Analyzing the Word-Problem Performance and Strategies of Students Experiencing Mathematics Difficulty

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Abstract

The purpose of this study was to examine the word-problem performance and strategies utilized by 3rd-grade students experiencing mathematics difficulty (MD). We assessed the efficacy of a word-problem intervention and compared the word-problem performance of students with MD who received intervention (n = 51) to students with MD who received general education classroom word-problem instruction (n = 60). Intervention occurred for 16 weeks, 3 times per week, 30 min per session and focused on helping students understand the schemas of word problems. Results demonstrated that students with MD who received the word-problem intervention outperformed students with MD who received general education classroom word-problem instruction. We also analyzed the word-problem strategies of 30 randomly-selected students from the study to understand how students set-up and solve word problems. Students who received intervention demonstrated more sophisticated word-problem strategies than students who only received general education classroom word-problem instruction. Findings suggest students with MD benefit from use of meta-cognitive strategies and explicit schema instruction to solve word problems.

Key words: learning disability, mathematics, problem solving, word problems

Analyzing the Word-Problem Performance and Strategies

of Students Experiencing Mathematics Difficulty

1. Introduction

In the United States (U.S.), approximately 20% of fourth-grade students fail to meet basic levels of mathematics proficiency with another 39% only exhibiting basic level performance, as determined by the National Assessment of Educational Progress (NAEP; National Center for Education Statistics, 2017). To demonstrate mathematics competency on the NAEP and other measures, students are largely assessed through their performance on solving word problems (Powell, Namkung, & Lin, 2019). Word problems involve interpreting a combination of words and numbers to develop a problem solution (e.g., *Sally has 8 stickers on her backpack. Lily has 12 stickers on her lunchbox. How many more stickers does Lily have?)*, and many students are inadequately prepared to set up and solve such word problems (García, Jiménez, & Hess, 2006).

Only 3% to 8% of school-age students receive a formal identification of a learning disability in mathematics (Devine, Soltész, Nobes, Goswami, & Szücs, 2013; Geary, 2004). Thus, many students experience persistent, low mathematics achievement without an official learning disability diagnosis (Nelson & Powell, 2018). Schools may identify these students as experiencing a mathematics difficulty and may elect to offer support by providing supplemental interventions. In this article, we use the umbrella term *mathematics difficulty (MD)* to describe both students with a specific learning disability in mathematics as federally outlined in the U.S. Individuals with Disabilities Education Act and students who demonstrate persistent, low achievement in mathematics without a formal disability diagnosis.

The focus of the present study was to examine the word-problem performance and strategies utilized by third-grade students with MD, who are at risk for low word-problem outcomes. We assessed the efficacy of a word-problem intervention and compared the word-problem performance of students with MD who received intervention to students with MD who received general education classroom word-problem instruction. Research has assessed the efficacy of word-problem interventions for students with MD (Fuchs et al., 2014; Jitendra et al., 2007) yet few, if any, studies have investigated the specific word-problem strategies utilized by third graders with MD. In this introduction, we summarize prior research on word-problem solving and various approaches to word-problem instruction in the general education classroom. Next, we describe elementary students with MD and their specific word-problem challenges. We highlight word-problem interventions tailored to address the targeted needs of students with MD. Then, we present the purpose and research questions of the study.

1.1 Word-Problem Solving and Instruction in the General Education Classroom

Word-problem proficiency proves critical for helping students connect mathematics to real life and to succeed in school and beyond (Wong & Ho, 2017), yet solving word problems remains one of the more difficult mathematics tasks for students in the elementary grades (Daroczy, Wolska, Meurers, & Nuerk, 2015). Word-problem solving requires students to comprehend text (Boonen, de Koning, Jolles, & van der Schoot, 2016; Fuchs, Fuchs, Compton, Hamlett, & Wang, 2015), as well as access and utilize long-term memory and working memory (Lee, Ng, & Bull, 2018; Schoenfeld, 1992; Thevenot & Oakhill, 2006). Word problems often overwhelm students due to their complexity; students often are required to follow multiple steps to develop a solution.

To best support U.S. students' word-problem understanding, the Institute of Education Sciences outlined several recommendations for general education teachers as they teach word-problem solving, including (a) preparing and using routine and non-routine word problems for whole-class instruction, (b) helping students monitor and reflect on their problem-solving processes, (c) using visual representations to model word problems, (d) exposing students to multiple problem-solving strategies, and (e) helping students use algebraic notation to understand word-problem concepts (Woodward et al., 2018).

In following these recommendations, general and special educators often teach students to solve word problems using a general approach or meta-cognitive strategy (Cornoldi, Carretti, Drusi, & Tencati, 2015; Jacobse & Harskamp, 2009; Özsoy & Ataman, 2009). For example, students may learn to: (1) read the problem; (2) construct a problem representation, including drawing a picture of important relationships; (3) make a plan for solution; (4) complete necessary calculations; (5) interpret and determine the answer; and (6) check the solution (Kajamies, Vauras, & Kinnunen, 2010). A similar approach encourages students to answer the following questions: (1) What information is given? (2) What are you asked to solve? (3) What strategy might help to solve? (4) What steps are needed to solve? (5) What calculations are needed to solve? (6) What is the solution? (Wilson, 2013).

Many general education teachers also instruct students to solve word problems by recognizing a word problem as belonging to a specific schema or situation and using a schema-specific model to determine a problem solution (Brissiaud & Sander, 2010; Carpenter, Hiebert, & Moser, 1981; Koedinger & Nathan, 2004; Verschaffel, Greer, & De Corte, 2000). With a focus on word-problem schemas, students may use a meta-cognitive strategy to: (1) determine the word-problem type (i.e., schema); (2) organize the information into a diagram (underline the label and circle, rectangle, or triangle important information); (3) write a number sentence; and (4) solve the number sentence and check their work (Griffin et al., 2018).

Several commonalities emerge across these word-problem approaches. First, students learn to read and interpret the word problem. In our experience, many students start solving a word problem before reading the problem; these approaches encourage students to first read the word problem. Second, students identify relevant content in the word-problem prompt and organize this information by schema. Third, students use visual or graphic organizers or equations to organize the information from a word problem (Bebout, 1990; van Garderen, Scheuermann, & Poch, 2013; Zahner & Corer, 2010). These general problem-solving approaches prove beneficial by lessening the working memory load activated during word-problem solving and by instructing students to approach problem solving in a consistent and organized manner.

1.2 Students with MD

Students with MD perform significantly lower than their typically developing peers and are at great risk for mathematics failure. For example, 70% of children who perform below the 10th percentile in mathematics at the end of kindergarten receive an identification of a specific learning disability in mathematics by fifth grade (Morgan, Farkas, & Wu, 2009), and over 95% of students with MD in fifth grade continue to demonstrate performance below the 25th percentile in high school (Shalev, Manor, & Gross-Tsur, 2005). Students with MD in the elementary grades experience challenges on measures of mathematics facts, computation, fraction understanding, and word-problem solving (Andersson, 2008; Fuchs, Fuchs, et al., 2008; Fuchs et al., 2013; Mabbott & Bisanz, 2008; Vukovic & Siegel, 2010).

1.2.1 Word problems and students with MD. Word-problem solving, with the complexity of reading the word problem, identifying important information, and solving the problem using mathematics, proves especially difficult for students with MD (Swanson, Orosco, & Lussier, 2014.) As described, word problems often require students to read a key and number a graph, understand the problem situation, build the situation model, determine the needed operation(s) for solving the problem, interpret and evaluate the problem, solve the problem correctly, and add a label corresponding to the number answer (Verschaffel et al. 2000). Without instruction on how to set up and solve word problems, many students attend to superficial cues in the word problem, such as keywords, and add or subtract without interpreting or considering a mathematical model (Van Dooren, De Bock, & Verschaffel, 2010; van Lieshout & Xenidou-Dervou, 2018; Verschaffel et al., 2000).

