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ABSTRACT

Adults use a variety of strategies to reason about fraction magnitudes, and this variability is adaptive. In two studies, we examined the relationships between mathematics anxiety, working memory, strategy variability and performance on two fraction tasks: fraction magnitude *comparison* and *estimation*. Adults with higher mathematics anxiety had lower accuracy on the comparison task and greater percentage absolute error (PAE) on the estimation task. Unexpectedly, mathematics anxiety was not related to variable strategy use. However, variable strategy use was linked to more accurate magnitude comparisons, especially among adults with lower working memory performance or those who use mathematics less frequently, as well as lower PAE on the estimation task. These findings shed light on the role of strategy variability in fraction problem solving and demonstrate a link between mathematics anxiety and fraction magnitude reasoning, a key predictor of general mathematics achievement.

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KEYWORDS Strategy; strategy variability; numerical cognition; fraction reasoning; mathematics anxiety

Introduction

People's behaviour is inherently variable. The strategies that one uses to solve a problem might change across time or even from problem to problem within a single time point (e.g., Alibali & Sidney, 2015a; Siegler, 1996, 2007; van der Ven, Boom, Kroesbergen, & Leseman, 2012). Variability within an individual's behaviour is common across a range of tasks, including calculating (e.g., Siegler, 1987; Siegler & Crowley, 1991; van der Ven et al., 2012), number conservation (Siegler, 1995), spelling (e.g., Rittle-Johnson & Siegler, 1999) and remembering (e.g., Coyle & Bjorklund, 1997). Furthermore, this variability may

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even be adaptive for learning and problem solving (e.g., Lemaire & Siegler, 1995; Siegler, 1995).

In this paper, we consider the role of strategy variability in one especially important domain of mathematics, fraction problem solving (Schneider & Siegler, 2010; Siegler, Fazio, Bailey, & Zhou, 2013; Siegler, Thompson, & Schneider, 2011) and examine the relationships between adults' strategy variability and their individual differences in mathematics anxiety and working memory. Mathematics anxiety is thought to affect performance on mathematics tasks due to the demands that anxiety processes place on students' working memory (e.g., Ashcraft & Kirk, 2001). Here, we further suggest that mathematics anxiety may play a role in students' variable strategy use. Although prior research has revealed connections across strategy variability, working memory and performance on fraction tasks (e.g., Fazio, DeWolf, & Siegler, 2016; Siegler & Pyke, 2013), this study is the first to explore the relationships between adults' mathematics anxiety and strategy variability during mathematics problem solving.

Fraction reasoning and strategy variability

When children and adults solve mathematical problems, they often rely on several different strategies for solving similar problems, which vary in their efficiency and likelihood of resulting in an accurate solution (e.g., Fazio et al., 2016; Siegler & Crowley, 1991; van der Ven et al., 2012). In contrast to viewing this variability as problematic for assessing knowledge, Siegler (1996, 2006) proposed that strategy variability during problem solving is adaptive for learning and problem solving. Variability allows solvers to adaptively tailor their strategy use to the demands of the current problem (Siegler, 1987, 1988; Siegler & Shrager, 1984), and increased strategy variability is associated with better performance (e.g., Coyle & Bjorklund, 1997; Fazio et al., 2016).

In the current study, we focus on adults' strategies while reasoning about the magnitudes of fraction symbols (i.e., numbers represented as A/B). Across studies (e.g., Bonato, Fabbri, Umilta, & Zorzi, 2007; DeWolf & Vosniadou, 2015; Meert, Grégoire, & Noël, 2010; Schneider & Siegler, 2010), adults appeared to employ several strategies for reasoning about fraction magnitudes, including considering the magnitudes of the components (i.e., the numerator or the denominator), estimating the magnitude of a fraction by considering a related fraction (e.g., estimating 2A/2B by considering the magnitude of A/B) and directly estimating the magnitude of the given ratio. Alibali and Sidney (2015b) noted that the variability in strategy use across studies may be due to variation in the specific fractions considered (e.g., 1/3 and 1/5 as compared to 13/17 and 3/25), in participants' fraction experiences, in the strength and activation of their related knowledge and in the contexts of the tasks. Some combinations of features afford efficient and accurate reasoning and other combinations may afford inaccurate strategies. For example, when adults are asked to compare fractions that all have one in the numerator (e.g., 1/3 and 1/5), they are likely to consistently make comparisons based only on the magnitude of the denominator component (Bonato et al., 2007). In contrast, when the problems vary across numerator and denominator components, adults do not rely only on the denominator to judge relative magnitude (Schneider & Siegler, 2010).

Recently, Fazio et al. (2016) directly examined variability in adults' strategies used to compare the magnitude of two fractions, demonstrating high levels of intra-individual variability across trials that afforded different strategies. Similarly, Siegler and Thompson (2014) found that 10- and 11-year-old children used a variety of strategies when asked to estimate the location of a given fraction on 0 to 1 and 0 to 5 number lines. Taken together, these findings demonstrate that children's and adults' reasoning about fraction magnitudes is a strategic process. Because adults use many strategies as they reason about fractions, the domain of fraction magnitude understanding is an apt test case to investigate strategy variability in greater depth.

Furthermore, we chose to test our hypotheses about strategy variability in the context of adults' reasoning about fraction magnitudes because fraction magnitude understanding is a crucial aspect of the development of students' mathematics knowledge. Students in the USA consistently fall behind in mathematics understanding as compared to their peers in other, similarly developed nations (e.g., OECD, 2014, 2016). Understanding the pathways to success in mathematics, and mathematics-related fields, is critically important in the current educational and economic climate. Several recent longitudinal studies of students' mathematics development have pointed to students' understanding of fraction magnitudes as a key predictor of later mathematics achievement (e.g., Siegler et al., 2012), even after controlling for earlier achievement, cognitive factors (e.g., working memory) and social factors (e.g., family income and education). In this study, we sought to better understand the role that students' strategic knowledge plays in their fraction magnitude performance.

Finally, we examine strategy variability in adults' fraction magnitude reasoning because strategy variability is advantageous in this domain. Fazio et al. (2016) found that indeed, adult college students who were more accurate in their comparisons also used significantly more strategies. Furthermore, not only did the number of strategies differ across higher-performing and lower-performing students, but their strategy choices differed as well. Participants who were more accurate at comparing fractions were also more likely to use strategies that were directly afforded by the specific fractions given in the problem (e.g., when reasoning about the magnitude of 7/13, using a strategy that involves comparison to the magnitude of 1/2). In contrast, many participants with lower accuracy relied on strategies that would not consistently result in accurate comparisons, and did not appear to tailor their strategy use to specific problems. In this study, Fazio and colleagues demonstrated that not only do adults have a high degree of intra-individual strategy variability across trials, but that this variability can afford adaptive strategy choices, and likely affects adults' accuracy.

To our knowledge, little direct evidence suggests that strategy variability may be similarly advantageous for adults' fraction magnitude *estimation*. In the number line task, some strategies are associated with high accuracy across all types of trials (e.g., transforming fractions into decimals) and some strategies are associated with low accuracy across all types of trials (e.g., relying only on the denominator magnitude; Siegler & Thompson, 2014). Thus, strategy variability may not necessarily improve accuracy over relying on a single, optimal strategy. However, the more advantageous strategies have a common feature: they involve reasoning about the fraction's holistic magnitude, by translating that magnitude into a more familiar number (i.e., a decimal or mixed number), relating the magnitude to a given landmark or segmenting the number line and relating the magnitude to the subjective landmarks (see Siegler & Thompson, 2014). Therefore, strategy variability in the fraction number line estimation task may be advantageous in that using more strategies may afford a greater variety of ways of thinking about holistic magnitude.

Despite these rich findings on students' strategy variability, it remains unclear *why* some students use richer strategy sets to reason about fractions than others, and also *why* some students are more adaptive in their strategy use. One hypothesis is that students with more mathematics knowledge use more problem-solving strategies. However, differences in mathematics knowledge may not fully account for differences in strategy use. For example, even though students from Fazio et al.'s (2016) highly selective university sample reported higher college admissions exam (SAT) scores than students in their high performing community college sample, strategy variability across these subsamples was comparable. These questions open up several avenues for examining the role of individual differences among students, apart from mathematical knowledge, that may contribute to differences in strategy use and result in differences in their fraction magnitude reasoning. In the current study, we examined the role of students' mathematics anxiety in their strategic behaviour in fraction magnitude tasks.

Mathematics anxiety

Broadly, mathematics anxiety has been characterised as fear, nervousness, discomfort or anxiety that some people feel when taking mathematics tests (e.g., Ashcraft, 2002; Ashcraft & Moore, 2009; Beilock & Maloney, 2015; Hembree, 1990), doing mathematical calculations in the context of everyday activities such as calculating a tip (e.g., Maloney & Beilock, 2012) or anticipating future mathematics activities (e.g., Lyons & Beilock, 2012). Both adults (e.g., Ashcraft & Kirk, 2001; Maloney, Ansari, & Fugelsang, 2011; Wang et al., 2015) and children (e.g., Hembree, 1990; Ramirez, Gunderson, Levine, & Beilock, 2013; Vukovic, Kieffer, Bailey, & Harari, 2013) with higher mathematics anxiety show decreases in mathematics performance relative to their lower anxiety peers. Mathematics anxiety can even affect basic numerical reasoning, such as estimating the size of whole numbers (Wang et al., 2015), comparing one-digit numbers (Maloney et al., 2011) and enumerating visual objects (Maloney, Risko, Ansari, & Fugelsang, 2010). Mathematics anxiety is related to, but distinct from, test anxiety (e.g., Hembree, 1990) and general anxiety (e.g., Eysenck, Derakshan, Santos, & Calvo, 2007) in that it appears to be specific to mathematics stimuli, such as symbolic numbers and numerical calculations (e.g., Ashcraft & Kirk, 2001).

