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Elementary Students' Understanding of the Equals Symbol: Do Florida Students Outperform Their Peers?

Guillermo Farfan and Robert C. Schoen

If you were to ask a person whether "1 = 1" is a true statement, many of us would expect the answer to be "yes." What may seem like a straightforward answer to us as mathematically literate adults, however, might not be so for children. Mathematics education and educational psychology journals, in fact, often signal a recurring concern that children do not understand the intended meaning of the equals symbol (=) in mathematics. Scholars report that children understand it to be a "do something" symbol rather than a symbol that indicates a bidirectional equivalence relation (Boggs et al., 2018; Hornburg et al., 2018). For example, elementary students often say that the unknown value in Equation 1—an item that is frequently used by researchers—is 12. (1) $8 + 4 = \square + 5$

That is, children commonly add the numbers on the left side of the equals symbol and write their sum (i.e., 12) in the box. Students also frequently determine the sum of all the given numbers and write 17 in the box (Falkner et al., 1999).

Many studies conducted in the years since Falkner et al.'s (1999) findings continue to paint a grim picture of elementary students' understanding of =. Scholars report low levels of understanding overall and limited increases in understanding as students enroll in higher grade levels—sometimes even reporting decreases in understanding. In our own work, we have encountered many elementary school children whose understanding does not conform to this picture, an experience that led us to wonder whether some of the published interpretations and conclusions have been limited by a reliance on small or biased samples. Here, to overcome some of these limitations, we use two large-scale sets of data with items intended to assess elementary students' understanding of =. We seek to answer three questions: (1) What percentage of Florida elementary students (grades K–5) respond correctly to test items designed to assess their mathematical understanding of =? (2) Do Florida elementary students in higher grades perform better than those in lower grades students on questions designed to assess their mathematical understanding of =? (3) How does the performance of Florida elementary students compare with that of their peers elsewhere, as represented in published studies?

In answering these questions, we refrain from formal hypothesis testing and limit our analysis to comparing the descriptive statistics provided by our two sets of data with some of the literature on this topic. For ease of comparison, we use students' responses to two equals-symbol items that resemble Equation 1, which are part of a category we call Operation on Both Sides (OBS) items (Schoen et al., 2016a, 2016b).

The Equals Symbol in Elementary Mathematics

In elementary arithmetic, the equals symbol plays a fundamental role in students' development of number sense, as seen in addition and subtraction problems, decomposition of two-digit numbers, and comparison between numbers (Bennett et al., 2016; Musser et al., 2014). In all these instances, the equals symbol is meant to denote a relationship where two mathematical expressions—the expressions on the left- and right-hand sides of the equals symbol—have the same value or are "equal to" each other. This definition is known in the literature as "relational understanding" of = (McNeil & Alibali, 2005; Molina & Ambrose, 2008), and it is seen as facilitating the transition from arithmetic to middle-school and high-school algebra (Bush & Karp, 2013; Knuth et al., 2006).

Recent school mathematics standards also underscore the importance of a relational understanding of =. For example, the Common Core State Standards for Mathematics (CCSSM) explicitly mention the equals symbol in two first-grade standards (CCSSM 1.OA.7; 1.OA.8) and allude to it once in the Standards for Mathematical Practice (National Governors Association, 2010). The Mathematics Florida Standards (MAFS), which were adopted by the state of Florida in early 2014, were very similar to the CCSSM, but differed in some key ways. The MAFS included the same standards as the CCSSM in first grade for the equals symbol, but they added standards to continue to address student understanding of the equals symbol in grades 2 (MAFS.2.OA.1.a) and 4 (MAFS.4.OA.1.a, MAFS.4.OA.1.b). Signaling the importance of student understanding of the mathematical meaning of this symbol, the recently adopted Benchmarks for Excellent Student Thinking (BEST) mathematics standards mention the equals symbol 16 times, starting in grade K and continuing through grade 7 (Florida Department of Education, 2020). The CCSSM, MAFS, and BEST standards all define = as a relational symbol.

