IES Institute of Education Sciences

# Program for the International Assessment of Adult Competencies (PIAAC)

## County-Level Estimation for Age and Education Groups Methodology Report

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## County-Level Estimation for Age and Education Groups Methodology Report

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# INTRODUCTION

This is the third methodology report on the Program for the International Assessment of Adult Competencies (PIAAC) Small Area Estimation (SAE) published by the National Center for Education Statistics (NCES) in PIAAC Cycle I. The first report was written to describe the methodology used for the purpose of creating model-based estimates for states and counties in the United States (Krenzke et al. 2020). The statistical modeling approach was used to produce hierarchical Bayes (HB) model-based estimates, which are available to the public on the Skills Map website at https://nces.ed.gov/surveys/piaac/skillsmap/. In 2020, the statistical modeling approach produced four different state- and county-level estimates for adult literacy and numeracy proficiencies: an average score (on the PIAAC scale of 0-500) and the proportion of adults at or below Level 1, at Level 2, and at or above Level 3. In 2022, the statistical modeling approach was adapted to produce HB model-based estimates for six age groups and four educational attainment groups for each state and the District of Columbia. The second report was a follow-up to the initial report, and described the methodology for creating HB model-based estimates for age and education groups by state (Li et al. 2022). Then, also in 2022, an allocation approach was developed to produce model-based estimates for domains defined by the cross-tabulations of the same six age groups and four education groups and all the U.S. counties. This approach consists of using the county-level and state by group-level HB model-based estimates and allocating them to the county by group domains of interest via a deterministic model. While this report can be read without having to read the first two reports, it makes reference to the content in the first two reports where appropriate if the reader is interested in more information.

The PIAAC study is a multicycle international survey of adult skills and competencies sponsored by the Organization for Economic Cooperation and Development (OECD). The survey examines a range of basic skills in the information age and assesses these adult skills consistently across participating countries. The first cycle of PIAAC included three rounds: 24 countries participated in 2011-12 (round 1), 9 additional countries participated in 2014-15 (round 2), and 5 additional countries participated in 2017-18 (round 3).

The United States has participated in all three rounds of the first cycle of PIAAC. The round 1 (PIAAC 2012) survey design was consistent with the international requirements (OECD 2016). In round 2 (PIAAC 2014), a supplemental sample was drawn to enhance the round 1 sample (Hogan et al. 2016). The combined PIAAC 2012/2014 sample is nationally representative of the U.S. adult population 16-74 years old. The round 3 (PIAAC 2017) data collection had two core objectives. First, it was designed to produce a nationally representative sample of the U.S. adult population 16-74 years old. Second, the sample was designed to arrive at a large enough sample size that, when combined with the 2012 and 2014

samples, can produce small area estimates for counties in the United States. The main advantage of using the combined samples is the increased reliability of the data. Like the 5-year estimates from the American Community Survey (ACS), the PIAAC small area estimates are "period" estimates.

This report is organized into five sections. Section 1 provides an overview of the SAE process and the data available in the Skills Map. Survey estimation at the level of interest is described in section 2. The availability and the quality of the survey data motivate the need for small area estimation. Survey estimates are also used to compare with the model-based estimates. Section 3 gives the technical description of the allocation approach. Various validation results are presented in section 4 and the document concludes with a summary in section 5.

## 1. SMALL AREA ESTIMATION PROCESS OVERVIEW

Making effective evidence-based policies and laws relating to adult education requires sound research based on reliable data that are most relevant to jurisdictions such as states and counties. As an international study involving over 30 countries under the leadership of the OECD, the first cycle of the PIAAC was designed to provide national estimates of the proficiency of adult literacy, numeracy and problem solving skills. In the United States, PIAAC is sponsored by NCES. The PIAAC survey provides high-quality national estimates through a multistage probability design with in-person data collections that include a screener questionnaire, a background questionnaire and an assessment of adult skills. From 2012 to 2017, some 12,330 U.S. adults ages 16 to 74 living in households were surveyed for PIAAC. Because the U.S. PIAAC sample size was too small to support the production of state and county estimates, model-based SAE methodology (Rao and Molina 2015) was used to produce state and county estimates of average scores for literacy and numeracy, and various proficiency levels. These HB model-based state and county estimates are available in the U.S. PIAAC Skills Map: State and County Indicators of Adult Literacy and Numeracy[1]. By using PIAAC survey data in conjunction with data from the ACS, the Skills Map provides reliable estimates of adult literacy and numeracy skills in all 50 states, all 3,141 counties, and the District of Columbia. Of importance to its stakeholders, the Skills Map allows for the comparison of states and counties. The *Program for the International Assessment of Adult Competencies (PIAAC): State and County Estimation Methodology Report* (Krenzke et al. 2020)[2] provides background on the PIAAC sample design and technical details about the model-based estimation process. Central to the SAE process was an area-level bivariate HB linear three-fold model for proportions, and a similar univariate model for averages.

The state and county estimates were produced so that policymakers can plan and allocate resources and target interventions as necessary at a more local level. PIAAC data are used by state and county adult education departments to plan interventions, allocate scarce resources and provide information to the general public.[3] To help further in targeting interventions, the set of model-based estimates in the Skills Map was expanded to include state-level HB model-based estimates for six age groups and four education groups. The age groups are 16-24, 25-34, 35-44, 45-54, 55-64, and 65-74. The education groups are less than high school, high school diploma or GED, some college (no degree or attained associate's degree), and bachelor's degree or higher. See the *PIAAC State-level Estimation for Age and Education Groups Methodology Report* (Li et al. 2022)[4] for details on the model-based estimation process. Finally, additional model-based estimates were included in the Skills Map for the

[1] Available at https://nces.ed.gov/surveys/piaac/skillsmap/.
[2] Available at https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2020225.
[3] According to the information compiled by American Institutes for Research PIAAC team for a meeting with NCES (January 2022).
[4] Available at https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2022050.

abovementioned age and education groups by county, and the methodology developed for producing them is described in this report. Irrespective of the geography (county or state), the quantities of interest for the age and education groups (hereafter sometimes referred to as "groups") are the same as for the state and county estimates produced in 2020: proportions at or below Level 1 (P1), at Level 2 (P2), and at or above Level 3 (P3), and averages.

The group estimates by county were constructed by allocating the group estimates by state to the counties nested within states using the ratios of county to state estimates. Tables and graphs were prepared as part of the allocation process to assess the quality of the resulting model-based estimates. Within this report, most of the results are illustrated for the literacy proportion at or below Level 1 for educational attainment lower than a high school diploma.

## 2. SURVEY ESTIMATES

The county-level survey estimates for groups can be generated directly from the available survey data. Similarly to the state-level survey estimates for groups of interest (see details in Section 2 of the *Program for the International Assessment of Adult Competencies (PIAAC): State-level Estimation for Age and Education Groups Methodology Report*; Li et al. 2022), the multiple imputation approach was implemented for calculating the survey estimates and the associated variance estimates, using ten plausible values. The plausible values were drawn from a posterior distribution by combining the item response theory (IRT) scaling of the cognitive items with a latent regression model using information from the background questionnaire (BQ) in a population model (Hogan et al. 2016). For the *m*-th plausible value for group *g,* in state *j*, county *k*, the survey estimate was computed as

$$\hat{y}_{jkg}^{(m)} = \sum_{a=1}^{n_{jkg}} w_{jkga} y_{jkga}^{(m)} \Big/ \sum_{a=1}^{n_{jkg}} w_{jkga} \tag{2a}$$

where $w_{jkga}$ is the weight for person *a* in state *j*, county *k*, group *g*; $y_{jkga}^{(m)}$ is the proficiency score (for average) or an indicator variable for the proficiency level (for proportions); and $n_{jkg}$ is the number of cases in state *j*, county *k*, group *g*. The weight $w_{jkga}$ has been raked to the 2013-2017 ACS control totals, defined as the adult population size (age 16 to 74), for raking dimensions comprised of age groups, education levels, gender, and race/ethnicity within state. More details can be found in section 2 of *PIAAC State-level Estimation for Age and Education Groups Methodology Report* (Li et al. 2022).

Then the county-level survey estimate for groups ($\hat{y}_{jkg}$) was calculated as

$$\hat{y}_{jkg} = \frac{1}{10} \sum_{m=1}^{10} \hat{y}_{jkg}^{(m)} \tag{2b}$$

The multiple imputation estimate of the variance is

$$\hat{\sigma}_{jkg}^2 = \hat{\sigma}_{Wjkg}^2 + (\tfrac{11}{10})\hat{\sigma}_{Bjkg}^2, \tag{2c}$$

where $\hat{\sigma}_{Wjkg}^2$ is the within-imputation variance and $\hat{\sigma}_{Bjkg}^2$ is the between-imputation variance. The theory for plausible value estimation in educational achievement assessments is due to Mislevy and Sheehan (1987, 1989). Their work is based on the more general procedure of multiple imputation described by Rubin (1987). A general review of multiple imputation with comments by several discussants is found in

Rubin (1996). The within-imputation variance component was computed as the average of the sampling variance for each of the 10 plausible values

$$\hat{\sigma}^2_{Wjkg} = \left(\sum_{m=1}^{10} \hat{v}^{(m)}_{jkg}\right)/10, \tag{2d}$$

where $\hat{v}^{(m)}_{jkg}$ is the sampling variance of the estimated mean or proportion for plausible value *m*. The between-imputation component was calculated as

$$\hat{\sigma}^2_{Bjkg} = \left[\sum_{m=1}^{10} \left(y^{(m)}_{jkg} - \hat{y}_{jkg}\right)^2\right]/9\ . \tag{2e}$$

Sampling variances were calculated using the Taylor series method (Wolter 2007), with secondary sampling units (SSUs), i.e., groups of census blocks, as variance units (clusters). The variances of the county-level survey estimates for groups are large on average due to very small sample sizes. The PIAAC data included 185 counties with at least one respondent, but some counties may have no respondents for one or more age and education groups. Table 2-1 shows the number of counties with sample and the distributions of sample sizes that were used to compute the county-level survey estimates for groups. With a small number of county-level survey estimates available for the groups, a model-based approach was deemed necessary to produce estimates for all the counties in the United States and for all the groups of interest. The medians of the sample sizes by age and education groups are between 6.5 and 16, indicating that for most of the counties the survey estimates by age and education groups might not be reliable.

Table 2-1. County-level sample size distributions for age and education groups: 2012/2014/2017 PIAAC

| Age and education groups | Number of counties with sample | Number of respondents | | | | | |
|---|---|---|---|---|---|---|---|
| | | Minimum | 10th percentile | Median | 90th percentile | Maximum | Mean |
| 16-24 | 172 | 1 | 2 | 10 | 31 | 82 | 15 |
| 25-34 | 177 | 1 | 3 | 11 | 33 | 96 | 16 |
| 35-44 | 180 | 1 | 2 | 9.5 | 21.5 | 56 | 11 |
| 45-54 | 178 | 1 | 2 | 10 | 21 | 72 | 11 |
| 55-64 | 180 | 1 | 2 | 10 | 18 | 45 | 10 |
| 65-74 | 176 | 1 | 1 | 6.5 | 14 | 31 | 7 |
| Less than high school | 175 | 1 | 2 | 8 | 25 | 67 | 12 |
| High school diploma or GED | 180 | 1 | 4 | 16 | 42.5 | 86 | 20 |
| Some college | 182 | 1 | 3 | 16 | 38 | 114 | 19 |
| Bachelor's degree or higher | 179 | 1 | 3 | 13 | 36 | 115 | 18 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Table 2-2 shows the distributions of coefficients of variation (CVs) for county-level survey estimates for age and education groups and for literacy proportion at or below Level 1; throughout the report, the coefficient of variation is constructed as the ratio of standard error to point estimate. Similar results are presented for the other quantities of interest in the appendix. The majority of the CVs for the estimates of proportions are 50 percent or higher. The literacy and numeracy average estimates are more precise, having median CVs ranging from 3.5 to 8.1 percent. In some cases the CVs are not estimable either because the estimated proportions are 0 or the respondents are all in one variance cluster (therefore the variances are not estimable). Model-based estimates and associated CVs were produced for all the counties in the United States and all the age and education groups of interest. For county and group combinations with available survey estimates, the model-based estimates were compared against the survey estimates; see section 4.

