Equations, Functions, and Modeling with Real-world Problems in Algebra I

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Abstract

This program focused on improving mathematics content and pedagogy for Algebra I teachers, including regular classroom teachers and exceptional education teachers, as they implemented the Tennessee Mathematics Standards. Concentrating work on a few topics allowed for tightening breadth while increasing depth of content, as well as devoting study to the mathematics required for career choices. A major focus of Algebra I content is working with equations, functions, and modeling of real-world problems. All activities were correlated to Tennessee Standards for Algebra I. The eight Mathematical Practices were woven throughout the activities.

The Tennessee Educator Acceleration Model was reviewed with teachers, and activities were presented and analyzed, with regard to the rubric. The 19 TEAM categories meshed with the program objectives.

Participants included 24 teachers from across southeast Tennessee. The program timeline included one spring Saturday session, a 5-day summer academy, and two follow-up Saturday sessions in the fall (50 contact hours), and online discussion throughout the spring, summer, and fall. Hands-on activities, using mathematics manipulatives and technology, and constructivist strategies for teaching and learning, were emphasized. Participants demonstrated a significant increase in algebra content knowledge.

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Introduction

The pairing of STEM and workforce development presented the opportunity to explore mathematics within real-world contexts, as detailed by Tennessee's Mathematics Standards for Algebra I (Tennessee Department of Education, n.d.a). UTC Chancellor Steven Angle noted that the "most critical need" in this geographic area is "educational attainment" (pers. comm., 09/15/15). STEM is one of 16 identified career clusters (Tennessee Department of Education, n.d.b) which "encompass virtually all occupations from entry through professional levels" (¶ 1). Prior to career selection, students complete the required high school Algebra I course. This program studied the topics of equations, functions, and modeling, and related these to real-world problems and career choices. This program focused on improving mathematics content and pedagogy for 24 Algebra I teachers, including regular classroom and exceptional education teachers, as they continued to implement the Tennessee Standards. Concentrating work on focused topics allowed for tightening breadth while increasing depth of content and devoting study to the mathematics required for career choices. The major focus of Algebra I content, with regard to equations, functions, and modeling, includes (a) quantitative reasoning; (b) seeing structure in expressions; (c) polynomials and rational expressions; (d) creating equations; (e) reasoning with equations and inequalities; (f) interpreting functions; (g) building functions; (h) linear, quadratic, and exponential models; and (i) interpreting categorical and quantitative data. Mathematical Modeling, one of the standards of Mathematical Practice, is embedded in many of these clusters. The program emphasized the importance of equations, functions, and modeling for solving contextual problems. The eight standards for Mathematical Practice were woven throughout the activities.

The Tennessee Educator Acceleration Model (TEAM, n.d.) was reviewed with teachers, and activities were presented and analyzed, with regard to the rubric. The 19 TEAM categories meshed with the program objectives. The four competencies with which teachers struggled most, and that needed the most focused attention in implementing higher standards, were questioning, thinking, problem solving, and academic feedback (E. Barton, pers. comm.., 05/20/14). Attention was focused on those four areas of TEAM, in relation to program content and pedagogy. In tying together the Tennessee Standards and TEAM, it was noted that teachers who taught the standards well fared better on the teacher evaluation, and students who wrote, at least once per month, in a subject other than English, fared better on assessments. English language arts skills relate to the TEAM categories with which teachers struggled, with questioning, thinking, and problem solving as foundational in literacy. The pairing of mathematics and English language arts was emphasized to assist in, both, teacher and student assessment, as well as in career choice.

Review of Literature

Noh and Webb (2015) investigated teacher knowledge of rate of change and found the context of the problem to be a factor in teacher ability to explore complex problems in algebraic functions. More experienced teachers were more successful in solving the problems and in recognizing similar and contrasting characteristics of types of problems. This suggests a need for improvement for less experienced teachers.

Dubinsky and Wilson (2013) stressed that, with appropriate pedagogy, algebra students from underrepresented groups were able to achieve a level of understanding of functions similar to that of beginning college students. Krupa (2011) found that North Carolina students who followed an integrated curriculum outperformed students who followed a subject-specific, Algebra I curriculum. This program drew sample problems from across the mathematics curriculum, as related to algebra.

Rust (2011) found that, in a community college pre-algebra course, stronger implementation of reading strategies positively impacted student achievement, and suggested that organized professional development is required for new strategies to be effective.

