

Intensive Intervention Practice Guide: Literacy Supports for Math Content Instruction for Students With Reading and Language Difficulty

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What Is It?

Recent research (Joyner & Wagner, 2020) notes that students with an existing math disability are twice as likely to have a reading disability. This additional impact in reading affects students' academic and functional skills. As early as prekindergarten, students with comorbid reading and math difficulties experience more severe math difficulty on numeracy measures than individuals at risk for math difficulty alone (Barnes et al., 2020). Cognitively, research has shown that students with comorbid reading and math difficulty do not perform as well on measures of attention, visual-spatial problem-solving, long-term memory, short-term memory, working memory, and executive functioning, when compared to their peers who only have a math difficulty (Peng & Fuchs, 2016; Swanson & Jerman, 2006; Swanson et al., 2009).

In math classes specifically, students with both reading and math difficulty perform lower in foundational numerical competencies and calculations than students with only math difficulty, but these differences are small (Cirino et al., 2015). Students with reading and math difficulty also experience higher levels of difficulty in word-problem solving and math language compared to students with math difficulty alone (Cirino et al., 2015; Powell et al., 2009). As word-problems and math language (in text and oral instruction) are used throughout K-12 math, students with comorbid reading and math difficulty may need additional support beyond what may be traditionally offered in a math class to students with math difficulties or disabilities.

This comorbid relationship between math and reading disabilities implies that a student with a math disability may face additional literacy barriers to accessing math content. A reading disability in math class could impact how a student understands math vocabulary, accesses words problems, and comprehends instructions. Educators should be armed with additional tools that support students with both reading and math difficulty in their math classrooms.

The purpose of this practice guide is to provide three concrete strategies that can be used to support students with comorbid reading and math difficulty. By using schemas to support problem solving, focusing intentionally on math language, and creating opportunities for math writing, math educators, from elementary to high school, can better support students with reading needs in the math classroom.

Strategy 1: Schema Instruction

Schemas identify consistent features and categories across word problems. All single and multi-step word problems can be broken into additive and multiplicative schemas. Each single step word problem fits into one schema category based on consistent features across problems. Alternatively, each step in a multi-step word problem fits into a schema category based on the features in that step. The three additive schema types include total, difference, and change problems (Fuchs et al., 2021; Powell et al., 2020). The three multiplicative schemas include equal groups, compare, and proportion/ratio problems (Alghamdi et al., 2020; Jitendra et al., 2016; Xin, 2008).

Strategy 2: Math Language

Academic language, which differs by discipline, includes the vocabulary, grammatical structures, and linguistic features that students must understand and use to acquire content knowledge in content-area subjects (Cummins, 2000). In math specifically, language is connected with symbolic representations (i.e., -, +, /, x representing: subtraction, addition, division, and multiplication) that support students' procedural and conceptual math knowledge (Capraro & Joffrion, 2006; Powell & Driver, 2015).

Strategy 3: Math Writing

Writing in math requires a student to have both math knowledge and writing skills. This guide defines math writing as when students perform writing of any kind that is related to math (Powell et al., 2017). As such, writing in math can address a variety of math topics such as operations, geometry, and algebra, and exist in different formats including formal assessments and informal student journals (Powell & Hebert, 2016; Powell et al., 2017). Table 1 explains the four main types and purposes of math writing found in instructional settings (Casa et al., 2016). Educators can first refer to informative or explanatory, as they are the most used types (Powell et al., 2017).

For Whom Is It Intended?

Although literacy supports embedded within math instruction can benefit all students, they are particularly helpful for students who have difficulty with reading and writing, who could include students with learning disabilities, including dyslexia, and students with speech and language impairments. The recommended schema instruction, math language, and math writing supports in this guide primarily discuss supporting address monolingual English speakers with reading and writing needs. Yet many of the recommendations align with the recommendations for emergent bilingual students (Arizmendi et al., 2021). As such, this guide is intended for educators of math, special education, English language acquisition, and speech language therapists who may support students in math. Further, the supports in this guide are intended to be used in math instruction across grade levels.

It should be noted that embedded literacy supports are simply a starting place for practitioners as one way to support students with reading difficulty in mathematics. Educators should consult with school or district personnel (i.e., special education teachers, speech language therapists, or bilingual specialists) regarding additional supports for students with more significant reading impacts.

How Does It Work?