As mentioned earlier, however, reports abound that many students in the United States read = not as a relational symbol but rather as marking an operation that must be performed. The frequently cited study by Falkner et al. (1999), for example, showed that fewer than 10% of first and second graders gave a mathematically correct response to the item "8 + 4 = ☐ + 5." More concerningly, their data indicated that the percentage of children who responded correctly to this item in fourth (n = 57) and fifth grades (n = 42) was lower than that in third grade (n = 208). Subsequent studies continued to reinforce the idea that children entering middle school interpret = operationally. In Table 1, we list eight such studies from outside Florida that made use OBS items.

As shown in Table 1, the mean percentage of correct responses initially increased from 15% in first grade to 24% in second grade and then decreased to 19% in third grade. Only in fifth grade (62%) did more than half of the students answer correctly. Table 1 reveals gaps in the literature, however; second grade is represented in seven out of the eight studies listed, but no other grade level appears more than three times. Although this list is not exhaustive, the studies on it were chosen because they reported the percentage of correct responses by grade level and used the OBS item "8 + 4 = ☐ + 5" or a similar item to assess children's understanding of =.

Table 1
Percentages (and Sample Sizes) of Students Who Responded Correctly to Equals-Symbol Items by Grade Level in Studies Conducted Outside Florida

	Grade level					
Study	1	2	3	4	5	*N*
Falkner et al. (1999)[a†]	0 (42)	6 (174)	10 (208)	7 (57)	7 (42)	668
Stephens et al. (2013)[‡]			2 (104)	24 (108)	56 (78)	290
Bennett (2015)[‡]		15 (213)	35 (233)	46 (267)	68 (469)	1,182
Powell et al. (2016)[b†]	16 (805)	27 (489)				1,294
Johannes et al. (2017)[c‡]		50 (49)				49
Matthews & Fuchs (2018)[a‡]		33 (153)				191
Johannes & Davenport (2019)[b‡]		28 (406)				406
Chow & Wehby (2019)[b‡]		24 (74)				74
Mean	15 (847)	24 (1,558)	19 (545)	35 (432)	62 (589)	

Note. Results for Operations on Both Sides (OBS) items (e.g., 8 + 4 = ☐ + 5), rounded to nearest percent. Sample sizes shown in parentheses.
[a]Results for other grade levels are not included in the table.
[b]Percentage available for Spring 2013 and Fall 2012 only.
[c]Randomized controlled trial. Percentage at pretest for control group only.
[†]CCSSM not implemented at the time of data collection.
[‡]CCSSM partially or fully implemented at the time of data collection.

Background

Here, we report findings from the administration of the Mathematics Performance and Cognition Interview (MPAC; Schoen et al., 2016a, 2016b), and the Elementary Mathematics Student Assessment (EMSA; Schoen et al., 2017, 2018a, 2021). These instruments were administered as part of the evaluation of a program called Cognitively Guided Instruction (CGI)

Cognitively Guided Instruction

Cognitively Guided Instruction is a professional-development program for mathematics teachers that focuses teachers' attention on their students' mathematical thinking and encourages them to use this knowledge to drive their instructional decisions (Carpenter et al., 2015). Research shows that CGI can improve students' mathematics achievement as well as teachers' mathematics knowledge and instructional practice (Jacobs et al., 2007; Schoen et al., 2018b). Teachers involved in our study were randomly assigned either to receive CGI training (typically eight days of professional development per school year) or to serve as the waitlist control group. We report data from the use of selected OBS items for Florida students in the classrooms of teachers who had participated in the CGI program, but we separate them from those whose teachers had not yet participated in the program. Here, we refer to the former as the Florida CGI group and the latter as the Florida BAU group, because the latter represents "business-as-usual" in Florida. We also note that CGI-trained teachers in our data varied in amount of participation in the program.

Data Sources

Mathematics Performance and Cognition Interview (MPAC)

The MPAC consists of a series of items designed to measure the mathematical thinking and achievement of first- and second-grade students with a focus on number, operations, and equality. MPAC is administered in a one-on-one interview format, where the interviewer poses mathematics problems, observes students solving the problems, and asks students to describe their thought processes when the key details of those processes are not observable. The MPAC was administered to a diverse sample of more than 1,400 students in 22 schools located in two Florida school districts during spring 2014 and spring 2015 (Schoen et al., 2016a, 2016b).

