ISBN: 978-989-8704-29-0 © 2021

BASIC MATHEMATICAL MODELLING COMPETENCIES FOR NON-STEM HIGHER EDUCATION

Marina Marchisio, Fabio Roman and Matteo Sacchet
Department of Molecular Biotechnology and Health Sciences, University of Turin
Via Nizza 52, 10126 Torino, Italy

ABSTRACT

The role of mathematical modelling pertains several disciplines, both STEM and non-STEM, and various fields: education, academy, work, everyday and social life. Despite its importance, it is not uncommon to see university students facing difficulties with the use of Mathematics to create models, even when mathematical entities that play a role in facing a problem belong to the study programs of secondary schools, and should thus be familiar also to students without a specific background in Mathematics. Difficulties can arise in various phases of modelling: in the comprehension of the problem, in the translation into mathematical formulas, in the resolution process or even in the interpretation of the results. In this paper, we give an analysis of an online test taken by 75 non-STEM students. The 10 questions of the test focused on specific items in mathematical modelling. During the test, students had to write down the reason why they chose a specific answer. The test allowed us to find and categorize the common errors students make and the phase in which it happens, suggesting actions in order to prevent them. Results show percentages of errors and discuss students' arguments.

KEYWORDS

Higher Education, Mathematical Modelling, Mathematics Education, Modelling Mistakes, Modelling Process, STEM Education

1. INTRODUCTION

Mathematical modelling plays a prominent role in various disciplines: it is important to many categories of students. Solving a problem contextualized in the reality is a task that occurs well beyond Mathematics: for this reason, every student of a scientific course could take advantage of mathematical modelling. These non-mathematical students need modelling the most since they are usually less fond of Mathematics. Furthermore, problems occur and need to be solved even outside STEM disciplines, thus making modelling an important component also in courses not belonging to a scientific area. Since mathematical modelling is a wide concept, it is generally not easy to state for it a precise and unique definition. Mathematical modelling is a cognitive activity in which the situation, the system, the phenomenon or the problem is understood, described in mathematical terms (objects, formulas, language, entities...) and this representation helps to mathematically solve problems, discuss solutions, interpret their results, predict future behaviors and build new models. The depth-complexity of the model depends on the theoretical background owned by the person that creates the model. In this study, we will focus our interest on the *basic* level, affordable by a large majority of secondary or university students since it largely relies on concepts acquired during the mandatory school years. Mathematical modelling consists of several phases. First, the "modeler" must understand the phenomenon and the related textual resources, and this is not always trivial since Mathematics requires a rigor that does not pertain the regular, simpler language. Secondly, a translation from common to mathematical language is necessary: words and sentences have to become formulas and equations, again by taking into account the need to be rigorous. Thirdly, the problem itself has to be solved, by using the proper mathematical tools and instruments close to the knowledge of the solver. The final step is the interpretation of the results. A correct interpretation may lack even at the end of the process, thus resulting in having "the right numbers" but the wrong meaning. Mathematical modelling is not only useful while studying, i.e. at school or at university: its importance also emerges during the working life. Furthermore, in the social and collective life of the present world, citizens need to be conscious in order to contribute to the development of a better society: in this sense, mathematical modelling is also useful to form a conscious citizen. A study on mathematical modelling could be food for thought for teachers, since it invites them to understand where the students encounter difficulties during the modelling process. In this paper we analyze results from a test targeted at finding evidence of difficulties during one or more of the mathematical modelling phases, in order to propose a proper action to overcome them. 75 students who will become non-STEM graduates took the online test remotely from home. Ten questions, which require a specific mathematical modelling skill, alternate with an open answer in which students had to write down their reasoning. The aim of this research is finding the most relevant mistakes that students face while modelling. Students were aware of the research purposes and were invited not to cheat. The paper is organized as follows: Section 2 presents the state of the art and some related works on the topic, while Section 3 is devoted to the research question and methodology. Section 4 contains the results of our research, and finally some concluding remarks are contained in Section 5.

