



APPENDIX A. COMMUNITY MATH NIGHT PROFESSIONAL LEARNING WORKBOOK

This workbook is a companion to the *Community Math Night Facilitators' Toolkit*, a comprehensive resource for elementary school educators to plan and implement a Community Math Night.

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Understanding the “why” of Community Math Nights

A Community Math Night brings together educators, students, and their families to learn about, talk about, and have fun with math; reinforce positive math mindsets; and help family members participate in their child’s learning. The Community Math Night program helps schools and communities create a shared understanding of math concepts and raise expectations for math knowledge and achievement, both of which promote children’s success in school (DeFlorio & Beliakoff, 2015).

The content and activities in the *Community Math Night Facilitators’ Toolkit* are based on key research related to supporting student success in math. Understanding the research foundations for the Community Math Night program can help educators feel confident about hosting a math night and implementing math night activities. A stronger understanding of the “why” behind the Community Math Night program can also help educators connect the math night activities to their own beliefs and practices, which in turn can strengthen their work with students and families beyond the event.

This workbook is designed as a professional learning resource for educators and other school staff, though other stakeholders may engage as they have interest. The workbook pairs with [Section 1–Learn](#) of the toolkit and includes important research on four key topics that form the basis of the Community Math Night program:

- The importance of learning math for future success.
- Supporting equitable family engagement in math.
- Building a growth mindset and positive math attitudes.
- Effective practices for math instruction in elementary school.

The workbook describes the research base and then provides examples of how to apply the evidence in practice. The workbook also includes questions prompting users to reflect on related teaching practice and to use while planning and implementing a math night. For each topic the workbook also includes a curated list of additional resources for deeper study.

Using this workbook

Below are considerations for a Community Math Night core planning team using this document for professional learning:

- **Frame the purpose.** Ensure that individuals understand that developing awareness and familiarity with the research underlying math night activities will help them reframe their own math mindsets in preparation for leading a school in implementing a successful event. The workbook is designed to help educators use research to reflect on their own mindsets, approach to math instruction, and family engagement.

- **Dedicate time.** Schedule dedicated time to review the topic summaries and address the Reflect and share questions together as a core planning team, and with other educators and school staff if possible. The team may choose to appoint a leader for the learning sessions or rotate leadership roles by topic.

A team should spend approximately 30 minutes to review and discuss each topic summary. The team can review all four topics during one long meeting (approximately two hours) or review one topic at a time during four shorter meetings or as part of other scheduled meetings (for example, during staff or department meetings). Consider asking team members to read and review the topic summaries before the meeting, to save time. Also consider inviting to the meetings other members of the school staff who would benefit from this information, such as school leaders. Including school leaders and other educators can build momentum and enthusiasm for the Community Math Night program as well as for ongoing math professional learning.

- **Provide sufficient time and space for all participants to think and share.** When discussing the Reflect and share questions, provide time for educators to gather their thoughts before sharing.
 - Consider a think-pair-share strategy, or provide a minute before discussing each question for team members to jot down a few thoughts.
 - Try to be comfortable with a bit of silence or lulls in conversation while participants are digesting information.
 - Include all team members in the discussion. Consider rotating who shares first for each question or more actively engaging team members who are less likely to speak up.
- **Keep the conversation focused and productive.** If conversation turns to negative experiences and you sense that educators are venting more than sharing and learning, try to refocus the conversation on the goal of supporting the Community Math Night program and how the research can inform future progress.



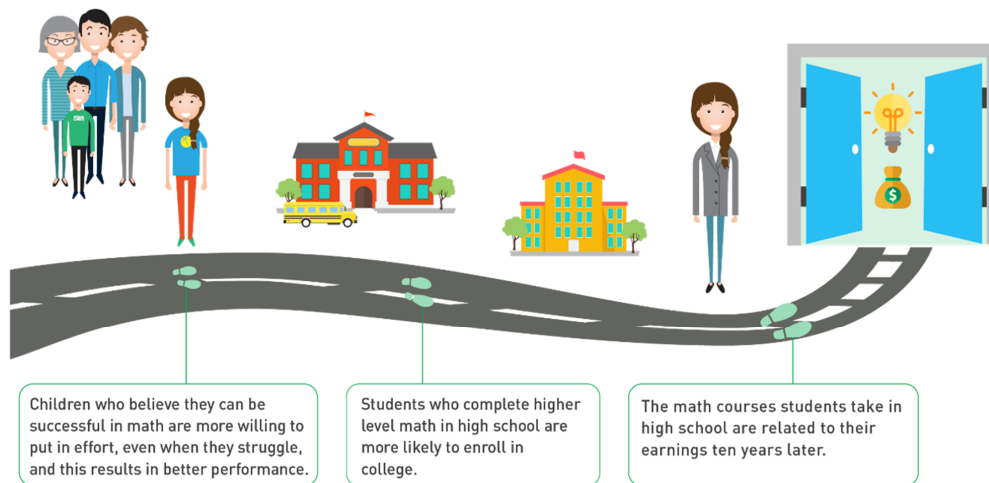
The importance of learning math for future success



