{2}$ SM, or $\frac{1}{4}$ the latus rectum.) Let a and b be the times in which the comet moves from A to M, and from A to P; then the areas described about S being proportional to the times, we have $a : b :: \frac{4}{3} : \frac{1}{3}t^3 + t$, therefore $at^3 + 3at = 4b$. Hence if a and the true anomaly be given, we have the time $b = \frac{1}{4} at^3 + \frac{3}{4} at$. And because $a : b :: \frac{4}{3} : \frac{1}{3}t^3 + t$, therefore if the true anomalies, and consequently t, be given in different parabolas, the times of describing those true anomalies from the perihelion, will be proportional to the times of describing 90° from the perihelion.

If the times a and b be given, the true anomaly may be found from resolving the cubic equation $t^3 + 3t = \frac{b}{\frac{1}{4}a}$ which may be done thus. Make a right angled triangle, one of whose sides is expressed by 1, and the other by $\frac{b}{\frac{1}{2}a}$ and find the hypotenuse (h) then find two mean proportionals between $h + \frac{b}{\frac{1}{2}a}$ and $h - \frac{b}{\frac{1}{2}a}$ and their difference will be the value of t.

Take the fluxion of $t^3 + 3t = \frac{4b}{a}$ and we have $3t^2\dot{t} + 3\dot{t} = \frac{4b}{a}$ (a being considered constant) ; hence $\dot{t} = \frac{4}{3a} \times \frac{b}{1+t^2}$; but $\dot{t} = \overline{1+t^2} \times \dot{z}$, therefore $2\dot{z} = \frac{8}{3a} \times \frac{b}{1+t^2} = \frac{8}{3a} \times \cos. z^4 \times b$, the variation of the true anomaly corresponding to any small variation b of time expressed in decimals of a day, a being expressed in days.

Let SA be the mean dist. of the earth from the sun = 1 ; then the area of the circle described with that radius will be 3.14159 ; also the area AMS = $\frac{4}{3}$. Now the velocity in the parabola : vel. in the circle :: $\sqrt{2}$; 1 ; for let Pp be an indefinitely small arc described by the body, S the place of the sun, SN a line drawn from the focus S perp. to a tangent to the parabola APD at the point P ; then 1st. *The vel. u in any point* P *of the parabola, is as the square root of the parameter to the axis, divided by* SN ; For the vel. is as the arc Pp, or $u = p$P : now pM being perp. to PS, in the similar rt. angled triangles pPM, PSN, SN : SP :: PM : pP $= \frac{pM \times SP}{SN}$. But the parameter is as the square of the sectors described ; hence put R = the parameter, we have R = pM² × SP², and $R^{\frac{1}{2}}$ = pM × SP ; and by substitution, pP or $u = \frac{R^{\frac{1}{2}}}{SN}$, or $u = \frac{\sqrt{4AS}}{SN}$ from the nature of the parabola.



2dly. *The velocity* u *in any point* P *of the parabola, is to the velocity* V *of a body running through the circumference of a circle, with a central force tending to its centre,* the rad. being = SP, *as* $2^{\frac{1}{2}}$: 1. For, since $u = \frac{\sqrt{4AS}}{SN}$, $u^2 = \frac{4AS}{SN^2}$; or as $SN^2 = SP \times SA$ (from the nature of the parabola) $u^2 =$

$\frac{4AS}{SP \times AS} = \frac{4}{SP}$; now the circle whose radius is SP being taken as an ellipse, its parameter is $= 2SP$; and the vel V being uniform, it is every where as $\frac{\sqrt{2SP}}{SP}$; hence $V^2 = \frac{2SP}{SP^2} = \frac{2}{SP}$: therefore $u^2 : V^2 :: \frac{4}{SP} : \frac{2}{SP} :: 2 : 1$; hence $u : V :: 2^{\frac{1}{2}} : 1^{\frac{1}{2}} :: 2^{\frac{1}{2}} : 1$.

The areas described in the same time will be in the same ratio as the velocities, because at A the motion in each orbit being perpendicular to SA, the areas described will be as the velocities; and this being the case in one instance, it must hold always so, because in each orbit respectively, equal areas are described in equal times. But the times of describing any two areas are as the areas directly, and the areas described in the same time inversely; therefore $\frac{3.14159}{1} :: \frac{4}{3\sqrt{2}} \left(\frac{\sqrt{8}}{3}\right)$ :: the time of the revolution in the circle $=$ 365d. 6h. 9′ : the time of describing AM $=$ 109d. 14h. 46′ 20″. Now, as the time of describing AM, is in a given ratio to the time in the circle, which varies as $AS^{\frac{3}{2}}$, therefore if $r =$ the perihelion dist. in any parabola, we have $1^{\frac{3}{2}} : r^{\frac{3}{2}}$ :: 109d. 14h 46′ 20″ : the time of describing 90° in that parabola from the perihelion. Hence the time corresponding to any true anomaly in that parabola whose perihelion dist. $= 1$, being given, we know the time corresponding to the same true anomaly in any other parabola, because the times of describing 90° are as the times corresponding to the same true anomaly. Hence if n $=$ the number of days corresponding to any given anomaly in that parabola, whose perihelion dist. is unity, then $nr^{\frac{3}{2}}$ will be the time t corresponding to the same anomaly in that whose perihelion distance is r, which may be readily found thus; multiply the log. r by 3 and divide by 2, and to the quotient add the log. n, and the sum will be the log. of the time required. Hence also $n = \frac{t}{r^{\frac{3}{2}}}$; therefore if from the log. t we subtract $\frac{3}{2}$ log. r, the remainder will be the log. n of the number of days corresponding to the same anomaly in the parabola whose perihelion dist $= 1$; hence the anomaly will be found from a table, which gives the times corresponding to the true anomaly for 200000 days from the perihelion, in that parabola whose perihelion dist. is unity. This table may be constructed by the preceding prob. by taking $a = 109.6155$, and assuming $b = 1, 2, 3, 4,$ &c. and finding the corresponding values of t. Dr. *Halley* first constructed a table of this kind.* De la

* See his synopsis of the astronomy of comets at the end of Dr. *Gregory's* Astr. where he also gives the elements of 24, from 1337 to 1698. The following is the method he makes use of in calculating his table. Let S be the sun (fig. pa. 402) ARP the orbit of a comet, A the perihelion, R the place where the comet is 90° distant from the perihel. P any other place. Let the lines AP, PS, be drawn, and make SH, SQ equal to PS; and having drawn the

Caille changed it into a more convenient form, by putting the areas for the times; but the most extensive and complete table is that computed by *Delambre*, and inserted in the tables annexed to the 3d ed. of La *Lande's* Ast. (tab. pa 204 to 234) See art. 3118, La Lande's Ast. The true anomaly in this tab. is calculated for days and quarters of a day to 200 days, then for days and half days to 400, afterwards for each day to 700, for 2½ days to 1200, for every 5 days to 1800, for 10 days to 3000, for 12½ days to 4000, for 25 days to 7000, for 50 days to 12000, for 100 days to 24000, for 200 days to 40000, for 250 days to 48000, for 500 days to 100000, and for 1000 days to 200000 days. He also gives a small table for reducing hours, minutes, and seconds, to decimals of a day. The following is an example of the use of this table. The perih. dist. of a comet being given = 0.5835 to find its true anomaly for 49d. 18h. 55′ 16″

rt. lines PQ, PH, one of which is tang. and the other perp. to the curve; let fall PO perp. to the axis AOQ. Now *any area, as* PRAS, *being given, the angle* PSA, *and the distance* PS *is required.* From the nature of the parabola, QO is always = ½ the parameter or *latus rectum* of the axis (Emerson's Con. Sect. b. 3, prop. 11) and hence if the param. = 2, then QO = 1. Let PO = z, then AO = $\frac{1}{2}z^2$, and the parabolic segment PRA = $\frac{1}{12}z^3$. But the triangle APS = $\frac{1}{4}z$, and hence the area ASPR = $\frac{1}{12}z^3 + \frac{1}{4}z = a$; whence $z^3 + 3z = a$. Wherefore solving this cubic equation, z or the ordinate PO will be known. Now let the area ARS be proposed to be divided into 100 parts, this area is $\frac{1}{12}$ of the sq. of the param. and therefore $12a$ = that sq. = 4. If therefore the roots of these equations $z^3 + 3z = 0.04$, 0.08, 0.09, &c. be successively extracted, there will be obtained so many values of z or the ordinate PO respectively, and the area PRS will be divided into 100 equal parts. In like manner the calculus is to be continued beyond the place R. Now the root of this equation (since OQ = 1) is the tabular tang. of the ∠ PQO or ½ ∠ ASP; hence the ∠ ASP is given. RC, also the secant of PRO, is a mean prop. between QO = 1, and HQ = 2HS. But if AS = 1, and latus rectum therefore = 4 (as in *Halley's* table) then HQ will be the dist. sought, that is 2PS in the former parabola. He then determines the place of a comet, by the numbers in his table, thus. The vel. of a comet in a parabola, being every where to that of a planet describing a circle about the sun, at the same distance from the sun, as $\sqrt{2} : 1$ (from what we have before dem. or *Newton's* prin. b. 1, prop. 16, cor. 7.) If therefore a comet in its perihelion were supposed to be as far distant from the sun as the earth is, then the diurnal arc, which the comet would describe, would be to the diurnal arc of the earth as $\sqrt{2} : 1$. And therefore the time of the annual rev. : the time in which such a comet would describe the quadrant of its orbit from the perih. :: 3.14159 : $\sqrt{\frac{8}{9}}$. Hence the comet would describe that quadrant in 109d. 14h. 46′; and so the parabolic area (analogous to the area ASR) being divided into 100 parts, to each day there would be allotted 0.912280 of those parts, the log. of which, that is, 9.960128, is to be kept for constant use. But then the times in which comets, at a gr. or less dist. would describe similar quadrants, are as the times of the rev. in circles, that is, in the *sesquiplicate* ratio of the distances; whence the diurnal arcs estimated in centessimal parts of the quadrant (which are put for the measures of the mean mot. like degrees) are in each, in the sesquialtera proportion of the distance from the sun in the perihelion. See the application of the above, in computing the appar. place of a comet, &c. with examples, in Dr. *Halley's Synopsis*, and which, except in the change in the tables, differs but little from the method given above.

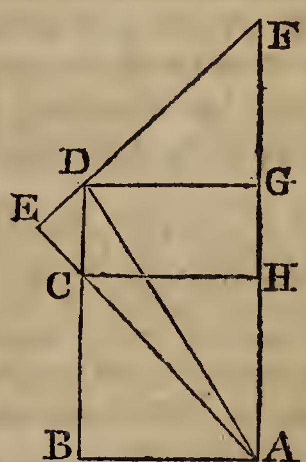
before or after its perihelion. (La Lande, art. 3121.) 18h. 55′ 16″ being reduced to decimals of a day, gives 0.7500. 0.03819, and 0.00018 respectively, the sum of which added to 49 days, gives 49.78837 days.

Log. dist. of the perihelion 0.5835	9.7660409
Half the same log. - - -	9.8830204
Log. of the perh. multiplied by $\frac{3}{2}$ -	9.6490613
Log. of the given time 49.78837 -	1.6971279
The remd. is the log. of 111.7034 days	2.0480666

Now 111.5 days correspond 90° 38′ 57″4 true anomaly; the difference given in the table between 111.50 and 111.75, is 5′ 6″1; hence .25 : .2034* :: 5′ 6″1 : 4′ 9″; therefore the true anomaly required is 90° 43′ 6″.

Vince has given *Delambre's* table in pa. 454, vol. 1, of his Astr.

Draw SY perp. to the tang. (fig. pa. 401) then SP : SY :: SY : SA; whence $SP^{\frac{1}{2}} : SA^{\frac{1}{2}}$ :: SP : SY :: rad. : cos. PSY, or $\frac{1}{2}$PSA the true anomaly; or SP : SA :: rad.^{2} : cos. square of $\frac{1}{2}$ the true anomaly. Hence if SA = 1, and $a + x = \frac{1}{2}$PSA, $a - x = \frac{1}{2}p$SA; then 1 : $SP^{\frac{1}{2}}$ :: cos. $(a + x)$: rad. and $Sp^{\frac{1}{2}}$: 1 :: rad. : cos. $(a - x)$; hence $Sp^{\frac{1}{2}} : SP^{\frac{1}{2}}$ :: cos. $(a + x)$: cos. $(a - x)$. Hence SP = SA ÷ the square of $\frac{1}{2}$ the true anomaly, rad. being = 1; therefore from log. SA, subtract twice the log. cos. $\frac{1}{2}$ true anomaly, the remainder is the log. of the dist. of the comet from the sun.



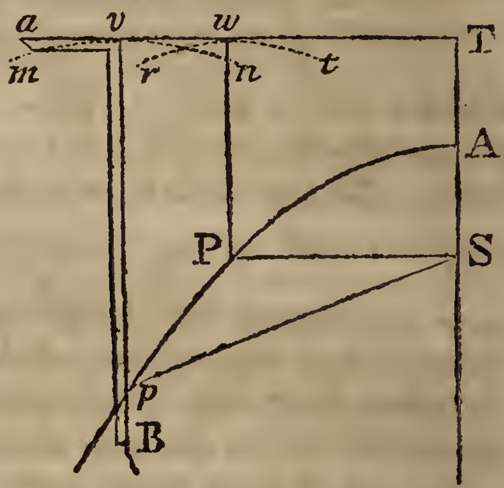
Now let BD be made perp. to AB, take BC = AB, join AC and produce it to E, from D let fall DE perp. to AE, and produce ED until it meets AF parallel to BD in F, join AD and draw DG, CH parallel to BA. Then as the angle EAF = 45°, EFA = 45°, (32 Eucl. 1) hence AE = EF (6 Eucl. 1); also FG = GD = AB (34 Eucl. 1); hence AF = BD + BA, and GH = BD — BA; also by similar triangles AF or BD + BA : CD = GH or BD — BA :: EF or EA : ED :: rad. : tang. DAE; but AB : BD :: rad. : tang. BAD, from which subtract 45°, and we have BD + BA : BD — BA :: rad. : tang. of that difference. If BD = $SP^{\frac{1}{2}}$ (see last fig.) and BA = $Sp^{\frac{1}{2}}$, then $SP^{\frac{1}{2}} : Sp^{\frac{1}{2}}$ :: rad. : tang. BAD $= \left(\frac{Sp}{SP}\right)^{\frac{1}{2}}$; hence to get this angle, take half the difference of the logarithms of SP and Sp, and add 10 to the index (because in the log. tangents, the index of log. tang. of 45°, or log.

* The above .2034 (.7034 — 50) is given in Delambre's tables .7034, this ex. being taken from there.

of rad. = 1, is 10, in place of 0) and this last sum will be the log. tang. of the angle, from which let 45° be taken, and we have $SP^{\frac{1}{2}} + Sp^{\frac{1}{2}} : SP^{\frac{1}{2}} - Sp^{\frac{1}{2}}$ :: rad. : tang of that difference.

Hence if two radii SP, Sp (fig pa. 402) and the contained angle pSP be given, the two anomalies will thence be given. For let a be $\frac{1}{4}$ of ASP + ASp, and x be $\frac{1}{4}$ of ASP — ASp; then $\frac{1}{2}$ ASP = $a + x$, and $\frac{1}{2}$ ASp = $a - x$; hence $Sp^{\frac{1}{2}} : SP^{\frac{1}{2}}$ :: cos. $a + x$: cos. $a - x$ :: (Trig.) cos. $a \times$ cos. x — sine $a \times$ sin. x : cos. $a \times$ cos. x + sine $a \times$ sine x; therefore $SP^{\frac{1}{2}} + Sp^{\frac{1}{2}} : SP^{\frac{1}{2}} - Sp^{\frac{1}{2}}$ :: cos. $a \times$ cos. x : sin. $a \times$ sin. x :: cos. $a \div$ sin. a : sin. $x \div$ cos. x :: cos. a : tang. x. Now the ratio of the two first terms is found from the last art. and as the angle PSp is given, the value of x will be given; hence a is also given, and therefore the sum, and difference of ASP, ASp is given, and hence the angles themselves. If p be on the other side of A, then a is given to find x.

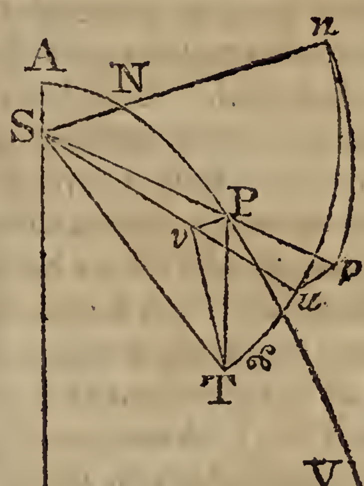
Given two distances SP, Sp, *from the focus to the curve of a parabola, and the angle between them to find the parabola.* With the centres P and p, and radii PS. pS. describe two circular arcs rwt, mvn, to which draw the tangent avwT; draw ST perp to it, and bisect it in A. and it will be the vertex of the parabola. For SA being = AT, and ST to Pw, or Sp to pv, the parabola will pass through A and P, p, &c. from a well known method of describing the parabola.* See a general solution to this prob. in Newton's prin. b. 1, prop. 19. Or prop. 24, b. 5, Dr. *Grigory's* Astronomy.



* If one end of a thread equal in length to the longest side vB of a ruler or square (similar to that which carpenters use) be fixed at the point S, and the other end at B the end of the square avB. If the side av of the squares be moved along the right line Ta, and always coincide with it; then, the string being always kept tight, and close to the side of the square, the point p will describe a *parabola.* For other methods consult the writers on conic sections.

If through either of the given points a circle be described with S *as a centre,* meeting again with the trajectory, and this point of intersection be joined to the former by a straight line; on this line let a perp. be let fall from S until it meets the parabola, this last point will be the *perihelium* of the trajectory; the dist. of which from the focus will be $\frac{1}{4}$ the *latus rectum.* The reason is evident; for the focus S being found in the axis of the parabola, a circle described about the centre S will cut the parabola, if it cuts it at all, in two points equally distant from the axis, and hence the rt. line joining the intersections, will be perp. to the axis, &c. If a time be taken whose interval, from the first obs. (the comet being then supposed in the point c of its orbit) is to the interval of time between the first and third obs. as the area cAS to the area cSd (c and d being the observed places of the comet, and A the perihelion) that will be the time of the comet's perih.

The elements of the orbit of a comet being given, to compute its place at any time. The elements of a comet's orbit are, 1. The time when the comet passes the perihelion, 2. The place of the perihelion, 3. The distance of the perihelion from the sun, 4. The place of the ascending node, 5. The inclination of the orbit to the ecliptic. From these elements the place at any time may be computed. The example given by M. de la *Caille* in his astr. is the comet of 1739, which passed its *perihelion* on June 17, at 10h. 9′ 30″ mean time; the *place* of the perihelion was in 3s. 12° 38′ 40″; the perihelion *distance* was 0.67358, the mean dist. of the earth from the sun being 1; the ascending *node* was in 0s. 27° 25′ 14″, and the inclination of the orbit 55° 42′ 44″; to compute the place seen from the earth on Aug. 17, at 14′ 20″ mean time.



Let APV be the parabolic orbit of the comet, N the ascending node, P the comet's place, T the corresponding place of the earth; draw Pv perp to the ecliptic; produce SN, Sv, SP, ST to n, u, p, and t the sphere of the fixed stars, and describe the great circles np, nu♈t and pu.

I. The interval of time from the perihelion to the given time, is 61d. 4h. 10′ 30″ = 61.174, whose log. = 1.786567; also the log. of .67358 is 9 828388, $\frac{2}{3}$ of which log. (from the nature of logar) is 9.742582, which sub. from 1.786567, leaves 2.043985, the log. of 110.6587 days, which by the table (see the ex. pa 405) answers to 3s. 0° 21′ 38″, the true anomaly PSA at the given time.

II. Subtract 3s. 0° 21′ 38″ from 3s. 12° 38′ 40″, the place of the perihelion, the comet being retrograde, and after passing the perihelion, and the remainder is 12° 17′ 1″ for the heliocentric place p of the comet in its orbit.

III. The longitude of n is 27° 25′ 14″, also np = 27° 25′ 14″ — 12° 17′ 1″ = 15° 8′ 13″; hence rad. : cos. pnu = 55° 42′ 44″ :: tang. pn = 15° 8′ 13″ : tang. un = 8° 39′ 53″, the distance of the comet from the ascending node measured upon the ecliptic.

IV. Take this value of un from the place of the node, and there remains 18° 45′ 21″ = ♈u, the true heliocentric place of the comet reduced to the ecliptic.

V. As rad. : sine pn = 15° 8′ 13″ :: sine pnu = 55° 42′ 44″ : sine pu = 12° 37′ 34″ the heliocentric latitude, or lat. seen from the sun, which is south.

VI. The true place T of the earth at the same time is 10s. 24° 34′ 36″; hence TS♈ = 35° 25′ 24″; therefore TS♈ + ♈Su = TSv = 1s. 24° 10′ 45″. Also TS = 1.0115.

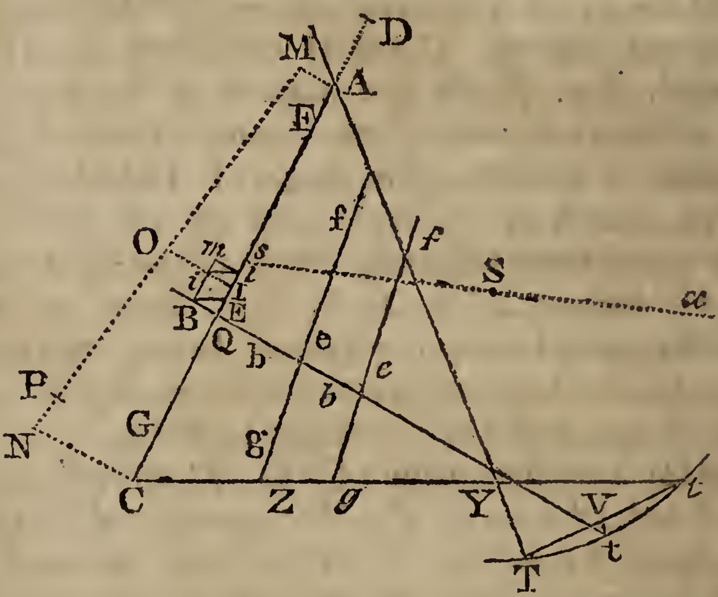
VII. By what is shewn in a preceding article (pa. 405) cos. square of 45° 10′ 49″ : rad.^{2} :: .67358 : SP = 1.3557.

VIII. As rad. : cos. PSv = 12° 27′ 34″ :: SP = 1.3557 : Sv = 1.32377.

IX. In the triangle TSv, TS, Sv, and the included angle TSv are given ; hence the angle STv is found = 77° 33′ 38½″, which being taken from 4s.24° 34′ 36″ the sun's place, leaves 2s. 7° 0′ 57½″ for the comet's true *geocentric longitude*.

X. Again, sine 54° 10′ 45″ : sine 77° 33′ 38½″ :: tang PSv = 12° 27′ 34″ : tang. PTv = 14° 54′ 4″, the comet's true *geocentric latitude.*

To determine the orbit of a comet from observation. Sir *Isaac Newton*, aided by his theory of the planets, and by those observations which shewed him that the comets descended from immense distances into the planetary regions, was the first who solved this important problem, which he called *problema longe difficillimum.* The following is the method given by him in his principia. Select three observations distant from one another by intervals of time nearly equal. But let the interval of time in which the comet moves more slowly, be somewhat greater than the other ; so that, for ex. the diff. of the times may be to the sum of the times, as the sum of the times to about 600 days. See *Newton* prop. 41, b. 3. If such observations be not at hand, a new place of the comet must be found. (See Lem. 6. b. 3. *Newton's* prin. or *Emerson's* differential method, prob. 10. ex. 17 and 18.)



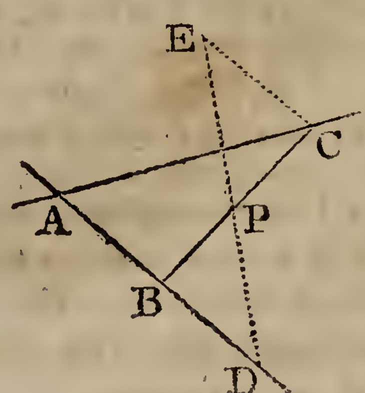
Let S represent the sun, T, t, *t*, three places of the earth in the *orbis magnus* ; TA, tB, *t*C, three observed longitudes of the comet ; V the time between the 1st obs. and the 2d. W the time between the 2d and 3d. X the length which in the whole time, V + W the comet might describe, with that velocity which it had in the mean dist. of the earth from the sun (which is found as directed pa. 402, or cor. 3. prop. 40. b. 3. *prin.*) and tV a perp. upon the chord T*t*. In the mean observed long. tB, take the point B at pleasure, for the place of the comet in the plane of the ecliptic ; and from thence towards the sun S, draw the line BE, which may be to the perp. tV as SB × St² to the cube of the hyp. of the rt. angled triangle, whose sides are SB, and the tang. of the lat. of the comet in the 2d obs. to the radius tB. Through the point E (see lemma 7 *prin.**) draw the right line AEC, whose parts AE and EC, terminating in the right lines TA, and *t*C, may be to each other as the times V and W; then A and C will be nearly the places of the comet in the plane of the ecliptic, in the 1st and 3d observations, if B was its place rightly assumed in the second.

Upon AC bisected in I, erect the perp. I*i*; through B draw the obscure line B*i* parallel to AC; join the obscure line S*i* cutting AC in *l*, and complete the parallelogram *i*I*lm*. Take I*s* = 3I*l*, and through the sun S draw the obscure line $sx = 3Ss + 3il$. Then cancelling the letters A, E, C, I, from the point B towards *x*, draw the new obscure line BE, which may be to the former BE in the duplicate proportion of the dist. BS to the quantity S*m* + $\frac{1}{3}$*il*. Again, through the point E draw the right line AEC as before, that is so that AE : EC :: the time V : the time W, between the observations. Then A and C will be the places of the comet more accurately.

Upon AC bisected in I, erect the perpendiculars AM, CN, IO, of which AM, CN may be the tangents of the latitudes in the 1st and 3d obs. to the radii TA and *t*C; join MN cutting IO in O; draw the rect. parallelogram *i*I*lm* as before; in *i*A produced take ID = S*m* + $\frac{2}{3}$*il*. Then in MN, towards N take MP to X, found above, in the subduplicate propor. of the earth's mean dist. from the sun (or of the semid. of the *orbis magnus*) to the dist. OD. If the point P fall on N; A, B, C, will be three places of the comet through which its orbit is to be described in the plane of the ecliptic. But if P do not fall on N; in the right line AC take CG = NP, so that the points G and P may be on the same side of the line NC.

By the same method as the points E, A, C, G, were found from the assumed point B, from other points b and *b* assumed at pleasure, find out the new points e, a, c, g, and *e*, *a*, *c*, *g*. Then through G, g and *g*, draw the circle Cg*g* cutting the rt. line *t*C in Z; Z will then be one place of the comet in the plane of the ecliptic. And in AC, ac, *ac*, taking AF, af, *af*, equal respectively to CG, cg, *cg*, through the points F, f and *f* draw the circle Ff*f* cutting the rt. line AT in X; the point X will be another place of the comet in the plane of the ecliptic. And at the points X and Z, erecting the tangents of the latitudes of the comet to the

* This lemma is as follows. *Through a given point* P *to draw a rt. line* BC, *whose parts* PB, PC, *cut off by two rt. lines* AB, AC, *given in position, may be to each other, in a given proportion.* From the given point P suppose any rt. line PD to be drawn to either of the right lines given, as AB (produced if necessary) and produce PD towards AC the other given right line to E, so that PE may be to PD in the given proportion.—(12 Eucl. 6) Draw EC parallel to AD; also draw CPB, and by sim. triangles PC : PB :: PE : PD.



radii TX and *t*Z, two places of the comet in its orbit will be determined. If therefore a parabola be described to the focus S through those two places, by the method given pa. 406, this parabola will be the orbit of the comet. See the demonstration of this construction in the lemmas (7, 8, 11 and 10) given by Newton, with a further explanation in Emerson's comment on the prin. pa. 105, 106, &c. This prob. is more fully explained by Dr. *Gregory* in his Astronomy, b 5, prob. 26. This explanation would be preferred were it not too tedious.

It will be convenient not to assume the points B, b, *b*, at random, but nearly true. If the angle AQt, at which the projection of the orbit in the plane of the ecliptic cuts the rt. line tB be rudely known ;* at that angle with Bt draw the obscure line AC, which may be to $\frac{4}{3}$ T*t* in the reciprocal subduplicate proportion of SQ to St. (See Emerson's Comment, pa. 106) or as $\sqrt{St}$ to $\sqrt{SQ}$.† And drawing the rt. line SEB so that its parts EB may be equal to the length V*t*, the point B will be determined, which we are to use for the first time. Then cancelling the rt. line AC, and drawing anew AC according to the preceding construction, and moreover finding the length MP ; in tB take the point b by this rule, that if TA and *t*C intersect in Y, the dist. Yb may be to YB in a proportion compounded of the proportion of MP to MN, and the subduplicate propor. of SB to Sb.‡ By the same method the 3d point b may be found if required. But if this method be followed, two operations will in general suffice. For if the dist. Bb happen to be very small after the points F, f, and G, g, are found, draw the right lines Ff and Gg, and they will cut TA and *t*C in the points X and Z. (See this also investigated in *Gregory's* ast. b. 5, prob. 27.)

Newton corrects the comet's trajectory found by the foregoing method as follows.

Operation 1. Assume that position of the plane of the trajectory which was determined according to the preceding method, and select three places of the comet, found from very accurate observations, and at great distances one from the other. Then sup-

* See the method of determining this in the Schol. prop. 13, b. 5. Greg. Ast. In what follows, in this chap. there are also given different methods, which are more convenient in practice.

† For vel. of a comet at Q in a parab. : vel. at Q in a circle : $\sqrt{2}$: 1, nearly as $\frac{4}{3}$: 1. Also vel. at Q in a circle : vel. at t in a circle :: $St^{\frac{1}{2}}$: $SQ^{\frac{1}{2}}$; therefore *ex equo*, vel. of the com. in Q : vel. of the earth at t :: nearly as AC : Tt; hence AC : Tt :: $\frac{4}{3}St^{\frac{1}{2}}$: $SQ^{\frac{1}{2}}$; or AC : $\frac{4}{3}$ T*t* :: $St^{\frac{1}{2}}$: $SQ^{\frac{1}{2}}$. Wherefore Q is nearly in the chord of the parabola, and B nearly a point of the comet's orbit.

‡ If MP = MN, or AG = AC, then Yb : YB :: Yc : YE :: ac : AC :: vel. in b : vel. in B :: $SB^{\frac{1}{2}}$: $Sb^{\frac{1}{2}}$. But if Sb = SB, and MP or AG invariable, it will be Yb : YB :: ac or AG : AC when G falls in CY. Therefore universally Yb : YB :: AG × $SB^{\frac{1}{2}}$: AC × Sb :: MP × $SB^{\frac{1}{2}}$: MN × $Sb^{\frac{1}{2}}$, to find b truly.

pose A to represent the time between the 1st obs. and the 2d, and B the time between the 2d and 3d. It will be convenient that in one of those times the comet be in its perigeon, or at least not far from it. From those apparent places find by trigonometry the three true places of the comet in that assumed plane of the trajectory, then through the places found, and about the centre of the sun as the focus, describe a conic section by arithm. operations (principia, b. 1, prob. 21, or pa. 406.) Let the areas of this figure which are terminated by radii drawn from the sun to the places found, be D and E, to wit, D the area between the 1st and 2d obs. and E the area between the 2d and 3d. Let T represent the whole time in which the whole area D + E should be described with the vel. of the comet found as in prob. 16, b. 1. prin. See the laws of Gravity, &c. at the end of this work.

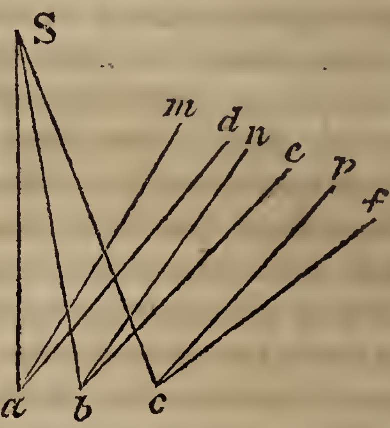
Oper. 2. Retaining the incl. of the plane of the trajectory to the plane of the ecliptic, let the longitude of the nodes of the plane of the trajectory be increased by the addition of 20′ or 30′, which call P. Then from the foresaid three observed places of the comet, let the three true places be found, as before, in this new plane, as also the orbit passing through those places, and the two areas of the same, described between the true observations, which call d and e, and let t be the whole time in which the whole area, d + e should be described.

Oper. 3. Retaining the long. of the nodes in the 1st operation, let the incl. of the plane of the trajectory to the plane of the ecliptic be increased by adding 20′ or 30′ to it, the sum of which call Q. Then from the foregoing three appar. places of the comet, let the three new places be found in this new plane, as well as the orbit passing through them, and the two areas of the same described between the observation, which call d and e, and let *t* be the whole time in which the whole area d + e should be described.

Then taking C : 1 :: A : B, and G : 1 :: D : E, also g : 1 :: d : e, and *g* : 1 :: *d* : *e* ; let S be the true time between the 1st and 3d. obs. and observing well the signs + and —, let such numbers *m* and *n* be found as will make 2G — 2C = *m*G — *mg* + *n*G — *ng* ; and 2T — 2S = *m*T — *m*t + *n*T — *nt*. If in the 1st oper. I represents the incl. of the plane of the trajectory to the plane of the ecliptic, and K the long. of either node, then I + *n*Q will be the true incl. of the plane of the trajectory to the plane of the ecliptic ; and K + *m*P the true longitude of the node. And lastly, if, in the 1st. 2d. and 3d. oper. the quantities R, r and *r* represent the parameters of the trajectory, and the quantities $\frac{1}{L}$, $\frac{1}{l}$ and $\frac{1}{l}$, the transverse diam. of the same ; then R + *m*r — *m*R + *nr* — *n*R will be the true parameter, 1 divided by L + *m*l — *m*L + *nl* — *n*L will be the true transverse diam. of the trajectory which the comet describes. And the transverse diam. being given, the periodic time is given. See the investigation of these expressions, and of computing them by the rule of false, in *Emerson's* comment, pa. 108, &c. The same investigations, &c. are also given in *Gregory's* astr. prob. 31, to which the reader is referred, as it would be rather tedious to insert them here; their application will be given in the following part of this chapter.

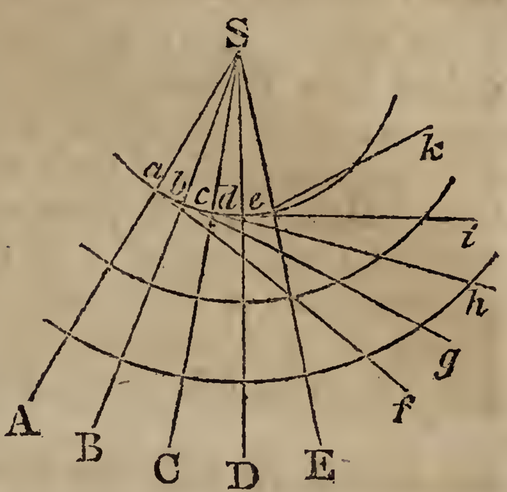
Newton remarks, that the periodic times of the rev. of comets, and the transverse diameters of their orbits, cannot be accurately enough determined but by comparing comets together, which appear at different times. If after equal intervals of time, several comets are found to have described the same orbits, we may thence conclude, that they are all but one and the same comet revolved in the same orbit, and then from the times of their revolutions, the transverse diameters of their orbits will be given; and from those diameters the elliptic orbits themselves will be determined. For this purpose an extensive table of the elements of the comets is given in this chap.

M. la *Lande* gives the following mechanical method of determing the orbit nearly. Let the dist of the earth from the sun be divided into equal parts, and let 10 parabolas be described, whose perihelion distances are 1, 2, 3, &c. of those parts; and divide these parabolas into days from the perihelion, answering to the motion of a body in each. Let S be the sun, *a*, *b*, *c* the places of the earth at the times of three observations of the comet. Then let three geocentric latitudes and longitudes of the comet be found, and set off the elongations S*ad*, S*be*, S*cf* in longitude. From *a*, *b*, *c*, extend three fine threads, *am*, *bn*, *cp*, vertical to *ad*, *be*, *cf*, making angles, with them equal the geocentric latitudes respectively. Then let any one of the parabolas be taken, place its focus in S,



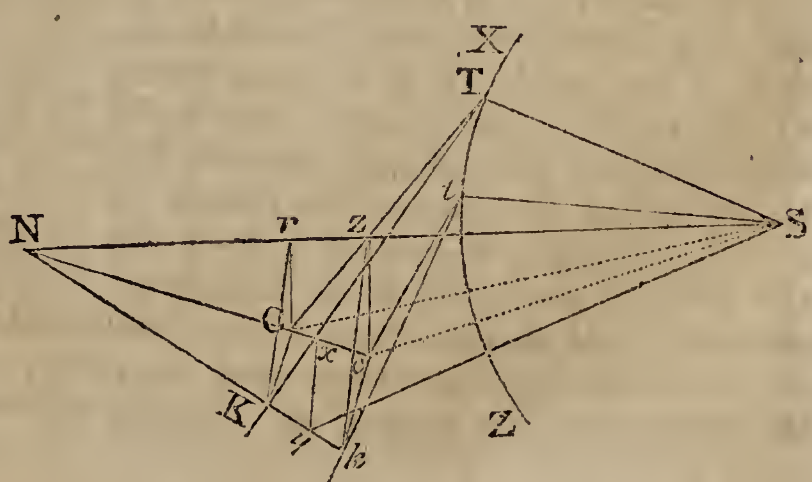
apply the edge to the threads, and observe whether it can be made to touch them all, and whether the intervals of time cut off by the threads upon the parabola, be equal to the respective intervals of the observations, or very nearly so; if this be the case, the true parabola, or very nearly the true one, is found. But if the parabola do not agree, let others be tried, until there be one found which agrees, or very nearly agrees, and the true, or nearly the true parabola will then be obtained, whose inclination, place of the node, and perihelion, are to be determined as accurately as possible from mensuration; also the projection upon the ecliptic. If none of the parabolas nearly answer, it shews that the perihelion dist. must be greater than the dist. of the earth from the sun, in which case other parabolas must be constructed; but this does not very often happen. As this method is rather troublesome, when only the elements of one comet is required, though useful when there are many, as it determines the elements very nearly, *Vince* proposes the following method by means of one parabola, without dividing it.

Take a firm board perfectly plane, and fix on paper for the projection; let a grove be cut near the edge, and five perpendiculars be moveable in it, so that they may be fixed at any distances. Let S represent the sun, and describe any number of circles about it; compute five geocentric latitudes and longitudes of the comet, from which you will have the five elongations of the comet at the times of the respective observations. Draw SA, SB, SC, SD, SE, making the angles ASB, BSC, CSD, DSE, equal to the sun's motion in the intervals of the observations; and on any one of the circles make the angles S*af*, S*bg*, S*ch*, S*di*, S*ek*, equal the respective elongations in longitude, and fix the five perpendiculars, so that the edge of each may coincide with *f*, *g*, *h*, *i*, *k*. From the points *a*, *b*, *c*, *d*, *e*, extend threads to the respective perpendiculars, making angles with the plane equal to the geocentric latitudes of the comet; then fix the focus of the parabola in S and apply its edge to the threads, and if it can be made to touch them all, it will be the parabola required, corresponding to the mean distance S*a* of the earth, which is here supposed to revolve in a circle, being sufficiently accurate for the present purpose. If the parabola cannot be made to touch all the threads, change the points *a*, *b*, *c*, *d*, *e*, to such of the other circles as will be judged from the present trial, most likely to succeed, and let the former trial be repeated again; by a few such repetitions, such a distance for the earth will be obtained, that the parabola will touch all the threads, in which position find the inclination, observe the place of the node, and measure the perihelion distance, compared with the earth's dist. and the elements of the comet's orbit will be nearly obtained.



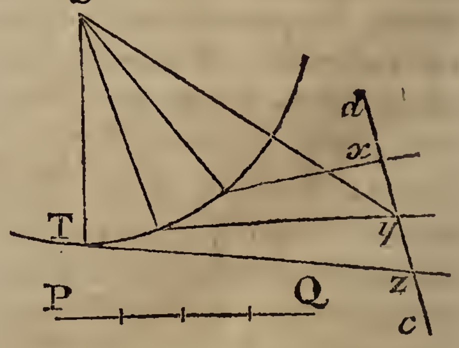
Boscovich gives the following method of approximating to the orbit of a comet.

Let S be the sun, XZ the orbit of the earth, supposed to be a circle; T the place of the earth at the first observation, and *t* at the third; draw TC, *tc* to represent the observed long. of the comet; and let L, *l*, λ, be the longitudes at the first, second, and third observations; *m* and *n* the geocentric latitudes of the comet at the first and third observations; and *t*, T, the intervals of time between the first and second, and



second and third observations. Assume C for the place of the comet at the first observation, reduced to the ecliptic; then to determine the point at the third observation, we have T × sin. $\overline{\lambda - l}$: t × sin. $\overline{l - L}$ :: TC : *tc*, and *c* will be nearly the place required (see Buscov. Opuscula vol. 3, or Sir H. Englefield's *determination of the orbits of comets*, pa. 27) join C*c*, and it will represent the path of the comet on the ecliptic, upon this assumption. Draw CK, *ck*, perp. to the ecliptic, and make CK : TC :: tang. *m* : rad. and *ck* : *tc* :: tang. *n* : rad. join K*k*, and it will represent the orbit of the comet, if the first assumption be true. Bisect C*c* in *x*, and draw *xy* parallel to *ck*, and *y* will bisect K*k*; join *y*S. Let SX = 1; then if *v* be put for the mean velocity of the earth in its orbit, the velocity of the comet at *y* will be $\frac{2^{\frac{1}{2}} \times v}{\sqrt{Sy}}$ hence taking $v = Tt$, let the value of $\frac{2^2 \times v}{\sqrt{Sy}}$ be found, and if this be equal to K*k* measured by the scale, the assumed point C was the true point. But if these quantities be not equal, assume a new point for C, in doing which the error of the first assumption will be a guide; if for instance the computed value of K*k* be greater than the true value, and the lines CK, *ck* are diverging from each other and receding from the sun, the point C must be taken further from T, and how much further, may be conjectured from the value of the error, and also from hence that the velocity of the comet diminishes as it recedes from the sun These considerations will lead us to make a second assumption near the truth. Having thus determined the true points C, *c*, very nearly, produce *c*C, *k*K to meet at N, join NS, and it will be the line of the nodes. Draw C*r*, *cz* perp. to SN, and the angles K*r*C, *kzc*, will measure the inclination of the orbit. From the two distances SC, S*c*, and the included angle CS*c*, the parabola may be constructed, and applied as in the preceding method, from which the time of passing the perihelion may be found.