2. STATE OF THE ART AND RELATED WORKS

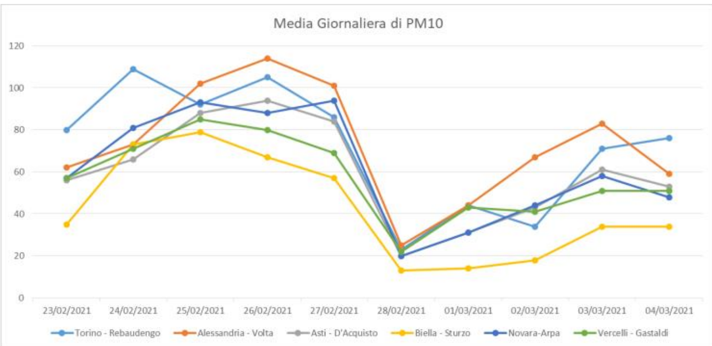
Problem solving and mathematical modelling concern various disciplines. As a general competence, problem solving includes mathematical modelling as a mean to solve problems (Noble, 1982). Nonetheless, the latter one can also imply other activities; it could be simply the description of a system or of a phenomenon, even without a proper solving. Within the problem solving and the mathematical modelling setting, students activate certain cognitive processes: it is therefore natural to ask which mistakes are made more frequently when activating such processes. Several studies have addressed the topic: (Clement et al., 1981) considered how, during the translation into an algebraic notation, a common error of college science students was to reverse the variables in an equation, by writing for example $2A = 3B$ instead of $3A = 2B$. The authors called this error the Reversal. The percentage of Reversal error was 37% over 150 calculus-level students, while 57% over 47 non-science students attending college algebra. Another source of error could even be students' previous knowledge and preconceptions, even when attending a hard STEM like Physics for engineers (Clement, 1981). In (Haines et al., 2001), the authors dealt with distractors in a multiple-choice, real-world setting. Moreover, the authors listed two more steps in the modelling process, namely evaluating the model and refining the model. Apart from the tests, a worker could also need to make a mathematically supported decision among a limited set of choices, with the risk of making a wrong one if the situation is not analyzed with care. (Houston and Neill, 2003) gave suggestions about how to assess modelling skills, basing them on a real experience at the University of Ulster, where students faced real-world problems, both in a third year industrial placement and in a return to university for their final year. This research emphasizes the connection of modelling with the real jobs. The mathematician Felix Klein already started a reform process at the beginning of the 20th century. The main focus was on functional thinking. In the context of the reform of Merano, a utilitarian principle was propagated: enhance our capability of dealing with real life with a mathematical way of thinking (Greefrath and Vorhölter, 2016). In (Schukajlow et al., 2018) a survey of various empirical studies in the field is presented. They state that researchers in modelling have paid more attention to the theoretical foundation of research questions and to using appropriate research methods in their studies. However, the authors suggest enhancing and promoting this kind of research. In (Marchisio et al., 2020) an application to a scientific track belonging to a *soft* STEM discipline is depicted by means of a blended mathematical course for biotechnologists, in which mathematical modelling is enhanced by proper digital resources. On the other hand, (Fissore et al., 2020) involves also students belonging to a course in Strategic Sciences, for which (apart for the name) science does not cover the majority of their program. In this context, the enhancement of mathematical modelling skills passes through students' submission of a worksheet, in which they state, analyze, solve and interpret a problem based on real-life context. In such case, students had to work with the Advanced Computing Environment (ACE) *Maple*. The ACE allows to propose different kinds of interactive resources, useful for learning and modelling (Barana et al., 2020). Moreover, in these courses, the Automatic Assessment System (AAS) *Mobius Assessment* was adopted to evaluate how much a student is prepared by using automatic, randomly generated questions with mathematical content, since the ACE behind the AAS is able to evaluate various types of mathematical expressions, stating in particular whether they are correct or not (Galluzzi et al., 2021). Furthermore, the AAS comes with several tools devoted to the analysis of what the students submitted: close and open-ended answers, even when automatically assessed (Barana et al., 2018), can be revised, and the teacher can change their evaluation manually (Barana et al., 2019). This is important especially when students are given the possibility to try an answer more than once: it is thus possible to see every attempt, gaining precious information about their reasoning. For teachers, analyzing errors allows to remodel the didactics on students' needs, since it can give a strong help in tracking difficulties. This is true for university teachers, but certainly also for secondary school teachers, since it is important for them to train their students according to what the university will require, in a vertical curriculum framework.

