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Clarissa A. Thompson, Charles J. Fitzsimmons, and Daniel A. Scheibe

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PEDAGOGICAL POINTS TO PONDER

Students' Ability to Calculate Their Final Course Grade May Not Be as Easy as You Think: Insights From Mathematical Cognition

Clarissa A. Thompson, Charles J. Fitzsimmons, and Daniel A. Scheibe
Department of Psychological Sciences, Kent State University

Is there an optimal grading scheme? Do psychology instructors prefer one grading scheme over another? These questions were recently posted on the Society for the Teaching of Psychology Facebook page. After reading the responses, we realized that research in the domain of math cognition might help to shed light on an optimal grading scheme and put some of the posters' comments into context. Posters often mentioned 100-point and 1,000-point grading schemes because of the ease with which students could convert course points to percentages. In this Pedagogical Points to Ponder article, we describe the quick development of whole number understanding in the 0–100 and 0–1,000 range relative to the slow development across the lifespan of rational number understanding. Although people struggle to understand fractions, percentages might serve as an intuitive bridge between familiar whole numbers and less familiar fractions. We encourage readers to ponder the fact that grading schemes are inherently relational and people of all ages, expertise levels, and cultural backgrounds fall prey to a common mathematical misconception in which they think about the components of rational numbers—the numerators and denominators—like independent whole numbers. This misconception, known as the whole number bias, may make any grading scheme challenging for students to comprehend. There are many open empirical questions about the optimal grading scheme that college instructors should adopt. Findings from the domain of math cognition can inform empirical research designs that may lead to improvements in students' comprehension of the course grading scheme and motivation, and may even diminish student requests for end-of-term grade bumps.

Keywords: grading schemes, Society for the Teaching of Psychology Facebook page, math cognition, whole number bias

What point system should instructors use in their courses: 100, 1,000, or some other amount? Why? The answer to these questions, recently posted in December 2020 on the Society for the Teaching of Psychology's Facebook page,[1] has practical implications. Students are stressed about grades. They may struggle to calculate their final course grade, and they often engage in "grade grubbing," or asking for additional extra credit opportunities even when they haven't completed

Clarissa A. Thompson https://orcid.org/0000-0001-8758-3218
Charles J. Fitzsimmons https://orcid.org/0000-0002-2529-2164
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Correspondence concerning this article should be addressed to Clarissa A. Thompson, Department of Psychological Sciences, Kent State University, 228 Kent Hall Addition, P.O. Box 5190, Kent, OH 44242, United States. Email: cthomp77@kent.edu

[1] We have paraphrased all quotes from the Facebook thread to preserve responders' anonymity.

the assigned work. Instructors announce their grading scheme in their syllabus, yet get bombarded with emails requesting grade adjustments at the end of the term. In this Pedagogical Points to Ponder article, we will draw on math cognition research about students' (mis)understanding about math to inform evidence-based decisions about optimal grading schemes in college courses.

Many instructors weighed in on the Facebook post. One instructor adopted a 100-point scheme because of personal difficulties with math. This comment elicited laugh emojis; it's not uncommon for people to declare that they aren't "math people" (Miller-Cotto & Lewis, 2020). Another instructor noted a different approach: Point totals can be "non-round" numbers (e.g., 483). If the instructor can do the math, so the argument goes, the onus should be on students to calculate their own final course grades.

These divergent views highlighted that instructors themselves have differing levels of math skills, and in turn may want to consider their students' math proficiency when choosing a grading scheme. Taking into consideration students' preexisting math skills may alleviate students' grading-related anxiety, make it easier for students to calculate their final grade, reduce grade gubbing, and perhaps induce more positive evaluations of the course and instructor.

One poster was onto something critical: Using a 100-point scale made it easy to convert points into a percentage resulting in a better "psychological response," such as more favorable attitudes toward the grading scheme. What might be the basis of this psychological response? Research in math cognition illuminates some underlying mechanisms.

Differential Experiences With Whole Numbers and Rational Numbers

Across a lifetime, people amass extensive experience with whole numbers. In fact, by second grade, the vast majority of children produce precise estimates of the relative location of numbers on 0–100 number lines, and by fourth grade, most children possess a precise understanding of numbers in the 0–1,000 range (Siegler et al., 2009; Siegler & Opfer, 2003; Thompson & Opfer, 2010). For instance, between second and fourth grade, children learn to correctly place their estimate for the number 150 about 15% of the way across a 0–1,000 number line from left-to-right.