The following is another method by which the orbit is readily, and very nearly obtained. Let S be the sun, T the earth, T, t, *t*, three places of the earth at the times of the three observations; extend three threads T*p*, t*n*, *tm*, in the direction of the comet, as before directed; assume a point *y* for the place of the comet at the second observation, and measure S*y*; then if ST = 1, and the velocity of the earth be *v*, the velocity of the comet at *y* will be $\frac{2^{\frac{1}{2}} \times v}{(Sy)^{\frac{1}{2}}}$; let *v* be represented by Tt, t*t*, and upon any straight edge PQ, set off $cc = \frac{2^{\frac{1}{2}} \times Tt}{(Sy)^{\frac{1}{2}}}$, and $ed = \frac{2^{\frac{1}{2}} \times tt}{(Sy)^{\frac{1}{2}}}$; then apply



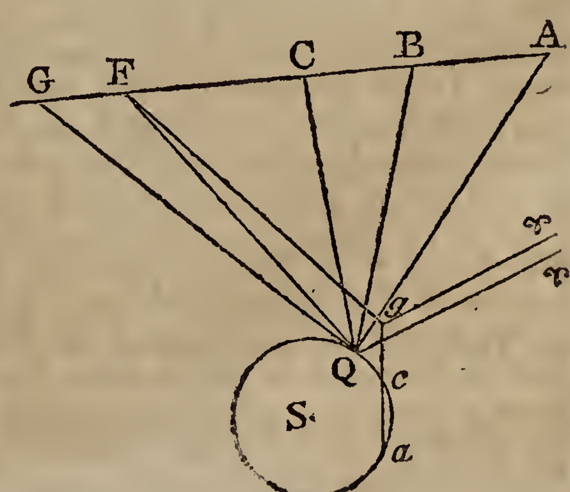
the point *e* to *y*, and by turning about the edge, try whether you can make the point C fall in T*p*, and the point *d* in *sm*; if this cannot be done, the error will be a guide to assume another distance, and by a few trials the point *y*, where the points *c* and *d* will fall in T*p*, *tm*. This method is very easy in practice, and sufficiently accurate to obtain a dist. S*y* from which to begin to compute, in order to find the orbit more correctly, when the comet is not too near the sun.

The parabola being determined nearly, let some quantity be assumed as known at the first and second observations, from which let the place of the comet be computed at those times, and also the time between; if that time agree with the observed interval, a parabola which agrees with the two first observations is obtained; if the times do not agree, let one of the assumed quantities be altered, and see how it then agrees: and then by the rule of false, the supposition which was altered may be corrected, and a parabola obtained which will agree with the two first observations. (See Dr. *Gregory's* Ast. b. 5, prop. 31 and 26.) In like manner by altering the other assumed quantity, another parabola is obtained, agreeing with the two first observations. Then if these do not agree with the third observation, a correction must be made by proportion, and the three observations will be answered.*

As the comets do not however move in parabolas, but in very eccentric ellipses, it is impossible to find a parabola that will accurately agree to all the *data*; it will therefore be sufficient when it nearly agrees. When great accuracy is required, we must take into consideration the effect of *aberration* and *parallax*; the former may be computed from the methods given in the following chap. and the latter by taking the horizontal parallax to the sun's horizontal parallax $= 8''75$ (see pa. 284) as the distance of the sun to the distance of the comet, and then finding the parallax in lat. and long. as directed pa. 331, &c.†

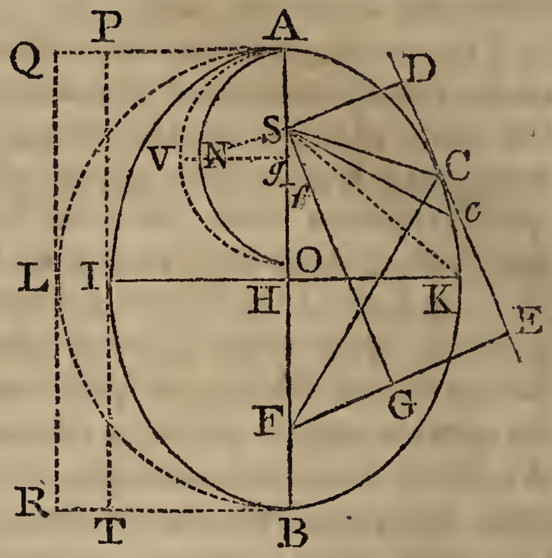
* For further particulars we must refer to *Vince's* ast. vol. pa. 428, &c. See also "An account of the discoveries concerning comets, with the way to find their orbits, and some improvements in constructing and calculating their places; to which are added new tables fitted for those purposes." By *Thomas Barker*, Gent. London, 1757.

† *Newton*, in determining nearly the dist. of a comet, determines also its parallax. The following is his method. Let ♈QA, ♈QB, ♈QC be three observed longitudes of the comet about the time of its first appearing, and ♈QF its last observed long. before its disappearing. Draw the rt. line ABC, whose parts AB, BC intercepted between the right lines QA and QB, QB and QC, may be to each other as the times between the three first observations respectively. Produce AC to G, so that AG may be to AB as the time



To ascertain the periodic time of a comet, and the axis of its orbit.

If comets, after receding from the lower regions of the Solar System, to vast distances beyond the orbits of the most distant planets, return again to the neighbourhood of the sun, the paths which they describe must be nearly elliptic: if then observations be made sufficiently accurate to be a basis of the operations, the requisites of the prob. may be determined in the following manner. Let AKBI be the trajectory of a comet, AB its major axis, IK the minor, S, F the two foci, the former of which represents the sun's place, C the comet's place, CS its dist. from the sun, Cc the space it passes over in a very small portion of time, DCE a tangent to the curve in the point C; SD, FE perpendiculars, let fall thereon from the foci; let SG be drawn parallel to the tangent, and join FC. Also let ALB be a circle, described on the tranverse axis AB; APTB a rectangle about the ellipse AIB, and AQRB a square about the circle ALB. Lastly, let ANO be the elliptic orbit of any planet, S, *f*, its foci; let SC $= a$, SD $= b$, Cc $= e$, the time in which e is described $= f$, the transverse or greater axis of the comet's orbit AB $= x$, that of the planet's orbit AO $= q$, the circumference of the circle AVO, described on the same axis $= p$, the periodic time of the comet $= t$, and that of the planet $= n$.



between the first and last obs. to the time between the 1st and 2d. and join QG. Now if the comet moved uniformly in a rt. line, and the earth either stood still, or was likewise carried forwards in a rt. line by an uniform motion, the long. ♈QG would be the comet's long. at the last obs. Hence the ∠ FQG, which is the diff. of long. proceeds from the unequal motions of the comet and the earth. If the earth and comet move in contrary directions, this angle is *added* to ♈QG, and accelerates the comet's appar. mot. but if they move in the same direction it is *subtracted*, and either retards the motion of the comet or renders it retrograde. This angle therefore proceeding from the earth's motion, is properly esteemed the *comet's parallax*; the small increment or decrement that may arise from the unequal mot. of the comet in its orbit being neglected. From this parallax the comet's *distance* is found thus. Let S represent the sun, *ac*T the earth's orbit, *a* the place in the 1st obs. *c* its place in the 3d obs. Q its place in the last, and Q♈ a rt. line drawn to the beginning of aries. Join *ac*, and produce it to *g*, so that *ag* : *ac* :: AG : AC, and *g*, will be the place at which the earth would have arrived at the time of the last obs. if it continued to move uniformly in *ac*. If therefore *g*♈ be drawn parallel to Q♈, and the ∠ at *g* made = ♈QG, (*g*F being drawn parallel to QG, meeting at F, or at any other point) the ∠ ♈*g*F will then be equal to the long. of the comet seen from *g*, and QF*g* will be the parallax which arises from the earth being transferred from the place *g* into the place Q; and therefore F will be the place of the comet in the plane of the ecliptic. This place F, *Newton* found to be commonly lower than the orb of Jupiter.

The space described Cc, the distance SC, and the angle SCD, are determined from observation. Let the mean dist. of the comet AH or its equal SK $= \frac{1}{2}x$, and that of the planet Ag or SN $= \frac{1}{2}q$; and the squares of the periodic times being as the cubes of the mean distances, we have $\frac{1}{8}q^3 : \frac{1}{8}x^3 :: n^2 : t^2$; hence $t^2 = \frac{\frac{1}{8}x^3 n^2}{\frac{1}{8}q^3}$, and $t = \frac{nx}{q}\sqrt{\frac{x}{q}}$.

It is however necessary to find another expression for the periodic time t, which may be thus found. Cc being a small portion of the orbit, may be considered as a straight line, and the sector CSc a rectilineal triangle, whose area $= \frac{1}{2}$ SD $\times$ Cc $= \frac{1}{2}be$ is given; then as $\frac{1}{2}be$: the area of the ellipse AKBI $=$ A $:: f : t$ $\frac{f}{\frac{1}{2}be} \times$ A. Now to determine the area A, the semiconjugate HK must be found; in order to which AB $=$ SC $+$ FC; hence FC $= x - a$; and the triangle SDC, FEC, being similar, we have SC : SD :: FC : FE; that is, $a : b :: x - a : \frac{bx - ab}{a} =$ FE; therefore FG $=$ FE $-$ GE $= \frac{bx - 2ab}{a}$. Again SC : CD :: FC : CE; or $a : (a^2 - b^2)^{\frac{1}{2}} :: x - a : \frac{x - a}{a} \times (a^2 - b^2)^{\frac{1}{2}}$; hence DE or SG$=$CE $+$ CD $= \frac{x-a}{a} \times (a^2 - b^2)^{\frac{1}{2}} + (a^2 - b^2)^{\frac{1}{2}}$ $= \frac{x}{a}(a^2 - b^2)^{\frac{1}{2}}$. But FG $= \frac{bx - 2ab}{a}$; therefore FS $=$ $(FG^2 + SG^2)^{\frac{1}{2}} = \left(\frac{b^2x^2 - 4ab^2x + 4a^2b^2 + a^2x^2 - b^2x^2}{a^2}\right)^{\frac{1}{2}}$ $= \left(\frac{a^2x^2 - 4ab^2x + 4a^2b^2}{a^2}\right)^{\frac{1}{2}}$; hence SH $= \frac{1}{2}$FS $=$ $\left(\frac{a^2x^2 - 4ab^2x + 4a^2b^2}{4a^2}\right)^{\frac{1}{2}}$. Moreover, as SK $=$ AH $= \frac{1}{2}x$, HK $= (SK^2 - SH^2)^{\frac{1}{2}} = \left(\frac{1}{4}x^2 - \frac{a^2x^2 - 4ab^2x + 4a^2b^2}{4a^2}\right)^{\frac{1}{2}}$ $= \frac{b}{a} \times (ax - a^2)^{\frac{1}{2}}$, and $\frac{xb}{a} \times (ax - a^2)^{\frac{1}{2}} =$ area of the rectangle APTB. Let P $=$ the periphery of the circle whose diam. $= x$, then its area will be $\frac{1}{2}$ LH $\times$ P $= \frac{1}{4}x$P; hence $x^2 : \frac{1}{4}x$P $:: \frac{1}{2}x^2$ $: \frac{1}{8}x$P :: AQRB : ALB :: APTB : AIB $:: q^2 : \frac{1}{4}qp$; that is $q^2 : \frac{1}{4}qp : \frac{xb}{a} \times (ax - a^2)^{\frac{1}{2}} : \frac{bpx}{4aq} \times (ax - a^2)^{\frac{1}{2}} =$ AIB; but 2AIB $=$ AIKB $=$ A $= \frac{bpx}{2aq} \times (ax - a^2)^{\frac{1}{2}}$. Let this value of A be substituted in the preceding expression, and we get $t =$ $\frac{fpx}{aeq} \times (ax - a^2)^{\frac{1}{2}}$, which being equated with that already given, then $\frac{nx}{q}\sqrt{\frac{x}{q}} = \frac{fpx}{aeq} \times (ax - a^2)^{\frac{1}{2}}$, from which x is found $=$ $\frac{af^2 p^2 q}{f^2 p^2 q - ae^2 n^2}$ $=$ AB, *the greater axis of the comet's elliptic trajectory.*

If this value of x be substituted in the above equation for t, we shall get $t = \frac{p^3 f^3 n^2 a^{\frac{3}{2}}}{(qf^2 p^2 - ae^2 n^2)^{\frac{3}{2}}} =$ *the periodic time.* Also, because the conjugate IK $= \frac{2b}{a} \times (ax - a^2)^{\frac{1}{2}} = c$, we have $x =$ $\frac{c^2 a^2 + 4b^2 a^2}{4b^2 a} = \frac{af^2 p^2 q}{f^2 p^2 q - ae^2 n^2}$, whence, by reduction, we find $c = 2ben \times \left(\frac{a}{f^2 p^2 q - ae^2 n^2}\right)^{\frac{1}{2}}$, *the lesser axis of the comet's orbit.*

From these equations it is evident, that when the velocity of the comet is such that $f^2 p^2 q = ae^2 n^2$, the axis x will be infinite, and therefore the trajectory will be a *parabola ;* if $ae^2 n^2$ be greater than $f^2 p^2 q$, the direction of the axis will be on the other side of the curve, which will be an *hyperbola ;* in either of which cases the comet can never return: but when $f^2 p^2 q$ is greater than $ae^2 n^2$, the comet will describe an ellipsis; among the ellipses we may comprise the circle where $x = 2a = \frac{af^2 p^2 q}{f^2 p^2 q - ae^2 n^2}$ and $f^2 p^2 q = 2ae^2 n^2$, whence $e = \text{C}c = \frac{fp}{a} \times \left(\frac{q}{2a}\right)^{\frac{1}{2}}$, the arc of the circle described in 1 day, 1 hour, &c. according as the value of n is given in days, or hours, &c. The solution of the above is also given in Simpson's Fluxions, art. 240.

If the earth, for example, be the planet which is supposed to describe the ellipse ANO; and taking its mean dist. $\frac{1}{2}q = 100000$, or $q = 200000$, then $p = 628318$; the periodic time $n = 1$ year; hence, if Cc be the portion of the comet's orbit described in 1 day, we have $f = \frac{1}{365.2565} = 0.0027378$. The other expressions will become as follow: $x = \frac{591826599235 \times a}{591826599235 - ae^2}$, and $t =$ $\frac{4750560000 \times a^{\frac{3}{2}}}{(591826599235 - ae^2)^{\frac{3}{2}}}$.

To obtain the elliptic orbit of a comet from computation to any degree of exactness, is extremely difficult; for when the orbit is very eccentric, a small error in the observation will change the computed orbit into a parabola, or hyperbole. And, from the thickness and inequality of the atmosphere with which the comet is surrounded, it is impossible to determine with great precision when either the limb or centre of the comet pass the wire at the time of observation. This uncertainty in the observations will subject the computed orbit to a great error. Hence it happened that *Bouguer* determined the orbit of the comet in 1729, to be an hyperbola. *Euler* determined the same for the comet of 1744, but from more accurate observations, he found it to be an ellipse. The period of the comet in 1680 appears, from observation, to be 575 years, which *Euler* by his computation determined to be 166½ years. The only safe way to get the periods of comets, as *Vince* remarks, is to compare the elements of all those which have been computed, and where they are found to agree very well, it may be concluded that they are elements of the *same* comet, it being so extremely improbable that the orbits of two different comets should have the same inclination, the same perihelion distance, and the places of the perihelion and node the same. Thus, knowing the periodic time, we get the greater axis of the ellipse ; and the perihelion dist. being known, the lesser axis will be known. When the elements of the orbits agree, the comets may be the same, although the periodic times should vary a little ; as that may arise from the attraction of the bodies in our system, and which may also alter all the other elements a little.

It has been already observed, that the comet which appeared in 1759, had its periodic time increased considerably by the attraction of *Jupiter* and *Saturn*. This comet was seen in 1682, 1607, and 1531, all the elements agreeing except a little variation of the periodic time. Dr. Halley suspected the comet in 1680 to have been the same which appeared in 1106, 531, and 44 years before Christ. He also conjectured that the comet observed by *Apian* in 1532, was the same as that observed by *Hevelius* in 1661 ; if so, it ought to have returned in 1790, but it has never been observed. But M. *Mechain* having collected all the observations in 1532, and calculated the orbit again, found it to be sensibly different from that determined by Dr. *Halley*. The comet in 1770, whose periodic time M. *Lexell* has found to be 5 years 7 months, has not been since observed, owing probably to the disturbing force of Jupiter. From the elements calculated by *Lexell*, the comet would be in conj. with Jupiter on Aug. 23, 1779, and its distance from Jupiter would be only $\frac{1}{419}$ of its dist. from the sun ; hence the sun's attraction would be only $\frac{1}{234}$ part of Jupiter's attraction. What a change should this make in the orbit ! The comet would not be visible if it returned to its perih. in March, 1776. See *Lexell's* account in *Phil. trans.* 1779. The elements of the orbits of the comets in 1264 and 1556, were so nearly the same, that it is probably the same comet which appeared at each time ; if it be, it ought to appear again about the year 1848.

The number of comets that, from the most accurate accounts, are stated to have appeared, since the commencement of our æra, is about 500 ; and before that æra, about 100 others are recorded to have been seen.

The elements of the comet for 1770, with the trajectory of its path, may be found in the transactions of the American *Phil.* Society, vol. 1.

In order to obtain the *course of a comet*, its distance from two known fixed stars must be observed; or its alt. taken when in the same azimuth with any two stars; from either of these observations its place may be calculated by spher. trigonometry,* or laid down on a *globe*. If several places of the comet be thus found, and marked on the globes, the great circle passing through them will be the way of the comet. This great circle may be drawn by the quad of alt. or the poles may be elevated or depressed, until all the places marked are, at the same time, found in the horizon; for then the circle denoted by the hor. on the surface of the globe will be that required. Hence its intersections with the ecliptic will be the nodes of the orbit of the comet, and the angle which the ecliptic makes with the horizon, measured by the alt. of the *nonagesimal degree*, will be the *incl.* of its orbit to the ecliptic. The *long. latitude*, &c. of a comet, may therefore be easily found on the globes.

Let a circle be described the diam. of which is equal to that of the globe (or reduced proportionally to a smaller scale if necessary) as ABD, whose centre is T, and A a point in its circumference, representing its place among the fixed stars, in which the comet was first observed. Let the arcs AB, AD be taken equal to the dist of the place of the comet, marked on the surface of the globe, from the place first observed, and let TB, TD be drawn. Draw the right line through the point A, so that AE : EC :: R : S (see the note pa. 409) that is, by construction, as the time between the 1st obs. and 2d to the time between the 2d and 3d. From T let fall TP perp. to AO, and

* If its dist. from two known fixed stars be taken, its place may be found thus: Let S be the comet (see fig. pa. 208) *s* one of the stars, Z the other, and P the pole of the equator; then ZP, *s*P, are the co. decl. of the stars, and the angle *s*PZ the diff. of their rt. ascensions, which are given; therefore the dist. between the stars *s*Z, and the angle Z*s*P are given. Now in the triangle SZ*s*, SZ, S*s*, the *correct* dist. of the comet from each of the given stars, and also *s*Z are given; hence the angle S*s*Z, and therefore S*s*P are given. Now in the triangle SP*s*, the dist. S*s* and *s*P, and the ∠ S*s*P are given, hence SP the co. decl. of the comet, and the angle SP*s*, which is the diff. between the rt. ascen. of the star and comet; therefore the comet's rt. ascen. and decl. are given, from which its lat. and long. is found, as shewn in the note to prob. 3, pa. 195.

When exactness is required, the apparent distances must be first corrected; thus, the places of the stars and the hour being given, their alt. may be found; and as the appar. place of the comet is given, and the hour, its appar. alt. may be also found, for which the refraction for that alt. will be nearly the refraction for its true alt. and hence from the appar. alt. the true alt. may be nearly found. If the refraction for this last alt. be again found (see the table, pa. 155) and taken from the appar. alt. the true alt. of the comet will producing it if necessary as far as the circumference at G; an arc equal to AG being transferred from the place first observed, to the way of the comet described above on the surface of the globe, the point G will shew the place among the fixed stars, in which the comet will be in its *perigæum*.

If the place of the comet can be observed when it has no latitude, the place and time of being in one of its nodes will then be exactly known; but as this can seldom be actually observed, these elements are generally observed by approximation from other methods. The appar. diam. of the comet must also be often observed; as by this means a judgment may be formed of its *relative distance* at different times. Its degree of motion, its brightness, &c. must also be regarded; for when it moves with the greatest velocity, or appears most bright, it may be inferred that it is near its *perihelion*.

If four stars round the comet be observed, such that the comet may be in the intersection of the rt. lines which join the two opposite, which are easily found, by extending the thread, placed before the eye, over the stars and comet; let the thread be extended in like manner, over those stars found on a globe, and the point of intersection will shew the place of the comet.*

Although the orbit of a comet may be computed from three observations, yet from these data the *direct* solution of the prob. is impossible. We have therefore given several indirect methods to find the orbit very near the truth, by mechanical and graphical operations (as did *Newton* himself for the comet of 1680, see his *prin.*) then by computation it may be corrected by what is given, until a parabola be found to satisfy the observations very nearly.—The result of these methods as given by *Clairault*, *Fenn* and others, and as pointed out in the preceding part of this chap. is as follows:

Let the rt. ascension and decl. of the comet be found, and from thence its long. reduced to the eclip. and its lat. corresponding to each obs. as shewn in the preceding note. Let the sun's long. be

then be obtained very nearly; the appar. place of the comet found on a good globe, will be sufficiently exact for trial. Now from the apparent and true altitudes, and the appar. distances, the true distances may be found, as shewn in the note pa. 224.

If the quantity of the comet's parallax be known, which may be estimated from its dist. from the earth, or as shewn in the note pa. 120, it may be allowed for, and also an allowance may be made for the aberration of light.

* Whoever wants more information on this subject, besides the works already quoted, may consult the following. *Sejour* Essai sur les cometes, 1775; *Pingre's* Cometographie, 2 vols. 4to. 1781; Sir H. *Englefield's* work "on the determination of the orbits of comets;" M. *Bode's* General Considerations on the situations of the orbits of the planets and comets which have hitherto been calculated, inserted in the memoirs of the Academy of Sciences of Berlin; O. *Gregory's* treatise on astronomy, 1803; De la *Lande* Theorie des Cometes, 1759; and Astronomie, vol. 3. An account of the discoveries concerning comets with the way to find their orbits, &c. by *Thomas Barker*, 1757, &c.

computed at each observation (selecting three best calculated for that purpose) and the diff. (A, a, *a*) between the comet's long. and that of the sun, corresponding to each obs. will be the *elongation* of the comet or its dist. from the sun reduced to the ecliptic. Let the dist (B, b, *b*) of the earth from the sun at the time of each obs. be computed (as shewn in the theory of the earth, ch. 4, or more readily by the Nautical Alm.* or *Delambre's* tables.)

Then let y and z be the assumed distances of the comet from the sun reduced to the ecliptic (found as nearly as possible by some of the foregoing methods) at the 1st and 2d obs. then the true distance may be determined as follows:

As the assumed dist. y *or* z : *sine* A *or* a *the elong.* :: *dist.* B *or* b *of the earth at the* 1*st or* 2*d. obs.* : *sine* ∠ C *or* c *contained by the rt. line drawn from the earth and sun to the comet.*† *This angle* C *or* c *being added to the elongation* A *or* a, *their sum will be the supplement of the angle of commutation* D *or* d.‡ Then (by the propor. pa. 351) *sine* A *or* a : *sine* D *or* d :: *tang. observed geocentric lat. of the comet corresponding to the* 1*st or* 2*d obs.* : *tang. corresponding heliocentric lat. of the comet* E *or* e.

Each of the *curtate distances* y and z divided by the cos. of the corresponding helioc. lat. E and e, will give the *true distances* of the comet V. v, from the sun.§ Now *to find the angle contained by those distances*, add to or subtract from the places of the earth (according to the comet's pos with respect to the signs) the corresponding ∠'s of com. D, d, the sum. or diff. will be the *heliocentric longitudes* L, l, of the comet whose diff. F, is the *helioc. motion* of the comet in the plane of the ecliptic. Then as *rad.* : *cos.* F :: *cot. greatest of the two hel. lat.* : tang. X. Let this arc X be taken from the compl. of the least hel lat. and the rem. call x. Then cos. X : *cos.* x :: *sine gr. of the two lat.* : *cos.* ∠ *contained by the two vector rays or distances* V, v, *of the comet.*

Now by what is shewn pa. 405, the *place of the perihelion* may be found by this rule: take the log. of the least vector ray from that of the greatest, add 10 to the characteristic of half the remainder, it will be the tang. of an angle, from which 45° being subtr. the log. tang of the remainder added to log cot. $\frac{1}{4}$ of the mot. of the comet in its orbit, will be the log tang. of an ∠ to which $\frac{1}{4}$ of

* In the Naut. Alm. pa. 3, the log. of the dist. of the earth from the sun is given every 6th day in the month, the earth's mean dist. being 1.

† In the triangle ESC, pa. 399, let SC represent y and ES, B, then the ∠ SEC will be the elongation of the comet from the sun, and SCE that found by lines from the earth to the sun and comet, whence from plane trig. the propor. is evident.

‡ In this fig. pa. 351, if B represent the plane of the comet reduced to the ecliptic, then (32 Eucl. 1) TBS + STB the elong. = 180° — TSB; hence TSB the commut. = 180° — TBS — STB.

§ In the fig. pa. 351, *rad.* : cos. PST :: PS : SB the curt. dist. B being supposed as before the comet's place, &c.; hence rad. being taken = 1, PS the comet's true dist. (P being its true place) $= \frac{\text{SB}}{\text{cos. PST}}$.

the mot. of the comet in its orbit being added, the sum will be $\frac{1}{2}$ the *greatest true anomaly*, and their difference will be $\frac{1}{2}$ the *least* of the two true anomalies. These quantities doubled will be the two true anomalies, which will be both on the same side of the *perihelion*, when their diff. is the whole motion of the comet ; but on different sides, when it is their sum, which is equal to the whole motion of the comet.

Let the *perihelion dist.* be found by adding twice the log. cos. of the greatest of the halves of the two true anomalies, to that of the greatest of the two distances, the sum will be the log. of the perihelion distance required. See pa. 405.

The time in which the comet describes the two vector rays may be thus determined. To the constant log. 1.9149328,* add the log. tang. of $\frac{1}{2}$ each true anom. add the triple of this same log. tang. to the constant log. 1.4378116, the sum of the two numbers corresponding to those two sums of logs. will be the exact number of days corresponding to each true anomaly, in a parabola whose perihelion dist is 1. (by what is shewn pa. 403.) Find the log. of the diff. or sum of those two numbers, according as the two anom. are situated on the same, or on different sides of the perihelion ; to this log. add $\frac{3}{2}$ of the log. of the perihelion dist. the sum will be the log. of the time in which the comet describes the angle contained between the two vector rays ; as shewn pa. 406, which see.

The above is called the 1*st hypothesis ;* the following is the 2*d supposition* of this hypothesis. If the time thus found does not agree with the observed time, another value of the curt. dist. z is to be assumed, corresponding to the 2d obs. the val. of y corresponding to the 1st being still retained, and the helioc. long. and lat. from thence deduced, and all the operations indicated in the foregoing articles being repeated ; another expression will be found for the interval of time between the two observations. If this time approaches nearer the observed time, the 2d val. assumed for z is to be preferred to the 1st ; if not, a 3d val. for z is to be assumed, and by the increase or decrease of the errors, the value to be assumed, so that the interval of time calculated may agree with the observed one, will be easily discovered ; and therefore a parabola will be found which answers the two first observations, or the *first hypothesis*.

The parabola answering the two first obs. would be the true orbit if it also answered the 3d obs. but as this seldom or never happens, a *second hypothesis* becomes necessary, in which another parabola is to be found which answers the two first observations, by increasing or diminishing the curt. dist. y, preserved constant in the 1st hypot. and preserving it still constant, but varying the 2d assumed

* By what is shewn pa. 401, $b = \frac{1}{4} at^3 + \frac{3}{4} at$, and $a = 109.6155$, $\frac{3}{4}$ of which or $\frac{3}{4} a = 82.2116$, the log. of which is 1.9149328 ; also $\frac{1}{4}$ of $a = 27.4038$, the log. of which is 1.4378116 as above. Hence this rule is taken from the value of b.

dist. z until this 2d parab. is obtained. The 3d obs. calculated in those two parab. will shew which of them approaches nearest the true orbit sought To calculate this 3d obs in each hypot. the *time of the comet's passing the perih.* the *incl. of its orbit*, and the place of the nodes of each parabola, is first to be determined.

To determine the comet's passage at the perihelion. Find the number of days corresponding to one of the true anom. for ex to that which corresponds to the 1st obs. in the parab. whose perih. dist. is 1. (by Delambre's table, or as shewn pa. 403, &c. where the method of constructing the table is given) the log. of this number of days being added to $\frac{3}{2}$ of the log. of the perih. dist. (found above) will be the log. of the interval of time elapsed between the 1st obs. and the comet's passing the perih. which is to be added to or subtracted from the time of the obs. according as it was made before or after the passage of the comet at the perihelion.

To determine the place of the node we have this proportion ; sine of the 2d arc x : sine 1st arc X :: tang. comet's mot. in the eclip. : tang. of an angle r ; then rad. : sine least lat. :: tang. r :: tang. *dist. from the node.* (See also pa. 407.) From this dist and the helioc long. of the comet, found as shewn above, the *heliocentric long. of the node* is obtained. With this hel. long. and the dist measured on the orbit of the comet, the *place of the perihelion* is determined. *To find the dist. measured on the comet's orbit*, we have this proportion ; sine r : rad. :: dist. measured on the ecliptic : dist. required. *To determine the inclination* it will be rad. : sine r :: cos. least lat. : cos. ∠ of incl.

The elements of each parabola being determined, the *geocentric place of the comet* answering to the 3d obs. is computed in each by the following rules : 1st. Let the log. of the diff. between the time of the 3d. obs. and the time of the comet's passing the perih. be taken, from which take $\frac{3}{2}$ of the log. of the perih. dist. the rem. will be the log. of the diff. between the time of the 3d obs. and that of the comet's passage at the perih. of a parab. whose perih. dist. is 1. 2d. Let the true anomaly corresponding to this time be found, by solving the equation $t^3 + 3t = \frac{b}{\frac{1}{4}a} = \frac{b}{27.4038}$, as shewn pa. 402,* in which t = tang. $\frac{1}{2}$ the true anom. and b the time of describing it. 3d. When the mot of the comet is *direct*, add the true anom. to the place of the perih. but subtract it if the

* The mean proportional required here between $h + \frac{b}{54.8077}$ and $h - \frac{b}{54.8077}$ to find t (see pa. 402) will be obtained by finding the cube root of the ratio between the two quantities ; the root thus found will be the ratio between the four quantities ; hence this ratio multiplied by the least extreme, will give the next term, or one of the mean proportionals, and this multiplied again by the same ratio, will give the other mean proportional. The equation may be also solved by any of the known methods for solving cubic equations.

obs. was made before the comet's passing the perih. When the comet's mot. is *retrograde*, add the true anom. to the place of the perih. if the obs. was made before the passage, at the perih. but subtract it if the obs. was made after the time of perih. the *true helioc. long. of the comet in its orbit* will be thus obtained. See also pa. 407. 4th. The diff. between this long. and the long. of the ascending node will be the argument of the lat. of the comet. 5th. As rad. : cos. incl. :: tang. argum. of lat. : tang. of this arg. measured on the ecliptic, which added to the true place of the node, gives the *helioc. long. reduced to the ecliptic*. 6th. Rad. : sine arg. of lat. :: sine incl. of comet's orbit : sine of its hel. lat. which when the motion of the comet is direct, is north or south according as the argument of lat. is less or greater than 6 signs; but when retrograde, is north or south, according as the arg. of lat. is greater or less than 6 signs. 7th. Log. cos. hel. lat. + log. perih. dist. — log. of twice cos. $\frac{1}{2}$ the true anom. = log. of the *curt. dist. answering to the 3d. obs.* 8th. Log. curt. dist. — log. dist. of the earth from the sun + 10 added to the characteristic, or index = log. tang. of an angle, from which subtract 45°, and to log. tang. remd. add log. tang. compl. of $\frac{1}{2}$ the angle of commutation, the sum = log. tang. of an arc, which added to this compl. if the curt. dist. of the comet from the sun exceed the earth's dist. but subtracted if the comet's dist. be less than the earth's, the sum or diff. will be the angle of *elongation*; this angle added to, or subtracted from the sun's true place or long. according as the comet seen from the earth is east or west of the sun, will give the *geocentric long. of the comet*. 9th. Sine ∠ commut. : sine elongation :: tang. helioc. lat. of the comet : tang. of its geocentric lat. See these different cases exemplified in pa. 407 and 408, and in what follows. The long. and lat. thus found, ought to agree with those observed, if the parabola obtained were really the comet's orbit.

As these rules without sufficient examples may, especially to beginners, be rather difficult in their application, the following *example* of the comet, which was observed in *Europe*, about the beginning of *March*, 1742, is given.

1742. *Mean Time.*	*Obs. long. of the comet.*	*Obs. lat. north of the comet.*	*Long. of the sun calculated.*	*Log. of the earth's dist. from the sun.*	*Elong. of the comet from the sun.*
4th Mar. 16h. 9′50″	9s.16° 0′40″	34°45′37″	11s.14°27′44″	9.996910	58°27′4″w.
28 . . at 13 39 0	2 18 52 45	63 3 55	0 8 11 28	9.999840	
24 Apr. 9 39 0	3 1 5 53	50 32 50	1 4 27 16	0.003092	56 38 17E

1st. *Supposition.* y = 0 879 and z = 0.957, of the earth's mean dist. from the sun, which is taken equal 1; then C = 105° 42′ 8″, c = 61° 31′, C + A = 164° 9′ 52″, and c + a = 118° 9′ 17″; hence D = 15° 50′ 8″, and d = 61° 50′ 43″; therefore E = 12° 31′ 42″ north, and e = 52° 3′ 38″, and log. V = 9.954455, and log. v = 0.192159.

The places of the earth at the 1st and 2d. obs. being 5s. 14° 27′ 44″, and 7s. 4° 27′ 16″ respectively, hence D + 5s. 14° 27′ 44″ (the comet being *east* of the earth's place) = L = 6s. 0° 17′ 52″, and 7s. 4° 27′ 16″ — d (the comet being *west* of the earth) = l = 5s. 2° 36′ 33″ ; then L — l = F = 27° 41′ 19″ the comet's mot. in the eclip. Also X = 34° 37′ 11″, and x = 42° 51′ 7″, the angle contained by the two vector rays = 45° 22′ 8″, the comet's motion in its orbit.

Log. v 0.192159 — log. V 9.954455 = 0.237704, half of which, together with 10 added to its characteristic = 10.118852 = tang. 52° 44′ 38″, from which 45° being taken, leaves 7° 44′ 38″ ; whence log. tang. of 7° 44′ 38″ + log. cot. of 11° 20′ 32″ ($\frac{1}{4}$ of 45° 22′ 8″) = log. tang. of 34° 8′ 5½″ ; therefore 34° 8′ 5½″ — 11° 20′ 32″ = 22° 47′ 33½″, half the least true anomaly ; and 34° 8′ 5½″ + 11° 20′ 32″ = 45° 28′ 37½″ half the greatest true anom. Hence the least true anom. = 45° 35′ 7″, and the greatest 90° 57′ 15″, and their diff. being equal the comet's mot. they are both therefore on the same side of the perihelion. Hence log. perih. dist. is found = 9.883835.

To find the time in which the comet described the angle contained by the two vector rays, we have log. 1.9149328 + log. 0.007233 (log. tang. of 45° 28′ 37½″) = 1.922166, and log. 1.438112 + 0.021699 (triple log. same tang.) = 1.459512, the numbers corresponding to which are 83.592 and 28.808 respectively ; hence 112.400 days is the time corresponding to the true anomaly 90° 57′ 15″ in a parabola, whose perih. dist. is 1. In like manner the number of days, in the same parab. corresponding to the true anom. 45° 35′ 7″, is 36.579. Now taking the diff. of those times = 75.821 days (the two anom. being on the same side of the perih.) the log. of which is 1.879789 added to 9.825752 (½ log. perih. dist.) gives log. 1.705541 corresponding to 50.762 days, the time required.

Comparing this time with the interval 50.728½ between the two observations, it is found to exceed it by 0.033, if therefore a variation of 0.001 be made in the dist. z, in order to discover which way, and by how much the elements of the corresponding parabola will be changed.

2d. *Sup.* y = 0.879 and z = 956, and repeating the same calculations as in the foregoing sup. we find E = 12° 31′ 42″, e = 52° 1′ 54½″, log of the dist. V = 9.954455, and v = 0.191424, the helioc. long. L = 6s. 0° 17′ 52″, l = 5s. 2° 43″ 11″ ; the mot. of the comet in the eclip. = 27° 34′ 41″, and in its orbit = 45° 18′ 13″, the true anomalies 45° 32′ 3″ and 90° 15′ 16″, the corresponding days 36.529 and 112.056, log. perih. dist. = 9.883997, and the reduced time of describing the angle contained by the two vector rays 50.594 days. Hence increasing z by 0.001, the time is diminished by 0.168; therefore 0.168 : 0.001 :: 0.033½ : 0.0002 ; hence z is diminished by 0.0002 to obtain a parabola answering the required conditions.

3d. *Suppos.* y = 0.879, z = 0.9568, from which the helioc. lat. E = 12° 31′ 42″ and e = 52° 3′ 16½″; log. V = 9.954455 and log. v = 0.192009 ; L = 6s. 0° 17′ 52″ and l = 5s. 2° 37′ 53″ ; comet's mot. in eclip. = 27° 39′ 59″, motion in its orbit 45° 21′ 22″; the true anomalies 45° 34′ 28″ and 90° 55′ 50″; corresponding times 36.568½ and 112.330 days ; log. perih dist. = 9.883870 ; reduced time 50.728½ days, agreeing with observation.

Having therefore found a parab. corresponding to the two first obs. another must be found answering the same obs. by making a variation in the dist. y preserved constant in the 1st hyp.

Second Hyp. 4th *Suppos.* y = 0.878, and z = 0.957, from which E = 12° 42′ 11″, e = 52° 3′ 38″, log. V = 9.954257, log. v = 0.192159, L = 6s. 0° 31′ 54″, l = 5s. 2° 36′ 33″, comet's mot. in the eclip. = 27° 55′ 21″, angle contained by the two vector rays = 45° 17′ 56″, true anomalies 45° 44′ 56″ and 91° 2′ 52″, corresponding times 36.743 and 112.680 days, log. perih dist. = 9.883115, reduced time = 50.714, differing 0.014½ from the observed interval ; hence diminishing y by 0.001, the time is diminished 0.048 ; therefore 0.048 : 0.001 :: 0.014½ : 0.0003.

5. *Sup.* y = 0.8783, z = 0.957, hence E = 12° 39′ 2″, e = 52° 3′ 38″, log. V = 9.954316, log. v = 0.192159, L = 6s. 0° 27′ 40″, l = 5s. 2° 36′ 33″, comet's mot. = 27° 51′ 7″, angle contained by the vector rays = 45° 19′ 20″, true anomalies 45° 41′ 45″ and 91° 1′ 5″, corresponding times 36.689 and 112.590, log. perih. dist. = 9.883344, reduced time 50.729, agreeing with observation.

Two parabolas being therefore obtained answering the 1st and 2d observations, the 3d obs. must be calculated in each to find which of them approaches nearest the real orbit of the comet ; hence the place of the perih. the time of the comet's passage at it, the incl. to the eclip. and the place of the nodes of each parabola, must be first found.

In the first parab. R = 23° 40′ 15″, comet's dist. from the ascending node reduced to the eclip. at the 1st obs. 5° 25′ 45″, which added to its helioc. long. 4th March, 6s. 0° 17′ 52″ (its mot. being retrograde as 6s. 0° 17′ 52″ is greater than 5s. 2° 36′ 33″ at the 2d obs. and the comet after passing its perih.) gives 6s. 5° 43′ 37″ for the place of the node. Dist. of the comet from its node measured on its orbit = 13° 38′ 14″, which taken from the place of the node, gives the place of the comet in its orbit at the 1st obs. and its true anomaly being then 45° 34′ 28″, therefore these being added to the comet's place in its orbit, give 7s. 7° 39′ 51″ for the place of the perihelion ; $\frac{3}{2}$ of the log. of which added to log. of 36.568½ days, the time corresponding to the least true anomaly 45° 34′ 28″, gives 24.486 days for the interval of the elapsed time between the 1st obs. and that of the perih. which being taken from March 4d. 16h. 9′ 50″ or 4th

March 0.673½, the time of the 1st obs. fixes the passage of the perihelion on the 8th of Feb. at 0.187½. The incl. of the comet's orbit to the ecliptic is found = 66° 56′ 14″.

In the 2d parab. the same elements are, the ascending node in 6s. 5° 59′ 6″, place of the perih. 7s. 7° 53′ 42″, incl. 66° 47′ 14″, time of passing the perih. 8th Feb. 0.151½.

The geocentric long. for March 28, at 0.569 of the day in each parabola is thus calculated ; the interval between this time and the comet's passing the perih. is 48.381 days ; log. perih. distance 9.883870, its triple is 9.651610, its ½ = 9.825805, which taken from log. of 48.381 or 1.684675, leaves log. 1.858870 corresponding to 72.255 days, answering to 73° 11′ 7″ or 2s. 13° 11′ 7″ anomaly, this subtracted from the place of the perih. 7s. 7° 39′ 51″ (the comet being retrograde) gives 4s. 24° 28′ 44″ for the true helioc. place of the comet in its orbit, from which 6s. 5° 43′ 37″ the place of the ascending node being subtracted, leaves 10s. 18° 45′ 7″ the argum. of lat. which on the ecliptic is 11s. 11° 2′ 47″ ; hence the comet's helioc. long. is 5s. 16° 46′ 24″, and hel. lat. 37° 20′ 41″ north, the arg. of lat. of the comet, which is retrograde, being greater than 6s.

The sun's true place March 28th at 13h. 39′ is 8° 11′ 28″, and log. dist. from the earth is 9.999841, hence the true place of the earth seen from the sun is 6s. 8° 11′ 28″, from which 5s. 16° 46′ 24″ being subtr. gives 21° 25′ 4″ the angle of commut. Log. of the curt. dist. for 3d obs. is 9.974915, which being taken from 9 99841, leaves 0.024926, to the caracter. of which 10 being added, gives the sum is 10.024926 log. tang. of 46° 38′ 42″ ⅔, from which taking 45° the log. tang. remd. 1° 38′ 42″ + log. tang. 79° 17′ 28″ (compl. of ½ ∠ of commut.) = log. tang. 8° 37′ 39″, which taken from 79° 17′ 28″ (because the comet's dist. from the sun is less than the earth's) gives 70° 39′ 49″ or 2s. 10° 39′ 49″ the elongation. If the places of the sun, the earth, and the comet found as above, be marked on the ecliptic of an artificial globe (or any circle divided into 12 signs) it will be seen that the comet to an observer on the earth appears east of the sun. Hence the elongation is to be added to the sun's place to find the true geocentric long. of the comet, which is 2s. 18° 51′ 17″, which is less than the observed long. by 1′ 28″. In like manner the comet's geocentric long. in the 2d parabola March 28, is 2s. 18° 45′ 14″, which is less than the observed long. by 7′ 31″ ; hence neither of the two parabolas is the comet's orbit.

