Predicting Mathematics Achievement from Subdomains of Early Number Competence: Differences by Grade and Achievement Level

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Abstract

This study investigated the relative importance of three subdomains of early number competence (number, number relations, and number operations) in predicting later mathematics achievement in cross-sequential samples of pre-K, kindergarten (K), and first graders (N=150 at each grade). OLS-regression analyses showed that each subdomain predicted mathematics achievement at each grade level, controlling for the other two subdomains as well as background variables. All of the subdomains explained a significant amount of variance in later mathematics achievement. Unconditional quantile regression analyses examined relations between number competencies and mathematics achievement at quantiles representing low (.2), intermediate (.5) and high (.8) achievement. The subdomain of number operations was highly related to mathematics achievement for high achievers. For low achievers, number and number relations abilities were most highly related to future mathematics achievement in the pre-K sample, and number relations abilities in the K and first grade samples. Findings highlight the unique importance of all three subdomains of early number competence for later mathematics achievement, but show some of the relations are contingent upon achievement level.

Predicting Mathematics Achievement from Subdomains of Early Number Competence: Differences by Grade and Achievement Level

Early mathematics competencies predict later achievement (Duncan et al., 2007). In particular, early number competence is uniquely important to mathematics outcomes in elementary school, when considering multiple variables (e.g., Jordan et al., 2009; Passolunghi & Lanfranchi, 2012). Early number competence can be operationalized by three relatively distinct subdomains: number, number relations, and number operations (National Research Council, 2009; Milburn et al., 2019; Purpura & Lonigan, 2013). Although past work has related some specific number competencies to later achievement, no studies have examined the relative predictability of each subdomain of early number competence to mathematics achievement over pre-K, kindergarten, and first grade, when children acquire critical foundations for mathematical learning (Frye et al., 2013). As such, there is no consensus concerning which of the subdomains of number competence are most strongly associated with future mathematics achievement during this crucial period. Moreover, no studies have examined these relations across different quantiles of the mathematics achievement distribution, so it is unknown how any predictive relations may differ for children at low, intermediate, and high levels of mathematics achievement. This information is key for targeting and differentiating early mathematical instruction so that limited instructional time and resources can be allotted appropriately. To address these issues, the present study assessed the relative predictability of number, number relations, and number operation in pre-K, kindergarten, and first grade children to general mathematics achievement one year later. These relations were estimated at the mean-level and at low, intermediate, and high quantiles of achievement.

Subdomains of Early Number Competence

The *number* subdomain refers to children's knowledge of whole numbers, especially the ability to enumerate sets (National Research Council, 2009). Number subdomain abilities in pre-K include subitizing (i.e., quickly ascertaining the quantity of small sets without counting; Clements et al., 2019), counting each item in a set only once, using the count words in a stable order, and knowing the cardinality principle (i.e., that the last number word said when counting indicates the quantity of counted items; Gelman & Gallistel, 1978; Wynn, 1992). Children also begin to recognize single-digit numerals. In kindergarten and first grade, the number subdomain includes more advanced abilities, such as counting on from a given number, counting by tens, and understanding basic place value concepts. The *number relations* subdomain involves reasoning about relationships between and among numbers (National Research Council, 2009). Core abilities include comparing the magnitude of sets of objects and number symbols (De Smedt et al., 2009) and representing quantities on the number line (Siegler & Booth, 2004). The *number operations* subdomain entails manipulation of numbers to add and subtract (National Research Council, 2009). It involves composing and decomposing small sets of objects (Fuson, 1992). Children increasingly engage in calculations involving symbols (e.g., story problems and small number combinations), with and without object referents (Clements & Sarama, 2007; Huttenlocher et al., 1994).

Evidence for these subdomains comes from studies using factor analysis. Jordan et al.'s (2006) exploratory factor analysis of early number competencies in a sample of kindergarten children resulted in a two-factor solution that included number (encompassing both number and number relations abilities) and number operations. Purpura & Lonigan (2013) assessed preschool children on a range of early number abilities including counting, subitizing, estimation, number and set comparison, sequencing, addition with objects, story problems, small number combinations, and early equivalence understanding. Using confirmatory factor analysis, they found that the data were best explained by a three-factor solution that represented the subdomains of number, number relations, and number operations. This three-factor solution was replicated by Milburn et al. (2019). Although these studies identified related but distinct dimensions of early number competence, the relative contributions of the subdomains to future mathematics achievement is largely unknown.

Previous studies have examined the relation between different number competencies and mathematics achievement, with many including abilities that span a single subdomain. For example, Sasanguie et al. (2012), studied the relation between various number relations abilities and mathematics achievement a year later in cross-sectional samples of kindergartners, first graders, and second graders. The researchers found that symbolic magnitude comparison and non-symbolic number line estimation predicted variance on a curriculum-based measure of mathematics achievement one year later (i.e., in first, second, and third grades, respectively).

Studies that have assessed abilities across all three subdomains of number competence point to developmental differences in the relative predictability of the subdomains for later mathematics achievement. There is evidence that the number subdomain explains more variance in mathematics achievement than the other subdomains when assessed in preschool. However, relations and operations may be better predictors once children enter formal schooling. Examining the contributions of competencies representative of all three subdomains of number competence at the start of pre-K to mathematics achievement at the end of the year, Geary and vanMarle (2016) found that counting and cardinality made significant contributions, while magnitude comparison and the number operations task did not. Titeca et al. (2014) found support in a preschool sample for the relative importance of the number subdomain (i.e., subitizing and counting) controlling for number relations (i.e., magnitude comparison and number line estimation) and operations when predicting later mathematics achievement in first grade. Number subdomain abilities before kindergarten may provide a critical foundation on which more advanced mathematics abilities may be built.

In contrast, studies where number competencies were assessed in kindergarten and first grade point to the unique importance of knowledge of number relations and number operations in predicting later mathematics achievement. For example, Locuniak and Jordan (2008) investigated how kindergarten number competence contributed to variance in second grade arithmetic fluency skill. Among a variety of number competencies, they found only number relations and number operations tasks explained a significant amount of variance in second grade fluency. Kindergarten facility with simple number combinations was the single strongest predictor of second-grade arithmetic fluency.

Similarly, Geary (2011) predicted first through fifth grade mathematics achievement and growth from first grade counting knowledge (number), number line estimation (number relations), composition of sets and addition strategy (number operations), along with several domain general variables. Number line estimation and quick retrieval of arithmetic responses in first grade predicted the growth slope, meaning the positive effects of early number relations and operations abilities increased over time. The ability to compose and decompose numbers and use advanced counting strategies to solve addition problems explained a significant amount of variance in mathematics achievement across all grade levels. It may be that older children’s ability to use basic number subdomain abilities such as counting when solving relations and operations tasks are more important for predicting later achievement than performance on simpler number subdomain tasks that assess those skills in isolation.

In summary, previous studies suggest that number, number relations, and number operations all are important to mathematics achievement, but they may be differentially predictive according to grade level. This pattern may speak to a developmental sequence from understanding of number and number relations to number operations. It has been suggested that the three subdomains of early number competence could in fact be developmental levels of numeracy acquisition (Purpura & Lonigan, 2013) by which number subdomain skills are necessary for the development of number relations and number relations for number operations, or both number and number relations abilities are causally related to skill with operations.