Elementary Mathematics Student Assessment (EMSA)

The EMSA is designed to serve as a mathematics achievement test administered in paper-and-pencil format to elementary students. The Florida data used in the current study were collected from tests administered to grades K–5 students near the end of the school year between spring 2016 and spring 2019. EMSA tests are designed to align with the core content domains described in the CCSSM and the MAFS (Schoen et al., 2021). EMSA test forms include items designed to measure student understanding of =.

Equals-Symbol Items

We chose two OBS items (i.e., $6 + 3 = \square + 4$, $5 + 3 = \square + 4$) from the MPAC and EMSA tests because they are the most similar to those that have been reported in the prior literature. Table 2 shows the total number of students (i.e., Florida BAU and Florida CGI together) and number of schools represented in the data. We note that we have a substantially larger sample size than what was reported for the studies outside Florida (Table 1) at most grade levels. We also provide results for students in Kindergarten, which was not reported in those studies.

Table 2
Total Number of Students Represented in the MPAC (Spring 2014–Spring 2015) and EMSA (Spring 2016–Spring 2019) Data by Grade

Data source	Grade level K	1	2	3	4	5	Total	Schools
Spring 2014 MPAC		332	278				610	22
Spring 2015 MPAC		442	420				862	22
Spring 2016 EMSA	950	1,821	1,764	1,057	744	953	7,289	66
Spring 2017 EMSA	610	1,077	1,021				2,708	77
Spring 2018 EMSA	109	55	37	46			247	3
Spring 2019 EMSA		638	655	690	19		2,002	29
Total	1,669	4,365	4,175	1,793	763	953	13,718	

Data Analysis

To determine the percentage of Florida students answering OBS items correctly, we first disaggregated our data by group (Florida BAU and Florida CGI), then calculated the percentage of correct responses to the two OBS items by grade level for each MPAC and EMSA test available. We then computed the global mean percentage (Mean) for each grade by adding the number of students who responded correctly to the two items across tests divided by the total number of students. This procedure was the same one used to calculate the global mean percentage of correct responses by grade level in Table 1. We also computed standard errors (SE) and 95% confidence intervals (CI) for our Florida BAU and Florida CGI group data and the data from outside Florida (Table 4).

Results

In our Florida BAU group sample, the average percentage of correct responses increased monotonically with grade level. The global mean percentage of correct responses in the early grades (K–2) was 10 percentage points higher than that in studies outside Florida (Table 1), starting with Kindergarten at 14%, followed by first grade at 25% and second grade at 34% (Table 3). Unlike the data in Table 1, however, average percentage of correct responses on these items did not decline in third grade but continued to increase, reaching 73% by fifth grade.

We note an analogous trend across grade levels in the average of correct responses to OBS items for the Florida CGI group. The global mean percentage of correct responses in the early grades (K-2) was approximately 20 percentage points higher than what was reported in Table 1. The mean percentage in Kindergarten was 17%, followed by 35% in first grade and 47% in second grade. Once again, unlike the data in Table 1, average performance on OBS items did not decline in third grade but continued upward, reaching 75% by fifth grade (Table 3).

Finally, we plotted a line of best fit using least-squares regression to aid visualization of the normative trends across grade levels and comparison of the mean performance of the three groups of students in Figure 1. In both cases, Florida elementary students performed better overall on equals-symbol items than did the groups for whom data are reported in Table 1, and average percentage of correct responses on these items appears strongly correlated with grade level for both the Florida BAU group and the Florida CGI group.

Table 3
Percentage of Students Who Responded Correctly to Equals-Symbol Items by Grade Level for Florida BAU, Florida CGI, and Groups Outside Florida