3d. *Hypothesis*. As the variations of the orbits are, however, sensibly proportional to those made in the curt. distances ; hence *to obtain the two curt. distances answering to the reqd. orbit*, we have from the rule of *false position* those two proportions ; as 6′ 3″ (diff.* of the two errors 1′ 28″ and 7′ 31″) : least of the

* If one of the errors was by excess and the other by defect, the *sum* of the errors would be here used. See this rule investigated, and different methods given in prob. 80. b. 1, and Corollaries Emerson's Algebra.

two 1′ 28″ :: 0.0007 and 0.0002 (corrections of y and z to obtain parabolas in the 1st and 2d obs.) : 0.000235 and 0.000065, corrections for those distances to obtain the orbit required.

Now as y, supposed = 0.879 gives an error of — 1′ 28″, and y, supposed = 0.8783, gives an error of — 7′ 31″, by diminishing y the error is increased ; hence the true value of y = 0.879 + 0.000235 = 0.879235. Reasoning in like manner we find z = 0.956735.

6, *Suppos.* y = 0.879235, and z = 0.956735, whence E = 12° 29′ 17½″ ; e = 52° 3′ 10½″, log. of the vector rays V = 9.954504 and v = 0.191963, L = 6s. 0° 14′ 37″ and l = 5s. 2° 38′ 19″, true anomalies = 45° 32′ and 90° 54′ 4″, the corresponding times 36.528 and 112.243 days, log. perih. dist. 9.884049 time of describing the angle contained by the two vector rays 50.729, place of the node 6s. 5° 38′ 29″, place of the perih. 7s. 7° 35′ 13″, inclination of orbit 66° 59′ 14″, and time of passing the perih. 8th Feb. at 4h. 48′. From those elements the geocentric long. on the 28th March, at 13h. 39′, is 2s. 18° 53′ 18″, and geoc. lat. 63 3′ 57″ north, agreeing with observation.

The following table of Mr. Lee's taken from Dr. Rees' New Cyclopedia, calculated on an extensive scale, and with immense labour, will be found extremely useful in comparing the computed orbits of new comets with those before observed, &c. The Elements of the foregoing comet determined as above, exactly agrees with that given in the following table by la *Caille*, except the time of the passage of the perihelion, which in the table is given Jan. 28th. 4h. 38′ 40″. The 11 days difference arises from the *old stile* being used in the table.

THE ELEMENTS OF NINETY-SEVEN COMETS.

No.	*Passage at per. mean time at Greenwich.*		*Long. of the per. on the orb. of the comet.*	*Per. dist. earth's being 1.*	*Long. of the ascending node.*	*Incl. of the orbit.*	*Motion.*
	Anno A. C.						
1	539 Oct. 20	15h. 0′ 0″	10s.13°30′ 0″	0.3412	1s.28°or7s.28′	10°+or—	D
	Anno Domini	*Old Stile.*					
2	837 March		9 19 3 0	0.5800	6s.26°33′ 0″	10° or 12°	R
3	1097 Sept. 21	21 36 0	11 2 30 0	0.7385	6 27 30 0	73°30′ 0″	D
4	1231 Jan. 30	7 12 40	4 14 48 0	0.9478	0 13 30 0	6 5 0	D
5	1264 July 6	8 0 0	9 21 0 0	0.445	5 19 0 0	36 30 0	D
	July 17	6 0 40	9 5 45 0	0.41081	5 28 45 0	30 25 0	
6	1299 Mar. 31	7 28 40	0 3 20 0	0.3179	3 17 8 0	68 57 0	R
7	1301 Oct	+ or —	9s. or 10s.	0.457	15° + or —	70°+or—	R
8	1337 June 2	6 25 0	1s. 7°59′ 0″	0.40666	2s.24°21′ 0″	32 11 0	R
	June 1	0 30 40	0 20 0 0	0.6445	2 6 22 0	32 11 0	
9	1351 Nov. 26	12 0 0	2 9 0 0	1.0000			D
10	1456 June 8	22 0 0	10 1 0 0	0.5855	1 18 30 0	17 56 0	R
11	1472 Feb. 28	22 23 0	1 15 33 30	0.54273	9 11 46 20	5 20 0	R
	1531 Aug. 24	21 18 30	10 1 39 0	0.56700	1 19 25 0	17 56 0	D
12	1532 Oct. 19	22 12 0	3 21 7 0	0.50910	2 20 27 0	32 36 0	D
13	1533 June 16	19 30 0	4 27 16 0	0.20280	4 5 44 0	35 49 0	R
14	1556 April 21	20 3 0	9 8 50 0	0.46390	5 25 42 0	32 6 30	D
15	1577 Oct. 26	18 45 0	4 9 22 0	0.18342	0 25 52 0	74 32 45	R
16	1580 Nov. 28	13 44 40	3 19 11 5	0.59553	0 19 7 37	64 51 50	D
		15 0 0	3 19 5 50	0.59628	0 18 57 20	64 40 0	
17	1582 May 7		8s.5° or 9s 11°	.23 or .04	7s. 5° or 21°	59° or 61°	R
18	1585 Sept. 27	19 20 0	0s. 8°51′ 0″	1.09358	1s. 7°42′ 30″	6° 4′ 0″	D
19	1590 Jan. 29	3 45 0	7 6 54 30	0.57661	5 15 30 40	29 40 40	R
20	1593 July 8	13 38 40	5 26 19 0	0.08911	5 14 15 0	87 58 0	D
21	1596 July 31	19 55 0	7 18 16 0	0.51293	10 12 12 30	55 12 0	R
	29	15 33 40	7 28 30 50	0.549415	10 15 36 50	52 9 45	R
*10	1607 Oct. 16	3 50 0	10 2 16 0	0.58680	1 20 21 0	17 2 0	R
22	1618 Aug. 7	3 2 40	10 18 20 0	0.51298	9 23 25 0	21 28 0	D
23	1618 Oct. 29	12 23 0	0 2 14 0	0.37975	2 16 1 0	37 34 0	D
24	1652 Nov. 2	15 40 0	0 28 18 40	0.84750	2 28 10 0	79 28 0	D
25	1661 Jan. 16	23 41 0	3 25 58 50	0.44851	2 22 30 30	32 35 50	D
26	1664 Nov. 24	11 52 0	4 10 41 25	1.025755	2 21 13 55	21 18 40	R
27	1665 Apr. 14	5 15 30	2 11 54 30	0.10649	7 18 2 0	76 5 0	R
28	1672 Feb. 20	8 37 0	1 16 59 30	0.69739	9 27 30 30	83 22 10	D
29	1677 Apr. 26	0 37 30	4 17 37 5	0.28059	7 26 49 10	79 3 15	R
30	1678 Aug. 16	14 3 0	10 27 46 0	1.23801	5 11 40 0	3 4 20	D
31	1680 Dec. 8	0 1 2	8 22 40 10	0.006030	9 1 57 13	61 22 55	D
	7	0 4 0	8 22 33 0		9 1 53 0	61 20 20	
	7	23 9 0	8 22 44 25	0.006170	9 2 2 0	61 6 48	
	7	20 38 39	8 23 26 48	0.006565	9 2 59 9	58 39 50	
	8	0 4 0	8 27 43 0	0.005920	9 1 53 0	61 20 20	
*10	1682 Sept 4	7 39 0	10 2 52 50	0.58328	1 21 16 30	17 56 0	R
		21 31 0	10 1 36 0	0.58250	1 20 48 0	17 42 0	
32	1683 July 2	3 50 0	2 25 29 30	0.56020	5 23 23 0	83 11 0	R
33	1684 May 29	10 16 0	7 28 52 0	0.96015	8 28 15 0	65 48 40	D
34	1686 Sept. 6	14 33 0	2 17 0 30	0.32500	11 20 34 40	31 21 40	D
35	1689 Nov. 21	14 55 40	8 23 44 45	0.016889	10 23 45 20	69 17 0	R
36	1698 Oct. 8	16 57 0	9 0 51 15	0.69129	8 27 44 15	11 46 0	R

No.	Passage at per. mean time at Greenwich.		Long. of the per. on the orb. of the comet.	Per. dist. earth's being 1.	Long. of the ascending node.	Incl. of the orbit.	Motion
37	1699 Jan. 3	8h.22′ 19″	7s. 2°31′ 6″	0.74400	10s.21°45′ 35″	69°20′ 0″	R
38	1702 Mar. 2	14 12 19	4 18 41 3	0.64590	6 9 25 15	4 30 0	D
39	1706 Jan. 19	4 22 39	2 12 29 10	0.42580	0 13 11 40	55 14 10	D
		4 56 4	2 12 36 25	0.426865	0 13 11 23	55 14 5	D
40	1707 Nov. 30	23 29 39	2 19 54 56	0.8597	1 22 46 35	88 36 0	D
		23 43 6	2 19 58 9	0.85904	1 22 50 29	88 37 40	D
41	1718 Jan. 3	23 38 39	4 1 30 0	1.02650	4 8 43 0	30 20 0	R
	4	1 14 55	4 1 26 36	1.02565	4 7 55 20	31 12 53	
42	1723 Sept. 16	16 10 0	1 12 15 20	0.998651	0 14 16 0	49 59 0	R
43	1729 June 14	11 6 40	10 22 40 0	4.26140	10 10 32 37	76 58 4	D
	12	6 32 2	10 22 16 53	4.0698	10 10 35 15	77 1 58	
44	1737 Jan. 19	8 20 0	10 25 55 0	0.22282	7 16 22 0	18 20 45	D
45	1739 June 6	9 59 40	3 12 38 40	0.67358	0 27 15 14	55 42 44	R
46	1742 Jan. 28	4 38 40	7 7 35 13	0.76568	6 5 38 29	66 59 14	R
		4 20 50	7 7 33 44	0.765555	6 5 34 45	67 4 11	
	27	4 14 39	7 10 49 23	0.7521	6 9 32 7	61 43 44	
47	1742 Dec. 30	20 25 40	3 2 41 45	0.83501	2 8 21 15	2 19 33	D
		21 15 16	3 2 58 4	0.838115	2 8 10 48	2 15 50	
48	1743 Sept. 9	21 16 18	8 6 33 52	0.52157	0 5 16 25	45 48 21	R
49	1744 Feb. 19	8 27 0	6 17 12 55	0.22206	1 15 45 20	47 8 36	D
50	1747 Feb. 20	7 10 40	9 7 2 0	2.19851	4 27 18 50	79 6 20	R
	17	11 44 38	9 10 5 41	2.29388	4 26 58 27	77 56 55	
51	1748 Apr. 17	19 25 0	7 5 0 50	0.84067	7 22 52 16	85 26 57	R
52	1748 June 7	1 24 15	9 6 9 24	0.65525	1 4 39 43	56 59 3	D
	New Stile.						
53	1757 Oct. 21	9 46 40	4 2 49 0	0.33800	4 4 0	12 48 0	D
54	1758 June 11	3 17 40	8 27 38 0	0.21535	7 20 50 0	68 19 0	D
*10	1759 Mar. 12	13 31 40	10 3 16 0	0.58349	1 23 49 0	17 39 0	R
		13 50 40	10 3 8 10	0.58490	1 23 45 35	17 40 14	
		13 48 16	10 3 16 20	0.58360	1 23 49 21	17 35 20	
55	1759 Nov. 27	0 2 37	1 23 34 19	0.80139	4 19 39 41	79 6 38	D
56	1759 Dec. 16	21 3 40	4 18 24 35	0.96599	2 19 50 45	4 51 32	R
57	1762 May 28	15 17 40	3 15 15 0	1.0124	11 19 20 0	85 45 0	D
		6 51 29	3 14 29 46	0.009856	11 19 2 22	85 3 2	
	29	0 18 28	3 15 22 23	1.01415	11 18 55 31	85 22 21	
58	1763 Nov. 1	19 43 18	2 24 51 54	0.49876	11 26 23 26	72 40 40	D
		20 58 14	2 25 1 6	0.49820	11 26 27 0	72 28 0	
59	1764 Feb. 12	13 42 16	0 15 14 52	0.55522	4 0 4 33	52 53 31	R
60	1766 Feb. 17	8 40 40	4 23 15 25	0.50533	8 4 10 50	40 50 20	R
61	1766 Apr. 22	20 46 20	8 2 17 53	0.33274	2 14 22 50	11 8 4	D
62	1769 Oct. 7	12 20 40	4 24 5 24	0.12376	5 25 0 43	40 37 33	D
		13 36 53	4 24 11 7	0.12272	5 25 6 33	40 48 49	
		14 56 39	4 24 16 0	0.12265	5 25 3 0	40 50 0	
		15 28 16	4 24 10 51	0.1227	5 25 4 41	40 49 33	
		12 34 9	4 24 11 8	0.1232852	5 25 2 24	40 48 29	
		15 42 2	4 24 15 53	0.12275	5 25 6 4	40 46 42	
		12 44 38	4 24 11 32	0.12327	5 25 3 40	40 47 56	
63	1770 Aug. 14	0 4 4	11 26 26 13	0.676893	4 12 17 3	1 34 30	D
	13	12 55 40	11 26 16 26	0.6743815	4 12 0 0	1 33 40	
	13	12 37 35	11 26 15 0	0.67435	4 11 54 50	1 34 31	

No.	Passage at per. mean time at Greenwich.		Long. of the per. on the orb. of the comet.	Per. dist. earth's being 1.	Long. of the ascending node.	Incl. of the orbit.	Motion.
64	1770 Nov. 22	5h.38′ 40″	6s.28°22′ 44″	0.52824	3s.18°42′ 10″	31°25′ 55′	R
65	1771 Apr. 18	22 5 7	3 13 28 13	0.90576	0 27 51 0	11 15 20	D
	19	5 1 21	3 14 2 54	0.90340	0 27 50 27	11 16 0	
66	1772 Feb. 18	20 41 13	3 18 6 22	1.01815	8 12 43 5	18 59 40	D
67	1773 Sept. 5	11 9 25	2 15 35 43	1.13390	4 1 15 37	61 25 21	D
		14 33 48	2 15 10 58	0.10120	4 1 5 30	61 14 7	
68	1774 Aug. 15	10 46 15	10 17 22 4	1.4286	6 0 49 48	83 0 25	D
69	1779 Jan. 4	2 2 40	2 27 13 11	0.71312	0 25 5 51	32 24 0	D
		2 15 10	2 27 13 40	0.7132	0 25 3 57	32 25 30	
70	1780 Sept. 30	18 3 30	8 6 21 18	0.09925	4 4 9 19	53 48 5	R
71	1781 July 7	4 32 0	7 29 11 25	0.775861	2 23 0 38	81 43 26	D
72	1781 Nov. 29	12 32 26	0 16 3 28	0.96101	2 17 22 52	27 13 8	R
		12 33 25	0 16 3 7	0.960995	2 17 22 55	27 12 4	
73	1783 Nov. 15	5 44 3	1 15 24 46	1.5655	1 24 13 50	53 9 9	D
74	1784 Jan. 21	4 47 40	2 20 44 24	0.70786	1 26 49 21	51 9 12	R
75	1784 Apr. 9	21 7 46	10 28 54 57	0.650531	2 26 52 9	47 55 8	R
76	1785 Jan. 27	7 48 44	3 19 51 56	1.143398	8 24 12 15	70 14 12	D
77	1785 Apr. 8	8 58 52	9 27 29 33	0.427300	2 4 33 36	87 31 54	R
78	1786 July 7	21 50 52	5 9 25 36	0.41010	6 14 22 40	50 54 28	D
79	1787 May 10	19 48 40	0 7 44 9	0.34891	3 16 51 36	48 15 51	R
80	1788 Nov. 10	7 25 40	3 9 8 27	1.06301	5 7 10 38	12 28 20	R
81	1788 Nov. 20	9 4 25	0 23 12 22	0.766911	11 21 42 15	64 52 32	D
82	1790 Jan. 15	5 5 39	2 0 14 32	1.7581	5 26 11 46	31 54 15	R
83	1790 Jan. 28	7 36 13	3 21 43 37	0.063286	8 27 8 37	56 58 13	D
84	1790 May 21	5 46 54	9 3 43 27	0.79796	1 3 11 2	63 52 27	R
85	1792 Jan. 13	13 35 9	1 6 29 42	1.293	6 10 46 15	39 46 55	D
86	1795 Dec. 14	23 17 53	5 13 37 0	0.227	11 29 11 0	24 17 0	D
	15	15 6 18	5 7 37 0	0.258	11 13 23 0	20 3 0	
	14	18 43 0	5 15 33 0	0.215	0 1 7 0	24 42 0	
87	1796 April 2	19 47 51	6 12 44 0	1.578	0 17 2 0	64 55 0	R
88	1797 July 9	2 43 31	1 19 34 48	0.52545	10 29 16 35	50 35 50	R
89	1797 April 4	11 32 21	3 14 59 0	0.48476	4 2 9 0	43 52 16	D
90	1798 Dec. 31	12 58 57	1 4 29 48	0.77968	8 9 30 44	42 23 25	
91	1799 Sept. 7	5 49 48	0 3 39 12	0.839865	3 9 31 59	50 55 37	R
		5 34 4	0 3 39 10	0.840178	3 9 27 19	50 57 30	
		4 24 39	0 3 36 0		3 9 34 0	50 52 30	
92	1799 Dec. 25	21 30 49	6 10 20 12	0.625810	10 26 49 11	77 1 38	R
93	1801 Aug. 8	13 22 39	6 3 49 0	0.2617	1 14 28 0	21 20 0	R
94	1803 Sept. 9	20 33 54	11 2 8 0	1.0942	10 10 17 0	57 0 0	D
95	1804 Feb. 13	13 31 7	4 28 44 51	1.07117	5 26 47 58	56 28 40	D
	13	15 30 39	4 28 53 32	1.072277	5 26 49 47	56 44 20	D
96	1805 Nov. 18	0 15 39	4 29 0 28	0.37567	11 15 6 51	15 58 12	D
97	1805 Dec. 31	6 1 36	3 19 23 40	0.89159	8 10 35 24	16 25 25	D

The following were calculated by M. *Burckhardt* ; 1, 3, 9, 58, 2d. 63, 3d in an elliptic orbit semimajor axis 3.1435, period, 65 Apr 19, in an hyperbolic orbit, 67, Sept. 14. 89, 90, 91, 3d. 93.

Those that follow by M. *Pingre*, No. 2, 4, 5, July 17, supposed to be the same as No. 14, 6, 8, 2d. 10, period 75½ years. 16, 17 (nearly) 21, 2d. 22 (nearly) 31, 35 (nearly) 53, 54 (see *mem. de l'Acad.* 1757 and 1759) 55, 58 (*mem. de l'Acad.* 1774) 59 (*mem.* 1774) 60 (nearly, *mem.* 1776) 61 (*mem.* 1778) 63, 6th. cal. in an elliptic orbit, *mem.* 1771. 63, 1st. in an elliptic orbit, mean dist. 3.08891, period 5.42886 years. 64, 65.

M. *Dunthorn* calculated the elements of the 5th, 1st supposed to be the same as No. 14.

Dr. *Halley* calculated the elem. of the following ; 8, 1st nearly 11, 1st 12, 13, 14, 15, the four last also nearly, No. 12 is supp. the same as No. 25, and No. 14 as 5. 16, 2d. 18, 19, 10,* (see *Newton's Prin.*) 23, 24, 26 (see the *prin.*) 27, 28, 29, 31, 1st. 2d. in an ellipse, mean dist. 138.2957, period 575 years (see *prin.*) 10,* 1st 2d in an ellipse, period 75 years. 32, 33, 34 and 36.

The following were calculated by M. *Douwes* ; 13, 30 (nearly) 41, 2d.

M. de la *Caille* calculated the following 20 (nearly) 37 (nearly) 38, 39, 1st 40, 1st 41, 1st 43, 1st see *Mem. de l'Acad. Roy.* 1763. 45, 46, 1st 47, 1st nearly. 50, 1st (*mem.* 1757) 10,* 1st 56.

M. *Euler* in his *Theoria motuum Planetarum et Cometarum*, calculates the following, 31, 4th an elliptic orbit. 46, 3d 62, 3d an elliptic orbit.

The 39, 2d 40, 2d 46, 2d 47, 2d. 52 (nearly) 57, 2d were calculated by M. *Struyck*.

The 42 (see *Newton's prin.*) and 44, were calculated very accurately by M. *Bradley*.

M. *Klinkenberg* calculated 48. M. *Betts* calculated 49 very accur. (*Phil. trans.* vol. 43.) M. *Chezeaux* 50, 2d. M. *Maraldi* 51, 10,* 3d. (*mem. de l'Acad.* 1759) 57, 3d. M de la *Lande* 10,* 3d 57, 1st (*mem. de l'Acad.* 1762 and 1763) 62, 1st (*mem.* 1769 and 1765.) M. *Prosperin* 62, 2d 86, 3d. M. *Lexell* 62, 4th an elliptic orbit. (See *mem.* 1795, pa. 430) 63, 2d an elliptic orbit, semimajor axis 3.14786, period 5.585 years. (*Phil. trans.* vol. 69.) Sir *Henry Englefield* 62, 5th. M. *Mechain* 68, 69, 2d 70, 71, 72, 73 (nearly) 74, 76, 77, 78 (*mem.* 1786) 80 (*mem.* 1788) 81, 83, 91, 1st (see *Connoissance des Tems*, an. 12) 92 (La *Lande* supposes this to be the same as No. 37) 94 (Con. des T. an. 14) *Chev. d'Angos* 69, 2d 75. M. *Legendre* 72, 2d (see *Nouvelle Methodes*, &c.) 96 and 97. P. de *Saron* 79. (*Mem.* 1787) 82 (*con. de T.*) M. *Zach* 86, 1st 91, 2d (*con. de T.* an. 12) M. *Bouvard* 86, 2d 88. Dr. *Olbers* 87. M. *Gauss* 95, 1st (*con. de T.* an. 15) 2d. con. an. 1808.

The preceding article being nearly written, when, from the politeness of Professor *Farrar* in Cambridge, Massachusetts, the author had been favoured with his observations on the present comet (1811) from Sep. 6 to Nov. 12.* From these observations those of Sept. 26, Oct. 17 and Nov. 10, have been selected for calculating the elements of this comet's orbit; as more nearly conforming with the directions given by *Newton*. As these sheets are immediately wanting for the press, there is therefore little time to enter into calculations so tedious as the present: the following results have, however, been hastily deduced; the time being changed to that of the meridian of Greenwich, allowing 71° 7′ for diff. long. and the correct distances† of the comet, for Sept. 26, 12h. 11′ 9″ being taken from Arcturus 30° 11′ 46″ from Lyra 60° 15′; on Oct. 17, 11h. 20′ 48″ from Arct. 34° 2′ from Lyra 27° 26′ 35″; Nov. 10, 12h. 34′ 58″, from Lyra 16° 30′ 5″, from Deneb 33° 10′.

* The following being, according to Mr. Farrar the most correct, are inserted here, and the Cambridge times, as given by him, retained. They may serve as an exercise for learners in making more accurate calculations, &c.

	ARCTURUS.		LYRA.	
	Appar. time.	*Appar. dist.*	*App. time.*	*Ap. dist.*
Sept. 6d.	7h. 52′ 14″	47° 43′ 20″	8h. 3′ 14″	82° 11′ 0″
9	7 59 20	45 38 20	7 54 40	79 32 18
10	8 19 38	44 53 16	7 47 31	78 30 46
14	8 38 18	42 0 15	8 28 20	74 33 50
18	7 39 23	38 58 45	7 27 2	70 18 50
26	7 26 41	30 8 15	7 32 16	60 17 10
30	7 12 0	30 49 30	7 19 20	54 38 30
Oct. 2	7 33 12	29 54 45	7 49 40	51 37 45
17	6 36 20	34 4 30	6 41 50	27 27 05
		POLAR STAR.		
(21) 19	8 6 30	49 0 0	7 59 25	24 13 15
21	8 58 10	50 34 15	8 55 0	21 7 15
23	7 23 30	52 20 30	7 17 30	18 27 45
29	7 3 50	57 40 15	7 1 20	12 24 45
Nov. 2	7 15 5	61 17 45	7 7 5	11 32 37
4	7 23 0	63 6 45	7 18 30	12 12 45
10	7 58 20	68 1 0	7 50 30	16 30 05
11	8 8 30	68 38 15	8 2 50	17 21 55
12			7 15 57	18 11 21

The distances were also taken from *Deneb* α Cygni (or α *Arided* as marked on Cary's Globes) as follow. Nov. 10d. 8h. 6′ 20″ ap. dist. 33° 10′ 25″ Nov. 11. 8h. 11′ 50″ dist. 33° 12′ 25″ and Nov. 12. 7h. 24′ 45″ dist. 33° 14′ 55″. It would have been better if the distances could have been taken at one instant by two observers and the altitudes given, as then the trigonometrical calculations, and allowances would become more simple. But the alt. in order to allow for refraction, &c. can be found nearly by the Globes.

† The distances corrected above, were not found from strict calculation, but when the star and comet were found to have nearly the same alt. a mean of the alt. was taken, and refraction being allowed, the dist. was increased as cos. variation of the altitudes from refraction, &c. When the star and comet were found to be nearly vertical, then the diff. of their refractions were subtracted. It being too tedious to give the trig. cal. not having 1 day to perform the oper. Some allowance was made for the comet's motion, but the aberrations and parallax were omitted, the calculation not being rigorously exact.

1811 Mean time at Greenwich.	obs. long. of the comet	obs. lat. north of the comet.	sun's long. calculated.	log. of the earth's dist. from the sun.	comet's elong. from the sun.
Sept. 26d. 12h. 2′ 32″	5s. 18° 2′	46° 17′	6s. 2° 49′ 57″	0.000715	14° 47′57″W.
Oct. 17 11 6 18	7 12 8	62 40	6 23 41 0	9.998334	18 27 0 E.
Nov. 10 12 19 14	9 12 30	45 15	7 17 44 33	9.995520	54 45 27 E.

Now the mean distances of the comet from the sun, by projection, was found 1.058 corresponding to the obs of Sept. 26.—And 1.422 corresponding to the obs. of Nov. 10, the earth's mean dist. being 1. From which data the elements of the comet may be found as before directed.*

Mr. *Farrar* remarks, that with a common night glass, he observed a dark ground of considerable extent immediately surrounding the coma, exterior to which a sort of halo, which after making a curve of about 180°, receded in a tangential direction forming the two branches of the tail, of which the convex, on that next the sun was somewhat the longest, and both a little incurvated ; the length of the tail, found by taking the dist. of a star at its extremity and the comet, was Sept. 10, in the evening 5°, breadth 2° ; Sept. 13, even. 7° 10′ ; Sept. 18, 12° ; Oct. 19, the evening being fine, the tail measured 14½°. Nov. 6, the diameter of the head including the coma, was measured with an object glass micrometer, fitted to a 3 feet Gregorian Reflector, made by Short, and found 2′ 46″. The nucleus had very much the colour and nearly the apparent mag. of Saturn. The exterior light surrounding the comet before mentioned, was judged to be five times the diameter of the head. Mr. *Farrar* further remarks, that the curved part of this light seemed very much to resemble in form, a current of water flowing round a stone or other obstacle placed in it.

* We had thus far proceeded when the sheets were wanting for the press, we may however resume the calculation in some of the following articles. Mr. N. *Bowditch* of Massachusetts, determines the elements of this comet as follow. Perih. dist. 1.032, time of passing the perih. Sept. 12d. 3h. Greenw. time, place of the perih. reckoned on the comet's orbit 2s. 15° 14′, long. of the ascending node, 4s. 20° 24′, incl. of its orbit to the eclip. 73°. Its motion he finds to be *retrograde*.

Mr. *Wood* of Richmond says that the mot. is *direct*, as it moves according to the order of the signs since its *appearing* ; but it is evident that its motion may be direct, as seen from the earth, though retrograde as seen from the sun, from which this motion is estimated in the cal. The *heliocentric longitudes* immediately shew whether the comet be direct or not, and not the geocentric.

CHAP. XI.

OF THE FIXED STARS.

HAVING given as comprehensive an account of the Solar System as an elementary work of this nature would admit, we shall now give a short description of those bodies which are situated beyond the limits of this System, and which are called *fixed stars*, from their not having any proper sensible motion of their own, except a few. Of their general division into constellations, &c. we have already given an account in the first part.

From the most accurate observations, the whole diameter of the earth's orbit, seen from them, is found to dwindle into a point, or in other words they are found to have no sensible parallax, and must therefore be immensely distant from us. From this circumstance, and that when examined by the most powerful telescopes, their disks appear but as luminous points, it is with reason inferred, that their magnitudes must be very great, as otherwise they would not be visible; and considering the weakness of reflected light, there can be no doubt but that they shine with their own light. They therefore differ from Planets in these two circumstances, that they do not borrow their light, or reflect it, as the planets do, and that their apparent diameters are not increased by telescopes. The smallness of their appar. diameters is also proved from their rapid disappearance in their occultations with the moon, the time of which not amounting to 1″ proves, as *Laplace* remarks, that their appar. diam. is less than (5″) 1″62. And as the smallest stars are subject to the same motions as the most brilliant, and constantly preserve the same relative positions, it is therefore probable that all those bodies are of the same nature and placed more or less distant from the planetary system. Whether they have planets revolving around them or not, is a question that can never be decided; but as they seem to be of the same nature with the sun, and of a mag. at least equal to him, analogy would lead us to suppose that they are destined to perform the same functions, and are, therefore, probably suns to other systems; for there are millions of them that are not at all visible to an inhabitant in our system.

The number of the stars visible to the naked eye is about 1000, as may be seen by reckoning all those to the 6th mag. incl. which are on a hemisphere of the celestial globe. But from the improvements in reflecting telescopes, Dr. *Herschel* and other astronomers have discovered that their number is great beyond conception. Every improvement of the telescope discovers stars not visible before, so that there seems to be no limits to their number or to the extent of the universe.

We have marked in the table of the constellations, in a separate column, all those stars that are not single, when viewed with good telescopes, but as some consist of 3, 4, or more stars, we shall give an account here of the most remarkable, following the order of the table, page 28.

In *Aries* α and β are double.

In *Taurus* Aldebaran is quadruple, the largest in the Pleiades double.

In *Gemini* α or Castor is double, β or Pollux, or Hercules consist of several, there are 11 marked on it, on Cary's Globe.

In *Leo* Regulus, γ, and some others are double.

In *Virgo* γ is double, and 9 quadruple.

In *Capricorn* α is double, one is much larger than the other, the largest is marked 1, the other 2.

In *Aquarius* β is double.

In *Ursa Minor* α is double, very unequal, the largest white, the smallest red.

In *Ursa Major* Mizar is double, or triple, two are easily distinguished, one larger than the other, the smallest is called Alcor.

In *Draco* υ 1, υ 2 appears single to the naked eye.

β in *Cepheus* is double. In *Cassiopeia* α or Shedir, and ε are doub. a small star marked *d* is triple. α or *Capella*, and β in *Auriga* are double. The star marked 2 in *Lynx* is double. *Cor Caroli* is double. In *Bootes* Arcturus is triple or quadruple, two of the stars in it appear of the 8th mag. ε or Mirach is double, very unequal, largest reddish, smallest blue, or faint blue, very beautiful δ, ζ, η, and others are double. In *Corona Borealis* ζ and σ are quadruple, υ is quintuple. In *Hercules* α, γ and others appear double, the stars marked 70 and 71, also 80 and 81 appear single to the naked eye. In *Lyra* α appears double, very unequal, the smallest appears with a power of 277. Dr. *Herschel* measured the *diameter* of this star and found it = 0″3553. β is quadruple, unequal, one white, three reddish, δ 1 and δ 2, appear as one, ε and others are double. β, σ, λ, 32, o2 and others, are double, γ triple, μ is also double, unequal, largest white, smallest blue. In *Delphinus* β and γ are double. In *Equuleus* γ, σ, ι, and others are double. In *Pegasus* ε or Enir and e are double. In *Andromeda* α and γ or Almaach, are double, the latter are unequal, the largest reddish white, the smallest blue, inclining to green, a beautiful object. In *Musca Borealis* the star marked 33 is double, 41 is triple, 3d mag. In *Perseus* ζ is triple, μ, H 31, and others, are double.— In *Serpens* β, δ and others are double. In *Ophicicus* α has two small stars of the 6th mag. nearly touching it, λ and others are double. In *Cetus*, Menkar is double, one of the 2d, and the other of the 6th mag. Mira, ι and others are double, Mira is changeable when greatest of the 2d, when smallest invis. period 344d. In *Orion*, Betelgeux, and Rigel, ζ, δ, η, τ, e and others, are double, ι, υ 1 and υ 2 contain each 6 or 7 small stars, σ has 6 or two triple stars. In *Canis Minor* Procyon has two stars of the 9th mag. very near its body, the star marked 31 is double. In *Hydra* Cor Hydra is triple, the star marked 2 is double and variable. In *Corvus* δ is double. In *Centaurus* α is double, the 1st is of the 1st and the 2d of the 4th mag. In *Piscis Australis* Fomalhaut has a star of the 6th mag. near its edge, ε is double, the 1st of the 3d, the 2d of the 5th mag. In the *Ship Argo* Canopus is double, the 1st is of the 1st, the 2d of the 6th mag. μ in the milky way has on its south side an innumerable multitude of stars, and in its body 9 or 10. The southern constellations have not been examined with the same care as the northern, which is the reason that so few of them are marked double, &c. Dr. *Herschel* has given a catalogue of the double stars in the Phil. trans. 1782 and 1785.

Many of the stars observed by ancient astronomers, do not appear at present, and others are at present observed which are not found in their catalogues. It was the appearance of a new star about 120 years before J. C. that caused *Hipparchus* first to undertake making a catalogue. There is however no account where this star appeared. A second is said to have appeared in the year 130 ; a 3d in 389 ; a 4th in the 9th century, in 15° of *Scorpio ;* a 5th in 945, and a 6th in 1264 ; the accounts we have of these are however imperfect. *Cornelius Gemma* on Nov 8, 1572, observed, in the chair of *Cassiopeia*, the 1st of which we have any regular account. It exceeded *Sirius* in brightness, and was seen at midday. It first appeared larger than Jupiter, then gradually decayed, and in 16 months vanished. Some suppose that it was this which appeared in 945 and 1264.

David Fabricius, on Aug. 13, 1596, discovered a new star in the neck of the whale in 25° 45′ of Aries, lat. 15° 54′ S. In Oct. the same year it disappeared. It was again seen in 1637 ; the periodic time between its greatest brightness is determined to be 333 days. Its greatest brightness is that of the 2d mag. and least that of a star of the 6th. But its greatest splendour and also its period, are found to be variable.

William Jansenius, in 1600, discovered a changeable star in the neck of the Swan. *Kepler*, who wrote a treatise on it, fixes its place in ♒ 16° 18′, with 55° 30′ or 32′ S. lat *Ricciolus* observed it in 1616, 1621, 1624, and 1629 ; and says that it was invisible in 1640 and 1650. *Cassini* observed it in 1655, from which it increased to 1660, then grew less, and at the end of 1660 disappeared. In Nov. 1666, it again appeared, and disappeared in 1681. In 1715, it appeared of the 6th mag.

P. *Anthelme* on June 20, 1670, discovered another changeable star near the swan's head. In Oct. it disap. and appeared again on March 17, 1761, and disap. Sep. 11. In March, 1672, it appeared again, disap in the same month, and was not observed since. Its long. was ♒ 1° 52′ 26″, lat. 47° 25′ 22″ N. The days are here as in the new stile.

In 1686 *Kirchius* observed χ in the swan to be a changeable star ; and from 20 years obs. it was found that the period of the return of the same phases is 405 days ; its magnitude is, however, subject to some irregularity. In 1604 *Kepler* discovered a new star in the right foot of Serpentarius, which exceeded even Jupiter in mag. Near the horizon it appeared white, in every other posit. it continually varied its colour into some of the colours of the rainbow. It disappeared in 1605. and was not seen since. Its longitude was ♐ 17° 40′, lat. 1° 56′ N. it had no parallax.

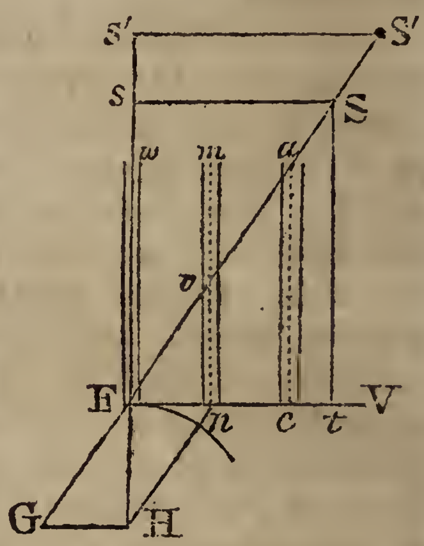
The stars β and γ in *Virgo* were found by *Montanari* to be wanting. They were visible in 1664, but were wanting in 1668. He found θ in the *serpent* visible from the time he observed until 1695. ψ in *Leo* disappeared, and was again seen in 1667. β in the *head of Medusa* also varied its mag.

Cassini discovered *one* new star of the 4th and *two* of the 5th mag. in *Cassiopeia;* he afterwards discovered *five* new stars in the same constel. three of which disappeared. He discovered *two* new stars in Eridanus, one of the 4th and the other of the 5th mag. He observed that ε in the *Little Bear* disappeared: that α in *Andromeda* which had disappeared, had again appeared in 1695: that in place of υ there are two stars more northerly, and that ξ is diminished.

Maraldi says that κ which was of the 3d mag. in 1671, was of the 6th in 1676, Dr. *Halley* found it again of the 3d in 1692, it was almost imperceptible, but in 1693 and 1694 it was of the 4th mag. In 1704 he discovered a new star in *Hydra*, in a rt. line with π and γ. The period of its changes is about 2 years. J. *Goodricke* has found the periodic variation of Algol in *Perseus* to be about 2d. 21h. He has also discovered that β *Lyræ* completes all its phases in 12d. 19h. δ *Cephei*, according to him performs the periodic variation of its phases in 5d. 8h. 37½′. E. *Pigott* has discovered that η *Antinoui* is a variable star, and fixes the period of its variation at 7d. 4h. 38′. For a further detail, consult *Vince's Astr.* or *Phil. trans.* 1785, and *Herschel's* remarks and method of observing these changes, *Phil. trans.* 1796. In the *Phil. trans.* for 1783, in a paper on the *proper motion of the Solar System*, he has given a large collection of stars which were formerly seen, but are now lost; also a catalogue of variable stars and of new stars.

These variations in those stars, considered as fixed, have afforded ample scope for conjecture. *Maupertuis* supposes the variations to arise from their quick motion round their axis, which he thinks may reduce them to very oblate spheroids, like a mill stone, and that when the flat side is presented to the earth, they become nearly invisible. *Laplace* remarks, that the extensive spots which these fixed stars present to us periodically in turning round their axis, nearly in the same manner as the last satellite of Saturn, and the interposition of large opake bodies which circulate round them, are sufficient to explain their periodic variations; and further remarks, that as to those stars which suddenly appear with a very brilliant light, and then vanish, it may be supposed that this takes place by means of great conflagrations on their surface, occasioned by extraordinary causes. As *light* takes a considerable time to pass from us to the fixed stars, it may have considerable effect in changing the apparent place of those that become invisible, when they again reappear. And as the nature of the *aberration* of the stars is not only useful to be known in this inquiry, but also in correcting their apparent places to obtain the true, we shall here collect, principally from *Vince's Astronomy*, what is most practical and important on this subject.

The situation of any object in the heavens, is determined by the position of the axis of a telescope annexed to the instrument with which we measure ; for the telescope is so placed, that the rays of light from the object may pass through it in the direction of its axis or length, and then the index shews the angular dist. required. Now if light take a determined time in paſsing from one object to another, when a ray from any distant object as a star descends down the axis, the position of the telescope must be different from what it would have been, if light had been instantaneous, and therefore the place to which the telescope is directed, will be different from the true place of the object. For let S′ be a fixed star, VF the direction of the earth's motion, S′F the direction of a particle of light entering the axis *ac* of a telescope at *a*, and moving through *a*F while the earth moves from *c* to F, then if the telescope continue parallel to itself, the light will descend in the axis. For let the axis *nm*, F*w*, continue parallel to *ac*, then considering each mot. as uniform (the rotary mot. of the earth being here neglected, as producing no sensible effect) the spaces described in the same time will continue in the same proportion ; but *c*F : *a*F :: *cn* : *av*, and *c*F, *a*F are, by suppos. described in the same time ; *cn*, *av*, are therefore described in the same time ; hence when the telescope comes into the situation *nm*, the particle of light will be in the axis at *v*, and this is true for every instant, the position of the telescope remaining the same ; hence when the telescope is at F, the place of the star, as determined by it, will be *s*′ and the angle S′F*s*′ will measure the change made in the appar. position of the star from the motion of the earth, combined with the progressive motion of light, or the *aberration*. If we therefore take FS : F*t* :: the vel. of light : the earth's vel. and join S*t*, and complete the paral. F*t*S*s*, the ∠ FS*t* will represent the aberr. S will be the true place of the star, and *s* the place determined by the instrument.



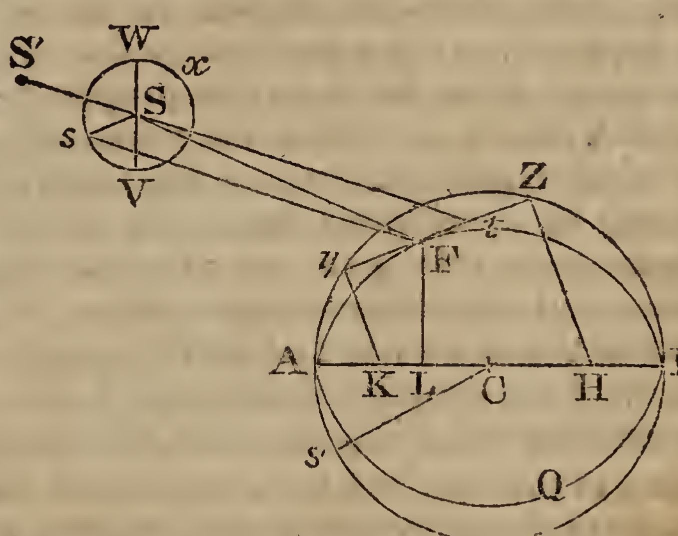
A similar change will be produced in the place of the star, seen with the naked eye For if a ray of light fall upon the eye in motion, its relative mot in respect to the eye, will be the same as if equal motions were impressed in the same direction upon each, at the moment of contact ; as equal motions in the same direction, impressed upon two bodies, will not affect their relative motions, and therefore the effect of one upon the other will not be allowed.