For many years, researchers have been interested in the relationship between mathematics anxiety, attitudes towards mathematics, course-taking and mathematics achievement (e.g., Hembree, 1990; Richardson & Suinn, 1972; Richardson & Woolfolk, 1980), without deeply considering the underlying cognitive mechanisms. More recently, several researchers, including Ashcraft and colleagues (e.g., Ashcraft, 2002; Ashcraft & Kirk, 2001; Ashcraft, Krause, & Hopko, 2007) and Beilock and Maloney and colleagues (e.g., Beilock, 2008; Maloney & Beilock, 2012; Ramirez et al., 2013), have brought together research on mathematics anxiety, general anxiety and mathematics cognition to develop a rich, and evolving, theory of the mechanisms and predictors of mathematics anxiety.

Prevailing theories propose that students' mathematics anxiety affects their performance due to its disruptive effects on students' available working memory resources for problem solving (e.g., Ashcraft, 2002; Ashcraft & Kirk, 2001; Ashcraft & Krause, 2007; Beilock, 2008). Similar to the theorised mechanisms of stereotype threat (e.g., Beilock, Rydell, & McConnell, 2007), and anxiety more generally (e.g., Eysenck & Calvo, 1992, Eysenck et al., 2007), students with mathematics anxiety experience negative thoughts before and during mathematical tasks which compete for students' available working memory resources, thus reducing the availability of those resources for the mathematical task at hand (e.g., Ashcraft, 2002; Beilock, 2008; Ramirez et al., 2013). More specifically, both Ashcraft and colleagues (Ashcraft & Kirk, 2001; Hopko, Ashcraft, Gute, Ruggiero, & Lewis, 1998) and Eysenck et al. (2007) have theorised that mathematics anxiety affects students' executive control functions in particular (see Miyake et al., 2000 for a discussion of executive control functions).

Strategy variability and anxiety

Adults' executive functioning, and their working memory resources more generally, have also been implicated in their ability to choose adaptively amongst strategies in mathematics tasks (e.g., Beilock & DeCaro, 2007; Hodzik & Lemaire, 2011), suggesting a potential link between students' mathematics anxiety and their strategy variability and strategy selection during mathematical problem solving. In particular, strategy variability during mathematics problem solving is associated with differences in executive function in both children (e.g., Lemaire & Brun, 2016; Lemaire & Lecacheur, 2011) and adults (e.g., Hodzik & Lemaire, 2011; Lemaire & Leclère, 2014). For example, Hodzik and Lemaire (2011) found that older adults used fewer strategies for addition and multiplication problems than younger adults and they were less likely than younger adults to choose the most efficient strategy for each problem. Furthermore, age-related differences in adults' executive functioning fully mediated these effects; older adults were less strategic due to lower executive functioning. In line with these findings, Lemaire and Brun (2016) suggested that young children perseverate on a specific strategy in part because of the limits of their executive functioning. This body of research connecting executive functioning to strategy variability in mathematics tasks provides a pathway through which mathematics anxiety might affect mathematics performance through adaptive strategy use.

Furthermore, mathematics anxiety has been linked to students' strategy choices during mathematics tasks, though not strategy variability specifically (Beilock & DeCaro, 2007; Ramirez et al., 2013). For example, Ramirez et al. (2013) found that children with higher working memory performance were more affected by mathematics anxiety than those with lower working memory performance, because they often relied on complex strategies that took advantage of their ability to manipulate a larger amount of information. These findings suggest that mathematics anxiety may be related to strategy choice, in that it may be more difficult to use computationally rich strategies while anxious, but that this effect may be moderated by students' working memory performance.

These studies suggest that mathematics anxiety may constrain the set of strategies that participants are able to use successfully in multiple ways. Participants with mathematics anxiety may perseverate on a smaller set of easier-to-implement strategies due to difficulty switching strategies or due to difficulty using both computationally rich and simpler strategies.

The current studies

In two studies, we explored the role of students' mathematics anxiety and working memory in their strategy variability and accuracy during two fraction reasoning tasks. First, given the importance of students' fraction magnitude understanding in mathematics achievement more generally, we examined whether there is a relationship between students' mathematics anxiety and fraction performance. In line with prior research linking students' mathematics anxiety to whole number magnitude estimation and fraction arithmetic proficiency (Wang et al., 2015), we expected that those with higher anxiety would be less accurate on fraction magnitude tasks.

Second, we examined whether students' mathematics anxiety was related to their strategy variability, and whether this relationship could account for the relationship between anxiety and performance. We expected this to be the case, given that students' mathematics anxiety affects their availability and control of working memory resources during mathematical problem solving (e.g., Ashcraft, 2002; Eysenck et al., 2007) and that differences in students' working memory resources have been linked to differences in mathematics strategy use (e.g., Hodzik & Lemaire, 2011) and fraction reasoning (e.g., Siegler & Pyke, 2013). In other words, we hypothesised that students' strategy variability on fraction tasks would mediate the relationship between mathematics anxiety and performance. In this study, we examined two facets of strategy variability: variability per se, measured by the total number of strategies that participants used, and adaptiveness of strategy use, measured by the frequency of using strategies that would most likely result in optimal performance on specific trials of a given task.

We chose to investigate the relationships between mathematics anxiety, strategy use and mathematics performance in the context of students' fraction magnitude reasoning. Importantly, our study is the first to examine whether students' mathematics anxiety negatively affects their fraction magnitude reasoning. Participants completed two types of fraction magnitude reasoning tasks: a fraction magnitude comparison task (e.g., Fazio et al., 2016; Siegler et al., 2011) and a fraction number line estimation task (e.g., Hamdan & Gunderson, 2017; Siegler & Thompson, 2014; Siegler et al., 2011). Although previous research has revealed variability in students' number line estimation strategies (e.g., Siegler & Thompson, 2014), no study has directly examined the effects of strategy variability on estimation accuracy.

Finally, we also included a working memory task, given the important role that students' working memory resources play in mathematics anxiety and strategy choice (e.g., Ashcraft & Kirk, 2001; Hodzik & Lemaire, 2011). Here, we used a task that had been used in prior research demonstrating a link between working memory performance and children's fraction reasoning (Siegler & Pyke, 2013). We had no a priori hypothesis about the role of working memory in our hypothesised mediation pathway. However, we anticipated one or more roles in our hypothesised pathway. We expected that working memory could (1) moderate the effects of mathematics anxiety, following similar findings from Ramirez et al. (2013), (2) have an independent effect on strategy variability, following findings from Lemaire and Brun (2016) or (3) moderate the effect of strategy variability on performance, as strategy variability may be more impactful in some students than others. Thus, to test these hypotheses, we first examined whether strategy variability mediated the relationship between students' mathematics anxiety and fraction performance, and then conducted exploratory analyses to determine the specific role of working memory.

We tested our hypotheses in two studies with parallel designs (see Open Science Collaboration, 2015 for a discussion of the importance of replication). In both studies, we measured students' mathematics anxiety, working memory, mathematics background and strategy use and performance on two fraction magnitude tasks. However, given that we did not have an a priori hypothesis concerning the specific role of working memory in the pathway across mathematics anxiety, strategy variability and performance, we conducted exploratory analyses in Study 1. Then, we sought to replicate our data-driven model in Study 2. Our final conclusions are informed by both our exploratory findings in Study 1 and confirmatory findings in Study 2.

Study 1

Method

Participants

Participants were recruited through their psychology courses and participated in exchange for partial course credit. In total, 124 participants responded to at least one question in the survey. One participant was excluded from data analysis because of completing only one task. The final sample included 123 college students at a public university in the mid-western United States. The sample was representative of general psychology courses at this university (*M* age = 20.32 years, SD = 2.03; 81.3% women, 15.5% men, 0.8% "gender fluid," 2.4% not specified; 81.1% Caucasian, 7.4% African-American, 4.1% Asian, 3.3% Hispanic, 3.3% other and 0.8% not specified). Students from a variety of majors participated, with the largest group represented by psychology majors (35.4%). Most participants (93.5%) were native English speakers.

Design and procedure

Participants performed all experimental tasks via Qualtrics on their own computers at their convenience. All participants were given the same tasks in a random order, except for the mathematics anxiety measure and the demographic questionnaire, which were consistently given last to mask the purpose of the study during the other tasks.

Tasks

Participants completed five tasks.

Magnitude comparison. Participants were given 32 pairs of fractions with magnitudes less than 1, and asked to choose the larger fraction by clicking on it (see Table A1). There were eight types of problems which were adapted from Fazio et al. (2016). Within each problem type, the fractions in some pairs were multiplied by 3/3 to increase the difficulty (e.g., 10/17 vs. 13/15 and 30/51 vs. 39/45). In Study 1, an error was made whereby all but two of the larger fractions were positioned on the right side of the screen; this error was addressed in the design of Study 2. After each trial, participants were prompted to report their strategies in a text box. For each participant, the percentage accuracy was calculated as the number of trials on which an accurate response was given divided by the number of trials on which any response was given.

Number line estimation. Participants completed 20 number line estimation problems in the 0–5 range. The given fractions, based on Fazio et al. (2016), are listed in Table A2. An equal number of fractions spanned each fifth of the number line and included proper and improper fractions. Participants moved their cursor along the number line to indicate the given fraction's magnitude. After each trial, participants were prompted to report their strategies in a text box. To determine accuracy, we calculated the percentage absolute error (PAE) for each trial. PAE was calculated using the following formula: ((|Participant's Answer – Correct Answer|)/Number Line Scale) $\times$ 100. For example, if a participant was tasked with placing $1\frac{1}{2}$ on the number line and selected 8/11, then their PAE would be calculated as

$((|8/11 - 1\frac{1}{2}|)/5) \times 100$. PAE is inversely related to accuracy, such that responses that are closer to the correct magnitude have lower PAE. For each participant, we averaged the PAE across trials on which there was a response given.