Saucedo (2017) examined how high school mathematics teachers felt about the level of support they had received from professional development experiences. The findings revealed that teachers felt differently based on, both, their amount of experience and the level at which they taught. Teachers who had been teaching for longer amounts of time, or were teaching at the Algebra 2 level or higher, felt much more negatively about professional development, indicating it was a waste of time because it did not pose much of a challenge. Teachers with less experience (10 years or less), or who taught math at the lower level (Algebra 1 or Geometry), viewed these professional development programs more positively and felt that the professional development was designed to be relevant to the teachers.

Using MathForward, a program that integrates technology and professional development concerning algebra-related concepts, teachers received increased amounts of professional development that resulted in them integrating technology into their lessons. The study ultimately found that this method of professional development was effective in decreasing the gap in teacher efficacy between newer teachers and experienced teachers (Hill et al., 2017).

Mills and Harrison (2020) focused on a professional development program that was designed to help middle and high school algebra teachers better understand the practice of formative assessment and improve their utilization of it in the classroom. The program allowed teachers to work on planning formative assessments, reflecting on them, and then revising their assessments.

Methods

This partnership was designed to increase teacher content knowledge, as defined by the Standards, and subsequent student mathematics achievement, as well as pedagogical change associated with the Mathematical Practices. According to 2015 data (Tennessee Department of Education, 2015), many of the partnering school districts had made little or no progress, or posted losses, regarding student growth in Algebra I, English I, and English II. Across the school districts, the percent of students proficient or advanced ranged from 48% to 67% for Algebra I, 64% to 74% for English I, and 54% to 69% for English II, with loss in all partnering school districts from English I to II. Algebra I proficiency gap data for historically underperforming groups (Black/Hispanic/Native American, Economically Disadvantaged, English Language Learners, Students with Disabilities) showed that the school districts included one to three subgroups for which the gap had increased, ranging from 0.6% to 4.3%. For English II, the gap had increased, ranging from 1.8% to 19.2% (significantly so, for Economically Disadvantaged).

Concepts of functions are crucial for students to learn but may be challenging for teachers to teach. Students need to understand these concepts to be successful in courses that "build on quantitative thinking and relationships" (NCTM, 2010, p. 1). The selected texts and activities provided needed work, especially through modeling real-world problems. Trends that emerged during the summer academy and through the NEON portal were revisited during the fall sessions. The National Governors Association (2008, abstract) found that teachers who attended all professional development sessions, and implemented the project materials, saw the greatest gain in student achievement. Program sessions and discussion allowed teachers the necessary time to study and implement new content and strategies.

Through an inquiry-based, concentrated study of mathematics concepts, this program focused on professional development as an agent to effect change in the Algebra I classroom. The Tennessee Standards and Mathematical Practices were emphasized through the selected instructional resources. See Figures 1 and 2.

Through teacher professional development, this program had the potential to increase student achievement in mathematics, and decrease achievement gaps between subgroups in mathematics, as reported by the Tennessee Department of Education (2015). Further, *The Nation's Report Card* (National Center for Education Statistics, n.d.) reported that Tennessee eighth-grade mathematics test results remained lower than the national average, on the 2013 test. This program provided numerous pedagogical strategies to improve teaching for diverse groups of students, through varied hands-on activities, real-world problems, career connections, and the incorporation of literacy strategies. Another strategy, aimed at closing gaps, was the learning community (DuFour, DuFour, & Eaker, 2008). The NEON site served as a learning community for the teacher to ask questions, comment on activities, and post a summary of the required presentation to colleagues.

Registration information was sent to each school district's curriculum coordinator for distribution to school principals. School district partners were committed to recruitment of teachers who could effect positive change in student assessment outcomes, and agreed to provide the opportunity for teachers to present an aspect of the program to colleagues, as they returned for the academic year. Including teachers from regular and exceptional education addressed some of the mathematics proficiency gaps to boost mathematics proficiency on formal assessments for students from historically underperforming subgroups. Effort was made to recruit teachers from underrepresented groups. Effective recruitment was evidenced in seven previous THEC programs, as 66 of 295 teachers (22%) were from underrepresented groups. The

Chattanooga Chamber of Commerce (n.d.) reported 2014 area ethnicity data as 81% White and 19% from underrepresented groups. For this program, 1 of the 24 participants was from an underrepresented group.