Let VF be a tangent to the earth's orbit at F, which will represent the direction of the earth's mot. at F, S a star, join SF and produce it to G, and make FG : Fn :: vel. of light : vel. of the earth in its orbit; complete the paral. and draw the diag. FH Now as FG represents the mot. of light, and nF that of the earth in its orbit, let a mot. Fn, equal and oppos. to nF, be conceived to be impressed upon the eye at F, and upon the ray of light, then the eye will be at rest, and the ray, by the two motions FG, Fn, will describe the diagonal FH; this is therefore the rel mot. of the ray in respect to the eye itself. The object will therefore appear in the direction HF, and the angle GFH = FSt will measure the diff. between its appar. and true place, as before. The place, therefore determined by the instrument, is properly called its apparent place.

Now sin. FSt : sine FtS :: Ft : FS :: vel. of the earth : vel. of light; hence sine of aberration = sin. FtS $\times$ vel. of the earth $\div$ vel. of light. If we therefore consider the vel. of the earth and light constant, sine aber. or the aber. itself, as it never exceeds 20$''$, varies as sine FtS, and is therefore greatest when FtS = 90°; taking s = sin. FtS, it will be rad. : s :: 20$''$: $s \times 20''$ the aberration.

By obs. ∠ FSt = 20$''$; hence when FtS = 90°, vel. of the earth : vel. of light :: sin. 20$''$: rad. :: 1 : 10314. The aber. S$'s'$ lies from the true place of the star, in a direction paral. to the direction of the earth's motion, and towards the same part.

While the earth makes one revolution in its orbit, the curve described by the appar. place of a fixed star, parallel to the ecliptic, is a circle. For let AFBQ be the earth's orbit, K the focus in which the sun is, H the other focus, on AB the greater axis let a circle be described in the same plane; to the point F draw the tangent yFZ, and ky HZ perp. to it, then the points y and Z will be always in the circumference of the circle (*Vince's* conic sect. prop. 5, el. or Emerson's, b. 1, prob. 20) Let S be the true place of the star, out of the plane of the ecliptic, and therefore elevated above the plane AFBQ, and let tF be to FS as vel. of the earth to vel of light; complete the paral. FtSs, and by what is shewn in the first part of this art. s will be the star's appar. place. Let FL be drawn perp. to AB, and let WsVx be the curve described by the point s; draw WSV parallel to FL. Now from a well known principle in physics, the

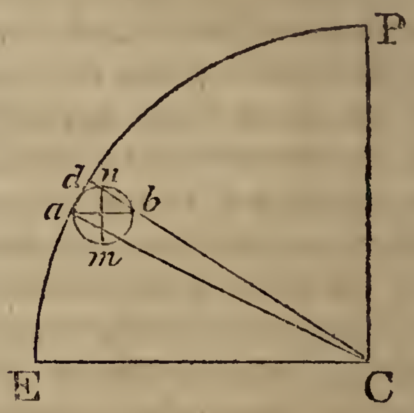


vel. of the earth varies as $\frac{1}{Ky}$, or as HZ (*Vince's* con sect. pr. 6 el.) but tF, or Ss, represents the earth's vel. hence Ss varies as HZ. And as Ss, SV are paral. to Fy, FL, the angle sSV = yFL = ZHL; for LFZ added to each makes two rt. angles, the angles at

L and Z being right angles Hence as Ss varies as HZ, and sSV = ZHA, the figures described by the points s and Z must be similar ; but Z describes a circle in the time of one rev. of the earth, s must therefore describe a circle about S in the same time. As Ss is always paral. to tF in the plane of the ecliptic, the circle WsVx is therefore paral to the ecliptic. Also as S and H are two points similarly situated in WV and AB, it appears that the true place of the star divides the diam. (which although in a different plane, may be considered as perp. to the greater axis of the earth's orbit) in the same ratio as the focus divides the greater axis. But as the earth's orbit is nearly a circle, S may be considered in the centre of the circle without any sensible error.

The earth's orbit AFBQ being considered circular, and therefore coinciding with AZB, draw Cs' paral. to Ss or yF, s' will then be the point in that circle corresponding to s in the circle WsV ; and as Fs'= 90° the appar. place of the star in the circle of aber. is always 90° *before* the earth's place in its orbit, and hence the appar. angular vel of the star and earth, about their respective centres, are always equal. Moreover S' being at an indefinitely great dist. the true place S being supposed not altered from the earth's mot. or the star to have no parallax, and FS being considered as always parallel to itself, it will be always directed to S'. Hence also the appar place of the sun being oppos to the earth's, the star's appar. place, in the circle of aberration, is 90° *behind* that of the sun

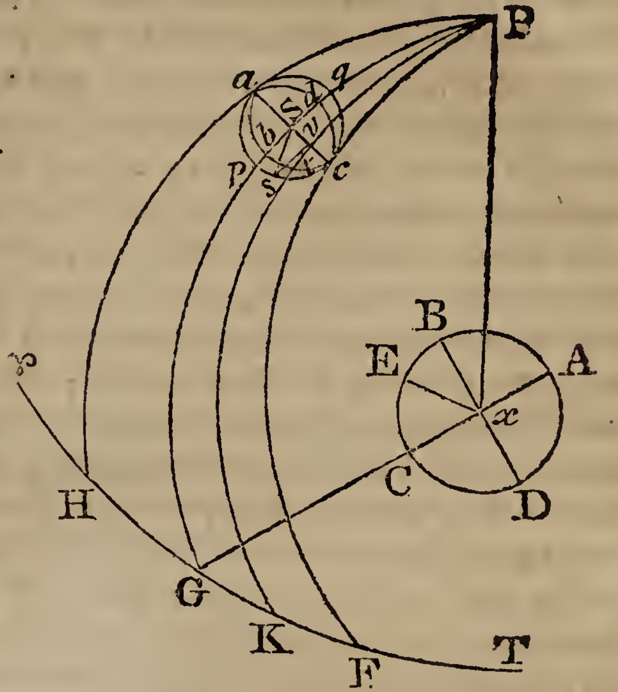
A small portion of the heavens being considered as perp. to a line joining the earth and star, the circle *anbm*, paral. to the ecliptic, described by the appar. place of the star, projected on that plane, will (from the principles of orthographic project. see Emerson's tracts) be an ellipse ; hence the star's appar. path will be an ellipse, and the trans. will be to the conjugate as rad. to sine of the star's lat. For let CE be the plane of the ecliptic, P its pole, PE a secondary to it, PC perp. to EC, C, the



place of the eye, and let *ab* be paral. to CE, then *ab* will be the diam. of the circle *anbm* of aberration, which is seen most obliquely, and therefore that diam which is projected into the lesser axis of the ellipse ; let *mn* be perp. to *ab*, and it will be seen directly, being perp to a line drawn from it to the eye, and will therefore be the greater axis ; let C*a*, C*bd*, be drawn, and *ab* will be the projection of *ab ;* and as *ad* may be considered as a straight line, it will be, *mn* or *ab* the greater axis : *ad* the lesser axis :: rad. : sine *abd*, or EC*d* the star's lat. Hence the circle is projected on a plane making an angle with it equal to the comp. of the star's lat. for *bad* is the comp of *abd*, or of the star's lat.

The lesser axis *da* coinciding with a secondary to the ecliptic, is therefore perp. to it ; and the greater axis *mn* is parallel to it, its posit. not being altered by projection. Hence in the pole of the ecliptic the sine of the lat. being rad. the ellipse becomes a circle; and in the plane of the ecliptic sine star's lat being = 0, the lesser axis vanishes, and the ellipse becomes a straight line, or rather a very small circular arc.

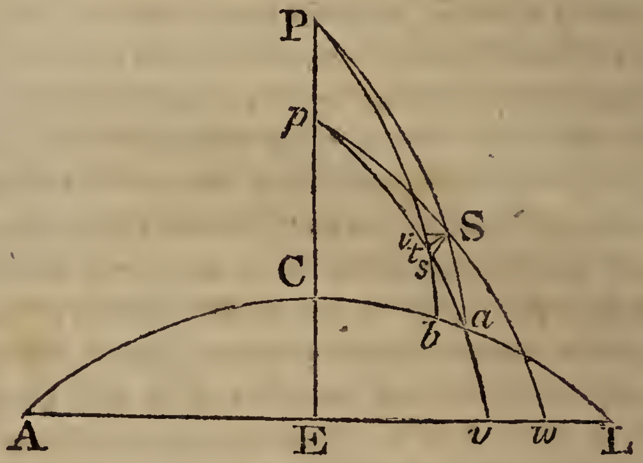
To find the aberration in *Latitude* and *Longitude*; let ABCD be the earth's orbit, supposed a circle with the sun in the centre at x, let P be in a line drawn from x perp. to ABCD so as to represent the pole of the ecliptic; let S be the star's true place, and let $apcq$ be the circle of aber paral. to the ecliptic, and $abcd$ the ellipse into which it is projected; let ♈T be an arc of the ecliptic, and draw the secondary PSG to it, and, by one of the foregoing articles, it will coincide with the lesser axis bd, into which the diam. pq is projected; let G CxA be drawn,



and it will be parallel to pq, and BxD, perp. to AC, is paral. to the greater axis ac; then, when the earth is at A, the star is in conj. and when the earth is at C, it is in oppos. Now the star's place in the circle of aber. being always 90° *before* the earth in its orbit, as shewn before, when the earth is at A, B, C, D, the appar. places of the star in the circle will be at a, p, c, q, and in the ellipse at a, b, c, d. To find the place of the star in the circle when the earth is at any point E, take the angle pSs = ExB, and s will be the corresponding place of the star in the circle; to find the projected place in the ellipse, draw sv perp to Sc, and vt perp. to Sc in the plane of the ellipse, then t will be the appar. place of the star in the ellipse; let st be joined. and it will be perp. to vt (the project. of the circle into the ellipse being in lines perp. to the ellipse) draw the secondary PvtK, which, as to sense, will coincide with vt, unless when the star is very near P; hence, except in this case, the rules here given will be sufficiently accurate.

Now as cvS is paral. to the eclip. S and v have the same lat. hence vt is *the aberration in lat.* and G being the true and K the appar. place of the star in the eclip. GK is the *aberration in long.* To determine these quantities, let m and n be the sine and cos. of sSc, or CxE, the dist. of the earth from syzygies, rad being = 1; and as ∠ svt = star's co. lat. as we have before shewn, vst = the lat. for the sine, and cos. of which let v and w be taken, let r = Sa or Ss; then in the rt. angled triangle Ssv, $1 : m :: r : sv = rm$; hence in the triangle vts, $1 : v :: rm : tv = rvm$ the *aber. in lat.* Also in the triangle Ssv, $1 : n :: r : v\text{S} = rn$. Now, as the similar arches of circles contain the same number of degrees $w : 1 ::$ $rn : GK = \frac{rn}{w}$ the *aber. in long.* When the earth is in syzygies $m = o$; and hence there is no aber. in lat. and n being then greatest, there is the greatest aber. in long. if the earth be at A or the star in conj. the star's appar. place is at a, and reduced to the ecliptic at H; GH is therefore the aber which diminishes the star's long. the order of the signs being ♈GT; but when the earth is at C, or the star in oppos. the appar. place c reduced to the eclip. is at F, and the aber. GF increases the long. the long. is therefore the greatest when the star is in oppos. and least when in conj. When the earth is in quadratures at D or B, then $n = o$, and m is greatest; hence there is no aber. in long. but the greatest in lat. when the earth is at D, the appar. place of the star is at d, and the lat. is there increased; but when at B the appar. place of the star is at b, and the lat. is diminished; hence the lat. is least in quadrature before oppos. and greatest in quad. after oppos. From the mean of a great number of observations, Dr. *Bradley* determined the value of r to be $20''$.*

To find the aber. in rt. ascen. and decl. Let AEL be the equator, p its pole; ACL the ecliptic, P its pole; S the star's *true place*, and s its appar. place in the ellipse; draw the great circles Psa, Psb, pSw, pSv, and Sv, st, perp. to Pb, pv. Now, as we have before shewn, $sv = rvm$; and $Sv = rn$; hence



rvm (vs) : rn (Sv) :: rad. : tang. Ssv $= n \div vm =$ cot. earth's dist. syzy divided by sine of the star's lat. = cos. star's lat. × cot. earth's dist. from syzy. Thus ∠ Ssv is immediately computed; in like manner Psp the angle of posit. is computed from the three sides of that triangle being given, the angle Ssp is given, being the sum or

* *Example* 1. What is the greatest aber. in lat. and long. of β *Ursa Minoris,* whose lat. the beginning of 1812 will be 74° 55′ 28″? Here $m = 1$, v = sine 74° 55′ 28″ = .9655836, which mult. by 20″ = 19″31 nearly, the greatest aber. in *latitude.* For the greatest aber. in long. $n = 1$, $w = .2600927$, which divided into 20″ gives 76″9 nearly, the gr. aber. in lat. and long.

2. When the earth is 30° from syzygies, what is the aber. in lat. and long. of the same star?

Here m = sine 30° = .5, hence 19″31 × .5 = 9″65 nearly, the *aber. in lat.* If the earth were 30° past conj. or before oppos. the lat. is diminished; but increased if the earth be 30° after oppos. or before conj. Also n = cos. 30° = .866; hence 76″9 × .866 = 66″59, the *aber. in long.* If the earth be 30° from conj. the long. is diminished: but increased if 30° from oppos.

3. For the *sun,* $m = o$, $n = 1$, and $w = 1$; hence the sun has no aber. in *lat.* and the aber in *long.* = r = 20″ constantly. This aberration answers to the sun's mean mot. in 8′ 7″ 30‴, which is therefore *the time in which light moves from the sun to the earth* at its mean dist. Hence the sun always appears 20″ more backward than his true place.

diff. of S*sv* and P*sp*. Let a = the sine, and b = the cos. of S*sv*; c = sine and d = cos. of S*sp*. z = cos. star's decl. then (*sv*, *st*, being the co. sines of S*sv*, S*st*, to rad. *s*S) $b : d :: sv\ (= rvm) : st = rvm \times \frac{d}{b} = 20'' \times vm \times \frac{d}{b}$ the *aber. in declination*; and (as S*v*, S*t*, are the sines of S*sv*, S*st*, rad. being *s*S) $a : c :: Sv\ (= rn) : St = \frac{rnc}{a}$; hence, from the property of similar arches, v (S*t* ÷ cos. decl.) $= 20'' \times \frac{ne}{az}$ the *aber. in rt. ascension*. Or the correct lat. and long. being given, the corresponding correct rt. ascen. and decl. may be found, and hence the aber. in rt. as. and decl. Before we conclude this chap. we shall collect a few remarks on *the nature of the nebulous appearances observed in the heavens*, which from the improvements in telescopes, have lately become so interesting.

The greater part of the fixed stars are collected into clusters, of which it requires a large magnifying power, with a great quantity of light, to be able to distinguish the stars separately. With a telescope of small magnifying power, and light, these clusters appear like small whitish spots, and thence were called *Nebulæ*; the *Milky Way* is a continuation of such nebulæ or perhaps the one in which we are situated, as Herschel observes. Allowing an observer (says *Herschel*) the use of a common telescope, he begins to suspect that all the milkiness of the bright path which surrounds the sphere may be owing to stars. He perceives a few clusters of them in various parts of the heavens, and finds also that there are a kind of nebulous patches: still his views are not extended to reach so far as the end of the stratum in which himself is situated, so that he looks upon these patches as belonging to that system which to him seems to comprehend every celestial object. He now increases his power of vision; and applying himself to a close observation, finds that the milky way is in reality no other than a collection of very small stars. He perceives that those objects which had been called *nebulæ*, are evidently nothing but clusters of stars. Their number increases upon him, and when he resolves one *nebula* into stars, he discovers ten new ones which he cannot resolve, he then forms the idea of immense *strata* of fixed stars, of *clusters* of stars, and of *nebulæ*; till going on with such interesting observations, he now perceives, that all these appearances must naturally arise, from the confined situation in which he is placed. *Confined* it may justly be called, though in no less a space than what appeared before to be the whole region of the fixed stars, but which now has assumed the shape of a crooked, branching nebula; not indeed one of the least, but, perhaps, very far from being the most considerable of those numberless clusters that enter into the construction of the heavens.

There are some *nebulæ*, however, which do not receive their light from stars. For in 1656, *Huygens* discovered a nebula in the middle of *Orion's sword*; it contains only seven stars, the other part being a bright spot upon a dark ground, and appears like an opening into brighter regions beyond. *Simon Marius* in 1612, discovered a nebula in the *Girdle of Andromeda*. Dr. *Halley*, when observing the southern stars, discovered one in the *Centaur*, and in 1714 another in *Hercules*, in rt ascen. above 248½° and decl. 37° N. this is visible to the naked eye when the sky is clear and the moon is absent. M. *Cassini* discovered one between the *Great Dog* and *Argo*. M. de la *Caille* gives an account of 42.

Messier and *Mechain*, in the *con. de Temps* for 1783 and 1784, has given a catalogue of 103 nebulæ ; but Dr. Herschel, from his own observations, has given a catalogue of 2000 *nebulæ* and *clusters*. Some of them form a round, compact system ; others are more irregular, of various forms ; some are long and narrow ; others are hollow in the middle as that in the constel. *Telescopium*, and others are thicker in the middle or more condensed towards the centre ; the globular systems of stars are of the latter kind On *Cary's* large globes there are described, including clusters, clusters and nebulæ, and nebulæ, exclusive of the milky way, at least 311. And as their figures and places are marked on the globes, the learner will have no difficulty in finding their places in the heavens.

That the stars should be thus accidentally disposed, is a supposition too improbable to be admitted. Dr. *Herschel*, therefore, supposes that they are thus brought together by their mutual attractions, and that the gradual condensation towards the centre is a proof of a central power of that kind. He also observes that there are additional circumstances in the appearance of extended clusters and nebulæ. that favour the idea of a power lodged in the brightest part. He supposes the milky way to be a nebula of which our sun is one of its component parts See his account in the *Phil trans*. 1786 and 1789, or in Low's or the Philadelphia ed of the Encyclopedia, art astr

Dr Herschel has discovered other phenomena in the heavens which he calls *Nebulous stars;* that is stars surrounded with a faint luminous atmosphere of a considerable extent He has given an account of seventeen of these stars, one of which he describes thus. "Nov 13, 1790, A most singular phenomenon ; a star of the 8th mag. with a faint luminous atmosphere of a circular form, and of about 3′ diameter. The star is perfectly in the centre, and the atmosphere is so diluted, faint, and equal throughout, that there can be no surmise of its consisting of stars ; nor can there be a doubt of the evident connection between the atmosphere and the star. Another star not much less in brightness, and in the same field of view with the above, was perfectly free of any such appearance."

Herschel therefore draws the following conclusions : that the central star must be immensely greater than those which give the nebulous appearance, if this consist of stars very remote and connected with the star which it surrounds ; or that if the central star be not larger than common the other luminous points must be extremely small and compressed to form the nebulosity. Hence according to the former supposition, as the central point must far exceed the standard of what we call a star, there must exist a central body which is not a star. That this may be, and is very probably the case, we have shewn in the note pa. 296 If the latter supposition be granted, there must exist a shining fluid surrounding a star of a nature entirely unknown to us. Dr. Herschel adopts the latter opinion, and says, that the existence of this shining matter does not seem to be so essentially connected with the central points, that it might not exist without them. The great resemblance there is between the chevelure, or the beams or hairy appearance, of these stars, and the diffused nebulosity about the constellation *Orion*, which occupies a space of more than 60 sq. degrees, renders it extremely probable that they are of the same nature. This being admitted, the separate existence of the luminous matter is fully proved This is also extremely probable from what is shewn in the note pa. 296 ; moreover light reflected from the star could not be visible at this dist. and besides the outward parts are nearly as bright as those next the star. *Herschel* further observes in confir of this supposition, that a cluster of stars will not account for the milkiness or soft tint of the light of those nebulæ, as a self luminous fluid.— There is a telescopic milky way extending in rt. ascen. from 5h. 15′ 8″ to 5h. 39′ 1″, and in polar dist. from 87° 46′ to 98° 10′. Dr. Herschel thinks that this is better accounted for by a luminous matter, than from a collection of stars. He observes that some may account for those nebulous stars, from a star being accidentally placed nearer, which appears in the centre of a collection of stars placed at an immense distance. but he is of opinion that this milky appearance does not at all favour the suppos that it is produced by a great number of stars. But as *Vince* remarks, when we reflect that nothing but a solid body is self luminous, or at least, that a fixed luminary must immediately depend upon such, as the flame of a candle upon the candle itself, it is extremely difficult to admit this suppos. Our knowledge of the nature of these phenomena must however be very imperfect, as we are, as yet, but imperfectly acquainted with the nature of light and its various modes of existence. See Dr. *Herschel's* account in the *Phil. trans.* 1791.

The *distance* of the fixed stars are great beyond conception ; for at the dist of the nearest, the whole diameter of the earth's orbit does not amount to, or at most much exceed, a single second. If this angle, or their parallax could be accurately determined their disk might be found in the same manner as that of the superior planets For other methods see Dr. *Gregory's* ast. sect. 9, b 3. Dr *Bradley* estimates the dist of the nearest at 80000 times that of the sun, and of γ *Draconis* 400000 times the earth's mean dist. from the sun, its *parallax* not amounting to 1″. How great then must be the dist of the nebulous stars ! Dr Herschel remarks that a nebula whose light is perfectly milky, cannot be supposed at less than 6000 or 8000 times the dist. of *Sirius*, considered the nearest of the fixed stars ; so that a ray of light, which traverses the immense space between the earth and the sun in 8′ 7″30‴ (pa 444) would take 36000 or 38000 years, to arrive from one of these nebula to us, according to the distances which Herschel assumes.

CHAP. XII.

OF SOLAR AND LUNAR ECLIPSES.

An *eclipse of the moon* is evidently caused by the interposition of some opake body which deprives it of the light of the sun; and it is equally evident that this opake body is the earth, as an eclipse of the moon never happens but at the full moon or oppositions, at which time the earth is between her and the sun; and projects behind it relatively to the sun, a conical shadow, the axis of which is the straight line which joins the centres of the sun and the earth, and terminates in a point where the diameters of these two bodies are the same. Hence the cone of the terrestrial shadow is at least three times the length of the moon's dist. from the earth, and its breadth at the points where it is crossed by the moon is more than double her diameter. Hence there would be a lunar eclipse at every oppos. if the plane of the moon's orbit coincided with the ecliptic. But from the incl of these planes, the moon in oppos. is often elevated above or depressed below the lunar shadow, and does not enter it but when she is near the nodes. If the whole of the disk be immersed in the shadow, the eclipse is *total;* if only a portion of the disk be obscured, it is *partial.*

In *calculating an eclipse of the moon*, the first thing to be found is the time of the *mean oppos.* or the time when the oppos. would have taken place were the motions uniform. To obtain which, from the table of *Epacts* (see *Delambre's* or *Burg's* tables as published by *Vince*) take out the epact for the year and month, and take this sum from 29d. 12h. 44′ 3″, one synodic rev. of the moon, or two if necessary, so that the rem. be less than a rev. this rem. will be the time of *mean conjunction*. If 14d 18h. 22′ 1″5, half a revolution be added to the time of mean conj. the sum will be the time of the next *mean oppos* or if it be subtracted, the rem. will be the time of the preceding *mean* oppos. If it be bissextile, *one* day is to be taken from the sum of the epacts in Jan. and Feb. before the above subtr. is made. When the day of the *mean* conj. is 0, it denotes the last day of the preceding month.*

To determine whether an eclipse may happen at oppos. let the earth's mean long. at the time of mean oppos. be found, and also the long. of the moon's node; then according to M. Cassini, if the diff. between the mean long. of the earth and the moon's node be less than 7° 30′, there will be an eclipse, if this diff. be greater than 14° 30′, there will be no eclipse; but between 7° 30′ and

* Ex. To find the times of the *mean* new and ful moons in Feb. 1813. Here 27d. 16h. 17′ 18″ + 1d. 11h. 15′ 57″ (the epact for Feb.) = 29d. 3h. 33′ 15″, which subt. from 29d 12h. 44′ 3″, leaves 0d. 9h. 10′ 48″ for *mean new moon* on January 31, the day being = 0. 14d. 18h. 22′ 1″5 added to this, gives 15d. 3h. 32′ 49″5 the time of *mean full moon.* (See pa. 19.)

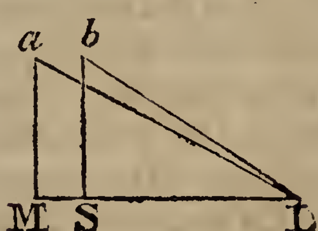
14° 30′, there may or may not be an eclipse. M. *Delambre* makes these limits 7° 47′ and 13° 21.*

The new and full moons for a month before and after the time at which the sun comes to the place of the nodes of the lunar orbit, being thus examined, no eclipse will be omitted. Or if the eclipses for the preceding 18 years be given, and to the times of the middle of these eclipses 18y. 10d. 7h. $43\frac{3}{4}'$ or 18y. 11d. 7h. $43\frac{3}{4}'$ (see note pa. 176) be added, the times at which the return of the eclipses may be expected will be given.

Next compute by the tables the *true* long. of the sun and moon, and the moon's true lat. for the time of *mean* oppos. and also their horary motions in long. the diff. (d) of the horary mot. is the moon's relative hor. mot. in respect to the sun, or the mot. with which the moon *approaches to*, or *recedes from* the sun ; let the moon's hor. mot. in lat. be also found ; and suppose the moon is at the dist. (m) from oppos. at the time (t) of *mean* oppos. then as the moon's access or recess from the sun may be supposed uniform $d : m ::$ 1 hour : the time (w) between t and oppos. which added to or subtr. from t, according as the time is before or after the moon's oppos. gives the time of the ecliptic oppos.

To find the moon's place in oppos. let n be the moon's hor. mot. in long. then 1h. : $w :: n$: the increase of the moon's long. in the time w, which applied to the moon's long. at mean oppos. gives the moon's true long. at the ecliptic oppos. The opposite point to this is the *sun's true place or long.* Let the moon's true lat. at the time of oppos. be also found, by this propor. 1h. : $w ::$ the hor. mot. in lat. : mot. in lat. in the time w, which applied to the moon's lat. at the time of the mean oppos. gives the true lat. at the time of the true oppos.† In like manner the true time of the ecliptic conjunction may be computed, and the places of the sun and moon for that time, when a solar eclipse is calculated.

From the sun's hor. mot. in long. and the moon's in long. and lat. *the incl. of the relative orbit*, and the *horary mot.* on it, may be thus found ; let LM be the moon's hor. mot. in long. SM that of the sun ; let Ma perp. to LM = the moon's hor. mot. in lat. take Sb = and parallel to aM, and join La, Lb, then La is the *moon's true orbit*, and Lb her *relative* orbit in respect to the sun. Hence

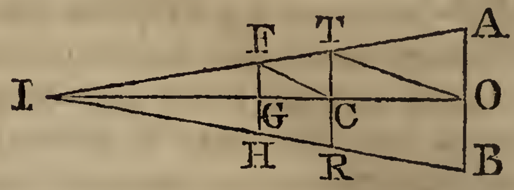


* To find whether there will be an eclipse at the full moon on Feb. 15, 1813. Sun's mean long. at 15d. 3h. 32′ 49″ 5 = 10s. 15° 14′ 19″ 6, hence the earth's mean long. = 4s. 15° 14′ 19″ 6, and long. of the moon's node = 4s. 19° 23′ 28″ 6, the diff. of which is 4° 9′ 9″, hence there must be an eclipse, because this diff. is less than the limit given above. The *ecliptic limit* being found, as will be shewn afterwards, to which if the greatest diff. of the true and mean places be applied, the above limit will be obtained.

† *Vince* directs that for greater certainty the places of the sun and moon may be computed again from the tables, and if they be not exactly in oppos. which may happen not to be the case, as the moon's long. does not increase uniformly, the operation may be repeated. This accuracy is however generally unnecessary in eclipses, unless where very great accuracy is required.

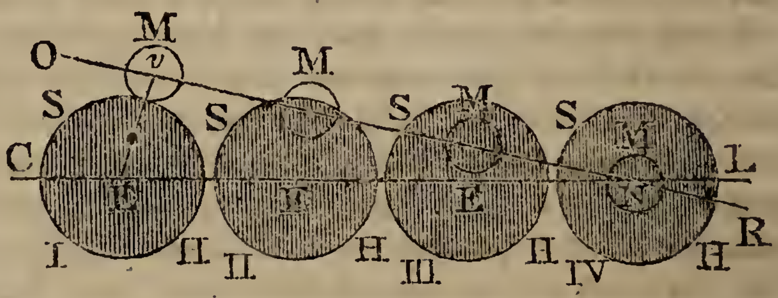
LS (diff. hor. mot. in long.) : S*b* (moon's hor. mot. in lat.) :: rad. : tang. *b*LS the incl. of the rel. orbit, and cos. *b*LS : rad. :: LS : L*b*, the hor. mot. in the *relative* orbit. The moon's hor. *parallax*, her *semidiam.* and the *semidiam. of the sun*, the hor. parallax, of which may be here taken = 9″, must also be found from the tables at the time of oppos.

To find the semidiam. of the earth's shadow at the moon seen from the earth. Let AB be the sun's diam. TR the diam. of the earth, O and C the centres; let AT, BR, be produced to meet at I, and draw OCI; let FGH be the diam. of the earth's shadow at the dist. of the moon, and join OT, CF. Now the ∠ FCG = CFA — CIA (32 Eucl. 1) but CIA = OTA — TOC; hence FCG = CFA — OTA + TOC; that is *the angle under which the semidiam. of the earth's shadow, at the moon, appears, is equal the sum of the horizontal parallaxes of the sun and moon less the sun's appar. semidiam.* From the earth's atmosphere, the shadow, in lunar eclipses, is found to be a little greater than this rule gives it. According to M. *Cassini* the augmen. is 20″; according to M. *Monnier* 30″, and to M. de la *Hire* 60″. *Mayer* makes the correction $\frac{1}{60}$ of the semidiam. of the shadow. Some computers always add 50″, but this must be subject to uncertainty.



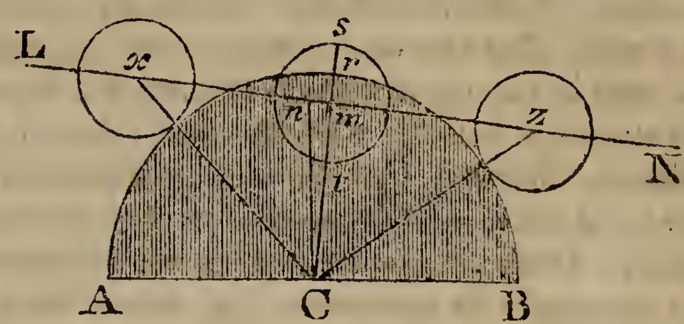
The ∠ CIT (= OTA — TOC) being known, we have sin. TIC : cos. TIC :: TC : CI the length of the earth's shadow. If the sun's mean semid. or the ∠ ATO be taken = 16′ 3″, his hor. parallax TOC = 9″, we have CIT = 15′ 54″; hence sin. 15′ 54″ : cos. 15′ 54″, or 1 : 216,2 :: TC : CI = 216,2 TC.

The different eclipses which may happen of the moon is thus explained by different authors. Let CL represent the plane of the ecliptic, OR the moon's orbit cutting the ecliptic in the node N; and let SH represent a section of the earth's shadow, at the dist. of the moon from the earth, and M the moon when she passes nearest the centre of the earth's shadow. Hence if the oppos. happen as in pos. I, the moon will touch the earth's shadow, without entering it, and hence there will be no eclipse. In posit. II, a part of the moon will pass through the earth's shadow, and there will be a *partial* eclipse.— In posit. III, the whole of the moon passes through the earth's shadow, and there is a *total* eclipse. In posit. IV, the moon's centre passes through that of the shadow, and there is a *total* and *central* eclipse. Hence it is evident, that whether an eclipse will happen at the time of oppos. or not, depends upon the moon's



dist. from the node at that time ; or the dist. of the earth's shadow, or of the earth, from the node. In lunar eclipses the angle at N may be taken $= 5° 17'$, and in posit. I the value of Ev, according to *Vince*, is about $1° 3' 30''$; hence sin. $5° 17'$: rad. :: sin. $1° 3' 30''$: sine QN $= 11° 34'$; when EN is therefore greater than QN at oppos. there can be no eclipse. This quantity $11° 34'$ is called the *ecliptic limit.*

Let ArBb be that half of the earth's shadow which the moon passes through, NL the moon's relative orbit; let Cmr be drawn perp. to NL, and let z be the centre of the moon at the beginning of the eclipse, m at the middle, x at the end ; also let AB be the ecliptic, and Cn perp. to it.



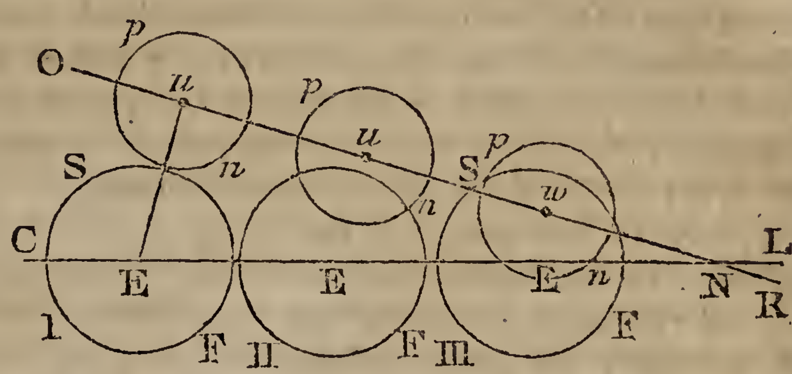
Now in the rt. angled triangle Cnm, Cn the moon's lat. at the time of the eclip. conj is given (as shewn in the beginning) and the ∠ Cnm, the comp. of the angle which the moon's rel. orbit makes with the ecliptic ; hence rad. : cos. Cnm :: Cn : nm, which is called the *reduction ;* and rad. : sin. Cnm :: Cn : Cm. The moon's horary motion (h) in her rel. orbit being known, *the time of describing mn* is thus found ; h : mn :: 1h. : the time of descr. mn. The time of the ecliptic conj. at n being known, we therefore know *the time of the middle of the eclipse* at m. Again, in the rt. angled tri. Cmz, Cm, and Cz, the sum of the semidiameters of the earth's shadow and the moon are given : hence mz is given. (47 Eucl. 1.) For $mz = \sqrt{Cz^2 - Cm^2} = (\overline{Cz + Cm} \times \overline{Cz - Cm})^{\frac{1}{2}}$, and therefore log. $mz = \frac{1}{2} \times$ log. $(\overline{Cz + Cm} +$ log. $\overline{Cz - Cm})$ The moon's hor. mot. being therefore known, we know the time of describing zm, which *subtracted* from the time at m, gives the time of the *beginning*, and *added*, the time of the *end* of the eclipse. The *magn. of the eclipse* at the middle is represented by tr ; which is the greatest dist. of the moon within the earth's shadow, and is measured in terms of the moon's diam. conceived to be divided into 12 parts called *Digits*, or *Parts deficient ;* to find which the diff. between Cm and Cr gives mr, which added to mt, or if m fall *without* the shadow, the diff. between mr and mt, and we have tr ; hence *to find the digits eclipsed* we have mt : tr :: 6 digits $360'$ (the digits being usually divided into 60 equal parts, and these parts called minutes) : the *digits eclipsed.* If the moon's lat. be N. the *upper* semicircle is used ; if S, the *lower*. And in the fig. if the moon at n have N. or S. lat. increasing, the ∠ Cnm is to be set off to the *right ;* otherwise to the left of Cn.

If the earth had no atmosphere, the moon would be invisible when totally eclipsed ; but from the refraction of the atmosphere, some rays will fall on the moon's surface, upon which account the moon will be visible at that time, and appear of a red, dusky colour. The earth's umbra in general, at a certain dist. is divided by a kind of penumbra from this refraction. And hence in some total eclipses the moon will be more visible than in others.

An eclipse of the sun is caused by the interposition of the moon between the sun and the spectator, or by the shadow of the moon falling on the earth at the place of the observer; for it is only in the conjunction of the sun and moon that we can observe a solar eclipse. Let the sun and moon be observed in the same straight line with the eye of the observer, he will then see the sun eclipsed, and if the appar. diameter of the moon be greater than that of the sun, the eclipse will be *total;* but if less, a luminous ring will be seen, formed by that part of the sun's disk which extends beyond the disk of the moon, in which case the eclipse will be *annular.* If the moon be not in the rt. line which joins the centre of the sun and the observer, the moon may then conceal only a part of the solar disk, and hence the eclipse will be *partial.* Thus the circumstances of a solar eclipse is subject to great variety, as well from the difference in the distances of the sun and moon, and the proximity of the moon to the node, as from the elevation of the moon above the horizon, which changes the angle under which her appar. diam. is seen, and which by the effect of the lunar parallax, may so augment or diminish the appar. distances of the sun and moon, that an eclipse of the sun which is visible to one observer, may be totally invisible to another. The length of a solar eclipse is also affected by the earth's rotation about its axis. M. du *Sejour* determines that an eclipse can never be annular longer than 12′ 24″, nor total longer than 7′ 58″.

A *total* eclipse of the sun is thus described by *Laplace.* We often see the shadow of a cloud transported by the winds, rapidly pass over the hills and valleys, depriving those spectators which it reaches of the light of the sun, which others are enjoying; this is the exact image of a total eclipse of the sun; a profound night, which under favourable circumstances may last five minutes, accompanies these eclipses; the sudden disapparition of the sun, with the sudden darkness that succeeds, fills all animals with dread; the stars which had been effaced by the light of day, shew themselves in their full lustre, and the heavens resemble the most profound night. Dr. *Halley* in his remarks on the total eclipse of the sun which happened on April 22, 1715, says, that a few seconds before the sun was totally obscured, he observed round the moon a luminous ring, about a digit, or perhaps a tenth part of the moon's diam. in breadth: that it was of a pale whiteness, or rather pearl colour, and seemed a little tinged with the colour of the *iris*, and to be concentric with the moon; and hence he concluded that it was the moon's atmosphere. But, says he, the great height of it far exceeding that of the earth's atmosphere; and the observations of some one who found the breadth of the ring to increase on the west side of the moon as the emersion approached, together with the contrary sentiments of those whose judgment I shall always revere, make me less confident, especially in a matter to which I paid not all the attention requisite. *Laplace* therefore concludes, that it must be the solar atmosphere, its extent not agreeing with that of the moon, as we are assured from the eclipses of the sun and stars that the lunar atmosphere is nearly insensible, if any.

The different eclipses of the sun may be thus explained. Let CL be the orbit of the earth, OR the line described by the centres of the moon's umbra and penu. at the earth; N the moon's node, SF the earth, E its centre; *pn* the moon's penumbra, *u* the umbra.



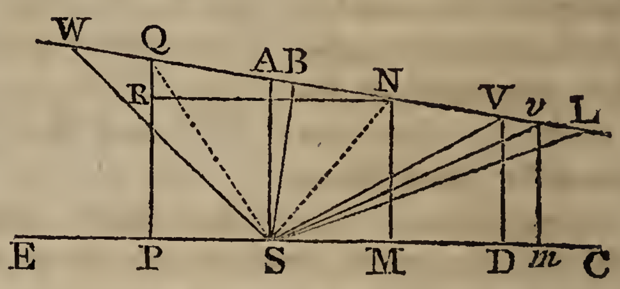
Then in pos. I, there will be no eclipse, as no part of the earth enters into the penumbra. In pos. II, the penumbra *pn* falls upon the earth, but the umbra *u* does not; there will, therefore, be no total eclipse, but there will be a *partial* eclipse where the penumbra falls. In pos. III there will be both a *partial* and *total* eclipse, as the umbra and penumbra both fall on the earth; where the umbra *w* falls, the eclipse will be *total;* where only the penumbra falls, it will be but *partial*, and where neither falls, there will be no eclipse. Now we may find the ecliptic limit thus; the $\angle$ N may be taken $= 5^\circ\ 17'$, and in pos. I, the value of E*u* (*u* being the centre of the umbra) is about $1^\circ\ 34'\ 27''$; hence sin. $5^\circ\ 17'$: rad. :: sine $1^\circ\ 34'\ 27''$: sine EN $= 17^\circ\ 21'\ 27''$ the ecliptic limit; hence if the earth be within this dist. of the node at the time of conjunction, there will be no eclipse.

We may calculate a solar eclipse, or rather an eclipse of the earth, without respect to any particular place, in the same manner as a lunar eclipse, that is, the times when the moon's umbra or penumbra first touches and leaves the earth; but to obtain the times of the beginning, middle, and end, at any particular place, is attended with more difficulty, as the *apparent* place of the moon, as seen from thence must be determined, and hence the parallax in lat. and long. must be computed, which renders the calculation of a solar eclipse extremely long and tedious.

To calculate the eclipse of the sun for any particular place, the first oper is to determine that there will be an eclipse somewhere upon the earth, or that the earth at the time of conj. is not further than $17^\circ\ 21'\ 27''$ from the node; the *true* long. of the sun and moon must be computed, by the astrom. tables, and the moon's true lat. at the time of mean conj. (determined as shewn before.) The horary mot. of the sun and moon in long. and the moon's hor.

mot. in lat. must also be found in the same manner as the time of the ecliptic oppos. was computed. At the time of the ecliptic conj. let the sun and moon's lat. be computed (as shewn in the beginning of this ch.) and also the moon's lat. let the horizontal parallax of the moon be also found, from the tables of the moon's mot. from which let the sun's horiz. parallax be subtracted, the rem. is the horiz. parallax of the moon from the sun.

Let the moon's parallax in lat. and long. from the sun, be computed (pa. 331) at the time of the ecliptic conj. to the lat. of the given place, and the moon's horiz. parallax from the sun;* the paral. in lat. applied to the true lat. gives the apparent lat. (L) of the moon from the sun, and the paral. in long. shews the appar. diff. (D) of the long. of the sun and moon.



Let S be the sun, CE the ecliptic, according to the order of the signs; let SM = D, and MN be perp. to MS and = L; then N is the moon's appar. place, and SN $= (D^2 + L^2)^{\frac{1}{2}}$ = moon's appar. dist. from the sun If the moon be *east* of the nonages. deg. the parallel *increases* the long. if *west*, it *diminishes* it (see pa. 331) hence if the true long. of the sun and moon be equal, in the former case, the appar. place will be from S towards E, in the latter towards C.

Find the true long. of the sun and moon, and also the moon's true lat from their horary motions, for some time, as an hour *after* the true conj. if the moon be to the *west* of the nonagesimal deg. or *before*, if east; and let the parallax in lat. and long. from the sun be found; the parallax in lat. being applied to the true lat. gives the appar. lat. (l) of the moon from the sun. The appar. dist. (d) of the moon from the sun in long. is also found by taking the diff between the sun and moon's true long. and applying the parallax in long. Now from S let SP be taken $= d$, and draw PQ perp. to EC and equal to l, Q will then be the moon's appar. place 1h. from the true conj. and SQ $= (d^2 + l^2)^{\frac{1}{2}}$ = the moon's appar. dist. from the sun; hence the rt. line NQ being drawn, will represent the *relative appar.* path of the moon,† and its value will also represent the rel. horary mot. of the moon in the appar. orbit, the rel. mot. in long. being = MP.