Prior research on the timing of acquisition of these abilities has demonstrated that cardinality knowledge (a number subdomain skill) in 4-year-old to 7- year-old children precedes number ordering knowledge (a number relations skill; Knudsen et al., 2015). Symbolic magnitude understanding, a number relations ability, also mediates the relation between cardinality knowledge and simple arithmetic in preschoolers (Scalise & Ramani, 2021) providing some evidence that specific number abilities may be important for the development of specific number relations abilities, and in turn help children to develop an understanding of operations. However, developmental progressions within and between subdomains are also likely related to shifts from non-symbolic to symbolic demands and working with smaller to larger set sizes. For example, within the number subdomain, mapping between number words and quantities (i.e., cardinality) and number words and written numerals generally develops prior to mapping between notations and quantities (Lira et al., 2017).

Predictability at Different Performance Quantiles

The aforementioned studies estimated the average relations between predictors and outcomes using the sample means. These analyses may fail to capture nuances in predictive relations that depend on the level of achievement. That is, it is unclear whether the subdomains of number competence relate to mathematics achievement in the same way across the achievement distribution. The present study uses *quantile regression* to explore this issue. Rather than split up the sample based upon arbitrary cut-points into low and high achievers and perform individual regressions within these smaller samples, quantile regression uses all data in the sample. These data are weighted differentially based on their proximity to the targeted points of the distribution of the outcome of mathematics achievement (Fuchs et al., 2020). Using all of the data in the analyses decreases the chances of producing biased estimates (Koenker, 2015).

Quantile regression has begun to inform developmental studies in mathematics as well as in reading. Purpura and Logan (2015) found that the ability to discriminate between non-symbolic quantities (i.e., ANS acuity) was a significant predictor of mathematics achievement only at the lower end of the achievement distribution when prior mathematics achievement and mathematical language were also included in the model. Fuchs et al. (2020) demonstrated that second graders' calculation fluency, language comprehension, and working memory differentially predict mathematics word problem solving, depending on their level of word problem solving skill. Specifically, working memory and calculation fluency were stronger predictors of word problem solving for low- and intermediate-level problem solvers than for high-level problem solvers. Another study showed that executive function (EF) skills have a larger positive relation with early mathematics abilities in pre-K children with low mathematics abilities than in their peers with intermediate or high abilities (Dong et al., 2020). In the area of reading, McIlraith (2018) found that letter knowledge strongly predicts poor and average, but not high reading skill. Conversely, rapid automatized naming of colors and shapes was found to be a strong predictor of reading skill for good readers, but not those at the lower end of the distribution. Predictive relations that are inconsistent across the outcome distribution matter when the goal is to identify predictors of later achievement and inform focused instruction. Gaining knowledge of these relations potentially allows for individualized intervention based upon level of achievement.

The Present Study

To build on earlier work, the present study assessed the relative predictability of children's abilities in the three subdomains of early number competence for mathematics achievement one year later. Cross-sectional samples of pre-K, kindergarten, and first-grade children were included within the same longitudinal study to explore developmental relations of the three subdomains with future mathematics achievement. We further explored whether the relative predictive relations between the subdomains of number competence and mathematics achievement identified at the sample mean-level were dependent on mathematics achievement quantile.

Drawing from previous findings, we explored whether basic number skills were more strongly related to mathematics achievement earlier in development, followed by number relations and operations. Specifically, we examined whether the number subdomain in pre-K relates more to achievement than number relations or operations and whether number relations and operations contribute more to later mathematics achievement in the kindergarten and first grade samples. We also expected basic number skills would relate to mathematics achievement more strongly for children near the bottom of the achievement distribution and relations and operations for children near the top of the distribution. However, as each subdomain included items assessing a range of difficulty in various set sizes and symbolic representations (e.g., dots and numerals), we also expected that each subdomain would explain a significant amount of variance in mathematics achievement at the mean-level, despite their shared relations.

In sum, three primary research questions guided the present study: (1) Do the subdomains of early number competence explain a significant amount of variance in mathematics achievement a year later? (2) Are there differences in the relative importance of each subdomain for predicting later mathematics achievement at the three grade levels studied? (3) Does the predictive strength of the subdomains of number competence estimated for the sample average differ from those estimated at specific mathematics achievement quantiles?

Method

Participants

Four school districts and five preschool programs in California participated as part of a larger study. The schools and programs were selected because they served a socioeconomically and ethnically diverse population of students. All sampled pre-K children were required to be eligible to attend kindergarten the following fall. Consent forms in English and Spanish were distributed to all participating classrooms (24 classrooms at pre-K; 29 at kindergarten; 17 at first grade). A sample of 700 children (200 pre-K; 300 K; and 200 first grade) participated at time 1 (T1). A subsample of 450 children was randomly selected to be assessed a year later at time 2 (T2). Using school district as a blocking factor and balancing by gender, 150 children were chosen at each grade level. The final sample for the present analyses included the 150 children from each grade-level.

Fifty percent of the final sample at each grade level was comprised of children who were designated as coming from low-income families. At pre-K, these children attended programs with income-eligibility requirements. For kindergarten and first graders, income-level was determined by eligibility for free/reduced priced lunch. About half of the sample at each grade level was female (49% in pre-K; 51% in K and first grade, respectively). In pre-K, children's ages ranged from 4 years, 1 month to 5 years, 1 month, with a mean age of 4 years, 6 months. In kindergarten, ages ranged from 4 years, 11 months to 6 years, 11 months, with a mean age of 5 years, 6 months. In first grade, ages ranged from 5 years, 11 months, to 7 years, 6 months, with a mean age of 6 years, 7 months.

The sample at each grade level was ethnically and racially diverse based on parent report (pre-K: 39.7% Latino, 29.8% White, 13.2 % Asian, 2% Black, and 15.3% multiple or other race/ethnicities; kindergarten: 36.7% Latino, 20.7% White, 23.3% Asian, 5.3% Black, and 14% multiple or other; first grade: 40% Latino, 23.3% White, 23.3% Asian, 5.3% Black, and 8.1% multiple or other). All samples had a number of multilingual learners, or children who come in contact with and/or interact in languages other than English on a consistent basis (WIDA, 2020). In pre-K, 39.33% of the sample were identified as multilingual learners. Parents indicated that English and another language (8% of sample) or another language only (31.33% of sample) was spoken at home. In the kindergarten sample, 28% of sample were multilingual learners (8.7% of the sample's parents listed English and another language and 19.33% of the sample's parents listed a language other than English as their home language). Finally, in the first grade sample, 28.7% were multilingual learners, with 8.7% of the sample's parents listing English and another language and 20% listing a language other than English as their home language. Across samples, Spanish was the primary non-English language spoken at home (79.5%, 70%, and 63.6% of multilingual learners in pre-K, kindergarten, and first grade samples, respectively).

Procedure

We used a cross-sequential design, assessing the three samples at two time points. All children were assessed individually by a trained and certified assessor. Children were assessed on the three subdomains of number competence during the fall of the school year in one session (~25 minutes). In pre-K, verbal directions were given in the dominant language (English or Spanish, as determined by parent consent form). A Spanish language version of the directions was translated by a native Spanish-speaker. English language learners were assessed by a bilingual assessor trained to code-switch between English and Spanish, and children were allowed to answer in either language. The majority of the pre-K sample was assessed in English only (84.7% of children), but 12% of children were assessed in Spanish and 3.3% were assessed in a mix of English and Spanish. All participants were followed to the next grade level (kindergarten, first grade and second grade, respectively) and administered a measure of broad mathematics achievement (~20-30 minutes) in the fall of the next school year.