Schema Instruction

Schema instruction paired with attack strategies has led to positive word problem solving outcomes (Fuchs et al., 2021; Powell et al., 2020). During schema instruction, teachers and students use a meta-cognitive attack strategy to work through the word problem (Powell et al., 2020). When using attack strategies, the exact mnemonic is not important; instead, knowing what steps the mnemonic requires is essential. Examples of mnemonics include DOTS, UPS-Check, TONS, and RUN (Fuchs et al., 2021; Griffin et al., 2018; Montague, 2008; Powell et al., 2020). In the RUN attack strategy, students first *Read* the problem. Then, the students *Underline* the relevant information. Last, the students *Name* the schema type. After naming the problem type, students then use the available information on the schema type to fill in the relevant information into the schema equation and solve for the unknown (Fuchs et al., 2021; Powell et al., 2020).

Along with explicitly teaching students how to use an attack strategy, students must be explicitly taught each schema. Table 2 lays out all the schemas with the equations and example word problems. The additive schemas include total, difference, and change schemas (Fuchs et al.,

2021; Powell et al., 2020). In total problems, parts are combined for a total. The unknown can be a part or the total ($P1 + P2 = T$). Students can be prompted to determine if a problem is a total problem with the question: “Are parts put together for a total?” In difference problems, a greater amount and a lesser amount are compared to find the difference. The unknown can be the greater amount, the lesser amount, or the difference ($G - L = D$). Difference problems pair with the prompt question: “Are amounts compared for a difference?” In change problems, a starting amount changes to increase or decrease over time and results in a new end amount ($ST +/- C = E$). For change problems, students can be prompted with the question: “Does an amount increase or decrease?” (Fuchs et al., 2021; Powell et al., 2020).

The three multiplicative schemas include equal groups, compare, and proportion/ratio problems. In equal group problems, multiple equivalent groups of items make up a quantity of items. The unknown can be the groups, number, or product ($G \times N = P$). Students can be prompted to determine if a problem is an equal groups problem with the question: “Are there groups with an equal number in each group?” In compare problems, a reference set is compared a number of times. The unknown can be the set, the times the set has been compared, or the product ($S \times T = P$). The prompt for comparison problems is the question: “Is a set compared a number of times?” (Griffin et al., 2018; Xin, 2008). In proportion/ratio problems, two proportional relationships are examined. For proportion/ratio problems, the unknown can be a part of either of the proportions/ratios ($\frac{Q1}{Q2} = \frac{Q3}{Q4}$). To determine if a problem is a proportion/ratio problem, students can be prompted with the question: Are there relationships among quantities? (Jitendra et al., 2016; Powell & Fuchs, 2018).

Math Language

As educators, explicitly teaching the link between math language to math concepts can help students have a better overall understanding of math content (Powell & Driver, 2015). Math language can be broken down into four categories (Monroe & Panchyshyn, 1995): (a) technical words with one definition or purpose (e.g., “pentagon”), (b) subtechnical words that carry multiple meanings (e.g., “degrees”), (c) words that are used in everyday language that are meaningful in math language (e.g., “product”), and (d) symbolic words where amounts can be represented with symbols or abstract numerals (e.g., “times” as “x”).

Math language is not practiced in isolation, but rather incorporated into daily math activities. In the example of word problems, research shows that when students learn the vocabulary in context instead of as key words, students understand the word problem more and the instruction is more effective (Fuchs et al., 2021). Additionally, when teachers directly explain the linguistic connections and construction of a word problem (e.g., creating parallel structure of word

problems) within the context of the problem, it helps to support students with utilizing the correct math operation associated with a math vocabulary term (Koning et al., 2017). In both studies, teachers did not take the math language out of the math problem, but rather taught the math language within the problem. When utilizing math language, teaching students to seek out key words to the isolation of syntactic context of the word problems is inappropriate and can often lead students to make more errors down the line in their math careers (Karp et al., 2019). Instead, math language should be incorporated into the context of the specific problem to help students acquire the right vocabulary to support them in and outside of the classroom.

Math Writing

A recent survey of teachers found that the majority used math writing at least monthly but only half of them reported teaching how to do it (Powell et al., 2021). Both the National Council of Teachers of Mathematics and the Common Core Standards require that students be able to effectively communicate their mathematical ideas, including through the written word (National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). As a result, writing in response to math instruction has become a more frequent feature of routinely administered assessments such as the Smarter Balanced Assessment (Hebert et al., 2019).

When writing is used as a way to assess student's math understanding, Powell and Hebert (2016) argue that three assumptions are made:

1. Students have the required writing skills for the given task.
2. Students can transfer their writing skills to communicate their mathematical thinking.
3. Students' math writing is therefore an accurate portrayal of their math knowledge.