The appar. hor. mot. (r) in long. of the moon from the sun is found from the diff. between the moon's appar. dist. in long. from the sun at the time of the ecliptic conj. and at the interval of an

* The horizontal parallax of the moon from the sun is here used, instead of the moon's horiz. paral. in order to determine what effect the parallax has in varying their appar. relative situations.

† The small portion of this path here considered is taken as a straight line, it being in general very nearly so.

hour; and the moon's appar. dist. in long. from the sun at the true time of the ecliptic conj. is = the diff. (D) between the true long. at the ecliptic conj. and the moon's appar. long. hence, r : D :: 1h. : the time from the true to the appar. conj. the time of the appar. conj. is therefore given. To find whether this time be accurate, let the true longitudes of the sun and moon (from their horary motions) be computed, and also the moon's paral. in long. from the sun, which applied to the true long. gives the appar. long. if this be the same as the sun's long. the time of appar. conj. was rightly determined; if they do not agree, the true time must be found from thence as before. For the true time of appar. conj. let the moon's true lat. from its horary mot. and her paral. in lat. be found, from which her appar. lat. at the time of the appar. conj. is obtained. Let SA be drawn perp. to CE and equal to this appar. lat. then as the point A will not probably fall in NQ, let us suppose it to fall in QN, to which let SB be perp. and draw NR parallel to PM. Then NR (= PM) and QR (= QP — MN) being given, we have by trig. NR : RQ :: rad. : tang. QNR, or ASB; and sin. QNR : rad. :: QR : QN. The time of describing NQ in the appar. orbit, being = the time from M to P in long. QN is therefore the hor. mot. in the orbit. Moreover, rad. : sin. ASB :: AS : AB and rad. : cos. ASB :: AS : SB.

The moon appears at A at the appar. conj. the time of which is known from the preceding article; when the moon appears at B, she is then at her nearest dist. from the sun, and the time corresponding is therefore that of the *greatest obscur.* or the time of the *middle* of the eclipse.* Now the *quantity* of the eclipse; its *beginning*, and *end*, are thus found. The mot. being considered uniform, it will be, QN : AB :: time of describing NQ : time of describing AB, which added to or subtracted from the time at A (according as the appar. lat. is decreasing or increasing) will give the time of the greatest obscuration.

To find the *digits* eclipsed; take BS from the sum of the appar. semid. of the sun and moon, and the rem. will shew how much of the sun is covered by the moon, or the parts deficient; hence sun's semid. : parts deficient :: 6 digits : the digits eclipsed. If SB be less than the *diff.* of the semid. of the sun and moon, and the moon's semid. *greater* than the sun's, the eclipse will be *total*; if the moon's semid. be *less*, the eclipse will be *annular*, the edge of the sun appearing like a ring round the moon's disk; but if B and S coincide, the eclipse will be *central*.

Let QN be produced if necessary, and let SV, SW = the sum of the appar. semid. of the sun and moon, at the beginning and end of the eclipse respectively; then $BV = (SV^2 - SB^2)^{\frac{1}{2}}$, and $BW = (SW^2 - SB^2)^{\frac{1}{2}}$; to find the times of describing those we have NQ : BV :: 1h. : the time of describing BV; and NQ : BW :: 1h. : the time of descr. BW, which times respectively subtr. from

* This is provided there be an eclipse, which will always be the case, when SB is less than the *sum* of the appar. semidiameters of the sun and moon.

and added to the time of the greatest obscur. will give the times of the beginning and end nearly. A different method must however be adopted where accuracy is required; for in supposing VW to be a straight line, there will arise errors too considerable to be neglected. It will however serve as a rule to assume the beg and end. It therefore follows that the time of the greatest obscur. at B is not necessarily equidistant from the beg and end. If the eclipse be total, let SV, SW, be taken equal the diff. of the semid. of the sun and moon, then $BV = BW = (SW^2 - SB^2)^{\frac{1}{2}}$, from whence the times of describing BV, BW, may be found as before; these times may be considered as equal, and if applied to the time of the middle of the eclipse, or gr. obscur. will give the beg. and end of total darkness.

To determine the time of the beg. and end of the eclipse more accurately, we must proceed thus. From the horary motions, and computed parallaxes, let the appar. lat. VD of the moon be found at the estimated time of the beg. and also her appar. long. DS from the sun, and we get $SV = (SD^2 + DV^2)^{\frac{1}{2}}$; if this be equal to the appar. semid. of the moon, added to that of the sun (which sum call S) the estimated time is the beginning; if it be not equal, let another time be assumed (as the error directs) at a small interval from it, *before* if SV be *less* than S, but *after* if *greater*; let the moon's appar. lat. mv, and appar. long. Sm from the sun, be again computed for that time, and we find $Sv = (Sm^2 + mv^2)^{\frac{1}{2}}$, which if not = S, say, diff. of Sv and SV : diff. Sv and SL (= S) :: the assumed interval, or time of mot. through Vv : the time through vL, which added to or subtr. from the time at v, according as Sv is greater or less than SL, will give the time at the beginning.* In the same manner the end of the eclipse may be computed.

* The reason of this oper. is, that as Vv, vL are very small, they will be very nearly prop. to the diff. of SV, Sv, and Sv, SL. But the var. of the appar. dist. of the sun and moon, not being exactly proportional to the var. of the diff. of the appar. long. and lat. where great accuracy is required, the time of the beg. thus found (if not correct) may be corrected by assuming it for a 3d time and proceeding as before. This correction is however never necessary, unless where extreme accuracy is required in order to deduce some consequences from it. But the time thus found is to be considered as accurate, only so far as the tables of the sun and moon can be depended on for their accuracy; the lunar tables of Mr. *Burg* and solar tables of *Delambre*, republished and corrected by *Vince*, are, as before remarked, by far the most correct. If however there remains any error in the tables, and some small errors are unavoidable, accurate observations of an eclipse compared with the computed time, furnish the best means of correcting the lunar tables. The above directions principally collected from *Vince* will, together with good tables, which the young astronomer should always be furnished with, be found sufficient in calculations of this nature. For examples, &c and more information, this author may be consulted. See also Rees Cyclopedia, the Philadelphia ed. of the Encyc. art. Astr. *Lead better*, *Ewing*, *Ferguson*, and other practical writers on this subject. Dr. *Gregory* treats this subject at large in B. 4. of his Astronomy, and also *Kiel* in his Astr. Lectures 11, 12, 13 and 14, where besides the calcul. he gives various graphical methods, for computing both solar and lunar eclipses, &c.

The duration of an eclipse of the sun can never exceed 2 hours; nor of the moon, from the first touching the earth until her leaving it, cannot exceed $5\frac{1}{2}$ hours. The moon cannot remain in the earth's umbra longer than $3\frac{3}{4}$ h. in any eclipse, nor be totally eclipsed for a longer period than $1\frac{3}{4}$ hours. (Emerson's ast. sec. 7, pa. 347 & 339.)

If a conj. of the sun or moon happen at, or very near the node, there will then be a great solar eclipse; but in this case, at the preceding oppos. the earth was not within the lunar ecliptic limits, and next oppos. it will be beyond it; hence it may happen that at each node there may be but one solar eclipse, and therefore in a year there *may* happen but *two*; and this is the *least* that can happen in a year, as there must be one conj. in the time in which the earth passes through the solar eclip. limits, and hence there *must* be one solar eclipse at each node. If there be an oppos immediately before the earth enters the ecliptic limit, the next may not happen until the earth is beyond the limit on the other side of the node; hence there *may not* be a lunar eclipse at the node, and not therefore in the course of a year. There can be at most but *three* lunar eclipses in a year; for when there is a lunar eclipse, as soon as the sun gets within the lunar eclip lim. it will be out of this lim. before the next oppos. and hence there can be but one lunar eclipse at each node; but as the moon's nodes have a retrograde mot. of about $19\frac{1}{3}°$ in a year (see ch. 4) the earth may come again within the lunar eclip. lim. at the same node in the course of a year. There may happen at each node two eclipses of the sun and one of the moon; for when a lunar eclipse happen at, or near the node, a conj. may take place before and after, while the earth is within the solar ecliptic limits; the eclipses of the sun in this case will be small, and that of the moon large. Hence when the eclipses do not happen a second time at either node, there may be *six* eclipses in a year, four of which will be of the sun, and two of the moon. But if, as in the last case, an eclipse happen at the *same* node a second time in a year, there may be *six* eclipses, three of the sun and three of the moon. These six may take place during 12 lunations or 354 days, or 11 days less than a common year; hence if an eclipse of the sun should happen before Jan. 11, and the last cases should also take place, there may be *seven* eclipses in a year, five of the sun and two of the moon; but there can be no more; the mean number is however *four*, and seven can seldom happen.

As the earth describes $19\frac{1}{3}°$ in about $19\frac{1}{2}$ days, hence the middle of the seasons of the eclipses is about 19 days sooner each year than the preceding. The solar ecliptic limits being greater than the lunar, in the ratio of 17° 21′ 27″ to 11° 34′ (as shewn before) or nearly of 3 : 2, there will therefore be more solar than lunar eclipses in about the same proportion; but, as a lunar eclipse is visible to a whole hemisphere at once, and a solar only to a part, there is greater probability of seeing a lunar than a solar eclipse, and hence more lunar than solar eclipses are seen at any place.

CHAP. XIII.

OF THE TIDES.*

A TIDE is that motion of the waters in the seas and rivers, by which they are found to rise and fall in regular succession. *Newton* shews (Prop. 66, b. 1, cor. 19 and 20; and prop. 24, b. 3) that the waters of the sea ought to rise and fall twice every day, as well lunar as solar, from the combined attraction of the sun and moon. In their conjunction or oppos. their forces being joined, or acting on the earth in the same straight line, will produce the *greatest flood* and *ebb*, and these tides are called *spring tides*. In the quadratures, or about the time of the first and last quarters of the moon, the sun's action will tend to raise the waters which the moon depresses, and depress those which she raises, and hence from the diff. of their actions the *least* tide will follow: these are called *neap tides*. And as there is an oppos. and conj. with the sun once in every lunation, there will be two spring, and two neap tides, in that period. A mean lunation or synodic rev. is 29d. 12h. 44′ 2″8 (pa. 325) the mean retardation of the tides, or of the moon's coming to the merid. in 24h. is therefore 48′ 45″7, hence the interval between two successive tides is 12h. 25′ 14″2, and the daily retardation of *high water* is 50′ 28″4, at a medium. But this retardation is considerably altered from the variation in the respective *dist.* of the sun and moon, and their different *declinations*, as also the change of the lat. of the place. (See *Prin.* b. 3, prop. 26, or *McKay's* pr. Nav. pa. 18.) Newton remarks that there will also arise some variation from the force of *reciprocation*, which the waters retain after being put in motion, &c.

Laplace makes the mean interval of the return of the tides, between two consecutive returns of the moon to the same meridian, equal 1.035050 days, or 1d. 0h. 50′ 28″32. The mean value of a *total tide* (or half the sum of the heights of two successive high tides above the level of the intermediate low tide) at Brest is 5.888 metres = 19.318528 feet at its maximum about the syzygies, and

* We have sometimes in the preceding parts of this work referred to this chap. as if given after the laws of motion, &c. this was our intention, as a subject so interesting merited a full investigation, and that this investigation could not be entered into without previously laying down the principles of gravity, &c. on which it depends. But our time at present not being sufficient to discuss a subject, the most intricate in physical astronomy, and the work already swelled beyond our intended plan, we have only inserted extracts from *Newton*, *Laplace* and others, sufficient to give the learner a comprehensive idea of this phenomenon. Convinced that the knowledge which only touches at the surface can be attended with no real utility, and can only nourish that vanity, too common in the present age, of appearing learned in matters of which we know nothing, we have without deviating from the simplicity of an elementary introduction, all along joined the theory with the practical part, and entered as deep into each subject, as the nature of a contracted School book would permit.

2.789 me. = 9.150709 feet, from whence *Laplace* concludes that the mean lunar tide, which corresponds to the constant part of the parallax of the moon, is three times less than the mean solar tide, or in other words, that *the action of the moon to elevate the waters of the ocean, is three times as great as that of the sun.** The height of the tides, all other circumstances remaining the same, augment and diminish with the lunar parallax, but in a greater ratio; so that if this paral. increase $\frac{1}{18}$, the total tide will increase $\frac{1}{8}$ in the syzygies, and $\frac{1}{4}$ in the quadratures; and as the tide is nearly twice as great in the first as in the second case, its increase in the two cases is the same

The greatest var. in the moon's diam. being about $\frac{1}{13}$ of the whole, the corresponding var of the total tide in the syz. is $\frac{3}{20}$ of its mean height, or about 2.997 feet: thus the entire effect of the change of dist. between the earth and moon is (1.766 me.) 3.794 feet nearly. The var. in the sun's dist. influences the tides in a much less degree.

The decl. of the sun and moon diminish the total tides of the syz. At Brest the dimin. is (0.8 me.) 2.62 feet nearly, less in the solstices than at the equinoxes; and in the quadratures, they are less by the same quantity in the equinoxes than at the solst. The greatest tide at Brest follows the syz. about $1\frac{1}{2}$ days, or is the 3d afte syz. and the dimin. of the total tides that are near it, is proportional to the squares of the times elapsed from that instant to the time of the intermediate low tide, to which the total tide is referred, it is (0.1064 me.) 0.349 feet nearly, when this interval is a lunar day. The following var. from the decl. of the sun and moon are also found at Brest. In the syz. of the sum. solstice the morning tides the 1st and 2d day after it, are less than the evening tides by (0.183 me.) or .6 feet nearly. They are gr. by the same quantity in the syz. of the winter solstice. In the quadratures of the autumnal equinox, the morning tides the 1st and 2d day after, exceed the evening tides by (0.131 me.) 0.429 feet, and are less by the same quantity in the quadr. of the ver. equinox.

* The power of a celestial body to raise a particle of water placed between it and the centre of the earth, is equal the diff. of its action on the centre and on the particle; let b be put for the mass of the body, r the semid. of the earth, d = the dist. between the body and the earth, then the above diff. $= br \div d^3$ = relatively to the sun, the one hundred and seventy-ninth part of the force of gravity acting on the moon mult. by the proportion of the terrestrial rad. to the moon's dist. this force of gravity is nearly = sum of the masses of the earth and moon div. by the sq. of the lunar dist. hence the power of the sun to raise the waters of the sea is $89\frac{1}{2}$ times less than the sum of the masses of the earth and moon mult. by r and div. by the cube of the lunar dist. this force, as shewn above, being only $\frac{1}{3}$ that of the moon, which is equal to double its mass mult. by r and div by the cube of its dist. thus the mass of the moon is to the sum of the masses of the earth and moon, as 3 : 179; from whence it follows, that this mass is very nearly $\frac{1}{58.7}$ of that of the earth, its volume being only $\frac{1}{49.316}$ of the earth's, its density is 0.8401, that of the earth being 1; and the weight which on the earth's surface is 1, would on the moon's surface be reduced to 0.2291.

The interval of the tides offer other phenomena. At Brest the high tide the moment of syz. follows midday or midnight at 0.14822d. or 3h. 33′ 26″2, this is called the *hour of the port*, and differs in different harbours. At quadr. the high tide in Brest follows midnight or midday at 0.35464 days, or 8h. 30′ 40″ 9. The tide of the syz. advances or retards (264″) 3′ 48″, for every hour that it precedes or follows the syz. and the tide of the quadr. advances or retards (416″) 5′ 59″4 for every hour before or after the quadr. In the syz. also, every minute of increase or dimin. in the moon's appar. semid. advances or retards the hour of high water (354″) 5′ 6″ 6 ; this phenomenon is three times less at the quadr.

From the sun and moon's decl. the time of high water advances about (2′) 2′ 52″8 in the syz. of the solstices, and is equally retarded in the syz. of the equinoxes ; but in the quadr. of the equin. high water advances (8′) 11′ 31″2, and is equally retarded in the quadr. of the solstices.

The daily retardation of the tides varies also with the phases of the moon ; it is a minimum at the syz. when the total tides is at their max. and is only 0.02705d or 38′ 57″12, when the tides are at their min. it is then greatest, and amounts to 0.05207d or 1h. 14′ 58″8 Thus the diff. 0 20642d. (0.35464 — 0.14822) of the times of high water at the syz. and quadr. increases. for the tides which follow in the same manner these two phases, and become nearly a quarter of a day relatively to the max. and min. of the tides.

Every minute of incr. or dimin. in the moon's appar. semid. augments this daily retardation (258″) 3′ 42″9 about the syz. and is three times less in quadr. From the var. of the sun and moon's decl. it varies likewise in the syz. of the solstices about (155″) 2′ 13″9 greater than in its mean state, and equally less in the syz. of the equinoxes ; on the contrary in the quadr. of the equin. it exceeds the mean by (543″) 7′ 49″1, and is surpassed by this quantity in the quadr. of the solstices. Hence the var. in the heights and intervals of the tides have very diff. periods ; some of half a day, and a day, others of half a month, a month, half a year, and of a year ; others again vary with the rev. of the nodes and the perigee of the lunar orbit, as they vary the decl. and dist. from the earth. All these phenomena appear to have been the same in the new as in the full moon.

These phenomena equally take place in all the harbours along the sea shore ; but local circumstances, without changing the laws of the tides, have a considerable influence in changing the heights of the tides and the hour of high water for a given port.

Laplace gives the following *method of determining the time of high water* on any day. Considering each of our ports as the extremity of a canal at whose *embouchure* (its mouth or entrance) the partial tides happen at the moment of the passage of the sun and moon over the meridian, and employ a day and a half to arrive at its extremity (supposed eastward of its embouchure) by a certain number of hours, called the *fundamental hour of the port*, and may be easily computed from the hour of the establishment of the port, by considering this as the hour of the full tide, when it coincides with the syz. The daily retardation of the tides being (2705″) 38′ 57″1, it will be for $1\frac{1}{2}$ days = (3951″) 56′ 53″6, which quantity is to be added to the hour of the establishment to have the fundamental hour Now if the hours of the tides at the *embouchure* be augmented by (15 hours) 36h. *plus* the fundamental hours, we shall have the hours of the corresponding tides in our ports. We shall now make a few remarks on the cause of those interesting phenomena.

As the action of gravity decreases as the sq. of the distance increases, the waters that are on the side next the sun or moon, will be more attracted, by them respectively, than the central parts of the earth, and the central parts than the surface on the opposite side ; therefore the distances between the centre of the earth and the surface of the water under the *Zenith* and *Nadir*, by the laws of attraction, will be increased ; for that part of the surface which is nearest to the sun or moon, will move with greater vel. towards those bodies, and that part that is more dist. with less vel. than the centre ; from which it is evident, that high water must take place nearly at the same instant at opposite parts of the earth ; and from the earth's mot. on its axis in 24h. there will be two tides of *flood* and two of *ebb* in that time agreeable to experience. It appears from the foregoing explanation, that the fig. of the earth caused by the tides, would be an oblate spheriod, having its longer axis passing through the moon, on suppos. that the whole surface was fluid.*

Laplace thus determines the law by which the waters rise and fall. Let a vertical circle be conceived whose circumference represents half a day, and whose diam. is equal to the whole tide or the diff. between the height of high and low water, and let the arc of this circum. from the lowest point express the time elapsed since low water, the *versed sines* of these arcs will express the *heights* of the water corresponding to these, so that the ocean in rising, covers, in equal times, equal arcs of this circumference.† This law is exactly observed in the middle of the ocean, but in our harbours local circumstances produce some deviation. The sea is also found to employ a longer time to fall than rise ; this diff. is found at *Brest* about ($10\frac{1}{2}'$) 15′ 7″2. The greater the extent of the surface of the water, the more perceptible will be the phenomena of the tides, the motion which is communicated to a part of a fluid being communicated to the whole ; hence such remarkable effects are produced in the ocean, and the waters communicating with it, which are insensible in lakes and small seas. If we imagine at the bottom of the sea a curved canal, terminated

* See Simpson's Fluxions, art. 403.

† See Emerson's Fluxions, prob. 25, where the method of determining the height of the tides is investigated.

at one of its extremities by a vertical tube rising above the surface of the water, and which if prolonged would pass through the centre of the sun or moon, the water would rise in this tube by the direct action of the sun or moon, which diminishes the gravity of its particles, and particularly by the pressure of the particles enclosed in the canal, all of which make an effort to unite under the sun and moon, and from the integral of all their efforts arise the elev. of the water in the tube above its natural level. If we conceive a similar canal communicating with the sea and extending into the land, the undulations caused by the tides at its entrance, will be propagated through its whole length, in the interval of half a day; but the hours will be retarded, in proportion as the points are further from the entrance of the canal.* This reasoning may be applied to rivers, whose surfaces rise and fall by similar waves, notwithstanding their contrary motion.†

The solar and lunar tides do not happen at the same instant, their periods being different; hence the lunar tide will retard upon a solar tide by the excess of half a lunar day above half a solar day, that is (1752″) 25′ 13″7. See *Laplace's* astr b. 4, ch. 10. In further confirmation of the theory, it is found, that in Brest the solar tide follows the passage of the sun (18358″) 4h. 24′ 21″3, and the lunar the passage of the moon (13101″) 3h. 8′ 39″2; the greater (1′ 26″4) tide following the syz. being nearly $1\frac{1}{2}$ days: that (100″) 1′ 26″4 variat. in the moon's semid. answers to half a metre or 1.6405 feet of variat. in the total tide when the moon is in the equinoctial: that (1′) 1′ 26″4 retards the tide as given in the preceding theory (354″) 5′ 6″6 very nearly: that (1′) 1′ 26″4 var. in the moon's semid. produces a var. of (258″) 3′ 42″9 agreeable to obs The above is on suppos. that the sun and moon move in the equinoctial; the phenomena resulting from their change of decl. are also found to correspond with what is given in the preceding theory. See this subject further detailed in *Laplace*, b. 4, ch. 10.‡

* See this investigated in prob. 20, Emerson's Flux. in the Schol. of which he shews, that if a pendulum be made whose length is the breadth of a wave from top to top, then in the time that it will perform one vibration, the waves will advance forwards a space equal their breadth. See also Newton's prin. b. 2, prop. 46, 47, &c. Their velocity, according to Newton, is in the subduplicate ratio of their breadths, B. 2, prop. 45.

† The action of the sun and moon is usually found separate, (*Principia*, b. 3, prop. 36 and 37) which by the composition of forces (see the next chap.) are combined, and from the resulting force result the tides, which are observed in our ports.

‡ We have, according to the remark pa. 308, changed Laplace's measures of time, in all this ch. allowing 10 hours to the day, 100′ to an hour, 100″ to a minute, &c. And we are further confirmed in this opinion, not only from the result agreeing with the known established hour in other ports, but also from *Laplace's* augmenting the hours of the tides at *embouchure* by 15 hours to find the time of high water, as this corresponds to 36 hours of our measures, or to $1\frac{1}{2}$ days, agreeable to theory. We have before remarked, that this was not among the least considerable errors in Mr. *Pond's* translation of Laplace's Astr.

CHAP. XIV.

OF THE GENERAL LAWS OF
MOTION, FORCES, GRAVITY, &c.

These laws being necessary in understanding those in the solar system, which we have given in the preceding chapters, and not only the foundation of Physical Astronomy, but the very basis of all Natural Philosophy, we have therefore given them a place in this chapter. But, as in the short compass of a single chap. it cannot be expected that we should enter into the detail of these laws, we shall therefore only insert what is necessary to give a general knowledge of the motions of bodies, particularly those in the Solar System, first premising the *rules of reasoning* in philosophy as delivered in the 3d. b. of the Principia.

*Rules of reasoning in Philosophy.**

1. We are to admit no more causes of natural things, than such as are both true and sufficient to explain their appearances.

2. Therefore to the same effects we must as far as possible assign the same causes.

3. The qualities which are found in all bodies upon which experiments can be made, and which can neither be increased or diminished, are to be esteemed as belonging to all bodies.

4. In experimental philosophy, propositions collected from phenomena by general induction, are to be admitted as accurately or

* In rule 3, Newton lays it down as a principle, that the properties of matter cannot be known otherwise than by the senses, from which we know that innumerable objects exist around us, and act upon us. Their powers, properties, causes, &c. is an interesting subject for our contemplation, and what is properly called *philosophy*. These objects may be divided into two general classes; the first is of those which have a self moving power, and several properties similar to those of our minds. The second is of those which never move of themselves, without the action of some external or invisible object. The former is called *Mind* or *Spirit*, and the latter *Body* or *Matter*. The properties of matter are *extension*, *figure*, *solidity*, *motion*, *divisibility*, *gravity* and *vis inertiæ*. These properties are therefore the foundation of all Nat. Phil. *Extension* is considered with regard to length, breadth and depth, and *Figure* the boundary of extension. *Solidity* or *impenetrability* is that property of matter by which it fills space, or excludes any other portion of matter from that space which it occupies. *Motion* is the change of place. *Divisibility* of matter the capacity of being separated into parts. *Gravity* is the force or tendency of a body to a centre, and *Vis Inertiæ* the innate force by which it resists any change. *Motion* is also absolute or relative; absolute when it is compared with a body at rest, and relative when compared with others in mot. The rate of this motion is called the *Velocity* of a body, causing the body to pass over a certain space in a given time, and whatever produces or changes this motion is termed *Force;* in moving bodies this is called their *Momentum*. The *Density* of a body is the proportional weight or *quantity of matter* in it, and is proportional to the *specific Gravity* which is the proportion of the weights of different bodies of equal magnitude.

nearly true, until from other phenomena they are rendered more accurate, or liable to exceptions.

Axioms, Laws, &c.

5. Every body perseveres in its state of rest, or of its uniform mot in a right line, unless compelled to change that state by other forces impressed on it.

6. The alteration of motion, or the motion generated or destroyed, in any body, is proportional to the force applied ; and is made in the direction of that straight line in which the force acts.

7. To every action there is always opposed an equal reaction ; or the mutual actions of two bodies upon each other, are always equal and directed to contrary points.

8. *The mass, or quantity of matter, or the volume in all bodies, is in the compound ratio of their magnitudes and densities.*

If b be put for the body, m its mag and d its density, b is as* md. For if the bodies be equal, the mass is as the density ; and if the densities be equal, the mass is as the mag. but when neither are equal, the mass is therefore in the compound ratio of the mag. and density

9. The volumes are also as the densities and cubes of the diam. the magnitudes being in this proportion.

10. The masses are also as the mag. and specific gravities, the density being as the spec. gravity.

11. *The quantities of motion in moving bodies are in the compound ratio of the masses and velocities.* That is m is as bv, where m represents the momentum and v the vel. For if the velocities be equal, the quantities of mot. will be as the quantities of matter, and if the masses be equal, the momentum will be as the velocities, hence, &c.

12. *The momentum generated by any momentary force is as the force.* For every effect is proportional to its adequate cause.

13. *In uniform motions the spaces are as the velocities and times of description.* For the vel being the same, the spaces are as the times, and the times being equal, the spaces are as the velocities ; hence, &c.

14. From this art. it appears that in uniform motions the time is as the space directly and vel. reciprocally, or as the space divided by the vel. and that the vel. is as the space directly and time reciprocally, &c. From art. 11, 12 and 13, many other general proportions may be deduced. (See Emerson's Mechanics, 4to. page 8, or Hutton's Mathematics, vol. 2, pa 134.

15. *The quantity of motion generated by a constant and uniform force, is in the compound ratio of the force and time of acting.* For if the time be divided into very small parts, the momentum in each is the same (art. 12) the whole momentum will therefore be as the sum of all the parts or the whole time ; but the momen-

* That is whatever variation is made in b, a proportional change will be made in md.

tum for each time is also as the force (art. 12) hence the whole momentum is in the compound ratio of the force and time.

16. From the preceding art. it is evident that the mot. lost or destroyed in any time is also in the same proportion. And that the vel. gener. or destroyed in any time, is as the force and time directly, and the body or mass reciprocally; hence if the body or force be given, the vel. will be as the time, &c.*

17. *The spaces passed over by bodies, urged by constant and uniform forces, are in the compound ratio of the forces and squares of the times of acting directly, and the body or mass reciprocally.* For let b be put for the body, s the space passed over with the vel. v in the time t. Then art. 16, $\frac{1}{2}v$ is the vel. at $\frac{1}{2}t$ the middle of the time; hence the increase of vel. being uniform, s will be described in the same time t by the uniform vel. $\frac{1}{2}v$, s is therefore as $\frac{1}{2}tv$, or $s = \frac{1}{2}tv$; but art. 16, v is as $ft \div b$, hence s or $\frac{1}{2}tv$ is as $ft^2 \div b$.

18. If b and f be given, s is at t^2; hence art. 16, s is as tv, or as v^2. s or $\frac{1}{2}tv$ is the space actually described, that is half the space that would be uniformly described with the last or greatest vel. in the same time t. From these general proportions. a table of all the particular relations of uniformly accelerated forces may be easily formed See Hutton's Math. pa. 136, vol. 2, or Emerson's Mechanics.

19. What is given in art. 17 and 18, hold equally true, for the spaces passed over by bodies freely descending by their own gravity, this force being considered uniform at all places at, or at equal distances from, the earth's surface. It is found that in the lat. of London a body falls $16\frac{1}{12}$ feet in $1''$, and that (art. 18) at the end of this time it has acquired a vel. that would carry it over $32\frac{1}{6}$ feet in $1''$; hence if $g = 16\frac{1}{12}$ feet the space passed through in $1''$, and $2g$ the vel. generated in that time; then art. 16 and 17 we have $1'' : t^2 :: 2g : 2gt = v$, and $1^2 : t^2 :: g : gt^2 = s$ the space; from which proportions if the different values of s, v, g, t, be found in general, we shall obtain the general equations for the descents of gravity, &c. Hence art. 16 and 18, if the time be as the numbers 1, 2, 3, 4, 5, &c. the velocities will be as the same, and the spaces as their squares 1, 4, 9, 16, 25, &c. and the spaces for each time the difference of these squares, or 1, 3, 5, 7, 9, &c.

20. These relations between the times, velocities, and spaces, may be represented by a rt angled triangle thus. If one side of the triangle (t) represent the time, and the other side (v) perp. to it, the vel. gained at the end of this time. Then if t be divided into any number of equal parts, and through these points lines be drawn parallel to the base to meet the hyp. these will represent the velocities in each corresponding interval of time, and by similar triangles will be proportional to the former. Also the area of the triangle being $= \frac{1}{2}tv$ will therefore represent the space s passed

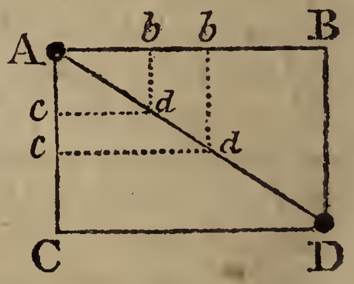
* In art. 8 and 9 *volume* was inadvertently used for the mass, the vol. is properly the size or mag.

over in the time t; and the smaller triangles, for the same reason, will represent the different spaces passed over in the corresponding intervals of time, and their other sides the velocities. Now these areas or spaces being as the squares of their sides, shews that s is as t^2 or v^2, as in art. 18.

21. The relations, &c. given in art. 20, may be more naturally represented by the abscissas and ordinates of a parabola. For if the ordinates represent the respective times from the beginning, or the velocities which are proportional to them, then the corresponding abscissas drawn parallel to the axis of the parabola, will represent the spaces described by a falling body in those times; the abscissas which represent the spaces, being as the squares of the ordinates which represent the time, by a well known property of the parabola.

22. As motions are destroyed in the same manner as produced, and by the same forces acting in contrary directions; hence 1. A body thrown directly upwards, will lose equal velocities in equal times. 2. If the body be projected upwards with the same vel. acquired in falling, it will lose all its motion in the same time in which it fell, and will have the same vel. in any point of the same line both in ascending and descending. 3. The respective heights ascended to, will be as the squares of the velocities with which they were projected, or as the squares of the times, until they lose all their motion These properties are accurately true in *Vacuo*, but near the earth's surface the resistance of the air, particularly in very swift motions, has considerable effect in changing the velocities, &c.

23. *If a body at* A *be acted upon by any two similar forces, so that they would separately cause the body to pass over the spaces* AB, AC *in an equal time; then if both forces act together, they will cause the body to move, in the same time, through* AD *the diagonal of the parallelogram* ABCD. Let cd, bd, be drawn parallel to AB, AC respectively; let t be the time in Ab or Ac, and T the time in AB or AC; then if the forces be impulsive or momentary, it will be Ab or cd : AB or CD :: t : T, and bd or Ac : BD or Ac :: t : T; therefore by equality (11 Eucl. 5) Ab : bd :: AB : BD; the parallelograms Abdc, ABDC are therefore similar, and consequently (26 Eucl. 6) they are about the same diam. hence d is in the diagonal AD. And this may be shewn of any other point d, the path of the body is therefore in AdD, the diagonal of the parallelogram.



If the forces be uniformly accelerated or retarded, the spaces will then be as the squares of the times (art. 18) in which case Ab or cd : AB or CD :: t^2 : T^2, and bd or Ac : BD or AC :: t^2 : T^2, hence Ab : bd :: AB : BD as before.

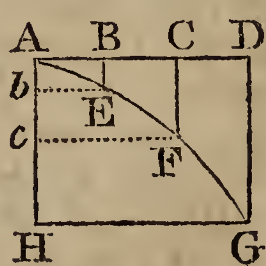
24 From the preceding art. it appears, 1. That the diag. AD by both forces, is described in the same time as AB, AC, by the

single forces impressed in these directions. 2. That the forces in the directions AB, AC, AD, are as these lines respectively. 3. That the single force AD is equal to the two AB, AC, and compounded of them; so that any single force as AD, may be resolved into two or more forces by describing any parallelogram whose diag. is AD, and each of these may be resolved in like manner. This is called the *resolution of forces*. 4. That any two or more forces may be compounded into one, by reducing any two of them to one, as in art. 23, and this again with the other; the force resulting will be that compounded of the whole, and hence this is called the *composition of forces*. 5. And hence the effect of any given force as AD in any other direction AB, CD, or AC, BD may be found, being as those sides respectively. 6. Hence also if the two forces act in the same line, in the same or contrary directions, their sum or diff. will be the resulting force, which will always act in the direction of the greater. 7. From the same principle it will appear, that if an elastic body impinge on a firm plane, the angle of incidence will be found equal the angle of reflection, by resolving the forces before and after the stroke, action and reaction being equal and contrary.

25. The forces of bodies acting on others, may be found from the same principle Thus the perp. force of AD or the body at D, on CD, is as BD or AC, that is as sine ∠ ADC the angle of incidence; AD which represents the force being radius. For the force AD may be resolved into AC, CD, the latter of which does not act on the plane, being parallel to it.

From this art. it appears, 1. That the action of any force and in any direction, is always perp. to the surface acted on. 2. That if the plane acted on be not fixed, it will move after the stroke, in a direction perpendicular to its surface.

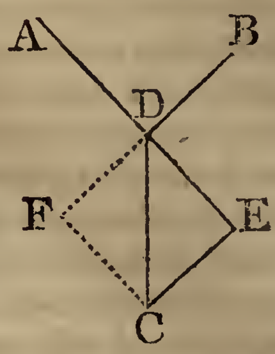
26. *If one of the forces* AD *be uniform, and the other* AH *uniformly accelerated, as the force of gravity, the motion resulting from these two forces will be in the curve* AEFG *of a parabola.* For if the body be projected from A in the direction AD with an uniform vel. then art. 14, it would describe the spaces AB, BC, CD, supposed equal, in equal times, when not acted on by any other force. Let BE, CF, DG, be drawn perp. to the horizon, so as to represent the spaces the body would fall through by the accelerated force, or



force of gravity, in the same time that by the uniform force it described AB, BC, CD; hence by the composition of motion the body will be in the points E, F, G respectively at the end of those times; therefore the real path of the body will be in the curve A, E, F, G. But the spaces in AD are as the times art. 13, and the spaces BF, CF, DG, as the squares of the times art. 18; hence AB, BC, &c. are as BE^2, CF^2, &c. which is the property of the parabola; therefore the projectile will move in the curve of the parabola. This demonstration holds whether AD be parallel to the horizon or in an oblique direction.

27. *If three forces acting together in the same plane, keep one another in equilibrio, they will be proportional to the three sides* DE, EC, CD *of a triangle which are drawn parallel to the directions of the forces* AD, DB, CD

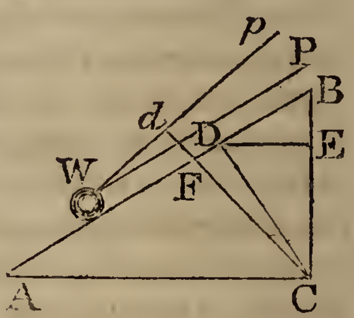
Let AD, BD be produced, and CF, CE drawn parallel to them; then by suppos the force in CD is equal to the two in AD, BD; but the force in CD is also equal to the two represented by ED, CE or FD; hence if CD represent the force C, ED, FD will represent the forces A and B; therefore the three forces A, B, C, are proportional to the three lines DE, CE, DC, parallel to the directions in which they act.



From this it follows, 1. That the three forces, when in equilibrio, are proportional to the sines of the angles of the triangle formed by their lines of direction. 2 That these three forces are also proportional to the three sides of any other triangle drawn perp to their lines of direction, or forming any angle with them; for this triangle will be similar to that whose sides are parallel to the lines of direction. 3 That if any number of forces acting against one another, be kept in equilibrio by these actions, they may be all reduced to two equal and opposite ones. For any two of the forces may, by composition, be reduced to one acting in the same plane; and this last force and any other may likewise be reduced to one force acting in the plane of these; and so on until they are all reduced to two equal and opposite forces.

28. *If a heavy body or weight* W, *be sustained on an inclined plane* AB, *by a power* P, *acting in a direction* WP *parallel to the plane; then if* AC *represent the weight* W, BC *will represent the power* P, *and the base* AC *the pressure (p) against the plane.*

Let CD be drawn perp to AB; then the weight W, or force of gravity acting perp. to AC or parallel to BC, the power P acting parallel to DB, and the pressure *p* acting perp. to AB or paral. to DC; will be to one another as BC, BD, CD respectively (art. 27.) that is, from similar triangles as AB, BC, AC, or as W, P, *p*



Hence W, P, *p*, are as rad. sine and cos. BAC the plane's elevation; or as AC, CD, AD perp. to their directions; hence also the relative weight down BA equal W × BC ÷ AB.

29. When the power P acts in any other direction as W*p*, let CF*d* be perp. to W*p*; then W, P, *p* are as AC, CF, AF perp. to the direction of those forces.

30. *If a heavy body descend freely down an inclined plane* AB, *its velocity in any time, is to the vel. of a body falling perpendicularly, in the same time, as* BC *the height of the plane, to* AB *its length.* For the force of gravity in the directions BA and BC is constant, and art. 28, as those sides; but art. 17, the velocities are as the

forces, that is, in the same time, as the force on BA to that on BC, or as BC : BA.

31. From the preceding art it follows, 1. That, as the mot. down an inclined plane is uniformly accelerated, being produced by a constant force, the laws which are given for accelerated forces in general, hold equally for motions on inclined planes; that is, vel. are as the times; the spaces as the sq. of the times or velocities; if the body be thrown upwards on the incl plane, it will lose its mot. and ascend to the same height in the same time, &c. 2. That the space descended on the incl. plane, is to the perp. descent in the same time as CB : AB, or as sine of the plane's incl. to rad. 3. That the velocities on inclined planes are as the sines of their elevations. 4. That if CD be perp. to AB, BD and BC will be described in the same time; for by sim. triangles BC : BD :: BA : BC. 5. That in a rt. angled triangle, whose hyp. is perp. to the horizon, a body will descend down any of its sides in the same time; hence if a circle be described on the hyp. the time of descending down any of its chords, drawn from either extremity of the hyp. or its perp. diam will be all equal, and also to the time of falling freely down the perp. diam.

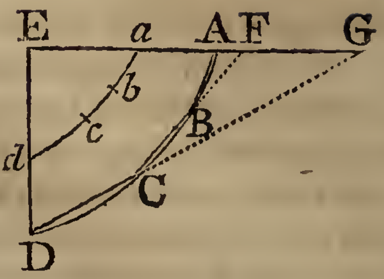
32 *The time of descending down the incl. plane* BA *is to that of falling through its height* BC *as* BA : BC. Let the time of describing BD or BC, which are equal (4 art. 31) be called t, and that of describing BA, T; then $t^2 : T^2 :: BD : BA$, the forces being constant; but BD : BC :: BC : BA; hence $BD : BA :: BC^2 : BA^2$; therefore by equality $t^2 : T^2 :: BC^2 : BA^2$, or $t : T :: BC : BA$.

Cor. Hence it follows, that the times of descending down different planes of the same height, are as the lengths of the planes.

33 *A body acquires the same vel. in descending down an inclined plane* BA, *as in falling perp. through its height* BC. For let F be the force of gravity in BC, f the force on AB, t the time of falling through BC, and T of descending down AB; then art. 28, $F : f :: BA : BC$, and art. 32, $t : T :: BC : BA$; hence by comp. $Ft : fT :: 1 : 1$; but Ft, fT, are as the velocities, art. 18, therefore the velocities are equal.

34. From the preceding art. it follows, 1. That the velocities acquired by bodies descending on any planes, from the same height to the same horizontal line are equal. 2. That if the velocities be equal at any two equal altitudes D, E, they will be equal at all other equal altitudes A, C. 3. That the velocities acquired in descending down any planes, are as the sq. roots of the heights.

35. *If a body descend from the same height through any number of contiguous planes* AB, BC, CD; *it will at last acquire the same velocity as a body falling perpendicularly from the same height; the vel. being supposed not altered in passing from one place to another.*



Let the planes DC, CB, be produced to meet the horizontal line EG in F and G; then (1 art. 34, the vel. at B is the same whether the body descends through AB or FB; and in C, for the same reason, the vel is the same whether the body descends through ABC, FC or GC; hence at D it will acquire the same vel. in des. through the planes AB, BC, CD as in des. through GD, that is, art. 33, as in falling through ED.

36 *The velocity acquired by a body descending along any curve surface is the same, as if it fell perp. through the same height.* This is evident from the last art. by supposing the lines, AB, BC, &c. indefinitely small, in which case they will form a curve.

37. Hence also it appears, 1. That the velocities acquired by bodies descending down any planes, or curves, or falling perp. from the same height, are the same. 2. That if the velocities be equal at any alt. they will be equal at any other alt 3. That the vel. are as the sq. roots of the perp heights 4. That a body after its descent through any curve will acquire a vel. that will carry it through the same height in any other curve, and in any direction, or by being retained in the curve by a string, and vibrating like a *pendulum.** 5. That the velocities will be equal, at equal altitudes, and also the times of ascending and descending will be the same, if the curves be of equal altitudes.