Rational numbers, like fractions, percentages, and decimals, are not as easily learned as whole numbers. They are introduced formally in the elementary school curriculum between third and sixth grades (Common Core State Standards Initiative, 2019), yet despite years of formal instruction on rational numbers, eighth graders (Siegler et al., 2011) and adults (Schneider & Siegler, 2010) still struggle to understand them. Part of this difficulty stems from overgeneralizing what is known about whole numbers to all rational numbers: a mathematical misconception known as the whole number bias (WNB). WNB is a common mathematical misconception that occurs across ages, expertise levels, and cultures (Alibali & Sidney, 2015; Alonso-Diaz et al., 2019; Braithwaite & Siegler, 2018a; DeWolf & Vosniadou, 2015; Fazio et al., 2016; Gómez et al., 2015; Ni & Zhou, 2005; Obersteiner et al., 2013; Opfer & DeVries, 2008; Van Hoof et al., 2020). It is characterized by a focus on the numerator and/or denominator in a fraction without regard for the relation between the two. For instance, because people have so much experience with whole numbers, they can easily say that 15 is less than 16. However, when they are confronted with the challenge of comparing two fractions, such as 1/15 versus 1/16, they have to inhibit their whole number knowledge of the denominators (number on the bottom of the fraction), to correctly answer that 1/16 is *smaller than* 1/15. Similarly, people might think that 10/15 is larger than 2/3 even though the fractions are equivalent in size (i.e., 66.7%; Braithwaite & Siegler, 2018b; Fitzsimmons et al., 2020b).

Despite the extensive experience that people have with whole numbers, there are a number of reasons why students may struggle to understand their grades. People hold more negative math attitudes about fractions and percentages relative to whole numbers (Sidney et al., 2021). To underscore the affective response that our students have to rational numbers, it is important to consider a recent open-ended response that a participant in one of our subject pool experiments typed to us. In the experiment, undergraduates estimated the size of fractions and then were asked to tell us the strategy that they used to estimate the fraction on the number line. Instead, the student typed, "Fractions are my worst nightmare!" Moreover, students often rate themselves as math anxious (Ashcraft, 2002; Berkowitz et al., 2015; Ramirez et al., 2018; Sidney, Thalluri, et al., 2019; Sidney, Thompson, et al., 2019), which may lead to math avoidance (Choe et al., 2019).

Adults' difficulties with rational numbers are alarming because proficiency with rational numbers is critical to success inside (e.g., comprehending grading schemes, access to higher education; National Mathematics Advisory Panel [NMAP], 2008) and outside of the classroom (e.g., succeeding at one's job, understanding interest rates and health communications; Handel, 2016; Peters et al., 2019; Thompson et al., 2020).

Bridging the Gap

Percentages may be the ideal bridge between whole numbers and rational numbers given that percentages are intuitive (Moss & Case, 1999). For example, people have experience watching a file load from 0% to 100% or their cell phone battery decrease from 100% to 0%. Instructors who commented on the Facebook post noted the value of percentages in helping students track their course grades. Approximately 35 unique posts contained the word, "percentage," or the symbol, "%." This aligns with recent findings from our lab: Even though adults were presented with health statistics represented as fractions (2/7) or whole number frequencies (2 out of 7), they often converted them to an easier-to-handle percentage to solve the health decision-making problem (Thompson et al., in press). When children and adults are asked to estimate the location of fractions on a number line or compare the size of two fractions, they similarly translate the fraction into a decimal or percentage. Those who do so make more precise estimates and comparisons of fractions (Fazio et al., 2016; Fitzsimmons et al., 2020b; Sidney, Thalluri, et al., 2019; Siegler et al., 2011; Siegler & Thompson, 2014).

As another example, health communication experts recommend that health statistics should be conveyed with percentages for optimal patient understanding (Waters et al., 2016). The benefit of percentages may be that they look (i.e., perceptual similarity) and behave (i.e., conceptual similarity) like whole numbers. These similarities likely allow people to disregard the percentage symbol and think about them like familiar whole numbers (Dehaene, 2011; Dehaene & Mehler, 1992; Fitzsimmons et al., 2020a).