Students with MD often misuse irrelevant information presented within the word-problem problem (Jarosz & Jaeger, 2019; Wang, Fuchs, & Fuchs, 2016). Students also may select the incorrect operation(s) for solving the word problem and make computational mistakes (Haghverdi, Semnani, & Selfi, 2012; Sharpe, Fults, & Krawec, 2014). Word problems become increasingly challenging for students with MD when multiple steps or operations are required to solve the problem (Boonen et al. 2016). Because of these word-problem challenges, students with MD may require intensive and specialized intervention to address their needs (Gersten et al., 2009; Schumacher, Zumeta Edmonds, & Arden, 2017). Without the support of evidence-based word-problem interventions for students with MD, mathematics performance gaps persist and widen across grade levels (Koponen et al., 2018; Stevens, Schulte, Elliott, Nese, & Tindal, 2015).

1.2.2 Word-problem interventions for students with MD. In special education, several research teams have contributed to a robust literature base of efficacious strategies that demonstrate improved word-problem outcomes for students with MD. Evidence-based word-problem strategies for supporting students with MD include self-regulation (Case, Harris, & Graham, 1992), meta-cognition (Krawec, Huang, Montague, Kressler, & Melia de Alba, 2012), explicit instruction (Swanson et al., 2014), and graphic organizers (van Garderen, 2007), as well as combinations thereof (Flores, Hinton, & Burton, 2016; Fuchs et al., 2014).

Over the past two decades, word-problem intervention research in special education has focused on using schemas to solve word problems (Cook, Collins, Morin, & Riccomini, 2019), described widely in mathematics education (Bebout, 1990; Carpenter et al., 1981; Cummins, Kintsch, Reusser, & Weimer, 1988; De Corte & Verschaffel, 1987; Kintsch & Greeno, 1985), and interpreted for use with students with MD (Fuchs et al., 2014; Jitendra et al., 2007). Developing schemas for categorizing word problems proves beneficial for helping students identify novel problems as belonging to familiar categories (Ng & Lee, 2009). Schemas also greatly influence whether students answer problems correctly (Kintsch & Greeno, 1985) and have been reported as more effective than other techniques for teaching word-problem solving to students with MD (Jitendra et al., 2015; Zhang & Xin, 2012).

With schema word-problem instruction, students learn to identify word problems by schema and apply a specific strategy to the schema. In the early elementary grades, students learn three different schemas: Total, Difference, and Change (Fuchs, Seethaler, et al., 2008; Riley & Greeno, 1988*)*. In Total problems, students combine parts together for a total. In Difference problems, students learn to compare an amount that is greater and an amount that is less to find the difference. In Change problems, students start with an amount and the amount increases or decreases to a new end amount.

1.4 Purpose and Research Questions

In this study, we compared the word-problem performance and strategies of students with MD who received 16 weeks of Pirate Math, a word-problem intervention originally developed by Fuchs and colleagues (Fuchs et al., 2014), to students with MD who received general education classroom word-problem instruction. Our intervention expanded on the original Pirate Math word-problem intervention research (Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs, Zumeta, et al., 2010) with an explicit focus on setting up and solving of equations used to represent word-problem schemas. We named our expanded intervention program Pirate Math Equation Quest (PMEQ). Analyzing the word-problem performance of students with MD offers an important way to recognize the mathematical processes utilized by students and may help educators alleviate future word-problem difficulty (van Lieshout & Xenidou-Dervou, 2018). We asked the following research questions:

(1) What is the word-problem performance growth of students with MD between conditions on double-digit additive word problems?
(2) What are the common word-problem strategies employed by students with MD who received general education classroom word-problem instruction?
(3) What are the common word-problem solving strategies employed by students with MD who received the 16-week word-problem intervention?

2. Method

2.1 Context and Setting

We recruited 13 elementary schools from a large urban school district in the Southwest of the United States that serves over 80,000 students. In 2017, the district reported 55.5% of students as Hispanic, 29.6% as Caucasian, 7.1% as African American, and 7.7% as belonging to another racial or ethnic category. In the district, 27.1% of students qualified as English learners, 12.1% received special education services, and 52.4% qualified as economically disadvantaged.

2.2 Participants

During the 2017-2018 school year, we worked in 51 classes with 44 teachers. Several schools used departmentalization (i.e., the same teacher taught multiple mathematics classes), which accounted for the differences in the numbers of teachers and classes. From these 51 classes, interventionists screened 818 third-grade students. We administered *Single-Digit Word Problems* (Jordan & Hanich, 2000) as a screener for MD in the area of the intervention (i.e., word problems). For study eligibility, we identified 236 students who answered 7 or fewer items correctly (out of 14) as experiencing MD.

Before randomization, interventionists administered individual pretesting across four weeks. During this time, we deemed 77 students as ineligible for intervention for the following reasons: limited English proficiency, disability and receiving other services, relocation to another school, teacher-identified behavior issues, too many students with MD in a classroom, or unable to schedule. We randomly assigned, blocking by classroom, the 159 remaining students to one of three conditions: Pirate Math Equation Quest (PMEQ; n = 60), Pirate Math without Equation Quest (n = 38), and business-as-usual (BAU; n = 61).

2.3 General Education Classroom Word-Problem Instruction

We surveyed all general education classroom teachers about their word-problem instruction and strategies, and teachers reported the following. Of the 51 participating teachers, the overwhelming majority relied on the *GO Math!* curriculum by Houghton Mifflin Harcourt, which utilizes a comprehensive approach to engage students and assist teachers in differentiating instruction through building and reinforcing of foundational skills. Several teachers identified *Investigations in Number, Data, and Space* by Pearson as their primary curriculum. *Investigations* embeds the standards for mathematical practice and aligns with U.S. Common Core standards to promote students' active thinking and exploration of mathematical concepts (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). A few teachers also reported using *Total Motivation Math* by Mentoring Minds, a curriculum directly aligned with the Texas Essential Knowledge and Skills standards.

When asked the question "*How do you teach students word problems?*," most teachers commonly reported using mnemonic devices to assist students in setting up word problems. Many of the mnemonic devices embedded a meta-cognitive strategy to guide students through the problem-solving process. Common mnemonic devices employed by general education classroom teachers included CUBES (i.e., Circle key numbers and units, Underline the question, Box math "action" words, Evaluate and eliminate, and Show your work and check), UPS Check (i.e., Understand, Plan, Solve, and check), and WIK and WINK (i.e., What I know and What I need to know). A few teachers reported using STAR (i.e., Search the word problem; Translate the problem; Answer the problem; and Review the solution).

Notably, only 3 of the 51 teachers reported using schemas to teach students to solve word problems. Instead, teachers relied primarily on keywords to help students identify the problem type. For example, teachers instructed students to underline and circle words like "altogether," "increase," and "total" to identify an addition problem and words like "more than," "decrease, and "left" to indicate a subtraction problem. When asked about additional strategies or prompts to support students' word-problem solving, teachers described using manipulatives, such as strip diagrams, arrays, highlighters, and cubes, as well as the drawing of pictures and models to represent word-problem stories.

Consistent with previous research on effective word-problem strategies, all participating general education teachers utilized a general approach or meta-cognitive strategy to help students organize the word-problem information (Cornoldi et al., 2015; Jacobse & Harskamp, 2009). However, teachers scarcely used schemas to help students set up and solve word problems, which was surprising given that schemas prove more effective than other techniques for teaching word-problem solving (Jitendra et al., 2015; Zhang & Xin, 2012). Rather than helping students to identify word problems as belonging to a specific problem type, teachers explicitly instructed students to tie keywords to operations and directed students to displayed classroom posters with keywords to assist with problem solving, which is an ineffective word-problem strategy not support by research (Karp, Bush, & Dougherty, 2019; Powell et al., 2019).

Students in the BAU condition did not receive supplemental word-problem instruction from our research team. BAU students only participated in general education classroom word-problem instruction. The BAU students provided a lens through which to examine the word-problem instruction provided in the general education classroom to determine how students responded to such instruction.