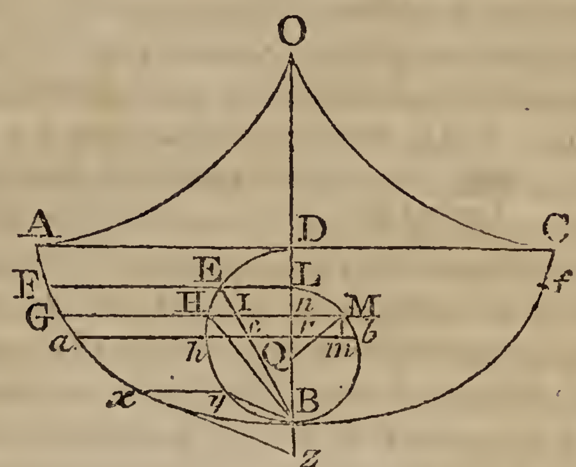
38. *The times in which bodies descend through similar parts of similar curves, in similar positions, are as the sq. roots of their lengths.* Let ABCD, *abcd*, in the foregoing figure, be two similar curves, and let any corresponding parts as AB, *ab*, be taken, these will be proportional to the whole; and as they are similarly situated, they will be parallel to each other; hence the times of describing these corresponding parallels are as the square roots of their lengths, art. 31, that is, as $\sqrt{AD} : \sqrt{ad}$. In the same manner it is proved that the times of describing any other two

* A simple pendulum consists of a ball or any other heavy body suspended by a fine string or thread, moveable about a fixed centre. If the pendulum be moved from its vertical situation, and then let fall, the ball, from its gravity, in descending, will describe a circular arc, in the lowest point of which, or where the pend. regains its vertical position, it will have that vel. that it would acquire by falling perpendicularly the length of the pendulum (1 art. 37) and this vel. will be sufficient to cause the ball to ascend through an equal arc to the same height (4 art. 37) from whence it fell; having there lost all its mot. it will again fall, by its own gravity, in the same manner as before, and will thus perform continual vibrations. Hence if the mot. of a pendulum suffered no resistance from the air, or from the friction at the centre of motion, its vibrations would never cease. But from these obstructions the vel. of the ball is a little diminished in every vibration, and hence the arcs described must become continually shorter, until at length they vanish with the mot. of the pendulum. To prevent this taking place in clocks, there is a mechanical contrivance called a maintaining power. See *Helsham's Lectures*, lecture 10.

similar parts, are as $\sqrt{AD} : \sqrt{ad}$; hence by compos. the whole times of describing are in the same ratio.

39. Hence it appears, 1. That the times of descent in curves are as the sq. roots of their axes, or as $\sqrt{ED} : \sqrt{Ed}$, the axes of similar curves being as the lengths of the similar parts. 2. That as the vibrations of pendulums are similar to the descent of bodies in curves; therefore, *the times of the vibration of pendulums in similar arcs of any curves, are as the square roots of the lengths of the pendulums.* 3. That the *velocity* of a pendulum at its lowest point is as the chord of the arch it descends through. See art. 37. 4. That pendulums of the same length vibrate in the same time.

40. *When a pendulum vibrates in a cycloid,* the time of one vibration, is to the time a body falls through half the length of the pendulum, as the circumference of a circle to its diameter.* Let ABC be the cycloid, DB its axis, or the diameter of the generating semicircle DB, OB or 2DB the length of the pendulum, or radius of curvature at B (see Simpson's Fluxions, art. 72.) Let the ball descend from F, and in vibrating describe the arc F B f; let FB be divided into innumerable parts, and let Ga be one of those parts; draw FEL, GM, ab, perp. to DB; on LB describe the semicircle LMB whose centre is Q; draw Mm parallel to DB, and also the chords BE, BH, EH and rad. QM. Now the triangles BEH, BHI are similar; hence BI : BH :: BH : BE (6 Eucl. 4) or $BH^2 = BI \times BE$, or $BH = \sqrt{BI \times BE}$. Also in the sim. tria. Mmb, MQn, Mm : Mb :: Mn : MQ, and by the nature of the cycloid Hh = Ga. If another body descend down the chord EB, it will acquire the same velocity as the ball in the cycloid falling from F, art. 37.



* If a circle BED (supposed complete) be rolled on a rt. line AC, until the fixed point B, which at first touched the line at A, arrives at C; the point B will then describe the curve ABC, which is called a cycloid. See its properties investigated in Emerson's properties of curve lines at the end of his Conic Sections, sect. 3, some of which are the following. 1. The rt. line AD = the cir. DEB. 2. Any rt. line FE paral. to AD = the arc EB. 3. If xy be drawn paral. to AD, the tang. xz is parallel to the chord. 4. The length of the arc Bx is double the chord By. 5. The length of the semicycloid BA = 2DB the diameter of the generating circle, &c.

The contrivance by which a pendulum is made to vibrate in the curve of a cycloid is the following. Let the semicycloids OA, OC be described each = $\frac{1}{2}$ABC, their vertices being at A and C. If then OA, OC be supposed to be two plates of some breadth, and the pendulum OB to vibrate between these plates, the upper part of the string will constantly apply itself to that plate towards which the body moves, and will thus describe the cycloid ABC. Here AO or OC is called the *evolute*, and OB becomes the *radius of curvature* of the cycloid. *Huygens* is the author of this contrivance.

Hence Ic and Ga are passed over with the same vel. and therefore the time in passing them will be as their lengths, or as Hh : Ic, or by sim tri. as BH (or BI × BE)$^{\frac{1}{2}}$: BI, or as $\surd$ BE : $\surd$BI, or sim. tri. as $\surd$BL : $\surd$Bn. That is, the time in Ga : time in Ic :: $\surd$BL : $\surd$Bn Again vel. at I : vel. at B :: $\surd$EK : $\surd$EB (3 art. 34) or $\surd$Ln : $\surd$LB. Now the uniform vel. for EB is equal half the vel. at the point B, and the time of describing any space with a uniform motion, being directly as the space and reciprocally as the velocity ; hence the time in Ic : EB :: Ic ÷ $\surd$Ln : EB ÷ $\frac{1}{2}\surd$LB :: (sim. tri.) nr ÷ $\surd$Ln : LB ÷ $\frac{1}{2}\surd$LB :: nr or Mm : 2 (BL × Ln)$^{\frac{1}{2}}$. That is time in Ic : time in EB :: Mn : 2 (BL × Ln)$^{\frac{1}{2}}$. But it was shewn that time in Ga : time in Ic :: $\surd$BL : $\surd$Bn ; hence by comp time in Ga : time in EB :: Mn : 2 (BL × Ln)$^{\frac{1}{2}}$ or 2nM (35 Eucl 3.) And by sim. tri. Mn : 2QM or BL :: Mm : 2nH ; hence time in Ga : time in EB :: Mm : BL. Therefore the sum of all the times in all the Ga's : time in EB or in DB :: sum of all the Mm's : BL ; that is time in Fa : time in DB :: Lb : LB, and time FB : time in DB :: LMB : LB, or time FBf : time in DB :: 2LMB : LB, or as 3.1416 : 1 *Q. E D.*

41. Hence all the vibrations of a pendulum in a cycloid, whether great or small, are performed in the same time. If p = 3.1416, l = length of the pend. and s the space fallen by a heavy body in 1″ ; then $\surd s$: $\surd\frac{1}{2}l$ :: 1″ : $(l \div 2s)^{\frac{1}{2}}$ the time of falling through $\frac{1}{2}l$; therefore 1 : p :: $(l \div 2s)^{\frac{1}{2}}$: $p \times (l \div 2s)^{\frac{1}{2}}$, the time of one vibration.

42. The time of vibration in the small arc of a circle is nearly equal to that in the cycloid, as both arcs nearly coincide at B ; hence $p \times \sqrt{l \div 2s}$ is the time of vibr. in a small circular arc, where l is the radius. If s or l be given, the rest is therefore given. The easiest way is to find by experiment the length of the pendulum vibrating seconds ; this in the lat. of London is found = 39$\frac{1}{8}$ inches ; hence we have $p \times (39\frac{1}{8} \div 2s)^{\frac{1}{2}}$ = 1″, from which $s = \frac{1}{2}p^2 l$, is found = 193 07 in or 16$\frac{1}{12}$ feet.

43 From art 41 and 42, if n = the number of vibrations performed in the time t ; then the lengths of pendulums describing similar arcs being as the squares of the times, we have 39$\frac{1}{8}$: 1^2 :: l : $l \div 39\frac{1}{8}$ = the sq of the time of one vibration ; hence t divided by $(l \div 39\frac{1}{8})^{\frac{1}{2}} = n$, from which $t^2 \times 39\frac{1}{8} = n^2 l$, and therefore n^2 : t^2 :: 39$\frac{1}{8}$: l. Thus to find the length of a half-seconds pendulum, then 1 : $\frac{1}{4}$ :: 39$\frac{1}{8}$: 9$\frac{3}{4}$ inches, &c.

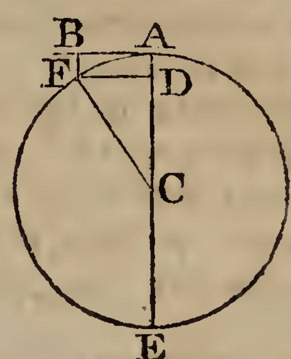
The reverse of this prop being also true, that is l : 39$\frac{1}{8}$ :: t : n^2, the number of vibrations made by a pendulum of a given length may from thence be found.

44. *The lengths of pendulums vibrating in the same time, in different places on the earth, will be as the forces of gravity.* For the vel. being as the force of gravity (the quantity of matter being the same in both pendulums) the space is as the vel. or as the gravity. Now as pendulums of the same length will vibrate in the

same time (4 art. 39) and the lengths of pendulums are as the spaces fallen through in equal times, that is as the forces of gravity.

45. By a similar reasoning it appears, 1. That the time in which pendulums of the same length will vibrate, by different forces of gravity, are reciprocally as the sq. roots of those forces respectively. 2. That the lengths of pendulums in different places are as the forces of gravity, and the sq. of the times of vibr. 3. That times of vibr. are as the sq. roots of the lengths of the pend. directly, and the sq. roots of the gravitating forces reciprocally.—4. That the forces of gravity in different places are as the lengths of pend. directly, and the sq. of the times of vibr. reciprocally.*

46. *If a body revolving in a circle, be retained in it, by a centripetal force tending to the centre of the circle; then its periodic time, or the time of one rev. will be* $pt \times (2r \div s)^{\frac{1}{2}}$, *and the vel or space it describes in the time t will be* $\sqrt{2rs}$. Where $r =$ rad. AC, s the space fallen through in the time t, by the force at A; and $p = 3.1416$.



Let AB be a tang. at A, AF an indefinitely small arc; let FB, FD, be drawn perp. to AB, AC, respectively. Let the body descend through AD or BF in the time 1; then AF will be described in the time 1.—The whole circum. $= 2pr$, and AF $= (2r \times \text{AD})^{\frac{1}{2}}$. Now art. 18, $s^{\frac{1}{2}} : t :: \text{AD}^{\frac{1}{2}} : t\sqrt{\text{AD} \div s} =$ time of moving through AD or AF; and AF : $t\,(\text{AD} \div s)^{\frac{1}{2}}$ :: circum. AFEA : the time of one rev. by subst. $= pt \times (2r \div s)^{\frac{1}{2}}$. And by uniform mot. time of descr. AF : AF or $\sqrt{2r \times \text{AD}} :: t : \sqrt{2rs} =$ vel. of the body *(b)* or the space descr. in the time t.

47. Vel. of $b =$ vel. acquired in descending through $\frac{1}{2}r$, by the force *(f)* at A uniformly continued. For $s^{\frac{1}{2}} : 2v$ (the vel.) :: $\sqrt{\frac{1}{2}r} : \sqrt{2rs} =$ vel. acquired in falling through $\frac{1}{2}r$.

48. Hence the arc described by b in any time is a mean proportional between $\frac{1}{2}r$ and s. For $2r : \sqrt{2rs} :: \sqrt{2rs} : s$.

49. Hence if AFE be any curve, and AC, or r its radius of curvature in any point A, and its centre of force be any other point (S) then vel. in A $= \sqrt{2rs}$. For this is the vel. in the circle, and therefore in the curve which coincides with it.

50. *The periodic times* (P) *of several bodies revolving in circles round the same or different centres, are as the sq. roots of their*

* In these articles the rod of the pendulum, or the thread, is supposed very fine, or of no weight; and that the ball is very small, or has its matter united in a point. Hence as this cannot be so, the length of the pend. is nearly its dist. from the point of suspension to the centre of the ball, or rather to the centre of oscillation of the pend. See Emerson's Tracts, sect. 2, prop. 28. The Methods of finding the centres of Gravity, Percussion, Oscillation, &c. with a further detail of the properties of the pend. are given in Emerson's Mechanics.

radii directly, and the sq. roots of the cent. forces reciprocally. For (art. 45) $P = pt \times (2r \div s)^{\frac{1}{2}}$, and s is as f, the force; hence $P = pt \times (2r \div f)^{\frac{1}{2}}$; but 2, p, and t are given, therefore P is as $(r \div f)^{\frac{1}{2}}$.*

51. From the preceding art. it appears, 1. That the periodic times are as $r \div v$. For $v = (2rs)^{\frac{1}{2}} = \sqrt{2rf}$, hence $v^2 = 2rf$; and $P^2 = p^2 t^2 \times 2r \div f$, therefore $P^2 v^2 = p^2 t^2 \times 4r^2$, from which $P = 2ptr \div v$; whence P is as $r \div v$. 2. That the periodic times are as $v \div f$ For $v^2 = 2rs = 2rf$, and $r = v^2 \div 2f$, also $r \div v$ is as $v \div f$; but P is as $r \div v$, that is as $v \div f$. 3. That v and f are as r. For if P be given, then $r \div f$, $r \div v$, and $v \div f$ are each given. 4. That if P be as $\sqrt{r}$, v will be as $\sqrt{r}$, and the centripetal forces equal. For (art. 49 and 50, cor. 1.) taking $\sqrt{r}$ for P, we have $\sqrt{r}$ is as $(r \div f)^{\frac{1}{2}}$, that is as $r \div v$. Hence 1 is as $1 \div \sqrt{f}$, or as $\sqrt{r} \div v$, and $\sqrt{r}$ is as v; the $\sqrt{f}$ is therefore a given quantity. 5. That if P be as r the velocities will be equal, and f as $1 \div r$. For taking r for P, we have r is as $(r \div f)^{\frac{1}{2}}$ or as $r \div v$; hence $\sqrt{r}$ is as $1 \div \sqrt{f}$, and 1 is as $1 \div v$, therefore v is a given or constant quantity (that is V is as v) and r is as $1 \div f$. 6. That if the periodic times be in the sesquiplicate ratio of the radii, or if P be as $r^{\frac{3}{2}}$, then v will be as $\sqrt{r}$, and f as $1 \div r^2$. For taking r^3 for P, $r^{\frac{3}{2}}$ is as $(r \div f)^{\frac{1}{2}}$, or as $r \div v$; and r is as $1 \div \sqrt{f}$, or r^2 is as $1 \div f$, also $\sqrt{r}$ is as $1 \div v$. 7. That if P be as r^n, then v will be as $1 \div r^{n-1}$, and f as r^{2n-1}. For taking r^n for P, then r^n is as $(r \div f)^{\frac{1}{2}}$, or as $r \div v$; whence r^{2n} is as $r \div f$, and r^{2n-1} is as $1 \div f$. Also r^{n-1} is as $1 \div v$.

52. *The velocities of several bodies revolving in circles round the same or different centres are as the radii directly, and periodic times reciprocally;* or v is as $r \div t$. For (art. 46) $v = 2rs = \sqrt{2rf}$, and P is as $v \div f$ (2 art. 51) and Pf is as v; also f is $v \div P$; hence $v = \sqrt{2rf} = (2r \times v \div P)^{\frac{1}{2}}$, and $v^2 = 2rv \div P$, and $v = 2r \div P$, that is v is as $r \div P$.

53. From the preceding art. it follows, 1. That v is as Pf. 2. That v^2 is as rf. For $v = \sqrt{2rf}$. 3. That the velocities being equal, P is as r, and r as $1 \div f$. For $r \div P$ is a given ratio, r being given; and as $\sqrt{rf}$ is given, r is as $1 \div f$. 4. That if v be as r, the periodic times will be the same, and f as r. For then v or r is as $r \div P$, and 1 is as $1 \div P$. Also $r = \sqrt{2rf}$, hence r is as f. 5. That if v be as $1 \div r$, then f will be as $1 \div r^3$, and P as r^2. For taking $1 \div r$ for v, then (cor. 2) $1 \div r = \sqrt{2rf}$, or $1 \div r = 2rf$; whence f is as $1 \div r^3$; also $1 \div r$ is as $r \div P$, and P is as r^2.

54. *The centripetal forces are as the radii directly, and the squares of the periodic times reciprocally.* For (art. 46) $P = pt \times$

* When any quantity is divided into another, the reciprocal of that quantity may be taken. The reciprocal of any quantity is 1 divided by that quantity. Thus the reciprocal of a is $1 \div a$.

$(2r \div s)^{\frac{1}{2}} = pt\ (2r \div f)^{\frac{1}{2}}$, and $P^2 = p^2\ t^2 \times 2r \div f$; also $p^2 f = 2p^2\ t^2 r$; hence $f = 2p^2 t^2 r \div P^2$ is as $r \div P^2$.

55. From the last art. it follows, 1. That f is as $v \div P$. For (art. 52) v is $r \div P$, and f is as $r \div P^2$, that is as $v \div P$. 2. That f is as $v \div r$. For f is as $v \div P$, and fP is as v; but (1 art. 44) P is $r \div v$; hence fP is as $fr \div v$; therefore $fr \div v$ is as v, and f is as $v^2 \div r$. 3. That, the centripetal forces being equal, v will be as P, and r as P^2 or as v^2. 4. That if f be as r, the periodic times will be equal. For if f is as $r \div P^2$, and $f \div r$ is as $1 \div P^2$; and if $f \div r$ be a given ratio, $1 \div P^2$ will be given, as also P. 5. That *if f be as $1 \div r^2$; then P^2 will be as r^3, and v as $1 \div \sqrt{r}$.* For taking $1 \div r^2$ for f, then $1 \div r^2$ is as $r \div P^2$, and $r^3 \div P^2$ a given quantity. Also $1 \div r^2$ is as $v^2 \div r$, and $1 \div r$ as v^2, or $(1 \div r)^{\frac{1}{2}}$ as v.

56. *The radii are directly as the centripetal forces and the squares of the periodic times.* For (art. 46 or 54) $P^2 = p^2\ t^2 \times 2r \div f$, and $P^2 f = 2p^2\ t^2\ r$; hence $P^2 f$ is as r, 2, p, and t being given.

57. Hence it follows, 1. That r is vP. For Pf is as v (1 art. 53) and $P^2 f$ is as r; hence Pv is as r. 2. That r is $v^2 \div f$. For (2 art. 51) P is as $v \div f$; but r is as Pv, by the preceding; hence r is as $v^2 \div f$. 3. That the radii being equal, f is as $v^2 \div P^2$, and that v is as $1 \div P$. For in this case $v^2 \div f$, $P^2 f$, and Pv are all given; and f is as v^2, or f is as $1 \div P^2$; hence v is as $1 \div P$.

Note. The converse of all these articles, &c. are also true; and what is shewn of cent. forces is equally true of centrifugal forces, they being equal and contrary. Moreover whatever is demonstrated in these articles, concerning the forces, velocities, and periodic times of bodies moving in circles, hold equally true in ellipses, taking the mean distances, or half the transverse axis, instead of the radii. The truth of this is fully shewn in Emerson's Centripetal forces, where the different cases of bodies revolving in the conic sections are investigated. The reader is therefore referred to this, or to *Newton's* prin. for more information on this subject. In *Gregory's* Ast. the Laws of Centripetal and Centrifugal forces are fully discussed. But for some recent improvements, consult *Laplace's System of the World*, B. 3, or his *Celestial Mechanics*. *Simpson, Emerson,* Mc *Laurin*, &c. in their respective treatises on Fluxions, have given the analytic investigation of these laws. See also pa. 402.

58. *The quantities of matter* (M) *in all attracting bodies, having others revolving about them in circles, are as the cubes of their distances directly, and the squares of the periodic times reciprocally.* For as observations have fully proved that the squares of the periodic times are as the cubes of the distances, both of the planets and satellites from their respective centres Hence (6 art. 51) f is as $1 \div r^2$; and the attractive forces at the same dist. being as the quantity of matter M, hence the absolute force of M is as $M \div r^2$, and (art. 54) since f is as $r \div P^2$, if we take $M \div r^2$ in place of f, then $M \div r^2$ is as $r \div P^2$, and therefore M is as $r^3 \div P^2$.

Cor. 1. Hence $M \div r^2$ (which is the force of the attracting body C at A) may be substituted for f in any of the foregoing articles, Cor. 2. Hence also *the attracting force of any body is as the quantity of matter directly, and the sq. of the dist. reciprocally.*

Note. If the mean dist. be taken for the radii of the circles, the same properties hold also in ellipses, &c.

59 *The densities* (D) *of central attracting bodies are reciprocally as the cubes of the parallaxes of the bodies revolving round them (as seen from those central bodies) and reciprocally as the squares of the periodic times.* For D $\times$ d^3 (the cube of the diam.) is as the quantity of matter (art. 9) that is as m^3 (cube of the mean dist.) or $r^3 \div P^2$ (art. 58) hence D is as r^3 (or m^3) $\div d^3 P^2$. But $d \div r$ or m, is as the angle of the parallax (*a*) therefore D is as $1 \div a^3 P^2$.

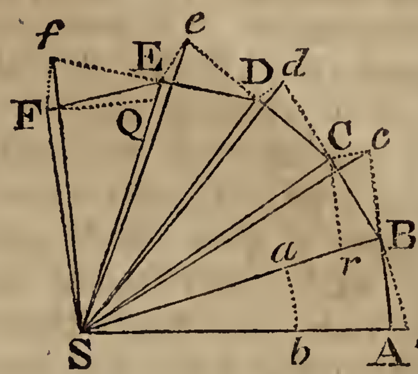
Cor. Hence D is as $1 \div d^3 P^2$.*

* From this and the foregoing article the *masses* and *densities* of the planets are thus found. The result in art. 58, being applied to Jupiter and his 4th satellite, we have the angle subtended by mean rad. of the orbit of this satel. at Jup. mean dist. from the sun = (1530″36 according to *Laplace*) 8′ 15″99 nearly, this angle at the earth's mean dist. from the sun = (7964″75) 43′ 0″57 the rad. of the circle = (636619″8) 57° 17′ 44″8. Hence the mean rad. of the orbit of the satel. : the mean rad. of the earth's :: 43′ 0″57 : 57° 17′ 44″8. The sidereal rev. of the 4th satel. is 16.689 days, and of the earth 365.2564 days; from which the mass of *Jupiter* is found = $\frac{1}{1066.08}$ that of the sun being 1. *Laplace* remarks that the denom. of this frac. must be augmented by 1, as the force which retains Jup. in his relative orbit round the sun is the sum of the attractions of the sun and of *Jupiter*. The mass of Jup. will then be $\frac{1}{1067.08}$. The mass of *Saturn* determined in the same manner is found to be $\frac{1}{3359.40}$, and of *Herschel* $\frac{1}{19504}$. The mass of the earth may be found in the same manner. *Laplace* gives a different method in B. 4, ch. 2, Ast. from which he determines that the masses of the sun and earth are as 1479560.5 and 4.48855, from whence it follows that the mass of the *earth* is $\frac{1}{329630}$ taking the sun's parallax (27″2) 8″8126. If the sun's parallax differ any thing from this, the value of the earth's mass should vary as the cube of this parallax compared with that of 8″8. *Laplace* determines the masses of *Venus* and *Mars* from the *secular* diminution of the obl. of the ecliptic supposed (154″3) 50″ nearly, and from the acceleration of the Moon's mean motion fixing it at (34″36) 11″13 nearly, for the 1st century comm. with 1700. The mass of Venus is thus found = $\frac{1}{383137}$, and that of Mars $\frac{1}{1846082}$. He has found the mass of *Mercury* from his mag. supposing the densities of this planet and the earth's inversely as their mean dist. from the sun, = $\frac{1}{2025810}$.

The *densities* of bodies being proportional to their masses divided by their volumes or mag. that is by the cubes of their radii when spherical. Their densities are therefore as their masses divided by the cubes of their radii; but to obtain greater accuracy that radius of a planet must be taken (as *Laplace* remarks) which corresponds to that parallel the square of the sine of whose latitude is $\frac{1}{3}$, and which is equal to $\frac{1}{3}$ of the sum of the radius of the pole added to twice the radius of the equator. It is thus that Laplace determines the densities of the Earth, Jup. Saturn and Herschel to be 3.9393, 0.8601, 0.4951 and 1.1376 respectively, the sun's mean density being taken 1.

As the force of gravity at the equator of the planets is as their masses divided by the squares of their diameter, supposing them spher. and deprived of their rotary motion. Now the equatorial diam. of Jup. is (626″26) 3′ 22″9;

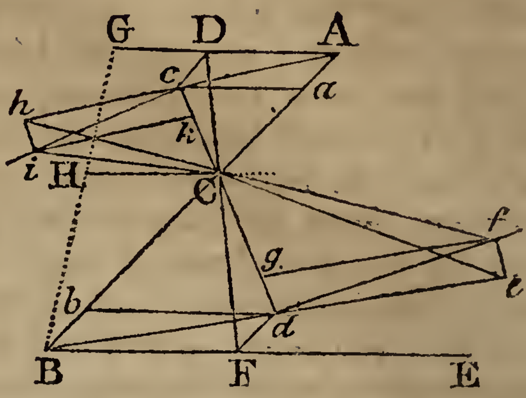
60. *The areas which a revolving body describes, by radii drawn to a fixed centre of force, are proportional to the times of description; and are all in the same immoveable plane.* Let S be the centre of force; and let the time be divided into very small equal parts. In the first part of that time let the body describe the line AB, then (art. 5) if no other force be impressed the body will proceed towards *c*, and describe B*c* = AB in the 2d part of time, so that the area ASB = BS*c* (38 Eucl 1) But when the body comes to B, suppose that a centripetal force act by a single, but strong impulse, then the body will describe the diagonal BC of a parallelogram whose sides B*c*, B*r* or *c*C parallel to SB represent those forces (art. 27) hence the triangle SBC = SB*c* = also SBA, all of which are in the same plane. In like manner it may be shewn that if the centripetal force act successively in C, D, E, &c. and make the body in each single particle of time describe the lines BC, CD, DE, EF, &c. they will all be in the same plane, and the triangles CS*d*, CSD, DS*e*, DSE, ES*f*, ESF, &c. are all equal. Hence in equal times, equal areas are described in one immoveable plane; and by comp. any sum of those areas are as the times of description. Now let the number of these triangles be increased, and their breadth diminished *ad infinitum*; the centripetal force will then act constantly, and the figure ABCD, &c. will be a curve.



61. From the preceding art. it appears, 1. That if a body describe areas proportional to the times about any fixed point, it is urged by a centripetal force directed to that point. For a body cannot describe areas proportional to the times about two fixed centres. 2. That the vel. of a body, in any point of a curve, is reciprocally proportional to the perp. on the tang. at that point.—The base of any of the triangles (which represents the vel.) being reciprocally as the perp. 3. That the angular velocity at the centre of force is reciprocally as the square of the dist. from that centre. For ABCD, &c. being considered as a curve, the small triangle SFE = SAB, they are descr. in equal times, and area of the 1st = SF × FQ ÷ 2, and of the 2d SB × B*p* ÷ 2; hence SF × FQ = SB × B*p*. But ∠ FSE : ASB :: FQ :: *ab* (S*a* being = SF) :: SF × FQ : SF or S*a* × *ab* :: SB × B*p* : S*a* × *ab* :: area SBA : area S*ab* :: $SB^2 : Sa^2$ or SF^2. See *Newton's prin.* B. 1. sect. 2.

and of the earth (54″5) 17″6, at the earth's mean dist. from the sun. Hence if the weight of a body at the terrestrial equator be 1, this body transported to the equator of Jup. would weigh 2.509. But this must be diminished $\frac{1}{9}$ from the effects of the centifugal force at Jupiter's eq. this effect may be determined by the preceding articles. The same body at the sun's eq. would weigh 27.65, and heavy bodies would fall 100 metres the 1st second of their descent, according to Laplace. The second here must, no doubt, be that adopted in the French measures.

62. *If two bodies* A, B, *revolve about each other, they will both revolve about their centre of gravity.* Let C be the centre of gravity of A, B acting on each other by any centr. forces. Let AD be the direction of A's mot. draw BE paral. to AD for B's direction. Let the time of descr. AD, BF be very short, so that AD : BF :: AC : BC, C will then be the centre of grav. of D and F, the tri. ACD, BCF being sim. (see the note pa. 253.) Hence AC : CB :: DC CF. Let A*a*, B*b*, be the spaces through which A and B will advance towards each other in the same time by their mutual attractions, these spaces will be reciprocally as the bodies, or directly as the dist. from C the centre of gravity; that is, A*a* : B*b* :: AC : BC. Complete the paral. A*c*, B*d*; *c*, and *d* will then be the corresponding places of the bodies, instead of D, F. (art. 23.) Now as AC : BC :: A*a* : B*b*, by div. AC : BC :: *a*C : *b*C; but AC : BC :: AD : BF :: *ac* : *bd*. Hence *a*C : *b*C :: *ac* : *bd*; the tri. *c*C*a*, and *d*C*b* are therefore similar, whence C*c* : C*d* :: *ac* : *bd* :: AC : BC :: B : A. Therefore C is still the centre of grav. of the bodies, at *c* and *d*.



If B*d* and A*c* be now produced until *df* = B*d*, and *ci* = A*c*, and if *ck*, *dg*, be the spaces drawn through from their mutual attr. and *ci*, *df*, be compl. then it will appear, in like manner, that C is the centre of grav. of the bodies at *h* and *e*, and also *i* and *f*; and A*c*, *ci*, &c and B*d*, *df*, &c. are therefore the paths of A and B round C.

If B be at rest while A moves towards G; then C will move uniformly along CH paral. to AG. Hence if the space the bodies move in, moves in the direction CH with the vel. of the centre of grav. this centre will then be at rest in that space, and B will move in the direction BF paral. to CH or AD, which comes to the same as the former case. Hence the bodies will always move round the centre of grav. which is either at rest, or moves uniformly in a rt. line.

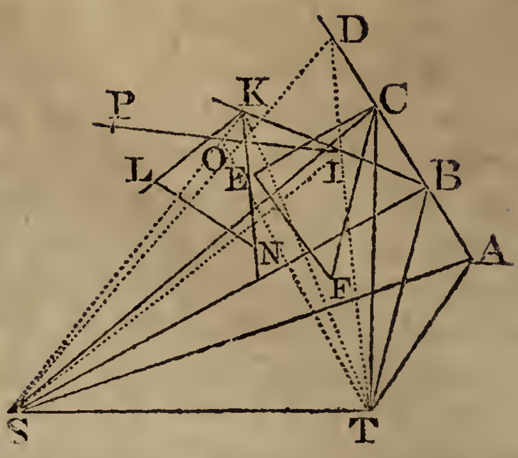
In the same manner it may be proved, that if A and B repel each other, they will also constantly move round their centre of gravity.

63 From the preceding art. it appears, 1. That in estimating the motions of a system of bodies among themselves, their motions round their common centre of grav. should be taken. 2. That the directions of the bodies in oppos. points of their orbits are always parallel to each other. For as AD : D*c* :: BF : F*d*, AD, D*c* is therefore paral to BF, F*d*; hence ∠ DA*c* = HB*d*; B*d* is therefore paral to A*c*. For the same reason *df* is paral. to *ci*, &c 3. That two bodies acting on each other by any forces, describe sim. figures about their centre of gravity. For A*c*, B*d*, &c. are parallel, and always proportional to AC, BC. 4. That if

the forces be directly as the dist. the bodies will describe concentric ellipses round the centre of grav. 5. That if the forces be reciprocally as the squares of the dist the bodies will describe similar ellipses, or some conic sections, about each other, having the centre of gravity in the focus of both, &c.*

64. *If a body be projected from* A, *in a given direction* AD, *and be attracted to two fixed centres* S, T *not in the same plane with* AD, *the revolving triangle* SAT *formed by lines drawn from the two fixed centres to the body, will describe equal solids in equal times, about* ST *the line joining the fixed centres.* Let the time

* Our limits would not permit us to enter into an investigation of any of the properties of the centre of grav. however interesting: the following observations of *Laplace* deserve, however, to be mentioned. He remarks, that when a body receives an impulsion in the direction passing through the centre of grav. all its parts move with equal vel. That if the direction pass on one side of this point, the different parts of the body acquire unequal velocities, from which results a mot. of rotation of the body about its centre of grav. together with its progessive mot. He then remarks that this is the case with the earth and the planets. He makes the dist. of the prim. impulse from the centre of grav. of the earth $= \frac{1}{160}$ of its rad. supposing it homogeneous. Sir *Isaac Newton* makes a similar suppos. in accounting for the centrifugal forces of the planets. But neither he, or any other person since his time, has ever shewn whence proceeded, or what was the cause of this primitive impulse, for it is certain that there is no effect without a cause. It is infinitely more probable that both the motion of rotation, and also that in the orbits of the planets, depend upon the sun, and are regulated by him. For as we have remarked before, we find no other active principle in matter, or emanating from it, capable of producing such an effect, except *light*, which modified, reflected, refracted, and varied in a thousand different manners, is very probably the cause of gravity, electricity, &c. and innumerable phenomena of which no rational account can be given, from our ignorance of the nature of this subtle fluid. If its action produces the rotary motion of the planets, it could not be from the situation of their centres of grav. For if the centre of motion be the same as the centre of the body, and the centre of gravity a little distant from it, let the line joining these centres be perp. to the direction of the rays of light, &c. from the sun; then the unequal action on the body may produce a rotary mot. but when this line is parallel to the direction of the rays, they have no effect on turning the body; and when the line passes this paral. posit. the action of the rays will tend to turn the body in a contrary direction; and hence their effect would be to produce a vibratory mot. which with a little resistance would subside, and the same side remain turned towards the sun. This may be the case with the secondary planets with respect to their primaries, whose action on them is greater than that of the sun's. If the unequal action of the sun's rays on the planets, from a rarification of their atmospheres, exceeded the former, a rotary motion would ensue. This may be the case with the primary planets. For if we suppose the former case, it may be asked why has not the secondary planets a rotary mot. Why does not the change of the sun's decl change that in the primaries? Until these questions be solved, *Laplace's* obs. can have no force.



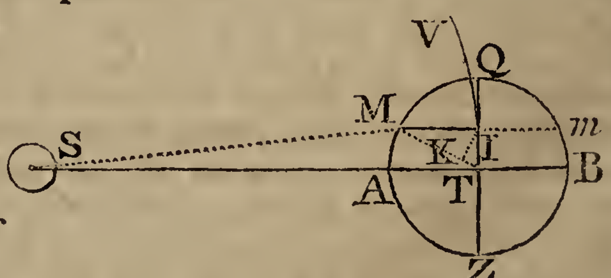
be divided into infinitely small equal parts, then in those equal times it is evident that the lines AB, BC, CD, &c. will be described, and hence that the solids STAB, STBC, STCD, &c. described in the same equal times, would be equal, if the focus at S and T did not act on the moving body Let S and T be now supposed to act at the end of each interval of time, let T act at B in the direction BT, in which direction the body is always drawn by T, so that instead of being at C, it is drawn in the direction CF; and by the force S, for the same reason, it is drawn from C in a direction CE parallel to SB. Hence from both forces the body at the end of the time must be somewhere in the plane ECF paral. to SBT as at I.* Now as pyramids upon the same base and of equal altitudes are equal (Emerson's Geom. b. 6, prop 17) the pyramid STBI = STBC, being between the paral. planes ECF, SBT, and therefore of equal altitude; hence pyr. STBI = STAB.

If BI be now continued to K so that IK = BI, the body in the next part of time would advance to K describing the pyr. STIK = STBI, but at the end of the time the body being drawn by the forces S, T parall to their directions KL, KN, it will be found somewhere in the plane LKN, as suppose at O. Then the solid STIO = STIK = STBI = pyr. STAB

If IO be now produced making OP = IO the body attracted by S, T at P will descr another equal pyr. Hence equal pyramids will be described in equal times: and therefore the whole described as the times of description.

It may be here remarked, 1. That the orbit becomes a *curve* when the number of the lines AB, BI, IO, &c. is increased, and their mag. dim. *ad infinitum.* 2. That any lines AB, BI, IO, &c. are tangents at A, I, O, &c. being corresponding points in the curve. 3. That the curve is not in the same plane except when the forces on each side of it are equal.

65. *If the body* T *revolves above* S *at a great dist. in the orbit* TV, *and* M *a lesser body about* T *which is near* M. *Then if* F = *the centr. force of* S *upon* T, *the disturbing force of* S *upon* M = F × 3MI ÷ ST, MI *being paral. and* IT *perp. to* ST. *And* F × MT ÷ ST *will be the increase of the centr. force from* M *towards* T. Let



* This point may be found by resolving the three forces in the directions BC, CF, CE, as shewn art. 27. And any number of forces may be resolved in like manner.

ST $= r$, MT $= a$, MI $= x$, $g =$ the force of gravity, s the space fallen by this force in the time 1. $h =$ the space descended by the force F in the time 1. P $=$ the period. time of T about S, and $t =$ the period. time of M about T, $f =$ the centr. force of T at M, $p = 3.1416$. Now as attraction is reciprocally as the sq. of the dist. then the force of S at T : its force at M $:: 1 \div r^2 : 1 \div SM^2 :: 1 \div r^2 :: 1 \div (r - x)^2 :: r : r + 2x + 3x^2 \div r + 4x^3 \div r^2$, &c. And force of S at T : diff. of the forces $:: r : 2x + 3x^2 \div r$, &c. that is $r : 2x + 3x^2 \div r :: F : 2Fx \div r + 3Fx^2 \div r^2$ nearly, the diff. of the forces, or the single force by which M is drawn from its orbit in the direction IM or MS. Let the forces MS be divided into the two MT, TS, which substituted for it and proceeding as before, we have force of S at T : force at M $:: 1 \div r^2 : 1 \div (r - x)^2$. And force of S on M in direct. MS : force on M in direct. TS $:: r - x : r :: 1 \div r : 1 \div (r - x)$. Hence from equality of propor. force of S at T : force on M in direct. TS $:: 1 \div r^3 : 1 \div (r - x)^3 :: 1 \div r^3 : 1$ divided by $r^3 - 3r^2x + 3rx^2 - x^3 :: r : r + 3x + 6x^2 \div r$, &c. And force at T : diff. of the forces $:: r : 3x + 6x^2 \div r$, &c or $r : 3x + 6x^2 \div r :: F :: 3Fx \div r + 6Fx^2 \div r^2 =$ the disturbing force of P paral. to TS. Also $x : a ::$ increase of the disturbing force in direct. MI ($Fx \div r + 3Fx^2 \div r^2$, &c.) : $Fa \div r + 3Fax \div r^2$, &c. the addition of the centripetal force in direction MT. For in the former disturbing force $2Fx \div r + 3Fx^2 \div r^2$, there was a diminution of centr. force at T, as appears from the next art.

66. From the preceding art. it appears, 1. That the simple disturbing force at M towards S $= 2Fx \div r$ nearly (rejecting the other terms of the series as inconsiderable) and dimin. of centr. force of M towards T $= Fv \div r$. Also accelerating force of M in the arc MA $= Fz \div r$. z and v being the sine and versed sine of 2MQ. For TI $= y$, and let IK be perp. to MT ; then sim. trian. $a : x :: x :$ MK $= x^2 \div a ::$ force MI ($2Fx \div r$ nearly) : force in direct. KM or TM $= zF\, x^2 \div ar = Fv \div r$. Also $a : y :: x :$ IK :: force MI : force in direct. IK or MA $= 2Fyx \div ar =$ (Trig. *Emerson's*, prop. 2. Schol.) $Fz \div r$. 2. That if M be the moon, and S the sun, the disturbing force at M $= fq \div 59.574$, q being sine dist. from the quadrature Q. For (art. 54) F $= ft\, r \div P^2a$, and $3Fx \div r$, &c. $= 3ft^2x \div P^2a$ (nearly) $= 3fq \div 178.724$ (because $t^2 \div P^2 = 1 \div 178.724$ see pa. 308 or 325, and 304 or 350, and $x \div a = q \div 1$) $= fq \div 59.6$ nearly. 3. That if M be a body in the equator the disturbing force of the sun at M $= qg \div 1306767$l. For M being the moon, the force is then $fq \div 59.6$; but $g = 60.3 \times 60.3f$ (see pa. 308) or $f = g \div 3636$ nearly ; hence the force becomes $qg \div 59.6 \times 3636$, and at the earth $= qg \div 59.6 \times 60.3^3 = qg \div 1306767$l nearly. 4. That the disturbing force of the moon on the equinoctial $= qg \div 3595802$. For the general perturbating force was nearly $3Fx \div r$, where F must be the centr. force at the moon. But the centr. force of the earth at the dist. of the

moon $= 1 \div 60.3^2 g$. And the moon being 49.2 times less than the earth (see pa. 327 or 344) the centr. force of the moon at the same dist $= g \div 49.2 \times 60.3^2$, which being substituted for F, then the force of the moon in the equator $= 3x \div r$ mult. by $g \div 49.2 \times 60.3^2 = 3gx$ divided by $60\ 3a \times 49.2 \times 60.3^2 = qg$ divided by $20\ 1 \times 49.2 \times 60.3^2 = 3595802$ nearly. 5. That *the disturbing force of the sun to that of the moon upon the equator* is as 1 : 3.6 nearly For $1 \div 1306767$ and $1 \div 3595802$ is nearly in this proportion. 6. That if D be the apparent diam. and d the density of the perturbating body, then the disturbing force will be as dD^3x. For that force $= 3Fx \div r$, or as $Fx \div r$. If the diam. $= b$, its quantity of matter $= m$; then F is as $db^3 \div r^2$, and the disturbing force is therefore $db^3x \div r^3$, or as dD^3x. 7. That the centrifugal force of M (at the equator) : perturbating force MT :: $P^2 : t^2$, t being here the time of one rev. of the earth on its axis. For $t : 2pa$ (circum) :: $1'' : 2pa \div t =$ arc descr in $1''$; and versed sine $= 4p^2a^2 \div 2at^2 = 2p^2a \div t =$ the ascent or descent from the earth's centrifugal force. But forces are as the effects produced; hence $s : g :: 2p^2a \div t^2$ (ascent) : $2p^2ag \div t^2s$ the centrifugal force itself Now as the perturb force $= Fa \div r = ahg \div rs$, we have centrif force : perturb. force :: $2p^2ag \div t^2s : ahg \div rs :: 2p^2 \div t^2 : h \div r :: 2p^2r : t^2h :: 2p^2r \div h : t^2$. But $2p^2r \div h = P^2$; for $\sqrt{2rh} : 1'' :: 3pr : P = 2pr \div \sqrt{2rh}$; hence $P^2 = 4p^2r^2 \div 2rh = 2p^2r \div h$ 8. That the body M is therefore accelerated from the quadr. Q, Z, to the syzygies A, B; and retarded from syzy. to the quadr. And also that the vel. and area descr. in the syz. are greater than in the quadr

67 *The linear error gen. in* M *in any time, is as the disturb. force and sq. of the time. And the angular error, seen from* T, *is as the force and sq. of the time directly, and the dist.* TM *reciprocally.* For the mot. genr. in any portion of time is as the force, and in any other time as the force and sq. of the time; this mot. is the linear error of M, being carried out of its proper orbit by the force $3Fx \div r$. This error as seen from T is as the angle under which it appears, that is as the linear error divided by the dist TM; and hence is as the force and sq. of the time divided by the dist.

