Show the Flow: Visualizing Students' Problem-Solving Processes in a Dynamic Algebraic Notation Tool

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Erin Ottmar is a designer of From Here to There! and owns 10% equity in Graspable Inc. All other authors have no interests to declare.

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Abstract: We apply an advanced data visualization technique, *Sankey diagram*, to explore how middle-school students ($N = 343$) solved problems in a game-based algebraic notation tool. The results indicate that there is a large variation in the types of students' strategies to solve the problems, with some approaches being more efficient than others. The findings suggest that Sankey diagrams can be used both in research and practice to unpack our understanding of variability in mathematical problem-solving.

Keywords: math learning, problem-solving strategies, data visualization, Sankey diagram

Introduction

Teachers' knowledge of their students' learning is one of the important factors contributing to classroom practices, and ultimately, what students learn in mathematics (Asquith et al., 2007). However, teachers often have difficulties in monitoring students' learning progress or identifying their misconceptions. In response to these issues, advances in data analytics and visualizations have enabled teachers and researchers to identify complex data on student learning in an easier and faster way. However, there has been little research that has employed advanced data visualization techniques on students' mathematical learning processes (Vieira et al., 2018). Over the past several years, our team has designed and developed a dynamic algebraic notation tool, *Graspable Math* (GM), which helps develop students' conceptual and procedural learning in algebra. The tool allows students to dynamically manipulate and transform numbers and mathematical expressions using various touch or mouse-based gesture-actions. Using the data collected in GM, students' problem-solving processes can be represented as a series of time-based steps that form the mathematical derivation as they transform mathematical expressions and equations. Thus, the purposes of this study are to 1) present visualizations of students' algebraic problem-solving processes in GM using an advanced and novel data visualization technique, called *Sankey diagram*, and 2) investigate how we can use these visualizations to provide meaningful and comprehensive information to researchers and teachers.

Methods

Our sample ($N = 343$) was drawn from a randomized controlled study conducted in 2019. Of the 343 students (54% male, 43% female, 3% not reported), most students (96%) were in sixth grade, and the remaining students (4%) were in seventh grade. *From Here to There!* (FH2T, https://graspablemath.com/projects/fh2t), is a gamified version of the GM that was developed to help students' algebra learning. It allows students to dynamically manipulate and transform numbers/mathematical expressions using various gestures. The problems in the game consist of two mathematically equivalent mathematical expressions, a start state and a goal state. Students must transform the starting expression into the target goal state using algebraically permissible actions. Among the 252 problems that cover a variety of mathematical concepts, this study focuses on two multiplication problems.

Results

Figure 1 presents the students' problem-solving processes for the square numbers problem (turning "9×4" into "3×6×2"). Each node in the diagram represents a different mathematical expression made by students, and the thickness of each path represents the number of students who made that expression. Colors in the diagram represent the productivity of students' first steps (blue: productive, red: non-productive). As shown in Figure 1, more than half of the students made productive first steps for this problem (e.g., 3×3×4, 9×2×2), and most of them solved the problem with the fewest possible steps to reach the goal state. On the contrary, the students who made non-productive first steps (e.g., 36, (3+6)×4) did not solve the problem in the most efficient way. Thus, noticing the underlying structure of the problem equation and how to transform it played a critical role in students' subsequent transformations as well as their overall efficiency of algebraic problem-solving.

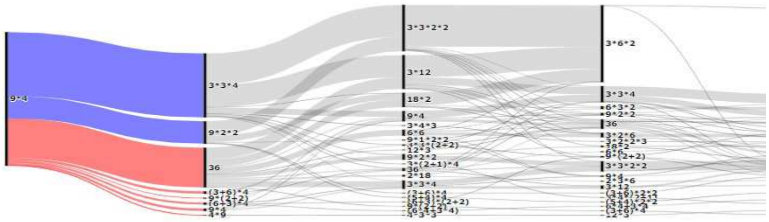

Figure 1. Section of the Sankey diagram showing productivity of initial solution strategies for the square numbers problem (for full image: http://tiny.cc/pd51tz)

Figure 2 presents the students' problem-solving processes for the non-square numbers problem (turning "6×10" into "2×15×2"). Most students who made the numbers in the goal state (e.g., 2) or the factors (e.g., 3, 5) of the numbers in the goal state on their first steps solved the problem in the most efficient way. However, the students who did not attend to the structure of the problem and simply combined two numbers failed to solve the problem in the most efficient way. Specifically, many of the students who made 60 on their first step did not solve the problem in the most efficient way, indicating that factoring 60 into small numbers was an obstacle point to the students. Thus, as demonstrated by the Sankey diagram, students' first mathematical transformation played a significant role in students' subsequent and overall efficiency of problem-solving.

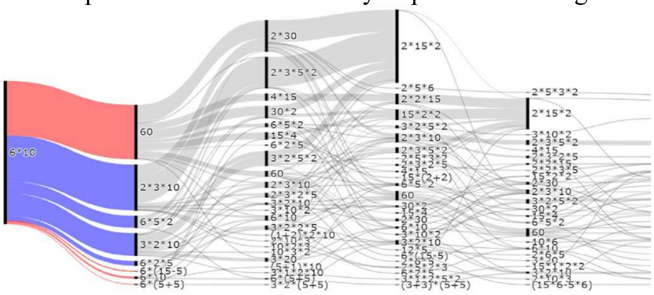

Figure 2. Section of the Sankey diagram showing students' problem-solving processes for the non-square numbers problem (for full image: http://tiny.cc/wd51tz)

Discussion

This work presents a new way to visualize students' problem-solving processes in a dynamic algebraic notation tool. The results showed that Sankey diagrams could provide a lot of information to help teachers and researchers efficiently understand students' algebraic problem-solving processes. They presented the variability of problem-solving processes, the most common pathways used, the obstacle points of problem-solving, and the efficiency of different solution strategies. In particular, the diagrams revealed that noticing the underlying structure of the problem equation and how to transform it on their first step played a critical role in students' subsequent transformations as well as their overall efficiency of problem-solving (Stephens et al., 2013). Visualizing students' problem-solving processes for a given mathematical task would help teachers and researchers perceive students' individual differences and their mathematical thinking processes. This work has clear implications for practice as students' strategies are often invisible to teachers. In combination with the GM, teachers could use these visualizations in several ways to inform their instruction.

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