Percentages, however, may not be the ideal solution for instructors who are trying to help their students better understand their grading scheme. Percentages can be misleading because not all percentages are created equal. Percentages that are weighted in a grading scheme might pose an obstacle for students who are trying to calculate their current grade to predict their future course performance (see Dunlosky & Metcalfe, 2009 for arguments about the value of metacognitive monitoring, control decisions, and self-regulated learning). For example, if a student earned 90% on two assignments worth 10% of their total grade, and an 80% on an assignment worth 20% of their total grade, what is their current course grade? This example illustrates that it is not uncommon for instructors to expect students to do quite complex calculations to understand their up-to-the-moment performance in a course.

This pitfall of percentages illustrates how percentages may elicit whole number bias errors. Intuitively, people may believe that 50% is always more than 10% because 50 is greater than 10, but this is not necessarily a true statement regarding percentages. Imagine two classes with different grading schemes. Class A has 10,000 total points. An assignment in Class A is worth 10% which equals 1,000 points. Compare this to Class B which has a 1,000-point scheme. A final paper or exam in Class B is worth 50% of the grade which equals 500 points. In absolute terms, 1,000 points in Class A is more than 500 points in Class B. However, in relative terms, 1,000/10,000 = 10% of the total points in Class A, and that is less than 500/1,000 = 50% of the total points in Class B. Grading schemes are inherently relational and likely not as transparent to students as instructors hope, especially because students must figure out unique grading schemes for many classes across many semesters. A failure to consider the relational nature of grading schemes is a real-world example of WNB at work in college classrooms.

Whole Number Bias and Student Motivation

WNB might be leveraged to motivate people inside and outside of the classroom. Numerical biases may make items appear cheaper than they are. For example, people might be more motivated to buy something that is $2.99 compared to $3.00 (e.g., Lai et al., 2018; Thomas & Morwitz, 2005). Furthermore, adults were more likely to cooperate with one another if they were offered a 300 cent reward as compared to a $3 reward (Furlong & Opfer, 2009), despite the fact that $3 = 300 cents.

Bringing our focus back to grading schemes, a 20-point paper in a 100-point course might feel "wrong" to students and instructors, even though a 200-point paper in a 1,000 point course might feel "right." Although *objectively* equivalent, these two assignments might feel *subjectively* different. Maybe it feels "wrong" because 20 is not a very big number in most scenarios, even though 20/100 = 200/1,000. Numbers cannot be considered in isolation.

Maybe instructors can use WNB to motivate students: A 200-point assignment in a 1,000-point course might elicit more effort than a 20-point assignment in a 100-point course, even if both assignments are worth the same percentage of the final grade. This may occur because students take minor quizzes more seriously when they are worth more absolute points (e.g., Cullen et al., 1975). If students perceive the point total for an assignment to be low, they might not put in as much effort to complete it. This could be part of the reason why instructors who commented on the Facebook thread were over two times more likely to endorse adopting a 1,000-point rather than a 100-point grading scheme. Instructors commented that the larger point range made it less likely for students to ask for missing points. It just feels absurd for a student to ask a professor for a 200-point grade bump, whereas requesting 20 points feels like a more reasonable "ask." This, of course, is an empirical question. Other instructors noted that using a 1,000-point scheme allowed them to offer their students several extra-credit opportunities. Prior research has shown that small point incentives were effective at motivating students to complete end-of-term course evaluations (Sundstrom et al., 2016). However, it is an empirical question as to whether these small incentives might motivate students to complete assignments involving more work, or if the assignment might be dismissed by students who realize that completing it will not make all that much difference in their final grade. That is, are high achievers always the ones who choose to complete extra credit assignments? Might low-achievers' math skills prevent them from fully comprehending the value of completing the extra credit assignments?