	Grade level						
Data Source	K	1	2	3	4	5	*N*
Florida BAU Group							
Spring 2014 MPAC		9	15				334
Spring 2015 MPAC		13	15				503
Spring 2016 EMSA	10	34	46	62	71	73	3,036
Spring 2017 EMSA		19	66				100
Spring 2018 EMSA	23	40	48	83			247
Spring 2019 EMSA		22	34	47	85		2,002
Mean	14	25	34	55	72	73	
Florida CGI Group							
Spring 2014 MPAC		21	15				276
Spring 2015 MPAC		21	15				359
Spring 2016 EMSA	14	39	46	63	67	75	4,253
Spring 2017 EMSA	21	36	66				2,608
Mean	17	35	47	63	67	75	
Groups Outside Florida							
Falkner et al. (1999)[a]	–	0	6	10	7	7	668
Stephens et al. (2013)	–			2	24	56	290
Bennett (2015)	–		15	35	46	68	1,182
Powell et al. (2016)[b]	–	16	27				1,294
Johannes et al. (2017)[c]	–		50				49
Matthews & Fuchs (2018)[a]	–		33				191
Johannes & Davenport (2019)[b]	–		28				406
Chow & Wehby (2019)[b]	–		24				74
Mean		15	24	19	35	62	

Note. Results for OBS items, rounded to nearest percent. No CGI-group data available for EMSA Spring 2018 and EMSA Spring 2019.
[a]Results for other grade levels are not included in the table.
[b]Percentage available for Spring 2013 and Fall 2012 only.
[c]Randomized controlled trial. Percentage at pretest for control group only.

Table 4

Standard Errors and 95% Confidence Intervals for Outside-Florida, Florida, and CGI Data by Grade

Variable	Outside Florida						Florida group						CGI group					
Grade	K	1	2	3	4	5	K	1	2	3	4	5	K	1	2	3	4	5
n	–	847	1558	545	432	589	355	2012	1663	1277	418	497	1314	2353	2512	516	345	456
Mean	–	.152	.244	.192	.354	.621	.141	.249	.344	.546	.715	.731	.168	.354	.469	.628	.672	.749
SE	–	.012	.011	.017	.023	.020	.018	.010	.012	.014	.022	.020	.010	.010	.010	.021	.025	.020
Lower 95% CI	–	.128	.223	.159	.308	.581	.105	.230	.321	.519	.671	.692	.148	.335	.450	.586	.622	.709
Upper 95% CI	–	.176	.266	.225	.399	.660	.177	.268	.367	.574	.758	.770	.189	.374	.489	.670	.722	.789

Note. Results for OBS items. We converted the global mean percentage of correct responses (Mean) to a sample proportion ($\hat{p}$) to calculate standard errors (e.g., 15.2% = .152).

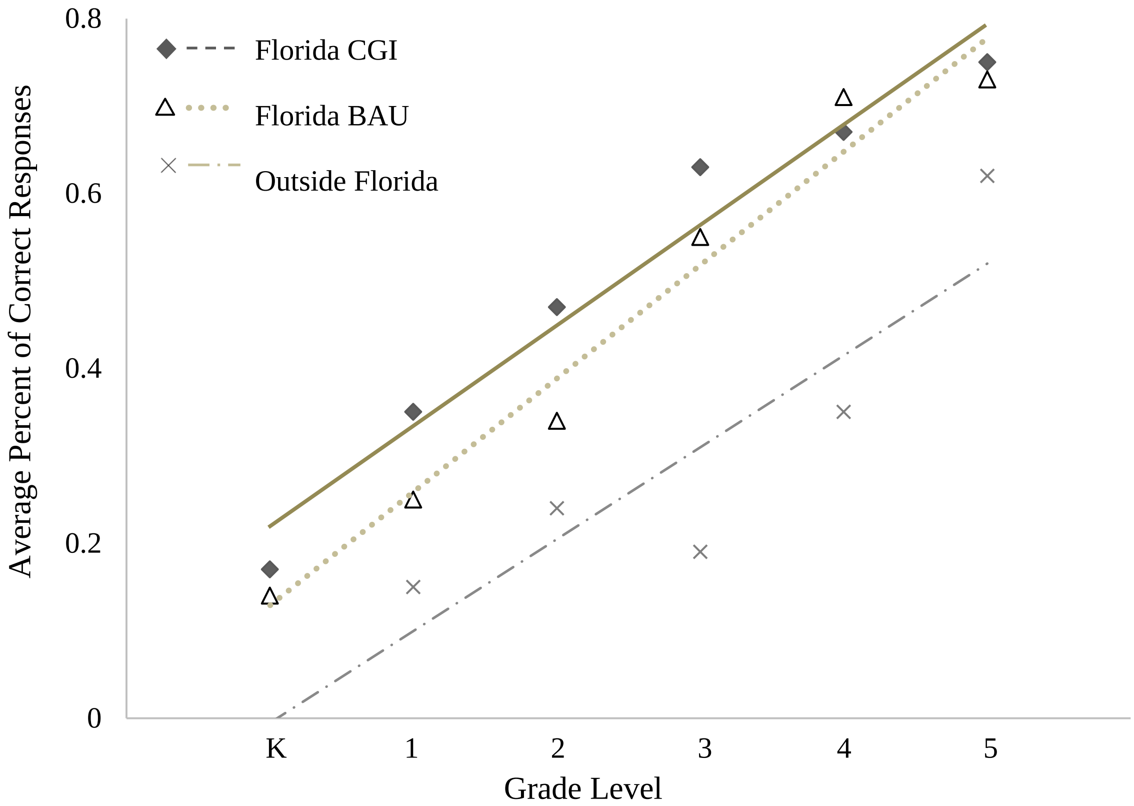
n = total number of students.

