Received: 30 May 2019 | Revised: 12 March 2020 | Accepted: 17 April 2020

DOI: 10.1002/jee.20354

JEE
Journal of Engineering Education

ORIGINAL ARTICLE

How drawing prompts can increase cognitive engagement in an active learning engineering course

Sally P. W. Wu[1] | **Barry Van Veen**[2] | **Martina A. Rau**[2]

[1]Northwestern University, Evanston, Illinois

[2]University of Wisconsin–Madison, Madison, Wisconsin

Correspondence
Sally P. W. Wu, Northwestern University, 2120 Campus Drive, Evanston, IL 60208.
Email: sally.wu@northwestern.edu

Funding information
Division of Undergraduate Education, Grant/Award Number: DUE1611782; Institute of Education Sciences, Grant/Award Number: R305B150003; Lynn H. Matthias Professorship; National Science Foundation Division of Undergraduate Education, Grant/Award Number: 1933078

Abstract

Background: Recent engineering education research has found improved learning outcomes when instructors engage students actively (e.g., through practice problems) rather than passively (e.g., in lectures). As more instructors shift toward active learning, research needs to identify how different types of activities affect students' cognitive engagement with concepts in the classroom. In this study, we investigate the effects of prompting novice students to draw when solving problems, a professional practice of engineers.

Purpose: We investigate whether implementing instructional prompts to draw in an active learning classroom (a) increases students' use and value of drawing as a problem-solving strategy and (b) enhances students' problem-solving performance.

Method: We compared survey data and exam scores collected in one undergraduate class that received prompts to draw in video lectures and in-class problems (drawing condition) and one class that received no drawing prompts (control condition).

Results: After drawing prompts were implemented, students' use and value of drawing increased, and these effects persisted to the end of the semester. Students were more likely to draw when provided drawing prompts. Furthermore, students who received prompts outperformed students who did not on exam questions that target conceptual understanding.

Conclusions: Our findings reveal how implementing drawing prompts in an active learning classroom may help students engage in drawing and solve problems conceptually. This study contributes to our understanding of what types of active learning activities can improve instructional practices in engineering education. Particularly, we show how prompts that foster authentic engineering practices can increase cognitive engagement in introductory-level engineering courses.

KEYWORDS

active learning, conceptual learning, drawing, educational technology, flipped classroom

1 | INTRODUCTION

Recent engineering education research has advocated for instructors to implement active learning activities that promote "hands-on" and "minds-on" engagement with content in contrast to traditional lectures where students listen passively (Chi & Wylie, 2014; Freeman et al., 2014). Much evidence in engineering education has shown that active learning activities are more effective than passive activities, but prior work has not focused on how and when specific types of activities are most effective (Streveler & Menekse, 2017).

One common approach to promote active learning in engineering is to focus class time on solving practice problems through reducing in-person lectures or replacing them with video lectures (Bishop & Verleger, 2013; Kerr, 2015). This approach allows students to solve problems with the support of instructors and peers in the classroom instead of listening passively to a lecture. Some evidence has shown that this method of reducing lectures can increase student performance on exams or grades in engineering classrooms (Kerr, 2015; Lo & Hew, 2019). However, much of this work does not identify which activities are most effective, particularly toward enhancing students' conceptual understanding (Bishop & Verleger, 2013; Kerr, 2015).

It is important to understand how active learning classrooms can support conceptual understanding because only focusing on practice problems in the classroom may encourage students to engage in ineffective problem-solving strategies that rely too strongly on procedural knowledge. Particularly in introductory-level engineering courses, students tend to solve problems procedurally using a "plug-and-chug" or "trial-and-error" method in which they immediately apply a formula or algorithm to given problems (Bergqvist, 2007; Lithner, 2003). This process tends to be a purely mechanical procedure of symbol and number manipulation. In so doing, they may not engage with the underlying concepts (Bergsten, Engelbrecht, & Kågesten, 2017; Higley, Litzinger, Van Meter, & Masters, 2007; Leppävirta, Kettunen, & Sihvola, 2011). Furthermore, students may not develop the professional practices of engineers who engage in conceptual problem solving (de Vere, Melles, & Kapoor, 2011; McCracken & Newstetter, 2001).

When engineers solve problems, prior research shows that they first translate problems into visual–spatial drawings that help them qualitatively "see" and reason with concepts (Cardella, Atman, & Adams, 2006; McCracken & Newstetter, 2001; Vale, Pimentel, & Barbosa, 2018). The drawings serve as a model of the problem and depict how the underlying concepts relate to one another (Kothiyal, Murthy, & Chandrasekharan, 2016; Schwarz et al., 2009). Then, engineers review and evaluate their drawings by quantitatively applying formulas or using a calculation tool to determine numerical parameters, which then helps them revise their drawings. Studies with engineers consistently show that they engage in this visual problem-solving strategy rapidly and flexibly to align with the specific problem they are solving (Kothiyal et al., 2016; McCracken & Newstetter, 2001). However, studies with students show that senior engineering students engage in this strategy and view it as valuable, but novice students often do not (McNeill, Douglas, Koro-Ljungberg, Therriault, & Krause, 2016). Novice students may not choose to engage in drawing as a problem-solving strategy if they often prefer to use the plug-and-chug method in engineering courses.

This study addresses the gap between professional engineers and novice students in problem-solving strategies by leveraging prior research in cognitive sciences. This research shows that novice students may struggle to draw without instructional support (Leutner & Schmeck, 2014; Van Meter & Garner, 2005). Hence, we provided instructional support via drawing prompts, which are instructional prompts (e.g., text presented under a problem) that ask students to draw freehand, typically with pen and paper (Leutner & Schmeck, 2014; Prain & Tytler, 2012). Prior research suggests that drawing prompts can help students make connections among concepts when solving problems and achieve higher levels of understanding (Fan, 2015; Wu & Rau, 2018). In engineering research, short-term studies with interviews and in workshops show that drawing prompts are effective in helping engineering students engage with conceptual relationships and solve problems (de Vere et al., 2011; McNeill et al., 2016). However, prior work has not tested whether drawing prompts can promote active learning and conceptual problem solving in a semester-long engineering classroom, such that they counteract students' preferred plug-and-chug method.

We address this gap by conducting a quasi-experimental study in two offerings of a semester-long active learning course for undergraduate engineering students. We implemented drawing prompts in one course and compared differences between the courses on students' (a) perceptions of drawing as a problem-solving strategy and (b) exam performance. We investigate the effects of drawing prompts to provide insight into whether drawing prompts can enhance active learning activities in the engineering classroom and help novice students draw to solve problems as professionals do.

In the following, we briefly review prior research on how students engage with different active learning activities and how drawing may help novice engineering students solve problems conceptually. Then, we discuss how instructors can implement drawing prompts to enhance active learning activities and facilitate conceptual problem solving in their classrooms.

1.1 | How students engage with active learning activities in engineering courses

One key goal of this study is to identify how and when specific types of active learning activities are effective for students in an authentic engineering classroom. To this end, we differentiate activities using the interactive-constructive-active-passive (ICAP) framework as suggested by engineering education researchers (Streveler & Menekse, 2017). The ICAP framework suggests that higher learning outcomes are associated with higher levels of cognitive engagement, which increase from passive to active to constructive to interactive activities (Wylie & Chi, 2014). Passive activities (e.g., lectures, videos) may not engage students cognitively. Active activities (e.g., highlighting, transcribing) help students physically interact with concepts but may not cognitively engage students. Constructive activities (e.g., making concept maps, summarizing) increase cognitive engagement because students must make connections and construct their own knowledge. Interactive activities (e.g., discussions, collaborative projects) further increase cognitive engagement because students construct ideas using input and feedback from others, which creates deeper cognitive connections to concepts. Prior work in engineering education has not differentiated between active, constructive, and interactive levels of engagement, research which may explain differences in learning outcomes between specific types of activities implemented in the classroom (Streveler & Menekse, 2017).

In the engineering classroom, students often learn content by solving practice problems, which may only engage them actively. Particularly in the introductory-level courses, students are expected to learn concepts and formulas by solving simplified problems with specified constraints and singular goals (McNeill et al., 2016). These types of problems, also called “well-structured problems” or “classroom problems,” are easy to solve by identifying the givens of the problem, finding a formula that involves those givens, and plugging in the givens to find the numerical solution. Because the formulas are typically provided or derived for students, they do not have to constructively organize and relate the underlying concepts but can actively plug-and-chug numbers. However, students who engage *actively* with concepts when problem solving may only gain procedural knowledge, but not conceptual understanding (McNeill et al., 2016; Streveler, Litzinger, Miller, & Steif, 2008).

To develop conceptual understanding, students may need active learning activities that increase cognitive engagement, such as drawing. Prior research shows that drawing can enhance students' conceptual understanding because it engages higher-order thinking (e.g., elaboration, synthesis, and integration of concepts; Leutner & Schmeck, 2014; Van Meter & Garner, 2005). When students generate drawings, they must select, organize, and integrate relevant features into a coherent form (Van Meter & Firetto, 2013; Van Meter & Garner, 2005). In this process, students must determine how concepts are related structurally or analogically (e.g., real and imaginary components are two perpendicularly related dimensions, like x and y on x–y axes). Importantly, they must specify how they understand these relationships in an externally generated representation (Kavakli & Gero, 2001; Tversky, 2011). The generated drawings then allow students to reflect, revise, and further explore related visual–spatial concepts (Bobek & Tversky, 2016; Cardella et al., 2006).

In accordance with the ICAP framework, drawing can engage students constructively or interactively. As an individual activity, drawing helps students constructively engage in higher-order thinking about how visual–spatial concepts relate. As a collaborative activity, drawing helps students *i*nteractively engage with others in their classroom (White & Pea, 2011; Zhang & Linn, 2011). When students draw, they generate external drawings that serve as shared artifacts for students to discuss (White & Pea, 2011). Students who discuss their drawings may realize that they hold divergent views, which can prompt them to clarify relationships between features in their drawings and engage more deeply in making sense of the underlying concepts (Johri, Roth, & Olds, 2013; White & Pea, 2011; Zhang & Linn, 2011). After discussing and comparing their drawings, students may revise their drawings to be more clear, parsimonious, and explanatory representations of the concepts they depict (Greeno & Hall, 1997; Nathan, Eilam, & Kim, 2007). Through this process, students may also learn how to use drawing as a communication tool in a way that resembles professional practices (McCracken & Newstetter, 2001; Ullman, Wood, & Craig, 1990).

1.2 | Challenges for drawing as an instructional practice in engineering education

Although drawing is a valued professional practice, current *i*nstructional practices in undergraduate engineering, particularly in introductory-level courses, typically do not focus on drawing as a problem-solving strategy to reason with concepts. Rather, engineering courses have been found to overly emphasize quantitative skills and algorithmic thinking (Litzinger et al., 2011; Nelson Laird, Shoup, Kuh, & Schwarz, 2008). In such courses, engineering instructors help students build a foundation of knowledge by defining relevant concepts, deriving formulas, and demonstrating how to apply formulas to solve well-structured problems that target specific concepts (Litzinger et al., 2011; Streveler et al., 2008). Students can excel in these courses by memorizing and applying formulas procedurally on exams or homework problems (Bergqvist, 2007; Lithner, 2003). Consequently, students may emerge from introductory-level courses without an understanding of how underlying concepts relate to one another or develop the problem-solving strategies used by professionals.