Mathematics received renewed attention through the implantation of the Tennessee Standards. School districts focused attention on building the foundation of knowledge and skills required for students to advance in mathematics. Program activities emphasized multiple approaches to teaching and learning, were written to be used with students in the classroom, and provided implementation information. The program timeline is presented in Figure 3.

Teachers focused on standards through real-world scenarios, collecting data, making connections across topics in mathematics, and communicating. Texts, Internet sites, and resources promoted important mathematics for all students and innovative practices. Constructivist strategies were emphasized, and included learning through posing problems, exploring possible answers, and focusing on global goals that specify general abilities such as problem solving, completing group work, and exploring open-ended questions (Roblyer, 2003). The summer academy daily schedule included activities in the morning and afternoon, and a formative assessment. Sample activities are presented in Figure 4.

The program included one spring Saturday session, a 5-day summer academy, and two follow-up fall Saturday sessions (50 contact hours), and online discussion throughout the program. Hands-on activities, using a variety of mathematics resources, and constructivist strategies for teaching and learning, were emphasized. Emphasis was placed on solving real-world problems. Relevant Internet resources were explored. An online group, through the NASA Educators Online Network (NEON, 2015), was established for communication. The goal was to provide high-quality, teacher professional development to Tennessee teachers to increase content knowledge and instructional skills aligned with the Tennessee Standards. Measurable objectives included the following:

1. There will be a statistically significant increase in teachers' scores on a 39-item mathematics quiz, between pre-test and post-test assessments (items correlated to Tennessee Standards).
2. There will be a statistically significant increase in teacher growth on observed instances of teachers' problem-solving skills, as the program progresses (*Survey on Mathematics*, n.d.).
3. There will be a statistically significant improvement in teacher attitude toward mathematics, as the program progresses (Remmers, 1960; qualitative instrument).
4. Teacher reporting of perception of student learning will be greater than 50% on all measures (University of Minnesota, Morris, 2000; qualitative instrument).

Results

Data collection and analysis was concerned with change in mathematics content knowledge, problem-solving skills, and attitude toward mathematics, and perception of student learning. Program evaluation was aligned with the measurable objectives and Algebra I served as the mathematics content.

A 39-item pre-test/post-test was administered in April at the outset of the professional development sessions, in June at the close of the summer academy, and in September, allowing time for initial learning, classroom implementation, and reflection. Item sources included Engage NY (2013), Engage NY (2014), the New York State Education Department (2015), the Louisiana Department of Education (2013), The University of Iowa, (2010), and the California Department of Education (2015). All items were aligned to the Tennessee Mathematics Standards for

Algebra I. Coefficient alpha was calculated as 0.789. A one-tailed *t*-test showed a significant increase in knowledge from the spring to the end of the summer academy administration (n=24, $p<.01$) and from the spring to the fall administration (n=19, $p<.05$). There was no significant difference from the end of the summer academy to the fall administration. See Figure 5.

For selected daily activities, observational data quantified teacher problem-solving skills, through the *Survey on Mathematics* (n.d.). The survey served as a checklist of problem-solving behaviors, and instances of observed problem-solving behavior were tallied and compared over time. No significant differences were found in observed instances of problem-solving skills.

The mathematics attitude survey was administered in April, June, and September, to determine if an attitude change had occurred over the course of the program. Remmers' (1960) scale is interpreted through a score assigned to the item which is the median of the list to which the teacher agreed. The 17-item scale is scored from 10.3 (high) to 1.0 (low). The three means were consistent, at 8.4, for the three administrations of the survey. No significant differences were found for change in attitude toward mathematics.

Prior to the October session, 20 teachers completed a survey regarding perception of student learning (University of Minnesota, Morris, 2000). Teacher-reported perception of student learning was lower than the set level of 50% for each of the 10 items. Average question ratings ranged from 32% to 79.5%. For the 10 measures, 7 measures were scored above the set level of 50%, with the highest rating given to student demonstration of knowledge and skills as basic, proficient, or advanced. Other ratings above the 50% level were demonstration of proficient or advanced knowledge and skills (56.32%), ability to model real-world problems (53.5%), ability to solve real-world problems (53.5%), ability to formulate a problem mathematically (53%), ability to determine an appropriate approach toward a solution to a problem (62.5%), and ability to interpret results of a mathematical or statistical analysis (50.75%). The three measures that were lower than the 50% level were demonstration of advanced knowledge and skills (32%), ability to solve problems whose solutions did not fit into existing mathematical knowledge (39.25%), and ability to communicate mathematical ideas effectively (47.25%).