2.4 Intervention

Pirate Math is an intervention focused on developing student knowledge of the additive schemas; Equation Quest is a pre-algebraic reasoning component designed to help students understand the equal sign as relational and solve different types of equations. Students receiving word-problem intervention in our study either participated in Equation Quest embedded within Pirate Math or participated in a non-Equation Quest activity designed to maintain a comparable length of the intervention sessions. Our preliminary data indicated Pirate Math Equation Quest (PMEQ) provided students with MD with a word-problem advantage over students who participated in Pirate Math without Equation Quest (Powell, Berry, & Barnes, in press); therefore, in this manuscript, we compare the word-problem performance and strategies of students in PMEQ and BAU. We do not report any data for the Pirate Math without Equation Quest condition.

PMEQ included 45 individual sessions, implemented three times a week, with each session lasting about 30 min. PMEQ was supplemental to each student's mathematics program. That is, all PMEQ students participated in general education classroom word-problem instruction *and* received the full PMEQ intervention. BAU students only participated in general education classroom word-problem instruction. Each PMEQ session included the following activities: (1) Math Fact Flashcards, fluency fact practice; (2) Equation Quest, interventionist-led activities about the equal sign, (3) Buccaneer Problems, interventionist-led word-problem practice featuring schema instruction, (4) Shipshape Sorting, schema sorting practice, and (5) Jolly Roger Review, cumulative review.

2.4.1 Math Fact Flashcards. Interventionists started every session with two, 1-min timings involving addition and subtraction fluency fact practice. We included math fact flashcard practice to increase students' addition and subtraction fact knowledge and alleviate working memory difficulty attributed to complex computation in third-grade word problems. Because students with MD benefit from repetitive practice opportunities to successfully understand a concept (Fuchs, Powell, et al., 2010), we included flashcard practice as the warm up activity for every session. During the first minute, students answered addition and subtraction facts presented on flashcards with minuend and subtrahends ranging from 0 to 9. With a correct response, interventionists placed the card in a pile on the desk. When students answered incorrectly, interventionists prompted them to use a counting strategy (i.e., counting up) to arrive at a correct answer; then, placed the card in the correct pile. For counting up with addition, interventionists taught students to start with the greater addend and count on the lesser addend to calculate the sum. For counting up with subtraction, students learned to start with the subtrahend and count up to the minuend to determine the difference. After the initial 1-min timing, the tutor and student counted the number of flashcards answered correctly. Interventionists also provided immediate, corrective feedback by reviewing the counting-up strategy. Prior to starting the second 1-min timing, interventionists challenged students to beat their previous scores. At the end of the second minute, students graphed the highest score from the two trials. The graph served as a self-regulation tool for setting future goals and monitoring students' progress (Montague, 2007). Figure 1 shows an example of the intervention materials used during PMEQ.

2.4.2 Equation Quest. During the second activity of each intervention session, which lasted 2 to 5 min, interventionists provided instruction on solving equations and the meaning of the equal sign through an activity called Equation Quest. Interventionists encouraged students to think about the equal sign as a relational symbol rather than as a symbol solely signaling a calculation. Students learned to interpret the equal sign as meaning "the same as" and read equations with that language: "Four plus seven is the same as eleven." To understand the equal sign as a relational symbol, students first solved standard (e.g., 4 + __ = 13 or __ – 40 = 55) equations with manipulatives (e.g., balance scale and blocks). For example, for 4 + __ = 13, a student placed 4 red blocks on the left side of a balance scale and 13 yellow blocks on the right side of a balance scale. Then, the interventionists asked students to determine how many blue blocks needed to be added to the left side of the scale to balance the scale or to make the sides of the equation the same. After practicing with standard equations, students also used manipulatives to solve nonstandard equations (e.g., 15 = 8 + __ or 10 – __ = 8 – 2). In addition to using manipulatives, during some sessions, students drew pictures to balance equations (see Figure 1 for an example worksheet from a session in which students drew pictures). As the intervention sessions progressed, students learned to solve equations presented with numbers and symbols using variables (e.g., "X") to represent missing numbers. Students learned to draw a vertical line down from the equal sign to separate two sides of the equation. Interventionists helped students learn how to isolate the variable (X) by removing the constant from the side of the equation with the variable. Equation Quest followed the concrete-representational-abstract (C-R-A) framework in special education (Miller & Hudson, 2006) or the enactive-iconic-symbolic framework in mathematics education (Bruner, 1966) to teach students with MD about the equal sign. The combination of standard and nonstandard equations assisted students with MD to develop better pre-algebraic reasoning (McNeil & Alibali, 2005; Powell et al., in press).

2.4.3 Buccaneer Problems. During the third activity of each PMEQ session, the tutor provided scaffolded instruction to set up and solve additive word problems. For every word problem, the tutor utilized a meta-cognitive attack strategy to aid students in thinking through the problem. For students with MD who present with lower word-problem performance, meta-cognitive strategies prove useful for organizing processes of setting up and solving word problems (Montague, Enders, & Dietz, 2011). Using the meta-cognitive strategy of a mnemonic device, students first checked whether the word problem included a graph or table. If the problem included a graph or table, students numbered the graph or table for easier access to the numbers. Second, students learned to RUN through a word problem by Reading the problem, Underlining the label and crossing out the irrelevant information, and Naming the problem type. Interventionists used the term *problem type* as a substitute for *schema.*

Our intervention focused on the three additive word-problem schemas identified by Carpenter et al. (1981) and Kintsch and Greeno (1985), among others. To help students remember the names of the schemas, we used the terms Total, Difference, and Change to describe the three problem types. The renaming of the schemas, led by Fuchs, Seethaler, et al. (2008), used words that did not all begin with the letter C (i.e., Combine, Compare, Change), which may have confused students with MD. Total, Difference, and Change also allowed students to use T, D, and C for identifying or labeling word problems by schema and to make the letters T, D, or C with their hands to explain a word problem's schema.

2.4.3.1 Total problems. Interventionists introduced the Total schema, also known as Part-Part-Whole and Combine problems (Carpenter, Fennema, Franke, Levi, & Empson., 2015), on Day 5 of the intervention. In Total problems, parts are put together for a total. The missing information (i.e., "X") may be the total or one of the parts. After checking for a table or a graph and RUNning through the problem, students used five steps to solve a Total problem: (1) Write P1 + P2 = T (i.e., Part 1 + Part 2 = the Total), (2) Find T, (3) Find P1 and P2, (4) Write the signs, and (5) Find X. For Total problems with more than two parts, interventionists taught students to expand the Total equation (i.e., P1 + P2 + P3 = T).

2.4.3.2 Difference problems. Interventionists introduced the Difference schema, also called Compare problems (Carpenter et al., 2015) on Day 17 of the intervention. In Difference problems, students learned to compare an amount that was greater and an amount that was less to find the difference between the two amounts. The missing information (i.e., "X") for Difference problems may be the amount that is greater, the amount that is less, or the difference. Interventionists taught students to focus on the compare sentence within each Difference word problem. A compare sentence featured a compare word (e.g., *more*, *less*, or *fewer*, or other words like *older*, *shorter*, or *faster*). Students identified the compare word, interpreted the compare sentence to determine which quantities were greater and less, and identified if the difference was given or missing. Students used six steps to solve a Difference problem: (1) Write G – L = D (i.e., Amount that is greater – Amount that is less = Difference), (2) Put brackets around the compare sentence and label G and L, (3) Find D, (4) Find G and L, (5) Write the signs, and (6) Find X.

***2.4.3.3 Change problems*.** Interventionists introduced Change problems, also known as Join and Separate problems (Carpenter et al., 2015), on Day 34 of intervention. In Change problems, there is a starting amount, then at a later time something happens to increase or decrease the starting amount, so the ending amount is changed. The missing information (i.e., "X") may be the starting amount, the change amount, or the end amount. Interventionists taught students six steps to solve a Change problem are: (1) Write ST +/– C = E (i.e., Start amount +/– Change amount = End amount), (2) Find ST, (3) Find C, (4) Find E, (5) Write the signs, and (6) Find X. When there was more than one change within the problem, students learned to expand the equation to reflect the information in the problem (ST + C – C = E).