3. RESEARCH QUESTION AND METHODOLOGY

In several hard science study courses, students have to deal with Mathematics in order to learn about tools that they are asked to use in relation with the central topics of their courses. Mathematics and mathematical modelling characterize large parts of these courses. It is important to keep in mind that they have been studying the subject since long before entering university, for 12 or 13 years of school (from elementary to high school, in a K-12 like setting). Thus, it is licit to assume that they have already acquired a certain amount of knowledge, competencies and some modelling skills. With regard to Mathematics, the university study program of soft sciences often consists of a simple first-year basic mathematical course. In this research, we want to verify what the students' skills are in solving problems through mathematical modelling, and, in case of poor modelling, why did they not perform better. The research question is: *how is it possible to find and understand the difficulties in basic mathematical modelling, encountered by non-mathematical students?* Our sample consists of 75 students, which are attending the third year of a program devoted to the training of military officers, in a collaboration between the University of Turin and the Italian Army. These students already attended several basic mathematical courses, from school to former university years, even with different backgrounds since they come from all over the country and there are even some foreign students. They just attended, almost completely online, a short modelling-oriented course with a duration of 21 hours (1 ECTS), for which they are going to take the exam, too. We constructed an online test consisting of ten questions, in which the prerequisites and competences required were lower than those needed for these students, since the skills acquired in secondary school were sufficient for the task. They read as:

- Q1: *By law, for every 15 square meters of living space, there have to be 3 square meters of windows, in order to enlighten the environment. Write a relation between S (for the living space) and F (for the windows' surface) which represents the described* situation.
- Q2: *A runner is doing a 2-kilometers race on a 400 meters long track; at the end of every lap, the time elapsed is detected. After a distance of 400 meters, 2 minutes have elapsed; after 800 meters, 4 minutes; after 1200 meters, 6 minutes; after 1600 meters, 8 minutes; after 2000 meters, 10 minutes. Write a formula between T (for time) and D (for distance) allowing to predict distance as a function of time. [data were shown in a table]*
- Q3: *In a clothing store, every five clothes bought, two gadgets are given for free. Write a formula between V (for the number of clothes) and G (for the number of gadgets) which represents the described relation.*
- Q4: *We want to measure the height of a building, given that an observer, placed 25 meters far from its base, sees its top under an angle of 67 degrees. Which geometrical object is necessary in order to compute the height of the building? [answer: triangle]*
- Q5: *During the emergency due to the SARS-CoV-2 pandemic, with the level of restrictions orange, from the municipalities constituted by less than 5,000 inhabitants it is possible to move without valid reason up to 30 kilometers from home. Which geometrical object could approximate the area in which a citizen residing in a municipality constituted by 3,500 inhabitants can move without valid reason? [answer: circle]*
- Q6: *While performing a goal kick, the goalkeeper kicks the ball in order to resume playing. Which geometrical object can describe the trajectory of the ball? [answer: parabola]*

- Q7: *A university student obtained the following grades: 24 for an exam of 6 ECTS, 30 for an exam of 3 ECTS, 25 for an exam of 12 ECTS, 28 for an exam of 6 ECTS. Which grade must the student obtain for the last exam of 9 credits, in order to achieve a weighted average of 28? [data were shown in a table]*
- Q8: *Figure 1 depicts on the left the observations of the pollution agent PM10 (mg/m3) in some provinces of Piedmont during the last week of February and the first days of March 2021.*
  - Which province has the highest number of days below the 40 mg/m3 threshold?
  - In which day the sum of the PM10 values, in the various provinces, is minimum?
  - In which day the sum of the PM10 values, in the various provinces, is maximum?
  - Can we state that a province performed always worse than another one in terms of pollution? Select all the right choices between:
  (i) Alessandria w.r.t. Asti; (ii) Torino w.r.t. Alessandria; (iii) Asti w.r.t. Novara;
  (iv) Novara w.r.t. Biella; (v) Biella w.r.t. Novara; (vi) Torino w.r.t Biella;
  (vii) Alessandria w.r.t. Vercelli; (viii) Novara w.r.t. Torino; (ix) Alessandria w.r.t. Biella;
  (x) Asti w.r.t. Biella.
- Q9: *5 printers print 5 copies of a book in 5 minutes. How much it would take 100 printers to print 100 copies of the same book?*
- Q10: *Figure 1 contains on the right a data frame about people detained in jail relative to the Italian regions. Select all the right choices between:*
  (i) Toscana has the highest number of detained which are foreign citizens ("di cui stranieri");
  (ii) Lombardia has the highest number of detained people ("detenuti presenti – totale");
  (iii) It exists at least one region with more women detained ("donne") than foreign citizens;
  (iv) It exists at least a region with less detained people w.r.t. capacity ("capienza regolamentare");
  (v) Regions having more jails ("numero istituti") have also higher capacity;
  (vi) The percentage of women is about 4% of the total number of detained people;
  (vii) At least ten regions have overcrowded jails.