Engineering education research suggests shifting instructional practices to focus on a minimal-mathematical approach that relies on drawings and other visuals. This approach helps students engage with the underlying concepts and gain insight into the relationships between concepts as professionals do (Bergsten et al., 2017; Otung, 2001). In line with this work, mathematics education research has found that visual–spatial representation of problems can enhance conceptual problem solving (Rittle-Johnson, Siegler, & Alibali, 2001). However, little research has investigated how to support drawing in the undergraduate engineering classroom as a means to promote conceptual problem solving (Litzinger et al., 2011; Streveler et al., 2008).

Furthermore, prior research shows that engineering instructors ask students to draw in order to visualize and communicate ideas in engineering (McNeill et al., 2016), but novice students often do not draw to solve problems (Atman, Cardella, Turns, & Adams, 2005). When novice engineering students solve well-structured classroom problems, they tend to choose nonvisual, algebraic strategies even when instructors believe drawing would be more useful (Presmeg, 2014; Vale et al., 2018). Students may choose such strategies because they require less cognitive effort and may reduce time spent on problems (Bergqvist, 2007). This preference for procedural problem solving may be particularly prevalent in a classroom setting where students focus on completing practice problems within a limited time. Hence, students in introductory-level courses may require more explicit support from instructors to engage in drawing and other types of active learning activities that increase cognitive engagement (Van Meter et al., 2016).

1.3 | How to implement drawing prompts that help students engage in drawing

In this study, we combine prior research on active learning with research on problem-solving to develop instructional practices that engage students in drawing to solve well-structured problems in an introductory-level engineering classroom. First, we use the ICAP framework to identify practice problems as an activity that may only promote active engagement and drawing as an activity that promotes constructive and interactive engagement. Second, we use prior research on problem solving to differentiate between formulaic plug-and-chug methods as a procedural problem-solving strategy that may only enhance procedural knowledge and drawing as a conceptual problem-solving strategy that may enhance conceptual understanding. Taken together, this suggests that supporting drawing as an instructional practice may *increase cognitive engagement* and promote conceptual problem solving.

Because we identified several challenges to drawing as an instructional practice above, we reviewed research in education and cognitive sciences to identify instructional practices that can help students engage conceptually in drawing. Below, we propose two levels in which these are implemented through drawing prompts.

First, prior work shows that students require support to “see” concepts in drawings as professionals do (Vale et al., 2018). When solving problems in engineering, students tend to fixate on one aspect of the problem and fail to identify all relevant pieces of information (Douglas, Koro-Ljungberg, McNeill, Malcolm, & Therriault, 2012; Kumsaikaew, Jackman, & Dark, 2006). Hence, when drawing, students overly focus on irrelevant features and not on the underlying relationships between relevant features (Jee et al., 2014; Van Meter & Firetto, 2013). However, instructors often do not explain how to distinguish relevant features from irrelevant or aesthetic features, or how to use relevant features for conceptual problem solving (Fiorella & Mayer, 2016; Valanides, Efthymiou, & Angeli, 2013). Thus, we provided drawing prompts in video lectures that model professional practice by providing specific cues regarding what features to draw (e.g., labeled axes and patterns) and how to “read” their drawings to “see” underlying concepts and solve a problem (e.g., extrapolate a trend and predict a resultant). Such drawing prompts can demonstrate *how* to use drawings and *why* it is effective for specific problems so that students use their drawings effectively as a problem-solving strategy.

Second, prior research shows that students do not draw spontaneously unless repeatedly prompted to do so (Uesaka & Manalo, 2012; Wu & Rau, 2018). Because students often choose nonvisual strategies (Presmeg, 2014; Vale et al., 2018), they need additional guidance on when to use drawing as a problem-solving strategy and what relevant information to focus on in their drawing (Leutner & Schmeck, 2014; Uesaka & Manalo, 2012). Thus, we also provided drawing prompts that target specific problem*s* to help students generate a drawing that depicts the underlying concepts and relationships. Such prompts can help them determine *when* to use drawing as a problem-solving strategy and *what* to draw for specific problems.

1.4 | The current study on the effects of drawing prompts in an engineering classroom

Our brief review of active learning activities and problem solving in introductory-level engineering classrooms identified drawing as an instructional practice that can increase students' cognitive engagement and promote conceptual problem-solving strategies used by professional engineers. However, prior work in engineering has only investigated drawing prompts in interview studies (Douglas et al., 2012; McNeill et al., 2016), not in a semester-long active learning classroom in which students solve well-structured problems to learn concepts. Thus, our goal is to investigate the effects of drawing prompts in the context of a realistic engineering class. Specifically, we investigate the effects of drawing prompts in an active learning classroom on students' (a) use and value of drawing and (b) exam performance.

First, we investigate whether providing drawing prompts as an instructional practice will help students use and value drawing as a problem-solving strategy because novice students often do not draw to solve problems as professionals do (Cardella et al., 2006; McCracken & Newstetter, 2001). Because we designed the prompts to help students determine how, why, when, and what to draw, we expect this instructional practice to affect students' engagement in drawing as a problem-solving strategy (Fan, 2015; Uesaka & Manalo, 2012). Specifically, we investigate:

RQ1a: Do drawing prompts affect students' use of drawing as a problem-solving strategy throughout the active learning course?

RQ1b: Do drawing prompts affect students' perceived value of drawing as a problem-solving strategy throughout the active learning course?

Exploratory question 1c: How do drawing prompts on specific practice problems affect students' use of drawing on those problems?

Furthermore, we investigate whether providing drawing prompts in a classroom context affects students' exam performance on problems that rely on conceptual understanding. Prior work suggests that students may gain a deeper conceptual understanding because drawing helps professionals and senior engineering students engage with the underlying concepts (Cardella et al., 2006; McCracken & Newstetter, 2001; Vale et al., 2018). However, novice students may not engage in drawing conceptually as professionals do. Hence, we investigate:

RQ2a: Do drawing prompts affect students' performance on exam questions that assess conceptual understanding?

We also investigated whether drawing prompts affect students' procedural knowledge. Although we draw upon prior research to design drawing prompts that increase cognitive engagement, students may choose to draw actively. That is, they may procedurally follow instructions to draw relevant features but use nonvisual strategies (e.g., formulas) and solve problems conceptually by "reading" or "seeing" relationships in the drawing. Furthermore, because prior work shows that drawing particularly targets higher-order thinking and increases students' cognitive engagement with content (Leutner & Schmeck, 2014; Van Meter & Garner, 2005), we do not know how drawing will affect students' procedural knowledge. Hence, we also investigate:

RQ2b: Do drawing prompts affect students' performance on exam questions that assess procedural knowledge?

2 | METHOD

2.1 | Experimental design and study setting

To investigate our hypotheses, we conducted a quasi-experimental study with a class in Spring 2018 ($n = 129$ students) that received drawing prompts (drawing condition) and a class in Fall 2017 ($n = 189$ students) that did not receive prompts (business-as-usual control condition). The larger class size in Fall 2017 resulted from limited enrollment capacity in the prior semester due to the availability of classrooms. No students repeated the course (i.e., were included in both the drawing and control conditions).

All students were enrolled in an introductory-level electrical engineering course on signal processing at a large U.S. Midwestern university. The course was held in a technology-enhanced classroom with laptop computers, TV monitors, whiteboards, and tables that seat three to six students. The course was designed as a “flipped” active learning classroom. Prior to class, students watched video lectures and completed a comprehension quiz about the lecture. During the class periods, students solved engineering problems in an educational technology as described below.

2.2 | Materials

2.2.1 | Educational technology

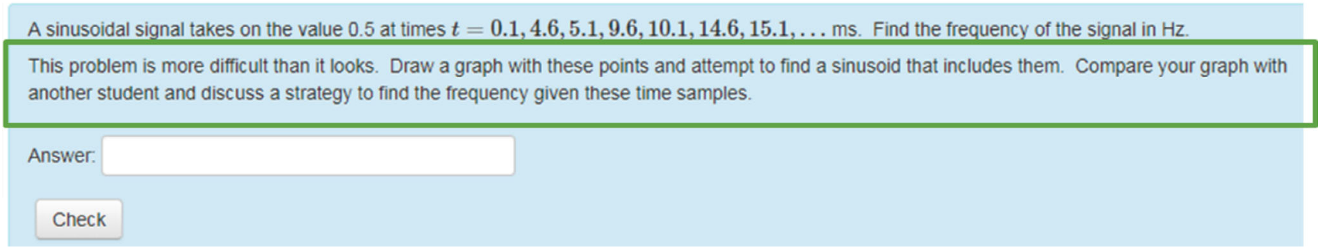
In each class period, students in both conditions used an educational technology to complete a problem set on an electrical engineering topic that was covered in the video lectures. Each problem set included 25 problems on average, for a total of 590 problems over the semester. For each problem, students entered a numerical answer and received correctness feedback for up to three attempts (see Figure 1 for an example problem). Students were encouraged to work with peers at their table to solve the problems or ask for help from the instructional team (instructor, teaching assistant, and three undergraduate peer coaches) who floated around the classroom.

2.2.2 | Drawing prompts

For students in the drawing condition, we made two changes to the course materials in accordance with the instructional practices discussed above. First, the instructor created new *video lectures* that focus on drawing. He demonstrated how professionals draw to “see” concepts and solve specific engineering problems. Furthermore, he encouraged students to draw when they solve similar problems and emphasized how “drawing graphical representations is *very* useful” in both written text and speech. By contrast, students in the control condition watched video lectures that focused on formulas. The instructor demonstrated how to derive and apply formulas to solve engineering problems without demonstrating or encouraging students to draw when solving problems. Appendix A shows example screenshots from video lectures for both conditions.

Second, we added drawing prompts to specific in-class problems, as shown inside the green box in Figure 1. Each drawing prompt immediately followed the problem text and asked students to draw a graph using the specific information provided in the problem. Furthermore, it asked students to share their drawings with peers and discuss how to solve the problem, which aimed to engage students with concepts interactively and encourage the use of drawings (Uesaka & Manalo, 2012).