2.4.3.4 Identifying the word problem's schema. Interventionists applied two types of prompts to help students determine the schema of each word problem. First, interventionists used verbally-presented questions. To determine if a problem was a Total problem, interventionists asked: *Are two or more parts being put together for a total?* For Difference problems, interventionists asked: *Are two amounts being compared for a difference?* For Change problems, interventionists asked: *Is there a starting amount that increases or decreases to a new amount?* Second, interventionists used hand gestures to accompany each of the questions. For Total problems, interventionists started with two hands apart and brought both hands together to indicate two parts coming together. For Difference problems, interventionists held two hands apart and moved their hands back and forth to show the difference between the hands. For Change problems, interventionists held out one hand in front of the forehead and moved the hand up (for a Change increase) or down (for a Change decrease).

2.4.3.5 Equations to represent the word problem's schema. With the pirate theme embedded within the PMEQ intervention, students used an X to represent missing information.

For every word problem, students learned to write an equation to represent the word problem's schema. For example, for a Change decrease problem with the change missing, a student might write 58 – X = 19. The content about the equal sign and solving equations learned during Equation Quest guided students in solving such equations.

2.4.4 Shipshape Sorting. The fourth activity within PMEQ, Shipshape Sorting, allowed students to practice identifying word-problem schemas learned during the Buccaneer problems. We included this timed activity to help students with MD focus exclusively on word-problem schemas without the additional task of solving problems (Fuchs et al., 2013). Beginning on Day 7 of intervention, interventionists read aloud word problems printed on cards during a 1-min timing. Before the sorting activity began, a mat with four squares was placed in front of students. Each square was labeled with a word problem type letter (i.e., T for Total, D for Difference, or C for Change) or the question mark symbol. Interventionists reviewed the three word-problem schemas and explained the directions to place each word-problem card on the square with the corresponding problem type letter (i.e., T, D, or C). If students were uncertain about the problem type, they could place the card on the square with the question mark symbol. Interventionists reminded students to sort the word-problem cards and to not solve any of the word problems. Interventionists then set the timer for 1 min and read the first word-problem card aloud before handing it to the students. Interventionists waited for students to place the card on the mat before reading the next word-problem card. After 1 min, interventionists provided immediate, corrective feedback by reviewing at least three of the word-problem cards. During the review, students were required to explain why a problem belonged to a specific problem type.

2.4.5 Jolly Roger Review. The final activity during each intervention session, the Jolly Roger Review, included a brief, timed paper-and-pencil review of the session content. Because low-stakes practice testing outside of the classroom has improved test scores for students with MD across grade and achievement levels (Burns, Ysseldyke, Nelson, & Kanive, 2015), the Jolly Roger Review served as an independent practice activity that incorporated components of low-stakes practice testing. Practice testing directly improves test-taking ability by providing students exposure to the test-taking environment and indirectly by promoting knowledge of the material (Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013). Low-stakes practice testing also proves especially beneficial for students with MD when provided in conjunction with immediate feedback (Dunlosky et al., 2013).

On the front side of the Jolly Roger Review, students answered up to nine computational math problems (i.e., single and double-digit addition and/or subtraction problems) or wrote appropriate equations for the three word-problem schemas (e.g., Total equation: P1 + P2 = T). Students were timed for 1 min. On the back of the Jolly Roger Review, students completed a word problem using the appropriate schema steps learned during the Buccaneer Problem activity. Students were timed for 2 min. Students performed the timed review autonomously and then received content-rich feedback from the interventionists, which reinforced mastered content.

2.4.6 Interventionists. We recruited 15 interventionists to conduct the pretesting, intervention, and posttesting. All interventionists were pursuing or had obtained a Master's or doctoral degree in an education-related field. Of the 15 interventionists, 100% were female (*n* = 15), and 73% of the interventionists identified as Caucasian (*n* = 11), 13% as Hispanic (*n* = 2), 7% as Indian American (*n* = 1), and 7% as African American (*n* = 1). Throughout the year, interventionists participated in trainings to ensure strong preparation for all intervention components. In late August and early September, interventionists participated in three, 3-hr pretesting trainings. In early October, the team participated in two, 1.5-hr trainings about the content of the intervention and Total problems. Two subsequent 1.5-hr trainings followed in November to introduce Difference problems and in January to introduce Change problems. Lastly, interventionists participated in one, 1.5-hr posttesting training meeting.

2.5 Fidelity of Implementation

We collected fidelity of implementation in several ways. First, for pretesting and posttesting, the interventionists recorded all testing sessions. We randomly selected >20% of audio recordings for analysis and measured fidelity to testing procedures against detailed fidelity checklists. We measured pretesting fidelity at 99.0% and posttesting fidelity at 99.6%. Second, we measured fidelity of implementation of the interventions. We conducted in-person fidelity observations once every three weeks for every tutor. We also measured fidelity of intervention implementation through analysis of >20% of audio-recorded sessions. Fidelity averaged 98% (SD = 0.041) for in-person supervisory observations and 98% (SD = 0.038) for audio-recorded intervention sessions. Third, all 15 interventionists tracked the number of sessions for their PMEQ students. We designed the intervention for students to finish at least 45 sessions with a maximum number of sessions at 51.

2.6 Measures

We used *Single-Digit Word Problems* as the primary measure for identifying students with MD (Jordan & Hanich, 2000). *Single-Digit Word Problems* included 14 one-step word problems involving sums or minuends of 9 or less categorized into the Total, Difference, and Change schemas (α = .89). We also administered three measures of double-digit word problems: *Texas Word Problems-Brief, Texas Word Problems-Part 1,* and *Texas Word Problems-Part 2* (Powell & Berry, 2015). *Texas Word Problems-Brief* included eight word problems requiring double-digit computation, with one Total, three Difference, and four Change problems, respectively. Cronbach's α was .85. We also administered *Texas Word Problems-Part 1*. Students solved nine double-digit word problems: two Total problems, one Difference problem, four Change problems, and two multi-schema problems (i.e., Difference and Change; Total and Difference). Two problems featured the interpretation of graphs (α = .81). Finally, interventionists administered *Texas Word Problems-Part 2*. Students solved nine double-digit word problems: two Total problems, two Difference problems, three Change problems, one multi-schema problem (i.e., Total and Change), and one multiplicative problem (i.e., Equal Groups schema). Three problems featured the interpretation of graphs and one problem included irrelevant information (α = .81).

Two interventionists independently entered scores on 100% on the test protocols for each outcome measure on an item-by-item basis into an electronic database, resulting in two separate databases. We compared the discrepancies between the two databases across each outcome measure and rectified them to reflect the original response. Two interventionists and the Project Manager resolved all discrepancies. Then, we converted students' responses to correct (1) and incorrect (0) scores using spreadsheet commands, which ensured 100% accuracy of scoring. Original scoring reliability was 99.8% for pretesting and 99.7% for posttesting.

2.7 Procedure

During the first week of September, we administered whole-class pretesting in one, 55-min session. Identification of students with MD occurred shortly thereafter, with four weeks of individual pretesting during the last two weeks of September and the first two weeks of October. During the third week of October, approximately 4 to 6 days after pretesting, intervention began and occurred three times per week for 16 weeks, concluding the third week in March. Students in PMEQ received supplemental intervention for approximately 90 min each week. We did not schedule PMEQ sessions to occur during general education classroom mathematics instruction. Therefore, PMEQ students received the same amount of general education mathematics instruction as students in the BAU with additional time for the PMEQ intervention. Approximately 4 to 6 days after the last intervention session, posttesting occurred in five, 45-min small group sessions with four students or fewer. We administered posttesting across three weeks, beginning the last week of March and ending the second week of April. We pre- and posttested all BAU students in the same time frame as the PMEQ students.

2.8 Data Analysis

For data analysis, we created a *double-digit word problems* score by combining *Texas Word Problems-Brief*, *Texas Word Problem-Part 1,* and *Texas Word Problems-Part 2* (maximum score = 52). We calculated Cronbach's α at .92. We used ANOVAs to identify differences among conditions at pretest and posttest. Then, we conducted a post-hoc pairwise comparison with a Bonferroni correction to examine differences between conditions at posttest. We calculated effect sizes (ES) using Cohen's *d* by subtracting unadjusted means and dividing by the pooled standard deviation.