However, the authors acknowledge that students with reading and writing difficulty may not be able to meet these assumptions. As such, explicit instruction should be used to help students transfer their general writing skills to math writing tasks (Hebert et al., 2019).

Self-regulated Strategy Development (SRSD) is an explicit and systematic approach that has shown to increase student's writing abilities and content knowledge (Hughes et al., 2019; Hughes & Lee, 2020; Kiuahara et al., 2020). This approach has six steps: (a) developing background knowledge, (b) discussing the strategy, (c) modeling the strategy, (d) memorizing the strategy, (e) supporting the strategy, and (f) independent performance of the strategy.

One recent study successfully used SRSD to improve argumentative writing about fractions for fourth-, fifth-, and sixth-grade students with and at-risk for learning disabilities (Kiuahara et al., 2020). Students were given an explicit framework for crafting their written response so that it restated the question, provided accurate reasoning, addressed counter arguments, and finished with a concluding thought; gains were demonstrated in both math knowledge and reasoning. Another recent study used SRSD to successfully improve middle school students' math writing through a six-step process: (a) understanding the problem, (b) making a plan to solve the problem, (c) drawing a visual representation of the problem, (d) explaining the problem-solving process and reasoning, (e) concluding by stating the answer, and (f) checking all components of the work (Hughes et al., 2019). Finally, Hughes and Lee (2020) used SRSD to help sixth-grade students improve their math writing and understanding through the PRISM-Check strategy. PRISM stands for *Problem, Representation, I do, State, say, share, My answer*, followed by a final *Check*. Following the SRSD approach, students were first taught to understand why math writing was important as part of developing background knowledge. The PRISM-Check strategy was then discussed, modeled, memorized, and practiced. Students who used this approach wrote responses that were longer, more structured, and contained more math complex language.

Other research has found positive effects for providing students with on-going informal opportunities for writing about math. In particular, the use of journal writing can increase students' math engagement and reasoning, including for students with disabilities (Powell et al., 2017). Teachers can vary the types of prompts that students respond to in their journal as a form of formative assessment of students' mathematical knowledge.

How Can Families Support Implementation?

Although these strategies were designed in the context of the classroom, this section provides some strategies and activities for families to use to engage with their child(ren) in math at home. These strategies and activities can be shared at meetings, conferences, back-to-school nights, or in classroom newsletters.

When children are young, families can facilitate early numeracy growth (see Table 3). Outside of early numeracy development, early research suggests that families can support the development of a child's math language and early numeracy skills through math picture books. Purpura and colleagues (2021) developed a set of family-implemented math picture books that contained embedded mathematical language (e.g., terms such as *more than*, *different*, or *least*) within the text along with pictures for the children to interact (i.e., count objects) with. Children who participated in the picture book intervention significantly outperformed children who were not exposed to the math picture books on a test of math language (*effect size* = 0.50) and early

numeracy (*effect size* = 0.22) following the intervention. Eight weeks later, children exposed to the math books continued to substantively outperform children who did not receive the picture book intervention on the early numeracy test (*effect size* = 0.23) and the math language test (*effect size* = 0.25), although the math language test was no longer significant.

Although families can foster math concepts outside of school, such as early numeracy, as children continue to develop their math knowledge, families may not possess the skills necessary to continue to support their child's math development (Ginsburg et al., 2008). Furthermore, if families carry anxiety around math, their personal experiences could impact their child's engagement with math (Maloney et al., 2015). To support families, schools and/or educators can provide resources such as family math nights or supply video tutorials with homework. When considering how to support families, educators should consider any relative cultural implications of a family related to their math background and home language prior to making recommendations. Overall, there are many ways for families to engage with their child(ren) in math conversations at home as well as support with math instruction.

How Adequate Is the Research Knowledge Base?

Overall, the research knowledge base for literacy supports for math instruction is a growing field, and much learning remains to be done. Of the three strategies listed in this guide, schema instruction has the strongest research base. In fact, schema instruction appears to be the most studied and most effective instructional method for word problem solving for students with math difficulty (Lein et al., 2020). Most studies of schema instruction began in the early 2000s (Lein et al., 2020; Peltier & Vannest, 2017; Peltier et al., 2018). Research more commonly focuses on additive schema instruction than multiplicative schema instruction (Peltier & Vannest, 2017; Peltier et al., 2018). The research knowledge base for additive schema instruction demonstrates a strong foundation for the effectiveness of additive schema instruction as a method for improving word problem solving for students with math difficulty (Peltier & Vannest, 2017). Alternatively, a limited base of knowledge currently exists on multiplicative schema instruction, but recent publications suggest an expanding research base (Alghamdi et al., 2020; Griffin et al., 2018; Peltier & Vannest, 2017; Peltier et al., 2018).