Measures

Number Competence Predictor Variables

The Screener for Early Number Sense (*SENS;* Jordan et al., 2021) was used to assess number knowledge in the subdomains of number, number relations, and number operations. The *SENS* has three vertically linked forms, one each for pre-K, kindergarten, and first grade. Although all subdomains were represented at each grade level, content was varied to align with grade-level benchmarks set by the Common Core State Standards (2010) for the kindergarten and first grade forms and benchmarks set by the National Research Council (2009) for the pre-K form. Examples of items from each subdomain can be found in the Appendix. Given the highly related nature of the subdomains, it is likely that items in one subdomain occasionally measured aptitude for a sub-construct from another subdomain. However, the items were designed to focus specifically on the sub-construct of interest. For example, object counting (a number subdomain skill) could have been used as a strategy for solving number combinations (in the number operations subdomain), by modeling the addends on fingers. However, object counting was not *required* to correctly solve the number combinations.

Number. Items assessed knowledge of whole numbers. In pre-K, there were 12 items assessing the number core (subitizing: n = 2; counting and cardinality: n = 7; numeral naming: n = 3). This subdomain had an internal consistency of α = .89. IRT standard error scores were also calculated to provide reliability information across the distribution of latent ability (theta). A standard error of .548 is equivalent to a classical test theory internal consistency statistic of .70 (Purpura & Lonigan, 2015). The pre-K number subdomain had a standard error of less than .548 from theta scores of -1.82 to 1.41, indicating acceptable reliability across the range of ability. In kindergarten, there were 11 number subdomain items (counting and cardinality, n = 6; numeral naming: n = 4; place value: n = 1). Internal consistency was good (α = .89), and there were standard errors of less than .548 from theta scores of -1.07 to 1.48. In first grade, there were 9 number subdomain items (counting: n = 3; numeral naming: n = 3; place value: n = 3). Internal consistency was good (α = .84), and standard errors were less than .548 from theta scores of -1.50 to 1.10.

Number Relations. Items assessed knowledge of the relationships between whole numbers. In pre-K, there were 11 items assessing the number relations subdomain (magnitude comparison with sets and numerals: n = 4; number ordering: n = 4; linear representation of number: n = 3) and an internal consistency of α = .80. There were standard error scores of less than .548 from theta scores of -1.70 to 1.00. In kindergarten, there were 9 items (magnitude comparison with numerals: n = 1; number ordering: n = 4; linear representations of number: n =

4) and α = .77. There were standard errors of less than .548 from theta scores of -1.30 to 0.99. In first grade, there were 8 number relations items (magnitude comparison with numerals: n = 2; number ordering: n = 3; linear representation of numbers: n = 3), and α = .79. Standard errors were less than .548 from theta scores of -1.10 to 0.97.

Number Operations. Items assessed knowledge of the ability to manipulate sets and perform calculations with small numbers. In pre-K, there were seven items (composition/decomposition: n = 2; nonverbal arithmetic: n = 2; story problems: n =2; number combination: n = 1) and an internal consistency of α = .64. There were standard errors of less than .548 from theta scores of -0.55 to 0.59. In kindergarten, there were ten number operation items (composition/decomposition: n = 4; story problems: n = 3; number combinations: n = 3) and α = .78. There were standard errors of less than .548 from theta scores of -1.10 to 1.40. In first grade, there were thirteen number operations items (composition/decomposition: n = 3; story problems: n = 4; number combinations: n = 4; fluency: n = 2), and α = .86. There were standard errors of less than .548 from theta scores of -1.40 to 1.40.

Mathematics Achievement Dependent Variable

Broad mathematics achievement was assessed at T2 using the *Test of Early Mathematics Ability, third edition* (TEMA-3; Ginsburg & Baroody, 2003), a standardized measure of general mathematics achievement. This test has high internal consistency (α = .94 for 5-year-olds, α = .95 for 6-year-olds, and α = .95 for 7-year-olds).

Data Analysis

We employed both ordinary least-squares (OLS) and unconditional quantile regression (UQR) methods to address our research questions. Based on past literature, we controlled for income-level (coded as 1 for low-income), gender (coded as 1 for female), and age in all models as variables that may influence predictive relations. As our samples had a plurality of children identified as Latino and an appreciable number of multilingual learners, we also planned to control for ethnicity (coded as 1 for Latino) and multilingual learner status (coded as 1 for exposure to languages other than English). Conventional OLS regression was used to investigate mean-level relations between the subdomains and mathematics achievement one year later in order to address the first two research questions.

OLS regression allows only for the estimation of an average relation between a predictor and outcome. Therefore, we also used unconditional quantile regression analysis to estimate the relations between predictor variables and later mathematics achievement at low, intermediate, and high quantiles of achievement and to compare these estimates to those at the mean-level. When quantile regression is used in the field of developmental science, the type most often employed is conditional quantile regression (CQR). CQR method assesses the impact of a predictor on an outcome distribution that is conditional on all variables in the model, making interpretation difficult. This is especially true when there are covariates included in the model, as results often will not generalize across all quantiles of performance (Wenz, 2019). Unconditional quantile regression (UQR) estimates the values of a relation over other variables in the model, by defining quantiles prior to regression (Borah & Basu, 2013). This procedure eases interpretation by estimating the relation between a predictor variable on the unconditional quantile of the outcome variable, holding other covariates constant. To clarify, in this context the term *unconditional* references that the outcome's distribution is not conditional on the model's covariates, not that there are no covariates included in the model. UQR has recently been utilized to predict educational achievement in Fuchs et al. (2020) and Hajovsky et al. (2020).

We used the two-step unconditional quantile regression method, introduced by Firpo et al. (2009). First, a re-centered influence function (RIF) of each quantile of mathematics achievement performance was calculated. Then, we conducted a regression with a quantile's RIF as the outcome. We chose to present UQR results at quantiles representative of low (.2), intermediate (.5), and high (.8) mathematics achievement for ease of interpretation and to follow the practice of other studies focused on educational achievement constructs (e.g. Dong, et al., 2020; Fuchs et al., 2020). Note that the estimates generated by UQR analyses for the selected quantiles would remain the same if additional quantiles (i.e. .10, .30, .70, .90) were also estimated. All analyses were conducted in Stata 16.1. The *rifreg* command was used for creating RIFS of quantiles of interest.

We considered two points when examining the relative relation between each subdomain and mathematics achievement a year later at low (.2), intermediate (.5), and high (.8) quantiles of performance. We first examined whether the subdomain emerged as a significant independent predictor of mathematics achievement at each quantile, controlling for the other variables, and studied the pattern of the strength of the coefficients. This allowed us to estimate each subdomain's relation with mathematics achievement and to view the pattern of how it differed at the low, intermediate, and high quantiles of the distribution.

We next compared the relative relation at each quantile to the mean-level estimated relation from the corresponding OLS regression model. We noted quantile coefficients that fell outside of the OLS 95% confidence interval. When a quantile coefficient fell outside of the OLS 95% confidence interval, it demonstrated that the mean-level estimated relation from the OLS model did not well represent the predictive relation at the specified quantile of mathematics. achievement.