We only added specific drawing prompts to 14 of the total 590 in-class problems (1 prompt in Class 2, 8 in Class 4, and 8 in Class 6). We selected these problems based on log data from the previous semester showing poor student performance as well as classroom observations showing that the instructional team used drawings to explain these problems when answering student questions one-on-one. Each problem targeted foundational concepts that help students depict electrical signals and build on mathematics concepts typically taught in high school: trigonometric functions (Classes 2 and 6) and complex numbers (Class 4). Because these are foundational concepts, students must use them to solve problems in later class periods and in future courses. In particular, Mercorelli (2015) and Meyer and Land (2003) refer to complex numbers as a threshold concept in engineering that may lead to troublesome knowledge for future courses. While prior engineering



A sinusoidal signal takes on the value 0.5 at times $t = 0.1, 4.6, 5.1, 9.6, 10.1, 14.6, 15.1, \ldots$ ms. Find the frequency of the signal in Hz.

This problem is more difficult than it looks. Draw a graph with these points and attempt to find a sinusoid that includes them. Compare your graph with another student and discuss a strategy to find the frequency given these time samples.

FIGURE 1 Example in-class problem with a prompt to draw in the rectangle [Color figure can be viewed at wileyonlinelibrary.com] research has sought to help students learn these foundational concepts through instruction focused on graphs, diagrams, and other visuals (Mercorelli, 2015; Shearman, Hong, & Pérez, 2009), it has not examined the effects of prompting students to draw when learning these concepts.

2.3 | Assessments

2.3.1 | Surveys

We assessed *RQ1a* and *RQ1b* using survey questions that asked students to rate on a 5-point scale how often they used drawing to solve problems in this class (1 = never, 5 = 3 or more times a week) and how much they valued drawing as a problem-solving strategy (1 = not valuable at all, 5 = extremely valuable), among other strategies. We also asked for demographic information about students' major, year in school, gender, prior math courses taken, and prior experience in a flipped classroom.

2.3.2 | Strategy checkbox

We further assessed students' use of drawing to explore *Q1c* by embedding checkbox questions that asked students in the drawing condition how they solved (a) problems with drawing prompts or (b) problems in which the instructor expected students to solve the problem with drawing. As shown in Figure 2, students selected all that apply from four strategies: applying a formula, drawing, using a calculation/computational tool, or another strategy.

2.3.3 | Exams

We assessed *RQ2a* and *RQ2b* using students' exam performance on conceptual understanding or procedural knowledge. The instructor first identified 33 exam questions that assessed students' understanding of underlying visual–spatial concepts. We then identified 39 exam questions that assessed procedural knowledge such as definitions and use of formulas (see Appendix B for examples of conceptual and procedural exam questions). Each set of questions was highly reliable (Cronbach's α = .796 and .847, respectively).

2.4 | Procedure

As shown in Table 1, students in both conditions met for 26 class periods over the semester. We conducted one survey in Class 25 near the end of the semester for students in the control condition. For students in the drawing condition, we conducted surveys at three time points: a pre-survey in Class 1, a mid-survey in Class 9, and a post-survey in Class 25 to match the timing of the survey in the control condition. All surveys were voluntary, but students received class time to complete them. For the post-survey in the drawing condition, the instructor offered one point toward students' final grade to increase the response rate.

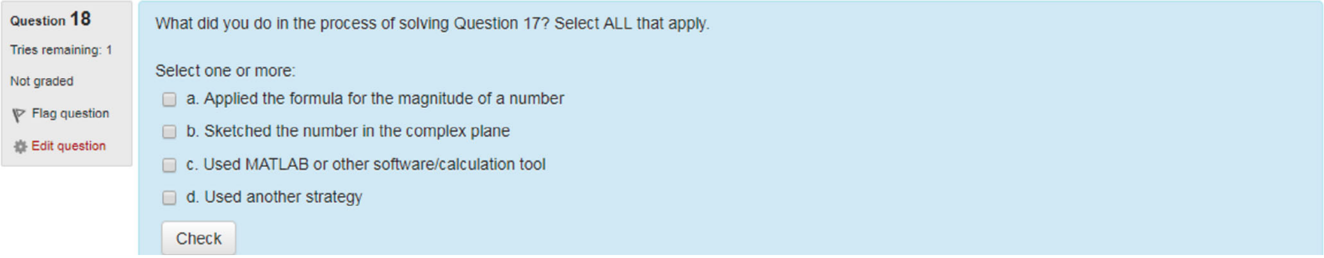
Furthermore, recall that students in the drawing condition received drawing prompts during video lectures and 14 prompts during in-class problems presented early in the semester as described previously. Table 1 shows when students in the drawing condition received drawing prompts during in-class problems and responded to strategy checkbox questions in Classes 4, 5, and 6.

In addition, students in both conditions received seven drawing prompts during the in-class problems in Class 18, near the end of the semester. These drawing prompts were designed by the instructor prior to the study. They were provided for problems on convolution, a difficult concept that requires a visual strategy (Buck et al., 2005). For students in the drawing condition, we added a self-report strategy checkbox question after these prompts.

3 | RESULTS

3.1 | Prior manipulation checks

Because students are not randomly assigned to conditions in quasi-experiments, we examined prior differences in demographics between conditions. Demographics were not normally distributed (see Table 2 for descriptive statistics). Therefore, we conducted Mann–Whitney tests with condition as the independent factor and students' major (electrical/



What did you do in the process of solving Question 17? Select ALL that apply.

Select one or more:

☐ a. Applied the formula for the magnitude of a number
☐ b. Sketched the number in the complex plane
☐ c. Used MATLAB or other software/calculation tool
☐ d. Used another strategy

FIGURE 2 Example strategy checkbox question asking students how they solved a specific problem from four strategies: applying a formula, sketching (drawing), using a calculation/computational tool, or another strategy [Color figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Overview of study procedure for the control (C) and drawing (D) conditions across 26 class periods

	Class period																									
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Survey																									C	
	D								D																D	
Drawing prompt																		C								
		D		D		D												D								
Strategy checkbox				D	D	D																				

Note: Drawing prompts at the instructor level are not shown because they are embedded in the video lectures of all class periods for the drawing condition.

computer engineering majors vs. nonmajors), year in school, gender, number of prior math courses, and prior experience in a flipped classroom (flipped experience) as the dependent measure, with the results showing significant differences only between conditions in year, $U = 13{,}370$, $p < .000$, and flipped experience, $U = 8{,}745$, $p = .020$. As shown in Table 2, students in the control condition had more years in school and less prior experience in a flipped class, compared to students in the drawing condition. We also conducted a correlation analysis that showed year and flipped experience were not correlated with exam performance or survey responses, $p < .05$.

Next, we checked for differences in students with missing survey data ($n = 19$ in the control condition and $n = 23$ in the drawing condition). Mann–Whitney tests showed no significant differences between students with and without missing data on students' reported value of drawing, demographics, or exam performance, $p > .05$. However, there were significant differences on condition, $U = 5{,}623$, $p = .026$, and on reported use of drawing, $U = 3{,}277$, $p = .028$. That is, students in the control condition and students who reported lower drawing use were more likely to have missing data.

3.2 | Differences in use and value of drawing as a problem-solving strategy

Because drawing prompts only guided students on how and when to draw, we investigated whether drawing prompts affected students' use (*RQ1a*) and value of drawing (*RQ1b*). For both research questions, we first compared differences between the two conditions for students who responded to the survey in the control condition ($n = 173$; 92.0%) and students who responded to the post-survey in the drawing condition ($n = 124$; 96.1%). Recall that both surveys were conducted at the end of the semester in Class 25. Then, we compared differences over time for the students in the drawing condition who completed all three surveys ($n = 106$; 82.2%). See Figures 3 and 4 for a summary of students' reported drawing use and value, respectively.

To investigate *RQ1a* (whether prompts affect students' use of drawing), we first compared reported drawing use between conditions using a Mann–Whitney test because survey results were not normally distributed (Figure 3). Results showed a higher frequency of drawing use, $U = 9{,}076$, $p = .018$, for the drawing condition ($M = 3.895$, $SD = 0.909$) than the control condition ($M = 3.572$, $SD = 1.068$). Second, we compared differences over time for students in the drawing condition. A repeated-measures ANOVA with survey time (pre-survey, mid-survey, and post-survey) as the independent factor and reported drawing use as dependent measures showed a main effect of drawing use, $F(2, 210) = 23.314$, $p < .001$, $\eta_p^2 = .182$. Compared to the pre-survey ($M = 3.104$, $SD = 1.077$), students reported drawing more at the mid-survey ($M = 3.821$, $SD = 1.058$) and post-survey ($M = 3.840$, $SD = 0.907$). Hence, in regard to *RQ1a*, we found that students in the drawing condition were more likely to use drawing on a regular basis after receiving drawing prompts compared to their use of drawing before prompts and compared to students in the control group.

To investigate *RQ1b* (whether prompts affect students' value of drawing), we first compared reported drawing value between conditions. A Mann–Whitney test showed a higher frequency of drawing value, $U = 8{,}623$, $p = .003$, for the

TABLE 2 Descriptive statistics of demographics organized by condition

	Control condition ($N = 189$)		**Drawing condition ($N = 129$)**	
	N	**%**	***N***	**%**
Year in school – first-year	1	0.5	3	2.3
Year in school – sophomore	70	37.4	83	64.3
Year in school – junior	74	39.6	25	19.4
Year in school – senior	26	13.8	6	4.6
Gender – male	141	75.4	101	78.3
Gender – female	29	15.5	14	13.1
Prior experience in a flipped classroom	122	65.2	95	73.6
Major – electrical/computer engineering	154	82.4	98	76.0
	Mean	*SD*	Mean	*SD*
Number of prior math courses taken	4.0	1.3	4.3	1.5

Note: Percentages do not add up to 100% due to missing survey data.

drawing condition ($M = 3.952$, $SD = 1.058$) than the control condition ($M = 3.602$, $SD = 1.021$). Then, we compared differences in drawing value over time for the students in the drawing condition. A repeated-measures ANOVA showed a main effect of drawing value, $F(2, 210) = 3.860$, $p = .023$, $\eta_p^2 = .035$. Compared to the pre-survey ($M = 3.660$, $SD = 0.925$), students reported valuing drawing more at the mid-survey ($M = 3.924$, $SD = 1.044$) and post-survey ($M = 3.906$, $SD = 1.083$). Hence, in regard to *RQ1b*, we found that students in the drawing condition reported valuing drawing as a problem-solving strategy more after receiving drawing prompts compared to their reported value of drawing before prompts and compared to students in the control group.