Working memory updating. The working memory task was adapted from one used by Siegler and Pyke (2013). Participants saw 12 sequences of letters, ranging from 7 to 12 letters each, and were asked to recall the last 3 letters of each sequence. For example, when participants saw the sequence, "QDXRMTZ," they should have recalled "MTZ." Letters appeared on screen one at a time for 1.5 s each. All participants were presented the same initial sample task question. The remaining 11 sequences were presented in a random order. Accuracy was calculated as the percentage of correctly recalled letters out of a total of 36 letters (3 letters in each of 12 sequences).

Demographic questionnaire. Participants were asked to self-report their age, gender, race/ethnicity, academic major and minor, class rank and native language. Furthermore, we asked when participants took their last math class (from "currently enrolled" to "more than three years ago"), the total number of math classes they have taken in college so far from "none" to "6 or more", and how often they use mathematics skills in their majors from "0 (Never)" to "5 (Always)". We focused only on our measure of the frequency of participants' use of mathematics skills in their majors as an index of participants' *mathematics background*. Participants' last math class and total number of classes were likely related to class rank, and potentially confounded with participants' high school mathematics experience, and thus these variables were excluded from analyses.

Mathematics anxiety. Participants completed the Abbreviated Mathematics Anxiety Rating Scale developed by Alexander and Martray (1989). Mathematics anxiety was rated on a scale of 1 ("Not at all") to 5 ("Very Much") across 25 scenarios. For example, participants were asked to rate their level of anxiety when, "Taking a math section on the college entrance exam" or "Studying for a math exam." For each participant, we calculated the average score across all scenarios.

Strategy coding

For the fraction magnitude comparison task and the fraction number line estimation task, participants were asked to give immediate retrospective strategy reports. Participants' written responses were coded to identify the strategies that participants used on each trial and to characterise the strategy variability across trials. Our coding schemes were adapted from prior research on students' strategies for fraction magnitude comparison (Fazio et al., 2016) and fraction number line estimation (Siegler & Thompson, 2014). A complete list of coding categories and examples can be found in Tables A3 and A4, respectively. For both tasks, strategies on each trial were coded independently by the second and third authors of this manuscript who coded without regards to participants' accuracy. Disagreements were resolved through discussion amongst the second, third and fourth authors, until agreement was reached. Percentage agreements for strategy coding on the magnitude comparison task in studies 1 and 2 were 83% and 84%, respectively. Percentage agreement for the number line estimation task in both studies 1 and 2 was 85%. For each participant, we counted the number of unique strategies across all of the trials.

Furthermore, we coded for adaptive strategy use by coding the frequency with which participants used optimal strategies in the fraction magnitude comparison and number line estimation tasks. For the fraction magnitude comparison task, 12 items afforded clearly optimal, *logical necessity* strategies. Logical necessity strategies only require simple comparisons of numerator and denominator magnitudes (Fazio et al., 2016) and should result in perfect accuracy every time. We coded whether participants reported using the optimal logical necessity *strategy* afforded by the given information in each logical necessity *item*. These logical necessity items had equal numerators (four items), equal denominators (four items) or both a larger numerator and a smaller denominator (four items), thus affording specific logical necessity strategies, "Equal numerators," "Equal denominators" or "Larger numerator/Smaller denominator," respectively.

For fraction number line estimation, we considered any correct reference to the fraction's holistic magnitude (see Table A4) to indicate optimal strategy use. Siegler and colleagues (Siegler & Thompson, 2014; Siegler et al., 2011) have suggested that reasoning about the holistic magnitudes of given fractions, rather than their whole number components, is most likely to lead to accurate reasoning. Importantly, this theoretical analysis suggests that referencing the holistic magnitude is most optimal on every trial, rather than on a subset of trials as with fraction magnitude comparison. There are many ways in which participants can reason about fractions' holistic magnitudes, including through *transformation*, *segmentation of the number line* or *reference to landmarks*. As such, these strategies often co-occur with holistic magnitude strategies. Thus, our coding of *optimal strategy use* captures any trial on which a participant either directly referenced the holistic magnitude correctly (e.g., for 1/19: "1/19 is very small, so I moved the marker as little as possible while not leaving it at 0"; *very small* indicates magnitude) or reported another strategy that facilitated a magnitude reference (e.g., 13/3: "number of times 3 goes into 12 with the remainder"; division operation indicates transformation; *number of times 3 goes into 12* indicates correct magnitude, approximately 4). In contrast, when participants referenced a strategy that does often co-occur with magnitude, but did not explicitly reference the fraction's magnitude on that trial (e.g., for 13/3: "divide", division operation indicates transformation; no reference to magnitude) were not coded as reflecting optimal strategy use. In this way, optimal strategy use on both tasks capture a class of strategies, but does not reflect every trial on which each of those individual strategies are used.

Results

Analytic overview

Our primary goal was to examine the role of participants' strategy use in the relationship between participants' reported mathematics anxiety and their fraction magnitude reasoning. We examined this question in the contexts of our two fraction tasks, magnitude *comparison* and number line *estimation*, separately, in this order. Critically, we hypothesised that students with higher mathematics anxiety would use *fewer strategies* for comparing and estimating fraction magnitudes, which in turn would result in reduced accuracy on these tasks.

In Figure 1, the hypothesised mediation is represented by pathways *a* and *b* combined. Thus, we then tested whether the indirect, mediated effect of participants' self-reported mathematics anxiety on performance via strategy variability, path *ab* in Figure 1, was significantly different from zero through a nonparametric bootstrapping procedure (see Preacher, Rucker, & Hayes, 2007 for a discussion of this approach). This is equivalent to testing for a significant reduction in the total effect, as *ab* is equivalent to the difference between the total effect and the direct effect, *c*′. We tested the indirect effect for each outcome using the *mediation* package (Tingley, Yamamoto, Hirose, Keele, & Imai, 2014) in R (R Core Team, 2015). The *mediate* function in this package directly estimates the indirect effect, or *Average Causal Mediated Effect* (*ACME*; path *ab* in Figure 1) and its 95% confidence interval using a nonparametric bootstrapping procedure. Missing data was handled through case-wise deletion of participants who were missing data for any one of the tasks in the mediation model.

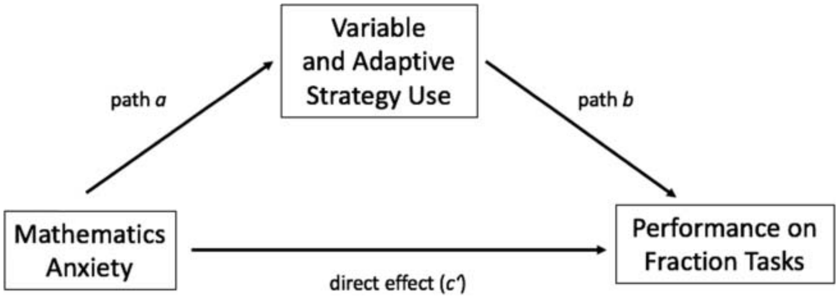

Figure 1. The hypothesised mediation model. The *ab* path corresponds to the indirect effect of participants' self-reported mathematics anxiety on performance via strategy variability.

For both outcomes, we followed the same analytic strategy. First, we checked to see whether mathematics anxiety was related to performance, as hypothesised. Second, we examined whether strategy variability mediated this effect in a simple mediation model, controlling for the effects of participants' mathematics background, as measured by participants' frequency of using mathematics skills in their majors, in both paths (*a* and *b*, Figure 1). Third, we explored whether working memory and mathematics skills in major moderated the relationship between mathematics anxiety and strategy use (path *a*) or the relationship between strategy use and performance (path *b*). Fourth, we tested for mediation in a final model, including the significant effects of working memory and mathematics skills in major from Step 3. In all models, the predictors were mean-centred, so that lower-order effects can be interpreted as simple effects at the mean of other predictors. We report results of each intermediate model we tested for each outcome (Steps 2 and 3) in the Supplementary Materials. Here, for brevity and clarity, we report the correlation between mathematics anxiety and performance and the results of our final mediation models for each outcome (Steps 1 and 4).

Preliminary analyses

In preliminary analyses, we examined whether our demographic information (age and gender) were significantly related to accuracy on either task, in order to determine whether these variables should be included in the analyses. Neither age, $r = 0.05$, $p = 0.56$; $r = -0.01$, $p = 0.88$, nor gender, $r = -0.13$, $p = 0.14$; $r = -0.05$, $p = 0.60$, were significantly correlated with accuracy on the magnitude comparison task or magnitude estimation task, respectively, therefore these variables were not included in the reported analyses.[1]

[1]We tested whether the exclusion of these variables in the analyses changed the nature of our conclusions, and it did not.

Fraction magnitude comparison

Overall, participants were highly accurate at the magnitude comparison task, $M = 91\%$, SD = 14%, and reported a variety of unique strategies, $M = 7.07$, SD = 3.29. Participants in the sample varied in their mathematics anxiety scores, $M = 2.44$, SD = 0.87. See Table 1 for descriptive statistics for each variable. As expected, participants' accuracy on the magnitude comparison task was significantly related to their self-reported mathematics anxiety, $r = -0.19$, $p = 0.04$, $n = 122$, such that participants with higher mathematics anxiety were significantly worse at comparing fraction magnitudes. Out of 123 participants, 114 participants provided codeable data on all the tasks in our final model: mathematics anxiety, strategy variability, working memory, mathematics skills in major and magnitude comparison accuracy.

As expected, participants with higher mathematics anxiety used fewer strategies for comparing fraction magnitudes, $b = -0.78$, $t(111) = -2.10$, $p = 0.04$, and using fewer strategies was in turn related to reduced accuracy, $b = 0.02$, $t(105) = 5.42$, $p < 0.01$. Indeed, strategy variability mediated the effect of mathematics anxiety on magnitude comparison accuracy, controlling for mathematics skills in major in path *a* and mathematics skills in major, working memory and their interaction in path *b*, ACME = −0.01, CI [−0.03, −0.001] (see Figure 2).