Table 2-2. Distribution of coefficients of variation (CVs) for county-level survey estimates for age and education groups for literacy proportion at or below Level 1: 2012/2014/2017 PIAAC

| Age and education groups | Percentile | | | | | Number of counties with missing CVs |
|---|---|---|---|---|---|---|
| | 20 | 40 | 50 (Median) | 60 | 80 | |
| 16-24 | 52.5 | 67.1 | 77.4 | 89.0 | 122.2 | 25 |
| 25-34 | 47.3 | 71.7 | 78.6 | 87.4 | 112.0 | 17 |
| 35-44 | 48.9 | 69.3 | 77.7 | 85.4 | 116.6 | 23 |
| 45-54 | 44.1 | 61.8 | 73.3 | 80.5 | 108.7 | 19 |
| 55-64 | 47.9 | 64.8 | 71.2 | 80.4 | 105.6 | 16 |
| 65-74 | 39.1 | 56.0 | 63.1 | 74.2 | 109.8 | 33 |
| | | | | | | |
| Less than high school | 21.5 | 32.1 | 36.6 | 42.2 | 61.5 | 12 |
| High school diploma or GED | 31.8 | 41.0 | 47.4 | 50.7 | 65.3 | 1 |
| Some college | 49.8 | 63.0 | 75.4 | 86.8 | 123.3 | 5 |
| Bachelor's degree or higher | 75.2 | 110.2 | 132.2 | 148.9 | 241.0 | 42 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

## 3. MODEL-BASED ESTIMATION

The county-level estimates for groups are constructed using an allocation method. This method consists of using the county-level and state by group-level HB model-based estimates and allocating them to the county by group domains of interest via a deterministic model. Similarly to the SAE modeling process developed for county, state, and state by age/education group estimation, model-based estimates for P1 and P3 are constructed independently, and then subtract their sum from one to obtain the model-based estimates for P2. Also, literacy and numeracy measures are estimated independently. The model output from the SAE models developed for the estimation of county, state, and state by age/education group proficiency measures serve as input into both the allocation and the validation and evaluation processes. The survey estimates constructed for the county by age/education groups serve as input into the validation and evaluation process, along with selected ACS population estimates and other ACS estimates related to proficiency.

Following the notation from the previous section, let $k$ denote the county, $j$ denote the state, and $g$ denote the age/education group. For either literacy or numeracy, let $Y$ denote any of the following three quantities of interest: P1, P3, and average score. Then, pseudo-posterior distributions are constructed for the county by age/education group quantities of interest as represented by the samples

$$Y_{g,jk}^{(b)} := Y_{g,j}^{(b)} \frac{Y_{jk}^{(b)}}{Y_j^{(b)}},$$

where $b$ is an index for the posterior samples, $Y_{g,j}^{(b)}$ are the posterior samples for the state by age/education group quantities, $Y_{jk}^{(b)}$ are the posterior samples for the county-level quantities, and $Y_j^{(b)}$ are the posterior samples for the state-level quantities. Note that the implicit working assumption is that the county-to-state ratio of model-based estimates is constant across age/education groups.

Pseudo-posterior distributions are constructed for the county by age/education group P2 as represented by the samples

$$P2_{g,jk}^{(b)} := 1 - P1_{g,jk}^{(b)} - P3_{g,jk}^{(b)}.$$

Posterior summaries, such as means, variances, and credible intervals are constructed using the pseudo-distributions described above. Posterior means for P1, P2, and P3 that are below 0 or above 1 are

set equal to 0 or 1, respectively. For this, individual adjustments are applied to the posterior samples $P1_{g,jk}^{(b)}, P2_{g,jk}^{(b)}$, and $P3_{g,jk}^{(b)}$ as follows:

$$Y_{g,jk}^{(b),adj} := Y_{g,jk}^{(b)} + (1 - Y_{g,jk}), \text{ if } Y_{g,jk} > 1,$$

$$Y_{g,jk}^{(b),adj} := Y_{g,jk}^{(b)} + (0 - Y_{g,jk}), \text{ if } Y_{g,jk} < 0,$$

where $Y_{g,jk}$ is the mean of the posterior samples $Y_{g,jk}^{(b)}$ and $Y$ represents any of the three proportions, P1, P2, or P3. The posterior means constructed using the adjusted posterior samples $Y_{g,jk}^{(b),adj}$ are greater than or equal to 0 and smaller than or equal to 1. Among the 175 counties with sample data for the less-than-high-school education group, there were 3 P1 model-based estimates greater than 1. Among the 2,967 counties without sample for the less-than-high-school education group, there were 42 P1 model-based estimates greater than 1. None of the county-level P1 model-based estimates for the less-than-high-school education group were below 0. Also, none of the average score model-based estimates were below 0 so no adjustment was needed to these estimates.

Note that the aggregations of county by age/education group estimates agree closely, but not exactly, to the state by age/education group model-based estimates. Using the ACS totals $T_{\cdot}$ corresponding to the population of interest at the various levels of aggregation (county, county by age/education group, state, and state by age/education group), benchmarking checks are outlined as follows.

State-level checks:

$$\begin{aligned}
\frac{\sum_k Y_{g,jk}^{(b)} T_{g,jk}}{\sum_k T_{g,jk}} &= \frac{\sum_k \left( Y_{g,j}^{(b)} \frac{Y_{jk}^{(b)}}{Y_j^{(b)}} \right) T_{g,jk}}{\sum_k T_{g,jk}} \\
&= \frac{\frac{Y_{g,j}^{(b)}}{Y_j^{(b)}} \sum_k \left( Y_j^{(b)} T_{g,jk} \right)}{T_{g,j}} \\
&= Y_{g,j}^{(b)} \frac{\sum_k \left( Y_{jk}^{(b)} T_{jk} \frac{T_{g,jk}}{T_{jk}} \right)}{Y_j^{(b)} T_{gj}} \\
&= Y_{g,j}^{(b)} \frac{\sum_k \left( Y_{jk}^{(b)} T_{jk} \frac{T_{g,jk}}{T_{jk}} \right)}{\frac{\sum_k Y_{jk}^{(b)} T_{jk} \sum_k T_{g,jk}}{\sum_k T_{jk}}} \\
&\neq Y_{g,j}^{(b)}
\end{aligned}$$

County-level checks:

$$\begin{aligned}
\frac{\sum_g Y_{g,j}^{(b)} T_{g,jk}}{\sum_g T_{g,jk}} &= \frac{\sum_g \left( Y_{g,j}^{(b)} \frac{Y_{jk}^{(b)}}{Y_j^{(b)}} \right) T_{g,jk}}{\sum_g T_{g,jk}} \\
&= \frac{\frac{Y_{jk}^{(b)}}{Y_j^{(b)}} \sum_g \left( Y_{g,j}^{(b)} T_{g,jk} \right)}{\sum_g T_{g,jk}} \\
&= Y_{jk}^{(b)} \frac{\sum_g Y_{g,j}^{(b)} T_{g,jk}}{Y_j^{(b)} \sum_g T_{g,jk}} \\
&= Y_{jk}^{(b)} \frac{\sum_g Y_{g,jk}^{(b)} T_{g,jk}}{\frac{\sum_g Y_{g,jk}^{(b)} T_{g,j}}{\sum_g T_{g,j}} \sum_g T_{g,jk}} \\
&\neq Y_{jk}^{(b)}
\end{aligned}$$

As reported in table 3-1, the median credible interval width is 24.5 percent for county-level estimates of literacy proportion at or below Level 1 for the less-than-high-school education group, where the median is taken over all the county by group domains in the United States. When these county by

group domains are categorized by sample availability, the median credible interval widths are 22.7 percent and 24.7 percent for in-sample and not-in-sample domains, respectively.

The CVs for the county-level model-based estimates of literacy proportion at or below Level 1 for the less-than-high-school education group are also summarized in table 3-1 by sample availability. For most of these county by group domains, the CVs are lower than 20 percent. Also, as expected, the CVs reported in table 3-1 are larger than the CVs reported in table 3-5 in the PIAAC State-level Estimation for Age and Education Groups Methodology Report (Li et al. 2022) because the level of aggregation is finer (same outcome and group, but different geography: county by group versus state by group); the state-level estimates by group have a median CV of 11.8 percent compared to 12.9 percent for the county-level estimates by group.

Table 3-1. Distribution of credible interval widths and coefficients of variation for county-level model-based estimates for less than high school for literacy proportion at or below Level 1: 2012/2014/2017 PIAAC

| | Percentile | | | | |
|---|---|---|---|---|---|
| Statistics for less than high school | 20 | 40 | 50 (Median) | 60 | 80 |
| County estimates for all domains | | | | | |
| 95 percent credible interval width (percent) | 21.1 | 23.4 | 24.5 | 25.9 | 29.4 |
| Coefficient of variation (percent) | 10.7 | 12.1 | 12.9 | 14.0 | 17.7 |
| County estimates for in-sample domains | | | | | |
| 95 percent credible interval width (percent) | 19.6 | 21.5 | 22.7 | 24.0 | 27.9 |
| Coefficient of variation (percent) | 10.3 | 11.6 | 12.3 | 12.8 | 16.3 |
| County estimates for not-in-sample domains | | | | | |
| 95 percent credible interval width (percent) | 21.2 | 23.5 | 24.7 | 25.9 | 29.5 |
| Coefficient of variation (percent) | 10.7 | 12.1 | 13.0 | 14.1 | 17.7 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

## 3.1 Alternative Model-Based Estimation

Alternative model-based estimates were constructed using the exact definition of the quantities of interest. For this, the posterior samples $Y_{g,jk}^{(b)}$ were constructed as

$$Y_{g,jk}^{(b)} := Y_{g,j}^{(b)} \frac{Y_{jk}^{(b)}}{Y_j^{(b)}} \frac{T_{jk}}{T_j} \frac{T_{g,j}}{T_{g,jk}}.$$

Note that the aggregations of county by age/education group estimates agree exactly to the state by age/education group model-based estimates. Using the ACS totals $T_{\cdot}$ corresponding to the population of interest at the various levels of aggregation (county, county by age/education group, state, and state by age/education group), benchmarking checks are outlined as follows.