Results

Descriptive Statistics and Multicollinearity Assessment

Means, standard deviations, and bivariate correlations are presented in Table 1. As the three subdomains of number competence are theoretically and practically related, it was important to rule out issues of multicollinearity. Although there is no universally accepted criterion for a level of correlation that constitutes a serious multicollinearity issue, a general rule of thumb is that correlations of .80 or higher between independent variables are problematic (Berry et al., 1985). No correlations between the number competence subdomain variables in the present study were above this level. Multicollinearity diagnostics also showed adequate variance inflation factors (i.e., all less than 4) and tolerance values (i.e., greater than .10) for regression analyses (Akinwande et al., 2015).

However, the planned control variables of income-level and Latino ethnicity were highly correlated, especially in the pre-K sample (r = .82; .63, and .61 in the pre-K, kindergarten, and first grade samples, respectively). In order to avoid multicollinearity issues, and to keep models consistent across grades, we only considered income-level, age, gender, and multilingual learner status as control variables. Estimated relations between the number competence subdomains and mathematics achievement are unchanged when ethnicity is used as a control variable instead of income-level.

Table 1

Descriptive Statistics of Raw Scores and Correlations by Grade

Grade	Variable			Correlations			
Pre-K *N*=150		*M*	*SD*	TEMA	N	NR	NO
	TEMA-3	26.04	9.34	—			
	Number	7.09	3.56	.80	—		
	Number Relations	6.73	2.93	.79	.77	—	

	Number Operations	4.16	1.80	.69	.68	.69	—
	Age	58m	3m	.30	.28	.24	.19
Kindergarten *N*=150				TEMA	N	NR	NO
	TEMA-3	39.89	10.6	—			
	Number	5.17	3.5	.74	—		
	Number Relations	5.07	2.4	.70	.76	—	
	Number Operations	3.70	2.60	.70	.71	.66	—
	Age	66m	4m	.17	.29	.30	.18
First *N*=150				TEMA	N	NR	NO
	TEMA-3	50.51	11.45	—			
	Number	5.20	2.74	.75	—		
	Number Relations	4.70	2.45	.71	.75	—	
	Number Operations	5.99	3.91	.79	.77	.70	—
	Age	79m	3m	.09	.10	.13	.02

Pre-K Predictors of Kindergarten Mathematics Achievement

OLS Regression

Results of OLS-regression predicting kindergarten mathematics achievement collected at the second time point from pre-K number, number relations, and number operations, controlling for age, income-level (low-income = 1), gender (female = 1) and multilingual learner status (exposure to languages other than English = 1) are presented in Table 2. All pre-K number competence subdomains made independent contributions to mathematics achievement a year later, and the overall model accounted for 74% of the variance, F (7,142) = 57.00, $p < .001$. For every one *SD* increase in number in pre-K, there was an estimated .42 *SD* increase in kindergarten mathematics achievement, holding the other variables constant. Every one *SD* increase in number relations and number operations were associated with a .34 *SD* and .20 *SD* increase in kindergarten mathematics achievement, respectively, controlling for the other variables.

Table 2

OLS Regression and Unconditional Quantile Regression Results for Pre-K Sample with Kindergarten Mathematics Achievement as the Dependent Variable

Variable	OLS β	CI 95% low	CI 95% high	Quantile Coefficients 0.2	0.5	0.8
Number	**.42**	.25	.55	**.36**	**.26**	$.13^-$
Number Relations	**.34**	.19	.48	**.32**	**.40**	**.19**
Number Operations	**.20**	.08	.30	$.05^-$	**.18**	**$.35^+$**
Female	-.08	-.16	.01	-.06	-.06	-.10
Age	.07	-.01	.14	.002	.02	**$.21^+$**
Low-income	.04	-.08	.16	.12	.02	-.01
Multilingual	.02	-.07	.12	-.07	-.02	.12

Note. OLS Model $R^2 = .74$.
Bolded coefficients are significant at $p < .05$.
$^+$ Exceeded upper bound of OLS 95% CI. $^-$ Exceeded lower bound of OLS 95% CI.

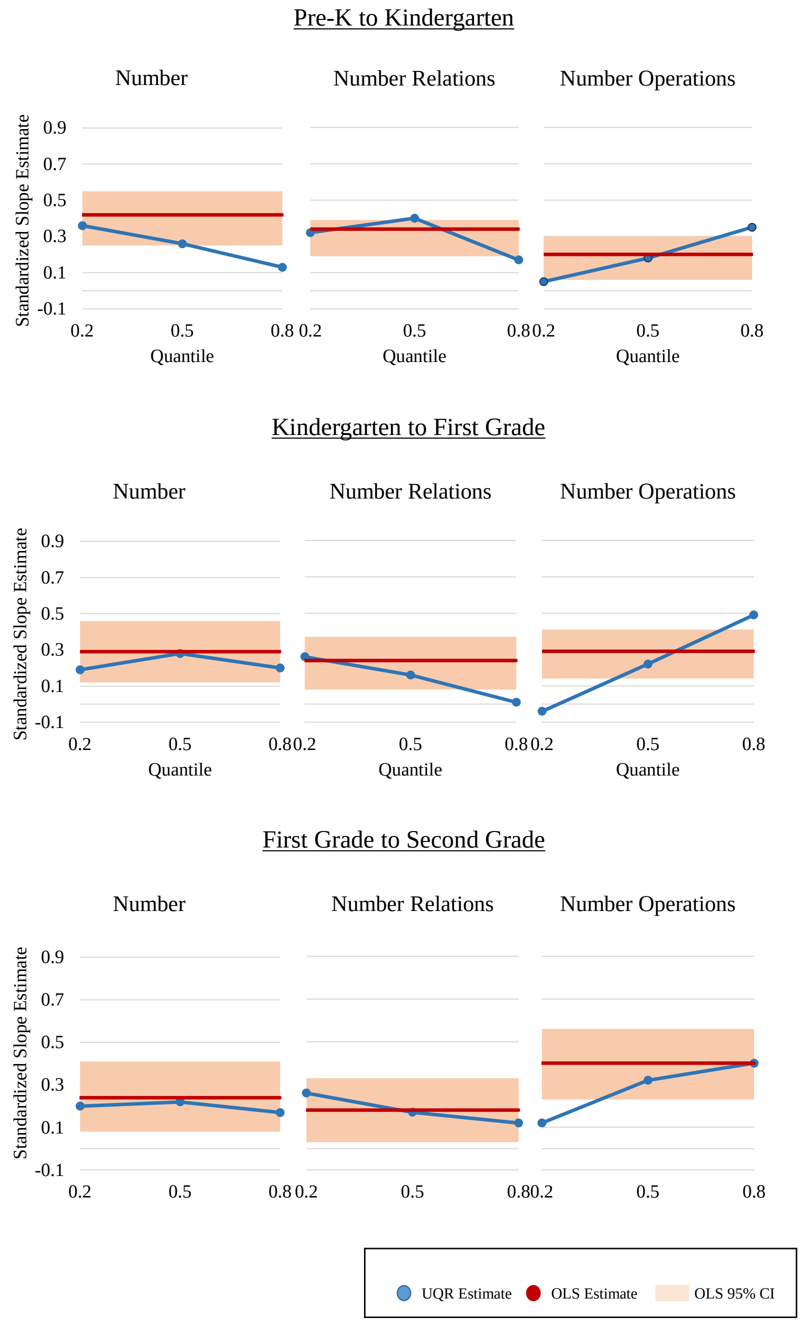
Quantile Regression

We next used unconditional quantile regression to predict kindergarten mathematics achievement at the .2 (low), .5 (intermediate), and .8 (high) quantiles of the performance distribution from pre-K number, number relations, and number operations, controlling for the background variables. Results of the unconditional multiple quantile regression analyses are presented to the right in Table 2.

