Issues and affordances in studying children’s drawings with a mathematical eye

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In this third consecutive MERGA symposium focused on young children’s drawings, three separate groups of researchers discuss the benefits and issues of using drawings as a source of data in their studies. Although drawings are ubiquitous in early years classrooms and in studies of children’s learning, there is no comprehensive framework for analysing children’s drawings in mathematical contexts. The overarching purpose of these symposiums has been to explore the qualitative methods that researchers have developed in their distinct projects and advance our critical perspectives on interpreting drawings and understanding the role they can play in children’s learning of mathematics.

Broadly, the researchers view drawings as an external representation of mathematical concepts, mathematical thinking, or perceptions of mathematical contexts. Typically, researchers trust that children’s drawings express to some extent the developing internal systems of the child, including the affective domain. In studying the interplay between children’s internal and external representations, researchers must grapple with the ambiguities of interpreting representational drawing, as explained in quotation below.

> “Internal systems, … include students' personal symbolization constructs and assignments of meaning to mathematical notations, as well as their natural language, their visual imagery and spatial representation, their problem-solving strategies and heuristics, and (very important) their affect in relation to mathematics. The *interaction* between internal and external representation is fundamental to effective teaching and learning. Whatever meanings and interpretations the teacher may bring to an external representation, it is the nature of the student's developing internal representation that must remain of primary interest.” (Goldin & Shteingold, 2001, p.2).

In this symposium, as well as sharing results from recent research, the authors reflect on some of the issues and affordances in studying children’s drawings with a mathematical eye.

Goldin, G. & Shteingold, N. (2001). Systems of representation and the development of mathematical concepts. In Cuoco, A. (Ed.), *The roles of representations in school mathematics,* NCTM 2001 Yearbook, (pp.1-23). Reston VA: NCTM.

Chair & Discussant: Jennifer Way

Paper 1: Jill Cheeseman, Ann Downton, Anne Roche & Sarah Ferguson *Drawings reveal young students’ multiplicative visualisation*

Paper 2: Katherin Cartwright, Janette Bobis & Jennifer Way *Investigating students’ drawings as communication and representation modes of mathematical fluency.*

Paper 3: Kate Quane, Mohan Chinnappan & Sven Trenholm *Children’s drawings as a source of data to examine attitudes towards mathematics: Methodological affordances and issues*

2021. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.), *Excellence in Mathematics Education: Foundations and Pathways (Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia)*, pp. 109. Singapore: MERGA.

Investigating students' drawings as a representational mode of mathematical fluency

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In sharing solutions of mathematical tasks, students may use various modes of representation such as: language (oral/written), numerical and symbolic, or drawings (pictures, diagrams or markings). In this paper we explore the potential of student drawings to provide evidence of mathematical fluency. Examples of young students' (5-8 years old) solutions to mathematical tasks are examined through the lens of drawing representations. The investigation suggested that students' drawings are valuable data when analysing work samples for evidence of mathematical fluency alongside other representations.

Drawn representations are a window into students' thinking and are worthwhile to explore in a mathematical context. Cai and Lester (2005) assert that representations not only help students make sense of mathematical problems but allow for communication of thinking to others. Bakar et al. (2016) agree that students use drawings to share solutions and suggest that "drawing was a *translation* from other types of representations, used [by students] to confirm and explain their answers" (p. 92). Within Way's (2018) research she utilised drawing to "reveal the variety in … drawings, and to explore similarities and differences across the age range" (p. 98). There exists an important transitional point during the early years of schooling for students between drawing (personal expression) and mathematical representation (function and purpose) (Bakar et al., 2016; Way, 2018). These representations require further analysis in observing students' mathematical fluency.