To investigate the word-problem strategies of students in the BAU (i.e., general education classroom word-problem instruction) versus PMEQ (i.e., general education classroom word-problem instruction plus supplemental word-problem intervention), we randomly selected 15 PMEQ and 15 BAU posttests and evaluated students' written work for solving three different word problems: (1) A Total problem with a part as the unknown; (2) a Difference problem with the difference as the unknown and data presented in a graph; (3) a Change problem with the end amount as the unknown and irrelevant information in the problem's text. We only analyzed three problems because of the complexity of analyzing and describing the word-problem approaches of students. We selected these three specific problems to represent the three additive schemas and because of their higher-level of difficulty (i.e., part unknown in Total problem; retrieving information from graph; irrelevant information). We analyzed how students marked the word-problem prompt and solved the problem using their written work. This information provided cues about how students approached and interpreted the problem. As part of the test directions, the examiners asked students to circle their answers, which explains why many students circled the number in their written work.

3. Results and Discussion

From the start of intervention through posttesting, nine PMEQ students and one BAU student left the study because of moving, extreme behavioral challenges, 30-day suspension, and protective custody due to abuse in the home. The resulting attrition rate was 15.0% for PMEQ and 1.7% for BAU. Table 1 presents the demographic information for the students who completed posttesting. At pretest, we calculated the average age of students as follows: 8 years, 9 months for PMEQ and 8 years, 8 months for BAU.

3.1 Word-Problem Performance Growth

Students in the BAU demonstrated word-problem performance growth of 3.91 points from pre- to posttest on *double-digit word problems.* When provided with general word-problem instruction from the general education teacher, students demonstrated some improvement across the school year. Students participating in PMEQ, however, demonstrated a word-problem performance growth of 20.90 points from pre- to posttest on *double-digit word problems.*

We compared the word-problem performance of the PMEQ students to that of the BAU students. At pretest, and as expected given random assignment, we detected no significant differences between PMEQ and BAU students on *Single-Digit Word Problems* (p = .427) or *double-digit word problems* (p = .114). We did not administer *Single-Digit Word Problems* at posttest. At posttest on *double-digit word problems*, however, we identified a significant difference between PMEQ and BAU students, $F(1, 110) = 120.846$, $p < .001$. We calculated Cohen's d at 2.04.

3.2 Strategies of BAU Students Receiving General Education Classroom Word-Problem Instruction

We randomly selected 15 BAU students and analyzed the word-problem strategies they utilized to solve three additive problems. We describe their strategies in the following three sections (see Figure 2 for examples). As we exclusively analyzed students' written work, all descriptions and statements about students' thinking reflect our own hypotheses.

3.2.1 Total problem. Students solved the following Total problem, "*Donna and Natasha folded 96 paper cranes. Donna folded 25 paper cranes. How many paper cranes did Natasha fold?*" We analyzed students' written work at posttest, at the end of their third-grade year after they participated in general education classroom word-problem instruction. Only two students (13.3%) solved the Total problem correctly. One of these students circled the numbers, underlined important information, and generated an equation for problem solution (see A in Figure 2), whereas the other student (not pictured) merely wrote an equation below the problem. These two students subtracted 25 from 96 to identify the number of paper cranes Natasha folded.

Two BAU students provided evidence of marking the word problem without a correct solution. For example, one student circled the numbers and word-problem label and underlined the sentence with the question, and then added 96 plus 25. Adding together the numbers presented in a word problem is a common strategy for students (Brissiaud, 1994). We hypothesized students developed this strategy (i.e., adding all presented numbers) because the first word problems they solved in kindergarten or first grade exclusively required adding two numbers together. Another student underlined the numbers and word-problem label as well as the question sentence; this student, however, provided no additional word-problem work. This student demonstrated a process for underlining important information and the question sentence, yet we hypothesized he or she did not know how to use the information in the word problem. Marking the word problem was easy; identifying the important information and correctly solving the word problem was difficult.

A total of seven BAU students did not mark the word problem at all; instead, these students wrote an equation of 96 + 25 to solve the problem (see B in Figure 2 for an example). Again, we noted that 8 of the 15 BAU students added the two provided numbers together, which indicated an incorrect interpretation of the word-problem question. Importantly, all students answered 96 + 25 correctly, which demonstrated adequate computation (with regrouping) skill for many of these students. This finding was positive as many students with MD often experience difficulty with computation (Tolar et al., 2016). One student (example B in Figure 2) exhibited a process of checking his or her work, as indicated by the drawing of the checkmark next to the answer. We hypothesized this student's teacher encouraged her class to check their work after solving a problem. We questioned, however, whether this student actually checked his or her work or merely drew a checkmark.

Four other BAU students did not mark-up the word-problem prompt or write an equation to set up the problem to find a solution. Each of these four students provided a unique incorrect answer. For example, students provided responses of "43" (see C in Figure 2) and "20" (see D in Figure 2). Another student drew tally marks not corresponding with their final response of "95" (see E in Figure 2), and we noted the student did not draw all tally marks correctly (i.e., in sets of

5). One student provided a semi-correct word-problem label (“peper fold”) and repeated one of the numbers presented in the problem (see F in Figure 2). We could not hypothesize how students arrived at these responses, but these results were unsurprising given that students with MD frequently make errors with no visual clues to indicate how they arrived at an incorrect response (Knifong & Holton, 1976).

3.2.2 Difference problem. The Difference problem read as follows, “*The graph shows the favorite subject of third-grade students. How many more students chose Math than chose Writing?*” Of the 15 BAU students, only one student (6.7%) answered the Difference problem correctly (see G in Figure 2). This student numbered the graph, circled important information in the graph, and circled information about determining “how many more.” The student then wrote an equation and solved the equation correctly. This student learned or individually developed effective word-problem strategies. However, the student demonstrated a novice skill with place value by retaining the “0” in the response of “09,” but this would likely fade with additional computation practice.

Another BAU student also numbered the graph and circled (and underlined) the sentence with the question. This student, like the student in G of Figure 2, demonstrated a sophisticated word-problem process and interpretation of the word-problem question. Although this student wrote an equation, he or she used an incorrect number (12 instead of 13) based on an incorrect reading of the graph (see H in Figure 2).

We noted several BAU students marking or numbering the graph (see I in Figure 2 for an example). Two of these students wrote equations but added incorrect amounts of 22 + 13 or 14 + 13. Another student colored in the bar graph for the information about Science and Writing to round 13 to 14 and 15 to 16. The work samples indicated these students have received instruction and practiced interpreting charts and graphs with their classroom teacher. This labeling of the graph, however, only served as a beginning step in the word-problem solving process. Without a clear interpretation of the question of the word problem, students did not solve the problem correctly.

We noted one BAU student underlined the question sentence and provided a written response (see J in Figure 2 for the written response). We have observed classroom teachers directing students to "underline the important information." This student likely received this instruction by his classroom teacher but developed limited skill in providing a response to the underlined question.

The remaining eight BAU students did not mark on the graph or the word-problem prompt; all of these students answered the problem incorrectly. These student work samples represented a diverse set of incorrect responses. Some students added (see K in Figure 2 for an example), and one student used tally marks to arrive at a response of "60" (see L in Figure 2). Two students subtracted amounts but subtracted incorrect amounts (see M and N in Figure 2). This collection of 8 unique incorrect responses demonstrated the level of difficulty for solving word problems for third-grade students with MD.

3.2.3 Change problem. The Change problem read as follows, "*Last year, there were 11 trumpet players in the band. This year, 14 new trumpet players and 4 tuba players joined the band. How many trumpet players are in the band now?"* We identified four BAU students (26.7%) who solved the Change problem correctly. One student (see O in Figure 2) circled important information and underlined the word-problem question. All other students only wrote equations to solve the problem (see P in Figure 2) or wrote an answer (see Q in Figure 2).