Relationships between OLS and UQR results are graphically displayed in Figure 1, where the *x* axis represents the selected quantiles of mathematics achievement performance in kindergarten, and the *y* axis represents the strength of the relation between each pre-K predictor and mathematics achievement a year later, while controlling for the other predictors (i.e., the slope estimates). The coefficients can be interpreted as the estimated increase in *SD* units in mathematics achievement at the specified performance quantile, associated with a one *SD* increase in the predictor subdomain score for a male, middle-income, English-speaking child of average age who had an average score on the other two subdomains. The blue dots represent UQR estimates, and the red dots represent OLS estimates (placed at the same estimate across quantiles for comparison). The shaded area shows the 95% confidence interval of the OLS estimate. A quantile point that falls outside of this shaded area shows that the estimate of the conditional relation at the specified achievement quantile differs from the estimate of the average relation, meaning that the mean-level estimate from the OLS model does not well-represent the relation at that quantile of achievement.

Figure 1

Early Number Competence Subdomains Predict Mathematics Achievement a Year Later, Controlling for Background Variables



Note. As an example of interpretation, a 1 *SD* increase in pre-K number operations ability was associated with a .35 standard deviation increase in kindergarten mathematics achievement at the .8 quantile of achievement (estimate in blue), controlling for the other variables. The estimate fell outside of the OLS estimate's (β = .20, in red) 95% CI (shaded area), demonstrating the estimated relation at this quantile of the distribution was not well-represented by the mean-level estimate; it was higher than estimated.

Number. The pre-K number subdomain explained a significant amount of variance in kindergarten mathematics achievement at the .2 (β = .36, *p* = .01) and .5 (β = .26, *p* = .009), but not the .8 quantile (β = .13, *p* = .47), when controlling for the other variables. The estimate at the .8 quantile fell outside the lower OLS 95% confidence interval [.25,.55], demonstrating that the relation between pre-K number ability and kindergarten mathematics achievement at the high achievement quantile (.8) was not well-represented by the mean-level estimate from the OLS model.

Number Relations. Pre-K number relations explained a significant amount of variance in mathematics achievement at all of the tested quantiles (β = .32, *p* = .007; β = .40, *p* <.001; β = .19, *p* = .04 at the .2, .5, and .8 quantiles, respectively), controlling for the other variables. All estimates fell within the OLS 95% confidence interval [.19, .48], demonstrating that the mean-level relation well-represented the relation across the achievement distribution.

Number Operations. The number operations subdomain in pre-K explained a significant amount of variance in kindergarten mathematics achievement at the .5 and .8 (β = .18, *p* = .03; β = .35 *p* = .004), but not the .2 quantile (β = .05, *p* = .60), when controlling for the other variables. The estimate at the .2 quantile fell outside of the lower OLS 95% confidence interval [.08, .30] and the estimate at the .8 quantile fell outside of the upper OLS 95% confidence interval, demonstrating that the relations between pre-k number operations and kindergarten achievement estimated for children at the .2 and .8 quantiles of kindergarten were not well-represented by the mean-level estimate from the OLS regression.

Kindergarten Predictors of First Grade Mathematics Achievement

OLS Regression

The model predicting first grade mathematics achievement from the kindergarten subdomains, controlling for background variables accounted for 66% of the variance in first grade mathematics achievement, F (7, 142) = 39.23, $p < .001$ (see Table 3). All kindergarten subdomains explained a significant and similar amount of variance in mathematics achievement a year later when holding the other variables constant.

Table 3

OLS Regression and Unconditional Quantile Regression Results for Kindergarten Sample with First Grade Mathematics Achievement as the Dependent Variable

	OLS			Quantile Coefficients		
Variable	β	CI 95% low	CI 95% high	0.2	0.5	0.8
Number	**.29**	.12	.46	.19	**.28**	**.20**
Number Relations	**.24**	.08	.39	**.26**	.16	.01⁻
Number Operations	**.29**	.14	.41	-.04⁻	**.22**	**.49⁺**
Female	-.10	-.21	-.01	-.003	-.11	-.06
Age	-.03	-.14	.07	-.006	-.04	-.10
Low-income	**-.14**	-.25	-.03	-.13	-.12	-.13
Multilingual	-.01	-.04	.04	.05	.04	-.11

Note. OLS Model R^2 = .66.
Bolded coefficients are significant at $p < .05$.
⁺ Exceeded upper bound of OLS 95% CI. ⁻ Exceeded lower bound of OLS 95%.

Quantile Regression

Next, unconditional quantile regression was used to predict first grade mathematics achievement at the .2, .5, and .8 quantiles of the performance distribution from kindergarten number, number relations, and number operations, controlling for background variables (see Figure 1 and Table 3).

Number. Kindergarten number performance explained a significant amount of variance of first-grade mathematics achievement at the .5 ($\beta = .28$, $p = .01$), but not the .2 or the .8 quantile ($\beta = .19$, $p = .06$; $\beta = .20$, $p = .07$), when controlling for the other variables. However, none of the quantile slope estimates for kindergarten number fell outside of the 95% confidence interval [.12, .46] of the OLS regression model. Although kindergarten number ability was not a statistically significant predictor of mathematics achievement at all quantiles, the relation was similar to the sample average relation across the distribution.

Number Relations. Kindergarten number relations explained a significant amount of variance in first grade mathematics achievement at the .2 quantile ($\beta = .26$, $p = .04$), but not at the .5 or .8 quantiles ($\beta = .16$, $p = .12$; $\beta = .01$, $p = .94$), when controlling for the other variables. Additionally, the estimate at the .8 quantile fell outside of the lower OLS 95% confidence interval [.08, .39], demonstrating that the relation between kindergarten number relations and first grade mathematics achievement at the .8 quantile differed from the OLS mean-level estimate; the relation at this point was weaker than the mean-level estimated relation.

Number Operations. Kindergarten number operations explained a significant amount of variance in first-grade mathematics achievement at the .5 ($\beta = .22$, $p = .02$) and .8 ($\beta = .49$, $p < .001$), but not at the .2 quantile ($\beta = -.04$, $p = .71$). Estimates at both the high (.8) and low (.2) quantiles fell outside of the OLS 95% confidence interval [.14, .41].

First Grade Predictors of Second Grade Mathematics Achievement

OLS Regression

All three of the first-grade number competence subdomains explained a significant amount of variance in mathematics achievement a year later, controlling for background variables. The full model accounted for 70% of the variance, F (7, 142) = 48.10, $p < .001$ (see Table 4).