All activities were correlated to the Tennessee Mathematics Standards, with activities being delivered through 50 contact hours, and cross-curricular connections being made between mathematics and English language arts. In addition, a daily evaluation was administered through a writing prompt, serving as a formative assessment of learning. The daily plan was modified as determined necessary to best meet needs. The program was assessed through a brief survey administered in October.

Discussion

With regard to the pre-test/post-test, the significant increase in mathematics content knowledge from the spring to end of the summer academy administration and from the spring to the fall administration demonstrated learning across the timeframe of the program. As there was no significant difference from the end of the summer academy to the fall administration, this may indicate that knowledge was both gained and retained through the timeframe of the program. Retention could be due, in part, to teachers enhancing content and pedagogy over the summer, and working with students, upon returning to school, with new teaching strategies, new content examples, and new materials to enhance content and pedagogy.

The 25-item checklist was used to assess changes in problem-solving strategies among the participants. This was done through observation as activities occurred and did not provide a significant result. If used in the future, the items might be grouped so a smaller number of problem-solving behaviors would be tallied by group.

Attitude toward mathematics did not change. All teachers were mathematics teachers; therefore, an attitude change might not be expected.

Teacher-reported perception of student learning was lower than the set level of 50%. For the 10 measures, 7 measures were above the set level of 50%. Teacher-reported perception of student learning was that a high percentage of students demonstrated basic, proficient, or advanced mathematics knowledge and skills, but student level was not, necessarily, advanced. Many of the mean responses were close to the 50% level, indicating that, in general, teacher perception was that half of the students were able to demonstrate a particular level of proficiency or an ability.

Overall, participants were highly satisfied with the professional development program. Participants expressed the need for continued sessions.

It was hoped that, upon program completion, teachers would have a sharpened focus toward identifying and implementing Algebra I, standards-based activities, and would have the demonstration resources necessary for successful implementation. Teachers would understand the most critical areas of subject-specific mathematics, be able to employ the targeted instructional practices of TEAM, and be able to address student needs through the integration of mathematics and related English language arts skills.

The books and resources were selected to have a long-term, useful life in the classroom. Activities were correlated to Standards, rather than to a particular publisher's curriculum. Activities and resources emphasized active learning, use of relevant tools, connections between mathematics and other content areas, deeper understanding of mathematical concepts, and improved problem solving. It was deemed important to a develop meaningful learning environment for teachers as a model to use for creating such an environment in their classrooms.

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Figure 1

A Summary of Program Texts, Internet Sites, and Manipulatives

- *Implementing the Common Core State Standards through mathematical problem solving: High school* (NCTM, 2012a). The text will be used for discussion and activities at the April meeting. It contains rich problems that tie Tennessee Standards content to problem-solving skills.
- *Mathematics lessons learned from around the world, grades 7-12* (NCTM, 2012b). The text contains algebraic and geometric problems that model real-world scenarios.
- Activities from the *Navigations* series for grades 9-12. Activities that best exemplify work with equations, functions and modeling will be selected from the series (NCTM 2001a, 2001b, 2003, 2005b, 2006a, 2006b, 2008a, 2008b), and from *Mission mathematics II, grades 9-12* (NCTM, 2005a), a collaboration between NCTM and NASA.
- *Illustrative mathematics* (n.d.). The site contains real-world problems correlated to standards.
- Books aligned with literacy activities. Books include *The number devil: A mathematical adventure* (Enzensberger, 1997); *Crimes and mathdemeanors* (Hathout, 2007); and *Math girls talk about equations and graphs* (Yuki, 2014).
- Interactive resources are available on Internet sites, such as *PhET interactive simulations* (University of Colorado Boulder, 2015); *Gizmos* (Explorelearning, 2015); *Illuminations* (NCTM, n.d.); *Interactivate* (Shodor, 2015); and the *National library of virtual manipulatives* (Utah State University, 2015).
- Participants received the following manipulatives: Pattern Numbers, two-speed cars, Zome Ice Crystals and Stars, customized package of additional materials.
- A demonstration of probeware that supports functions and modeling was provided.