68. From the preceding art. it appears, 1. That the linear error generated in one rev. of M, is as the distu. force and sq. of the period. time $3Fat^2 \div r$. And the angular error in one rev. is as the force and sq. of the period. time divided by the dist. 2. That the mean error of M in any given time will be as the force and period. time $Fat \div r$ And the mean angular error is as the force and period. time divided by the dist. For let $1 =$ given time; then $t : Fat^2 \div r$ (whole error) :: $1 : Fat \div r$ error in the time 1. 3. That the mean linear error in any given time is as $at \div P^2$. And the mean angular error as $t \div P^2$. For (art 54) F is as $r \div P^2$; hence $Fat \div r$ is as $at \div P^2$. And ang. error as $t \div P^2$. 4. In any given time the linear error is as $at \div r^3$.

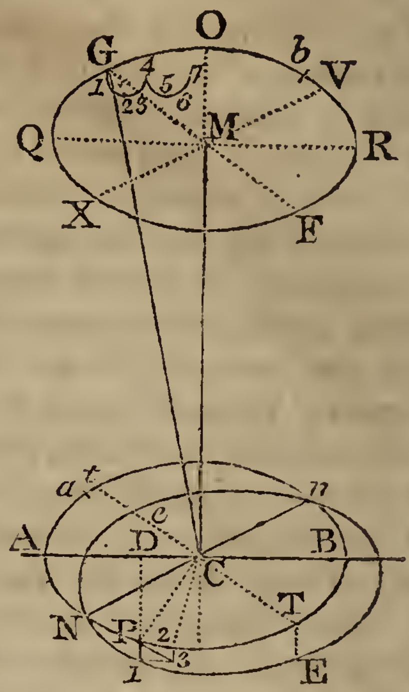
For (art. 58) P^2 is as r^3, hence $at \div P^2$ is as $at \div r^3$. 5. That the linear error in a given time is as $Fa^{\frac{5}{2}} \div r$. And the angular error as $Fa^{\frac{3}{2}} \div r$. For t^2 is as a^3, hence $Fat \div r$ is as $Fa^{\frac{5}{2}} \div r$. 6. That universally the angular errors in the whole rev. of any satellites are as $t^2 \div P^2$. And the mean ang. errors as $t \div P^2$. For (1) ang. error is as $Fa \div r \times t^2 \div a$, that is as $t^2 P^2$, because F is as $r \div P^2$. The latter case has been already proved. (3.)

69. *To determine the disturbing force of Jupiter or Saturn, upon the earth in its orbit; that of the sun upon the moon being given.* Let the quant. of matter in the sun and Jup. be as $m : 1$. P, p, t the periodic times of the earth, Jup. and the moon. r, b, the distances of the earth and Jup. from the sun, a the moon's dist. from the earth, F, f, the centr. forces of the sun and Jup. Now (art. 65) the distur. force of S the sun on M the moon is $3Fx \div r$, or as $Fa \div r$; but if S be Jup M the earth, and T the sun; the force is then $fr \div b$. That is the sun's distur. force on the moon : Jup distur. force on the earth :: $Fa \div r : fr \div b :: Fbr : fr^2$. But (2 art. 58) $F = m \div r^2$, and $F = 1 \div b^2$; hence the sun's force on the moon : Jupiter's on the earth :: $abm \div r^2 : r^2 \div b^2 :: ab^3m : r^4 :: aP^2m : rP^2$ (art. 65.) But the sun's disturbing force on the moon is given, and therefore that of Jup. If for P and m, Saturn's periodic time and quantity of matter be substituted, his disturbing force will be known.

70. From the preceding art. it appears, 1. That the angular errors generated in the moon by the sun are to those gen. by Jup. in the same time, as $p^2tm : P^3$. For (2 art. 68) these errors are as the forces and periodic times divided by the distances. Hence the sun's to that of Jup is as $ap^2mt \div a : rP^2 \times P \div r :: p^2tm : r^2$. 2. That the error in the moon from the sun's action, is to the earth's by Jupiter's as 11466 : 1; and to that gener. by Saturn as 222600 : 1. For let p = 4332.6 days, t = 27.32 days nearly, m = 1067.1 nearly. P = 365.25; then $p^2tm \div P^3 = 11466$. And taking p = 10759.07, and m = 3359.4 for Saturn; then $p^2tm \div P^3 = 222600$. 3. That the force of Saturn to that of Jup. to disturb the earth is as 1 : 19.4. 4. That *the secular motion of the nodes of the earth's orbit* by Jupiter's action is 10′ 9″2, and by Saturn's 31″4. For the annual mot of the moon's nodes = 19° 19′ 43″ (pa. 323) = 69583″, which divided by 11466 gives 6″0686, and mult. by 100 = 606″86; this being increased in the ratio of cos. of incl. of Jup. orbit 1° 19′ to that of the moon's 5° 8′ 48″ (pa. 324) gives 10′ 9″2, which divided by 19.4 gives 31″4 for Saturn. 5. That *the secular mot. of the earth's aphel.* by the action of Jup. is 21′ 16″ in consequentia, and by Saturn 65″77 For the annual mot. of the moon's apogee = 40° 39′ 50″ (pa 323) = 146390″ div by 11466 = 12″76, and mult. by 100 gives 1276″ = 21′ 16″. This div. by 19.4 gives 65″77.

71. *If a planet* P *(or the moon) perform its mot. round an immovable centre* C *in the orbit* N*Tnt*, *whose plane is inclined to that of the ecliptic* N*En*, *and is acted on by a force perp. to* AB *and paral.*

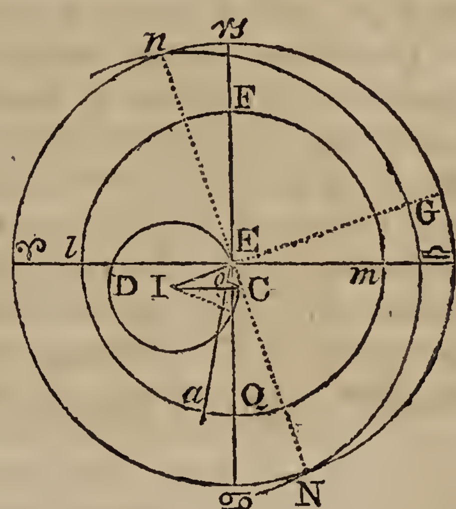
to the ecliptic, and always directed from the plane AB *to either side. To find the mot. of the nodes* N, *n, and the var. of the orbit's incl.* PNE. Let NTB*n* be half the orbit above the eclip. NE, N*n* the lines of the nodes, T, *t*, the tropics. Let CM be drawn perp. to the eclip. and CG perp. to the planet's orbit. Let the circle GORX be descr. round M as a pole ; then GC will be the axis of the orbit, and MC of the eclip. Through T, C, draw the plane GMC, and another through N, C, M, cutting the cir. GRF in X and V ; then GF is perp. to XV, and GRF paral. to NB*n*. Here GNX represents the upper surface. Let the planet at P describe the space P2 in any short interval of time, and let P*s* be the space it would be drawn in the same time by the force acting from the plane MAB. Compl. the paral. P3, then P3 will be the direction from both forces. Now as P*s* is paral. to the eclip. the point 3 is below the plane of the orbit, and the plane CP2 will be moved in the pos. CP3, about CP ; hence GC will be moved perp. to CP. The pole G will therefore be moved to some point between F and V. Hence the mot. of G will be known for all the places of P in its orbit. For about A, G moves perp. to MQ ; at N perp. to MX, or in the direct. GM ; at T paral. to MV, or in the curve RO ; about B it moves perp. from MQ ; hence in the passage of P from A to B, the pole of its orbit G describes the curve G1234. But in the other half of the orbit B*nt*A, G will return back at 4, and describe a sim. curve 4567, the force being directed the contrary way from the plane ABMR. Hence when P has made 1 rev. G will be at 7. In this posit. of the nodes 7 will be within the circle ; for the points G, 4, being equidistant from QR ; 4, 7 will be also equidist. If the force and plane ABRQ revolve round CM in the direct. ANT*n* ; then when the ascend. node N is as far on the other side of A, as at *a*, the pole G will be as far on the other side of O, as at *b*, and being equally dist. from RQ on the same side, the curves (12467) will approach OV there, by the same degrees as they receded from it at GO. Hence G will, by degrees, be brought to the circle again. Thus in every two corresp. points on each side of O, the forces and their effects balance one another, and G will be at the same dist. from the circle GOV. Hence after half a rev. of the plane AB to the nodes, the ∠ GCM, or incl. of orbit, becomes the same as at first. And as G moves forward or backward in the circle, the mot. of the nodes N, *n*, will be forward or backward.



72. From this art. it appears, 1. That in the pos. of the nodes at N and n, the inclination of the orbit will be dimin. every rev. of P, but at a increased. For then 4 and 7 approach M and recede at b. 2. That the incl. decreases when P is in AT or Bt, and increases in TB, tA For G moves to 3, while P moves through AT. At 3 it is nearest M; from 3 to 4, G recedes from M while P moves through TB. The same will happen in the other half of the orbit. 3. That when P is in AN and Bn, the nodes move forward, but backward in NB, nA. For while G describes G1, its mot. is forward, that is from G towards Q; at 1 it is stat. P being in N. G moves backwards or towards O through 1234; and then P is in NB. 4. That in general the nodes are always regressive except when P is between a node and its quadr. and then they are progr. wherever they are situated. 5. That the nodes move faster when P is in T and t. For then G is at 3 and 6. 6. That the incl. varies most when P is at N and n. For G is then at 1 and 5. 7. That therefore the incr. or decr. of the incl. may be easily found, the place of P, and diff. situations of N, n being given. 8. That hence the forces being given, the mot. of G may be delin. on the surface GXFV; and the incl. and place of the node at any time found.

For more information consult Emerson's Centr. forces, where this subject is fully investigated.

73. *To find the secular variat. of incl. of the earth's orbit, from the action of Jup. and the same for Saturn.* Let ♈♋♎♑ be the eclip. NGn the orbit of Jup. N Jup. ascending node, E, I, Q the poles of the eclip. Jup. orbit, and the equat. respectively. ECD a circle paral. to NG, and FmQ paral. to the eclip. Q moves regularly along the circle QlF, from the preces. of the equin. and as Jup. has no force to alter this regular mot. his force being only exerted on the whole body of the earth, and therefore in altering its orbit and the pole E of the ecliptic, which therefore moves in the circle ECD.



Hence the orbit of Jup. must be considered as fixed, and therefore the pole I and circle ECD; in which circle the mot. of E must therefore be computed. The precess. of the equin. in 100 years = 1° 23′ 45″ (pa. 244 and 305.) The secular motion of the nodes of Jupiter is 10′ 9″2* (4 art. 70) = 609″2 Jup. asc. node N in 1812 (∠ QEN) ♋ 8° 31′ 15″. Incl. of Jup. orbit (1812) 1° 18′ 49″. Hence making ∠QEa = 1° 23′ 45″ and EIC = 10′ 9″2. Make Co perp. to Ea, then Eo is the

* See pa. 360, where the true secular var. is given.

decrease of EQ or Ea, and is the same as that of the incl. of the eclip. and equinoctial. (art. 71.)

In the $\triangle$EIC, the $\angle$ EIC being very small, we have rad : s. IC 1° 18′ 40″ :: $\angle$ EIC 10′ 9″2 : EC = 13″94 nearly. Again $\angle a$EN or aEC = 8° 31′ 15″ + 1° 18′ 47″ = 9° 50′ 2″; then in the small rt $\angle$'d $\triangle$ ECo rad. : EC :: cos. oEC : Eo = 13″73 nearly, the secular decrease of the equin. by the action of Jup.

The same comput. being applied to Saturn, then EIC = 31″4, IC = 2° 29′ 45″, oEC = 21° 32′ + 1° 23′ 45″ = 22° 55′ 45″; hence the decr. by Saturn will be 1″26 nearly, and therefore by both the decr. will be 15″ nearly.

74. From the preceding art. it appears, 1. That the incl. will decrease until E and a be at their nearest dist. in the two circles, which will be about 6600 years, after which it will incr. again. It has been decreasing for more than 8000 years. For the diam. of the circles ED and FQ being nearly as 1 : 19.4. And the $\angle$IEQ = 81° 28′ 45″, and the diff of the mot. of E and Q being 1° 13′ 36″, it will decrease nearly as many centuries as 81° 28′ 45″ ÷ 1° 13′ 36″, which is 66. Also suppl 98° 31′ 15″ ÷ 1° 13′ 36″, gives 80 centuries it has been decreasing. The decrease for every century is not the same. For at its max. or min it is very slow, and is at a stand for a long time. 2. That the incl. can never be less than about 21°, or greater than about 26°. For the nearest and great dist. of the two circles EQ, FQ amount but to these. *Laplace* makes these limits (3°) 2° 42′.

To obtain these results more accurately, more terms of the series should be made use of; the above results are too small, but are however sufficient to give the learner an idea of this subject, the most interesting in modern astronomy. For more information the Celestial Mechanics of *Laplace*, or his Astr. vol. 2 may be consulted. See also *Newton's* prin. where different methods of performing this and the preceding articles, relative to the moon, are given. The above calculation being according to *Emerson's* method. Dr. *Gregory's* Astr. may also be consulted.

Declination of the Sun for the years 1812, 1816, 1820, 1824, *&c.*

BEING LEAP YEARS.

Days.	Jan. S.	Feb. S.	Mar. S & N.	April. N.	May. N.	June. N.	July. N.	Aug. N.	Sept. N & S.	Oct. S.	Nov. S.	Dec. S.
1	23° 5′	17°20′	7°31′	4°36′	15° 7′	22° 5′	23° 8′	18° 1′	8°16′	3°14′	14°29′	21°51′
2	23 0	17 3	7 8	4 59	15 25	22 13	23 3	17 46	7 54	3 37	14 48	22 0
3	22 55	16 46	6 45	5 22	15 42	22 20	22 59	17 31	7 32	4 0	15 7	22 9
4	22 49	16 28	6 22	5 45	16 0	22 27	22 54	17 15	7 10	4 24	15 26	22 17
5	22 43	16 10	5 59	6 7	16 17	22 34	22 48	16 59	6 48	4 47	15 44	22 25
6	22 36	15 52	5 36	6 30	16 34	22 40	22 42	16 42	6 25	5 10	16 3	22 32
7	22 29	15 34	5 13	6 53	16 51	22 46	22 36	16 26	6 3	5 33	16 20	22 39
8	22 22	15 15	4 49	7 15	17 7	22 52	22 29	16 9	5 40	5 56	16 38	22 45
9	22 14	14 56	4 26	7 38	17 23	22 57	22 22	15 51	5 18	6 19	17 55	22 51
10	22 5	14 37	4 2	8 0	17 39	23 2	22 15	15 34	4 54	6 42	17 12	22 57
11	21 56	14 17	3 39	8 22	17 55	23 6	22 7	15 16	4 32	7 4	17 29	23 2
12	21 47	13 58	3 15	8 44	18 10	23 10	21 59	14 58	4 9	7 27	17 45	23 7
13	21 37	13 38	3 52	9 6	18 25	23 14	21 51	14 40	3 46	7 49	18 1	23 11
14	21 27	13 18	2 28	9 27	18 39	23 17	21 42	14 22	3 23	8 12	18 17	23 15
15	21 17	12 57	2 4	9 49	18 54	23 20	21 32	14 3	3 0	8 34	18 33	23 18
16	21 6	12 37	1 41	10 10	19 8	23 22	21 23	13 44	2 37	8 56	18 48	23 21
17	20 54	12 16	1 17	10 31	19 21	23 24	21 13	13 25	2 14	9 18	19 3	23 23
18	20 43	11 55	53	10 52	19 35	23 26	21 2	13 6	1 50	9 40	19 17	23 25
19	20 31	11 34	29	11 13	19 48	23 27	20 52	12 46	1 27	10 2	19 31	23 26
20	20 18	11 13	6	11 34	20 0	23 27	20 40	12 26	1 4	10 24	19 45	23 27
21	20 5	10 51	N 18	11 54	20 13	23 28	20 29	12 7	40	10 45	19 58	23 28
22	19 52	10 29	42	12 14	20 25	23 28	20 17	11 46	17	11 6	20 11	23 28
23	19 38	10 8	1 5	12 34	20 36	23 27	20 5	11 26	S 6	11 28	20 24	23 27
24	19 24	9 45	1 29	12 54	20 48	23 26	19 53	11 6	30	11 49	20 36	23 26
25	19 10	9 24	1 52	13 14	20 58	23 25	19 40	10 45	53	12 9	20 48	23 25
26	18 55	9 1	2 16	13 33	21 9	23 23	19 27	10 24	1 17	12 30	21 0	23 23
27	18 40	8 39	2 39	13 52	21 19	23 21	19 13	10 3	1 40	12 50	21 11	23 20
28	18 25	8 16	3 3	14 11	21 29	23 18	19 0	9 42	2 4	13 11	21 21	23 17
29	18 9	7 54	3 26	14 30	21 38	23 15	18 46	9 21	2 27	13 31	21 32	23 14
30	10 53		3 49	14 48	21 48	23 11	18 31	8 59	2 50	13 50	21 42	23 10
31	10 37				21 56		18 16	8 38		14 10		23 6

Change of the Sun's decl. for periods of four years.

Periods of years.	JANUARY. Days. 1	7	13	19	25	FEBRUARY. Days. 1	7	13	19	25	MARCH. Days. 1	7	13	19	25	Periods of years.
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	+	
4	0′.1	0′.2	0′.3	0′.4	0′.4	0′.5	0′.5	0′.6	0′.6	0′.7	0′.7	0′.7	0′.7	0′.7	0′.7	4
8	.3	.4	.6	.7	.8	1.0	1.1	1.2	1.2	1.3	1.3	1.4	1.4	1.4	1.4	8
12	.4	.7	.9	1.1	1.3	1.5	1.6	1.7	1.9	2.0	2.0	2.1	2.1	2.1	2.1	12
16	.6	.9	1.2	1.4	1.7	2.0	2.2	2.3	2.5	2.6	2.6	2.7	2.8	2.8	2.8	16
20	.7	1.1	1.5	1.8	2.1	2.5	2.7	2.9	3.1	3.3	3.3	3.5	3.5	3.6	3.6	20

In the above tables of declination, the declination is given for the noon of each day, under the meridian of Greenwich, for four successive years. S and N shew when the decl. is north or south. These tables are principally calculated for the years 1810, 1811, 1812 and 1813.

The table of the change of the sun's decl. is intended to reduce this decl. to a subsequent period. The variation of decl. is given opposite the years, and under the 1st, 7th, &c. days of the month, which is to be added to or subtracted from the decl. in the table, according as the sine over it is + or —.

Declination of the Sun for the years 1813, 1817, 1821, 1825, *&c.*

BEING THE FIRST AFTER LEAP YEAR.

Days.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	S.	S.	S & N.	N.	N.	N.	N.	N.	N & S.	S.	S.	S.
1	23° 1′	17° 7′	7°37′	4°30′	15° 2′	22° 3′	23° 9′	18° 5′	8°21′	3° 8′	14°25′	21°49′
2	22 56	16 50	7 14	4 53	15 21	22 11	23 5	17 50	7 59	3 31	14 44	21 58
3	22 51	16 32	6 51	5 16	15 38	22 18	23 0	17 35	7 37	3 55	15 3	22 7
4	22 45	16 14	6 28	5 39	15 56	22 26	22 55	17 19	7 15	4 18	15 22	22 15
5	22 38	15 56	6 5	6 2	16 13	22 33	22 50	17 3	6 53	4 41	15 40	22 23
6	22 31	15 38	5 41	6 25	16 30	22 39	22 44	16 46	6 31	5 4	15 58	22 30
7	22 24	15 19	5 18	6 47	16 47	22 45	22 38	16 30	6 8	5 27	16 16	22 37
8	22 16	15 0	4 55	7 10	17 3	22 51	22 31	16 13	5 46	5 50	16 34	22 44
9	22 7	14 41	4 31	7 32	17 20	22 56	22 24	15 56	5 23	6 13	16 51	22 50
10	21 59	14 22	4 8	7 55	17 35	23 1	22 17	15 38	5 1	6 36	17 8	22 56
11	21 49	14 2	3 44	8 17	17 51	23 5	22 9	15 21	4 38	6 59	17 25	23 1
12	21 40	13 43	3 21	8 39	18 6	23 10	22 1	15 3	4 15	7 21	17 41	23 6
13	21 30	13 22	2 57	9 0	18 21	23 13	21 53	14 45	3 52	7 44	17 58	23 10
14	21 19	13 2	2 34	9 22	18 36	23 16	21 44	14 26	3 29	8 6	18 13	23 14
15	21 9	12 42	2 10	9 44	18 50	23 19	21 35	14 8	3 6	8 29	18 29	23 17
16	20 57	12 21	1 46	10 5	19 4	23 22	21 25	13 49	2 43	8 51	18 44	23 20
17	20 46	12 0	1 23	10 26	19 18	23 24	21 15	13 30	2 19	9 13	18 59	23 23
18	20 34	11 39	59	10 47	19 32	23 25	21 5	13 11	1 56	9 35	19 14	23 25
19	20 21	11 18	35	11 8	19 45	23 26	20 54	12 51	1 33	9 57	19 28	23 26
20	20 8	10 56	12	11 29	19 57	23 27	20 43	12 31	1 9	10 18	19 42	23 27
21	19 55	10 35	N 12	11 49	20 10	23 28	20 32	12 12	46	10 40	19 55	23 28
22	19 42	10 13	36	12 9	20 22	23 28	20 20	11 52	23	11 1	20 8	23 28
23	19 28	9 51	59	12 30	20 34	23 27	20 8	11 31	S 1	11 22	20 21	23 27
24	19 13	9 29	1 23	12 49	20 45	23 26	19 56	11 11	24	11 44	20 33	23 26
25	18 59	9 7	1 47	13 9	20 56	23 25	19 43	10 50	48	12 4	20 45	23 25
26	18 44	8 44	2 10	13 29	21 7	23 23	19 30	10 29	1 11	12 25	20 57	23 23
27	18 28	8 22	2 34	13 48	21 17	23 21	19 17	10 8	1 34	12 46	21 8	23 21
28	18 13	7 59	2 57	14 7	21 27	23 19	19 3	9 47	1 58	13 6	21 19	23 18
29	17 57		3 21	14 26	21 36	23 16	18 49	9 26	2 21	13 26	21 29	23 15
30	17 41		3 44	14 44	21 46	23 12	18 35	8 5	2 45	13 46	21 39	23 11
31	17 24		4 7		21 54		18 43	8 43		14 5		23 7

Change of the Sun's decl. for periods of four years.

Periods of years.	APRIL.					MAY.					JUNE.					Periods of years.
	Days.					Days.					Days.					
	1	7	13	19	25	1	7	13	19	25	1	7	13	19	25	
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
4	0′.7	0′.7	0′.7	0′.6	0′.6	0′.6	0′.5	0′.5	0′.4	0′.3	0′.3	0′.2	0′.1	0′.0	0′.1	4
8	1 .4	1 .4	1 .3	1 .3	1 .2	1 .1	1 .0	.9	.8	.7	.5	.4	.2	.0	.1	8
12	2 .1	2 .1	2 .0	1 .9	1 .8	1 .7	1 .6	1 .4	1 .2	1 .0	.8	.5	.3	.1	.2	12
16	2 .8	2 .7	2 .6	2 .5	2 .4	2 .3	2 .1	1 .9	1 .6	1 .3	1 .0	.7	.4	.1	.3	16
20	3 .5	3 .4	3 .3	3 .2	3 .0	2 .8	2 .6	2 .3	2 .0	1 .6	1 .3	.9	.5	.1	.3	20

Thus if the sun's declination for the 1st of May, 1824, be required. The given year being leap year, or 12 years after 1812. Hence

The sun's declination 1st of May, 1812, is	15° 7′ N.
Equation or change for 12 years, is	+ 1.7
Sun's declination, 1st of May, 1824,	15 8.7

Again, required the sun's decl. at noon, for the 30th of September, 1833?

Here the given year is the 1st after leap year, and is 20 years after 1813. Hence

Declination of the Sun for the years 1810, 1814, 1818, 1822, *&c.*

BEING THE SECOND AFTER LEAP YEAR.

Days.	*Jan.*	*Feb.*	*Mar.*	*April.*	*May.*	*June.*	*July.*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
	S.	S.	S & N.	N.	N.	N.	N.	N.	N & S.	S.	S.	S.
1	23° 3′	17°11′	7°42′	4°24′	14°58′	22° 1′	23°10,	18° 9′	8°27′	3° 2′	14°20′	21°47′
2	22 58	16 54	7 20	4 47	15 16	22 9	23 6	17 54	8 5	3 25	14 39	21 56
3	22 53	16 37	6 57	5 10	15 34	22 17	23 2	17 38	7 43	3 48	14 58	22 5
4	22 47	16 19	6 34	5 33	15 52	22 24	22 57	17 23	7 21	4 12	15 17	22 13
5	22 40	16 1	6 11	5 56	16 9	22 31	22 52	17 7	6 59	4 35	15 36	22 21
6	22 33	15 43	5 47	6 19	16 26	22 38	22 46	16 51	6 37	4 58	15 54	22 28
7	22 26	15 24	5 24	6 42	16 43	22 44	22 40	16 34	6 14	5 21	16 12	22 35
8	22 18	15 5	5 1	7 4	17 0	22 50	22 34	16 17	5 52	5 44	16 30	22 42
9	22 10	14 46	4 38	7 27	17 16	22 55	22 27	16 0	5 29	6 7	16 47	22 48
10	22 1	14 27	4 15	7 49	17 32	23 0	22 20	15 43	5 6	6 30	17 4	22 54
11	21 52	14 8	3 51	8 11	17 47	23 4	22 12	15 25	4 44	6 53	17 21	23 0
12	21 43	13 48	3 27	8 33	18 2	23 8	22 4	15 8	4 21	7 16	17 37	23 5
13	21 33	13 28	3 4	8 55	18 17	23 12	21 56	14 50	3 58	7 38	17 53	23 9
14	21 23	13 8	2 40	9 17	18 32	23 16	21 48	14 31	3 35	8 1	18 9	23 13
15	21 12	12 47	2 16	9 38	18 47	23 19	21 39	14 13	3 12	8 23	18 25	23 17
16	21 0	12 26	1 52	10 0	19 1	23 21	21 29	13 54	2 49	8 45	18 41	23 20
17	20 48	12 5	1 29	10 21	19 15	23 23	21 19	13 35	2 25	9 8	18 56	23 22
18	20 36	11 44	1 6	10 42	19 29	23 25	21 9	13 16	2 2	9 30	19 10	23 24
19	20 24	11 23	42	11 3	19 42	23 26	20 58	12 56	1 39	9 51	19 24	23 26
20	20 11	11 2	18	11 24	19 55	23 27	20 47	12 37	1 15	10 13	19 38	23 27
21	19 58	10 40	N 5	11 44	20 7	23 27	20 36	12 17	52	10 35	19 52	23 28
22	19 45	10 1[illegible]	29	12 5	20 19	23 28	20 24	11 57	29	10 56	20 6	23 28
23	19 31	9 57	52	12 25	20 31	23 28	20 12	11 37	5	11 17	20 19	23 27
24	19 17	9 35	1 16	12 45	20 42	23 27	19 59	11 17	S 18	11 38	20 31	23 27
25	19 2	9 12	1 40	13 4	20 53	23 26	19 46	10 56	42	11 59	20 42	23 26
26	18 47	8 50	2 4	13 24	21 4	23 24	19 33	10 35	1 5	12 20	20 54	23 24
27	18 32	8 28	2 28	13 43	21 14	23 22	19 20	10 14	1 29	12 41	21 5	23 22
28	18 17	8 5	2 51	14 2	21 24	23 20	19 6	9 53	1 52	13 1	21 16	23 19
29	18 1		3 15	14 21	21 34	23 17	18 52	9 32	2 15	13 21	21 27	23 16
30	17 45		3 38	14 40	21 43	23 14	18 38	9 10	2 39	13 41	21 37	23 13
31	17 28		4 1		21 52		18 24	8 49		14 1		23 9

Change of the Sun's decl. for periods of four years.

Periods of years.	JULY.					AUGUST.					SEPTEMBER.					Periods of years.
	Days.					*Days.*					*Days.*					
	1	7	13	19	25	1	7	13	19	25	1	7	13	19	25	
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	+	
4	0′.1	0′.2	0′.3	0′.4	0′.4	0′.5	0′.5	0′.6	0′.6	0′.7	0′.7	0′.7	0′.7	0′.7	0′.7	4
8	.3	.4	.6	.7	.9	1.0	1.1	1.2	1.3	1.3	1.4	1.4	1.4	1.4	1.4	8
12	.4	.7	.9	1.1	1.3	1.5	1.6	1.8	1.9	2.0	2.0	2.1	2.1	2.1	2.1	12
16	.6	.9	1.2	1.4	1.7	2.0	2.2	2.4	2.5	2.6	2.7	2.8	2.8	2.9	2.9	16
20	.7	1.1	1.5	1.8	2.2	2.5	2.7	3.0	3.2	3.3	3.4	3.5	3.5	3.6	3.6	20

The sun's declination 30th of Sept. 1813, is	2° 45′ S.
Equation or variation for 20 years, is	+ 3.6
Sun's declination 30th Sept. 1833,	2° 48′.6

The correction may be found independent of the table thus; From the given year take as many times 4 as will reduce it to one of the years to which the table is adopted, and take out the decl. answering the given time as before; find also the decl. for the following day, and multiply the difference between them by ¼th of the difference between the given and tabular years, and the

Declination of the Sun for the years 1811, 1815, 1819, 1823, *&c.*

BEING THE THIRD AFTER LEAP YEAR.

Days.	*Jan.*	*Feb.*	*Mar.*	*April.*	*May.*	*June.*	*July.*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
	S.	S.	S & N.	N.	N.	N.	N.	N.	N & S.	S.	S.	S.
1	23° 4′	17°16′	7°48′	4°18′	14°53′	21°59′	23°11′	18°13′	8°32′	2°56′	14°15′	21°44′
2	22 59	16 59	7 25	4 42	15 11	22 7	23 7	17 58	8 11	3 19	14 34	21 53
3	22 53	16 41	7 3	5 5	15 29	22 15	23 2	17 42	7 49	3 43	14 53	22 2
4	22 48	16 24	6 40	5 28	15 47	22 22	22 58	17 27	7 27	4 6	15 12	22 11
5	22 41	16 6	6 17	5 50	16 4	22 29	22 5[illegible]	17 11	7 4	4 29	15 31	22 19
6	22 35	15 47	5 53	6 13	16 21	22 36	22 47	16 55	6 42	4 52	15 49	22 27
7	22 27	15 29	5 30	6 36	16 38	22 42	22 41	16 38	6 20	5 15	16 7	22 34
8	22 20	15 10	5 7	6 58	16 55	22 48	22 35	16 21	5 57	5 38	16 25	22 41
9	22 12	14 51	4 43	7 21	17 11	22 53	22 28	16 4	5 35	6 1	16 42	22 47
10	22 3	14 32	4 20	7 43	17 27	22 59	22 21	15 47	5 12	6 24	16 59	22 53
11	21 54	14 13	3 57	8 5	17 43	23 3	22 13	15 30	4 49	6 47	17 16	22 58
12	21 45	13 53	3 33	8 27	17 58	23 7	22 5	15 12	4 26	7 10	17 33	23 3
13	21 35	13 33	3 9	8 49	18 14	23 11	21 57	14 54	4 4	7 32	17 49	23 8
14	21 25	13 13	2 46	9 11	18 28	23 15	21 48	14 36	4 41	7 55	18 5	23 12
15	21 14	12 52	2 22	9 33	18 43	23 18	21 39	14 17	3 17	8 17	18 21	23 16
16	21 3	12 32	1 59	9 54	18 57	23 21	21 30	13 58	2 54	8 40	18 36	23 19
17	20 52	12 11	1 35	10 15	19 11	23 23	21 20	13 39	2 31	9 2	18 51	23 21
18	20 40	11 50	1 11	10 36	19 25	23 25	21 10	13 20	2 8	9 24	19 6	23 23
19	20 28	11 29	47	10 57	19 38	23 26	21 0	13 1	1 45	9 46	19 21	23 25
20	20 15	11 7	24	11 18	19 51	23 27	20 49	12 41	1 21	10 7	19 35	23 27
21	20 2	10 46	0	11 39	20 3	23 28	20 38	12 22	58	10 29	19 48	23 27
22	19 49	10 24	N 24	11 59	20 16	23 28	20 26	12 2	35	10 50	20 2	23 28
23	19 35	10 2	47	12 19	20 28	23 27	20 14	11 41	11	11 12	20 14	23 28
24	19 21	9 40	1 11	12 39	20 39	23 27	20 2	11 21	S 12	11 33	20 27	23 27
25	19 6	9 18	1 35	12 59	20 50	23 26	19 50	11 1	36	11 54	20 39	23 26
26	18 51	8 56	1 58	13 19	21 1	23 24	19 37	10 40	59	12 15	20 51	23 24
27	18 36	8 33	2 22	13 38	21 12	23 22	19 24	10 19	1 23	12 35	21 2	23 22
28	18 21	8 11	2 45	13 57	21 22	23 20	19 10	9 58	1 46	12 55	21 13	23 20
29	18 5		3 9	14 16	21 32	23 17	18 56	9 37	2 9	13 16	21 24	23 17
30	17 49		3 32	14 35	21 41	23 14	18 42	9 15	2 33	13 36	21 34	23 13
31	17 32		3 55		21 50		18 28	8 54		13 55		23 9

Change of the Sun's decl. for periods of four years.

Periods of years.	OCTOBER.					NOVEMBER					DECEMBER.					Periods of years.
	Days.					Days.					Days.					
	1	7	13	19	25	1	7	13	19	25	1	7	13	19	25	
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	—	
4	0′.7	0′.7	0′.7	0′.6	0′.6	0′.5	0′.5	0′.4	0′.4	0′.3	0′.2	0′.2	0′.1	0′.0	0′.0	4
8	1.4	1.4	1.3	1.3	1.2	1.1	1.0	.9	.8	.7	5	.4	.2	.0	.1	8
12	2.1	2.0	2.0	1.9	1.8	1.6	1.5	1.3	1.2	1.0	.7	.5	.3	.1	.1	12
16	2.8	2.7	2.6	2.5	2.4	2.2	2.0	1.8	1.5	1.3	1.0	.7	.4	.1	.2	16
20	3.5	3.4	3.3	3.2	3.0	2.8	2.5	2.2	2.0	1.7	1.3	.9	.5	.2	.3	20

product divided by 33 will give the correction; which is to be added when the decl. is increasing or to be subtracted when decreasing. If the given time be anterior to that in the tables, the correction is to be applied in a contrary manner. Thus, if the sun's decl. for the 1st of May, 1824, be required. Here the given year is 12 years after 1812, the sun's decl. on the 1st of May, 1812, is 15° 7′, and for the 2d, 15° 25′, the difference is 18′. Now 18 × 3 (because 12 ÷ 4 = 3) gives 54; hence 54 ÷ 33 = 1.6 nearly, the correction, which added to 15° 7′, because the decl. is increasing, gives 15° 8′.6, the declination for the 1st of May, 1824.

A TABLE

OF THE

Latitudes and Longitudes

OF SOME OF THE

PRINCIPAL PLACES IN THE WORLD,

Collected from the most authentic Tables, Maps, Astronomical Observations, &c.

THE longitude, or difference of meridians, is reckoned from the meridian of Greenwich observatory, which is 5′ 37″ east of London, 2° 19′ according to Mayer (or 2° 19′ 42″ according to Delambre) west of Paris observatory, 9° 53′, according to Mayer, west of Gottingen observatory, 17° 33′ 45″ east of the town of Ferro, 75° 14′ 22″ east of Philadelphia, 74° 1′ east of New-York, 71° 3′ 37″ east of Boston, and 77° 14½′ east of Washington City, generally reckoned 77° 9′.

There is nothing in which authors disagree more than in the lat. and long. of places, as they generally copy each other, or the mistakes of those who have actually made observations, and whose errors must have arisen from the imperfection in their instruments, or want of the necessary knowledge in using them.

The *Tide-Table* annexed to the following latitude and longitude of places, gives the distance of the moon from the meridian, when it is high water at those respective places. As there are two tides in the day, it is evident that this table will only give the moon's distance from the meridian at one of them : if this time be however taken from 12, the dist. of the moon from the meridian at the other will be given. Thus, if when the moon is full, high water at New-York be at 9 o'clock, P. M. it is evident that the next tide will be about 9 o'clock in the morning, when the moon will be about 3 hours distant from the meridian ; and this is the reason that some make the time of high water in New-York 9 hours, while others make it 3. The same observation will hold for any other place.

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
A.					
Acre,	Syria,	32 42 N.	35 10 E.		
Abbeville,	France,	50° 7′ N.	1° 49′ E.	10h.30′	
Aberdeen,	Scotland,	57 9 N.	2 8 W	45	
Abo,	Finland,	60 27 N.	22 13 E.		
Acapulco,	Mexico,	17 10 N.	101 26 W		
Achen,	Sumatra, I.	5 22 N.	95 35 E.		
Adrianople,	Turkey,	41 12 N.	26 28 E.		
Adventure Bay,	Van Dieman's, L	43 22 S.	147 30 E.	4 36	
Agra,	Hindoostan,	26 43 N.	78 45 E.		
Air,	Scotland,	54 25 N.	4 26 W	10 30	
Aix,	France,	43 32 N.	5 26 E.		
Albany,	N. America,	42 39 N.	73 46 W	3 24	
Aleppo,	Syria,	36 11 N.	37 10 E.		
Alexandretta,	Syria,	36 35 N.	36 15 E.		
Alexandria,	Egypt,	31 12 N.	29 55 E.		
Alexandria,	Virginia,	38 45 N.	77 16 W		
Algiers,	Africa,	36 49 N.	2 12 E.		
Alicant,	Spain,	38 21 N.	0 30 W		
Altorf,	Switzerland,	46 53 N.	8 37 E.		
Amboy,	N. Jersey,	40 33 N.	74 20 W	8 9	
Amboyna, I.	Moluccas,	3 36 S.	128 15 E.		
Amiens,	France,	49 54 N.	2 18 E.		
Amsterdam,	Holland,	52 22 N.	4 51 E.	3 0	7
Ancona,	Italy,	43 38 N.	13 30 E.		
Angers,	France,	47 28 N.	0 34 W		
Angra,	Tercera I. Azores	38 39 N.	27 12 W	11 45	5½
Annapolis,	Maryland,	39 2 N.	76 45 W	10 0	
Antigua, I. St John's town,	Carib. Sea,	17 4 N.	62 9 W		
Antioch,	Syria,	35 55 N.	36 15 E.		
Antrim,	Ireland,	54 58 N.	6 27 W		
Antwerp,	Netherlands,	51 13 N.	4 24 E.	6 45	
Archangel,	Russia,	64 34 N.	38 55 E.	6 0	
Arica,	Peru,	18 27 S.	71 13 W		
Ascension, I.	S. Atlant. Ocean,	7 56 S.	14 21 W		
Astracan,	Russia,	46 21 N.	48 3 E.		
Athens,	Turkey Eur.	38 5 N.	23 52 E.		
Ausburg,	Germany,	48 19 N	10 56 E.		
Augusta,	Georgia, U. S.	33 20 N.	81 4 W		
St. Augustine,	E. Florida,	29 58 N.	81 40 W	4 30	
Ava,	East India,	21 56 N.	95 15 E.		
Avignon,	France,	43 57 N.	4 48 E.		
Avranches,	France,	48 41 N.	1 22 W	6 30	
Auxerre,	France,	47 48 N.	3 34 E.		