Detenuti presenti e capienza regolamentare degli istituti penitenziari per regione di detenzione
Situazione al 28 febbraio 2021

Regione di detenzione	Numero Istituti	Capienza Regolamentare (*)	Detenuti Presenti		di cui Stranieri	Detenuti presenti in semilibertà (**)	
			Totale	Donne		Totale	Stranieri
ABRUZZO	8	1.658	**1.622**	48	281	12	3
BASILICATA	3	401	**390**	0	44	1	0
CALABRIA	12	2.704	**2.520**	60	448	27	0
CAMPANIA	15	6.054	**6.519**	317	867	149	0
EMILIA ROMAGNA	10	2.993	**3.270**	146	1.541	56	12
FRIULI VENEZIA GIULIA	5	467	**637**	20	264	10	4
LAZIO	14	5.230	**5.692**	375	2.131	45	3
LIGURIA	6	1.120	**1.318**	71	711	24	11
LOMBARDIA	18	6.143	**7.766**	370	3.499	80	13
MARCHE	6	846	**857**	20	319	20	5
MOLISE	3	271	**344**	0	100	6	1
PIEMONTE	13	3.918	**4.144**	138	1.613	67	18
PUGLIA	11	2.686	**3.538**	182	509	72	2
SARDEGNA	10	2.609	**2.019**	33	475	29	0
SICILIA	23	6.447	**5.793**	191	1.026	91	4
TOSCANA	16	3.098	**3.124**	87	1.532	85	25
TRENTINO ALTO ADIGE	2	498	**379**	18	235	3	2
UMBRIA	4	1.324	**1.301**	43	390	4	0
VALLE D'AOSTA	1	177	**172**	0	100	1	0
VENETO	9	1.907	**2.292**	133	1.221	24	7
Totale	**189**	**50.551**	**53.697**	**2.252**	**17.306**	**806**	**110**

Figure 1. Data relative to questions Q8 and Q10

This allowed us to consider all of them at the same level, regardless of how much Mathematics they did at university during the first two years, in which they were in another city with different teachers. The test was mandatory, but it did not grade students, in order to have them answer calmly, without distractions and avoiding cheating behaviors. Since the test has not been designed as summative, it possesses a formative significance, which extends its usefulness also beyond the scope of our research. We delivered the test electronically, by means of the AAS *Mobius Assessment*, which collected all the data we needed for our analyses. The response areas required answers in different forms: formulas, text, numbers, and check boxes, in order to allow the exploration of various kinds of reasoning; in particular, they were not limited to multiple choices (Wiggins, 1993; Barana et al., 2015). For every question, they had two attempts: on one side, this reduced the risk of accidental errors, but on the other side, this allowed to detect cases in which the reasoning leading to the first attempt was conceptually wrong, and where the second time the student answered

ISBN: 978-989-8704-29-0 © 2021

correctly by changing the reasoning. This happened possibly thanks to the information given by the error made in the first instance, in a simple example of “learning through errors” (Scriven, 1967). Halfway between two questions, the students had to write in detail, in an open answer setting, why they gave a certain answer, and how they reasoned in order to choose it. The test referred to all knowledge, competencies and abilities in Mathematics according to the Italian educational system: Numbers (arithmetic), Space and Figures (geometry), Relations and Functions (logic, algebra and analysis), Data and Forecasts (probability and statistics). After the test, we analyzed all the results with descriptive statistics and discussed the reasoning process, especially where errors occurred.

4. RESULTS

As a first result, it is interesting to note how many students would have passed the test, if the test had been graded. Even with a generous evaluation, for example by giving full score to only partially corrected answers (some questions were multipart), and setting the passing score to 5 out of 10 (while usually more than the 50% is required), only 76% of students would have obtained a positive outcome. This means that a quarter of them was not capable of developing basic mathematical models, although they all graduate, and they are going to obtain a Bachelor’s Degree at the end of the present year. Entering into the detail of the 10 questions, almost all the students answered correctly to the three geometric questions (Q4, Q5, Q6), in which they were asked to apply elementary shapes to reality. Although the number of correct answers is very high (98.6% for Q4, 93.0% for Q5, 97.2% for Q6), some remarks still need to be made. For example, almost half of the students was overabundant in the reasoning about Q4: most of them brought up extensive trigonometric computations, for instance, while the question simply asked them to determine the geometrical object, not to compute the height of the building. In real life, the completion of a task properly and as fast as possible is a key success factor, so it would have been better had unnecessary aspects been put aside. In Q6, almost half of the students answered reasoning by comparison (43%): they are military, so they know very well the parabola as the trajectory of a projectile, thus being able to compare it explicitly to the trajectory of a ball (with standard modelling assumptions, even if not elicited). If on one side this is undoubtedly positive, since it shows their capabilities in generalizing concepts learned for specific contexts, on the other side we could argue whether they would have been able to still answer correctly without this knowledge; this can be tested in the future with students belonging to other courses.