Figure 2

Working List of Internet Resources

- PhET interactive simulations - https://phet.colorado.edu/
- Gizmos - https://www.explorelearning.com/index.cfm?method=cResource.dspBookCorrelation&id=598
- Illuminations - http://illuminations.nctm.org/
- Interactivate - http://www.shodor.org/interactivate/
- National library of virtual manipulatives - http://nlvm.usu.edu/en/nav/vlibrary.html
- Algebra virtual manipulatives - http://www.teachmathematics.net/page/2958/algebra-virtual-manipulatives
- Rossman/Chance applet collection - http://www.rossmanchance.com/applets/
- Algebra applets - http://www.saltire.com/HTML5/HTML5%20apps/Algebra/Algebra%20Index.html
- Geometry Java applet gallery - https://saltire.com/HTML5/gallery.html
- Java Applets, Boston University - http://math.bu.edu/DYSYS/applets/
- Math and physics applets, Paul Falstad - http://www.falstad.com/mathphysics.html
- Mathematics, Science, and Technology Education, University of Illinois - http://mste.illinois.edu/resources/
- Spirograph - http://www.wordsmith.org/~anu/java/spirograph.html
- Tower of Hanoi - https://www.mathsisfun.com/games/towerofhanoi.html
- Probability, mathematical statistics, stochastic processes (Apps) - http://www.math.uah.edu/stat/
- Math applets, Math Warehouse - http://www.mathwarehouse.com/interactive/html5-applets/
- Algebra 1 & 2, Geometry - https://www.geogebra.org/m/j6srv3jf
- Model algebra - http://www.mathplayground.com/AlgebraEquations.html
- Java applets on mathematics, Walter Fendt - https://www.walter-fendt.de/html5/men/
- Interactive mathematics activities - http://www.cut-the-knot.org/Curriculum/
- Virtual manipulatives, David Young - http://plaza.ufl.edu/youngdj/portfolio.htm

Figure 3

Program Timeline

Spring 2016

- January-March – Identify teachers, begin resource Web page. Order resources for April 9.
- April 9, Saturday session, 6.25 contact hours. Pre-test for content knowledge. Attitude survey. Formative evaluation. Begin standards-based activities, and online discussion through NEON.
- May – Order resources and prepare notebooks for the summer academy.

Summer 2016

- June 6-10 – 5-day summer academy, 31.25 contact hours. Activities and implementation strategies. Formative evaluation. Mid-point post-test for content knowledge. Attitude survey.
- June – December – Continue online discussion through NEON.

Fall 2016

- August – September – Each teacher will present a program aspect to colleagues.
- September 17 – Saturday session, 6.25 contact hours. Teachers report on information presented to colleagues. Continued work with standards-based activities and implementation strategies. Post-test for content knowledge. Attitude survey.
- October 22 – Saturday session, 6.25 contact hours. Continued work with standards-based activities. Focus on student assessment. Program evaluation. Collection of teacher perception survey.
- October to December – Classroom co-teaching and district professional development, as requested. Continue online discussion through NEON.

Figure 4

Sample Activities

- Quantities – Ice cream van. Modeling is used for an open-ended problem.
- Seeing Structure in Expressions – Animal populations. Variables are considered within context.
- Arithmetic with Polynomials and Rational Expressions – Graphing from roots. The remainder theorem is used to deduce a linear factor of a cubic polynomial.
- Creating Equations – Starbucks expansion. Create a mathematical model of store expansion.
- Reasoning with Equations and Inequalities – Shadowy measurements. Collected data is used to determine a direct proportion between object height and shadow length.
- Interpreting Functions – Telling a story with graphs. Connections are made across graphs.
- Building Functions – Lake algae. Exponential growth is studied, from the ending value.
- Linear, Quadratic, and Exponential Models – Moore's Law and computers. The increase in hard drive storage capacity is modeled and used.
- Interpreting Categorical and Quantitative Data – Coffee and crime. Correlation and causation are studied within context.
- Books will highlight English language arts skills within mathematics tasks.
- Each of the eight professional development sessions will highlight 2 of the 16 Tennessee career clusters.

Figure 5

Pre-test and Post-test Data

	n	**Mean**	**Median**	***p***
April	24	32.17	34	$p<.01$
June	24	35.33	36.5	
April*	19	33.26	34	$p<.05$
October	19	35.47	36	
June*	19	35.95	36	NS
October	19	35.47	36	

*This includes the paired data for the participants who completed the three assessments.