Research roundup

An early emphasis on math is one way to set children up for future success in school and life; hosting a math night can help communicate this message to families and the broader community.

- Mastery of math skills at an early age is a strong predictor of later achievement in math and other subjects. For example, kindergartners' math skills (such as understanding quantity and one-to-one correspondence, comparing measurable attributes such as length and width, and classifying objects into given categories) can predict math, reading, and science achievement and the likelihood of grade retention in grade 8 (Claessens & Engel, 2013).
- Similarly, students' knowledge of fractions and division in elementary school is a predictor of their high school achievement in algebra, regardless of factors such as family income or education (Siegler et al., 2012). This suggests that having a strong foundation in math can help put all learners on the path to academic success.



- Research tells us that student learning is greatest when activities and tasks encourage high-level thinking and least when tasks are procedural (Boaler & Staples, 2008; Van de Walle, 2004). Thus, teachers should plan learning activities that balance procedural skill development (for example, acquisition of basic number facts, proficiency with math procedures—add, subtract, multiply, divide) with higher-level problem solving (math tasks that provide intellectual challenge and enhance math understanding; National Council of Teachers of Mathematics, 2014c).

Higher-level math courses lead to future success

- Taking higher-level math courses in high school has positive effects on students' math achievement and college enrollment, particularly for students from low socioeconomic backgrounds (Byun et al., 2015).

- Economists have found that the math courses students take in high school are strongly related to their earnings 10 years later, regardless of the person’s race/ethnicity, gender, family characteristics, college major, or occupation (Rose & Betts, 2004).
- One explanation for the link between math learning and progress in areas beyond traditional math fields is that learning math promotes habits of mind or critical thinking skills—such as understanding patterns and relationships and striving for accuracy—that can be applied to solving problems in other disciplines (Cuoco et al., 1996).

Application to practice

Explain your thinking

When students verbalize their mathematical thought processes, teachers gain insight into students’ conceptual understanding, and students gain confidence and build computational fluency, which supports skill mastery and sets students on a path to academic success. While the math night activities, with proper facilitation, offer opportunities for students and families to verbalize their math thinking, this concept of math talk could be integrated into classroom instruction as well. This can be done in many ways. One structure to reveal the value of explaining one’s thinking about math is the math clinic, highlighted in [this video from Edutopia](#). Math clinics push students’ conceptual understanding of math through read-alouds, conversation prompts, and symbolic representations. You can lead math clinics for different size groups (small, large, or one-on-one) and various formats such as read-aloud, games, diagrams, and pictorial representations.



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Discuss with your colleagues

What is your reaction to the video? How can you ensure that your facilitation of activities during the math night encourages math talk? And beyond a math night, how can you embed the spirit of math clinics, where students have opportunities to explain their thinking, into your instruction and class routines? Use the box below to record discussion points.



Reflect and share

Discuss with your colleagues

1. What are your key takeaways on the connection between developing early math skills and practices and students' future success in secondary school and beyond?

2. How does having a strong foundation in math open doors for students from different backgrounds?

3. How does the research align with or differ from your own experience as a math learner?
As an educator? How does it align with your goals for students?