Speaking of weaknesses, the question with the lowest performance was the problem on impossibility inside a certain context, namely Q7. Although not specified in the text, it was implicit that the highest grade was 30, since the Italian university system gives grades in thirtieths. By doing computations, it results that the value 34 arithmetically solves the problem: without solving in detail, we can directly compute the total number of credits as 36, and:

$$24\cdot(6/36) + 30\cdot(3/36) + 25\cdot(12/36) + 28\cdot(6/36) + \mathbf{34}\cdot(9/36) = 28$$

However, a student cannot obtain 34 as a grade since the maximum is 30. Nonetheless, more than half of the students did not take that into account, and answered 34: by excluding those answering 34 at the first attempt and correctly at the second one, we still have 66% of students. The majority of them (36%) did not even refer in the reasoning to the actual impossibility of reaching 28 as the average; the overall percentage of students forgetting the context is even higher (51%). Only a minority of them (30%) mentioned (directly or indirectly, e.g. answering – again wrongly, but at least respecting the context – 30 the second time) how it was impossible to reach 28, possibly after noticing the wrongness of the answer 34. Along with other errors, this resulted in only 27.1% of students answering correctly, even if they had two attempts.

About the other questions, Q1-Q3 resulted in middling percentages (70.4% for Q1, 84.6% for Q2, 64.2% for Q3), but two main errors arose. Some of them wrongly input the answer in the AAS (which is flexible, but especially when it comes to formulas, it requires some precision in the syntax), or exchanged the role of the two variables, by writing a wrong relation of proportionality, like the error described in (Clement et al., 1981). Last but not least, the three final questions Q8-Q10, concerning interpretation, resulted in variable percentages (77.6% for Q8, 90.4% for Q9, 50.8% for Q10), although for example in Q8 we considered as correct some partial answers. It is noteworthy to remark how in Q10 60% of the students gave a reasoning that was very short, such as "data comparison" or "analysis of values", or a bit longer but essentially without giving real information on how they reasoned. Table 1 shows the global situation: this is not necessarily an exhaustive classification, but it takes into account the main peculiarities we found in students' answers; more than one of them can refer to the same question for the same student, so the sums in the columns can exceed 100%. Note that following a correct approach did not result necessarily in answering correctly, as the example of Q7 shows, where the quota interpreting correctly the question exceeded the quota writing a correct answer, due to other errors. Furthermore, some students did not attempt one or more questions, and this is why every question totalized a number of answers lower than 75.

Table 1. Percentages of correctness and peculiarities, relative to the ten questions administered

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Total answers	71	65	67	70	71	72	70	58	73	61
Correct answers	50	55	43	69	66	70	19	45	66	31
Wrong answers	21	10	24	1	5	2	51	13	7	30
Correct answers (%)	70.42	84.62	64.18	98.57	92.96	97.22	27.14	77.59	90.41	50.82
Errors (%)										
Forgotten datum	1.41		1.49		1.41		1.43			
Input error	28.17	23.08	13.43							
Forgotten context							51.43			
Extra variables	1.41	4.62								
Reading error			2.99						10.96	
Exchanged proportionality	19.72	1.54	31.34							
Inverted proportionality	1.41									
Inadequate formula		1.54								
Correct approaches (%)										
Numerical example	8.45	6.15	14.93							
Correct interpretation	50.70	66.15	38.81	34.29	77.46	47.22	45.71	17.24	84.93	14.75
Comparison						43.06				
Overabundant reasoning				45.71	5.63	1.39				
Change between attempts	11.27	4.62	13.43		7.04	1.39	27.14		10.96	
Other (%)										
Incomplete reasoning	7.04	4.62	8.96	14.29	7.04	6.94	1.43	10.34	2.74	37.70
Absent reasoning	2.82		1.49	1.43	1.41		1.43		1.37	
Inconsistent reasoning			2.99		4.23					
Answer not making sense	5.63	9.23	8.96	1.43	4.23	1.39	2.86			
Answer only partially correct								70.69		47.54
Computation error	2.82						2.86		2.74	

