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FUZZY LINGUISTIC VARIABLES IN MATHEMATICAL ACTIVITIES IN KINDERGARTEN

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ABSTRACT

In this paper, the application of fuzzy logic in mathematical education is viewed from the perspective of pre-school education. The aim of the paper is to give a brief overview of examples from the literature related to fuzzy logic and to point out the presence of fuzzy linguistic variables in the everyday life of a preschool child, as well as the importance of developing and respecting the approximate reasoning of preschool children. Although crisp mathematics requires crisp arguments that characterize our cognitive development, preschoolers start from common argumentation and use vague words. Fuzzy logic is an excellent tool for supporting such approximate reasoning which allows preschoolers to model real-life situations using vague words.

Key words: fuzzy linguistic variables, fuzzy logic, preschool children, mathematical education, mathematical modeling.

INTRODUCTION

Classical mathematical logic deals with formal languages and is dichotomous; there are two values of True and False in it. When the language syntax is specified, that language is called the formal language. Classical logic presents real-world problems as either black or white, while fuzzy logic allows real-world problems to be represented as shades of gray. Propositions in fuzzy logic containing fuzzy linguistic variables. Values of linguistic variables are words of the natural language and they consist of: basic linguistic values (high, low, long, short, deep ...), linguistic modifiers (very, somewhat, medium, strong) and linguistic connectives (and , or not).

Fuzzy logic is widely applied at different levels of education, from kindergarten to college (Garcia-Honrado, 2013), but is also present in educational evolution, laboratory experiments, training education, student academic evaluation scoring, etc. (Ilahi, Widiaty, & Abdullah, 2018). In this paper, the application of fuzzy logic in education is viewed from the perspective of pre-school education. The aim of the paper is, in accordance with the importance of learning through examples highlighted by Polya (2009), to give a brief overview of examples from the literature related to fuzzy logic and to point out the presence of fuzzy linguistic variables in the child's everyday life as well as the importance of developing and respecting the approximate reasoning for preschool children.

The first part of the paper deals with integrated learning as a process of making connections between concepts and experience, allowing the child to analyze one problem from different angles. The second part of the paper presents the stages of mathematical modeling, ie. translating real-life problems into mathematical language. The third part of the paper provides a brief overview of examples from the literature that illustrate fuzzy sets, fuzzy linguistic variables, fuzzy set operations, fuzzy relations, as well as fuzzy logical reasoning.

INTEGRATED LEARNING

The Basics of Preschool Education Program (2018) emphasize that a child is a unique and holistic being, a being of play, committed to learning, and rich in potential and competencies. Therefore, an integrated approach to learning and development is based on the connected experience of the child and not through the isolated content of particular areas. In planned learning situations within the project, children explore natural and physical phenomena while developing different languages and modes of expression. In this way, the child develops mathematical, scientific and technological competences, develops logical-mathematical thinking when solving problems in life-practical situations.

The learning of preschool children is integrated, children learn by looking at one topic from several angles and thus the child learns together by networking knowledge from different fields. A child as a creature of the game, possesses his or her own competences and develops them through learning by exploring real-world problems, translating them into his own language, testing hypotheses, finding solutions, making his own decisions and putting them into action. This approach to learning enables the overall development of the child (Milošević, et. al. 2019).

The math is integrated into real-world problems. Thus, integrated activities allow children to be mathematicians-researchers who recognize and solve tasks in the world around them, translating them into their own language and mathematics language, analyzing and naming phenomena depending on the context, gaining a comprehensive picture of the problem and experimenting. Children must go to their own way to the solution, and along that path they need to make decisions, developing their own language, computing with words, and using the approximate reasoning inherent in preschool children, which indicates the presence of a fuzzy logic in mathematics education of preschoolers. At this age, children also develop crisp reasonig, if, for example, the goal is to adopt the concept of a circle or concepts that are precisely defined in crisp mathematics.

FUZZY LOGIC IN MATHEMATICAL EDUCATION

Crisp Mathematics is based on a well-defined system of axioms and theorems that are very precise and consistent. However, when modeling real-life situations, it is very common to use vague terms that need to be translated into mathematics. Zadeh (1996) exposes a computing with words methodology that uses the theory of fuzzy logic and fuzzy sets. Therefore, fuzzy set theory can be very useful in mathematics education and a tool for modeling spoken language and uncertain situations from everyday life as well as for making conclusions (Voskoglou, 2000).

Problems that children notice in everyday life can be "translated" into mathematical language using mathematical modeling. Voskoglou (2011: 49) states the stages of mathematical modeling:

- „ Analysis of a given real world problem, i.e. understanding the statement and recognizing limitations, restrictions and requirements of the real system.
- Mathematization, i.e. formulation of the real situation in such a way that it will be ready for mathematical treatment, and construction of the model.
- Solution of the model, achieved by proper mathematical manipulation.
- Validation (control) of the model, usually achieved by reproducing through it the behavior of the real system under the conditions existing before the solution of the model (empirical results, special cases, etc.).
- Implementation of the final mathematical results to the real system, i.e. “translation” of the mathematical solution obtained in terms of the corresponding real situation in order to reach the solution of the given real problem.”