In contrast, there is limited, empirical research regarding the evaluation of math language and what language components influence, and can improve, math language acquisition. Although reading ability affects word problems and verbal calculations (Jordan et al., 1995) and phonological processing influences math achievement (Fuchs et al., 2005; Vukovic & Lesaux, 2013), there

are no vocabulary interventions that have been directly tested on math outcomes in an experimental study. However, research has demonstrated effective vocabulary instructional methods that improve reading comprehension that math teachers may consider using (Wright & Cervetti, 2017).

For math writing, adaptations of SRSD have demonstrated positive effects on math writing in four controlled studies (Kiuvara et al., 2020; Hebert et al., 2019; Hughes et al., 2019; Hughes & Lee, 2020). A meta-analysis of other math writing instructional practices demonstrate small but positive effects ($k = 0.32$) in well-controlled studies (Graham et al., 2020). These instructional practices include explaining how to solve certain math problems, developing an oral argument and writing an accompanying explanation, and paraphrasing key terms and other information (Powell et al., 2017).

How Practical Is It?

As students with math difficulty are more likely to also have reading difficulty (Joyner & Wagner, 2020), it is important that literacy instruction is incorporated into math instruction so that students with both reading and math difficulty can be supported. All three strategies incorporated into this practice guide can be implemented in the K-12 general education or special education setting through one-on-one, small group, or whole group instruction. Additionally, all three practices can be taught by a special education teacher, general education teacher, or support specialist or modeled by a literacy coach.

Schema instruction specifically can be utilized for both additive and multiplicative word problems with single and multiple steps to improve word problem solving accuracy (Alghamdi et al., 2020; Fuchs et al., 2021; Griffin et al., 2018; Powell et al., 2020). Students with and without disabilities make growth on word problem solving accuracy after exposure to schema instruction (Peltier & Vannest, 2017).

As 95% of students identified with a math disability prior to 5th grade continue to have difficulty with math at the high school level (Shalev et al., 2005), it is key for teachers to include instruction on math language in the classroom, as supporting language at an early level could support math performance later (Powell & Driver, 2015). Educators can practice explicitly connecting math vocabulary and sentence structure to math concepts to further incorporate language in the classroom. To feasibly incorporate math writing into instruction, journal writing may be a notable and sustainable addition to teachers' workload (Baxter et al., 2005) by incorporating a math writing question into a daily lesson or homework assignment.

Finally, all three strategies are simple to execute with minimal materials or training needed. Training students on schema types can take place during any student exposure to a word problem and students can use the schema type identification when exposed to word problems in any situation. Teachers can help students practice math language by using precise and correct language when they model or teach. To incorporate writing, teachers can utilize additional writing supports (e.g., graphic organizer) or routines (e.g., a math writing question is always last on homework) in the classroom to help students acclimate to writing in math. To build confidence implementing unfamiliar approaches, such as SRSD, teachers may seek support from a literacy coach, interventionist, or district personnel.

How Effective Is It?

The implementation of schema instruction, math language, and math writing supports literacy in math instruction. Although the authors recommend all three supports, the research base and effectiveness of each support varies. Table 4 provides greater detail on the research base and effectiveness of each recommended literacy support in math.

What Questions Remain?

The current recommendations for schema instruction, math language, and math writing can support students with and without comorbid reading and math difficulty across academic settings. Yet, they do not answer all questions about language in math. A greater research base exists for schema instruction than for math language and writing. The research base for schema instruction provides answers to specific questions, it allows for specific questions for future research and implementation. The research base for math language and writing is just beginning. Therefore, the questions for math language and writing are broader than those for schema instruction.

For schema instruction, much of the schema instruction research surrounds additive schema types for single-step word problems (Peltier & Vannest, 2017; Peltier et al., 2018; Powell, 2011). Less research currently exists on multiplicative schema instruction and multi-step word problem instruction with the use of schemas (Alghamdi et al., 2020; Griffin et al., 2018). Additionally, limited research exists on using schema instruction at the high school level (Jitendra et al., 2018). Therefore, questions remain unanswered on how supportive schema instruction can be for students who need support with multiplicative word problems and multi-step word problems. Additionally, questions remain on how supportive schema instruction is for students in high school, performing at grade level and below grade level.