Table 4

OLS Regression and Unconditional Quantile Regression Results for First Grade Sample with Second Grade Mathematics Achievement as the Dependent Variable

	OLS			Quantile Coefficients		
Variable	β	CI 95% low	CI 95% high	0.2	0.5	0.8
Number	**0.24**	.08	.41	.20	**.22**	.18
Number Relations	**0.18**	.03	.33	**.26**	.17	.12
Number Operations	**0.40**	.23	.56	.12⁻	**.32**	**.40**
Female	-0.07	-.16	.02	-.06	**-.13**	-.07
Age	0.04	-.06	.13	.02	.04	.001
Low-income	**-0.12**	-.23	-.02	-.02	-.08	-.06
Multilingal	-0.02	-.12	.08	-.13	-.04	-.03

Note. OLS Model $R^2 = .70$.
Bolded coefficients are significant at $p < .05$.
$^{+}$ Exceeded upper bound of OLS 95% CI. $^{-}$ Exceeded lower bound of OLS 95% CI.

Quantile Regression

Finally, unconditional quantile regression was used to predict second-grade mathematics achievement at the .2, .5, and .8 quantiles of the performance distribution from first grade number, number relations, and number operations, controlling for background variables (see Figure 1 and Table 4).

Number. First grade number knowledge explained a significant amount of variance in second-grade mathematics achievement at the .5 (β =.22, p = .04) but not the .2 or .8 quantiles (β =.20, p = .10; β =.18, p = .11, respectively). However, none of the estimates fell outside of the OLS 95% confidence interval [.08, .41], demonstrating that the relation between first grade number abilities and second grade mathematics achievement was similar across the achievement distribution, and was well-represented by the sample mean-level estimate.

Number Relations. First grade number relations knowledge explained a significant amount of variance in second grade mathematics achievement, when controlling for the other variables, at the .2 quantile (β =.26, p = .02) but not the .5 or .8 quantiles (β =.17, p = .08; β =.12, p = .26, respectively). However, none of the estimates fell outside of the OLS 95% confidence interval [.03, .33], demonstrating that the relation between first grade number relations abilities and second-grade mathematics achievement was similar across the achievement distribution, and was well-represented by the sample mean-level estimate.

Number Operations. As in the pre-K and kindergarten results, the domain of number operations in first grade explained a significant amount of variance in mathematics achievement a year later at the .5 (β = .32, p = .003) and .8 (β = .38, p = .007) quantiles, but not at the .2 quantile (β = .12, p = .34), when controlling for the other variables. Additionally, the estimate at the .2 quantile fell outside of the lower OLS 95% confidence interval bound [.23, .56]. As in the two younger samples, the predictive relation at the .2 quantile was weaker than the estimated mean-level relation.

Discussion

The predictive relation between early number competence and later mathematics outcomes is well established (Jordan et al., 2009; Passolunghi & Lanfranchi, 2012). However, within the core construct there are three relatively distinct but related subdomains of number, number relations, and operations (National Research Council, 2009; Purpura & Lonigan, 2013; Milburn et al., 2019). The present study shows that the relative contribution of these subdomains to future mathematics achievement in the crucial pre-K to first grade period varies. Furthermore, the present findings demonstrate that the relations between the number competence subdomains and future mathematics achievement sometimes differ based upon mathematics achievement quantile.

Predictive Relations Between the Subdomains of Early Number Competence and Future Mathematics Achievement at the Mean-level

Our first two research questions addressed whether each of the subdomains would explain a significant amount of variance in mathematics achievement a year later at the mean-level, and whether these predictive relations would vary by grade level. We examined the relative importance of number, number relations, and number operations in predicting mathematics achievement across the pre-K to first grade period, while controlling for the other domains as well as age, income-level, gender, and multilingual learner status. As expected, our models at each grade level accounted for a large share of the variance in mathematics achievement a year later (pre-K to kindergarten 74% of variance; from kindergarten to first grade 66% of variance; and from first grade to second grade 70% of variance). Across grade levels, each subdomain made significant contributions to mathematics achievement a year later, supporting the importance of all three subdomains of early number competencies for later mathematics achievement, despite their shared relations.

Predicting Mathematics Achievement at Low, Intermediate, and High Levels of Achievement

Our next research question asked whether the relative predictive strength of the number sense subdomains at specific quantiles of mathematics achievement differed from the mean-level estimate. To determine whether the relations were similar to the mean-level estimated relations across the achievement distribution, we examined predictive relations of the subdomain at performance quantiles representing low, intermediate, and high levels of future mathematics achievement at each grade. We found that the mean-level estimates from the OLS models did not well-represent the predictive relations for the number or number operations subdomains in the pre-K sample, for the number relations or number operations subdomains in the kindergarten sample, and for the number operations subdomain in the first grade sample. These analyses demonstrate that although all three subdomains of number competence explain a significant amount of variance in mathematics achievement a year later at the mean-level, these predictive relations, in some cases, are contingent upon achievement level. For example, the strong predictive relation between pre-K number and kindergarten mathematics achievement identified at the mean-level appears to be mostly driven by relations at the low and intermediate points of the mathematics achievement distribution, as the estimate at the .8 quantile was not significant and fell outside of the 95% confidence interval of the mean-level estimate.

The most robust finding from the unconditional quantile regression analysis was that across grades, the conditional relation between number operations and mathematics achievement a year later followed a similar pattern. The relation became stronger at higher levels of the achievement distribution, with no significant relation near the bottom of the distribution at the .2 quantile, and strong relations near the top of the distribution, at the .8 quantile. These findings echo grade-based differences in the OLS regressions where examination of the standardized coefficients revealed relatively larger coefficients for the number subdomain than the others in pre-K and for the number operations subdomain than the others in first grade. It may be that children who grasp basic aspects of counting and cardinality (measured in the number subdomain) and relative magnitude (measured in the number relations subdomain) leverage these skills to reason about operations, resulting in the pattern of number and number relations being the strongest independent predictors of achievement for lower achievers, and skill with operations the strongest predictor for the higher achievers. Previous work has shown that cardinality usually develops prior to specific number relations abilities (Knudsen et al., 2015) and may in turn help children to solve basic number operations (Scalise & Ramani, 2020).

However, it is also likely that there are more basic and more advanced abilities within each subdomain that are developing concurrently (Frye et al. 2013), supported by the finding that all three subdomains were significant contributors to variance in future mathematics achievement at the mean- level. Developmental changes within and between subdomains are likely related to shifts from working with smaller to larger set sizes and non-symbolic to symbolic stimuli. For example, a child in pre-K who can subitize and understand the cardinality of small set sizes may be able to engage in the number operations skill of the addition and subtraction of small sets of objects before being able to engage with the number relations skills of comparing the magnitude of two numerals.

The quantile regression results also shed light on the relative importance of each subdomain for children who performed near the chosen quantiles of achievement, and the results at the .2 quantile are of particular interest, as these children may be at risk for mathematics learning difficulties. In the pre-K sample, for children at the .2 quantile of mathematics achievement in kindergarten, number skills including counting and cardinality and number relations skills including magnitude comparison and linear representations of number were the only subdomains predictive of achievement. However, once children entered formal schooling, the number relations subdomain was the only significant independent predictor of later achievement for children near the lowest quantile of the distribution. The items included in the number relations core at each grade-level form of the *SENS* were primarily focused on assessing an understanding of relative numerical magnitude, with forms including more symbolic representations in the older grades. Several studies have noted a primary deficiency in numerical magnitude processing in children with mathematical disabilities, particularly when working with symbolic magnitudes (Geary et al., 2008; Rousselle & Noël, 2007). Individual differences in relative magnitude understanding, then, may be most predictive for children in the lower end of the distribution. These items could be differentiating children with underlying symbolic magnitude-processing difficulties and mathematical learning difficulties from other low-achieving children.