Furthermore, in our exploratory analyses, we found that working memory and mathematics skills in major both moderated the effect of strategy variability, $b = -0.06$, $t(105) = -3.84$, $p < 0.01$, $b = -0.02$, $t(105) = -3.89$, $p < 0.01$, respectively. There was a significant three-way interaction between strategy variability, working memory and mathematics background, $b = 0.08$, $t(105) = 3.62$, $p < 0.01$. In order to probe this interaction, we examined the

Table 1. Descriptive statistics for all variables.

Individual difference variables	Mean	SD	Range	*n*
Mathematics anxiety scores	2.44	0.87	1.00–5.00	122
Working memory scores	0.75	0.27	0.00–1.00	115
Mathematics skills in major	2.53	0.98	1–5	122
Magnitude comparison variables	**Mean**	**SD**	**Range**	***n***
Overall accuracy	91%	14%	12%–100%	123
Number of unique strategies	7.07	3.29	1–14	122
Accuracy on logical necessity problems	95%	13%	17%–100%	123
Frequency of optimal strategies on logical necessity problems	15%	22%	0%–92%	122
Magnitude estimation variables	**Mean**	**SD**	**Range**	***n***
Average PAE	12%	8%	1%–41%	120
Number of unique strategies	4.37	1.56	0–7	120
Frequency of optimal strategy use	74%	32%	0%–100%	120

Note: Given *n*s indicate the number of participants who provided codeable data for each measure. In the table and the text, we present accuracy, frequency and PAE as percentages to facilitate interpretation; however, these values were entered as decimals in our analyses.

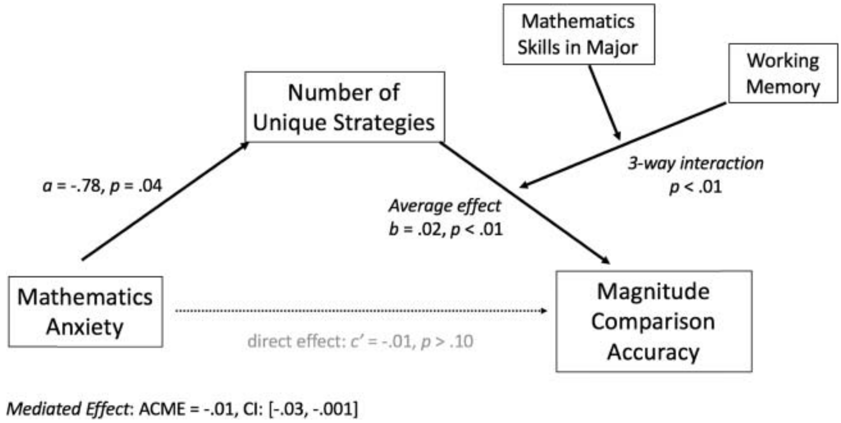

Figure 2. The final model included the three-way interaction between mathematics skills, working memory and strategy variability, in the mediation. Strategy variability significantly mediated the relationship between mathematics anxiety and accuracy on the magnitude comparison task. Furthermore, effect of strategy variability was moderated by both working memory and mathematics skills in major.

simple interactions and simple effects at high (1 SD above the mean) and low (1 SD below the mean) levels of working memory performance and reported mathematics skills in major.

Among students who reported using mathematics somewhat infrequently in their academic major (1 SD below the mean; Figure 3, Panel A), working memory capacity moderated the effect of strategy variability on accuracy, $b = -0.14$, $t(105) = -4.84$, $p < 0.01$. For students with lower mathematics skills in

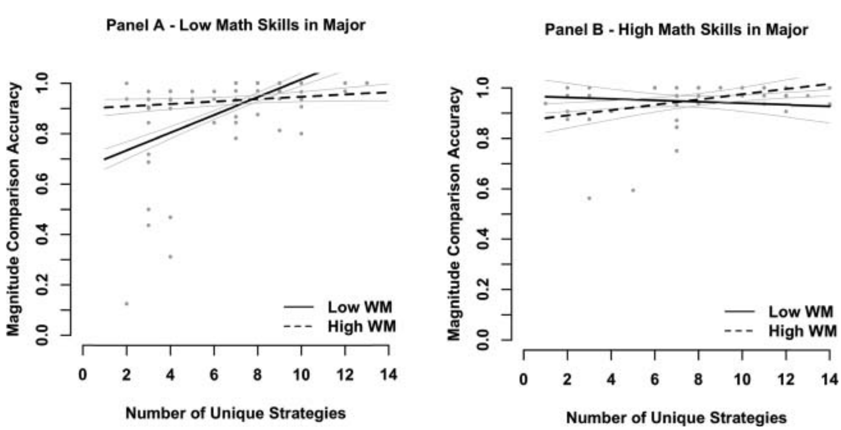

Figure 3. There was a significant interaction between working memory capacity and strategy variability among participants who use mathematics less frequently in their academic major (Panel A). In contrast, participants who use mathematics frequently in their majors were highly accurate on the magnitude comparison task, regardless of working memory score or strategy variability (Panel B). Confidence bands represent +/−1 SD.

major but higher working memory, strategy variability was not related to accuracy, $b = -0.001$, $t(105) = -0.16$, $p = 0.88$. However, among students with *lower mathematics skills in major and lower working memory*, there was a strong positive effect of strategy variability on accuracy, $b = 0.07$, $t(105) = 6.39$, $p < 0.01$. In other words, for students with lower working memory and infrequent mathematics usage in their majors, every additional strategy in their repertoire was associated with a 7% increase, or 0.5 SD increase, in accuracy.

In contrast, among students who reported using mathematics skills very frequently in their academic major (1 SD above the mean; Figure 3, Panel B), there was no significant interaction between working memory and strategy variability, $b = 0.02$, $t(105) = 0.94$, $p = 0.35$, or overall effects of working memory or strategy variability, $b = -0.01$, $t(105) = -0.1$, $p = 0.92$ and $b = 0.00$, $t(105) = 0.87$, $p = 0.39$, respectively. Students who used mathematics skills more frequently were highly accurate on the magnitude comparison task, regardless of working memory or strategy variability, $M = 94\%$ correct, SE = 2%. Across these results, it appears that using a variety of strategies to compare two fractions was particularly important for those students who did not report using mathematics skills very often in their academic majors *and* have lower working memory capacity. In contrast, students who either frequently use mathematics skills *or* have higher working memory capacities were highly accurate at comparing fraction magnitudes, regardless of strategy. It is important to note that the model controls for mathematics anxiety, and students who use mathematics skills more frequently are less anxious ($M = 2.25$, SE = 0.11, point estimate at 1 SD above the mean) than those who report using mathematics less frequently ($M = 2.53$, SE = 0.11, point estimate at 1 SD below the mean), however, mathematics anxiety was not significantly correlated with mathematics skills in major, $r = -0.17$, $p = 0.07$.

Adaptive strategy use. Next, we explored whether adaptive strategy use mediated the relationship between mathematics anxiety and accuracy on logical necessity problems (see the “Strategy coding” section). For each participant, we calculated the frequency of optimal strategy use as a percentage of the total number of logical necessity trials (see Table 1). Out of 123 participants, 115 participants provided codeable data on all the tasks in this model: mathematics anxiety, frequency of optimal strategy use, mathematics skills in major, working memory and magnitude comparison accuracy on logical necessity problems.

Although participants with higher mathematics anxiety had a lower frequency of optimal strategies on logical necessity fraction comparison problems after controlling for mathematics skills in major, $b = -0.04$, $t(112) = -1.76$, $p = 0.08$, this relationship was not statistically significant. Furthermore, optimal strategy use was not significantly related to accuracy on the fraction comparison task, $b = 0.08$, $t(110) = 1.36$, $p = 0.18$, after controlling for working memory, mathematics skills in major and mathematics anxiety. Thus, the indirect effect of mathematics anxiety on accuracy via optimal strategy use was not significantly different from zero, ACME = −0.003, CI [−0.009, 0.00003]. It may be the case that we were unable to properly test for this mediated pathway due to the low frequency of optimal strategy use in our data-set, M = 15% (see Table 1). We will discuss this further in the "General discussion" section.

Fraction number line estimation

Overall, participants were fairly accurate at the magnitude estimation task, with moderately low PAE across trials, M = 12%, SD = 8%, and reported a variety of unique strategies, M = 4.37, SD = 1.56. As expected, participants' PAE on the number line estimation task was significantly related to their self-reported mathematics anxiety, $r = 0.30$, $p < 0.01$, $n = 120$, such that participants with higher mathematics anxiety also had significantly higher average PAE. Out of the total sample, 118 participants provided data on all the measures included in the final model.

In contrast to our primary hypotheses, mathematics anxiety was not related to strategy variability on the estimation task, $b = -0.27$, $t(115) = -1.69$, $p = 0.09$, controlling for mathematics skills in major, and consequently, there was no indirect effect of mathematics anxiety on estimation PAE via strategy variability, ACME = 0.01, CI [−0.002, 0.02]. Instead, strategy variability, $b = -0.02$, $t(114) = -5.72$, $p < 0.01$, and mathematics anxiety, $b = 0.02$, $t(114) = 2.65$, $p < 0.01$, both predicted PAE on the number line estimation task, while controlling for mathematics skills in major, $b = -0.005$, $t(114) = -0.67$, $p = 0.49$ (Figure 4). Students who used more strategies to estimate the magnitude of a fraction on a number line had lower PAEs, whereas students who reported higher mathematics anxiety had higher PAEs.