State-level checks:

$$\begin{aligned}
\frac{\sum_k Y_{g,jk}^{(b)} T_{g,jk}}{\sum_k T_{g,j}} &= \frac{\sum_k \left( Y_{g,j}^{(b)} \frac{Y_{jk}^{(b)} T_j}{Y_j^{(b)} T_j} \frac{T_{g,j}}{T_{g,jk}} \right) T_{g,jk}}{\sum_k T_{g,jk}} \\
&= \frac{\frac{Y_{g,j}^{(b)} T_{g,j}}{Y_j^{(b)} T_j} \sum_k \left( Y_{jk}^{(b)} T_{jk} \right)}{T_{g,j}} \\
&= \frac{Y_{g,j}^{(b)}}{Y_j^{(b)} T_j} \left( Y_j^{(b)} T_j \right) \\
&= Y_{g,j}^{(b)}
\end{aligned}$$

County-level checks:

$$\begin{aligned}
\frac{\sum_g Y_{g,jk}^{(b)} T_{g,jk}}{\sum_g T_{g,jk}} &= \frac{\sum_g \left( Y_{g,j}^{(b)} \frac{Y_{jk}^{(b)} T_{jk}}{Y_j^{(b)} T_j} \frac{T_{g,j}}{T_{g,jk}} \right) T_{g,jk}}{\sum_g T_{g,jk}} \\
&= \frac{\frac{Y_{jk}^{(b)} T_{jk}}{Y_j^{(b)} T_j} \sum_g \left( Y_{g,j}^{(b)} T_{g,j} \right)}{T_{jk}} \\
&= \frac{\frac{Y_{jk}^{(b)} T_{jk}}{Y_j^{(b)} T_j} \left( Y_j^{(b)} T_j \right)}{T_{jk}} \\
&= Y_{jk}^{(b)}
\end{aligned}$$

This alternative method was not adopted as the final method because of three reasons: (1) the resulting model-based estimates were unreasonable for many county by group domains, for example literacy average estimates above 500 or initial proportion estimates (before the adjustment) greater than 1;

(2) there was not an exact consistency between the ACS population totals at various levels of aggregation (state SAE estimates are based on an approximated population total of adults 16 to 74 year olds, because the publicly-available data contain only the 15-74 year olds age group; state by age group SAE estimates are constructed using an exact population total for the 16-74 year olds age group because special tabulations were later provided by the U.S. Census Bureau and were used); and (3) the ACS population totals at fine levels of aggregation are subject to uncertainty which would contribute to error in the estimates.

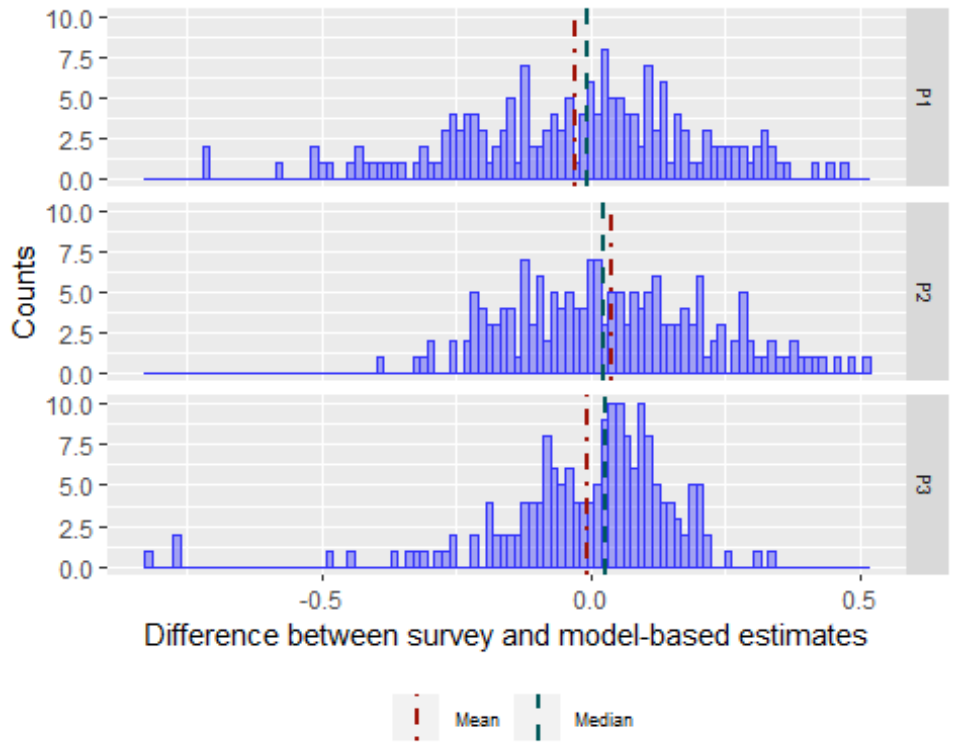
# 4. VALIDATION AND EVALUATION

This chapter includes a discussion of the extensive validation and evaluation process for the model-based estimates after the truncation was applied to the point estimates for proportions (i.e., proportions between 0 and 1). The process comprised six parts and the subsections of this section correspond to these parts. Specifically, the following visual and tabular checks were adopted: (1) histograms of the difference between survey estimates and model-based estimates, (2) bubble plots of survey estimates versus model-based estimates (point estimates and associated standard errors), (3) shrinkage plots with arrows showing the direction from survey estimates to model-based estimates, (4) interval coverage plots and tables with counts of nonoverlapping uncertainty intervals, (5) aggregation plots of model-based estimates versus HB model-based estimates, and (6) scatterplots of selected ACS estimates versus survey estimates or model-based estimates.

## 4.1 Histograms of Differences in Estimates

The differences between survey estimates and model-based estimates are shown in the histograms in figure 4-1. This plot shows the distribution of the difference as well as possible outliers. The means and medians of the differences are around 0. The majority of the differences are within 50 percentage points. The outliers in the plots show that a few model-based estimates could deviate from the survey estimates by about 50-80 percentage points. Given the small sample sizes for the domains of interest, one can expect the model estimates to deviate from the survey estimates in many domains, especially the ones with smaller sample sizes. The information presented using these histograms of differences is supplemented by the information presented in the plots from subsections 4.2, 4.3, and 4.4, where sample size is one of the graphical features.

Figure 4-1. Literacy proportion (less than high school) - Histograms of differences between survey estimates and model-based estimates: 2012/2014/2017 PIAAC

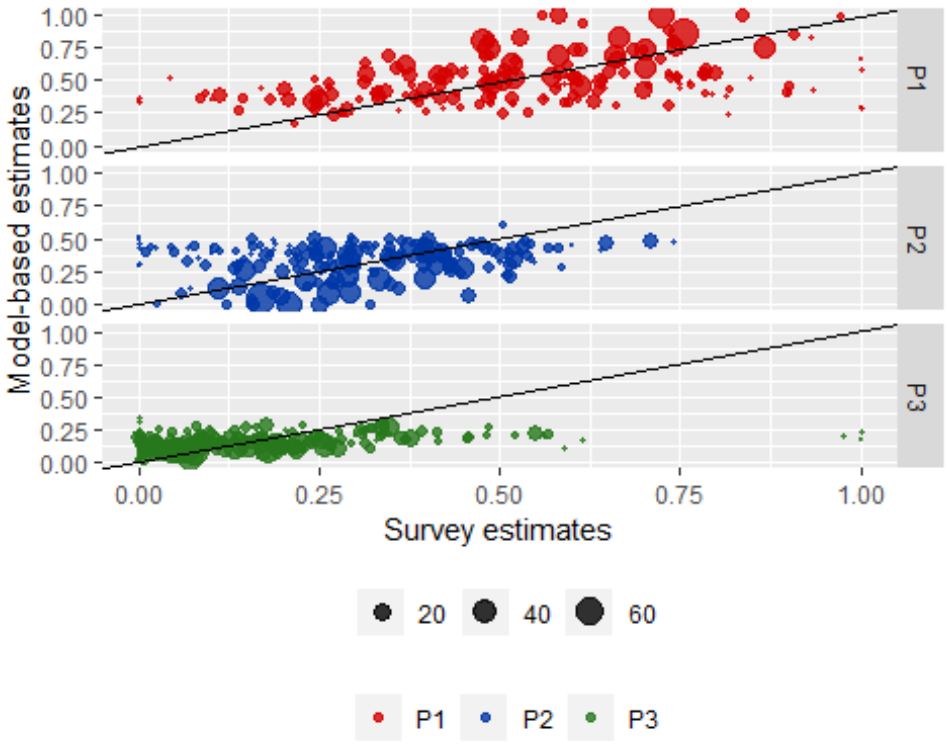



SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

## 4.2 Bubble Plots of Survey Estimates and Model-Based Estimates

Figure 4-2 shows the survey estimates versus model-based estimates. The size of the bubble is proportional to the domain sample size: large bubbles correspond to county by group domains with large sample sizes, and small bubbles correspond to county by group domains with small sample sizes. The majority of the points are on both sides of the 45-degree line, indicating that the model-based estimates are closer to the survey estimates for some county by group domains and farther away from the survey estimates for other county by group domains, without any outstanding pattern. The model-based and the survey estimates seem to be closer to each other for county by group domains with larger sample sizes (larger bubbles) than for county by group domains with smaller sample sizes (smaller bubbles)-some of the small bubbles are farther away from the 45-degree lines than the majority of the large bubbles. Similarly to the discussion above, this is expected due to higher sampling errors for the survey estimates for domains with small sample sizes.

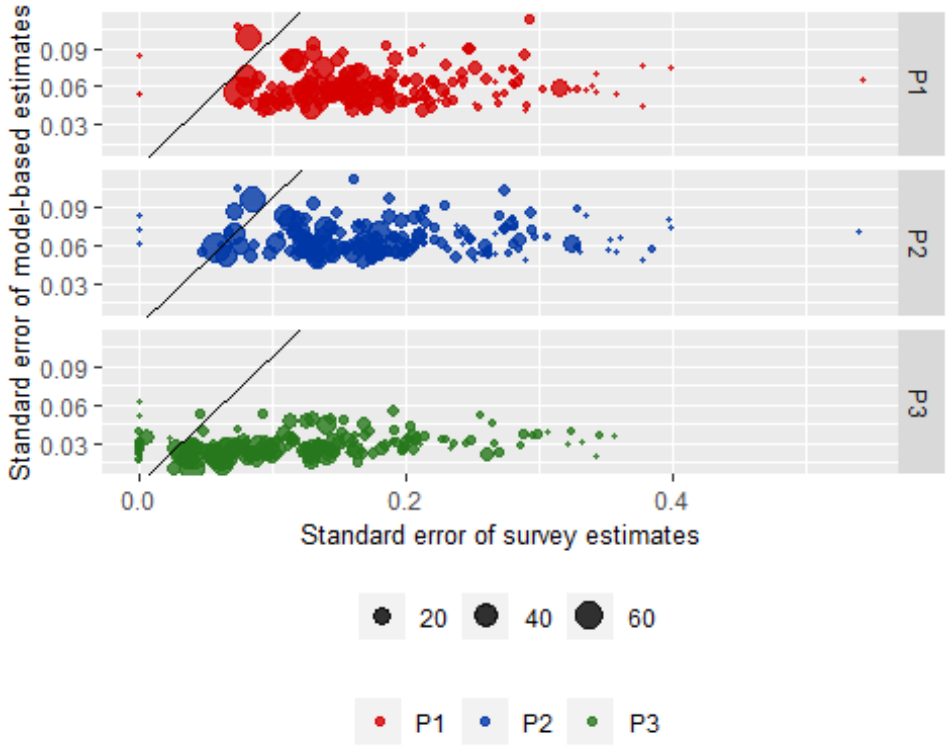
Figure 4-2. Literacy proportion (less than high school) - Comparison between survey estimates and model-based estimates: 2012/2014/2017 PIAAC




SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Figure 4-3 shows the standard errors of the survey estimates and the model-based estimates. The majority of the points are below the 45-degree line, indicating that the model-based estimates are more precise than the survey estimates. The model-based and the survey standard errors are closer to each other for county by group domains with larger sample sizes (larger bubbles) than for county by group domains with smaller sample sizes (smaller bubbles)-some of the small bubbles are farther away from the 45-degree lines than the majority of the large bubbles.