4. What do you want families to understand about the connection between early math skills and academic and life success?



Learn more

- [Supporting Your Child in Developing Math Skills for Future Success](#). This infographic from Regional Educational Laboratory (REL) Appalachia (n.d.) displays research findings on the importance of math success from elementary school to college and careers. It helps family members understand how math learning influences their child's future and provides simple ways that they can encourage positive math attitudes and learning.
- [Using Math Talk to Support Learning](#). This short video from the Center for Early Childhood Education (2013) at Eastern Connecticut State University presents researchers discussing the importance of early math learning interspersed with clips of teachers engaging students in math learning through play.
- [Building Educators' Understanding of Early Mathematics to Promote Students' Later Mathematics Success](#). This webpage (Regional Educational Laboratory West, n.d.) offers resources, including webinar recordings, discussion guides, and PowerPoint presentations from in-person and web-based events hosted by REL West. The content focuses on the foundational preK–grade 2 math knowledge and skills necessary for students to achieve mastery of the Common Core State Standards and meet with success across grade levels.
- [The Institute for Habits of Mind](#). This infographic from the Institute for Habits of Mind (Kallick & Costa, n.d.) elaborates on the 16 habits that characterize problem-solving behavior and shows how to incorporate them into teaching and learning.



Supporting equitable family engagement in math



Research roundup

Engaging families in their children’s math education validates them as partners in learning and contributes strategic benefits for students. Educators are experts in their content and pedagogy, and parents are experts in their children. By partnering, families and educators can leverage their strengths for maximum benefits to students. Events such as a math night offer an engaging opportunity to establish or cement family–educator partnerships.

Why engage families? Research affirms that families are powerful partners to support learning.

- More than four decades of evidence confirm the family’s powerful influence on children’s development. Family involvement is a strong predictor of school success (Harris et al., 2017; Weiss et al., 2009), particularly with literacy and math skills (Van Voorhis et al., 2013).
- Well-designed parent–family–community partnerships that involve parents and family members in their children’s learning are associated with increased student self-confidence and achievement generally and in math specifically (Epstein et al., 2018).
- Urban youth identified their family members (parents, siblings) as providing them with the greatest amount of support throughout their schooling (emotional encouragement, academic assistance, and college preparation help; Vega et al., 2015).

Engaging in extracurricular math activities can help students’ math development.

- Out-of-school-time family math activities, in which teachers, students, and families learn and talk about math together, can be one avenue to develop partnerships and can generate families’ high expectations for their children’s math learning (DeFlorio & Beliakoff, 2015; Meyer, 1996).
- Home-based activities, such as playing number board games, have been shown to improve the foundational math skills of economically disadvantaged students (Siegler & Ramani, 2008). Parents and students engaging in math-focused activities together is strongly and positively related to math achievement test scores in elementary and secondary grades (Sheldon & Epstein, 2005).

Traditional barriers to family engagement can be addressed with a holistic, strengths-based approach that recognizes and builds on positive attributes rather than focuses on potential deficits.

Holistic strategies for overcoming barriers to family engagement—among educators, in classrooms, with family members, and in the broader community—are listed below.

Among educators

- Develop self-awareness of negative biases about families and of assumptions that families have “problems” that the school needs to “fix.” Instead, focus on families’ strengths that you can leverage to support students’ success (Kim, 2009).
- Internalize strengths-based attitudes and develop communication skills that allow you to convey positive assumptions to family members. See the [Application to practice](#) section for examples.
- Recognize the many ways that families can offer support to their children (such as providing emotional encouragement) that may not look like traditional family involvement (such as volunteering in the classroom), and acknowledge these strengths to parents as important in meeting students’ needs (Vega et al., 2015).

In classrooms

- Invite the knowledge and perspectives of culturally diverse families to create respectful, engaging learning environments that empower students’ identities as learners (Ishimaru et al., 2015). Strategies include inviting students to present and share cultural heritage such as traditions or favorite foods, noting math in the recipes (Scott, 2016) and integrating family interviews, lived histories (plotting events on timelines, calculating elapsed time in years), and folktales (which often incorporate counting, patterns, and repetition) into the curriculum (Eisenbach et al., 2016).
- Connect in-school math activities to everyday family routines and culture (Harris et al., 2017). Examples include investigating symmetry through traditional quilting, analyzing patterns in Navajo rug weaving, beading, and exploring measurement and fractions through family recipes.

With family members:

- Consider and minimize logistical challenges posed by many family engagement events (Koonce & Harper, 2005), which can make attendance challenging for some families because of work commitments and transportation difficulties.
- Consider strategies for eliminating barriers (transportation, scheduling, meeting locations) to attendance and increasing family participation.
- Communicate respect in every way possible: in written communications to families, spoken language at school and community gatherings, and nonverbal body language. Many parents have powerful, negative memories of their school experiences, and some may feel that school staff view them negatively (Koonce & Harper, 2005). Specifically, members of historically underrepresented groups often report feeling unwelcome, disrespected, and devalued when visiting their child’s school (Morton, 2017). Thus, they might be reluctant to engage with teachers in full partnerships or to attend school events. Remember that by making families feel valued and respected, educators are helping to build a powerful partnership that benefits everyone—particularly students (Morton, 2017).