Preliminary Recommendations Based on Math Cognition Research

When it comes to thinking about optimal grading schemes, instructors do not have to rely on their intuition or anecdotes from other instructors. Decades of research in the domains of math cognition and math education show that people struggle to understand rational numbers (Ma, 1999; Mack, 1995; Siegler, 2016; Siegler et al., 2013; Steffe & Olive, 2010). Therefore, whatever grading scheme instructors adopt, they should consider their students' varying levels of math proficiency to help all students comprehend the value of each assignment within the overall grading scheme. We argue that this is necessary even for courses that include specific objectives pertaining to math (e.g., statistics). Decades of research on the topic of transfer

Table 1

Recommendations for Instructors Inspired by Math Cognition Research

- Remember that people of all ages struggle with rational numbers, and grading schemes rely on rational numbers. Don't assume all of your students can "do the math" to figure out your grading scheme. Whatever grading scheme you choose, be sure to remind students how to translate among points and percentages for each assignment as well as for the final course grade. This will especially help students who have low pre-existing math skills.
- Students may benefit from seeing points for each assignment in the learning management system gradebook as opposed to percentages because raw points may be easier to reason with as whole numbers which *can* be added up and divided by the total number of course points and then converted into a percentage. Keep in mind that you can't add two percentages together in the same way that you can add two whole numbers together (e.g., 50% + 20% does not equal 70%).
- Use a number line to show students how to visualize their points earned relative to the total points in the course. Online number line generators like the one linked here and shown in Figure 1 are user friendly and could be easily included in the course syllabus (https://www.oliverboorman.biz/projects/tools/number_lines.php). The student can plug in the total number of course points and how many they have earned in the course so far, and then they can easily see how far away from the total number of points they are. Of course, they can also easily figure out their percentage by dividing their points earned by their point total, but the number line visualization also provides non-numerical cues (i.e., distance from the start of the number line to points earned; distance from the points earned to the total number of points) about the magnitude of the points that they have earned.
- Choose a "round" point total, such as 1,000, because it will be easier for students to translate their points earned to a percentage than a "less round" grading scheme. Picking a grading scheme with a large point total will likely also cut down on "grade grubbing" because each assignment subjectively feels like it is worth a substantial number of points.

Table 2
Fruitful Avenues for Future Research

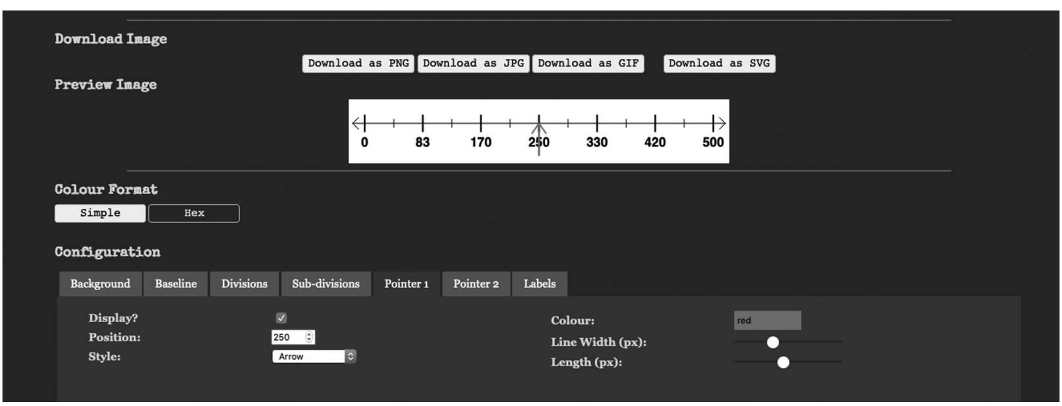
- Does performance on well-validated tasks that measure rational number understanding (e.g., fraction number line estimation, fraction magnitude comparison, fraction ordering, and fraction arithmetic) predict comprehension of course grading schemes? Specifically, are those who are better at transforming fractions in a pure numerical task also better at making transformations between points and percentages in their course grading schemes?
- How do different grading schemes affect students' attitudes towards their grades? Are some grading schemes more preferred by students (e.g., 100- and 1,000-point schemes) than others (e.g., 500-point or 872-point schemes)? Why?
- Are some grading schemes more motivating than others for assignments worth equal course value (e.g., a 20-point assignment in a 100-point course as compared to a 200-point assignment in a 1,000 point course)?
- Do some grading schemes compared to others help students better self-regulate their study behaviors? That is, the more transparent the grading scheme, the easier it is for students to know "where they stand" in the course?
- Might low-achievers' math skills prevent them from fully comprehending the value of completing extra-credit assignments?
- How do different grading schemes affect students' ability to calculate course grades?
- How can students' comprehension of grades be best supported? What types of visualizations (e.g., number lines) or instructional interventions (e.g., direct instruction, worked examples) are best to improve student comprehension of how their final grade is calculated?