Figure 4-3. Literacy proportion (less than high school) - Comparison between model standard errors and survey standard errors: 2012/2014/2017 PIAAC

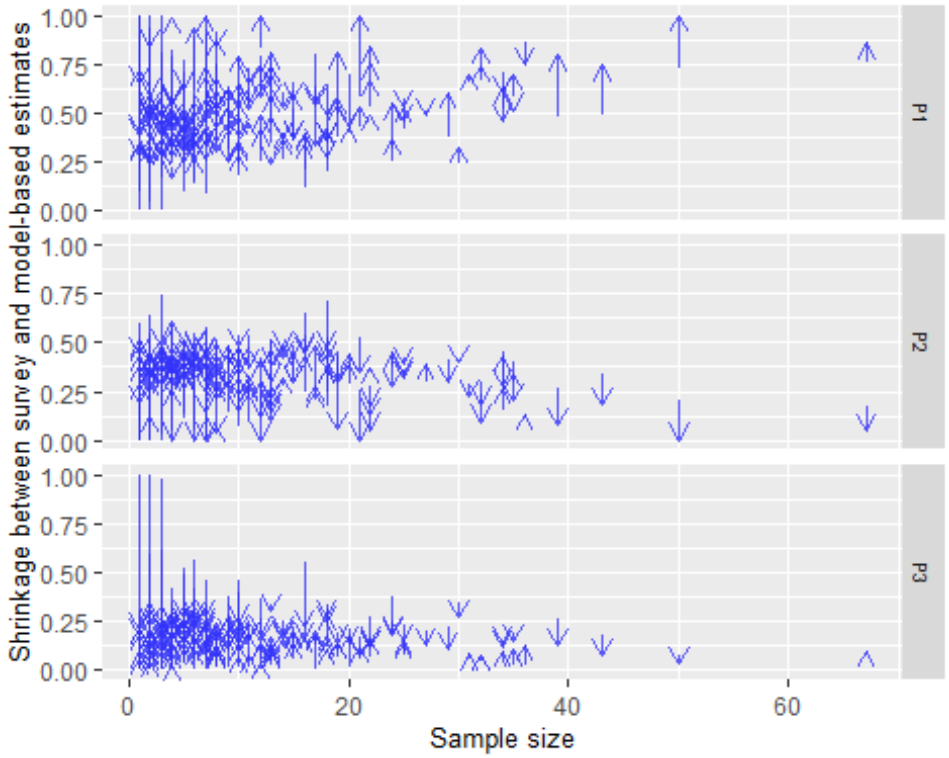



SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

## 4.3 Shrinkage Plots

Shrinkage plots show the magnitude and direction from survey estimates to model-based estimates, by sample size. The shrinkage can be observed in figure 4-4. Short arrows correspond to small differences between the survey and the model-based estimates and long arrows correspond to large differences between the survey and the model-based estimates. The shrinkage is more substantial in domains with smaller sample sizes than those in domains with larger sample sizes. Arrows pointing upward correspond to negative differences between the survey and the model-based estimates and arrows pointing downward correspond to positive differences between the survey and the model-based estimates.

Figure 4-4. Literacy proportion (less than high school) - Shrinkage plots of point estimates by sample size: 2012/2014/2017 PIAAC




SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

## 4.4 Interval Coverage Plots and Associated Tables

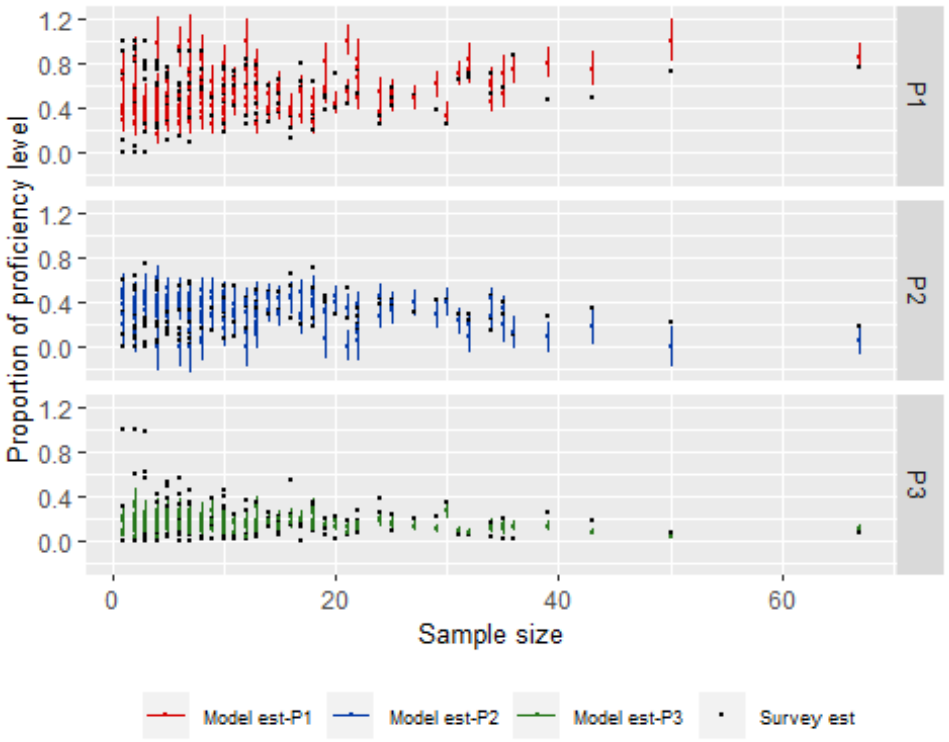
The interval coverage plots in figure 4-5 show that the credible intervals tend to be narrower for domains with larger sample sizes than for domains with smaller sample sizes. The credible intervals for P3 model-based estimates are narrower than the credible intervals for P1 or P2. Due to similar sample sizes for some county by group domains, it is not clear whether the 95 percent credible intervals of the model-based estimates cover the survey estimates or not. In addition, the survey estimates are subject to uncertainty, which is not represented in this figure. To complement the results in this figure, counts of nonoverlapping uncertainty intervals are presented in table 4-1, where the uncertainty interval for a survey estimate is defined as the survey estimate plus or minus two survey standard errors. Note that the number of nonoverlapping intervals decreases with an increase in the domain sample size. This result is expected because the survey estimates for larger domains are more reliable than the survey estimates for smaller domains, so the model-based estimates should not be significantly different from the survey estimates for large domains.

Table 4-1. Nonoverlapping uncertainty intervals for county-level survey and model-based estimates: 2012/2014/2017 PIAAC

| | | Domains with sample data broken down by sample size | | | | | |
|---|---|---|---|---|---|---|---|
| | Domains with sample data | Domains with 0 < sample size <= 10 | Domains with 10 < sample size <= 20 | Domains with 20 < sample size <= 30 | Domains with sample size > 30 | Domains with survey estimates = 0 | Domains with survey estimates = 1 |
| Total | 175 | 97 | 49 | 15 | 14 | 3 | 4 |
| Total with nonoverlapping uncertainty intervals | 9 | 8 | 1 | 0 | 0 | 1 | 2 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Figure 4-5. Literacy proportion (less than high school) - Indication of coverage by credible interval: 2012/2014/2017 PIAAC




NOTE: The legend for the survey estimates are point estimates shown as the black dots in the figure. Uncertainty intervals for the survey estimates are not illustrated.
SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

## 4.5 Aggregation Plots

County and state by group model-based estimates were constructed by aggregating the county by group model-based estimates using the county by group ACS totals as aggregation weights; see the PIAAC State and County Estimation Methodology Report (Krenzke et al. 2020) for details about the aggregation method. These newly constructed aggregated estimates were compared against the HB model-based estimates. Specifically, the two comparisons are as follows: 1) aggregated model-based estimates across the groups (i.e., age or education) within counties compared against the county-level HB model-based estimates, and 2) aggregated model-based estimates across counties within states, for each group, compared against the state by group-level HB model-based estimates. Example results from the first comparison are illustrated in figures 4-11 for Maryland. The results from the second comparison are illustrated in figure 4-12.

In general, the only noticeable discrepancies were between the county-level model-based estimates aggregated across education groups within counties and the county-level HB model-based estimates. For example, for literacy proportions at or below Level 1, some 93 of the 3,142 county-level model-based estimates aggregated across education groups were significantly different from their corresponding county-level HB model-based estimates. Further investigation into these discrepancies involved reproducing figure 4-6 using a function of ACS population totals on the *y* axis: the county-level ACS population totals, the standard errors of the county-level ACS population totals, the coefficients of variation of the county-level ACS population totals, or the range of the coefficients of variation of the county by group-level ACS population totals with the range taken over the education groups within counties. Note that measures of uncertainty apply only to county by education group ACS population estimates because they are not readily available for the county by age group ACS population estimates. Finally, the county-level measure