- Create a culture of partnership, openness, and hospitality at your school to help parents overcome any negative school experiences by promoting ongoing family engagement through frequent oral and written communication, home visits, and open houses. Establish and build trust for long-term relationships. See a math night event as but one element of a comprehensive plan to build relationships through sustained, positive, welcoming, affirming communication that builds trust.
- Engage parents and focus on shared goals for their children’s success, building their capacity to support their children’s learning, and develop relationships for the long term, beyond just a one-hour positive interaction.

In the broader community

- Build rapport and long-term connections with trusted partners such as nonprofits (Boys & Girls Club, YMCA); faith communities (churches, temples, mosques); and advocacy organizations committed to supporting families in your community (Jordan & Wilson, 2017; Latunde, 2016). These groups interact with students and families in nonacademic settings and can help educators serve families in holistic ways.
- Tailor outreach and partnership strategies to better engage underrepresented families in supportive programming. See [Promote enthusiasm and participation](#) in [Section 2–Plan](#) for sample strategies.
- Encourage and incentivize students (for example, through contests) to communicate positive math attitudes to friends and parents through social media and other Internet platforms (Latunde & Clark-Louque, 2016).
- Think about family involvement within the context of a linked system of other learning supports for a child’s school success—and build such a linked system, including the school, community, families, and educators—so that “if one type of support falters, others hold strong” (Weiss et al., 2009, p. 16).



Application to practice

Adopting and internalizing strengths-based attitudes toward family engagement

To meaningfully lead and support a Community Math Night, it is critical for educators to examine their beliefs and biases to ensure that they are operating from a strengths-based approach. That requires educators to meet students and families where they are along their own education paths and build on the positive attributes they bring to the math night. This approach differs from operating from a deficit approach, which views families as having problems that the school needs to solve or fix. For some, this is a significant shift in perspective.

Practice can be helpful in promoting this switch. Examine exhibit A1 on strengths-based attitudes about family engagement from U.S. Department of Health and Human Services (2018).

Exhibit A1. Strengths-based attitudes toward family engagement

Strength-based attitudes	Description
All families have strengths.	Each child and family has unique strengths that can be the foundation of our discussions and partnership. Always start with strengths, even when there are challenges
Families are the first and most important teachers of their children.	Families are the most important constant in children's lives, and children's healthy development relies on sensitive and nurturing interactions within the family and the community.
Families are our partners and have a critical role in their child's development.	Families make choices every day that affect a child's development and learning. These choices are rooted in their belief systems and cultural identities.
Families have expertise about their child and their family.	Families understand their children best and make decisions for their children's well-being. When families share what they know, children, families, and providers benefit.
Families' contributions are important and valuable	Being open to families' suggestions and requests helps us do our best on behalf of their children

Source: U.S. Department of Health and Human Services, 2018.

Keep the strengths-based attitudes and descriptions in mind as you complete a practice activity.

Below is a conversation between an educator and a family member. Read the conversation and look for evidence of strengths-based attitudes in their dialogue.

Educator: Thank you for meeting with me today to talk about Zara. Is there anything you would like to share with me about her so that I can get to know her better?

Family member: She is very quiet and shy in a large group, but she's not quiet once she trusts you. She loves music and math.

Educator: That's very helpful to know. Thank you for sharing that with me. I've conducted some initial assessments and I'll share the educational plan I've developed for her based on the results.

Family member: OK.

Educator: <Teacher shares results and plan> I will need your help to reinforce this plan and partner with me and Zara to make it successful. Do you have any questions or objections to what I've laid out here today?

Family member: No, I don't think so. You are the teacher, so I guess you know what to do.

Educator: Great. I'll let you know if there's anything else you should know as the year moves on. Thanks for taking the time to come in today.

Family member: Thank you. Take care.

How could the educator's communication be adjusted to reflect more of a strengths-based attitude?

Let's debrief. The educator reflects a strengths-based attitude by thanking the family member for taking time to meet, asking the family to share, and valuing what was shared. Also, the educator views the family member as a partner in implementing a plan for the child.

The educator could improve communication of a strengths-based attitude toward the family member by sharing strengths, asking the family member for observations of what occurs at home, co-developing a plan with the family member, communicating clearly that the family member is a valued and critical partner—not just someone who should receive a memo with updates as needed.