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
B.					
Babelmandebstr,	Red Sea,	12° 50′N.	43° 45′E.	0h. 0′	
Babylon, (anc.)	Syria,	33 0 N.	42 46 E.		
Badajoz,	Spain,	38 46 N.	6 45 W.		
Bagdad,	Syria,	33 20 N.	44 23 E.		
Balasore,	Hindoostan,	20 21 N.	86 45 E.	10 0	
Balbec,	Syria,	33 50 N	36 20 E.		
Baltimore,	Ireland,	51 16 N.	9 30 W.	4 30	
Baltimore,	Maryland,	39 20 N.	76 43 W.		
Banca, I. S. end,	Indian Ocean,	3 15 S.	107 10 E.		
Bantam point,	Java I.	5 50 S.	106 9 E.		
Bantry,	Ireland,	51 27 N.	9 46 W.	5 15	
Barbuda I.	Atlantic,	17 49 N.	62 0 W		
Barcelona,	Spain,	41 26 N.	2 12 E.		
Basil or Basle,	Switzerland,	47 34 N.	7 35 E.		
Basse Terre,	Guadaloupe,	15 59 N.	61 54 W.		
Bassora or Basra,	Turkey, A.	30 25 N.	47 30 E.		
Bastia,	Corsica,	42 42 N.	9 25 E.		
Batavia,	Java I.	6 11 S.	106 52 E.		
Bayonne,	France,	43 29 N.	1 29 W.	3 30	15
Beachy head,	England,	50 44 N.	16 E.	10 30	
Belfast,	Ireland,	54 43 N.	5 57 W.	10 0	
Belgrade,	Turkey, E.	45 0 N.	21 20 E.		
Bencoolen,	Sumatra,	3 49 S.	102 3 E.		
Bennington,	Vermont, U. S.	42 50 N.	73 6 W.		
Bergen,	Norway,	60 23 N.	5 12 E.	1 30	
Bergen-op Zoom	Holland,	51 30 N.	4 17 E.	1 30	
Berlin,	Germany,	52 32 N.	13 23 E.		
Bermudas I. N.	Atlantic,	32 35 N.	64 28 W.	6 30	4½
Berne,	Switzerland,	46 57 N.	7 26 E		
Berwick,	Scotland,	55 47 N.	2 5 W.	2 0	
Bethlehem,	Pennsylvania,	40 37 N.	75 25 W.		
Bilboa,	Spain,	43 26 N.	2 47 W.	3 45	15
Bologna,	Italy,	44 30 N.	11 21 E.		
Bologne,	France,	50 43 N.	1 36 E.	10 45	
Bombay I.	India, E.	18 56 N.	72 54 E.		
Boston,	Massachusetts,	42 23¼ N	71 0 W.	11 9	
Botany Bay,	N. Holland,	34 0 S.	151 20 E.	8 0	
Bourbon, I. N.	Ind. Ocean,	20 51 S.	55 30 E.		
Bourdeaux,	France,	44 50 N.	35 W.	7 14	15
Braganza,	Portugal,	41 53 N.	7 3 W.		
Breda,	Netherlands,	51 36 N.	4 46 E.		
Bremen,	Germany,	53 5 N.	8 49 E.	5 45	
Breslaw,	Silesia,	51 5 N.	17 6 E.		
Brest,	France,	48 23 N.	4 30 E.	3 45	19

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
Bristol,	England,	51° 28′N.	2° 35′ W	6h.45′	
Bruges,	Netherlands,	51 13 N.	3 13 E.		
Brunswick,	Germany,	52 25 N.	10 31 E.		
Brunswick,	Dist. Maine,	43 52 N.	69 59 W.		
Brunswick,	New-Jersey,	39 39 N	74 18 W.		
Brussels,	Netherlands,	50 51 N.	4 21 E.		
Breda,	Hungary,	47 30 N.	19 0 E.		
Buenos Ayres,	S. America,	34 35 S.	58 24 W.		
Bukarest,	Turkey,	44 27 N.	26 8 E		
Burlington,	New-Jersey,	40 5 N.	75 6 W.	9 14	
Burgos,	Spain,	42 20 N	3 30 W.		
C					
Cabello port,	S. America,	10 31 N.	67 32 W.		
Cadiz,	Spain,	36 31 N	6 17 W.	2 30	
Cærnarvon,	Wales,	53 6 N.	4 30 W.	7 0	24
Cagliari,	Sardinia I.	39 25 N	9 38 E.		
Cairo,	Egypt,	30 3 N.	31 17 E.		
Caithness point,	Scotland,	58 46 N.	3 22 W.	9 0	
Calais,	France,	50 57½ N	1 50 E.	11 45	
Calcutta, F. W.	Bengal,	22 35 N.	88 28 E.	3 5	
Callao,	Peru,	12 2 N	76 58 W.	6 30	2
Calmar,	Sweden,	56 41 N.	16 25 E.		
Cambray,	Netherlands,	50 11 N.	3 14 E.		
Cambodia,	East India,	13 1 N.	105 0 E.		
Cambridge,	England,	52 13 N.	5 E.		
Cambridge,	Massachusetts,	42 23½ N	71 7 W.		
Canary, I. N. E.	Canary, Is.	28 13 N	15 39 W.	3 0	
Candi,	Ceylon,	7 45 N.	80 46 E.		
Candia,	Candy, I.	35 19 N.	25 18 E.		
Canton,	China,	23 7 N	113 16 E.		
Cape Canso,	Nova Scotia,	45 16 N	60 55 W	8 30	
—Cantin,	Morocco,	32 44 N.	9 10 W.	1 0	9½
—Clear,	Ireland,	51 18 N.	9 30 W.	4 30	
—Ortegal,	Spain,	43 46 N	7 39 W	3 0	
—Finisterre,	Spain,	42 53 N.	9 18 W.	3 15	12
—St. Vincent,	Portugal,	37 2 N.	9 2 W.	3 0	
—Bajodor,	Africa,	26 13 N.	14 27 W.	0 0	
—Blanco,	do.	20 55 N.	17 10 W.	9 45	
—Verd,	do.	14 47 N.	17 33 W.	7 45	3
—Siera Leon,	do.	8 30	13 9 W.	8 15	
—Mount,	do.	6 46	11 48 W.		
—Palmas,	do.	4 30	7 41 W.		
—Good Hope. }	do.	34 29 S.	18 23 E.	3 0	3
—Town, }	do.	33 56 S.	18 23 E.	2 30	
—Comorin,	Hindoostan,	8 4 N.	77 34 E.		

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
Cape Lapotka,	Kamskatcha,	51° 0′ N.	156°42′ E.		
—Race,	Newfoundland,	46 40 N.	53 44 W.		
—Sable,	Nova Scotia,	43 24 N.	65 39 W.	8h. 15′	
—Cod, (light)	Massachusetts,	42 5 N.	70 14 W.	11 30	6½
—Charles,	Virginia,	37 12 N.	76 9 W.	7 0	
—Hatteras,	N. Carolina,	35 12 N.	75 5 W.	11 0	
—Francois (new)	do.	19 46 N.	72 18 W.	6 0	3
—Horn,	S. America,	55 58 S.	67 26 W.		
—Bianco,	Patagonia,	47 20 N.	64 42 W.		
—Farewell,	Greenland,	59 38 N.	42 42 W.		
—Florida,	America,	25 47 N.	80 35 W.	7 30	
—Capricorn,	N. Holland,	23 27 S.	151 6 E.	8 0	
—Diggs,	Labradore,	62 41 N.	78 51 W.		
—Henry,	Virginia,	36 57 N.	76 19 W.	10 54	4
—Lahogue,	France,	49 45 N.	1 57 W.	8 30	18
—May,	New-Jersey,	39 4 N.	74 54 W.	8 9	
Cardigan,	Wales,	52 2 N.	4 45 W.	7 15	
Carthage ruins,	Tunis,	36 35 N.	10 10 E.		
Carthagena,	Spain,	37 37 N.	1 1 W.	8 0	
Carthagena,	Terra Firma,	10 26 N.	75 21 W.	2 0	10
Casan,	Siberia,	55 44 N.	49 28 E.		
Cayenne,	Cayenne, I. S. A	4 56 N.	52 16 E.	4 0	6
Charleston,	South Carolina,	32 50 N.	80 1 W.	7 54	6
Charlestown,	Massachusetts,	42 22 N.	71 1 W.		
Christiana,	Norway,	59 55 N.	10 48 E.		
Christiansand,	Norway,	58 10 N.	8 2 E.		
Christianstat,	Sweden,	56 5 N.	14 2 E.		
Christmas sound,	Terra del Fuego,	55 22 S.	70 3 W.	2 30	
St. Christophers,	West Indies,	17 15 N.	62 43 W.		
Cologne,	Germany,	50 55 N.	6 55 E.		
Columbia,	South Carolina,	33 58 N.	81 5 W.		
Conception,	S. America,	36 43 S.	73 6 W.	3 0	
Constance,	Germany,	47 37 N.	9 13 E.		
Constantinople,	Turkey,	41 1 N.	28 55 E.		
Copenhagen,	Denmark,	55 41 N.	12 35 E.		
Corinth,	Turkey,	37 54 N.	22 55 E.		
Cork,	Ireland,	51 54 N.	8 28	4 45	
Corvo,	Azores,	39 43 N.	31 5 W.		
Cracow,	Poland,	50 11 N.	19 50 E.		
St. Cruz, I.	Atlantic,	17 49 N.	64 53 W.		
Curacoa, I. north point,	W. Indies,	12 16	69 7 W.		
Cusco,	Peru,	12 25 S.	73 35 W.		

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
D.					
Damascus,	Syria,	33° 16′ N.	36° 20′ E.		
Dantzick,	Poland,	54 22 N.	18 38 E.		
Dardanelles,	Turkey,	30 10 N.	26 26 E.		
St. David's head,	Wales,	51 55 N.	5 27 W.	6h. 0′	36
Delhi,	Hindoostan,	28 37 N.	77 40 E.		
Deseada, I.	W. Indies,	16 36 N.	61 10 W.		
Detroit,	United States,	42 31 N.	83 12 W.		
Deventer,	United Prov.	52 17 N.	6 13 E.		
Dieppe,	France,	49 56 N.	1 4 E.	10 30	
Dijon,	Burgundy,	47 19 N.	5 1 E.		
Dingle Bay,	Ireland,	51 55 N.	10 49 W.	4 0	
St. Domingo,	Hispaniola,	18 20 N.	69 46 W.		
Dort,	Holland,	51 47 N.	4 35 E.	3 0	
Douay,	Flanders,	50 22 N.	3 5 E.		
Douglas,	I. of Man,	54 7 N.	4 38 W.	10 30	
Dover,	England,	51 8 N.	1 19 E.	11 45	16
Dresden,	Germany,	51 3 N.	13 41 E.		
Drontheim,	Norway,	63 26 N.	10 22 E.		
Dublin,	Ireland,	53 22 N.	6 17 W.	9 0	
Dublin, obs.	Do.	53 23 N.	6 20½ W		
Dunbar,	Scotland,	56 1 N.	2 33 W.	3 30	
Dungarvon,	Ireland,	52 0 N.	6 50 W.	4 30	
Dungeness,	England,	50 52 N.	0 59 E.	9 45	
Dunkirk,	France,	51 2 N.	2 22 E.		
Dunnose,	I. of Wight,	50 37 N.	1 11 W.	9 45	
E.					
East Cape,	New Zealand,	37 44 S.	178 58 W.		
Eddystone light,	England,	50 8 N.	4 24 W.	5 30	18
Edenton,	N. Carolina,	36 6 N.	76 50 W.		
Edinburgh,	Scotland,	55 57 N.	3 12 W.	4 30	13
Embden,	Germany,	53 23 N.	7 10 E.	15	
Ephesus,	Natolia,	38 0 N.	27 53 E.		
Erzerum,	Natolia,	39 56 N.	41 10 E.		
Eustatia I.	West-Indies,	17 30 N.	63 2 W.		
Exeter,	England,	50 44 N.	3 34 W.	10 30	
F.					
Fair Island,	Orkney Is.	59 30 N.	1 46 W.	10 0	
Falmouth,	England,	50 8 N.	5 2 W.	5 30	18
False Cape,	Delaware,	38 38 N.	75 9 W.		
Fayal Town,	Azores,	38 32 N.	28 41 W.	2 20	
Fayetteville,	N. Carolina,	35 11 N.	78 50 W.		
Ferrara,	Italy,	44 50 N.	11 36 E.		
Ferro (Town)	Canaries,	27 47 N.	17 46 W.	3 0	

Names of places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
Ferrol,	Spain,	43° 29′N.	8°15′W.	3h. 0′	15
Fez,	Africa,	33 31 N.	5 0W.		
Florence,	Italy,	43 46 N.	11 3 E.		
Flores,	Azores,	39 26 N.	31 11W.		
Flushing,	United Prov.	51 27 N.	3 34 E.	45	
N. Foreland,	England,	51 23 N.	1 27 E.	10 20	
Fort Royal,	Martinico,	14 36 N.	61 10W.	7 30	2½
France, I. of S. W.	Indian Ocean,	20 27 N.	57 15 E.	30	3
Francfort on the Main,	Germany,	50 8 N.	8 35 E.		
Frankfort,	Kentucky,	38 4 N.	85 12W.		
Fribourg,	Switzerland,	46 48 N.	7 8 E.		
Fuego I.	Cape Verd Is.	14 57 N.	24 22W.		
Funchal,	Madeira,	32 38 N.	16 56W.	4	11
Galway,	Ireland,	53 10 N.	10 1W.	4 0	
Geneva,	Switzerland,	46 12 N.	6 8 E.		
Genoa,	Italy,	44 25 N.	8 50 E.		
Georgetown,	Columbia dist.	38 55 N.	77 14		
Georgetown,	S. Carolina,	33 32 N.	79 3W.		
Fort St. George,	or Madras,	13 5 N.	80 25 E.		
St. George's town	Bermudas I.	32 22 N.	64 33W.	5 30	
St. George's Isle, W.	Azores,	28 53 N.	28 10W.		
Ghent,	Netherlands,	51 3 N.	3 43 E.		
Gibraltar,	Spain,	36 5 N.	5 4W.	0 0	
Glasgow,	Scotland,	55 52 N.	4 15W.	3 0	18
Gluckstad,	Holstein,	53 48 N.	9 27 E.		
Goa,	Malabar,	15 28 N.	73 59 E.		
Gondar,	Abyssinia,	12 34 N.	37 28 E.	1 30	
Gottenburg,	Sweden,	57 42 N.	11 57 E.		
Gottingen (ob.)	Germany,	51 32 N.	9 54 E.		
Gravesend,	England,	51 28 N.	20 E.		
Greenwich (ob.)	England,	51 28⅔N.	0 0	2 40	
Groningen,	United Prov.	53 10 N.	6 22 E.		
Guadaloupe,	West-Indies,	15 59 N.	61 59W.		
Guernsey,	British ch.	49 30 N.	2 52W.	1 30	
H					
Haerlem,	Holland,	52 22 N.	4 36 E.	9 0	
Hague,	Holland,	52 4 N.	4 17 E.	8 15	
Halifax,	Nova Scotia,	44 44 N.	63 36W.	7 30	
Hamburg,	Germany,	53 34 N.	9 54 E.	6 15	
Hanghoo,	China,	30 25 N.	120 12 E.		
Hanover,	Germany,	52 22 N.	9 45 E.		
Hartford,	Connecticut,	41 50 N.	72 35W.	11 14	
Havannah,	Cuba I.	23 12 N.	82 18W.		

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
Havre de Grace,	France,	49° 29′N.	0° 6′ E.	9h. 0′	
St. Helena, James town,	Atlantic,	15 55 S.	5 49 W.	2 15	2½
Hervey's I.	Society Isles,	19 17 S.	158 56 W.		
Holla,	Iceland,	65 45 N.	19 44 N.		
Holyhead,	Wales,	53 23 N.	4 45 W.	9 45	20
Hull,	England,	53 48 N.	33 W.	6 0	
I.					
Jackson (Port)	New Holland,	33 52 S.	151 14 E.	8 15	
Jackutskoi,	Siberia,	62 2 N.	129 44 E.		
Jaffa,	Siberia,	32 5 N.	35 10 E.		
St. Jago,	Cuba I.	19 55 N.	75 35 W.		
Jassay,	Moldavia,	47 8 N.	27 30 E.		
Java head,	Java I.	6 49 S	105 7 E.		
Ice Cape,	Nova Zembla,	75 30 N.	67 30 E.		
Jeddo,	Japan Is.	36 30 N.	140 0 E.		
Jersey I. St. Aubins,	Eng. Channel,	49 13 N.	2 12 W.	30	
Jerusalem,	Syria,	31 45 N.	35 20 W.		
Ingolstadt,	Germany,	48 46 N.	11 25 E.		
Inverness,	Scotland,	57 36 N.	4 15 W.	11 50	
St. John's,	Newfoundland,	47 32 N.	52 26 W.	6 0	6
St John's,	Antigua,	17 4 N.	62 9 W.		
St. Joseph's,	California,	23 4 N.	109 42 W.		
St. Julian (Port)	Patagonia,	49 10 S.	68 44 W.	4 45	
Ispahan,	Persia,	32 25 N.	52 50 E.		

Isthmus of Corinth joins the Morea to Greece.
of Darien joins North and South-America.
of Suez joins Africa to Asia.

Ivica I.	Mediterranean,	35 50 N.	1 30 E.		
St. Juan,	Porto Rico I.	18 30 N.	66 29 W.		
K.					
Kamtschatka,	Siberia,	56 30 N.	161 0 E.		
Kiel,	Holstein,	54 20 N.	10 18 E.		
Kilkenny,	Ireland,	52 37 N.	7 15 W.		
Kingston,	Jamaica I.	18 15 N.	76 44 W.		
Kinsale,	Ireland,	51 32 N,	8 38 W.	4 45	
Kiow,	Russia,	50 27 N	30 27 E.		
Koningsberg,	Prussia,	54 43 N.	21 35 E.		
L.					
Laguna, Teneriffe I.	Canaries,	28 29 N.	16 27 W	3 0	7½
Lancaster,	England,	54 4 N.	2 50 E.	11 15	

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
Lancaster,	Pennsylvania,	40° 3′N	76°20′W.		
Lands End,	England,	50 6 N.	5 54 W.	4h 30′	
Leghorn,	Italy,	43 33 N.	10 16 E.		
Leuwarden,	United Prov.	53 9 N.	5 55 E.		
Leipsic,	Germany,	51 22 N.	12 20 E.		
Lexington,	Kentucky, U. S.	37 59 N.	84 46 W.		
Leyden,	United Prov.	52 8 N	4 28 E.		
Liege,	Netherlands,	50 39 N.	5 31 E.		
Lima,	Peru,	12 2 S.	76 50 W.	6 30	2
Limburg,	Netherlands,	50 40 N.	5 57 E.		
Limerick,	Ireland,	52 33 N.	8 42 W.	4 30	
Limoges,	France,	45 50	1 15 E.		
Lisbon,	Portugal,	38 42 N.	9 9 W.	3 30	
Liverpool,	England,	53 22 N.	2 57 W.	11 15	
Lizard,	England,	49 57 N.	5 13 W.	5 30	20
London(st.paul's)	England,	51 31 N.	6 W.	3 0	
Londonderry,	Ireland,	54 59 N.	7 15 W.	6 0	
Louisbourg,	Cape Breton I.	45 54 N.	59 59 W.	7 15	
Louisville,	Georgia, U. S.	32 54 N.	82 44 W.		
Louvain,	Netherlands,	50 53 N.	4 41 E.		
Lubec,	Germany.	53 51 N.	10 41 E.		
Lucerne,	Switzerland,	47 3 N.	8 18 E.		
St. Lucia I.	West-Indies,	13 24 N.	60 51 W.		
Lunden,	Sweden,	55 42 N.	13 12 E.		
Luxemburg,	Netherlands,	49 37 N.	6 11 E.		
Lyons,	France,	45 46 N.	4 49 E.	5 50	
M.					
Macao,	China,	22 13 N.	113 35 E.	5 50	5½
Macassar,	Celebes I.	5 9 S.	119 49 E.		
Madeira I. } Funchal, }	Atlantic,	32 38 N.	16 56 W.	0 4	11
Madras,	India,	13 5 N.	80 25 E.		
Madrid,	Spain,	40 25 N.	3 38 W.		
Mahon (Port)	Minorca I.	39 52 N.	3 48 E.		
Majorca I.	Mediterranean,	39 35 N.	2 30 E.		
Malacca,	E. India,	2 12 N.	102 9 E.		
St. Maloes,	France,	48 39 N.	2 2 W.	6 0	45
Malta I.	Mediterranean,	35 54 N.	14 28 E.	0 0	
Manilla,	Philippine Is.	14 36 N.	120 52 E.		
Mantua,	Italy,	45 8 N.	10 52 E.		
Marigalante,	W. India,	15 55 N.	61 11 W.		
Marietta,	Ohio, U. S.	39 8 N.	81 38 W.		
Marseilles,	France,	43 18 N.	5 22 E.		

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
Martha's vineyard I. Edgar's town,	Massachusetts,	41°22′ N.	70°26′W.		
Martinico I. Fort Royal,	W. Indies,	14 36 N.	61 10 W.		
Mecca,	Arabia,	21 45 N.	40 15 E.		
Mexico,	N. America,	19 54 N.	100 7 W.		
St. Michael's I.	Azores,	37 47 N.	25 42 W.		
Milan,	Italy,	45 28 N.	9 14 E.		
Milford,	Wales,	51 45 N.	5 21 W.	5h.15′	
Minorca (Port Mahon)	Mediterranean,	39 51 N.	3 54 E.		
Mocha,	Arabia,	13 44 N.	44 0 E.		
Modena,	Italy,	44 47 N.	10 55 E.		
Mons,	Netherlands,	50 27 N.	3 57 E.		
Montpelier,	France,	43 37 N.	3 52 E.		
Montreal,	Canada,	45 33 N.	73 18 W.		
Montserat, N. E.	West-Indies,	16 49 N.	62 27 W.		
Morocco,	Barbary,	31 0 N.	7 4 W.		
Moscow,	Russia,	55 45 N.	37 46 E.		
Munich,	Germany,	48 8 N.	11 35 E.		
N.					
Namur,	Netherlands,	50 28 N.	4 51 E.		
Nancy,	France,	48 42 N.	6 10 E.		
Nangasaki,	Japan,	32 45 N.	130 15 E.	6 0	
Nankin,	China,	32 5 N.	118 46 E.		
Nantes,	France,	47 13 N.	1 34 W.	3 45	
Nantucket,	Nantucket I.	41 18 N.	70 10 W.	3	6
Naples,	Italy,	40 50 N.	14 17 E.	9 30	
Newbern,	N. Carolina,	35 17 N.	77 18 W.		
Newburyport,	Massachusetts,	42 47 N.	70 52 W.	11 30	10
Newcastle,	England,	55 3 N.	1 30 W.	5 15	
New-Haven,	Connecticut,	41 18 N.	72 53 W.	10 44	8
Newport,	Rhode-Island,	41 25 N.	71 15 W.	7 37	5
New-Orleans,	Louisiana,	29 58 N.	90 6 W.		
New-York,	New-York,	40 42⅔N.	74 1 W.	8 54	5
N. York light h.	New-York,	40 28 N.	74 0 W.	7 30	
Niagara,	New-York,	43 16 N.	79 0 W.		
Nice,	Italy,	43 43 N.	7 15 E.		
Nieuport,	Flanders,	51 8 N.	2 45 E.	0 0	
Nootka,	N. America,	49 36 N.	126 43 W.	20	
Norfolk,	Virginia,	36 55 N.	76 22 W.		
North Cape,	Lapland,	71 10 N.	25 49 E.	3 40	

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
O.					
Ochotsk,	Russia,	59°20′ N.	143°13′ N.		
Odense,	Funen I.	55 24 N.	10 11 E.		
Ohitatoo I.	Society I.	9 55 S.	139 6 W.	2h.30′	
Olmutz,	Moravia,	49 37 N.	17 5 E.		
St. Omer,	Netherlands,	50 45 N.	2 15 E.		
Oporto,	Portugal,	41 10 N.	8 27 W.	3 15	
L'Orient, (port)	France,	47 45 N.	3 22 E.	3 30	
Ostend,	Netherlands,	51 16 N.	2 56 E.	0 0	
Otaheite,	S. Pacific Ocean,	17 20 S.	149 30 E.		
Oviedo,	Spain,	43 18 N.	5 50 W.		
Owhyhee, S. point	S. Pacific Ocean,	18 54 N.	155 48 W.	3 45	2½
P.					
Padua,	Italy,	45 23 N.	11 53 E.		
Palermo,	Sicily I.	38 7 N.	13 35 E.		
Palmyra,	Arabia,	33 58 N.	38 42 E.		
Panama,	Mexico,	8 58 N.	80 15 W.	3 0	6½
Paris (obs.)	France,	48 50 N.	2°19′42″ E		
Parma,	Italy,	44 47 N.	10 21 E.		
Pegu,	East India,	17 55 N.	96 45 E.		
Pekin (obs.)	China,	39 54 N.	116 27 E.		
Pensacola,	W. Florida,	30 30 N.	87 10 W.		
Perth Amboy,	New-Jersey,	40 33 N.	74 20 W.	8 9	
Petersburg,	Russia,	59 56 N.	30 18 E.		
Philadelphia,	Pennsylvania,	39 57 N.	75 14 W.	1 54	
Pico I.	Azores,	38 27 N.	28 28 W.		
Pittsburg,	Pennsylvania,	40 26 N.	80 0 W.		
Placentia,	Newfoundland,	47 26 N.	53 30 W.	9 0	6½
Plymouth,	England,	50 22 N.	4 12 W.	6 0	
Plymouth,	Massachusetts,	41 57 N.	70 40 W.	10 40	6½
Poitiers,	France,	46 35 N.	0 21 E.		
Pondicherry,	East India,	11 56 N.	79 52 E.		
Port au Prince,	St. Domingo I.	18 34 N.	72 28 W.		
Portland,	Dist. Maine,	43 39 N.	70 28 W.	10 45	9
Portland (light)	England,	50 31 N.	2 27 W.	7 30	
Porto Bello,	Terra Firma,	9 33 N.	79 50 W.	8 0	
Port Royal,	Jamaica,	18 0 N.	76 45 W.		
Portsmouth,	England,	50 47 N.	1 6 W.	11 15	
Portsmouth,	New Hampshire	43 4 N.	70 46 W.	11 15	10
Potosi,	Peru,	20 0 S.	66 15 W.		
Prague,	Bohemia,	50 6 N.	14 24 E.		
Presburg,	Hungary,	48 8 N.	17 10 E.		
Providence,	Rhode I. U. S.	41 47 N.	71 22 W.	8 11	

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
Q.					
Quebec,	Canada,	46°48′ N.	71° 6′W.	7h.30′	
Quimper,	France,	47 58 N.	4 7W.	2 30	
Quito,	Peru,	13 S.	78 10W.		
R.					
Ramsay,	I. of Man,	54 17 N.	4 26W.	10 30	
Revel,	Russia,	59 27 N.	24 39 E.		
Rheims,	France,	49 15 N.	4 2 E.		
Rhodes,	Rhodes I.	35 27 N.	28 45 E.		
Richmond,	Virginia,	37 35 N.	77 43 E.		
Riga,	Russia,	56 55 N.	24 0 E.		
Rio Janeiro,	Brazil,	22 54 S.	42 44 W.	4 30	
Rochelle,	France,	46 9 N.	1 10 W.	3 45	18
Rochester,	England,	51 26 N.	30 E.	45	
Rome (St. Pet.)	Italy,	41 54 N.	12 28 E.		
Rotterdam,	United Prov.	51 56 N.	4 28 E.	3 45	7
Rouen,	France,	49 27 N.	1 5W.	2 45	
Rugen I.	Baltic,	54 32 N.	14 30 E.		
S.					
Saba I.	W. Indies,	17 39 N.	63 17W.		
Sagan,	Silesia,	51 36 N.	15 13 E.		
Salem,	Massachusetts,	42 29 N.	70 52W.	11 30	12
Salonica,	Turkey,	40 41 N.	23 7 E.		
St. Salvador,	Brazil,	12 58 N.	39 0W.		
Samarcand,	W. Tartary,	39 35 N.	64 20 E.		
Samos I.	Archipelago,	37 46 N.	27 13 E.		
Sancta Cruz,	Teneriffe I.	38 39 N.	16 22W.		
Sancta Fee,	New Mexico,	36 54 N.	104 30W.		
Saragossa,	Spain,	41 43 N.	50W.		
Saratov,	Russia,	51 35 N.	46 0 E.		
Savannah,	Georgia, U. S.	32 4 N.	81 11W.	7 45	
Scanderoon,	Syria,	36 35 N.	36 14 E.		
Scaff-house,	Switzerland,	47 42 N.	8 37 E.		
Sigo,	Africa,	14 0 N.	2 15W.	10 30	
Senegal (Fort)	Africa,	15 53 N.	16 31W.	10 30	
Sion,	Switzerland,	46 14 N	7 22 E.		
Seringatapam,	Hindoostan,	12 22 N.	76 50 E.		
Siam,	E. India,	14 18 N.	100 49W.		
Sigan,	China,	34 16 N.	109 0 E.		
Sinope,	Natolia,	42 2 N.	35 0 E.		
Smyrna,	Natolia,	38 28 N.	27 7 E.		
Stans,	Switzerland,	46 57 N.	8 22 E.		
Stockholm,	Sweden,	59 21 N.	18 4 E.		
Strasburg,	France,	48 35 N.	7 45 E.		

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
Suez,	Egypt,	29° 50′ N.	33°27′ E.		
Surrinam,	S. America,	6 30 N.	55 30 W.		
Syracuse,	Sicily I.	36 53 N.	15 17 E.		
T.					
Tamarin town,	Socotra,	12 30 N.	52 9 E.	9h. 0′	
Tanjore,	Hindoostan,	10 46 N.	79 48 E.		
Tavira,	Portugal,	37 8 N.	7 40 E.	1 30	
Teflis,	Persia,	42 6 N.	45 15 E.		
Teneriffe Peak,	Canary I.	28 15 N.	16 45 W.	3 0	
Tercera I.	Azores,	38 39 N.	27 12 W.	11 45	5½
Texel I.	United prov.	53 10 N.	4 59 E.	7 30	15
Tobolsk,	Siberia,	58 12 N.	68 19 E.		
Toledo,	Spain,	39 50 N.	3 20 W.		
Tornea,	Lapland,	65 51 N.	24 14 E.		
Toulon,	France,	43 7 N.	5 55 E.	3 14	1¼
Toulouse,	France,	43 46 N.	1 26 E.		
Tours,	France,	47 24 N.	42 E.		
Trent,	Germany,	46 5 N.	11 6 E.		
Trenton,	New-Jersey,	40 13 N.	74 50 W.		
Trincomale,	Ceylon, I.	8 33 N.	81 21 E.	6 0	3
Tripoly,	Barbary,	32 54 N.	13 20 E.		
Troyes,	France,	48 18 N.	4 5 E.		
Tunis,	Barbary,	36 16 N.	10 40 N.		
Turin,	Italy,	45 5 N.	7 39 E.		
U.					
Upsal,	Sweden,	59 52 N.	17 43 E.		
Uraniburg,	Denmark,	55 54 N.	12 51 E.		
Ushant I.	Coast of France,	48 28 N.	5 4 W.	4 30	
Utrecht,	United prov.	52 5 N.	5 9 E.		
V.					
Valencia,	Spain,	39 25 N.	25 W.		
Venice,	Italy,	45 27 N.	12 4 E.	10 30	3
Vera Cruz,	Mexico,	19 10 N.	97 20 W.		6½
Vernon, (mount)	Virginia,	38 46 N.	77 11 W.		
Verona,	Italy,	45 26 N.	11 1 E.		
Versailles,	France,	48 48 N.	2 7 E.		
Vienna (obs.)	Austria,	48 12 N.	16 22 E.		
W.					
Wardhuys,	Lapland,	70 23 N.	31 7 E.		
Warsaw,	Poland,	52 16 N.	21 3 E.		
Washington city,	N. America,	38 53 N.	77 13 W.		
Waterford,	Ireland,	52 12 N.	7 6 W.	4 45	
Wells,	England,	51 12 N.	2 45 W.	6 0	
Wexford,	Ireland,	52 20 N.	6 24 W.	8 30	
Weymouth,	England,	52 40 N.	2 34 W.	7 20	18

Names of Places.	*Country or Sea.*	*Lat.*	*Long.*	*H. water*	*Feet*
Williamsburg,	Virginia,	37°14′ N.	76°49′ W.	11h.10′	
Wilmington,	N. Carolina,	34 11 N.	78 5 W.		
Wilna,	Poland,	54 42 N.	25 27 E.		
Wyburg,	Russia,	60 55 N.	30 20 E.		
Y.					
Yarmouth,	England,	52 55 N.	1 40 E.	9	0
York,	England,	53 58 N.	1 7 W.		
York Town,	Virginia,	37 14 N.	76 36 W.		
Z.					
Zug,	Switzerland,	47 10 N.	8 31 E.		
Zurich,	Switzerland,	47 22 N.	8 32 E.		
Zutphen,	United Prov.	52 12 N.	6 15 E.		

A table of the mean rt. ascen. and decl. of the principal Fixed Stars adapted to the beginning of 1800.*

Names of Stars.	*Mag.*	*Rt Ascen.*	*an.var.*	*Declination.*	*An. var.*
γ Pegasi *Algenib*,	2	0° 23′ 16″	45″9	14° 4′ 23″N.	+20″2
α Cassiopeiæ, *Schedar*,	3	7 18 38	49 7	55 26 20N.	+19 9
α Urs. Min. *Pole Star*,	2.3	13 8 46	173 2	88 14 26N.	+19 4
β Androm, *Mirach*,	2	14 38 35	49 9	34 33 25N.	+19 4
δ Cassiopeiæ,	3	18 12 44	57 0	59 11 28N.	+18 9
β Arietis,	3.4	25 54 12	49 4	19 49 27N.	+18 0
γ Androm. *Almaach*,	2	27 55 24	54 1	41 21 46N.	+17 7
α *Arietis*,	2.3	28 58 54	50 2	22 30 39N.	+17 5
α Ceti, *Menkar*,	2	42 57 33	46 6	3 17 54N.	+14 7
β Persei, *Algol.* var.	2.5	43 48 15	57 5	40 8 19N.	+14 5
α Persei,	2	47 31 45	62 9	49 8 13N.	+13 7
η Pleiadum, *Alcyone*,	3	53 55 19	53 0	23 28 38N.	+11 9
γ Tauri,	3	62 6 22	50 8	15 8 2N.	+ 9 5
α Tauri, *Aldebaran*,	1	66 6 53	51 2	16 5 43N.	+ 8 0
β Eridani,	3	74 30 21	43 7	5 21 18S.	— 5 4
α Aurigæ, *Capella*,	1	75 29 3	66 0	45 6 39N.	+ 4 6
β Orionis, *Rigel*,	1	76 13 59	43 0	8 26 30S.	— 4 9
β Tauri,	2	78 33 53	56 6	28 25 30N.	+ 3 9
η Orionis, *Bellatrix*,	2	78 35 23	45 1	2 35 33S.	— 4 0
δ Orionis,	2	80 26 54	45 5	0 27 27S.	— 3 5
ε Orionis,	2	81 31 3	45 4	1 20 28S.	— 3 0
ζ Orionis,	2	82 40 6	45 1	2 3 33S.	— 2 6
α Columbæ,	2	83 6 11	32 2	34 11 17S.	— 2 4
β Columbæ,	3	85 58 46	32 1	35 51 6S.	— 1 8

* The above is from the latest observations of La *Lande*, but fitted to the year 1800, the latest British globes being adapted to that year.

A table of the mean rt. ascen. and decl. of the principal Fixed Stars adapted to the beginning of 1800.

Names of Stars.	*Mag.*	*Rt. Ascen.*	*an.var.*	*Declination.*	*An. var.*
α Orionis, *Betelguese*,	1	86° 8 12″	48″5	7° 21′ 29″N.	+ 1″5
ζ Canis Majoris,	2.3	93 9 31	34 1	29 58 56S.	+ 1 0
β Canis Majoris,	2.3	93 28 22	39 4	17 52 2S.	+ 1 2
α Canis Maj. *Sirius*,	1	99 5 4	39 5	16 26 54S.	+ 4 5
ι Canis Majoris,	3	102 41 28	35 1	28 42 19S	+ 4 3
δ Canis Majoris,	2.3	105 3 53	36 2	26 5 2S	+ 5 1
η Canis Majoris,	2	109 2 41	35 2	28 55 16S.	+ 6 5
α Geminorum, *Castor*,	1.2	110 27 13	57 6	32 18 46N	— 7 0
α Canis Min. *Procyon*,	1.2	112 12 20	47 0	5 43 34N	— 6 7
β Geminorum, *Pollux*,	2.3	113 15 51	55 0	28 29 50N	— 8 0
ζ Navis	2	119 8 20	31 5	39 26 45S.	+ 9 8
ι Ursæ Majoris,	3	131 21 45	63 4	49 49 2N.	—13 5
α 2 Cancri, *Acubens*,	3	131 52 58	49 2	12 37 24N.	—13 4
α Hydæ, *Alphard*,	2	139 26 23	43 9	7 47 50S.	+15 0
μ Leonis,	3	145 20 26	51 6	26 56 24N	—16 5
α Leonis, *Regulus*,	1	149 25 38	47 9	12 56 25N.	—17 2
β Ursæ Majoris,	2	162 25 4	55 4	57 27 7N.	—19 1
α Urs. Maj. *Dubhe*,	2.1	162 48 57	57 2	62 49 40N.	—19 3
β Leonis, *Deneb*,	2.1	174 42 43	45 9	15 41 25N.	—20 0
β Virginis,	3	175 4 8	46 8	2 53 34N.	—20 3
γ Ursæ Majoris,	2	175 48 41	48 1	54 48 25N.	—20 2
δ Ursæ Majoris,	3	181 21 49	45 3	58 8 41N.	—20 2
γ Corvi, *Algorab*,	3	181 23 6	45 7	16 25 44S	+20 0
ε Ursæ Maj. *Alioth*,	3	191 17 53	40 4	57 2 55N.	— 9 7
α Virg. *Spica Virg.*	1	198 40 7	46 9	10 6 34S.	+19 1
ζ Ursæ Majoris,	2.3	198 57 43	36 8	55 58 29N	—19 0
η Ursæ Maj. *Benetnach*	2	204 54 45	35 2	50 19 1N.	—18 2
α Draconis,	3	209 24 45	24 6	65 20 8N.	—17 3
α Bootis, *Arcturus*,	1	211 38 8	40 7	20 13 54N	—19 0
γ Bootis, *Seginus*,	3	216 0 15	36 4	39 10 22N.	—16 1
α 2 Libræ, Zubenelch.	2.3	219 57 34	49 2	15 17 4S.	+15 5
β Ursæ Min. *Kochab.*	3	222 51 58	—4 8	74 58 27N.	—14 9
β Libræ, *Zubenelg.*	2 3	226 33 58	47 9	8 38 2S.	+13 7
α Coron. bor. *Alphacca*,	2.3	231 33 24	37 8	27 23 50N.	—12 5
α Serpentis,	2.3	233 36 24	43 9	7 3 59N.	—11 8
β Serpentis,	3	234 14 27	41 3	16 3 30N.	—11 7
δ Scorpii,	3	237 9 56	52 8	22 2 18 S.	+10 9
β Scorpii,	2	236 27 28	51 9	19 14 39 S.	+10 4
δ Ophiuchi,	3	241 58 10	46 7	3 9 58 S.	+ 9 7
α Scorpii, *Antares*,	1	244 17 3	54 6	25 51 16 S.	+ 8 8
β Herculis,	3	244 24 27	38 3	21 56 9N.	— 8 3
α Herculis, *Ras Algethi*	3	256 3 0	40 8	14 37 54N.	— 4 6
λ Scorpii, *Lesath*,	2	260 0 36	60 5	36 56 31 S.	+ 3 4
α Ophiuchi, *Ras Alhagus*	2	261 24 50	41 5	12 43 7N.	— 3 1
γ Draconis, *Rastaben*,	3	267 59 31	20 4	51 31 6N.	— 0 7

A table of the mean rt. ascen. and decl. of the principal Fixed Stars adapted to the beginning of 1800.

Names of Stars.	*Mag.*	*Rt. Ascen.*	*an.var.*	*Declination.*	*An. var.*
ε Sagitarii,	3	272°43′27″	59″5	34°27′41″N.	— 1 0
α Lyræ, *Vega*,	1	277 32 29	30 4	38 36 19N.	+ 2 9
β Lyræ,	2.3	280 40 28	33 0	33 8 35N.	+ 3 6
σ Sagitarii,	2.3	280 42 52	55 7	26 31 41 S.	— 3 7
γ Lyræ,	3	282 51 56	33 5	32 23 30N.	+ 4 5
δ Aquilæ,	3	288 51 9	45 2	2 43 44N.	+ 6 6
β Cygni, *Albirco*,	3	290 39 50	36 3	17 32 56N.	+ 7 1
α Aquilæ, *Altair*,	1.2	295 15 20	43 8	8 21 8N.	+ 8 9
β Aquilæ,	3	296 22 16	44 5	5 55 10N.	+ 8 6
α 1 Capricorni,	3.4	301 38 16	49 9	13 6 44 S.	—10 5
α 2 Capricorni,	3	301 44 11	49 9	13 9 3 S.	—10 8
β Capricorni,	3	302 26 23	50 6	15 23 56 S.	—10 9
α Delphini,	3	307 35 13	41 6	15 12 59N.	+11″4
α Cygni, *Deneb*,	2	308 39 13	30 5	44 34 18N.	+12 6
α Cephei, *Alderamin*,	3	318 26 51	21 5	61 44 29N.	+14 9
β Aquarii,	3	320 15 17	47 3	6 26 31 S.	—15 6
ε Pegasi,	3	323 35 22	44 5	8 57 57N.	+16 2
δ Capricorni,	3	323 59 43	49 8	17 1 32 S.	—16 3
α Aquarii,	3	328 52 36	46 0	1 17 6 S.	—17 3
γ Aquarii,	3	332 49 45	46 3	2 23 19 S.	—18 0
α Pisc. Aus. *Fomalhaut*	1	341 38 34	49 9	30 40 30 S.	—19 0
β Pegasi, *Scheat*,	2	343 31 22	42 9	27 0 7N.	+19 4
α Pegasi, *Markab*,	2	343 42 5	44 4	14 8 4N.	+19 2
α Androm. *Alpheratz*,	2.3	359 30 8	45 9	27 59 14N.	+20 0
β Cassiopæ,	2.3	359 38 41	46 7	58 2 48N.	+19 9

INDEX.

THE NUMBERS INDICATE THE PAGES.

D.

E.

G.

H.

I.

K.

L.

M.

N.

INDEX TO THE TABLES.

FINIS.

O. A. M. D. G.

ERRATA.

Page 1, line 3, for Definition, *read* Description.—p. 2, l. 37, for 23d, *r.* 25th.—p. 3, l. 40, omit has.—p. 4, l. 43, for decrease, *r.* decreases.—p. 7, l. 1, for follows, *r.* follow.—p. 11, l. 16, for *space, r. interval,* &c.—p. 14, l. 25, for their, *r.* these.—p. 14, l. 41, for follows, *r.* follow.—p. 15, l. 30, for 20° 24′, *r.* 20′ 24″.—p. 15, l. 51, for anticipations, *r.* anticipation.—p. 23, l. 42, for increases, *r.* increase.—p. 32, l. 6, for who, *r.* which.—p. 52, l. 27, after equally, *r.* as.—p. 54, l. 1, for places those, *r.* those places.—p. 54, l. 51, for in, *r.* on.—p. 56, l. 38, for 75° 8′ 45″, *r.* 75° 14′ 22″—p. 56, l. 47, for 30° 11′, *r.* 30′ 11″.—p. 57, l. 28, 30, 33, for 75° 15′ 22″, *r.* 77° 14′ 22″.—p. 57, l. 29, 30, for 1° 13′ 22″, *r.* 3° 13′ 22″.—p. 57, l. 33, for 14° 52′ 8″, *r.* 12° 52′ 8″.—p. 75, l 6, for brass meridian, *r.* meridian.—p. 111, l. 45, for towards the eastward, *r.* eastward.—p. 128, l. 33, for position, *r.* portion.—p. 140, l. 25, for place *r.* plane.—p. 140, l. 32, for where, *r.* when.—p. 147, l 24, for sun, *r.* sum.—p. 151, l. 4, for 13th, *r.* 32d.—p. 158, l. 18, for by *r.* from.—p. 185, l. 42, for place, *r.* plane.—p. 188, l. 50, 51, for set method above, *r.* above method.—p. 197, l. 37, after 16° 6′, place : —p. 230, l. 44, after similar, *r.* manner.—p. 251, l. 29, for ecliptical, *r.* elliptical.—l. 39, for 29° 36′, *r.* 39° 36′.—p. 255, l. 25, for foregoing fig. *r.* fig. pa. 250.—p. 261, l. 2, for near distance, *r.* mean distance.—p. 266, l. 21, for Venus, *r.* the planet.—p. 270, last line, for chap. 7, *r.* chap. 8.—p. 289, l. 29, omit act.—p. 353, l. 15, for ST, *r.* SB.—p. 367, l. 37, after the *r.* sun.—p. 373, l. 26, for he, *r.* the observer.—p. 392, l. 41, for thus, *r.* this.—p. 393, l. 25, for the *r.* a.—p. 399, l. 7, for on *r.* in.