Limitations

Several limitations should be considered when interpreting the present findings. The cross-sequential nature of the samples makes direct comparison by grade difficult; instead we report differences and similarities in patterns between grades. A second limitation is the lack of geographical variation in the present sample, although it was diverse in terms of ethnicity and

SES. In addition, reliability of the present measures should be considered when interpreting these results. In particular, the pre-K number operations subdomain measure had lower average internal consistency than the other subdomains and acceptable reliability across a smaller range of latent ability. Therefore, results using this measure should be interpreted cautiously. Other subdomain measures demonstrated higher reliability at different points of the ability spectrum that could have influenced results of the quantile regression patterns of significance. For example, adequate internal consistency for the number relations measures extended lower down the latent ability spectrum than higher up the ability spectrum. However, most of the measures of subdomain knowledge demonstrated adequate reliability across a large range of the ability spectrum. That is, all measures except the pre-K number operations subdomain had adequate reliability from theta scores that ranged from one standard deviation below to one standard deviation above the average.

It should also be noted that the outcome measure of the TEMA-3 included items that assess informal and formal mathematical concepts and the assessment generally moves from more informal to more formal content and from smaller to larger set sizes. Although content representing each of the three number competence subdomains is represented throughout the measure, we cannot fully rule out an alternate explanation for the present quantile regression findings that they are reflective of the types of items at different points of the outcome measure.

Finally, this study was limited to number specific skills, and thus no domain general skills (i.e., working memory) were measured. Research has demonstrated that domain general skills, such as executive function and language (Geary et al., 2011; Hornung et al., 2014; Passolunghi et al., 2007) as well as number-specific skills may independently contribute to later mathematics achievement. Future work should explore the relative importance of domain general skills in models with number competencies across the distribution of future mathematics achievement. Future works should also consider the role of environmental factors beyond controlling for the background variables of age, gender, income-level, and home language. Research has demonstrated the importance of home and school factors for predicting mathematics achievement (e.g., the home numeracy environment, Niklas & Schneider, 2014; quality and quantity of parent math talk, Berkowitz et al., 2015; in-class attention, Fuchs et al., 2006; and teacher math talk, Kilbanoff, et al., 2006). Past work has also shown that mathematics achievement is related to the level of English exposure in the home (Reardon & Galindo, 2009), a detail not captured by our multilingual learner variable. Future studies should explore how these environmental variables, along with early number skills, relate to mathematics achievement across the achievement distribution.

Educational Implications

The present study solidifies and broadens our understanding of the relation between subdomains of early number competence and later mathematics achievement. Results from the OLS regression analyses show the importance of the early three subdomains to mathematics achievement, on average. Quantile regression analyses highlight the heterogeneity of some of these relations according to achievement level, specifically for the number operations subdomain, as estimates followed a consistent pattern across grade-levels.

These results have implications for designing instructional programs. Typically, less instructional time is given in U.S. classrooms in the early years to mathematics than literacy (Litkowski et al., 2020; Piasta et al., 2014), and, when number competencies are the focus, early educators may underestimate children's abilities. Specifically, Engel's (2016) review of kindergarten classroom practices found that mathematics instruction time was most often focused on basic counting over arithmetic operations. Concentrating instruction on only the most basic skills in the number subdomain may be detrimental for higher achievers who may have already mastered those skills (Engel et al., 2016). As all three subdomains of number competence are related to later mathematics achievement at the mean-level, it may be important to engage in instructional approaches that simultaneously target abilities from various subdomains from early on.

On the other hand, the quantile regression results highlight specific areas to concentrate instruction based upon individual differences in mathematics achievement. Many school districts use multi-tiered systems of supports where interventions take place during a specified block of time in the school day in which children are grouped by low, intermediate, and high levels of mathematics achievement (Fuchs et al., 2020). The present study's results may point to specific concepts to focus instruction for each group. For example, results suggest that once children enter formal schooling, number relations concepts including symbolic quantity comparison and understanding of the number line are the most predictive number competencies of future achievement for those at the low quantile of achievement, and may be an area for concentrated instruction or assessment for struggling learners.

One strategy to support early childhood teachers would be to use assessments to provide a profile of the abilities that children in their classroom have developed in each of the subdomains. A recent study by Raudenbush et al. (2020) found positive effects of an assessment and instruction program that supported preschool teachers in tailoring instruction to their students' needs by assessing three times across the school year on early number competencies that included number, number relations, and number operations.

References

Akinwande, M. O., Dikko, H. G., & Samson, A. (2015). Variance inflation factor: as a condition for the inclusion of suppressor variable (s) in regression analysis. *Open Journal of Statistics*, *5*(07), 754-768. https://doi.org/ 10.4236/ojs.2015.57075

Berkowitz, T., Schaeffer, M. W., Maloney, E. A., Peterson, L., Gregor, C., Levine, S. C., & Beilock, S. L. (2015). Math at home adds up to achievement in school. *Science*, *350*(6257), 196–198.

Berry, W. D., Feldman, S., & Stanley Feldman, D. (1985). *Multiple regression in practice* (50). Sage.

Borah, B. J., & Basu, A. (2013). Highlighting differences between conditional and unconditional quantile regression approaches through an application to assess medication adherence. *Health Economics*, *22*(9), 1052-1070. https://doi.org/10.1002/hec.2927

Clements, D. H., & Sarama, J. (2007). Early childhood mathematics learning. *Second handbook of research on mathematics teaching and learning*, *1*, 461-555.

Clements, D. H., Sarama, J., & MacDonald, B. L. (2019). Subitizing: The neglected quantifier. In *Constructing number* (pp. 13-45). Springer.

De Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement. *Journal of Experimental Child Psychology*, *103*(4), 469-479. https://doi.org/10.1016/j.jecp.2009.01.010

Dong, Y., Clements, D. H., Sarama, J., Dumas, D., Banse, H. W., & Day-Hess, C. (2020). Mathematics and Executive Function Competencies in the Context of Interventions: A

Quantile Regression Analysis. *The Journal of Experimental Education*, 1-22. https://doi.org/10.1080/00220973.2020.1777070

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Pagani, L. S., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446. https://doi.org/10.1037/0012-1649.43.6.1428

Engel, M., Claessens, A., Watts, T., & Farkas, G. (2016). Mathematics content coverage and student learning in kindergarten. *Educational Researcher*, *45*(5), 293-300.

Firpo, S., Fortin, N. M., & Lemieux, T. (2009). Unconditional quantile regressions. *Econometrica*, *77*(3), 953-973. https://doi.org/10.3982/ecta6822

Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C., & McDowell, J. (2013). Teaching math to young children: A practice guide (NCEE 2014-4005).

Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., Schatschneider, C., & Fletcher, J. M. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology*, *98*(1), 29–43. https://doi.org/10.1037/0022-0663.98.1.29

Fuchs, L. S., Powell, S. R., Fall, A. M., Roberts, G., Cirino, P., Fuchs, D., & Gilbert, J. K. (2020). Do the processes engaged during mathematical word-problem solving differ along the distribution of word-problem competence? *Contemporary Educational Psychology*, *60*. https://doi.org/10.1016/j.cedpsych.2019.101811

Fuson, K. C. (1992). Research on whole number addition and subtraction. In *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 243–275). Macmillan Publishing Co, Inc.