How might you rescript the exchange to better reflect strengths-based attitudes?

To make this challenge more real, consider pulling out a recent parent newsletter or correspondence you have written and examining it through the same lens, without judgment, but with mindfulness and reflection. This could be a starting point for goal setting if a strengths-based communication change is in order.



Reflect and share

Discuss with your colleagues

1. What are your key takeaways from the research on the benefits or added value of involving family members in children’s math education versus only working with children during instructional time?

2. How does the research align with your own experience of involving families in the educational process?

3. According to a recent qualitative research study, urban youth identified their family members (for example, parents, siblings) as providing them with the greatest amount of support throughout their schooling (emotional encouragement, academic assistance, and college preparation help; [Vega et al., 2015](#)), while other research suggests educators perceive a lack of parental involvement. How can these two findings be reconciled? What are some implications for how educators can expand their thinking about family involvement?

4. In what ways do you already partner with families in their children’s education? Is math education a part of these efforts, or is it an area in which you have room to grow?

5. Assess your awareness of and sensitivity to reasons that family members might hesitate to engage in school-based math activities. To what degree can you continue to build awareness, and how might you tailor your efforts to engage families accordingly?
Example: What methods of communication might you use? What kinds of key messages might you try to communicate to build trust?

Learn more

General family engagement resources

- [Critical Practices for Anti-bias Education: Family and Community Engagement](#). Produced by Learning for Justice (n.d.), a project of the Southern Poverty Law Center, this professional development guide details critical practices of anti-bias education that address assumptions or stereotypes.
- [Strategies for Equitable Family Engagement](#). Funded by the U.S. Department of Education, this document (State Support Network, 2018) is organized around five categories of equitable family engagement strategies and includes rich examples of school system implementation from across the country.
- [The National Center for Family and Community Connections with Schools](#). This organization's website (n.d.) shares research-based information and resources that can effectively connect schools, families, and communities. The center highlights community connections that directly affect student achievement in reading and math as well as connections that contribute to students' overall success.
- [Toolkit of Resources for Engaging Families and the Community as Partners in Education Part 1: Building an Understanding of Family and Community Engagement](#). This REL Pacific toolkit (Garcia et al., 2016) provides resources for school staff to build relationships with families and community members and to support family well-being, strong parent-child relationships, and students' ongoing learning and development.

Math-specific family engagement resources

- [Teaching Math to Young Children for Families and Caregivers](#). This REL Central website (n.d.b), developed in partnership with REL Appalachia and REL Northwest, includes videos and activities to help families support children as they practice math skills at home.
- [Formula for Success: Engaging Families in Early Math Learning](#). This publication from the Global Family Research Project (2017) is a compilation of articles that offer important lessons about family engagement for children's learning of math anywhere and anytime.
- [Family and Caregiver Activity to Support Young Math Learners' Understanding of Fractions](#). Families and caregivers can use this activity resource from REL West (2020b) to help young children in grade 2–4 practice sharing cookies to learn about fractions.
- [Encouraging Girls in Math and Science: Three Powerful Female Role Models](#). To promote positive views about women in math and science, this activity from REL West (2020a), geared toward families and caregivers for use with children in grade 3–8, features three female role models who have made important contributions to math and computer science.



Building a growth mindset and positive math attitudes



Research roundup

The Community Math Night emphasizes that building a growth mindset and positive math attitudes for math learning can have immense implications for students' performance and academic path—now and into the future.

Power of caring adults

The power of a caring adult in the life of a child cannot be overstated. Developmental research suggests that the presence of one or more caring adults in a child's life—whether a family member, teacher, or other community member—can increase the chances that the child will flourish in adulthood (Murphey et al., 2013). Given the extraordinary position of influence these trusted adults hold in shaping the trajectory of children in their care, it is important to develop a sense of mindfulness and awareness of messages, both direct and indirect, intentional and unintentional, that adults send and children receive. Messages about math are one such area.

Math anxiety

One study of math attitudes found that over 90 percent of Americans report some level of math anxiety (Blazer, 2011). Many of us have received negative messages about math, and these messages and beliefs can affect success in math (Chang & Beilock, 2016). We live in a culture where it is oddly socially acceptable to say “I'm not a math person” or “I'm not good at math.” Even many preschool and elementary teachers hold this attitude. Math anxiety has been documented in children as young as first graders (Ramirez et al., 2013). Math anxiety interrupts working memory, which keeps even strong students from using complex problem-solving strategies, thus negatively affecting their math performance (Ramirez et al., 2016). And a 2015 study found an association between the math anxiety level of a parent who helped a child with math homework and their child's math outcomes in early elementary grades (Maloney et al., 2015).