However, all of these students ignored the irrelevant information of *4 tuba players*, which confused many other students.

Two BAU students circled or underlined important information, including the entire question sentence (see Figure R in Figure 2 for an example). This student, like student S in Figure 2 and three others, added all three provided numbers together (11 + 14 + 4). These students did not recognize the question of the word problem asked about *trumpet players* instead of all players, a common mistake in word-problem solving (Wang et al., 2016).

The other BAU students provided responses similar to student work displayed in T, U, and V of Figure 2. Three responses used some of the numbers presented in the problem, and in all three cases, students appeared distracted by the irrelevant information about the *4 tuba players*. We hypothesized that students worked quickly, did not read the problem (at all), or failed to focus on the details and the question of the word problem. We could not interpret the answer of "44," which showed no clear process of adding or subtracting two or three of the numbers in the problem.

3.3 Strategies of PMEQ Students Receiving Word-Problem Intervention

Similar to the BAU students, we randomly selected 15 PMEQ students and analyzed their performance on three word problems. Figure 3 provides examples.

3.3.1 Total problem. Of the 15 PMEQ students, 10 (66.7%) solved the Total problem correctly. Of the 10 students with correct responses, eight interpreted this problem as a Total problem (for an example, see A of Figure 3). During PMEQ, students learned to underline the label (i.e., what the problem is mostly about). We asked students to underline the label instead of the entire question sentence to ensure students identified the focus term(s) in the prompt. Students identified *paper cranes* as the label. Students also generated a Total equation (P1 + P2

= T) and used this equation to appropriately organize the total (96) and one of the parts (25); students solved for the unknown part. Students also showed evidence of pre-algebraic reasoning by identifying the variable and circling X, and isolating X by subtracting the constant of 25 from both sides of the equation.

Two PMEQ students who provided correct responses interpreted the problem as a Difference problem (see B in Figure 3 for an example). As students worked through the PMEQ sessions, students could use other schemas to solve a word problem if they could provide a verbal explanation as to why they thought another schema applied. Students may have applied the Difference schema because they realized early in the problem-solving process that they wanted to subtract to solve the word problem. In many cases, students were able to use another schema to solve a word problem. Encouraging students to explore and explain their reasoning served as an important component of PMEQ.

Five PMEQ students answered the Total problem incorrectly, but of these students, four provided evidence as interpreting the problem as a Total problem. Notably, these students identified the correct schema; however, students interpreted 96 and 25 at the parts in the Total problem rather than recognizing 96 represented the total and 25 one of the parts (see C in Figure 3 for an example). Another student added 96 + 25 and provided an incorrect sum of 131 (see D in Figure 3). We assumed this student correctly identified this problem with the Total schema but set up and solved the problem incorrectly. Because identifying a word problem's schema proves a difficult task, we expressed excitement about the correct schema identification. These findings also suggested more schema-related discussions and practice would be necessary to ensure students understand the word-problem question.

3.3.4 Difference problem. Of the 15 PMEQ work samples, eight students (53.3%)

solved the Difference problem correctly. See E of Figure 3 for a correct example. Students labeled the graph and crossed out the irrelevant information (i.e., Reading and Science). In PMEQ, we encouraged students to label any graph or chart before reading a word problem to alleviate any working memory difficulty once students started solving the word problem. Initial numbering of a graph allowed students to easily reference the labeled numbers from the graph rather than disrupting the word-problem process by pausing to determine how many students voted for Math or Writing, for example. This student exemplar (see I) circled and underlined *more students* in the question sentence to assist in identifying the problem as belonging to the Difference schema and determining the missing amount as the difference.

In F and G of Figure 3, PMEQ students made computational mistakes (see F) or selected an incorrect subject (i.e., Reading instead of Writing, see G), which led to incorrect responses. These students recognized the Difference schema, yet responses indicated further discussion and practice would be warranted to understand how to subtract or how to identify the important information.

Four PMEQ students identified the problem as a Total problem and added 22 + 13 (see H in Figure 3 for an example). One student (see I in Figure 3) solved the problem as a Change problem, also by adding 22 + 13. It was difficult to determine why or how these students interpreted the problem as a Total or Change problem. Perhaps students interpreted the word *more* as needing to add the numbers together, which is a common mistake associated with overused keywords (Powell & Fuchs, 2018) and a strategy many of our project's classroom teachers used in general education classroom word-problem instruction. The word-problem strategies employed by these students were sound, yet these students would require more time to develop knowledge related to the word problem's schema.

3.3.6 Change problem. We identified seven of the PMEQ students (46.7%) as solving the Change problem correctly (see J in Figure 3 for a correct example.) The written work of this sample student demonstrated that students used a similar word-problem approach as used with the Total and Difference problems. That is, students underlined the word-problem label (*trumpet players*) and crossed out the irrelevant information (*4 tuba players*). Students identified the Change schema, wrote a Change equation, and then organized the numbers from the word problem based on start amount, change amount, and end amount. Students used X to mark the unknown and then solved for X.

We identified one of the seven PMEQ students with a correct response interpreted the Change problem as a Total problem (see K in Figure 3). As with the prior examples in which students used a different schema to solve the problem, we encouraged the use of another schema *if* the student could explain *why* he or she thought this problem was a Total problem.

Two PMEQ students identified the problem as a Change problem with multiple changes (see L in Figure 3 for an example), leading to an incorrect solution. In PMEQ, students learned of two situations in which three numbers may be used within a schema: a Total problem with three or more parts or a Change problem with two or more changes. These students correctly identified the Change schema yet incorrectly interpreted this problem (with three numbers in the word-problem prompt) as a Change problem with multiple changes. These results suggested more discussion and practice related to identifying irrelevant information would be needed for students to successfully solve the word problem.

Four PMEQ students solved the Change problem with irrelevant information as a three-part Total problem (see M in Figure 3 for an example), and two students solved the problem as a Difference problem (see N in Figure 3 for an example). We hypothesized the four students with an interpretation of this problem as Total focused on the three numbers presented in the word problem and applied the Total schema with three parts. These students would require a deeper-level discussion related to schema questioning (i.e., Are parts put together for a total? Are two amounts compared for a difference? Is there a starting amount that increases or decreases to a new amount?) to conceptually understand the word problem. The two students who interpreted this problem as a Difference problem would also need additional schema practice. These students would require more time and explicit schema instruction to develop knowledge related to the word problem's schema.

4. Summary

Our BAU students with MD participated in general education classroom word-problem instruction. As reported by their teachers, this involved a large focus on the use of mnemonic meta-cognitive strategies (e.g., CUBES, UPSCheck, WIK and WINK, STAR), which has a strong evidence base (Cornoldi et al., 2015; Woodward et al., 2018). Teachers also reported using keywords to solve word problems, a strategy without any evidence (Powell et al., 2019), and visuals (e.g., strip diagrams) to represent word problems, a strategy suggested by Woodward et al. (2018) as helpful in problem solving. Very few teachers reported a focus on word-problem schemas, a strategy with a strong research base, especially for students with MD (Fuchs et al., 2014; Verschaffel et al., 2000).

In our examination of the word-problem work of BAU students, we did not see evidence of use of meta-cognitive strategies. We hypothesized some of the BAU students employed a meta-cognitive strategy without providing evidence (e.g., a box or quadrant drawn to delineate the steps of the meta-cognitive strategy) of use. Only student interviews could provide information about whether students used a meta-cognitive strategy and which strategies they used. We conjectured some BAU students did not use a meta-cognitive strategy, especially students who did not provide a word-problem answer or students who wrote answers disconnected from the word-problem prompt.

Two of the three word problems featured keywords (e.g., *more* in the Difference problem and *joined* and *now* in the Change problem). We identified a common BAU mistake with both of these problems was to add numbers together. Students could have been using keywords and tying the keywords to a specific operation (e.g., *more* means to add) or they could have reverted to the commonly-used strategy of adding all numbers together to solve a word problem without attention paid to any keywords (Brissiaud, 1994). Of the BAU students, we noted no drawings or visuals, even though classroom teachers reported using visuals in general education classroom word-problem instruction.