Data reported on in this paper is part of a larger research project (Cartwright, 2019) investigating students' characteristics of mathematical fluency and teachers' noticing of fluency. Within the study, many students produced drawings in their written work to convey their mathematical understanding in solving tasks. The drawings, as a mode of representation, became a vital aspect of analysis when observing a students' mathematical fluency. The purpose of this paper is to build on the drawing representational analysis conducted by Way (2018). In-depth analysis of the drawing work samples addresses the following research question: *How can students' drawing representations provide evidence of their mathematical fluency?*

Method

For the analysis, 39 Kindergarten to Grade 3 work samples were selected from schools involved in the research study. All students responded to the same problem: *The farmer saw 16 legs in the field. How many animals might he have seen?*

To analyse the drawings, previously researched drawing categories (Bakar et al., 2016; Way, 2018) pertaining to students' development of drawings within a mathematical context were employed. The drawing types *pictographic* and *iconic* (Bakar et al., 2016) were used to initially sort the data. Bakar et al. (2016) define drawing as pictographic "if it has realistic

2021. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.*), Excellence in Mathematics Education: Foundations and Pathways (Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia)*, pp. 114-117. Singapore: MERGA.

depictions of the objects stated in the problem” and iconic drawing as containing “only simple lines and shapes to embody the intended objects” (p. 89). Cartwright’s (2019) mathematical fluency characteristics were then used as an additional lens through which to view the drawings. Four fluency characteristics were used as deductive analysis categories: use of other representations (numerical or symbolic), correct process or solution, multiple solutions, and efficient strategy. Following the characteristics analysis, data were ordered into a developmental sequence based on Way’s (2018) drawing categories: picture, partial story, partition and solution.

Findings

Overall, 17 students (44%) used pictographic representations, 14 used iconic (36%), and 8 used no drawn representations (20%). Interestingly, a few students used both pictographic and iconic representations. During analysis it was necessary to split the iconic category further as a distinct difference between the way students used shapes and lines emerged. Instead of using shapes and lines to represent the animal or its legs, students used lines and circles to cordon off solutions. Some students also used lines, arrows, or circling to connect numerical solutions to symbolic or language representations (see Figure 1). The new category was named *iconic (as organisers)* to distinguish between the two uses of iconic drawings: in place of a picture, or as part of explaining the mathematical process.

The second level of analysis took the sorted work samples (pictorial features) and analysed the data using Cartwright’s (2019) mathematical fluency characteristics. All Kindergarten students (N=6) used pictographic representations. Most students also included a numerical representation. One sample included multiple solutions and the majority of students were able to use an efficient strategy to count the legs (see Figure 2). Most students obtained the correct number of legs (16) but did not mention the number of animals.

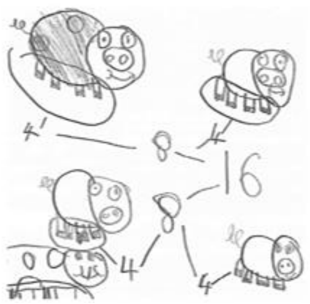


Figure 1. Example of using lines and circling to organise solution

Figure 2. Kindergarten example of counting by ones

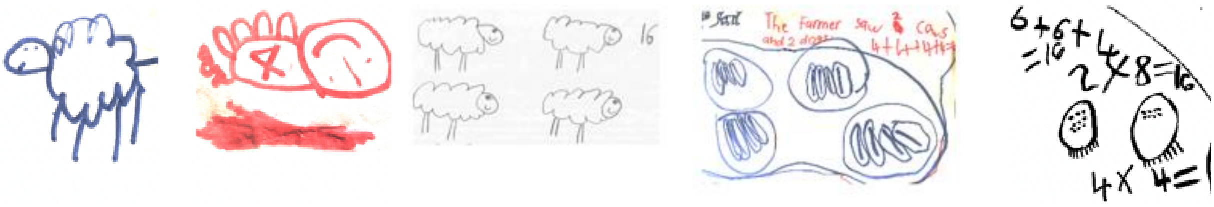
The Grade 1 samples have not been reported on in this paper as there were only three work samples, not enough to make significant statements. For Grade 2 (N=22) twenty of the students included a numerical representation to support their process or solution. Students used pictographic and iconic drawing types, however, there were significant differences in the mathematical features across the samples. One significant difference was the use of symbolic representation. Almost all students who used no drawings included symbols. Whereas only a few students who drew pictographic or iconic representations used symbols. Another significant difference was with solutions and types of efficient strategies. Most students who used pictographic representations did not produce multiple solutions and showed no strategy or an additive strategy. Compared with students who drew iconic representations or no drawings where multiple solutions and higher strategies (multiplicative) were observed. All but one Grade 3 sample (N=8) used numerical representations and six included symbolic representations as well. Most students recorded a correct process and solution and the majority of students used multiplicative strategies. Of the students who drew pictographic representations, none produced multiple solutions. Students who drew iconic representations or used no drawings were able to produce multiple solutions, often using their knowledge of number patterns to find different combinations.