3.2.1 | Exploratory analysis: Problem-solving strategies on specific problems

We additionally explored how drawing prompts for specific problems may affect use of drawing for those problems (*Q1c*). Specifically, we qualitatively compare trends in strategy checkbox data, which we only collected from students in the drawing condition. As shown in Figure 5, a higher proportion of students used drawing to solve the given problems when they received drawing prompts (left, in solid blue) than when they do not receive prompts (right, in blue stripes). When prompted to draw, most students reported using drawing as a problem-solving strategy (78.6–92.2%), except for Class 6 Q19 (31.8%), while under half of students reported using drawing to solve problems without prompts (14.0–45.0%), except for Class 4 Q9 (69.8%). Recall that even for problems without prompts, the instructor expected students to use

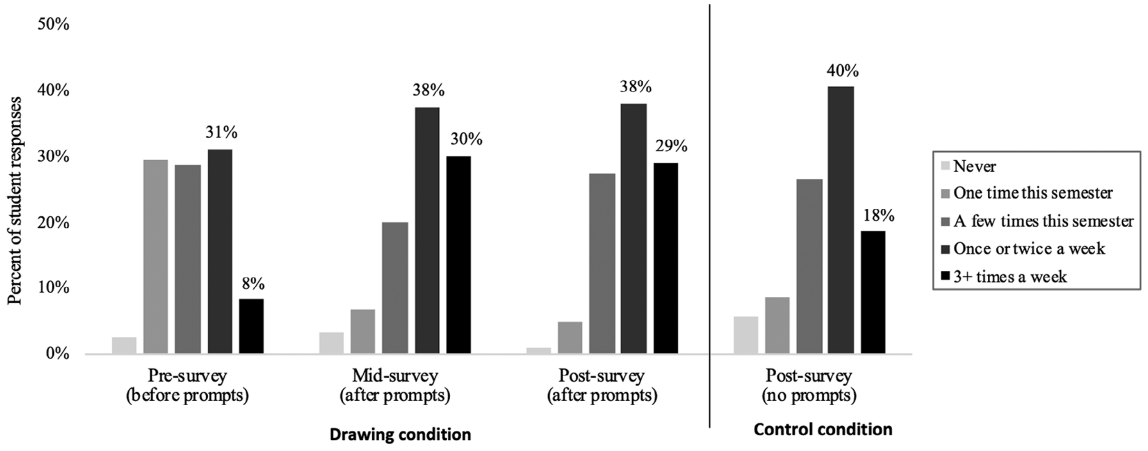


FIGURE 3 Students' reported drawing use on surveys by time of survey (pre, mid, post) and condition (drawing on left, control on right)

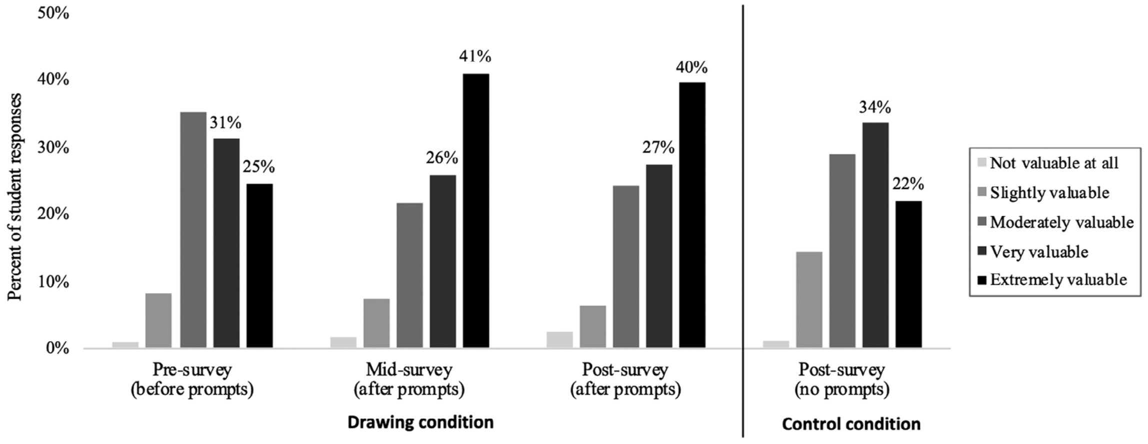


FIGURE 4 Students' reported value of drawing on surveys by time of survey (pre, mid, post) and condition (drawing on left, control on right)

drawing to solve the targeted problem. In regard to exploratory *Q1c*, these findings suggest that when prompted to draw in a given problem, students were more likely to use it as a problem-solving strategy than when unprompted.

3.3 | Differences in exam performance

Finally, we investigate differences between conditions in conceptual understanding (*RQ2a*) and procedural knowledge (*RQ2b*) by comparing exam performance on conceptual and procedural questions, as summarized in Figure 6. In our study, students in both conditions scored near 90% out of a total 100% and exam scores were not normally distributed. Hence, we used Mann–Whitney tests to compare differences between conditions below.

To investigate *RQ2a* (effects on conceptual understanding), we conducted a Mann–Whitney test with condition as the independent factor and score on conceptual exam questions as the dependent measure. Results showed significant differences between condition, $U = 10{,}455$, $p = .044$, such that the drawing condition ($M = 89.7\%$, $SD = 7.6\%$) outperformed the control condition ($M = 86.9\%$, $SD = 10.4\%$). In sum, drawing prompts enhanced conceptual understanding.

Next, we investigated *RQ2b* (effects on procedural knowledge) using a Mann–Whitney test with score on procedural exam questions as the dependent measure. We found no significant differences between conditions, $U = 11{,}299$, $p = .339$. That is, exam performance on procedural knowledge did not differ between the drawing condition ($M = 90.7\%$, $SD = 7.7\%$) and control condition ($M = 89.9\%$, $SD = 7.9\%$). In sum, drawing prompts did not affect procedural knowledge.

4 | DISCUSSION

This study investigated the effects of implementing drawing prompts in an active learning engineering classroom to help students engage in drawing, a practice used by professional engineers. Results showed that drawing prompts increased students' reported use and value of drawing as a problem-solving strategy (*RQ1a* and *RQ1b*). Specifically, we found that students who received prompts used drawing more frequently and perceived drawing as more valuable in

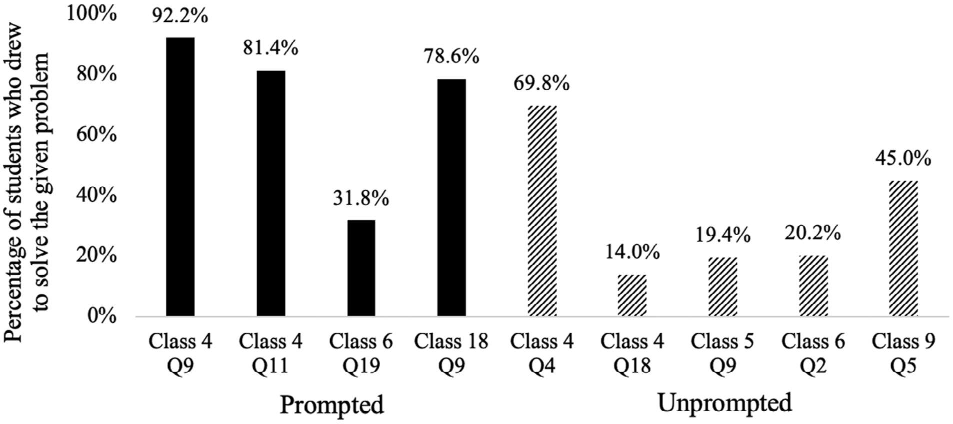


FIGURE 5 Percentage of student who responded that they used drawing to solve a specific problem, organized by problems with a drawing prompt (left) and problems without a drawing prompt (right)

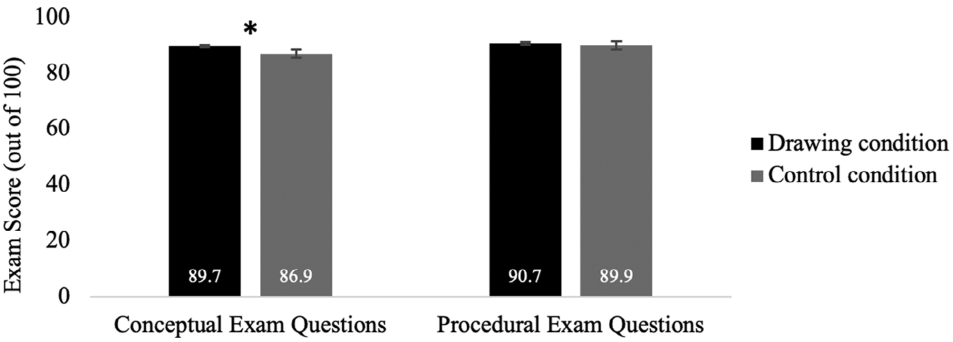


FIGURE 6 Average scores on conceptual and procedural questions by condition

the middle and end of the semester, compared to the beginning of the semester and to students who did not receive prompts. Recall that we only implemented drawing prompts in video lectures and a small subset of 14 of 590 problems, which were provided early in the semester to guide and encourage students to draw. However, students can use any strategy that they prefer. In fact, our exploratory analysis of students' problem-solving strategies for specific questions showed that, on average, most students in the drawing condition were likely to draw when prompted to draw and only some students drew when not prompted. Hence, it is more likely that across the 590 problems that students solved over the entire semester, most students would have chosen not to draw. The fact that we found large-sized effects for the increased use and value of drawing at the middle and end of the semester suggests that students chose to engage in drawing as a valued problem-solving strategy throughout the course.

Furthermore, results showed that the drawing condition outperformed the control condition on exam questions that assessed conceptual understanding (*RQ2a*), but not on questions that assessed procedural knowledge (*RQ2b*). This finding aligns with prior research on drawing, which shows that tests of conceptual understanding are more sensitive to the effects of drawing prompts than tests of simple knowledge (Leutner & Schmeck, 2014; Van Meter & Garner, 2005). These results suggest that implementing prompts to draw may engage students in conceptual problem-solving without affecting procedural knowledge.

Taken together, our findings suggest that drawing prompts in active learning classrooms may help students engage in drawing to solve practice problems and thereby enhance students' understanding of the underlying concepts. Building on the ICAP framework (Chi & Wylie, 2014; Streveler & Menekse, 2017), we designed drawing prompts so as to change students' cognitive engagement with concepts covered in the course. Drawing requires students to constructively generate coherent drawings that depict how concepts relate, which can help students clarify relationships that may be unclear in their minds (Bobek & Tversky, 2016). Furthermore, when implemented in a classroom that promotes active learning with peers and instructors, drawing prompts can ask students to compare their drawings with others, which allows them to interactively discuss and construct ideas with peers (Prain & Tytler, 2012; White & Pea, 2011). Hence, engaging in drawing individually or collaboratively may help students solve the given problems and learn the underlying concepts. This may explain why, even though we did not require students to draw, students reported increased use and perceived value of drawing as a problem-solving strategy later in the semester.

Furthermore, our exploratory results showed that implementing drawing prompts in specific problems helped students use drawing as a problem-solving strategy. Students were more likely to draw when a specific problem prompted them to draw compared to other problems where they received no prompts to draw even though the instructor expected students to do so. Recall that the instructor focused on drawing in video lectures throughout the semester but only added drawing prompts to 14 in-class problems that targeted difficult threshold concepts in electrical engineering (Buck et al., 2005; Mercorelli, 2015; Shearman et al., 2009). These prompts may have helped students draw to understand the difficult concepts instead of choosing to plug-and-chug numbers. Prior studies have not examined the effects of drawing prompts on learning outcomes in engineering courses even though students report that instructors often prompt them to draw in their courses (McNeill et al., 2016). Interview studies have investigated how students draw when solving problems (Cardella et al., 2006), but no prior work has examined how students draw in classrooms and how instructional prompts to draw may affect students' choice to draw or to use other strategies. Hence, to the best of our knowledge, this study is the first to show that drawing prompts are an effective instructional practice that may help students engage in drawing as a problem-solving strategy in engineering classrooms.