Adaptive strategy use. Finally, we explored whether *adaptive* strategy use mediated the relationship between mathematics anxiety and PAE, by testing

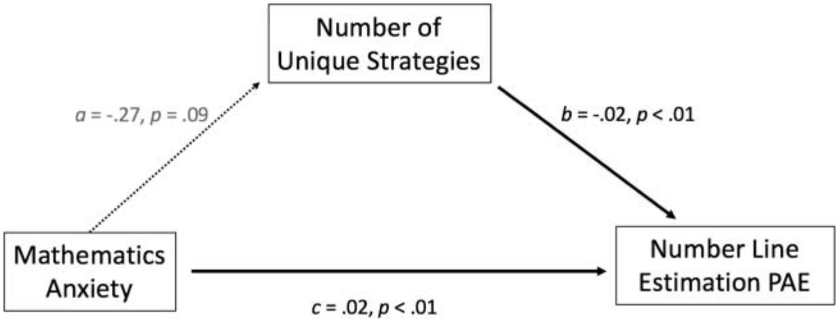

Figure 4. Both strategy variability and mathematics anxiety predicted participants' average PAE in the fraction number line estimation task.

whether the participants' use of the optimal, *magnitude* strategy mediated the effect of mathematics anxiety. We calculated the percentage of trials on which participants made a correct reference to the fraction's holistic magnitude (see Table 1). Out of the total sample, 111 participants provided data on all the measures included in these following models. As expected, participants with higher mathematics anxiety were less likely to report using the magnitude strategy to solve fraction magnitude estimation items than those with lower anxiety, $b = -0.05$, $t(104) = -1.93$, $p = 0.05$. Furthermore, participants who reported using magnitude strategies more often also had lower PAE, $b = -0.14$, $t(105) = -5.32$, $p < 0.01$. Indeed, the frequency with which participants reported using the magnitude strategy significantly mediated the effect of mathematics anxiety on PAE, ACME = 0.01, CI [0.001, 0.02]. Additionally, variable strategy use was also related to the frequency of using the magnitude strategy, $b = 0.11$, $t(104) = 7.01$, $p < 0.01$, and use of the magnitude strategy mediated the effect of strategy variability on PAE, ACME = −0.02, CI [−0.02, −0.01]. In other words, participants who reported using more strategies also reported directly relying on their knowledge of the holistic magnitude of a given fraction in order to estimate its magnitude on the number line, and in doing so, had reduced PAE on the task (Figure 5).

Finally, working memory significantly moderated both the effects of strategy variability, $b = -0.10$, $t(104) = -2.32$, $p = 0.02$, and mathematics anxiety, $b = 0.22$, $t(104) = 2.63$, $p < 0.01$, on the frequency of magnitude strategy use, controlling for mathematics skills in major (see Figures 6 and 7). Strategy

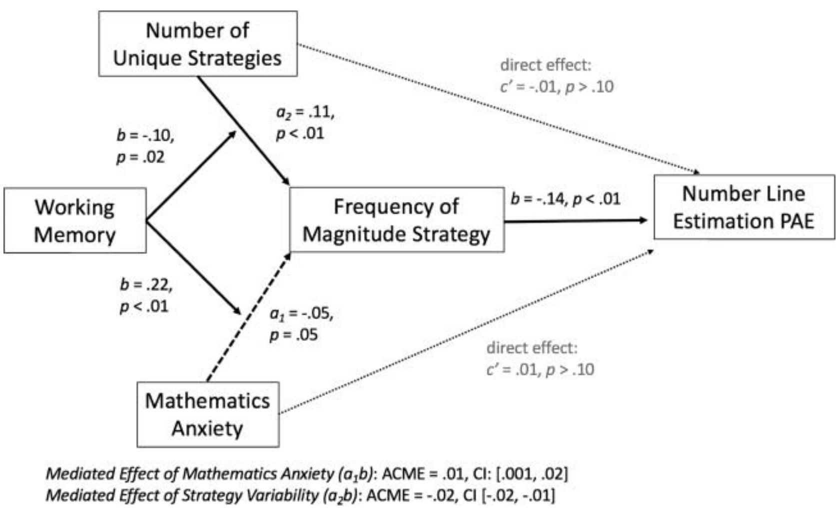

Figure 5. Participants' fraction magnitude estimation is related to mathematics anxiety, strategy variability and optimal strategy use. The effects of mathematics anxiety and strategy variability are mediated by the frequency with which participants used the optimal, magnitude strategy and moderated by participants' working memory.

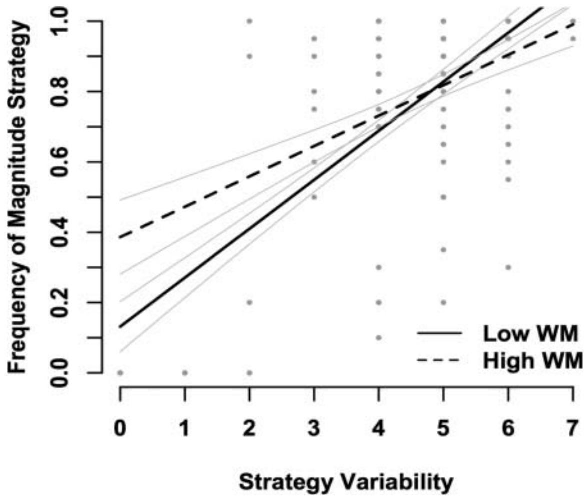

Figure 6. Participants' working memory scores moderated the effect of strategy variability on the frequency of using the optimal, magnitude strategy. The positive relationship between overall strategy variability and participants' use of the magnitude strategy was stronger among those with lower working memory scores.

variability and mathematics anxiety had larger effects on optimal strategy use among students with lower working memory scores, $b = 0.14$, $t(104) = 8.50$, $p < 0.01$, and $b = -0.11$, $t(104) = -3.50$, $p < 0.01$, respectively. Among students with higher working memory scores, the relationship between strategy

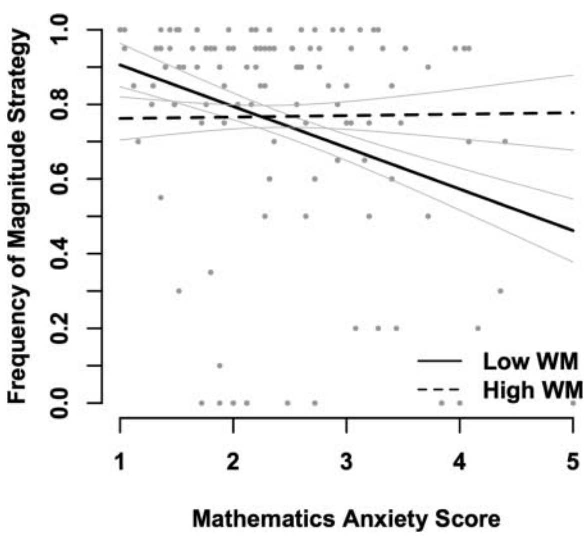

Figure 7. Participants' working memory scores moderated the effect of mathematics anxiety on the frequency of using the optimal, magnitude strategy. The negative relationship between mathematics anxiety and use of the magnitude strategy was stronger among those with lower working memory scores.

variability and the magnitude strategy was smaller, $b = 0.09$, $t(104) = 3.90$, $p < 0.01$, mathematics anxiety had no effect on their frequency of using the magnitude strategy, $b = 0.00$, $t(104) = 0.11$, $p = 0.92$.

Study 1 discussion

Among the participants in Study 1, mathematics anxiety was significantly correlated with performance on both fraction tasks. On the fraction magnitude comparison task, strategy variability mediated the effect of mathematics anxiety on performance accuracy. In contrast, on the fraction number line estimation task, participants' use of a single, optimal strategy mediated the relationship between mathematics anxiety and participants' performance on this task.

In both tasks, working memory moderated the effect of strategy variability. On the fraction magnitude comparison task, the interaction between working memory and strategy variability was further moderated by participants' frequency of mathematics usage, such that strategy variability was most important for those with lower working memory scores and less frequent mathematics usage. On the fraction number line task, working memory moderated strategy variability in a similar way, such that strategy variability was more important among those with lower working memory scores. Additionally, working memory moderated the effect of mathematics anxiety on optimal strategy use, such that students' mathematics anxiety negatively predicted optimal strategy use among those with lower working memory. Importantly, these analyses were largely exploratory. Thus, we ran a second study in order to test our data-driven model in a new sample.

Study 2

In Study 2, we sought to replicate Study 1. Importantly, in Study 1 we conducted an exploratory analysis in order to better understand the roles of working memory and participants' mathematics background in our hypothesised pathway models. In Study 2, we aimed to test whether the model we developed in Study 1 would fit data from a new, similar sample. All tasks and procedures were identical to Study 1, apart from our correction to the fraction magnitude comparison stimuli presentation.

Method

Participants

In total, 124 participants responded to at least one question in the survey. Six participants were excluded from data analysis due to completing only one task. The final sample included 116 college students at a public university in the mid-western United States (*M* age = 20.43 years, SD = 3.58; 69.0% women, 27.5% men, 3.5% not specified; 80.2% Caucasian, 6.0% African-American, 3.5% Asian, 1.7% Hispanic, 3.5% other and 5.1% not specified). Psychology was the most common reported major (26.4%). Most participants (94.0%) were native English speakers.

Design and procedure

The design and procedure were identical to Study 1 with one exception. In the fraction magnitude comparison task, half of the pairs were presented such that the larger fraction appeared on the right side of the screen and half of the pairs were presented such that the larger fraction appeared on the left side of the screen.

Results

First, we aimed to replicate our full mediation model of participants' fraction magnitude comparison accuracy in Study 1 (see Figure 2). Second, we aimed to replicate our double mediation model of participants' fraction number line estimation PAE in Study 1 (see Figure 5). As in Study 1, missing data was handled through case-wise deletion of participants who were missing data for any of the tasks used in the final model for each analysis. Descriptive statistics for all variables, based on the full set of data available for each variable, is shown in Table 2.

Fraction magnitude comparison

Overall, participants were highly accurate at the magnitude comparison task, *M* = 91%, SD = 14%, and reported a variety of unique strategies, *M* = 6.81, SD = 3.29. Participants in the sample varied in their mathematics anxiety scores, *M* = 2.57, SD = 0.94. As expected, participants' accuracy on the magnitude

Table 2. Descriptive statistics for all variables.