(see Klahr & Chen, 2011 for an overview) suggests that transfer of learning is difficult and should not be expected in all cases. Therefore, even if an instructor helps their students understand the formula to calculate a *t*-test by hand, for instance, this instruction does not ensure automatic transfer to students' understanding of how ratios play a role in their grades.

Transformations from course points to percentages might be transparent for the instructor and mathematically savvy students, but could be opaque and even anxiety provoking for others (Schiller, 2020). In an ideal world, all students should be able to do basic math to calculate their course grade, but that is not the reality (NMAP, 2008). Even undergraduates at a highly selective university reported some faulty strategies when reasoning about rational numbers (Fazio et al., 2016).

Many college students are enrolled in developmental math courses, so they may need extra support to comprehend the grading scheme. In fact, about 40% of adults sampled in one of our recent studies made WNB errors in a crucial health domain: deciding whether the flu was more fatal than COVID-19 (Thompson et al., 2020). Clearly,

Figure 1
Online Number Line Generator to Visualize the Number of Points Earned and the Total Number of Points Possible in a Course



Note. Instructors can use multiple number lines to show the points earned toward the total course grade as well as the points earned out of the total available points at any given point in the course. Landmarks (Siegler & Thompson, 2014) can be used to show the location of letter grades (e.g., C is farther to the left than B, and A landmarks).

rational numbers are hard. Instructors should do whatever they can to help students understand their course grading scheme. How best to do this is an open empirical question. In Table 1, we have included a list of preliminary practical recommendations for instructors to consider implementing in their courses. However, we want to underscore that more research is needed before we can make stronger prescriptive recommendations (see Robinson et al., 2013; Robinson & Levin, 2019 for arguments about going too far beyond one's data).

Our preliminary literature search on the Web of Science database including search terms such as "grading scheme," "grades and college courses," and "grade bumps" did not uncover much existing literature involving the impact of a specific numerical range on student understanding of course grades and overall grading schemes. There is, however, some existing literature in which researchers have investigated individual differences in the motivational nature of course incentives (Chulkov, 2006; Lei, 2013) and comparisons of grading schemes and incentive structures in which points are added to or taken away from students (Cullen et al., 1975; Docan, 2006). This suggests that the future research ideas listed in Table 2 are novel and could ultimately benefit both students and instructors by improving course attitudes, decreasing grade anxiety, and decreasing email burden.

One future research idea is to use worked examples (McGinn et al., 2015) that involve step-by-step instructions on how to calculate final grades and visualizations, like number lines, to help students track their grades relative to total course points (cf. Sidney & Thompson, 2019; Siegler et al., 2011). Worked examples have been used successfully in the math cognition literature to teach students of all ages how to complete procedures required to correctly solve math problems.

Another plan for future research could involve randomly assigning willing instructors to grading schemes of various point values and analyzing end-of-semester course evaluations. Even though all point totals can be equated with percentages, various schemes may result in differential levels of student motivation, positive instructor evaluations, and end-of-semester requests for grade adjustments.

In summary, we hope that this Pedagogical Points to Ponder article has provided some food for thought for college-level instructors who have not previously given extensive thought to their grading schemes. Table 2 indicates there are many open empirical questions worth answering, yet research from math cognition can inform some initial ideas for immediate implementation (Table 1). The main take-home message of our recommendations is for instructors to take a learner-centered approach to choosing a grading scheme. It is crucial for instructors to take students' rational number proficiency into consideration and how it might pose problems for students as they attempt to figure out their course grades. There are pros and cons to all grading schemes given that they all require relational understanding. Additional research is needed to crown a grading scheme the "winner." Regardless of which scheme instructors choose, they should be prepared to help students understand the conversions. Instructors can support students' rational number understanding of course grading schemes by illustrating how point totals, percentages, decimals, and letter grades are equivalent and by offering direct instruction on how to calculate grades throughout the term.

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