Our results further showed that drawing prompts may enhance conceptual understanding and address the gap between professionals and novice students in introductory-level engineering courses. In our study, the drawing prompts led to gains in conceptual understanding, suggesting that students may have solved problems conceptually rather than procedurally. In contrast to the plug-and-chug method often used by novices (Bergqvist, 2007; Lithner, 2003), professional engineers often draw on paper as a low-tech strategy to conceptually solve a variety of problems (Kavakli & Gero, 2001; Purcell & Gero, 1998). Drawing aligns with recommendations for engineering education to minimize mathematics and emphasize visual representations of the underlying concepts (Bergsten et al., 2017; Otung, 2001). Yet, prior work has not investigated whether novice engineering students would benefit from drawing to solve problems as professionals do because they struggle to draw without instructional support (de Vere et al., 2011; Leutner & Schmeck, 2014). Hence, our findings provide some preliminary evidence that implementing drawing prompts may be an effective instructional practice to facilitate conceptual problem solving in the introductory-level engineering classroom.

4.1 | Implications for engineering education

Our findings have several implications for research in engineering education. First, we showed that drawing prompts are a productive avenue of research, particularly in introductory-level engineering courses. Prior research shows that drawing is a valued professional practice, but traditional instruction rarely helps students to draw during problem solving in engineering courses (Litzinger et al., 2011; Streveler et al., 2008). Our study suggests that drawing prompts can provide this support by showing students how professionals use drawing to solve problems and guiding students on how to solve specific problems themselves by drawing. To further help students engage in such practices, we encourage additional investigations on the effects of drawing prompts in classrooms, which can inform how best to support drawing as a problem-solving strategy in engineering education.

Second, we showed how to use the ICAP framework to identify effective active learning interventions. Prior research on active learning has focused only on comparing active versus passive interventions and encouraged the use of active learning activities without identifying what aspects of a specific activity make it effective in engineering courses (Bishop & Verleger, 2013; Kerr, 2015; Streveler & Menekse, 2017). The ICAP framework can extend this work by identifying how engaging students in constructive and interactive activities can further enhance active learning classrooms. In our study, we isolated the effects of drawing prompts on students' learning outcomes by comparing an active learning engineering classroom with or without drawing prompts. Specifically, we differentiated active learning activities by levels of cognitive engagement in the ICAP framework as suggested by engineering education researchers (Streveler & Menekse, 2017). The ICAP framework helped explain how drawing may engage students constructively and interactively when combined with problem-solving activities that may only engage them actively. These findings extend research on active learning by demonstrating how to identify a specific instructional practice as effective in enhancing an existing classroom with active learning activities. Additional research should use the ICAP framework to test the effects of other constructive and interactive activities that increase cognitive engagement (e.g., self-explanation, teaching peers, and writing design reports) to determine which types of active learning activities are effective at enhancing learning in engineering courses.

Our study also has several practical implications for instruction in engineering education, particularly on the development of active learning activities. First, our study suggests that instructors can enhance active activities, such as practice problems in an engineering classroom, by implementing constructive and interactive activities that increase cognitive engagement, such as drawing prompts. Adding activities to increase cognitive engagement may help students engage with content conceptually, particularly in an introductory-level engineering class where students are likely to use plug-and-chug strategies to solve problems procedurally. Specifically, our findings suggest that, if instructors aim to increase conceptual understanding, they should not only augment or replace passive activities with active activities but also add constructive and interactive activities to their classrooms.

Second, this study showed how to design text-based prompts for an instructional practice that is relatively easy to implement in a variety of engineering courses. To replicate our drawing prompts in traditional or virtual lectures, instructors can demonstrate how they draw to solve a problem and end with verbal encouragement to do likewise. Then, they can add a sentence that prompts students to draw on problems in homework assignments, classwork, and/or exams to provide guidance on how, why, when, and what students should draw when solving problems (Leutner & Schmeck, 2014; Valanides et al., 2013). Because students only need a pen and paper, drawing prompts can be implemented in a variety of materials from paper worksheets to adaptive technologies that facilitate problem solving. Such drawing prompts can amplify course materials to promote instructional practices so that students receive support when they access the materials, even when an instructor is not immediately available.

Finally, we found that students benefit from multiple, targeted prompts to draw. Because students can choose to use any strategy in the classroom, novice students may choose the plug-and-chug method to solve problems quickly even though it can impede their understanding of the underlying concepts (Bergqvist, 2007; Lithner, 2003). Given that students often struggle to draw, instructors should provide not only brief announcements and reminders to draw before problem-solving sessions but provide guidance on what to draw when solving problems until students learn to use drawing as a problem-solving strategy. Our results suggest that students were more likely to draw when prompted in specific practice problems, and, thus, students may need repeated, targeted prompts to draw as they solve problems in engineering classrooms.

4.2 | Limitations and future directions

Our study should be considered in light of the following limitations. First, while we consider the ecological validity of our study a strength, it is limited by its use of a quasi-experimental cohort comparison, similar to many other studies that have compared active learning classrooms to other instructional designs (Bishop & Verleger, 2013). We controlled for differences using demographic information, but students enrolled in different semesters may have additional differences that were not measured in this study (e.g., motivations, cognitive abilities, and collaboration patterns). Thus, the results of this study may not be generalizable to other populations without additional experiments that use random assignment of students to conditions or use quasi-experimental designs that control for other possible a priori differences.

Second, we found nonrandom differences in missing survey data between semesters and students' reported use of drawing. Because surveys are voluntary, there may be attrition and nonresponse biases in missing data. However, recall that the instructor provided an extra incentive for the post-survey of the drawing condition, which increased the response rate of those students. This incentive likely increased this response rate, particularly from students who used drawing less often, as our missing data analysis indicated respondents were more likely to report higher drawing use. Therefore, the true difference between conditions on students' use of drawing may have been larger than the effects reported here.

Third, we found a relatively small difference in performance on conceptual exam questions between the drawing and control conditions. Because we conducted our study in a classroom that already implemented multiple active learning activities, the average exam scores for both conditions neared 90%, which suggests a ceiling effect. Hence, future studies with drawing prompts in other active learning classrooms or additional measures of conceptual understanding are needed to determine how drawing prompts facilitate conceptual problem solving.

Fourth, we did not investigate the effects of drawing prompts for specific populations of students. Prior work has found that certain instructional practices can particularly support females and minorities in engineering, such as building spatial skills that help students better learn with visuals (Hill, Corbett, & St. Rose, 2010). Students in our study were mostly male students, in line with the demographics of engineering majors at our university and in the broader population. Due to the small proportion of female students in our study, we were unable to identify significant differences between genders in perceptions of drawing or learning outcomes. However, there was a trend that females used drawing more and valued drawing as a problem-solving strategy more than males. The increased drawing use may align with exploratory studies that suggest females are more likely to follow instructions (e.g., Bairaktarova, 2017). However, additional research should explore whether drawing prompts address a gap in instruction, particularly for female students in introductory engineering courses who report drawing as valuable for problem solving.

Fifth, we only provided drawing prompts on selected problems during class. We selected problems based on log data from students in our study and classroom observations of how instructors helped students solve problems when answering questions one-on-one. However, students may need support on other types of problems or need additional drawing prompts to engage with problems without one-on-one instructor support. Hence, future work should determine whether drawing prompts for other types of problems are effective and what number of drawing prompts are optimal for novice engineering students in the active learning classroom.

Finally, our study did not determine how students interacted with prompts and with one another to identify mechanisms that may underlie students' learning outcomes. Particularly, we did not assess whether students constructed and reflected on drawings individually (i.e., constructive engagement) or discussed their drawings with peers or instructors (i.e., interactive engagement). Because students received feedback only on their final answers to problems but not on drawings, it is possible that drawing prompts were effective because students gave feedback to one another's drawings through interactive engagement with their peers. Hence, future studies should investigate the nature of classroom interactions when students draw and determine whether drawing prompts are more effective as an individual or collaborative activity.

5 | CONCLUSION

Our study makes several important contributions to instructional practices in engineering education. First, we shed light on which types of active learning activities are effective in engineering courses. Specifically, our study suggests that providing instructional prompts to draw can help students engage in drawing as a problem-solving strategy in the context of an active learning classroom. Second, we show that drawing prompts may address a gap between how professionals and introductory-level undergraduates solve problems. The findings suggest that drawing prompts helped students in an undergraduate engineering course engage in conceptual problem solving as professionals do throughout the semester, even after students stop receiving drawing prompts that guide them in solving specific problems. Finally, we provide theoretical insights into how increasing cognitive engagement makes an active learning activity more effective. Specifically, our study shows that prompting students to draw is an effective instructional practice that can enhance an engineering course that already implements other active learning activities. Our findings provide practical recommendations for engineering instructors on how to implement a low-tech instructional practice that can enhance their active learning classrooms in the digital age. Specifically, we suggest that providing a simple text-based prompt to draw in lectures and classroom problems can help students develop important engineering practices that help them solve problems on paper.

ACKNOWLEDGMENTS

This work was supported by Lynn H. Matthias Professorship and Institute of Education Sciences, U.S. Department of Education, through Award Number 305B150003 to the University of Wisconsin-Madison. This research was also in part supported by the National Science Foundation Division of Undergraduate Education 1933078. We thank Sarah Mason for initiating and supporting our interdisciplinary collaboration. We also thank Chung-Han Hsieh and Akhil Sundararajan as well as Kass Chupongstimun, Felipe Gutierrez, Roumen Guha, Henri Hasti, Kayla Huemer, Haley Massa, Alliot Nagle, Pragathi Praveena, Noah Rhodes, Andrew Slade, and Max Strange for their instructional support and perspectives on the courses.