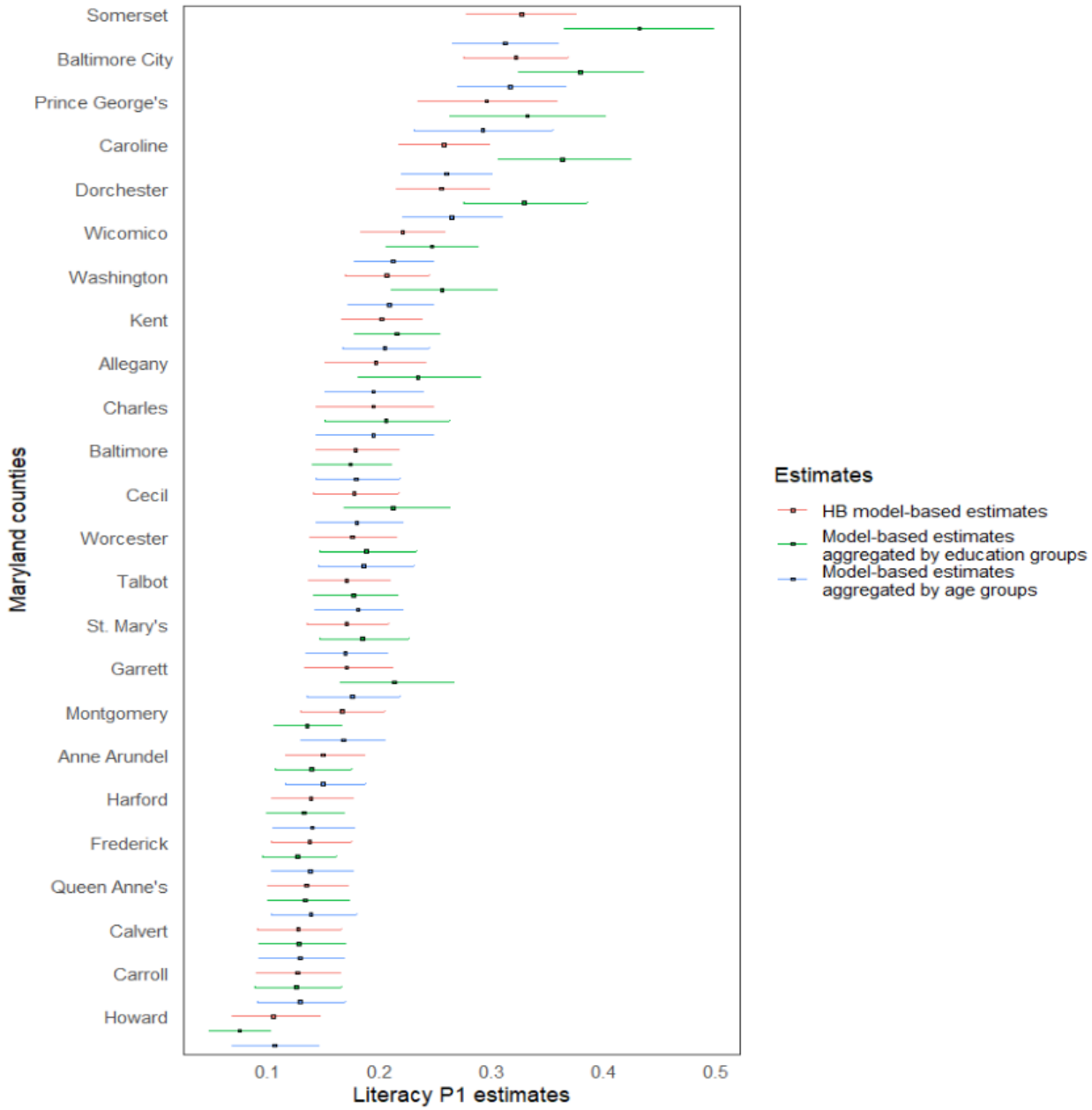
$$d_{jk} \coloneqq 1/\gamma_{jk} \sum_{g} \mid 1 - \frac{T_{jk}}{T_j} \frac{T_{g,j}}{T_{g,jk}} \mid,$$

where $\gamma_{jk}$ is the number of groups indexed by $g$ in county *k* and state *j*, and $\mid . \mid$ denotes the absolute value, was considered as another function of ACS population totals replacing the county identifier on the *y* axis in figure 4-6. This measure is the average of the absolute value of the difference between 1 and the product between the ratios of ACS county-level population total $T_{jk}$ to ACS state-level population total $T_j$ and ACS state by group-level population total $T_{g,j}$ to ACS county by group-level population total $T_{g,jk}$, with the average taken across the groups (six age groups or four education groups) within county. The closer the

values of this measure are to 0 the closer the model-based estimates are to the alternative model-based estimates.

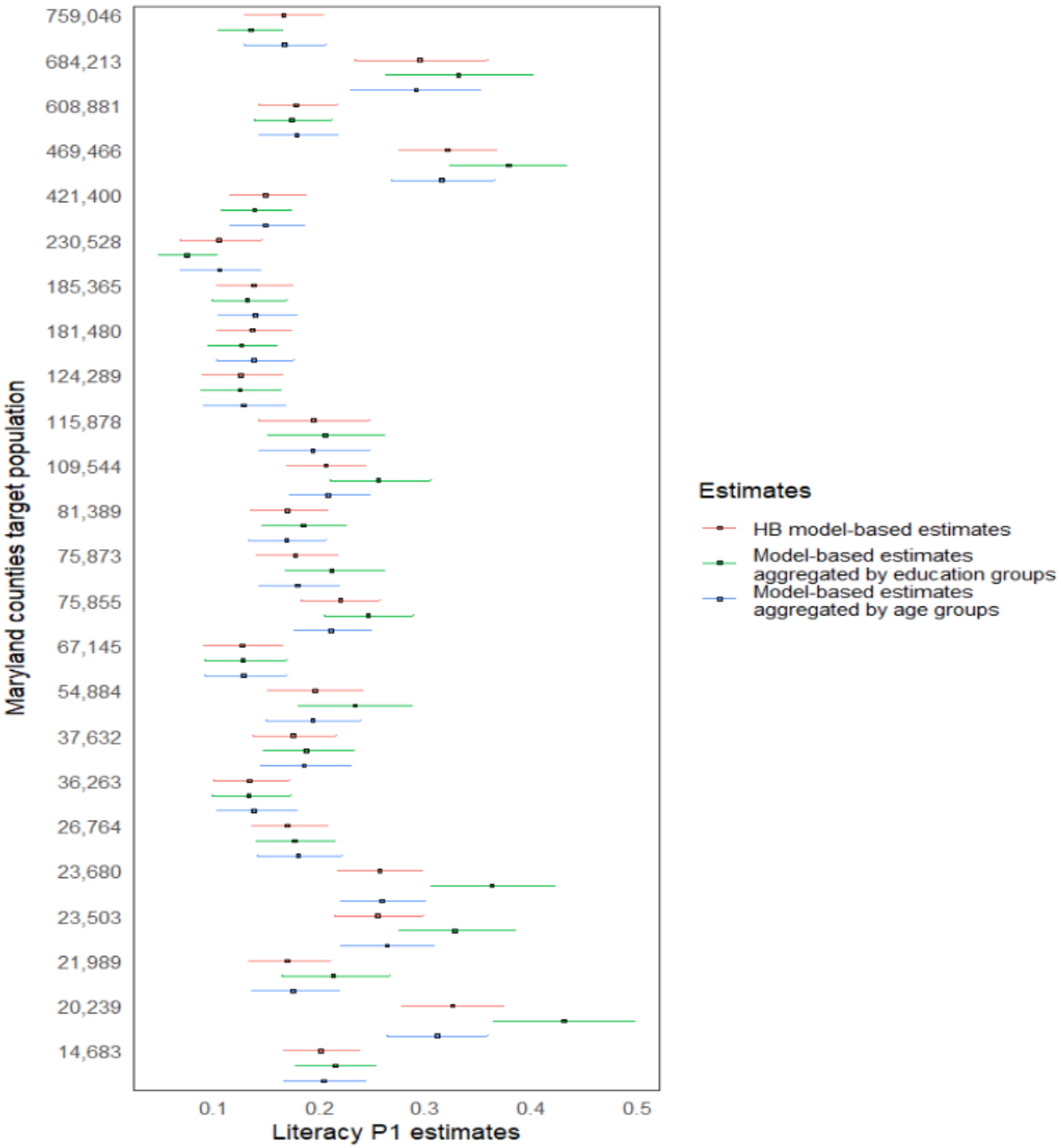
Instead of displaying the counties within a state sorted by their point estimates, as it is currently illustrated in figure 4-6 for Maryland, the counties are displayed within a state sorted by their values of the function of ACS population totals. See figures 4-7 to 4-11 for such examples for Maryland. From the visual exploration, it was observed that larger discrepancies between aggregated model-based estimates and HB model-based estimates corresponded to either larger ranges in the coefficients of variation of the county by group-level ACS population totals or larger $d_{jk}$ values.

Figure 4-6. Literacy proportion at or below Level 1 - County-level estimates in Maryland: 2012/2014/2017 PIAAC



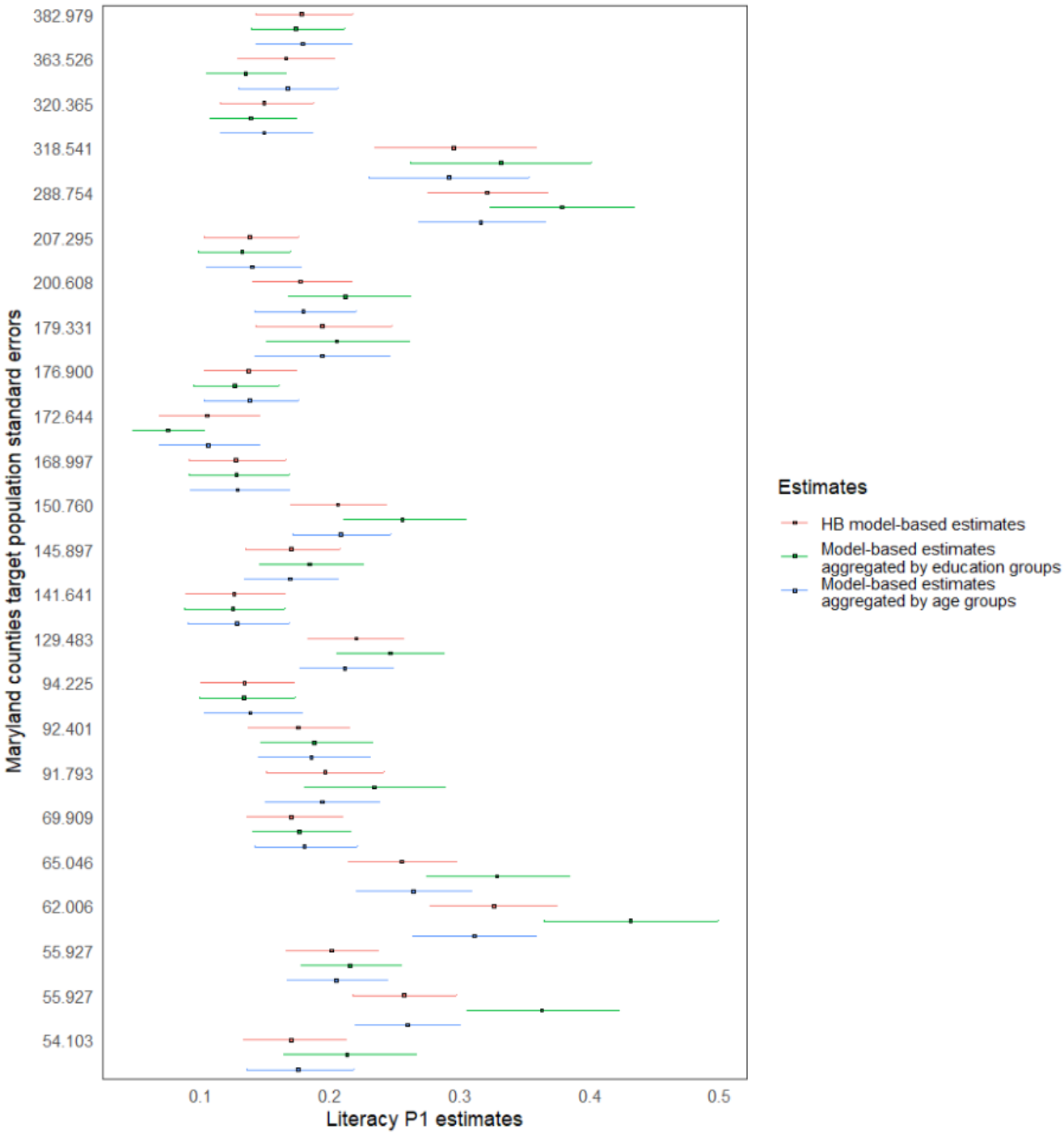

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Figure 4-7. Literacy proportion at or below Level 1 - County-level estimates in Maryland sorted by the county-level population total: 2012/2014/2017 PIAAC