As previously stated, limited research exists regarding the relationship between math language and math performance (Lin et al., 2021; Powell & Driver, 2015). Most suggested strategies for math language instruction were derived from reading-based vocabulary and language instruction (Powell & Driver, 2015). Studies are needed that empirically evaluate explicit math language instruction on math outcomes. For math writing, more research is needed to examine best practices in all grade levels and math subtopics. Furthermore, the interaction of math language and math writing instruction with students who have co-occurring math difficulty and reading difficulty is needed to inform instruction for students with complex or intensive instructional needs.

Where Can I Learn More?

A variety of free resources provide more information on math language, schema type instruction for word problems, and math writing. Table 5 lists videos, online modules, and readings with more information on these topics. All resources listed in this table are free of charge.

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Table 1

Types and Purposes of Math Writing

Type	Purpose	Example
Exploratory	Student makes sense of a given problem of their own ideas	Students use writing to explore the differences between two different multiplication strategies.
Informative or explanatory	Student describes or explains	Describe the difference between a circle and a sphere.
Argumentative	Constructs or critiques an argument	Jaliah claims that $\frac{1}{100}$ is greater than $\frac{1}{10}$ because 100 is greater than 10. Do you agree or disagree? Defend your answer.
Mathematically creative	Demonstrates original ideas and flexible thinking	After learning about different numeration systems, students create their own.

Table 2

Schema Types and Examples

Schema Category	Schema	Equation	Example Word Problem	Example Equation
Additive Schemas	Total	Part 1 + Part 2 = Total $P1 + P2 = T$	Annie and Grace set up a lemonade stand. Annie sells 32 cups of lemonade at the lemonade stand. Grace sells 24 cups of lemonade. How many cups of lemonade did they sell in all?	$32 + 24 = T$ $T = 56$ cups of lemonade
	Difference	Greater – Lesser = Difference $G - L = D$	Tim runs 4 miles. Alan runs 7 miles. How many more miles does Alan run than Tim?	$7 - 4 = D$ $D = 3$ miles
	Change	Start +/- Change = End $ST +/- C = E$	Emanuel brings 25 cupcakes to class for his birthday. His classmates eat some cupcakes. Now he has 4 cupcakes left. How many cupcakes did he give to his class?	$25 - C = 4$ $C = 21$ cupcakes
Multiplicative Schemas	Equal Groups	Groups × Number = Product $G \times N = P$	The three friends want to buy a pack of popsicles. They each have 4 dollars. How much money do they have all together?	$3 \times 4 = P$ $P = 12$ dollars
	Compare	Set × Times = Product $S \times T = P$	Eric works on his class project for 3 hours. Damion works on his class project twice as long. How long does Damion work on his class project?	$3 \times 2 = P$ $P = 6$ hours
	Proportion/ Ratio	Quantity 1 ÷ Quantity 2 = Quantity 3 ÷ Quantity 4 $\frac{Q1}{Q2} = \frac{Q3}{Q4}$	Shawn swims two laps in 50 seconds. How many seconds would it take him to swim	$50 \div 2 = Q3 \div 5$ $Q3 = 125$ seconds

Note. Adapted from Fuchs et al. (2021), Jitendra et al. (2016), and Powell et al. (2020).