The success rate of the BAU students on word problems was low. Two students answered the Total problem correctly, one student solved the Difference problem correctly, and four students solved the Change problem correctly. Because students in the U.S. demonstrate their mathematics competency on high-stakes tests with word problems, we were distressed by these low success rates. The current general education classroom word-problem instruction for these BAU students was not enough; these students require teachers to provide more targeted and elaborated word-problem instruction.

For the students with MD who received supplemental word-problem intervention (PMEQ), we noted greatly improved word-problem outcomes. That is, 10 students solved the Total problem correctly, 8 students solved the Difference problem correctly, and 7 students solved the Change problem correctly. In several cases, students who answered incorrectly used the correct schema but used incorrect numbers or added or subtracted incorrectly; therefore, we considered these students on the way to becoming successful word-problem solvers.

Almost all PMEQ students relied on word-problem schemas to aid with problem solution. See D in Figure 3 for an example of no schema use (and note the incorrect response). Our focus on schemas, which relied on literature from both general education (Carpenter et al., 1981; Carpenter et al., 2015; Van de Walle, Karp, & Bay-Williams, 2013; Verschaffel et al., 2000) and special education (Flores et al., 2016; Fuchs et al., 2014; Jitendra et al., 2015; Zhang & Xin, 2012), benefitted students. We explicitly taught students about the three additive schemas, and we realize explicit schema instruction may not be the approach favored in general education (Carpenter et al., 2015). For students with MD who often necessitate explicit modeling on mathematics concepts and procedures (Gersten et al., 2009), however, this explicit approach to teaching the schemas when combined with a meta-cognitive strategy led to much improved word-problem performance.

The PMEQ intervention combined many evidence-based practices described by Woodward et al. (2018) as necessary for word-problem instruction. That is, students learned to monitor and reflect using the meta-cognitive attack strategy of RUN (Reading the problem, Underlining the label and crossing out the irrelevant information, and Naming the problem type). PMEQ interventionists used visuals to show the differences among the three word-problem schemas and taught students how to use algebraic notation to represent word problems. Students also learned how to solve algebraic equations. In our analysis of students' written work, we noted many examples in which students used algebraic notation to represent the word problem and algebraic strategies for solving equations. PMEQ students also learned how to set-up and solve word problems based on schemas. The PMEQ intervention, however, only focused on routine word problems. Further research should investigate whether the PMEQ strategies translate to non-routine word problems.

Before concluding, we note several limitations to our study. First, we did not analyze the performance of students without MD. Such an analysis, especially if we compared students with MD to students without MD, could describe similarities and differences in the word-problem processes of a wide range of students. Second, the PMEQ students received PMEQ sessions as a supplement to the general education classroom mathematics instruction. Therefore, the PMEQ students had more time to practice word problems with an interventionist than students in the BAU. Third, we only analyzed the written work of students. We did not ask students to explain their thinking or provide justifications for using a specific schema or strategy. Future research should consider structured interviews and written assessments requiring students to explain their thinking orally and in writing to provide a more in-depth analysis of the word-problem approaches of students with MD.

In conclusion, our students with MD experienced difficulty solving word problems. Without supplemental word-problem intervention, general education classroom word-problem instruction did not meet the needs of students with MD and left these students with weak word-problem performance. After receiving supplemental intervention focused on combining a meta-cognitive strategy with schema and equations instruction, students with MD demonstrated greater conceptual and procedural knowledge for solving word problems. Not all classroom students may require such word-problem intervention, but it is clear that students with MD benefitted from focused word-problem efforts.

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Table 1

Demographic Information and Descriptive Data

Variable	PMEQ (n = 51) *n*	%	BAU (n = 60) *n*	%
Female	32	62.7	35	58.3
Race				
African American	10	19.6	8	13.3
Asian American	0	0.0	2	3.3
Caucasian	3	5.9	1	1.7
Hispanic	31	60.8	42	70.0
Multi-racial	5	9.8	5	8.3
Other	2	3.9	0	0.0
School-identified disability	8	15.7	4	6.7
Dual-language learner	31	60.8	37	61.7
Retained	4	7.8	8	13.3
	M	*SD*	*M*	*SD*
Single-Digit Word Problems				
Pretest	4.71	1.74	4.97	1.72
Double-digit word problems				
Pretest	7.71	5.50	6.11	5.06
Posttest	28.61	10.95	10.02	6.76

Note. PMEQ = Pirate Math Equation Quest; BAU = business-as-usual comparison.

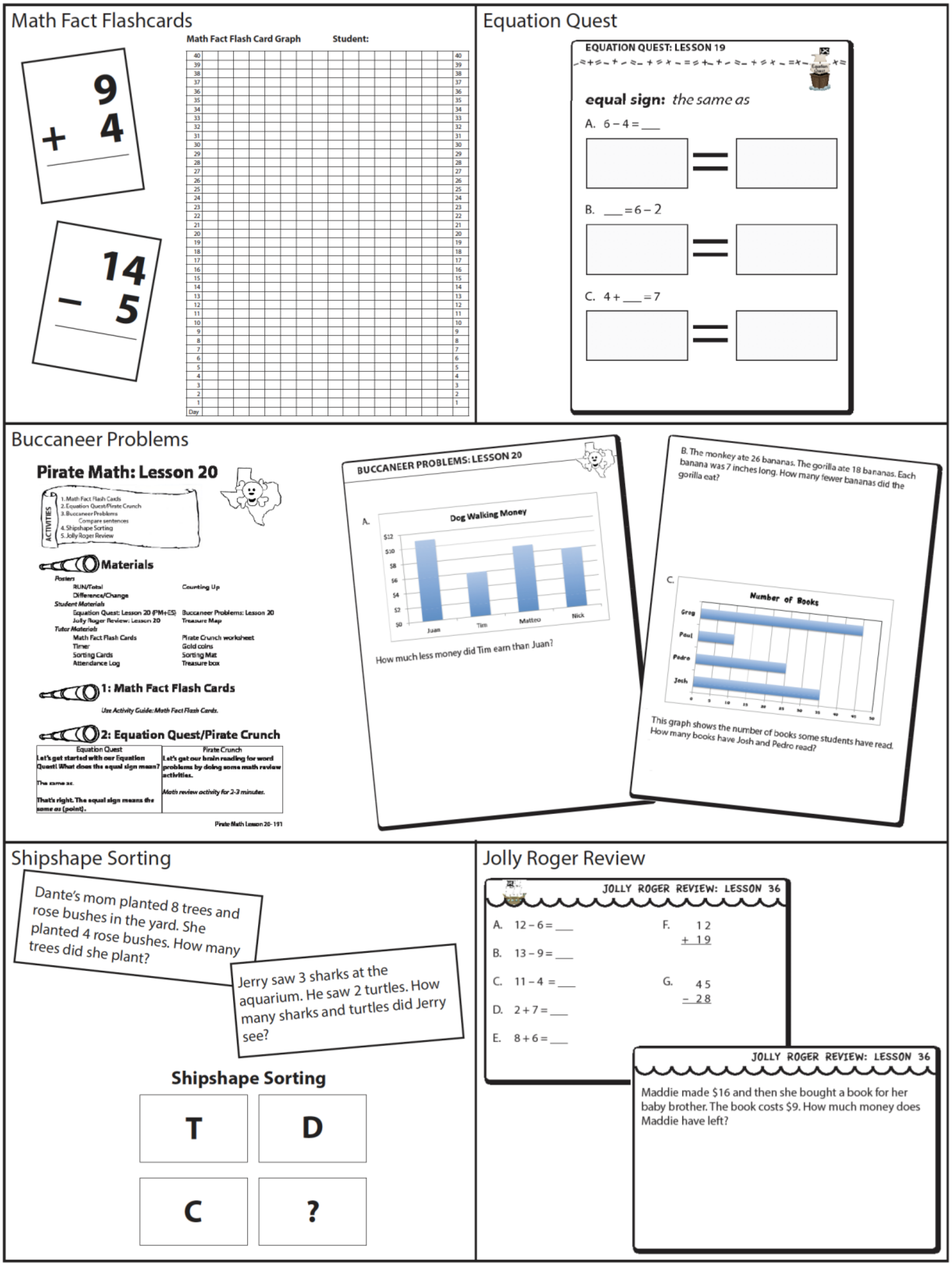

Figure 1. Pirate Math Equation Quest materials.