4.1 Categorization of Errors

We now want to categorize the errors in relation to the phases of mathematical modelling we presented in Section 1. First, it has to be noted how it is not always possible to precisely place an error within such a categorization: sometimes, for example due to steps that are not sufficiently explicated, it is hard to precisely determine where the mistake lies. Nonetheless, in certain cases, the form of the very answer (formula, number, etc.) is sufficient to give proper suggestions. A valid categorization can be given as:

- Category 0, technical difficulties: this includes errors due to wrong input in the AAS, which occurred not unfrequently in Q1-Q3.
- Category 1, misunderstanding of the text: the error lies in a not correct understanding of the problem, in a wrong identification or interpretation of significant data, or in relating them wrongly.
- Category 2, translation from common to mathematical language: the error lies in the wrong writing of formulas, equations, or other mathematical tools representing what the text states.
- Category 3, solving process: the error lies in the misuse of the mathematical concepts needed, in the wrong formulation of strategies, in the wrong consideration of formal tools, or in the wrong development of procedures and computations.

- Category 4, interpretation of results: the error lies in not correctly contextualizing the results in the environment the problem is defined in.

Table 2. Categorization of the errors depicted in Table 1

Error	Category	Error	Category
Forgotten datum	1	Inadequate formula	2
Input error	0	Incomplete reasoning	3
Forgotten context	4	Absent reasoning	
Extra variables	2	Inconsistent reasoning	3
Reading error	1	Answer not making sense	
Exchanged proportionality	2	Answer only partially correct	
Inverted proportionality	2	Computation error	3

The types of error depicted in Table 1, along with the items referred to as “Other” (which are often indicative of incorrectness too), can be categorized as in Table 2. Of course, it is not possible to categorize a reasoning that is completely absent, or an answer that does not make sense at all, while errors in only partially correct answers can depend on various categories. We see how the errors made by the students cover every category: if, as already stated, a Category 0 error occurred in Q1-Q3, a Category 1 error occurred in Q9, where the repetition of the number 5 three times could have triggered a similar repetition for the number 100, while in fact the answer was still 5. Category 2 errors were detected again in Q1-Q3, where students tend to get confused: for example, in Q1 the right relation was $S/15 = F/3$, or equivalently $3S = 15F$, but some of them wrote instead $15S = 3F$, possibly because 15 referred to the living space and 3 to the windows (Clement et al., 1981). Category 3 errors occurred in Q10, causing a large number of answers which were only partially correct, but also in Q4, involving even students answering correctly to the question (but not motivating their answer in a fully proper way). The latter occurrences are noteworthy, since they show how some errors can be made independently from the correctness of the answer: maybe the student is able to identify the right item, but not to give a precise motivation about their own choice. In all likelihood, this does not only mean that the student is unable to give a proper explanation, but it can also mean that he or she does not even know why that answer is the correct one. Possibly the student was helped by intuition, which is of course very useful in Mathematics, but should be accompanied by capabilities related to knowledge and rigor. Finally, a Category 4 error occurred often in Q7 where, as already stated, the arithmetical result was unfeasible due to a practical constraint: it is likely that the students simply proceeded mechanically, not thinking about the meaning of the quantities involved in the real context. A reflection going beyond this categorization, but interesting as well, concerns the capabilities of the students to argue. An argumentation should comment and justify with proper means the choice of the solving strategy, the fundamental steps of the executive process, and the consistency of the results with regard to the context of the problem. A case could occur when the students express themselves in a way that is not objectively very clear, but for which their teacher is able to understand them, because he or she followed their learning path. This is a double-edged sword: if on one hand this can be indicative of a good understatement between the two sides of the cathedra, on the other hand this makes their argumentation of scarce use outside the assessment of their teacher. The latter aspect collides with the aim of teaching them Mathematics and mathematical modelling in order to use it beyond educational contexts.