FUZZY LINGUISTIC VARIABLES IN KINDERGARTEN

Below is an overview of examples from the literature dealing with fuzzy linguistic variables. The examples that are present in the everyday life of preschool children as well as in mathematical activities integrated with other fields in kindergartens were selected. There are often situations in everyday life that if we want to model mathematically it is necessary to use fuzzy sets theory (Voskoglou, 2018). The following examples are in line with the integrated learning that is experienced in kindergartens, in which real-life situations are mathematically modeled using the concept of fuzzy sets, which is consistent with the ability of children to use approximate quantifiers and approximate reasoning.

Example 1 (Sobrino, 2013:76):
A child of three years may have problems with crisp reasoning, but he or she will certainly have the approximate reasoning ability. Approximate reasoning includes approximate quantifiers such as “fewer” and “more” and the arguments used by the child in decision making are more often implicit than explicit. Thus, a child of this age is fully capable of deciding whether it wants, for example, fewer vegetables and whether or not it wants more cookies.

Example 2 (Garcia-Honrado, 2013: 686):
Let the figures be given. Figures can be grouped according to whether they satisfy the property being defined, they do not satisfy the property, or they partially satisfy it.

Example 3 (Voskoglou, 2000: 1):
Fuzzy sets can be defined when referring to real-life objects like a river, a mountain, a city of a school, etc., for example "long rivers" or "high mountains" of a country, the "young people" of a town, the "tall pupils" of a school, e.t.c..

Example 4 (Sobrino, 2013: 81):
Fuzzy logic deals with fuzzy lingustic variables such as “young” and “tall” that are measurable, but not with variables like “happy” that are not measurable in the same way as the previous two. When “Athletes are tall” is said, then “tall” is a fuzzy linguistic variable, ie a vague predictor whose value depends on the context. The same man may be tall enough to play hockey and cycling but not tall enough to play basketball. Also, the value of the fuzzy linguistic variable “young” depends on whether we are talking about a teacher or chess player.

Example 5 (Garcia-Honrado, 2013: 686):
Preschoolers may be asked to single out non-tall friends, thus introducing the concept of negation.

Example 6 (Garcia-Honrado, 2013: 686):
A pre-school child may be asked to relate two elements that are of the same color or are in the same tone, thereby introducing fuzzy relations.

Example 7 (Janković, 2019: 56):
When talking to preschoolers about money, they talk about the price of the goods, what is cheap and what is expensive. Goods can be classified on the basis of the following requirements: expensive goods, cheap goods. Also, fuzzy relations between objects can be established by bringing in goods with a similar price.

Example 8 (Janković, 2019: 55):
Children in kindergartens measure weight, volume, length. Objects of similar mass can be classified, thus establishing the fuzzy relations between objects. Children can form two sets where one set will contain objects that are heavy (or very heavy) and another set will contain objects that are not heavy, thus introducing the concept of negation. In the activities of measuring mass, length, volume, fuzzy relations and linguistic modifiers and conjunctions are used.

Example 9 (Zadeh, 1996):
Fuzzy logic provides methodology for computing with words. Let *p1* and *p2* be the propositions:
p1 = Carol lives near Mary
p2 = Mary lives near Pat.
If the question arises "How far is Carol from Pat?", conclusion is the proposition *p3*:
p3 = Carol lives not far from Pat.

Example 10 (Sobrino, 2013: 83):
A cyclist can move very slowly, slowly, slowly, very quickly depending on the size of the freewheel and chainring. Let *p1* and *p2* be the propositions:
p1: On a bicycle, with a small freewheel and a large chainring, I go fast.
p2: I selected a very small freewheel and a very large chainring.
Conclusion is the proposition *p3*:
p3: I go very fast.

Ten examples are given that are suitable for the development of approximate reasoning and computing with words of preschool children. Examples include the formation of fuzzy sets, the use of fuzzy linguistic variables, the operations on fuzzy sets and fuzzy relations, as well as fuzzy logical reasoning.

CONCLUSION

This paper reviews examples from the literature relating to fuzzy linguistic variables that can be applied in pre-school education. Examples should help educators to encourage the approximate reasoning of preschool children, encourage children to compute with words, and evaluate the linguistic variable depending on context. Although crisp mathematics requires crisp arguments that characterize our cognitive development, preschool children start from common argumentation and use vague words (Beth, & Piaget, 1961). Fuzzy logic is an excellent tool for supporting such approximate reasoning and allows preschoolers to model real-life situations using vague words.

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