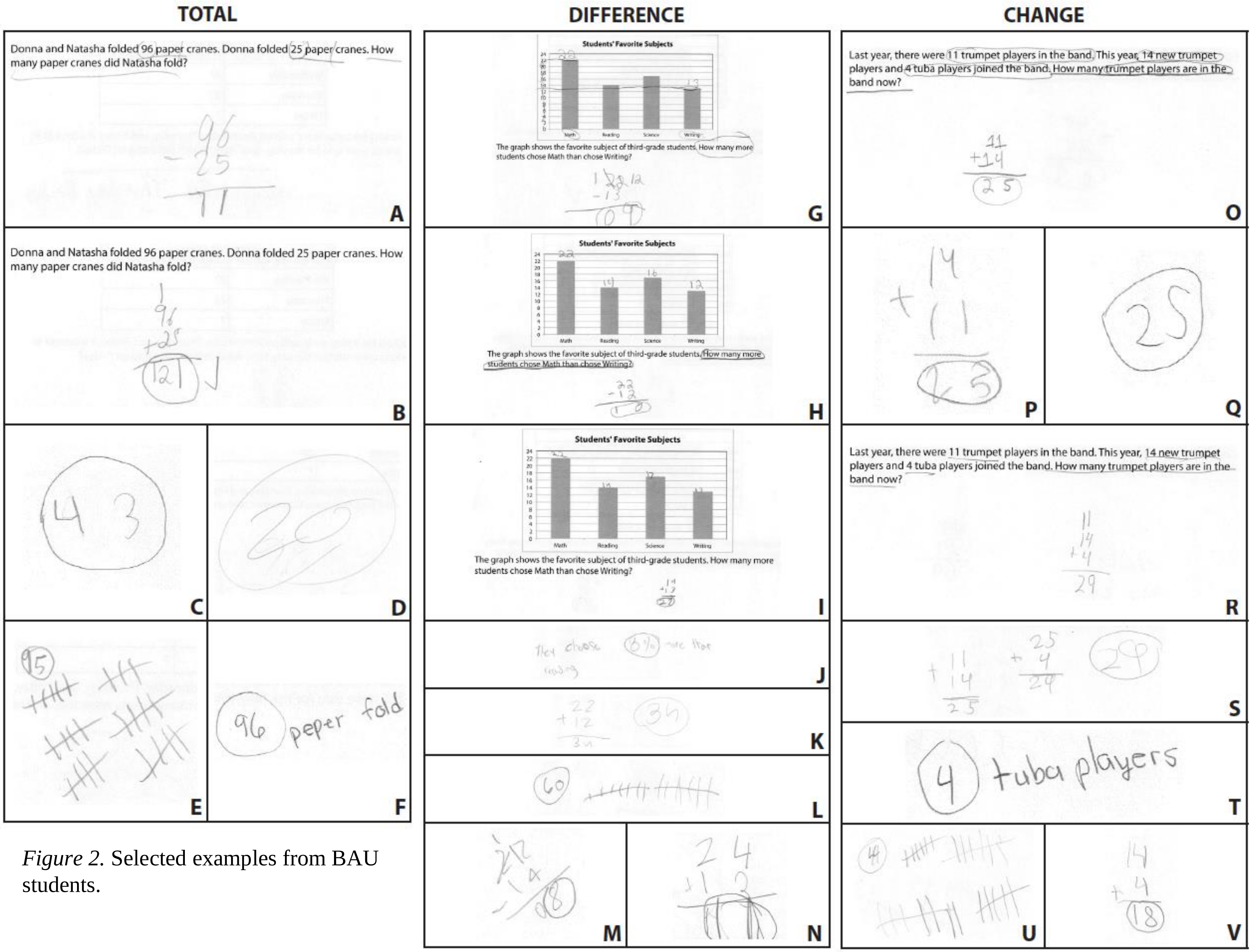

Figure 2. Selected examples from BAU students.

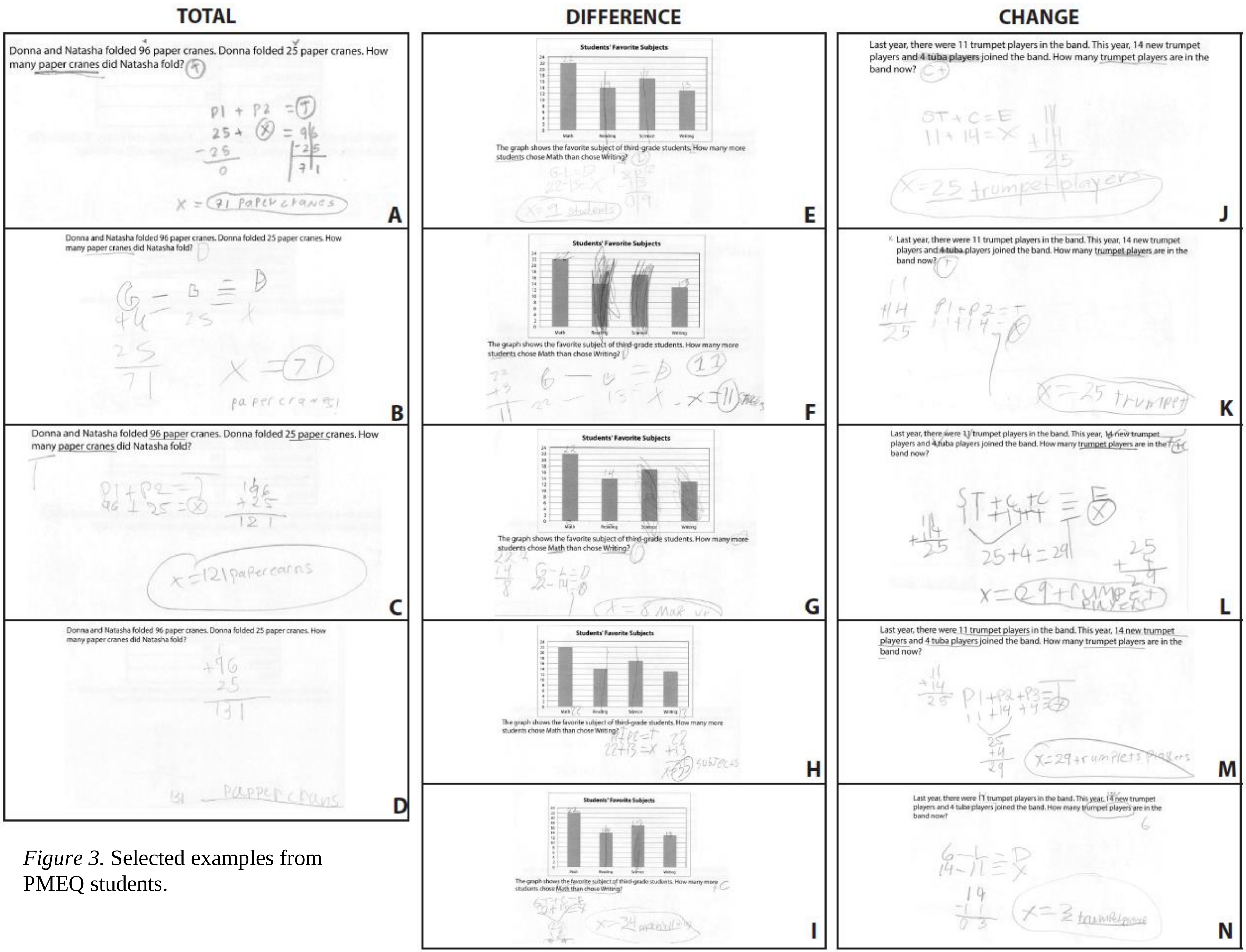

Figure 3. Selected examples from PMEQ students.