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: a 5-year longitudinal study. *Developmental Psychology*, *47*(6), 1539-1552.

Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology*, *33*(3), 277-299. https://doi.org/10.1080/87565640801982361

Geary, D. C., & vanMarle, K. (2016). Young children's core symbolic and nonsymbolic quantitative knowledge in the prediction of later mathematics achievement. *Developmental Psychology*, *52*(12), 2130. https://doi.org/10.1037/dev0000214

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Harvard University Press.

Ginsburg, H. P., & Baroody, A. J. (2003). TEMA-3: Test of Early Mathematics Ability–Third Edition.

Hajovsky, D. B., Villeneuve, E. F., Schneider, W. J., & Caemmerer, J. M. An Alternative Approach to Cognitive and Achievement Relations Research: An Introduction to Quantile Regression. *Journal of Pediatric Neuropsychology*, *6*, 83–95. https://doi.org/10.1007/s40817-020-00086-3

Hornung, C., Schiltz, C., Brunner, M., & Martin, R. (2014). Predicting first-grade mathematics achievement: The contributions of domain-general cognitive abilities, nonverbal number sense, and early number competence. *Frontiers in Psychology*, *5*, 1-18. https://doi.org/10.3389/fpsyg.2014.00272

Huttenlocher, J., Jordan, N. C., & Levine, S. C. (1994). A mental model for early arithmetic. *Journal of Experimental Psychology: General*, *123*(3), 284-296. https://doi.org/10.1037/0096-3445.123.3.284

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences*, *20*(2), 82-88.

Jordan, N. C., Kaplan, D., Nabors Oláh, L., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development*, *77*(1), 153-175. https://doi.org/10.1111/j.1467-8624.2006.00862.x

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*(3), 850-867. https://doi.org/10.1037/a0014939

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews*, *15*(1), 60-68. https://doi.org/10.1002/ddrr.46

Koenker, R. (2015). *Quantile regression.* International encyclopedia of the social & behavioral sciences (2nd ed., pp. 712–718).

Knudsen, B., Fischer, M., & Aschersleben, G. (2015). The development of Arabic digit knowledge in 4-to-7-year-old children. *Journal of Numerical Cognition*, *1*(1), 21-37.

Lira, C. J., Carver, M., Douglas, H., & LeFevre, J. A. (2017). The integration of symbolic and non-symbolic representations of exact quantity in preschool children. *Cognition*, *166*, 382-397. https://doi.org/10.1016/j.cognition.2017.05.033

Litkowski, E. C., Duncan, R. J., Logan, J. A., & Purpura, D. J. (2020). When do preschoolers learn specific mathematics skills? Mapping the development of early numeracy knowledge. *Journal of Experimental Child Psychology*, *195*, 1-25. https://doi.org/10.1016/j.jecp.2020.104846

Locuniak, M. N., & Jordan, N. C. (2008). Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities*, *41*(5), 451-459. https://doi.org/10.1177/0022219408321126

McIlraith, A. L. (2018). Predicting word reading ability: a quantile regression study. *Journal of Research in Reading*, *41*(1), 79-96.

Milburn, T. F., Lonigan, C. J., DeFlorio, L., & Klein, A. (2019). Dimensionality of preschoolers' informal mathematical abilities. *Early Childhood Research Quarterly*, *47*, 487-495. https://doi.org/10.1016/j.ecresq.2018.07.006

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). Common Core State Standards for mathematics.

National Research Council. (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. National Academies Press.

Niklas, F., & Schneider, W. (2014). Casting the die before the die is cast: The importance of the home numeracy environment for preschool children. *European Journal of Psychology of Education*, *29*(3), 327–345. https://doi.org/10.1007/s10212-013-0201-6

Passolunghi, M. C., & Lanfranchi, S. (2012). Domain-specific and domain-general precursors of mathematical achievement: A longitudinal study from kindergarten to first grade. *British Journal of Educational Psychology*, *82*(1), 42-63. https://doi.org/10.1111/j.2044-8279.2011.02039.x

Passolunghi, M. C., Vercelloni, B., & Schadee, H. (2007). The precursors of mathematics learning: Working memory, phonological ability and numerical competence. *Cognitive Development*, *22*(2), 165-184. https://doi.org/10.1016/j.cogdev.2006.09.001

Penner, A. M., & Paret, M. (2008). Gender differences in mathematics achievement: Exploring the early grades and the extremes. *Social Science Research*, *37*(1), 239-253.

Piasta, S. B., Pelatti, C. Y., & Miller, H. L. (2014). Mathematics and science learning opportunities in preschool classrooms. *Early Education and Development*, *25*(4), 445-468.

Purpura, D. J., & Logan, J. A. R. (2015). The nonlinear relations of the approximate number system and mathematical language to early mathematics development. *Developmental Psychology*, *51*(12), 1717–1724. https://doi.org/10.1037/dev0000055

Purpura, D. J., & Lonigan, C. J. (2013). Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal*, *50*(1), 178-209. https://doi.org/10.3102/0002831212465332

Purpura, D. J., & Lonigan, C. J. (2015). Early numeracy assessment: The development of the preschool early numeracy scales. *Early Education and Development*, *26*(2), 286-313.

Raudenbush, S. W., Hernandez, M., Goldin-Meadow, S., Carrazza, C., Foley, A., Leslie, D., ... & Levine, S. C. (2020). Longitudinally adaptive assessment and instruction increase numerical skills of preschool children. *Proceedings of the National Academy of Sciences*, *117*(45), 27945-27953. https://doi.org/10.1073/pnas.2002883117

Reardon, S. F., & Galindo, C. (2009). The Hispanic-White achievement gap in math and reading in the elementary grades. *American Educational Research Journal*, *46*(3), 853-891.

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition*, *102*(3), 361-395. https://doi.org/10.1016/j.cognition.2006.01.005

Sasanguie, D., De Smedt, B., Defever, E., & Reynvoet, B. (2012). Association between basic numerical abilities and mathematics achievement. *British Journal of Developmental Psychology*, *30*(2), 344-357. https://doi.org/10.1111/j.2044-835x.2011.02048.x

Scalise, N. R., & Ramani, G. B. (2021). Symbolic magnitude understanding predicts preschoolers' later addition skills. *Journal of Cognition and Development*, *22*(2), 185–202. https://doi.org/10.1080/15248372.2021.1888732

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, *75*(2), 428-444.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly*, *19*(1), 99-120. https://doi.org/10.1016/j.ecresq.2004.01.002

Titeca, D., Roeyers, H., Josephy, H., Ceulemans, A., & Desoete, A. (2014). Preschool predictors of mathematics in first grade children with autism spectrum disorder. *Research in Developmental Disabilities*, *35*(11), 2714-2727. https://doi.org/10.1016/j.ridd.2014.07.012

Wenz, S. E. (2019). What quantile regression does and doesn't do: A commentary on Petscher and Logan (2014). *Child Development*, *90*(4), 1442-1452. https://doi.org/10.1111/cdev.13141

WIDA. (2020). *WIDA English language development standards framework, 2020 edition: Kindergarten–grade 12.* Board of Regents of the University of Wisconsin System.

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, *24*, 220-251.

Xu, C., & LeFevre, J. A. (2020). Children's knowledge of symbolic number in grades 1 and 2: Integration of associations. *Child Development*. https://doi.org/10.1111/cdev.13473