Table 3

Family Activities for Early Numeracy Development

Activity	Instructions	Script examples
Counting objects	<ul style="list-style-type: none"> Count objects aloud with your child. Adult should point to, touch, or grab the object as the corresponding number is being counted. 	<ul style="list-style-type: none"> “Wow, look at the birds. I see 1 [point to first bird], 2 [point to next bird], 3 [point to next bird], 4 [point to final bird] birds.” “I need two potatoes. One [grabs potato from bowl] and two [grabs second potato].” “Let’s see how many toy horses you have: one...”
Sharing equally	<ul style="list-style-type: none"> Share objects equally between people or things. Adult can model or do the activity with their child. Children should feel comfortable counting independently, or, with support to do this activity. 	<ul style="list-style-type: none"> “Let’s divide this pizza up so that each person gets the same amount. We have X slices [count the slices and point to each as you count] and X people [count the people and point as you count]. Let’s give each person one slice – one, one, one [place each slice on a plate]. Let’s see if we have enough to give everyone another piece!”
Comparing amounts	<ul style="list-style-type: none"> Comparing can be numerical (e.g., having 4 carrots is more than having 2) or conceptual (e.g., one pile of laundry can be larger than another). When doing numerical comparisons, count out objects. When doing conceptual comparisons, compare by volume or size. 	<ul style="list-style-type: none"> “Which house has more windows? This one has one, two, three windows and that one has one, two, three, four, five windows. Five is more than 3, so this house has more windows.” “Looks at these stacks of books! Which stack has more? Let’s count and find out.” “Oops! I gave Jan more mashed potatoes than I gave myself. Looks how much bigger her pile is than mine!”
Baking	<ul style="list-style-type: none"> Baking provides opportunities to use measurements. Children can practice comparing, measuring, and counting. 	<ul style="list-style-type: none"> “I need $\frac{1}{4}$ cup of sugar. Can you fill the cup all the way to the top?” “We need two eggs – let’s count them out – one, two.” “Our cake needs $\frac{1}{2}$ teaspoon of cinnamon and 1 tablespoon of honey. Wow! Look how much more honey we have than cinnamon! 1 tablespoon is larger than $\frac{1}{2}$ teaspoon.”
Reading math storybooks	<ul style="list-style-type: none"> Teachers should share recommendations for math books. If you as an educator need recommendations, talk with a local librarian. 	<ul style="list-style-type: none"> Follow the text when reading - including all vocabulary words and cues (e.g., count the number of balloons on this page after reading).

Table 4

Research Base and Effectiveness

Topic	Research base	Effectiveness
Schema instruction <ul style="list-style-type: none"> Additive schemas Multiplicative schemas 	<p>Positive effects</p> <p>Mixed effects</p>	<p>The schema instruction knowledge base demonstrates that additive schema instruction produces large effects (<i>effect size</i> = 1.57) on word-problem solving accuracy (Peltier & Vannest, 2017).</p> <p>A smaller body of evidence exists on the effects of multiplicative schema instruction (Alghamdi et al., 2020; Griffin et al., 2018; Xin, 2008). With some producing positive effects (Alghamdi et al., 2020; Xin, 2008) and others producing no effect (Griffin et al., 2018).</p>
Mathematics language	Limited experimental studies	The majority of strategies for teaching and applying mathematical language are adapted from reading-based vocabulary research. Few studies exist that directly evaluate the effectiveness of explicit mathematical language instruction (Powell & Driver, 2015).
Mathematics writing	Positive effects	<p>SRSD adapted to math content showed positive effects in 4 controlled studies.</p> <p>Meta-analysis measuring the effects of 21 studies of math writing on learning demonstrated an effect size of 0.32 in math (Graham et al, 2020).</p>

Table 5

Further Information on Mathematics Language, Schema Type Instruction, and Mathematics Writing

Topic	Location	Platform	Link
Schema instruction	National Council of Teachers of Mathematics	Article	www.jstor.org/stable/10.5951/teacchilmath.25.7.0428
Schema instruction	NCII Mathematics Course	Online module	intensiveintervention.org/instructional-strategies-math-course#Module5Part2
Schema instruction	Pirate Math Equation Quest	Videos	www.piratemathequationquest.com/videos.html
Schema instruction	Teaching Exceptional Children	Article	journals.sagepub.com/doi/full/10.1177/0040059918777250
Schema instruction	YouTube	Videos	youtube.com/playlist?list=PLsIPXYHeky6tyRuGv90Gkt4NFnay75RX_
Schema instruction and math language	What Works Clearinghouse	Practice guide	ies.ed.gov/ncee/wwc/PracticeGuide/26
Math language	National Council of Teachers of Mathematics	Article	www.nctm.org/Publications/Teaching-Children-Mathematics/2014/Vol21/Issue1/13-Rules-That-Expire/
Math language	NCII Math Course	Online Module	intensiveintervention.org/instructional-delivery-math-course#Module4Part3
Math language	Reading and Writing Quarterly	Article	www.researchgate.net/publication/276443539_The_Language_of_Mathematics_The_Importance_of_Teaching_and_Learning_Mathematical_Vocabulary
Math language	Teaching Exceptional Children	Article	www.researchgate.net/publication/309335508_Supporting_Clear_and_Concise_Mathematics_Language_Instead_of_That_Say_This
Math writing	National Science Foundation	Practice guide	mathwriting.education.uconn.edu/wp-content/uploads/sites/1454/2016/04/Types_of_and_Purposes_for_Elementary_Mathematical_Writing_for_Web-2.pdf
Math writing	The Reading Teacher	Article	ila.onlinelibrary.wiley.com/doi/am-pdf/10.1002/trtr.1733