Overcoming anxiety with a growth mindset

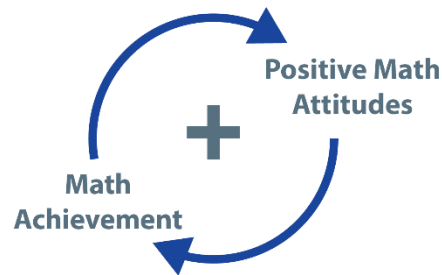


The good news is that math attitudes and beliefs can change and that teachers and family members, as natural role models and teachers for children, can promote positive attitudes toward learning and support student achievement in math (Epstein et al., 2018). In fact, the negative impact of parents' math anxiety can be counteracted by normalizing feelings about math (Boaler, 2015).

How can students and families experiencing math anxiety and other negative feelings toward math begin to change course? One way is by building an understanding of the growth mindset. People tend to believe that math ability is fixed (Boaler, 2015). With a fixed mindset, you either are a math person or you are not. In contrast, a person with a growth mindset believes that intellectual abilities can be increased with effort (Dweck, 2006). With this perspective, growth and change are possible. To clarify, embracing a growth mindset doesn't mean that there are no challenges or that change is simple, but rather a growth mindset introduces "the power of yet" (as in "I can't do this yet"), which unveils "a path into the future that creates persistence" (Dweck, 2014, 05:16). Helping children develop resilience when faced with disappointments provides life lessons beyond the classroom.

Power of positive math attitudes

Unsurprisingly, students who have embraced a growth mindset tend to persist and perform better (Blackwell et al., 2007; Dweck, 2008).



Additional research suggests that math attitudes and math skills have a reciprocal relationship—positive attitudes about math promote math achievement, which in turn fosters even more positive attitudes (Gunderson et al., 2018; Ma, 1997)—and this is true as early as grade 1 and 2 (Dweck, 2008). Think of math attitudes as creating a virtuous cycle of positivity that continues to yield benefits for children.

Math night activities include opportunities for families to reflect on how they feel and talk about math with their children and to build an understanding of the growth mindset and the role of positive math attitudes in supporting children's math learning. See [appendix B](#) for a handout that you can share with families at your math night.

Application to practice

Brainstorming ways to teach brain plasticity and the power of "yet"

Consider ways to demonstrate the growth mindset and the power of "yet" to families and students using either a math or a non-math activity. This can take place in an activity station during the math night, for the whole group as part of the [Mindsets and Math presentation](#), or as a follow-up in classrooms. Here are a few ideas:

- Write statements that demonstrate growth or fixed mindsets on index cards and have students sort them accordingly. Here is an example of [a statement sort activity](#) available online.
- Read a children's picture book featuring a character who struggles, persists, and overcomes a challenge or masters a skill, such as *Brave Irene* by William Steig (1988). Introduce the power of "yet" to students as a preservation technique that the book character employs to keep from becoming discouraged.

- Lead a brainstorming session and record thoughts using words and pictures of what the growth mindset looks, sounds, and feels like. Consider using a “[Looks like, feels like, sounds like](#)” graphic organizer.

Which activity resonates with you? How might you adjust or tweak it to make it work for your students? What other ideas come to mind?



Reflect and share

Discuss with your colleagues

1. What are your key takeaways about the importance of positive attitudes toward math for parents and students?

2. How does the research align with or differ from your own experiences surrounding math attitudes—either on a personal level or with your students?

3. To what degree are you familiar with the fixed mindset versus growth mindset dichotomy? Have you ever discussed the growth mindset with your students? If so, what was their reaction? If not, how might you explain it to them and help more students embrace a growth mindset?

4. How might embracing a growth mindset change the way you relate to students? The way you deliver instruction? The way you assess learning? The way you encourage students?