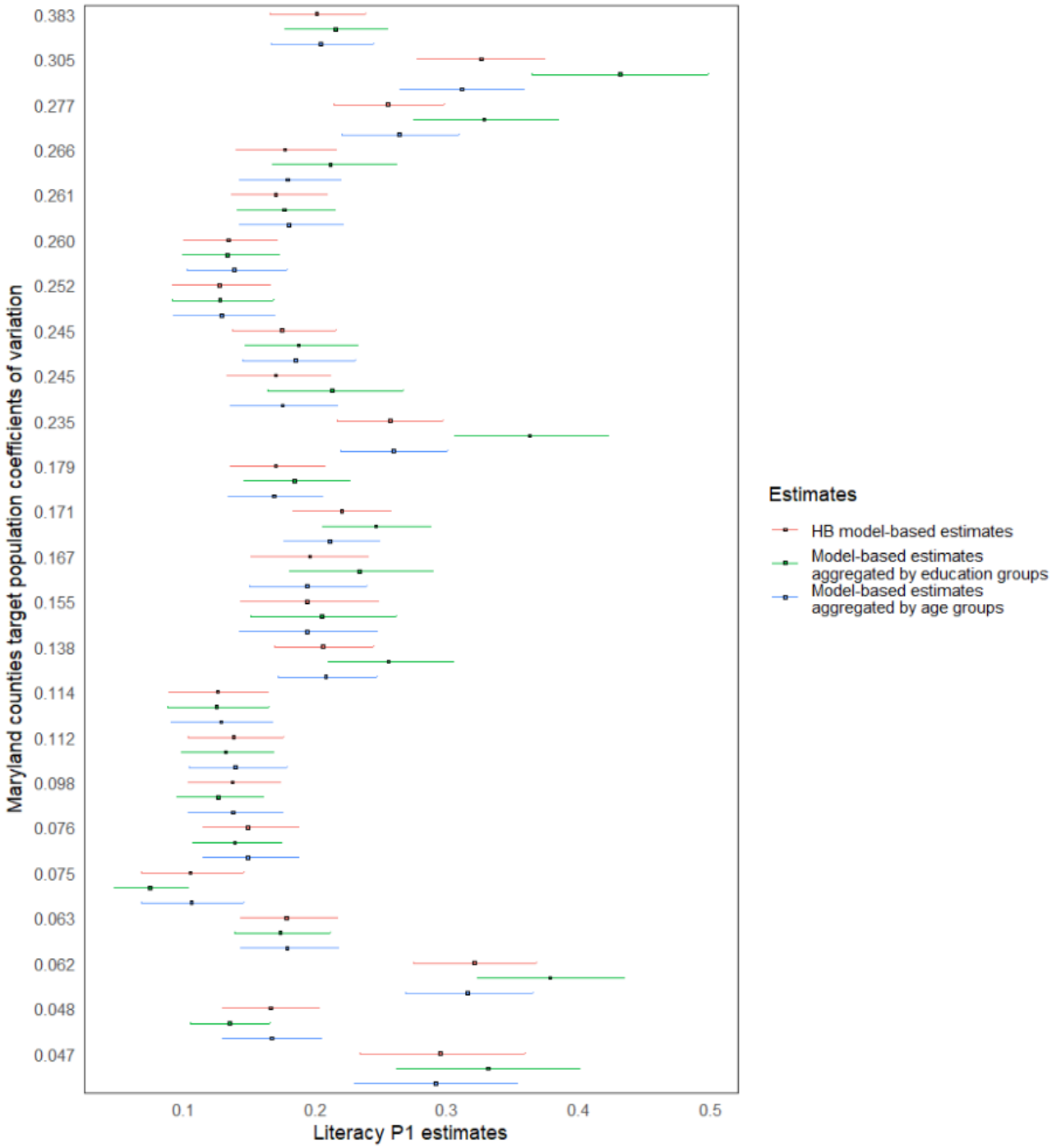

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Figure 4-8. Literacy proportion at or below Level 1 - County-level estimates in Maryland sorted by the county-level population total standard error: 2012/2014/2017 PIAAC



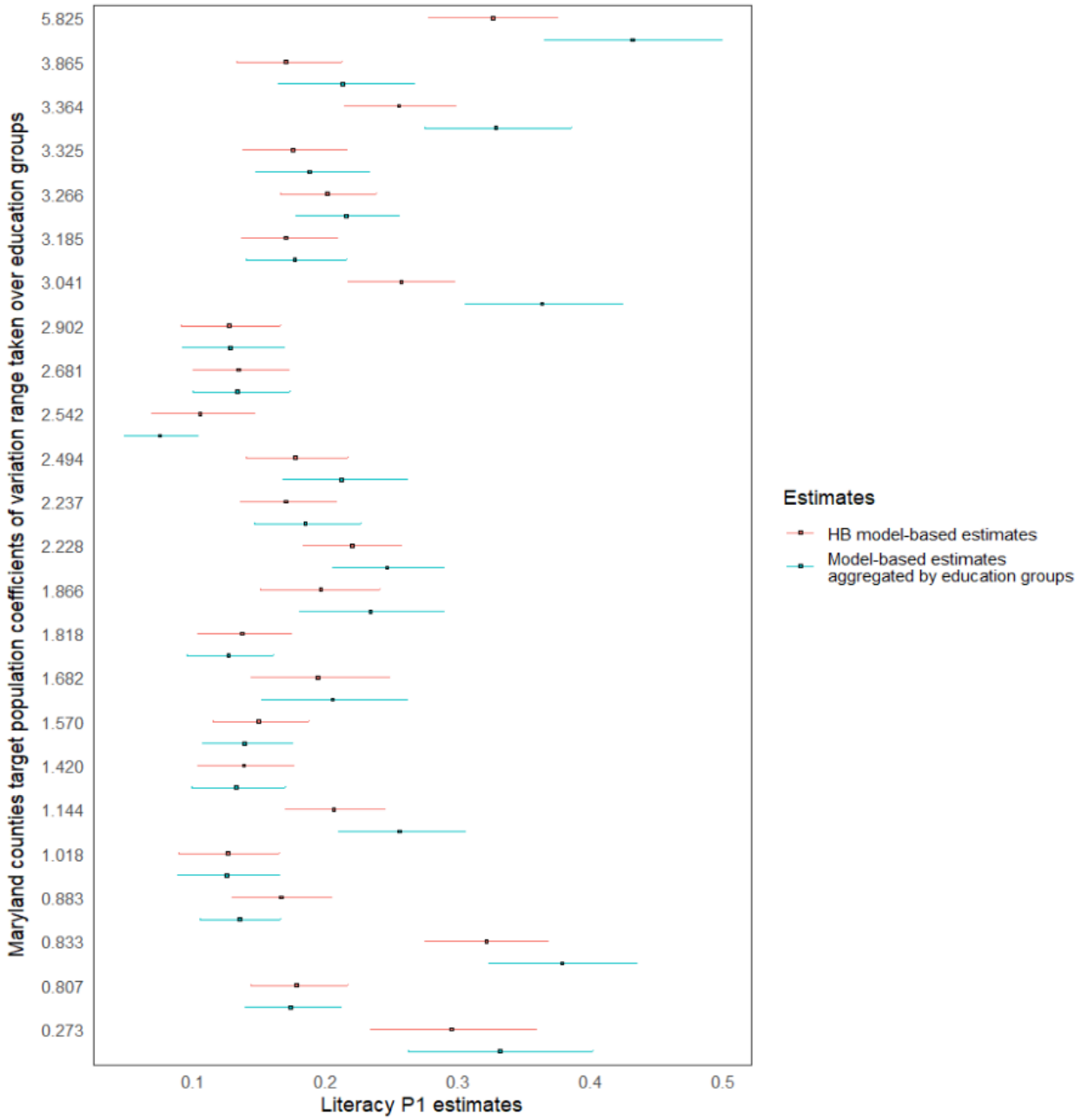

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Figure 4-9. Literacy proportion at or below Level 1 - County-level estimates in Maryland sorted by the county-level population total coefficient of variation multiplied by 100: 2012/2014/2017 PIAAC



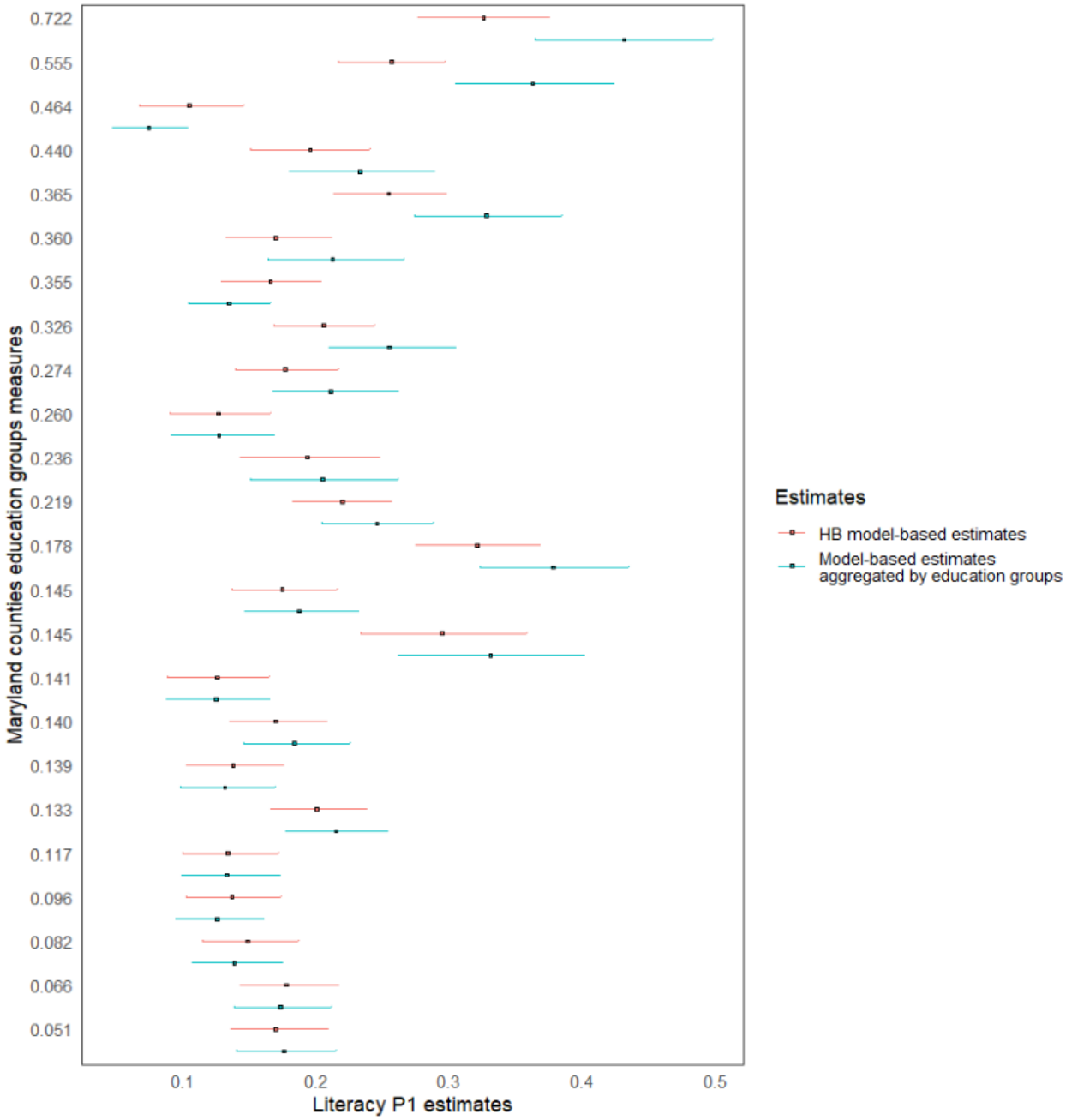

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Figure 4-10. Literacy proportion at or below Level 1 - County-level estimates in Maryland sorted by the range of county by education group population totals coefficients of variation multiplied by 100: 2012/2014/2017 PIAAC