SE = standard error: $\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{N}\right)}$.

CI = confidence interval: Mean $\pm$ (1.96 * SE).

Figure 1

Scatterplots for Average Percentage of Correct Responses to Equals-Symbol Items by Grade Level (K–5) for Outside-Florida, Florida BAU, and Florida CGI Data



Note. Results for Operations on Both Sides (OBS) items (e.g., 8 + 4 = ☐ + 5).

Discussion

Together, these results show that elementary students in Florida tend to perform better, on average, on items intended to assess their understanding of = than do their counterparts for whom results have been reported elsewhere in the literature. In our sets of data, the global mean percentage of correct responses in the early grades for Florida students is 10 to 20 percentage points higher than those in studies outside Florida. By third grade, more than half of Florida students in our sample responded correctly to the OBS items—and the CGI group performed even higher. We think it noteworthy that this higher level of performance appears in both the BAU and CGI groups, suggesting that these results are independent of teachers' CGI training. In addition, our findings clearly suggest that elementary students in higher grades outperform those in lower grades on the equals-symbol items—a phenomenon observed in all three groups of data. This results differs from the flatter (or up-and-down) trend shown in Table 1, though we cannot rule out this trend's being the result of the paucity of data available on third grade.

We suggest two plausible explanations of the differences between students' performance in our data and those from outside Florida listed in Table 1. The first is the possibility of publication bias—a well-known phenomenon in in all fields of research (Hopewell et al., 2009; Kühberger et al., 2014). In this case, publication bias could make it more likely for scholars to report results that are shocking or that confirm an ongoing narrative in the corpus of literature regarding elementary students' failure to understand the mathematical meaning of the equals symbol.

A second explanation could be the influence of the CCSSM and MAFS, which specifically address understanding of =. Some states, including Florida, have a state-level process of review of instructional materials and their alignment with the curriculum standards. School districts in Florida are required to spend at least 50% of their budget for instructional materials on resources that are listed as approved. This process provides some amount of assurance that textbooks align with the adopted curriculum standards. The specific references to student understanding of the equals symbol in grade 1 in the CCSSM and then in grades 1, 2, and 4 in the MAFS—and the subsequent updates to curriculum materials that followed—led to positive effects on student performance.

Limitations

We believe one strength of our set of data is its size (Table 2), but as is often the case in scientific literature, our data do not represent a random sample of the general population of elementary students, so we cannot claim that our results generalize to the Florida student population. In addition, we use student responses to OBS items on the EMSA to infer comprehension. We acknowledge that this inference has limitations, and these are also present in previously published findings. Because the MPAC data were obtained in an interview setting (which allows for student feedback), we have more confidence in inferences drawn from them than in those from the EMSA data or the data available from previously published studies that we used in the secondary analysis. We note that student performance on the MPAC does appear to be lower than performance on the EMSA.

Conclusion

We entirely agree with previous research that = is a fundamentally important topic in elementary mathematics and that room remains for improvement in children's understanding of it (Bush & Karp, 2013; Knuth et al., 2006). On the other hand, our results suggest that students' performance on items similar to "8 + 4 = ☐ + 5" is not as poor as the published literature suggests. These results may imply that policies and practices in Florida better support student understanding of = than do those in other states.

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