Individual difference variables	Mean	SD	Range	*n*
Mathematics anxiety scores	2.57	0.94	1.00–4.60	114
Working memory scores	0.72	0.29	0.00–1.00	102
Mathematics skills in major	2.70	1.07	1–5	113
Magnitude comparison variables	**Mean**	**SD**	**Range**	***n***
Overall accuracy	91%	14%	34%–100%	116
Number of unique strategies	6.81	3.29	1–14	114
Magnitude estimation variables	**Mean**	**SD**	**Range**	***n***
Average PAE	14%	10%	3%–49%	112
Number of unique strategies	4.33	1.85	0–7	110
Frequency of optimal strategy use	69%	36%	0%–100%	110

Note: Given *n*s indicate the number of participants who provided codeable data for each measure.

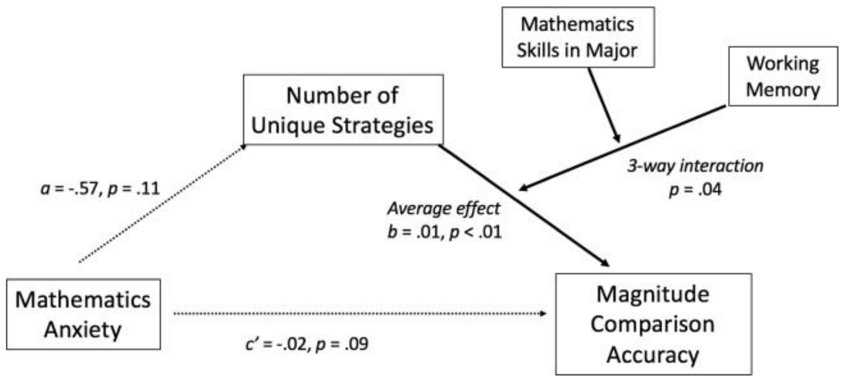

Figure 8. Strategy variability did not mediate the relationship between mathematics anxiety and magnitude comparison accuracy. As in Study 1, strategy variability did predict accuracy, and this relationship was moderated by both working memory and participants' mathematics skills in major.

comparison task was significantly related to their self-reported mathematics anxiety, $r = -0.23$, $p = 0.01$, $n = 114$, such that participants with higher mathematics anxiety were significantly worse at comparing the magnitudes of two fractions. These descriptive statistics were similar to those found in Study 1 (see Tables 1 and 2). Out of 116 participants, 96 participants provided codeable data on all the tasks in our final model: mathematics anxiety, strategy variability, working memory, frequency of mathematics skills used in major and magnitude comparison accuracy.

In contrast to Study 1, participants' mathematics anxiety was not related to their strategy variability on the magnitude comparison task, $b = -0.57$, $t(93) = -1.63$, $p = 0.11$, when controlling for mathematics skills in major. As such, strategy variability did not mediate the relationship between mathematics anxiety and accuracy, ACME = −0.01, *CI* [−0.02, 0.001]. However, in line with Study 1, strategy variability was related to accuracy, $b = 0.01$, $t(87) = 3.02$, $p < 0.01$, and further moderated by working memory and mathematics skills in major, $b = 0.03$, $t(87) = 2.04$, $p = 0.04$ (see Figures 8 and 9).

Fraction number line estimation

Overall, participants were fairly accurate at the magnitude estimation task, with moderately low PAE across trials, $M = 14\%$, SD = 10%, and reported a variety of unique strategies, $M = 4.33$, SD = 1.85. As in Study 1, participants' accuracy on the magnitude estimation task was significantly related to their self-reported mathematics anxiety, $r = 0.37$, $p < 0.01$, $n = 110$, such that participants with higher mathematics anxiety were significantly worse at estimating the magnitudes of given fractions. In total, 95 participants provided data on all measures in the model.

Strategy variability and mathematics anxiety were both related to PAE, controlling for mathematics skills in major. As in Study 1, participants' strategy

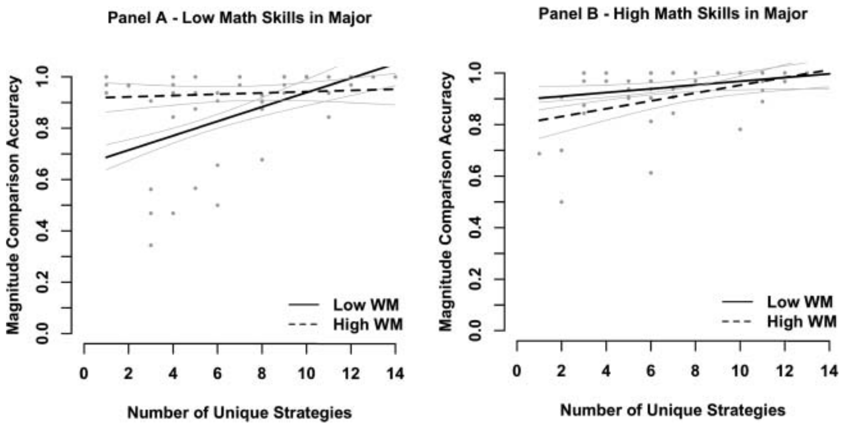

Figure 9. As in Study 1, among participants who use mathematics less frequently in their academic major, there is a significant interaction between working memory and strategy variability (Panel A). Participants who use mathematics skills frequently in their majors were highly accurate on the magnitude comparison task, regardless of working memory score or strategy variability (Panel B). Confidence bands represent +/−1 SD.

variability predicted the likelihood of using the optimal, magnitude strategy, $b = 0.13$, $t(88) = 8.05$, $p < 0.01$, and participants' use of the magnitude strategy was in turn related to reduced PAE, $b = -0.14$, $t(89) = -4.58$, $p < 0.01$. The use of the magnitude strategy mediated the effect of participants' strategy variability on PAE, ACME = −.02, *CI* [−0.03, −0.01].

In contrast to Study 1, participants' mathematics anxiety was not related to their use of the magnitude strategy, $b = -0.02$, $t(88) = -0.56$, $p = 0.58$. Furthermore, working memory did not moderate the effect of strategy variability, $b = -0.01$, $t(88) = -0.24$, $p = 0.81$, and there was no interaction between working memory and mathematics anxiety on participants' use of the magnitude strategy, $b = -0.19$, $t(88) = -1.86$, $p = 0.07$. Instead, we found that participants' mathematics anxiety had an independent effect on their number line estimation PAE, $b = 0.03$, $t(89) = 2.94$, $p < 0.01$, as did participants' working memory, $b = -0.07$, $t(89) = -2.40$, $p = 0.02$ (see Figure 10).

Study 2 discussion

A full comparison of results from Study 1 and Study 2 can be found in Table 3. We replicated many, though not all, findings from Study 1. As in Study 1, participants' mathematics anxiety was significantly related to their performance on both the fraction magnitude comparison task and the fraction number line estimation task. However, our findings from Study 1 linking mathematics anxiety, strategy variability and performance on these tasks were not replicated, largely due to non-significant relationships between mathematics

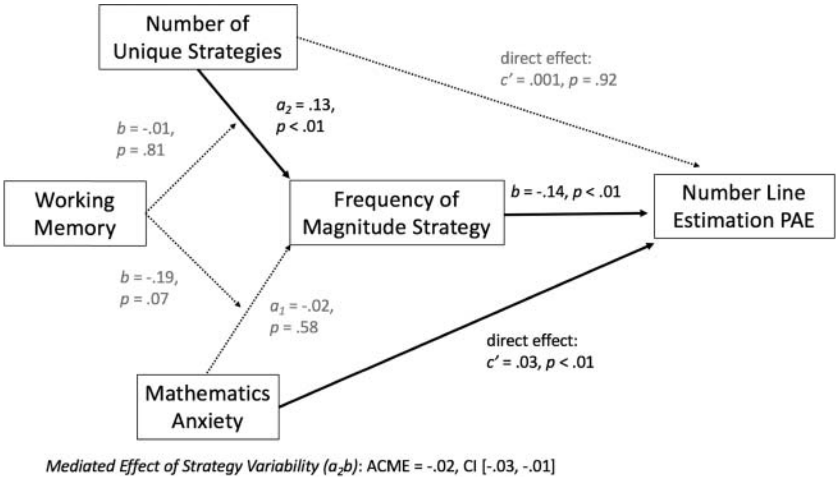

Figure 10. As in Study 1, the frequency of participants' use of the magnitude strategy mediated the effect of strategy variability on participants' average PAE in the fraction number line estimation task. However, participants' magnitude strategy use did not mediate the relationship between mathematics anxiety and PAE.

Table 3. Comparison of Study 1 and Study 2 findings.

	Study 1	Study 2
Fraction magnitude comparison: Overall		
Math anxiety and accuracy	✓	✓
Math anxiety and strategy variability	✓	ns
Strategy variability and accuracy	✓	✓
Strategy variability mediates math anxiety and accuracy	✓	ns
WM and MS moderate strategy variability	✓	✓
Simple effect of strategy variability at low WM and MS	✓	✓
Fraction magnitude comparison: adaptive strategy use		
Math anxiety and adaptive strategy use	ns	–
Adaptive strategy use and accuracy	ns	–
Adaptive strategy use mediates math anxiety	ns	–
Fraction number line estimation: overall		
Math anxiety and PAE	✓	✓
Math anxiety and strategy variability	ns	–
Strategy variability mediates math anxiety and PAE	ns	–
Strategy variability and PAE	✓	✓
Fraction number line estimation: adaptive strategy use		
Math anxiety and adaptive strategy use	✓	ns
Adaptive strategy use and PAE	✓	✓
Adaptive strategy use mediates math anxiety and PAE	✓	ns
Variability and adaptive strategy use	✓	✓
Adaptive strategy use mediates strategy variability and PAE	✓	✓
WM moderates strategy variability	✓	ns
WM moderates math anxiety	✓	ns

Note: Check marks indicate significant relationships and replications in Study 2, ns indicates non-significant results in Study 1 and failures to replication in Study 2. Empty cells indicate relationships not tested in Study 2 due to being non-significant in Study 1. Replicated effects are listed in bolded text. Working memory (WM) and mathematics skills in major (MS) are abbreviated.

anxiety and strategy use on both tasks. In general, the relationship between mathematics anxiety and strategy variability on the magnitude task was in the same direction as in Study 1 – participants with higher mathematics anxiety used fewer strategies – but this relationship was not significant. Similarly, students' mathematics anxiety was not related to either strategy variability or use of the optimal strategy on the fraction number line task. In sum, we were unable to replicate our findings from Study 1 which suggested that strategy variability and strategy use play key roles in the effect of mathematics anxiety on fraction magnitude reasoning. This may not be surprising, given the small indirect effects observed in Study 1, with confidence intervals close to 0, as well as the reduction in sample size due to missing data from the online tasks. Taken together, the results from these studies invite several questions, such as whether there is a true relationship between mathematics anxiety and strategy variability that we were unable to capture in our Study 2 data or whether mathematics anxiety may be affecting performance in other ways. We will discuss this further in the "General discussion" section.