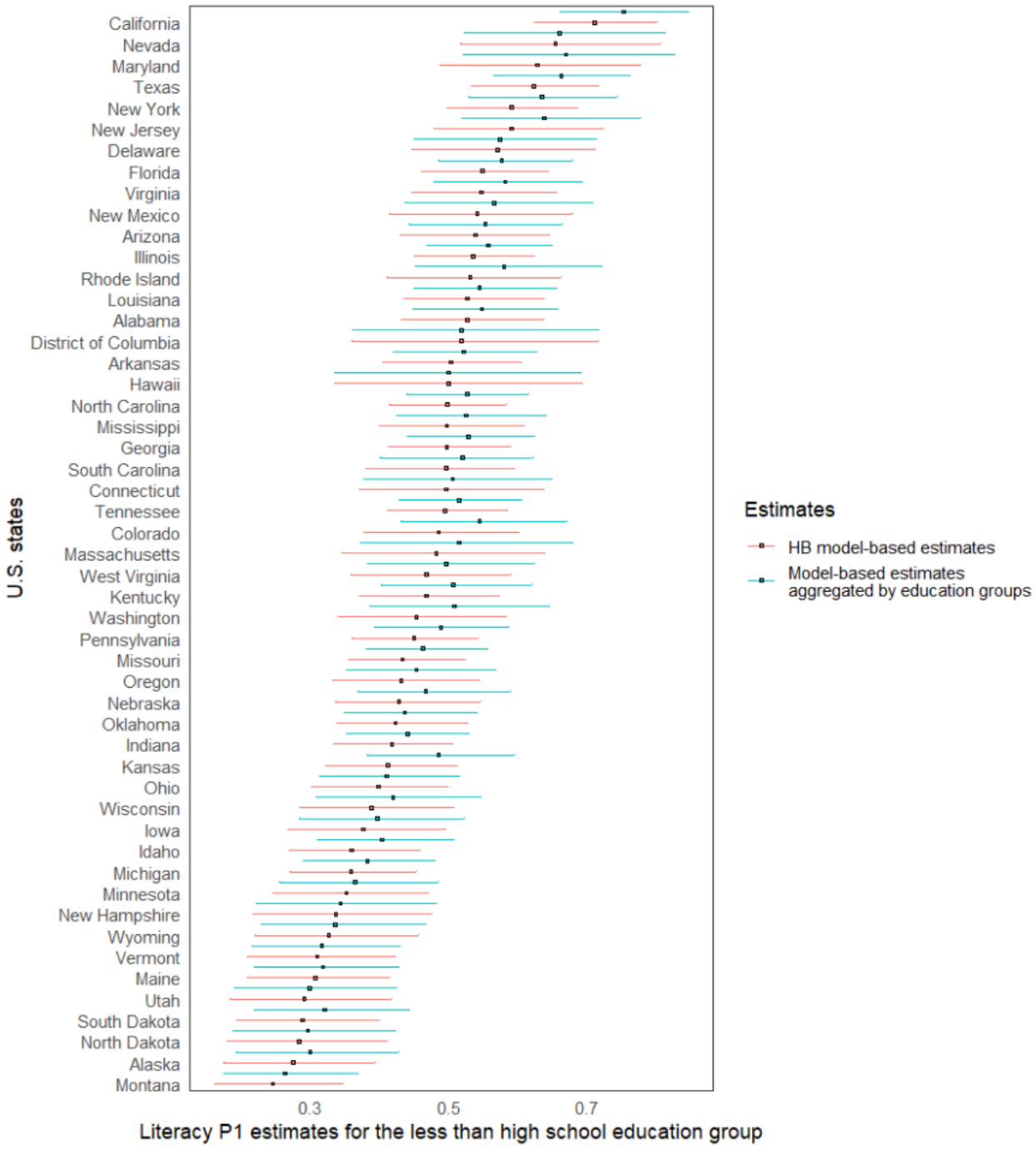

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Figure 4-11. Literacy proportion at or below Level 1 - County-level estimates in Maryland sorted by the county-level education measure: 2012/2014/2017 PIAAC




SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Figure 4-12. Literacy proportion at or below Level 1 for less-than-high-school education group - State-level estimates: 2012/2014/2017 PIAAC



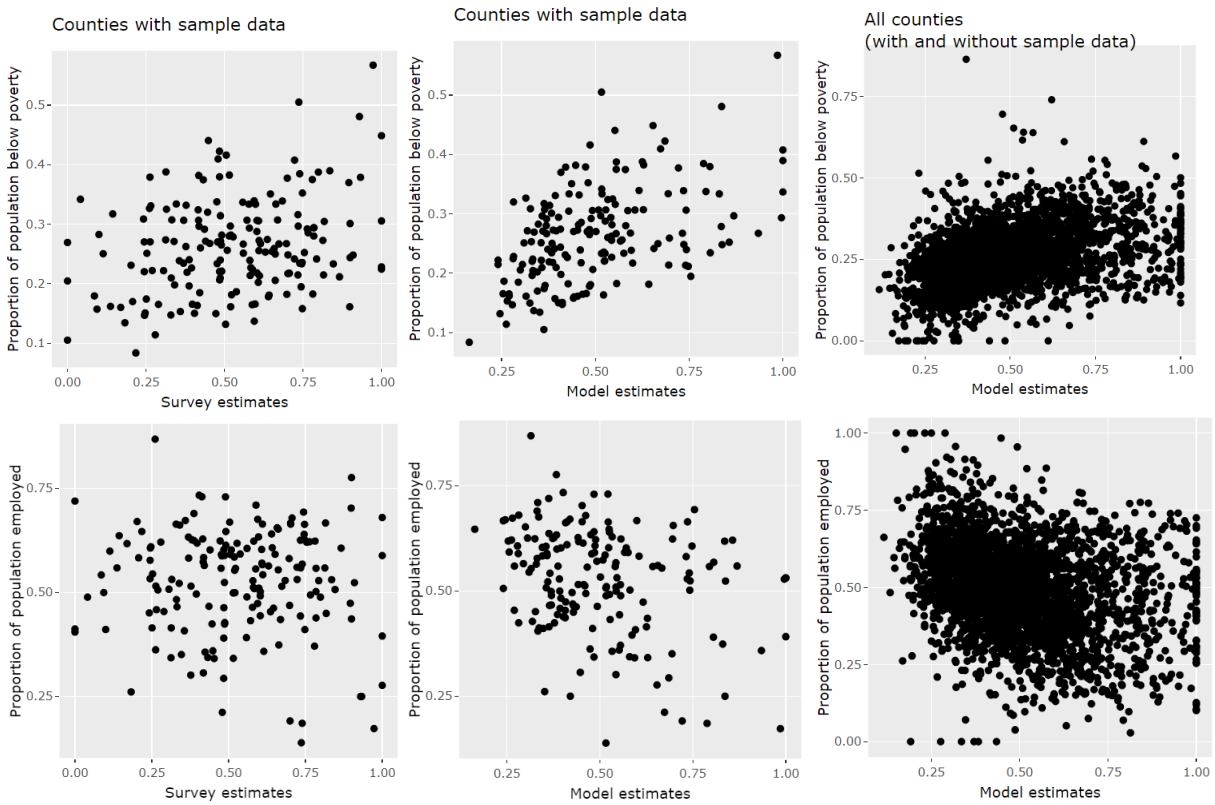

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

## 4.6 Scatterplots Using Selected ACS Variables

Comparisons of model-based estimates and selected ACS variables were also conducted. The selected ACS variables represent variables included in the initial pool of variables considered for the small area estimation models for county and state by group estimation (Krenzke et al. 2020; Li et al. 2022) and comprise the following list for age groups: proportion of population with less than high school education, the proportion of population below 100 percent of the poverty line, and the proportion of population with no health insurance coverage; and the following list for education groups: the proportion of population below 100 percent of the poverty line, and the proportion of population employed.

The scatterplots of estimates of proportions at or below Level 1 for the less-than-high-school education group are illustrated in figure 4-13. The ACS variables (proportion of population below poverty, proportion of population employed) are illustrated on the *y* axes and the estimates (survey estimates, model-based estimates for counties with sample, model-based estimates for all counties) are illustrated on the *x* axes. In general, similar range of the survey estimates and model-based estimates, and similar relationship between the ACS variables and the estimates were observed, with this relationship being clearer between the model-based estimates and the ACS variables than between the survey estimates and the ACS variables. This result is expected because the model-based estimates are functions of the HB model-based estimates, which are themselves functions of the ACS variables considered in this evaluation or related to the ones considered in this evaluation.

Figure 4-13. Literacy proportion at or below Level 1 for less-than-high-school education group versus selected ACS variables - County by group-level estimates: 2012/2014/2017 PIAAC




SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

## 5. SUMMARY

The PIAAC SAE process includes a statistical modeling approach that has taken into account significant enhancements made in SAE methodology. The statistical modeling approach was used to produce model-based estimates, which are available to the public on the Skills Map website at https://nces.ed.gov/surveys/piaac/skillsmap/. In 2020, the statistical modeling approach produced four state- and county-level estimates for adult literacy and numeracy proficiencies: an average score (on the PIAAC scale of 0-500) and the proportion of adults at or below Level 1, at Level 2, and at or above Level 3. The model-based estimates relied on the pooled 2012/2014/2017 PIAAC data as well as the ACS (2013-2017) data. The modeling depended on (1) PIAAC survey estimates, (2) area-level HB linear three-fold models (bivariate for proportions, univariate for averages); and (3) seven covariates relating to educational attainment, poverty, race/ethnicity, health insurance coverage, and occupation (service industry).

In 2022, the statistical modeling approach was adapted to produce state-level model-based estimates for six age groups and four educational attainment groups. Like the state and county estimates, the model-based state-level estimates for groups also relied on the combined 2012/2014/2017 PIAAC data and ACS (2013-2017) data. The modeling depended on (1) PIAAC survey estimates, (2) area-level HB linear univariate models with state-level random effects, and (3) four covariates for the age group models (percentage of population with less than high school education, percentage of population with more than high school education, percentage of population below 100 percent of the poverty line, and percentage of population age 16 and over with service occupations) and three covariates for the education group models (percentage of civilian noninstitutionalized population with no health insurance coverage, percentage of population below 100 percent of the poverty line, and percentage of population in the age group 16-24). Then, an allocation approach was developed to produce county-level model-based estimates for the same six age groups and four educational attainment groups. Previous model-based SAE estimates, constructed at the county, state, and state by group levels, served as inputs into the allocation approach. Overall, the county-level estimates by group are less precise than the state-level estimates by group, but most are still considered to be high confidence for publication purpose (CVs below 20 percent).