ORCID

Sally P. W. Wu https://orcid.org/0000-0003-0096-7898

REFERENCES

Atman, C. J., Cardella, M. E., Turns, J., & Adams, R. (2005). Comparing freshman and senior engineering design processes: An in-depth follow-up study. *Design Studies*, *26*(4), 325–357. https://doi.org/10.1016/j.destud.2004.09.005

Bairaktarova, D. (2017). Coordinating mind and hand: The importance of manual drawing and descriptive geometry instruction in a CAD-oriented engineering design graphics class. *Engineering Design Graphics Journal*, *81*(3), 1–16. Retrieved from http://www.edgj.org/index.php/EDGJ/article/view/655

Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics. *Journal of Mathematical Behavior*, *26*(4), 348–370. https://doi.org/10.1016/j.jmathb.2007.11.001

Bergsten, C., Engelbrecht, J., & Kågesten, O. (2017). Conceptual and procedural approaches to mathematics in the engineering curriculum—Comparing views of junior and senior engineering students in two countries. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(3), 533–553. https://doi.org/10.12973/eurasia.2017.00631a

Bishop, J. L., & Verleger, M. A. (2013). *The flipped classroom: A survey of the research*. Paper presented at the ASEE Annual Conference & Exposition, Atlanta, GA. Retrieved from https://peer.asee.org/22585

Bobek, E., & Tversky, B. (2016). Creating visual explanations improves learning. *Cognitive Research: Principles and Implications*, *1*(1), 27. https://doi.org/10.1186/s41235-016-0031-6

Buck, J. R., Wage, K. E., Hjalmarson, M. A., Buck, J. R., Wright, C. H. G., & Welch, T. B. (2005). The signals and systems concept inventory (SSCI). *IEEE Transactions on Education*, *48*(3), 448–461. https://doi.org/10.1109/TE.2005.849746

Cardella, M. E., Atman, C. J., & Adams, R. S. (2006). Mapping between design activities and external representations for engineering student designers. *Design Studies*, *27*(1), 5–24. https://doi.org/10.1016/j.destud.2005.05.001

Chi, M. T. H., & Wylie, R. (2014). The ICAP framework: Linking cognitive engagement to active learning outcomes. *Educational Psychologist*, *49*(4), 219–243. https://doi.org/10.1080/00461520.2014.965823

de Vere, I., Melles, G., & Kapoor, A. (2011). Developing a drawing culture: New directions in engineering education. *Proceedings of the 18th International Conference on Engineering Design: Impacting Society Through Engineering Design, Vol 8: Design Education*, *8*(August), 226–235.

Douglas, E. P., Koro-Ljungberg, M., McNeill, N. J., Malcolm, Z. T., & Therriault, D. J. (2012). Moving beyond formulas and fixations: Solving open-ended engineering problems. *European Journal of Engineering Education*, *37*(6), 627–651. https://doi.org/10.1080/03043797.2012.738358

Fan, J. E. (2015). Drawing to learn: How producing graphical representations enhances scientific thinking. *Translational Issues in Psychological Science*, *1*(2), 170–181. https://doi.org/10.1037/tps0000037

Fiorella, L., & Mayer, R. E. (2016). Effects of observing the instructor draw diagrams on learning from multimedia messages. *Journal of Educational Psychology*, *108*(4), 528–546. https://doi.org/10.1037/edu0000065

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences of the United States of America*, *111*(23), 8410–8415. https://doi.org/10.1073/pnas.1319030111

Greeno, J. G., & Hall, R. P. (1997). Practicing representation: Learning with and about representational forms. *The Phi Delta Kappan*, *78*(5), 361–367. https://doi.org/10.2307/20405797

Higley, K., Litzinger, T., Van Meter, P., & Masters, C. B. (2007). *Effects of conceptual understanding, math and visualization skills on problem-solving in statics*. Paper presented at the ASEE Annual Conference and Exposition, Honolulu, HI. Retrieved from https://peer.asee.org/2382

Hill, C., Corbett, C., & St. Rose, A. (2010). *Why so few? Women in science, technology, engineering, and mathematics* (Vol. 5, pp. 1–134). Washington, DC: American Association of University Women.

Jee, B. D., Gentner, D., Uttal, D. H., Sageman, B., Forbus, K. D., Manduca, C. a., … Tikoff, B. (2014). Drawing on experience: How domain knowledge is reflected in sketches of scientific structures and processes. *Research in Science Education*, *44*, 859–883. https://doi.org/10.1007/s11165-014-9405-2

Johri, A., Roth, W.-M., & Olds, B. M. (2013). The role of representations in engineering practices: Taking a turn towards inscriptions. *Journal of Engineering Education*, *102*(1), 2–19. https://doi.org/10.1002/jee.20005

Kavakli, M., & Gero, J. S. (2001). Sketching as mental imagery processing. *Design Studies*, *22*(4), 347–364. https://doi.org/10.1016/S0142-694X(01)00002-3

Kerr, B. (2015). *The flipped classroom in engineering education: A survey of the research*. Paper presented at the 2015 International Conference on Interactive Collaborative Learning, Florence, Italy. https://doi.org/10.1109/ICL.2015.7318133

Kothiyal, A., Murthy, S., & Chandrasekharan, S. (2016). "Hearts pump and hearts beat": Engineering estimation as a form of model-based reasoning. *Proceedings of International Conference of the Learning Sciences, 1*(2015), 242–249. Retrieved from https://repository.isls.org/handle/1/122

Kumsaikaew, P., Jackman, J., & Dark, V. J. (2006). Task relevant information in engineering problem solving. *Journal of Engineering Education*, *95*(3), 227–239. https://doi.org/10.1002/j.2168-9830.2006.tb00895.x

Leppävirta, J., Kettunen, H., & Sihvola, A. (2011). Complex problem exercises in developing engineering students' conceptual and procedural knowledge of electromagnetics. *IEEE Transactions on Education*, *54*(1), 63–66. https://doi.org/10.1109/TE.2010.2043531

Leutner, D., & Schmeck, A. (2014). The generative drawing principle in multimedia learning. In R. E. Mayer (Ed.), *The Cambridge handbook of multimedia learning* (pp. 433–448). Cambridge, England: Cambridge University Press. https://doi.org/10.1017/CBO9781139547369.022

Lithner, J. (2003). Students' mathematical reasoning in university textbook exercises. *Educational Studies in Mathematics*, *52*(1), 29–55. https://doi.org/10.1023/A:1023683716659

Litzinger, T. A., Lattuca, L. R., Hadgraft, R. G., Newstetter, W. C., Alley, M., Atman, C., … Yasuhara, K. (2011). Engineering education and the development of expertise. *Journal of Engineering Education*, *100*(1), 123–150. https://doi.org/10.1002/j.2168-9830.2011.tb00006.x

Lo, C. K., & Hew, K. F. (2019). The impact of flipped classrooms on student achievement in engineering education: A meta-analysis of 10years of research. *Journal of Engineering Education*, *108*(4), 523–546. https://doi.org/10.1002/jee.20293

McCracken, W. M., & Newstetter, W. C. (2001). *Text to diagram to symbol: Representational transformations in problem-solving*. Paper presented at the 31st Annual Frontiers in Education Conference, Reno, NV. https://doi.org/10.1109/FIE.2001.963721

McNeill, N. J., Douglas, E. P., Koro-Ljungberg, M., Therriault, D. J., & Krause, I. (2016). Undergraduate students' beliefs about engineering problem solving. *Journal of Engineering Education*, *105*(4), 560–584. https://doi.org/10.1002/jee.20150

Mercorelli, P. (2015). *Conceptual understanding of complex components and Nyquist-Shannon sampling theorem: A design based research in engineering*. Paper presented at IEEE Global Engineering Education Conference, Tallinn, Estonia. https://doi.org/10.1109/EDUCON.2015.7096011

Meyer, J. H. F., & Land, R. (2003). Threshold concepts and troublesome knowledge: Linkages to ways of thinking and practising within the disciplines. *Improving Student Learning – Ten Years On*, *4*(1), 1–16. https://doi.org/10.1007/978-3-8348-9837-1

Nathan, M. J., Eilam, B., & Kim, S. (2007). To disagree, we must also agree: How intersubjectivity structures and perpetuates discourse in a mathematics classroom. *Journal of the Learning Sciences*, *16*(4), 523–563. https://doi.org/10.1080/10508400701525238

Nelson Laird, T. F., Shoup, R., Kuh, G. D., & Schwarz, M. J. (2008). The effects of discipline on deep approaches to student learning and college outcomes. *Research in Higher Education*, *49*(6), 469–494. https://doi.org/10.1007/s11162-008-9088-5

Otung, I. E. (2001). Reassessing the mathematics content of engineering education. *Engineering Science and Education Journal*, *10*(4), 130–138. https://doi.org/10.1049/esej:20010401

Prain, V., & Tytler, R. (2012). Learning through constructing representations in science: A framework of representational construction affordances. *International Journal of Science Education*, *34*(17), 2751–2773. https://doi.org/10.1080/09500693.2011.626462

Presmeg, N. (2014). Creative advantages of visual solutions to some non-routine mathematical problems. In S. Carreira, N. Amado, K. Jones, & H. Jacinto (Eds.), *Proceedings of the Problem@Web International Conference: Technology, Creativity and Affect in Mathematical Problem Solving* (pp. 156–167). Faro, Portugal: Universidade do Algarve.

Purcell, A. T., & Gero, J. S. (1998). Drawings and the design process. *Design Studies*, *19*(4), 389–430. https://doi.org/10.1016/S0142-694X(98)00015-5

Rittle-Johnson, B., Siegler, R., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*(2), 346–362. https://doi.org/10.1037/0022-0663.93.2.346

Schwarz, C. V., Reiser, B. J., Davis, E. A., Kenyon, L., Achér, A., Fortus, D., … Krajcik, J. (2009). Developing a learning progression for scientific modeling: Making scientific modeling accessible and meaningful for learners. *Journal of Research in Science Teaching*, *46*(6), 632–654. https://doi.org/10.1002/tea.20311

Shearman, S., Hong, S., & Pérez, E. (2009). To improve signal processing concept demonstration software, consider the interactive experience. *Proceedings of the IEEE 13th Digital Signal Processing Workshop and 5th IEEE Signal Processing Education Workshop*, 227–231. https://doi.org/10.1109/DSP.2009.4785926

Streveler, R. A., Litzinger, T. A., Miller, R. L., & Steif, P. S. (2008). Learning conceptual knowledge in the engineering sciences: Overview and future research directions. *Journal of Engineering Education*, *97*(3), 279–294. https://doi.org/10.1002/j.2168-9830.2008.tb00979.x

Streveler, R. A., & Menekse, M. (2017). Taking a closer look at active learning. *Journal of Engineering Education*, *106*(2), 186–190. https://doi.org/10.1002/jee.20160

Tversky, B. (2011). Visualizing thought. *Topics in Cognitive Science*, *3*(3), 499–535. https://doi.org/10.1111/j.1756-8765.2010.01113.x

Uesaka, Y., & Manalo, E. (2012). Task-related factors that influence the spontaneous use of diagrams in math word problems. *Applied Cognitive Psychology*, *26*(2), 251–260. https://doi.org/10.1002/acp.1816

Ullman, D. G., Wood, S., & Craig, D. (1990). The importance of drawing in the mechanical design process. *Computers and Graphics*, *14*(2), 263–274. https://doi.org/10.1016/0097-8493(90)90037-X

Valanides, N., Efthymiou, I., & Angeli, C. (2013). Interplay of internal and external representations: Students' drawings and textual explanations about shadow phenomena. *Journal of Visual Literacy*, *32*(2), 67–84. https://doi.org/10.1080/23796529.2013.11674710

Vale, I., Pimentel, T., & Barbosa, A. (2018). The power of seeing in problem solving and creativity: An issue under discussion. In N. Amado, S. Carreira, & K. Jones (Eds.), *Broadening the scope of research on mathematical problem solving* (pp. 243–272). Cham, Switzerland: Springer. https://doi.org/10.1007/978-3-319-99861-9_11

Van Meter, P., & Firetto, C. M. (2013). Cognitive model of drawing construction. In G. Schraw, M. T. McCrudden, & G. Robinson (Eds.), *Learning through visual displays* (pp. 247–280). Charlotte, NC: Information Age Publishing, Incorporated.