Learn more

- [How You Can Be Good at Math, and Other Surprising Facts about Learning](#). This video (TEDx Talks, 2016) features Dr. Jo Boaler, a Stanford professor of math education, who shares brain research showing that everyone can be good at math with the right teaching and messages. Boaler highlights a successful pathway for teachers' approaches to teaching math, focusing on positive, growth mindset messaging.
- [Youcubed](#). This webpage (Stanford Graduate School of Education, n.d.) offers resources that support math success through growth mindset and innovative teaching practices—such as classroom posters and videos, teaching examples, cross-curricular lessons, apps and games, parent resources, and research.
- [Young Mathematicians](#). This webpage (n.d.) offers resources for fostering a growth mindset classroom culture, including positive mathematical mindsets. Resources include videos, books, and articles.
- [Mindset Kit](#). This website (The Project for Education Research that Scales, n.d.) presents a free set of online lessons and practices for teachers, parents, and mentors that foster and support students' growth mindset.



Effective practices for math instruction in elementary school

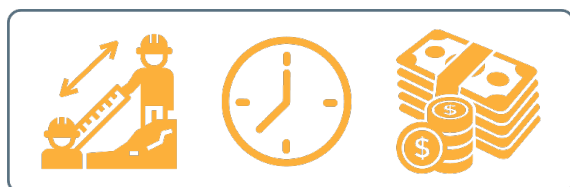


Research roundup

The evidence-based instructional practices and Community Math Night activities highlighted in this toolkit reflect real-world relevance, progression along a concrete–representational–abstract continuum, and the development of fluency.

Real-world relevance

Effective K–5 math activities focus on the meaningful learning of math skills and concepts by building on what students know from their everyday experiences. Research shows that math tasks that present problems in personally and socially meaningful contexts (for example, estimating/measuring distance, area, or volume or using money) are effective for engaging

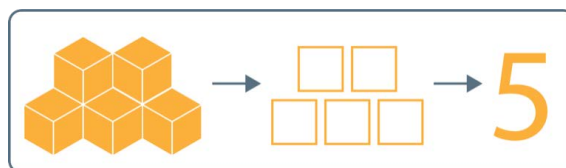


students, generating interest and curiosity, and building positive math attitudes (Clements, 2013; Frye et al., 2013). During math night activities, children describe their mathematical thinking in informal and formal ways and use concrete tools,

like counters or other manipulatives, to represent math ideas. Developing connections between math and the real world can be particularly effective for students who are underperforming in STEM fields (National Council of Teachers of Mathematics, 2014a).

Progression along a concrete–representational–abstract continuum

Methods for teaching math along a concrete–representational–abstract continuum begin by modeling math concepts using concrete materials (for example, buttons, blocks, string, and manipulatives); followed by a pictorial representation, such as shading an equivalent part of a whole; and then using abstract symbols, such as numbers or notation. These teaching methods are particularly successful for struggling students (Steadly et al., 2008). For this strategy to be effective, students must engage physically with the manipulatives, which helps them form a mental image of the concept. Manipulatives help children think and reason in meaningful ways and support interconnected understandings of math concepts (Stein & Bovalino, 2001). Practice with concrete objects prepares students to represent their mental image in pictures and abstract symbols.



Using the resources in the toolkit, teachers can help children relate their use of concrete materials to more formal math vocabulary and symbols by using activity prompts (questions to ask children during the activity) and resources and supports (for example, a picture glossary; Stein & Bovalino, 2001).

Development of fluency to support algebraic and higher-order mathematical thinking

Researchers reference two types of fluency when discussing algebraic and higher-order mathematical thinking. The first type, conceptual fluency or “number sense,” includes an understanding of place value and the relationships between the four operations (add, subtract, multiply, divide; Hiebert, 1984). The second type, procedural fluency, is described by the National Council of Teachers of Mathematics (2014b, p. 1) as “the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another.” Conceptual and procedural fluency extend beyond speed and automaticity of basic facts and apply to all strands of math. The foundations of these fluencies are laid in grades K–5.



Conceptual and procedural fluency are building blocks for algebra. According to a report from the National Mathematics Advisory Panel (2008), the critical foundations of algebra comprise three clusters of concepts and skills. These are:

- *Fluency with whole numbers:* number sense; place value; building and breaking down numbers through operations; commutative, associative, and distributive properties; automatic recall of math facts and fluency with algorithms; estimation.
- *Fluency with fractions:* locating positive and negative fractions on a number line; comparing and moving between fractions, decimals, and percentages; estimating size; using operations with fractions; applying real-world use of fractions such as rates, proportions, and probability.
- *Aspects of geometry and measurement:* identifying properties of similar triangles; identifying properties of two-dimensional and three-dimensional shapes using formulas for perimeter, area, volume, and surface area; finding unknown values for lengths, angles, and areas.