In the PIAAC Skills Map, comparisons are available between county by group domains on the eight outcomes, in addition to the comparisons described in the previous reports (Krenzke et al. 2020; Li et al. 2022). That is, for literacy and numeracy, comparisons can be conducted on the proportion at or below Level 1, the proportion at Level 2, the proportion at Level 3 and above, and the average, for each group (age groups 16-24, 25-34, 35-44, 45-54, 55-64, 56-74; education groups: less than high school,

high school, some college, and bachelor's degree or higher). For each group, the areas involved in the comparisons now cover the following:

- County-to-county within state
- County-to-state

Pairwise comparisons and multiple comparisons are also available as in the current Skills Map website at https://nces.ed.gov/surveys/piaac/skillsmap/. Details of the comparison methods can be found in the *Program for the International Assessment of Adult Competencies (PIAAC): State and County Estimation Methodology Report* (Krenzke et al. 2020, available at https://nces.ed.gov/pubs2020/2020225.pdf).

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## APPENDIX

The tables in this appendix show the distributions of coefficients of variation (CVs) for county-level survey estimates for age and education groups and for numeracy proportion at or below Level 1, literacy and numeracy proportions at Level 2 and at or above Level 3, and literacy and numeracy averages.

Table A-1. Distribution of coefficients of variation (CV) for county-level survey estimates for age and education groups for numeracy proportion at or below Level 1: 2012/2014/2017 PIAAC

| | Percentile | | | | | Number of counties with missing CVs |
|---|---|---|---|---|---|---|
| Age and education groups | 20 | 40 | 50 (Median) | 60 | 80 | |
| 16-24 | 37.7 | 50.5 | 55.9 | 63.6 | 92.9 | 31 |
| 25-34 | 40.4 | 58.0 | 65.2 | 73.5 | 102.4 | 30 |
| 35-44 | 46.5 | 61.9 | 68.6 | 78.3 | 101.9 | 28 |
| 45-54 | 40.9 | 55.0 | 64.8 | 71.3 | 97.3 | 27 |
| 55-64 | 42.2 | 54.9 | 61.4 | 72.1 | 95.1 | 22 |
| 65-74 | 35.3 | 46.3 | 53.8 | 61.6 | 93.0 | 40 |
| | | | | | | |
| Less than high school | 21.4 | 31.7 | 36.0 | 44.1 | 66.6 | 25 |
| High school diploma or GED | 25.0 | 32.3 | 36.5 | 40.5 | 54.3 | 19 |
| Some college | 37.8 | 49.3 | 55.5 | 65.9 | 88.4 | 21 |
| Bachelor's degree or higher | 66.2 | 90.2 | 109.1 | 123.6 | 172.0 | 42 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Table A-2. Distribution of coefficients of variation (CV) for county-level survey estimates for age and education groups for literacy proportion at Level 2: 2012/2014/2017 PIAAC

| | Percentile | | | | | Number of counties with missing CVs |
|---|---|---|---|---|---|---|
| Age and education groups | 20 | 40 | 50 (Median) | 60 | 80 | |
| 16-24 | 36.6 | 49.4 | 55.8 | 62.4 | 81.1 | 27 |
| 25-34 | 39.6 | 49.9 | 56.7 | 65.6 | 89.3 | 24 |
| 35-44 | 44.0 | 57.2 | 62.4 | 69.0 | 94.6 | 26 |
| 45-54 | 40.6 | 54.8 | 61.5 | 67.5 | 89.1 | 26 |
| 55-64 | 40.4 | 49.7 | 52.8 | 59.9 | 75.2 | 22 |
| 65-74 | 41.9 | 54.1 | 62.1 | 70.1 | 96.6 | 35 |
| | | | | | | |
| Less than high school | 43.0 | 55.4 | 62.5 | 68.9 | 108.0 | 26 |
| High school diploma or GED | 28.6 | 35.1 | 38.8 | 43.5 | 63.1 | 17 |
| Some college | 30.3 | 36.9 | 39.2 | 44.5 | 54.1 | 20 |
| Bachelor's degree or higher | 42.0 | 52.9 | 60.0 | 67.4 | 86.8 | 24 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Table A-3. Distribution of coefficients of variation (CV) for county-level survey estimates for age and education groups for numeracy proportion at Level 2: 2012/2014/2017 PIAAC

| Age and education groups | Percentile | | | | | Number of counties with missing CVs |
|---|---|---|---|---|---|---|
| | 20 | 40 | 50 (Median) | 60 | 80 | |
| 16-24 | 37.7 | 47.3 | 53.2 | 60.9 | 83.2 | 28 |
| 25-34 | 39.6 | 49.0 | 54.5 | 61.1 | 76.8 | 23 |
| 35-44 | 44.5 | 53.2 | 60.1 | 66.4 | 88.9 | 27 |
| 45-54 | 41.8 | 54.4 | 62.0 | 68.2 | 93.0 | 24 |
| 55-64 | 39.9 | 49.3 | 53.7 | 63.5 | 86.5 | 23 |
| 65-74 | 46.9 | 59.1 | 64.1 | 73.9 | 94.3 | 34 |
| | | | | | | |
| Less than high school | 49.2 | 64.4 | 74.6 | 85.3 | 130.4 | 29 |
| High school diploma or GED | 33.2 | 40.2 | 44.5 | 49.2 | 66.4 | 18 |
| Some college | 29.7 | 36.3 | 39.9 | 44.7 | 55.8 | 20 |
| Bachelor's degree or higher | 38.9 | 47.5 | 54.1 | 57.4 | 73.9 | 24 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Table A-4. Distribution of coefficients of variation (CV) for county-level survey estimates for age and education groups for literacy proportion at or above Level 3: 2012/2014/2017 PIAAC

| Age and education groups | Percentile | | | | | Number of counties with missing CVs |
|---|---|---|---|---|---|---|
| | 20 | 40 | 50 (Median) | 60 | 80 | |
| 16-24 | 27.6 | 39.1 | 43.0 | 48.7 | 67.8 | 29 |
| 25-34 | 20.3 | 26.0 | 28.4 | 32.4 | 53.2 | 24 |
| 35-44 | 26.0 | 35.9 | 40.0 | 46.0 | 65.3 | 27 |
| 45-54 | 26.8 | 36.6 | 40.6 | 49.7 | 68.7 | 24 |
| 55-64 | 30.9 | 43.2 | 47.5 | 55.2 | 71.9 | 27 |
| 65-74 | 37.8 | 52.9 | 65.3 | 77.0 | 102.5 | 36 |
| | | | | | | |
| Less than high school | 54.7 | 75.7 | 85.9 | 100.9 | 177.8 | 41 |
| High school diploma or GED | 34.3 | 43.4 | 49.4 | 60.8 | 79.6 | 19 |
| Some college | 21.6 | 26.8 | 29.7 | 34.2 | 54.7 | 20 |
| Bachelor's degree or higher | 10.7 | 14.7 | 17.1 | 20.0 | 30.4 | 24 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Table A-5. Distribution of coefficients of variation (CV) for county-level survey estimates for age and education groups for numeracy proportion at or above Level 3: 2012/2014/2017 PIAAC

| Age and education groups | Percentile | | | | | Number of counties with missing CVs |
|---|---|---|---|---|---|---|
| | 20 | 40 | 50 (Median) | 60 | 80 | |
| 16-24 | 35.7 | 48.2 | 52.8 | 59.8 | 88.7 | 33 |
| 25-34 | 23.4 | 30.0 | 35.9 | 43.3 | 64.7 | 23 |
| 35-44 | 27.4 | 36.0 | 41.4 | 49.7 | 70.4 | 30 |
| 45-54 | 31.9 | 40.4 | 45.1 | 52.2 | 72.7 | 28 |
| 55-64 | 33.5 | 45.9 | 52.4 | 59.2 | 83.1 | 28 |
| 65-74 | 39.4 | 54.5 | 59.9 | 67.9 | 95.1 | 46 |
| Less than high school | 38.4 | 52.2 | 68.6 | 75.4 | 105.4 | 42 |
| High school diploma or GED | 39.8 | 50.2 | 56.6 | 65.4 | 97.8 | 22 |
| Some college | 28.2 | 36.9 | 43.0 | 47.1 | 72.3 | 21 |
| Bachelor's degree or higher | 13.4 | 19.2 | 22.4 | 26.0 | 38.5 | 24 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Table A-6. Distribution of coefficients of variation (CV) for county-level survey estimates for age and education groups for literacy average: 2012/2014/2017 PIAAC

| Age and education groups | Percentile | | | | | Number of counties with missing CVs |
|---|---|---|---|---|---|---|
| | 20 | 40 | 50 (Median) | 60 | 80 | |
| 16-24 | 3.9 | 5.0 | 5.4 | 5.9 | 7.7 | 14 |
| 25-34 | 3.6 | 4.7 | 5.2 | 5.7 | 7.5 | 15 |
| 35-44 | 4.3 | 5.3 | 6.1 | 7.1 | 10.5 | 20 |
| 45-54 | 4.8 | 5.7 | 6.4 | 7.4 | 9.4 | 16 |
| 55-64 | 4.3 | 5.7 | 6.2 | 6.7 | 8.9 | 17 |
| 65-74 | 5.2 | 6.4 | 7.2 | 8.7 | 11.4 | 24 |
| Less than high school | 5.7 | 7.1 | 7.8 | 8.7 | 11.6 | 13 |
| High school diploma or GED | 3.8 | 4.7 | 5.1 | 5.7 | 7.6 | 12 |
| Some college | 3.0 | 3.6 | 4.0 | 4.4 | 5.9 | 17 |
| Bachelor's degree or higher | 2.5 | 3.1 | 3.5 | 3.9 | 5.1 | 18 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.

Table A-7. Distribution of coefficients of variation (CV) for county-level survey estimates for age and education groups for numeracy average: 2012/2014/2017 PIAAC

| Age and education groups | Percentile | | | | | Number of counties with missing CVs |
|---|---|---|---|---|---|---|
| | 20 | 40 | 50 (Median) | 60 | 80 | |
| 16-24 | 4.7 | 5.7 | 6.2 | 6.8 | 8.7 | 14 |
| 25-34 | 4.2 | 4.9 | 5.5 | 6.2 | 7.6 | 15 |
| 35-44 | 4.9 | 5.7 | 6.2 | 6.8 | 9.6 | 20 |
| 45-54 | 5.3 | 6.5 | 7.1 | 7.8 | 9.9 | 16 |
| 55-64 | 4.6 | 5.9 | 6.4 | 7.2 | 9.8 | 17 |
| 65-74 | 5.5 | 7.1 | 7.7 | 8.3 | 11.6 | 24 |
| | | | | | | |
| Less than high school | 5.7 | 7.4 | 8.1 | 8.9 | 11.5 | 13 |
| High school diploma or GED | 4.3 | 5.4 | 5.7 | 6.3 | 8.5 | 12 |
| Some college | 3.3 | 4.2 | 4.5 | 5.0 | 6.3 | 17 |
| Bachelor's degree or higher | 2.9 | 3.5 | 3.9 | 4.3 | 5.3 | 18 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, U.S. Program for the International Assessment of Adult Competencies (PIAAC), 2012/2014/2017.