In Study 2, we did replicate our earlier finding that students' strategy variability predicted their performance on both the magnitude comparison task and the number line task. This finding further replicates and extends results from Fazio et al. (2016), demonstrating that adults who are more accurate at magnitude comparison use more strategies to solve these problems. Additionally, we replicated our finding that strategy variability in magnitude comparison appears to be protective. That is, using a wide variety of strategies to compare fraction magnitudes was more important for students with both lower working memory and lower mathematics skills usage.

In the number line estimation task, we replicated our earlier finding that increased strategy variability is also associated with increased use of the optimal, magnitude strategy, and this is one mechanism by which variable strategy use improved students' magnitude estimations. We did not replicate the moderating effect of working memory scores on the relationship between strategy variability and optimal strategy use. However, our finding that strategy variability was strongly related to optimal strategy use among participants with higher and lower working memory scores is in line with our findings from Study 1 (see Figure 6). These results advance our understanding of the role of variable strategy use in fraction reasoning, and will be further discussed in the following section.

It is important to note that we were able to replicate effects where performance, rather than strategy variability, was the *dependent* variable (see Table 3), despite what might appear to be "ceiling effects" on performance. Although performance on these tasks is high, there is certainly room for improvement. For instance, Fazio et al.'s, 2016 sample of college students' PAE was 5% (SD = 2%) as compared to our 12%–14% PAE across both studies. Furthermore, because fraction problem solving is highly strategic, even when participants have high accuracy they still use a variety of strategies (see Tables 1 and 2). Thus, our inability to replicate our models of strategy variability as a mediator cannot be attributed to issues resulting from an attenuated range on the outcome.

General discussion

In these studies, we examined the relationships between adults' mathematics anxiety, working memory performance, variability in strategy use and performance on two tasks assessing fraction magnitude reasoning. In both studies, we found that college students with higher mathematics anxiety performed worse when asked to reason about the magnitudes of fraction symbols. This, in itself, is an important finding given the centrality of fraction magnitude reasoning to mathematics development more broadly (e.g., Siegler et al., 2012). By demonstrating this link, we add to the existing literature showing the many negative effects of mathematics anxiety on mathematical thinking in children (e.g., Ramirez et al., 2013; Vukovic et al., 2013; Wang et al., 2015) and adults (e.g., Ashcraft & Kirk, 2001; Maloney et al., 2010; Wang et al., 2015).

The primary goal of our studies was to examine the relationships between strategy variability, mathematics anxiety and fraction reasoning. First, we discuss the role of strategy variability in fraction reasoning. Then, we discuss the implications, and limitations, of our findings for the relationship between mathematics anxiety, strategy variability and working memory.

Strategy variability in adults' fraction reasoning

Adults' reasoning about fractions is highly strategic, and variability in adults' strategy use has been linked to higher performance on fraction magnitude tasks (e.g., Fazio et al., 2016). In this study, we replicate previous findings from Fazio and colleagues that strategy variability on a magnitude comparison task is advantageous. Furthermore, our findings refine and extend our understanding of the role of strategy variability in fraction magnitude reasoning in two productive ways: by elucidating the role of individual differences in the effect of strategy variability on performance and by suggesting a mechanism by which strategy variability may impact accuracy in students' fraction magnitude estimations.

First, we found that the advantage of strategy variability in *comparing* fraction magnitudes is moderated by two facets of students' individual differences: their working memory performance and their mathematics background. Although we did find that strategy variability was associated with accuracy in comparing fractions, we also saw that this relationship was strongest amongst students with lower working memory performance and lower frequencies of using mathematics skills in their academic majors. This finding replicated across both studies and suggests that variable strategy use is, indeed, an adaptive problem solving behaviour. Our empirical data cannot speak directly to the reasons why strategy variability matters most for those with lower working memory and lower mathematics usage; however, we propose that this finding may be related to differences in strategy choice among students with differential working memory and mathematics skills.

Beilock and DeCaro (2007) found that students with higher performance on working memory tasks also tended to use computationally rich, algorithm-based strategies for solving mathematics problems, whereas students with lower working memory tended to use strategies based on estimation or guessing. In our sample, students with higher working memory or more frequent mathematics usage may be similarly more likely to choose a single, computationally rich strategy that they know will work across every trial (e.g., translating fractions into decimals or finding common denominators) and executing it correctly every time. When one or two strategies can be implemented efficiently and effectively across all trials, there may be no need for highly variable strategy use. In contrast, students with lower working memory and mathematics skills may be less effective at choosing and implementing a single, computationally rich strategy that would lead to an accurate answer on each trial. Those students who do rely on only one strategy may be relying on strategies that are easier to execute, even when those strategies are less effective overall (e.g., guessing) or less effective given their own mathematics knowledge (e.g., relying on denominator magnitude without understanding how denominator magnitude is related to overall fraction magnitude). Students who rely on multiple strategies may still be relying on less computationally intense strategies, but take better "advantage" of the given information from trial to trial by deploying strategies that come to mind based on features of the task context and their own knowledge and experience, as proposed by Alibali and Sidney (2015b).

One possible limitation to this explanation is the low observed rate of adaptive strategy use in our sample during fraction comparison. In other words, we did not find that optimal strategy use, as we and Fazio et al. (2016) have defined it, occurred at a high rate. Overall, our rate of optimal strategy use is more similar to the rate of lower-performing students in Fazio et al.'s sample than their higher-performing samples. However, our criteria for coding an optimal strategy were fairly conservative; students had to explicitly state the optimal strategy for that trial. Furthermore, our definition of the optimal strategies does not take into account how students' own knowledge and experience may interact with problem features to afford certain strategies on certain problems (e.g., see Alibali & Sidney, 2015a, 2015b). Future research is needed to shed light on how differences in individual knowledge and experience interact with features of the mathematical problems and the problem-solving context to give rise to the rich strategy variability observed in mathematical problem solving.

Second, we found that strategy variability also plays a role in students' fraction magnitude *estimation*, though a different role than in magnitude *comparison*. The current studies are the first to directly examine the effect of strategy variability in fraction number line estimation, a task that has been linked to general mathematics achievement (e.g., Siegler et al., 2012). In a prior study, Siegler and Thompson (2014) documented a variety of strategies used by students to facilitate reasoning about the magnitude of fractions on a given number line. Here, we demonstrated that students who use more strategies to place fractions on a number line were more likely to make direct references to fractions' holistic magnitude, and had smaller errors, on average. When students estimate the location of a fraction on a number line, using a variety of strategies may facilitate thinking about fraction magnitudes in a variety of ways. Importantly, this mechanism contrasts with the role of strategy variability in the magnitude comparison task, in which strategy variability appears to facilitate making judgments about relative magnitude in ways that circumvent thinking directly about holistic magnitude, for example, by relying only on numerator or denominator magnitude when applicable (e.g., Fazio et al., 2016).

Mathematics anxiety and strategy variability

Given the theoretical role of working memory, and specifically executive control functions, in both mathematics anxiety (e.g., Ashcraft, 2002; Eysenck et al., 2007) and strategy choice (e.g., Hodzik & Lemaire, 2011; Lemaire & Brun, 2016), we hypothesised that students with higher mathematics anxiety would use fewer strategies to reason about fraction magnitudes, due to decreases in working memory resources during problem solving. Although we found that strategy variability significantly mediated the relationship between mathematics anxiety and performance on the magnitude comparison task in Study 1, this did not replicate in Study 2. Furthermore, across the two studies, the size of the indirect effect was similarly small.

Given this mixed finding across studies, it is possible that students' mathematics anxiety does *not* affect their strategy variability on fraction magnitude comparison, and the results of Study 1 were spurious. In both studies, the relationship between mathematics anxiety and performance on the fraction number line *estimation* task was independent of strategy variability. Given that mathematics anxiety is thought to decrease working memory resources during problem solving, it may be that anxiety affects students' *implementation* of strategies regardless of how many strategies are used across the whole task. This line of reasoning is in line with findings from Beilock and DeCaro (2007) demonstrating that students who used computationally rich strategies were more strongly affected by a high pressure context, due to difficulties in implementing these strategies. However, if this is the case, one might expect to find that students' working memory skills moderated the effect of mathematics anxiety, mirroring this earlier work.