5. CONCLUSION

This study allowed us to answer to the research question of Section 3: we were able to find specific difficulties students have in basic mathematical modelling, and to quantify them, by also taking into account their status as non-mathematicians. The sample was quite homogeneous, due to the origin of students from all over the country and the presence of some foreign students. Furthermore, the difficulties were numerous and of various kinds, thus configuring a situation which generally saw students being not too confident about mathematical modelling. In order to overcome these difficulties, it would be proper, in secondary schools (within a vertical curriculum framework) but also at university, especially during the first years, to devise actions with the aim of developing problem solving competences through mathematical modelling. As future work, it could be devised to perform starting and final assessment for a comparison of pre- and post- course status to evaluate teaching approaches and propose new ways for teaching mathematical modeling. While working on this, an idea is to consider also metacognitive and affective measures, since emotions, beliefs or motivation are worthy to be investigated further (Schukajlow et al., 2018). In addition, it would be possible to focus on the development of mathematical modelling and pedagogical skills of pre-service and in-service teachers, through country-wide actions that involve distance education and math teachers' training (Barana et al., 2017).

REFERENCES

Barana, A. et al, 2015. Automated Assessment in Mathematics. *Proceedings of IEEE 39th COMPSAC Conference*. Taichung, Taiwan, pp. 670-671.

Barana, A. et al, 2017. Teacher training: a model for introducing innovative digital Methodologies for learning Mathematics. *Proceedings of 3rd International Conference on Higher Education Advances (HEAd'17)*. Valencia, Spain, pp. 608-616.

Barana A. et al, 2018. Model of automatic assessment and interactive feedback. *Proceedings of IEEE 42nd COMPSAC Conference*. Tokyo, Japan, pp. 1016-1025.

Barana, A. et al, 2019. Advantages of Using Automatic Formative Assessment for Learning Mathematics. *Proceedings of the 2018 International Technology Enhanced Assessment Conference (TEA2018)*. Amsterdam, The Netherlands, pp. 180-198.

Barana, A. et al, 2020. The creation of animated graphs to develop computational thinking and support STEM education. *In: Gerhard J., Kotsireas I. (Eds.), Maple Conference 2019: Maple in Mathematics Education and Research*, Cham, Springer, pp. 189-204.

Clement, J. et al, 1981. Translation Difficulties in Learning Mathematics. *American Mathematical Monthly*, Vol. 88, No. 4, pp. 286-290.

Clement, J. 1981. Students' preconceptions in introductory mechanics. *American Journal of physics*, Vol. 5, No. 1, pp. 66-71.

Fissore, C. et al, 2020. Development of Problem Solving Skills with Maple in Higher Education. *Proceedings of Maple Conference 2020*, Waterloo, Canada (virtual), accepted.

Galluzzi, F. et al, 2021. Mathematics in higher education: a transition from blended to online learning in pandemic times. *Proceedings of IEEE 45th COMPSAC Conference*. Madrid, Spain (virtual), accepted.

Greefrath, G. and Vorhölter, K., 2016. Teaching and Learning Mathematical Modelling: Approaches and Developments from German Speaking Countries. *In the homonym book, part of the ICME-13 Topical Surveys book series (ICME13TS)*, pp. 1-42.

Haines, C. et al, 2001. Understanding Students' Modelling Skills. *In: Matos, J. F. et al (Eds.), Modelling and Mathematics Education,* Woodhead Publishing, pp. 366-380.

Houston, K. and Neill, N., 2003. Assessing Modelling Skills. *In: Lamon, S. J. et al (Eds.), Mathematical Modelling*, Woodhead Publishing, pp. 155-164.

Marchisio, M. et al, 2020. Teaching Mathematic in Scientific Bachelor Degrees Using a Blended Approach. *Proceedings of IEEE 44th COMPSAC Conference*. Madrid, Spain (virtual), pp. 190-195.

Noble, R.D., 1982. Mathematical modeling in the context of problem solving. *Mathematical modelling,* Vol. 3, pp. 215-219.

Schukajlow, S. et al, 2018. Empirical research on teaching and learning of mathematical modeling: a survey on the current state-of-the-art. *ZDM Mathematics Education*, Vol. 50, pp. 5-18.

Scriven, M. 1967. The Methodology of Evaluation. *In: Gagné R. M., Tyler R. W., Scriven M. (Eds.), American Educational Research Association Monograph Series on Curriculum Evaluation, vol. 1: Perspectives of Curriculum Evaluation*. Rand McNally, Chicago, USA (Illinois).

Wiggins, G.P., 1993. *Assessing student performance. Exploring the purposes and limits of testing*. Jossey-Bass Publishers, San Francisco, USA (California).