Van Meter, P., Firetto, C. M., Turns, S. R., Litzinger, T. A., Cameron, C. E., & Shaw, C. W. (2016). Improving students' conceptual reasoning by prompting cognitive operations. *Journal of Engineering Education*, *105*(2), 245–277. https://doi.org/10.1002/jee.20120

Van Meter, P., & Garner, J. (2005). The promise and practice of learner-generated drawing: Literature review and synthesis. *Educational Psychology Review*, *17*(4), 285–325. https://doi.org/10.1007/s10648-005-8136-3

White, T., & Pea, R. (2011). Distributed by design: On the promises and pitfalls of collaborative learning with multiple representations. *Journal of Learning Sciences*, *20*(530), 489–547. https://doi.org/10.1080/10508406.2010.542700

Wu, S. P. W., & Rau, M. A. (2018). Effectiveness and efficiency of adding drawing prompts to an interactive educational technology when learning with visual representations. *Learning and Instruction*, *55*, 93–104. https://doi.org/10.1016/j.learninstruc.2017.09.010

Wylie, R., & Chi, M. T. H. (2014). The self-explanation principle in multimedia learning. In R. E. Mayer (Ed.), *The Cambridge handbook of multimedia learning* (pp. 413–432). Cambridge, England: Cambridge University Press. https://doi.org/10.1016/0364-0213(89)90002-5

Zhang, Z. H., & Linn, M. C. (2011). Can generating representations enhance learning with dynamic visualizations? *Journal of Research in Science Teaching*, *48*(10), 1177–1198. https://doi.org/10.1002/tea.20443

AUTHOR BIOGRAPHIES

Sally P. W. Wu is the Director of Curriculum Development on the Computational Thinking in STEM (CT-STEM) Project at Northwestern University, 2120 Campus Drive, Evanston, IL 60208; sally.wu@northwestern.edu

Barry Van Veen is the Lynn H. Matthias Professor and Associate Chair for Graduate and Online Studies in the Electrical and Computer Engineering Department at the University of Wisconsin–Madison; 1415 Engineering Drive, Madison, WI 53706; vanveen@engr.wisc.edu

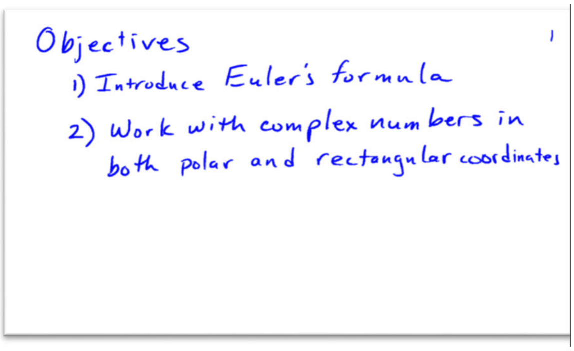
Martina A. Rau is an Associate Professor in the Department of Educational Psychology at the University of Wisconsin–Madison, 1125 W. Johnson Street, Madison, WI 53706; marau@wisc.edu

How to cite this article: Wu SPW, Van Veen B, Rau MA. How drawing prompts can increase cognitive engagement in an active learning engineering course. *J Eng Educ*. 2020;1–20. https://doi.org/10.1002/jee.20354

APPENDIX A

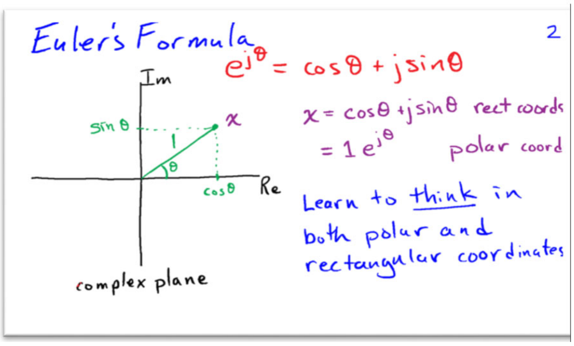
EXAMPLE SCREENSHOTS FROM VIDEO LECTURES FOR THE DRAWING AND CONDITION CONDITIONS

Drawing condition



Objectives

1) Introduce Euler's formula
2) Work with complex numbers in both polar and rectangular coordinates



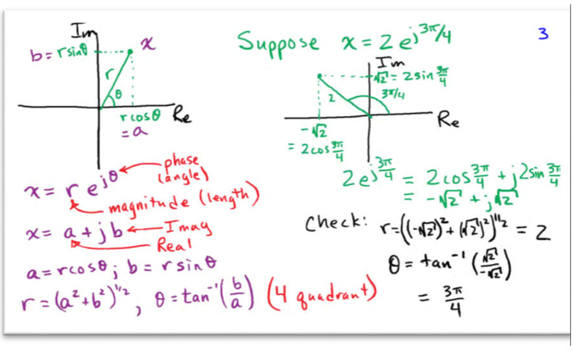
Euler's Formula

$e^{j\theta} = \cos\theta + j\sin\theta$

$x = \cos\theta + j\sin\theta$ rect coords

$= 1e^{j\theta}$ polar coord

Learn to think in both polar and rectangular coordinates



Suppose $x = 2e^{j3\pi/4}$

$x = re^{j\theta}$ ← phase (angle); r ← magnitude (length)

$x = a + jb$ ← Imag; a ← Real

$a = r\cos\theta;\ b = r\sin\theta$

$r = (a^2+b^2)^{1/2},\ \theta = \tan^{-1}\left(\frac{b}{a}\right)$ (4 quadrant)

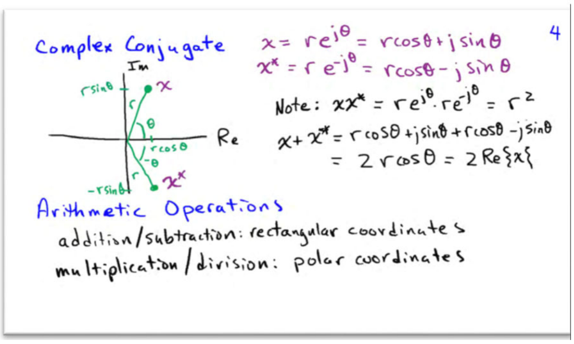
$2e^{j\frac{3\pi}{4}} = 2\cos\frac{3\pi}{4} + j2\sin\frac{3\pi}{4}$

$= -\sqrt{2} + j\sqrt{2}$

Check: $r = \left((-\sqrt{2})^2 + (\sqrt{2})^2\right)^{1/2} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{2}}{-\sqrt{2}}\right)$

$= \frac{3\pi}{4}$



Complex Conjugate

$x = re^{j\theta} = r\cos\theta + j\sin\theta$

$x^* = re^{-j\theta} = r\cos\theta - j\sin\theta$

Note: $xx^* = re^{j\theta} \cdot re^{-j\theta} = r^2$

$x + x^* = r\cos\theta + j\sin\theta + r\cos\theta - j\sin\theta$

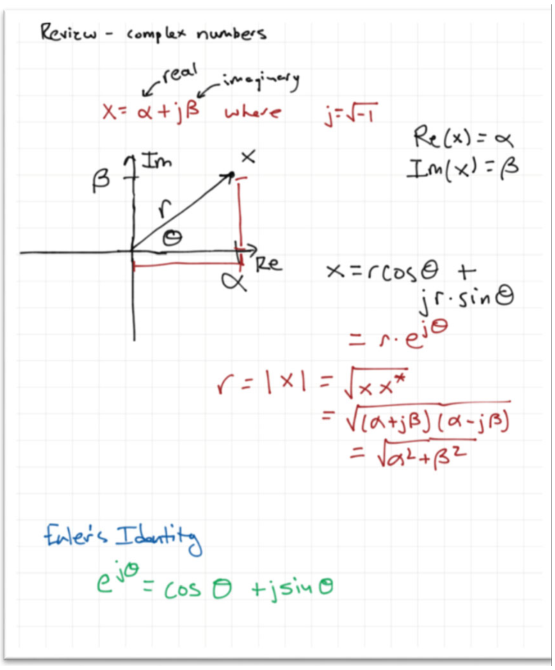
$= 2r\cos\theta = 2\mathrm{Re}\{x\}$

Arithmetic Operations

addition/subtraction: rectangular coordinates

multiplication/division: polar coordinates

Control condition



Review - complex numbers

$X = \alpha + j\beta$ where $j = \sqrt{-1}$ (α ← real, jβ ← imaginary)

$\mathrm{Re}(x) = \alpha$

$\mathrm{Im}(x) = \beta$

$x = r\cos\theta + jr \cdot \sin\theta$

$= r \cdot e^{j\theta}$

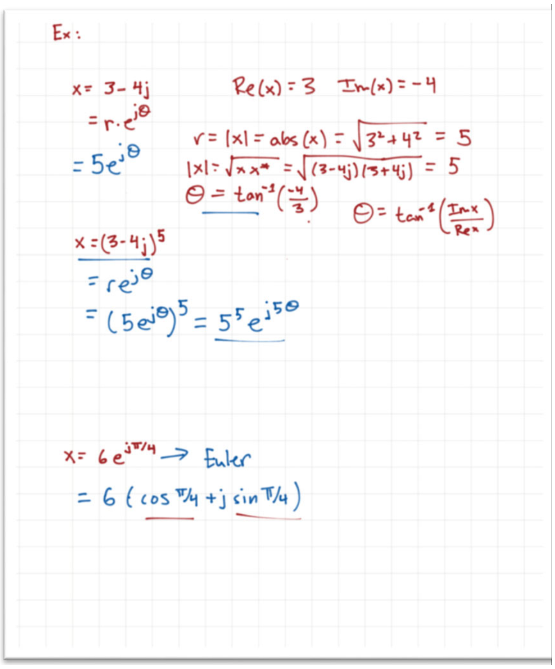
$r = |x| = \sqrt{xx^*}$

$= \sqrt{(\alpha+j\beta)(\alpha-j\beta)}$

$= \sqrt{\alpha^2+\beta^2}$

Euler's Identity

$e^{j\theta} = \cos\theta + j\sin\theta$



Ex:

$x = 3 - 4j$

$= r \cdot e^{j\theta}$

$= 5e^{j\theta}$

$\mathrm{Re}(x) = 3 \quad \mathrm{Im}(x) = -4$

$r = |x| = \mathrm{abs}(x) = \sqrt{3^2+4^2} = 5$

$|x| = \sqrt{xx^*} = \sqrt{(3-4j)(3+4j)} = 5$

$\theta = \tan^{-1}\left(\frac{-4}{3}\right)$

$\theta = \tan^{-1}\left(\frac{\mathrm{Im}\,x}{\mathrm{Re}\,x}\right)$

$x = (3-4j)^5$

$= re^{j\theta}$

$= (5e^{j\theta})^5 = 5^5e^{j5\theta}$

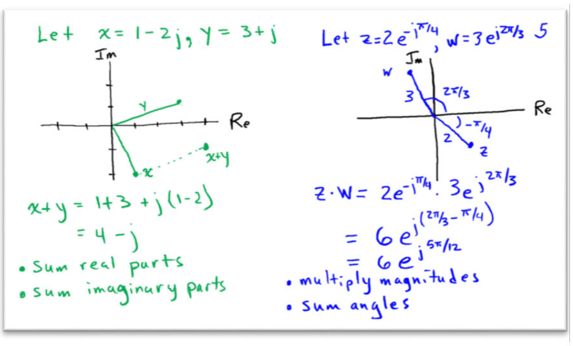
$x = 6e^{j\pi/4} \rightarrow$ Euler

$= 6(\cos\pi/4 + j\sin\pi/4)$

(Continues)

APPENDIX A (Continued)

Drawing condition



Let $x=1-2j$, $y=3+j$

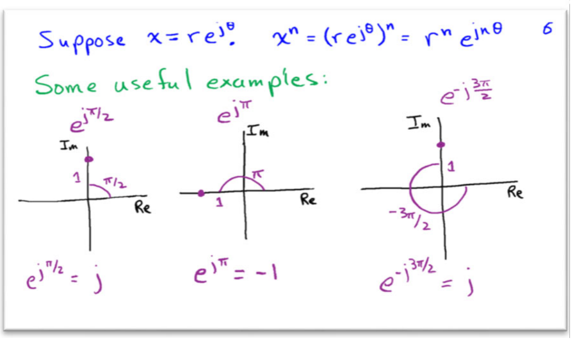
Let $z=2e^{-j\pi/4}$, $w=3e^{j2\pi/3}$ 5

$x+y=1+3+j(1-2)$
$=4-j$

- Sum real parts
- sum imaginary parts

$z\cdot w=2e^{-j\pi/4}\cdot 3e^{j2\pi/3}$
$=6e^{j(2\pi/3-\pi/4)}$
$=6e^{j5\pi/12}$

- multiply magnitudes
- sum angles

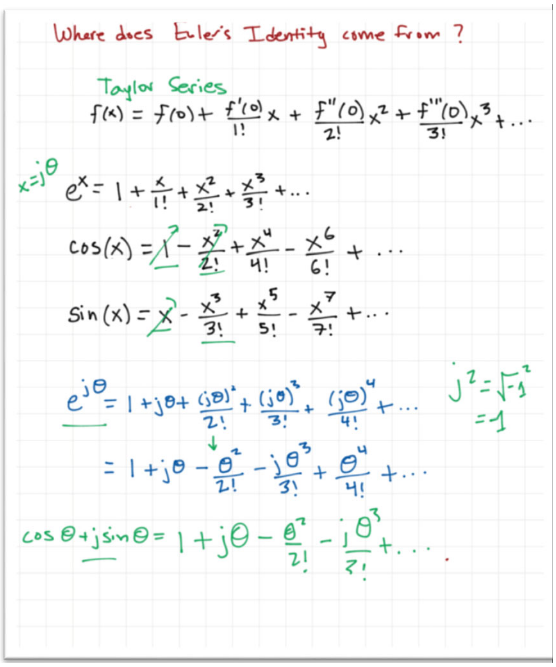


Suppose $x=re^{j\theta}$. $x^n=(re^{j\theta})^n=r^ne^{jn\theta}$ 6

Some useful examples:

$e^{j\pi/2}=j$ $e^{j\pi}=-1$ $e^{-j3\pi/2}=j$

Control condition



Where does Euler's Identity come from ?

Taylor Series

$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\ldots$

$x=j\theta$

$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$

$\cos(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$

$\sin(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots$

$e^{j\theta}=1+j\theta+\frac{(j\theta)^2}{2!}+\frac{(j\theta)^3}{3!}+\frac{(j\theta)^4}{4!}+\ldots$

$=1+j\theta-\frac{\theta^2}{2!}-\frac{j\theta^3}{3!}+\frac{\theta^4}{4!}+\ldots$

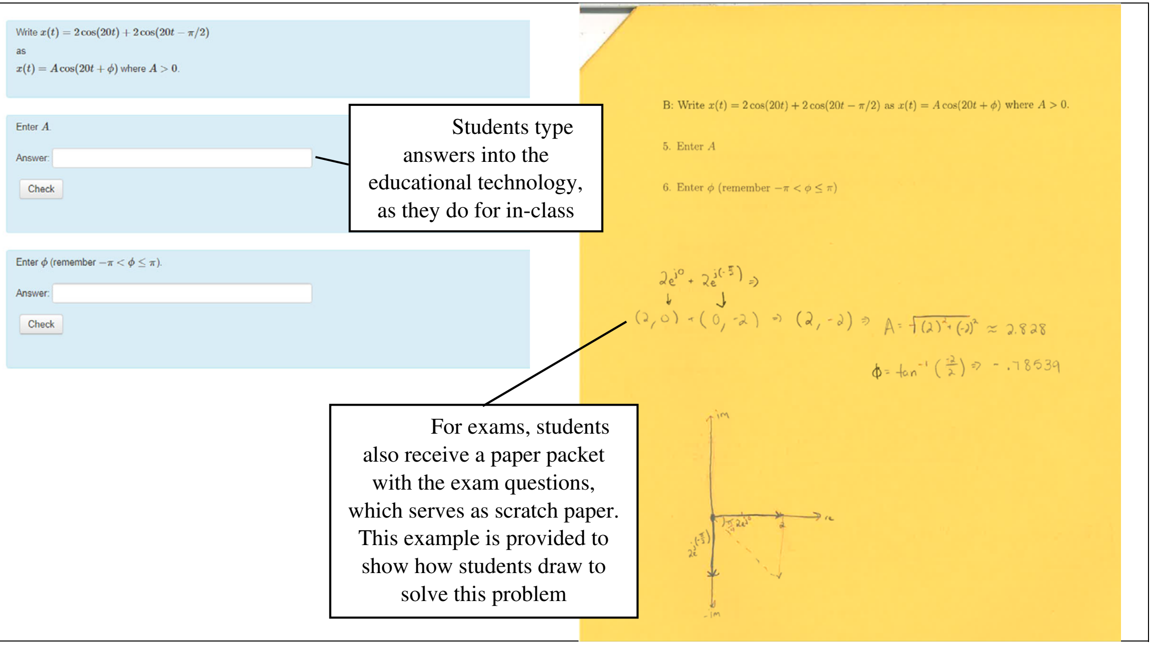
$j^2=\sqrt{-1}^2$
$=-1$

$\cos\theta+j\sin\theta=1+j\theta-\frac{\theta^2}{2!}-\frac{j\theta^3}{3!}+\ldots$

APPENDIX B

EXAMPLE EXAM QUESTIONS THAT FOCUS ON CONCEPTUAL UNDERSTANDING OR PROCEDURAL KNOWLEDGE

Conceptual



Write $x(t)=2\cos(20t)+2\cos(20t-\pi/2)$
as
$x(t)=A\cos(20t+\phi)$ where $A>0$.

Enter A.
Answer:
Check

Enter ϕ (remember $-\pi<\phi\le\pi$).
Answer:
Check

Students type answers into the educational technology, as they do for in-class

B: Write $x(t)=2\cos(20t)+2\cos(20t-\pi/2)$ as $x(t)=A\cos(20t+\phi)$ where $A>0$.

5. Enter A

6. Enter ϕ (remember $-\pi<\phi\le\pi$)

$2e^{j0}+2e^{j(-\frac{\pi}{2})}\Rightarrow$

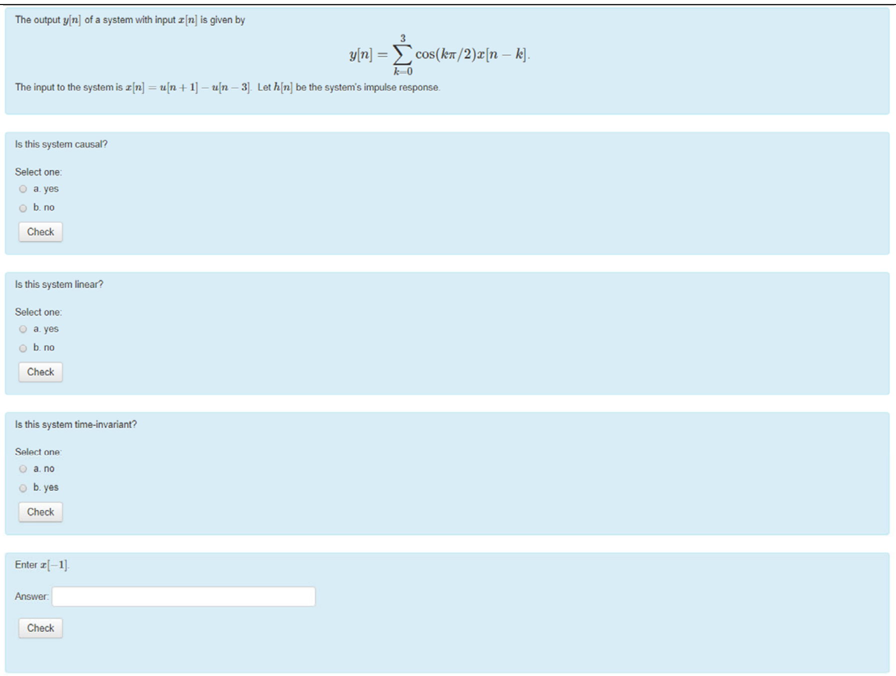
$(2,0)+(0,-2)\Rightarrow(2,-2)\Rightarrow A=\sqrt{(2)^2+(-2)^2}\approx 2.828$

$\phi=\tan^{-1}(\frac{-2}{2})\Rightarrow -.78539$

For exams, students also receive a paper packet with the exam questions, which serves as scratch paper. This example is provided to show how students draw to solve this problem

(Continues)

APPENDIX B (Continued)

Procedural	The output $y[n]$ of a system with input $x[n]$ is given by $y[n] = \sum_{k=0}^{3} \cos(k\pi/2)x[n-k].$ The input to the system is $x[n] = u[n+1] - u[n-3]$. Let $h[n]$ be the system's impulse response. Is this system causal? Select one: ○ a. yes ○ b. no Check Is this system linear? Select one: ○ a. yes ○ b. no Check Is this system time-invariant? Select one: ○ a. no ○ b. yes Check Enter $x[-1]$. Answer: Check