In our studies, we found mixed support for the hypothesis that students' working memory moderates the effect of mathematics anxiety on mathematics performance. Although this was the case in Study 1 for fraction number line estimation, we did not replicate this effect in Study 2 and did not find interactions between working memory and mathematics anxiety for fraction magnitude comparison in either study. This mixed result may be due in part to the specific task that we used to measure working memory in these studies. Anxiety is thought to affect the executive function component of students' working memory resources (e.g., Ashcraft & Kirk, 2001; Eysenck et al., 2007), and executive function is thought to be a multifaceted construct, including the ability to *update*, *inhibit* and *shift* attention (e.g., Miyake & Friedman, 2012; Miyake et al., 2000). In our study, we used a task thought to measure the *updating* facet of executive function, as it had been used in previous studies of students' fraction reasoning (e.g., Siegler & Pyke, 2013). However, Eysenck et al. (2007) suggest that inhibition and shifting are impaired when people experience anxiety. In our study, we may find moderating effects of working memory on mathematics anxiety only to the extent that updating, inhibition and shifting share a common executive function component (see Miyake & Friedman, 2012). To more clearly test the relationship between mathematics anxiety, executive function and strategy variability, a variety of executive function tasks may be needed in future research.

A second reason for our mixed result may be that limits on working memory affect performance only *under pressure* (e.g., Beilock & DeCaro, 2007). Although fraction tasks are thought to tax working memory (Siegler & Pyke, 2013), adults' working memory may not be sufficiently taxed to reveal working memory-related effects of mathematics anxiety unless an additional constraint (e.g., a time pressure) is introduced. However, we did find relationships between mathematics anxiety and fraction performance even without imposing an external pressure. Thus, we leave open the question concerning mathematics anxiety's mechanism of effect.

Conclusion

Reasoning about the magnitudes of fraction symbols is a challenging topic in mathematics development, and students' fraction magnitude understanding predicts later mathematics achievement. Adults use a variety of strategies to reason about fraction magnitudes, and this variability is adaptive. The current studies make two important contributions to our understanding of the role of strategy variability in adults' fraction reasoning. First, when asked to compare two fractions, we found variable strategy use to be especially adaptive for students with lower working memory performance or those who use mathematics infrequently in their academic lives. Second, we found that when asked to place fractions on a number line, students who use multiple strategies are more likely to use the most optimal strategy for precisely representing fraction magnitudes. Both findings were replicated.

Furthermore, these studies are the first to demonstrate that adults with higher mathematics anxiety are less accurate at comparing and estimating the magnitudes of fractions. However, we found little evidence that individual differences in students' mathematics anxiety and working memory affect variable strategy use. This finding leaves open the specific mechanisms through which anxiety affects adults' fraction reasoning. Mathematics anxiety may affect performance by impeding students' implementation of strategies, rather than affecting variability. Regardless of the specific mechanism, it is clear that mathematics anxiety plays a role in key facets of students' mathematics reasoning, such as fraction magnitude reasoning.

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Appendix

Table A1. Magnitude comparison stimuli.

Problem type	Larger fraction	Smaller fraction
Equal numerator	1/3	1/4
	3/4	3/5
	3/9	3/12
	4/13	4/15
Equal denominator	4/9	2/9
	3/7	2/7
	9/21	6/21
	13/17	9/17
Larger numerator/smaller denominator	7/8	5/9
	10/13	9/14
	39/45	30/51
	13/15	10/17
Halves reference	8/15	5/12
	11/16	6/13
	21/36	24/51
	7/12	8/17
Multiply for common denominator	5/6	2/3
	7/8	3/4
	3/7	5/14
	21/24	9/12
Multiply for common numerator	1/4	2/9
	6/19	3/11
	16/19	8/13
	48/57	24/39
Large distance	4/9	1/8
	8/17	2/15
	18/19	7/12
	24/51	6/45
Small distance	5/6	3/4
	4/11	5/19
	17/19	12/17
	12/33	15/57

Table A2. Magnitude estimation stimuli.

Numerical range	Fraction
Magnitude: 0–1	1/19
	3/13
	4/7
	8/11
Magnitude: 1–2	7/5
	13/9
	14/9
	12/7
Magnitude: 2–3	13/6
	19/8
	8/3
	11/4
Magnitude: 3–4	13/4
	10/3
	17/5
	7/2
Magnitude: 4–5	17/4
	13/3
	9/2
	19/4

Table A3. Fraction magnitude comparison strategy definitions.

Strategy category	Strategies	Example	Definition	Study 1 avg. use	Study 2 avg. use
Logical necessity *Strategies yield correct answers on all applicable problems*	Equal denominators	*Same denominator, the one with the larger numerator is larger.*	If both fractions have equal denominators, the fraction with the larger numerator is larger.	3.38	3.44
	Equal numerators	*The numerator is the same but the lowest denominator is greater than the one with the larger denominator.*	If both fractions have equal numerators, the fraction with the smaller denominator is larger.	1.48	1.39
	Larger numerator/smaller denominator	*10/13 has a larger top number and smaller bottom number than 9/14.*	The larger fraction has a larger numerator and a smaller denominator than the smaller fraction.	0.95	1.58
Intermediate Steps *Strategies yield correct answer on all applicable problems if intermediate steps are executed correctly*	Transformation	*1/3 = 0.333; 1/4 = 0.25*	Transform one or both of the fractions, often rounding, simplifying, or translating in order to make them easier to compare.	42.69	44.06
	General magnitude reference	*21/24 is closer to 1*	Compare one or both fractions to a nearby known magnitude. Examples include 0, 1/4, 1/3, 1/2, 2/3, 3/4 and 1.	21.26	22.08
	Multiply for a common denominator	*I multiplied 3/7 by 2 to get a common denominator. 6/14 is larger than 5/14.*	Multiply one fraction by 1 (in the form of fraction a/a) in order to get common denominator.	4.31	4.68
	Halves reference	*2/15 is less than 0.5 while 8/17 is more than 0.5*	The larger fraction is greater than $\frac{1}{2}$ and the smaller fraction is smaller than $\frac{1}{2}$.	2.92	2.89
	Multiply for a common numerator	*If you multiply 1/4 by 2 it would be 2/8. 2/8 is larger than 2/9.*	Multiply one fraction by 1 (in the form of fraction a/a) in order to get common numerator.	0.90	0.98
Usually correct *Strategies that yield better than*	Visualisation	*I imagined 4/13 of a cake pan and 4/15 of a cake pan and visualised which one would have more.*	Using a pie, pizza, or other visual representation (words like "pieces" or	13.58	8.07

(*continued*)

Table A3. (*Continued*)

Strategy category	Strategies	Example	Definition	Study 1 avg. use	Study 2 avg. use
chance results, but do not guarantee correct answers			"parts") of a fraction in order to compare magnitudes.		
	Difference between numerator and denominator within each fraction is smaller	*The difference between numerator and denominator within each fraction is smaller.*	The difference between the numerator and denominator of the larger fraction is smaller than that of the smaller fraction.	12.43	8.83
	Smaller denominator	*21/36 has a smaller denominator and less to divide into therefore it is the larger fraction.*	The larger fraction has a smaller denominator.	6.13	6.31
	Larger numerator	*9/21 simply has a larger numerator/uses more pieces of the whole.*	The larger fraction has a larger numerator.	5.82	5.71
	Parts missing from the whole	*There is only 2/19 left in the second fraction and there is 5/17 left in the other (regarding the fractions 17/19 and 12/17).*	If the fractions of interest are subtracted from a whole, the amounts remaining can then be compared.	1.40	1.14
Questionable *Strategies not guaranteed to yield above chance performance*	Intuition	*Just knew.*	Just knew that one was larger.	7.38	12.67
	Other	(Misc. responses).	Reserved for infrequently used strategies.	7.59	7.16
	Blank	(No response).	Participant leaves prompt blank.	6.53	12.90
	Guess	*I guessed on this one.*	States that they guessed.	3.59	4.25
	None	*AFLKSK223.*	Nonsensical or irrelevant response.	1.00	2.23

Note. Strategy template adapted from Fazio et al., (2016).

Table A4. Fraction number line estimation strategy coding.

Strategy category	Strategies	Example	Definition	Study 1 avg. use	Study 2 avg. use
Transformation *Transforming the presented fraction into a number that is easier to handle*	Mixed number strategy	*7/5 equals 1 and 2/5*	Translating the given fraction into a whole number and a fraction, providing it is greater than 1.	18.66	14.51
	Rounding/ simplifying/ translating	Rounding: *4/7 is a little more than ½* Simplifying: *8/16 = 1/2* Translating: *3/4 = 0.75*; into decimals, whole numbers, percentages, etc.	Rounding: Estimating the value of the given fraction by referencing a number nearby in magnitude. Simplifying: Converting (often by reducing) to an easier to interpret fraction. Translating: Changing the form of a fraction, e.g., to decimal form.	84.47	79.78
Segmentation *Dividing the line into segments; participants differ in the segments they use*	Halves	*The middle of the number line is 2.5 since that is half of five, and 19/8 is a little less than 2.5. By knowing that, I moved the dot to a little before what I saw as the centre of the number line.*	Dividing the number line in half.	4.45	6.46
	Fifths	*I divided the number line into fifths to find where 1 is and then made it close to one*	Dividing the number line into fifths. Reference to whole numbers (1, 2, 3 and 4) due to the range of the number line being 0–5.	42.77	45.68
	Other	*I mentally split the line into fifths then tried to visualise the space from 0 to 1 in 19ths* (for the fraction 1/19)	Dividing the number line into any other divisions besides halves and fifths.	1.71	3.65
Magnitude *Making reference to the magnitude of the fraction, whether it be a specific or vague reference*	Magnitude	*Large number, close to 5*	Placing given fraction on the number line based on its magnitude.	81.76	73.71
Independent components *Referencing numerator or denominator components independently*	Independent components	*12 − 7 = 5* (for the fraction 12/7)	Placing the fraction on the number line using a strategy that solely focused on the size of the numerator and/or denominator.	0.47	1.08
Landmark *The pre-marked landmarks (e.g., 0 or 5) that the participant referenced*	Near/far	Near: *very close to zero* Far: *very far from five*	Given fraction referenced as near or far from the "0" and/or "5" landmarks.	11.67	10.62

Note: Strategy template adapted from Siegler and Thompson (2014). In the "Example" column, italics are used to indicate participant responses.