Engaging with math night activities allows students to work with numbers and operations in different ways, to use diverse strategies, and to consider how their problem-solving approach differs from or is similar to that of others, supporting the co-development of conceptual and procedural fluency (Gersten et al., 2009).

Building both conceptual and procedural fluency helps students gain the skill and experience needed to solve multistep and complex problems, which become increasingly prevalent in upper-level math courses. By building a firm conceptual foundation for students now, educators set the stage for students’ later success. In short, sound instructional practices now yield great benefits later.

Application to practice

Like many of the learning experiences you plan for your students, the math station activities in this toolkit reflect the three evidence-based practices described in this section.

Real-world relevance

These activities portray math learning in real-world activities so that students can perceive their learning as relevant and useful. The following two activities can spur math-rich conversations and inspire families to bring math into their daily interactions.

- [Dinner Time](#), a grade 4–5 activity, is set in a café. Families determine a budget for dining out and then strive to stay within their budget as they select menu items.
- [How Many of Me?](#), an activity differentiated across K–5, engages families in measuring the dimensions of a room based on nontraditional units. Students make predictions about distance, practice their measuring skills, and compare the nontraditional units to traditional ones.

Progression along a concrete–representational–abstract continuum

Multiple math station activities employ manipulatives to help students build their conceptual understanding of shapes and numbers. All three Geometry math station activities and a K–1 activity in the Number and Operations in Base-10 station uses base-10 blocks:

- [Fill the Shapes.](#)
- [Hexagon Challenge.](#)
- [Symmetric Mosaics.](#)
- [Race to 100.](#)

As students manipulate concrete items, they build mental models that help them transfer these concepts to pictorial representations and symbols.

Development of fluency to support algebraic and higher-order mathematical thinking

These math station activities support students' conceptual understanding of numbers and operations and build fluency in applying arithmetic procedures. The activities prompt students to work with numbers and operations in different ways, use strategies that support conceptual understanding and fluency, and reflect on their methods and solutions.

- [Flip the Cards](#), a grade K–1 activity, is designed to promote fluency and flexibility of addition facts with sums up to 20 or less and their related subtraction inverses (such as $6 + 8 = 14$ so $14 - 8 = 6$).

- [Many Ways of Counting](#), a grade 2–3 activity, asks families to determine the number of various items in an array. The arrays vary from simple to challenging and allow families to arrive at solutions in multiple ways.
- [Game of 24](#), a grade 4–5 activity, asks families to select and apply a combination of the four operations (addition, subtraction, multiplication, and division) to reach 24 with up to five number cards.

The core planning team should take time to work through all the station activities in their meetings both to better prepare for the event itself and to begin to see the connections between the evidence-based practices and the activity designs.



Reflect and share

Discuss with your colleagues

1. What are your key takeaways about the recommended math strategies in recent research?

2. When you hear about the strength of the research showing that students' math foundations in elementary school predict math achievement in high school, how do you respond?

3. How does the research align with or differ from your own instructional practices?

4. As you reflect on these strategies, which of these is an area of strength in your instruction? Why? What might be an area for professional growth? Why?

5. As you reflect on your curriculum, what opportunities can you provide for authentic problem solving? How might such opportunities help students engage in their math learning?



Learn more

- [Teaching Math to Young Children](#). This What Works Clearinghouse practice guide (Frye, et al., 2013) offers five recommendations for teaching math to children in preschool, pre-kindergarten, and kindergarten. Each recommendation includes implementation steps and solutions for common roadblocks.
- [Early Childhood Math Videos](#). This webpage from REL Central (n.d.a) shares a series of videos that focus on actionable strategies to increase the quality of math instruction in the early school years by providing recommendations that address implementation steps and common roadblocks experienced by early learners.
- [What Works Clearinghouse: What Works in Math](#). This webpage (What Works Clearinghouse, n.d.) features practice guides, interventions, reports, and videos that support effective math instruction in K–12.
- [Effective Mathematics Teaching Practices: National Council of Teachers of Mathematics](#). This resource (PDF) from the National Council of Teachers of Mathematics (2014a) highlights eight teaching practices described as components of effective math lessons.
- [Standards for Mathematical Practice: Common Core State Standards](#). These standards (Common Core State Standards Initiative, n.d.) describe types of expertise that teachers at all grade levels should seek to develop in their students.