Way's (2018) developmental sequence was used in analysing both pictorial and mathematical features of the work samples. Levels (described in Table 2 and illustrated in Figure 3) were adapted as the analysis progressed.

Table 2.
Developmental Sequence of Mathematical Drawings (Adapted from Way, 2018)

Level	No.	Level description
1. Scribble	0	Incoherent, no representation of the mathematical story
2. Picture	2	Shows pictures from the story problem (i.e. animal, farm) but no numerical labels or symbolic representations attached
3. Emergent Story - incorrect process/ solution	2	Shows pictures or iconic representations of the story and includes numerical values. No correct mathematical process or solution are visible.
4. Partial Story - errors with process or solution	7	Uses pictures or iconic representations and numerical values to show process of solving the problem. Correct process but incorrect/incomplete solution. Or correct solution with incomplete/incorrect process.
5. Partition and Solution	7	Uses pictures or iconic representations and numerical values during the process. Shows a correct solution.
6. Advanced Partition and Solution	13	Uses pictures or iconic representations and numerical values during the process. May include multiple solutions or patterns to find solutions.

N=31 (students who did not use drawings have not been included within this analysis)



Level 2 Level 3 Level 4 Level 5 Level 6

Figure 3. Illustrations of mathematical drawing levels

The use of a developmental sequence was beneficial when analysing the mathematical fluency features. For example, both Ellen and Daniel (Figures 4 and 5) used pictographic representations and in the initial analysis were grouped together. However, once these student samples were analysed using the developmental sequence of drawing levels, differences in their use of the representations appeared. Ellen used pictographic and iconic representations in an advanced way compared to Daniel. She labelled her pictures numerically which aligned to her cumulative count by fours. Ellen also drew lines to explain her partitioning of 16 into eights, then fours to describe her process. Although Daniel used a correct process and found a correct solution, his pictographic and numerical representations were separate. It is unclear if Daniel made a connection between the animals' legs and his count of four. Both samples show characteristics at *Level 5: Partition and solution.* However, if we see the drawings along a continuum of development, Ellen's would be placed higher.

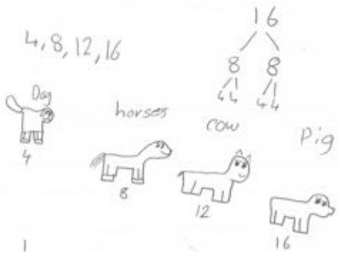


Figure 4. Ellen's work sample

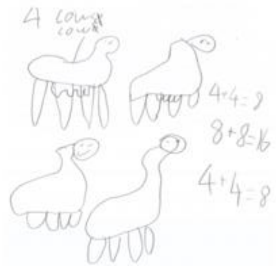


Figure 5. Daniel's work sample

Discussion and Conclusion

It was clear that drawing ability by itself did not always correspond to a student's mathematical understanding. However, students who made direct links between drawings, numerical, and symbolic representations, showed a higher level of mathematical fluency. The findings suggest that there are both affordances and issues with utilising students' drawings to analyse their mathematical fluency. One benefit was that drawings were a visual depiction of students' mathematical strategies. The way students grouped animal legs or drew arrays assisted in deciding if students were applying additive or multiplicative thinking, especially when the symbolic representations were not present. Some impacting factors emerged. Drawing ability *was* an issue for students unable to draw animals appropriately, i.e. incorrect number of legs. For students who drew pictographic representations time was a factor. The time it took to draw the animals resulted in only one solution being found, whereas students who used iconic representations generally found multiple solutions. Future research could explore iconic drawing further, specifically when students created array structures, and could be aligned to Mulligan and Mitchelmore's (2013) levels of Awareness of Mathematical Pattern and Structure (AMPS). Iconic representations revealed students' knowledge of number structure and provided scaffolding to efficiently solve the task.

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