International Society of the Learning Sciences

Collaborative Virtual Learning in the shAR Geometry Simulation Environment

Publication Date: June 2021

Candace Walkington, Jamie Gravell, José A. Velazquez, Tianyu He, George Hickey
cwalkington@smu.edu, jgravell@smu.edu, javelazquez@smu.edu, het@smu.edu, gihickey@smu.edu
Southern Methodist University
Mitchell J. Nathan, mnathan@wisc.edu, University of Wisconsin-Madison
Anthony Cuevas, acuevas@smu.edu, Southern Methodist University

Abstract: Augmented Reality (AR) and Virtual Reality (VR) platforms now support shared, immersive experiences that enable people to directly physically and perceptually engage with mathematical objects, including shared objects. These new forms of AR/VR technology – which we call *shared holographic AR/VR* (shAR) - enable multiple learners to manipulate and reason about the same mathematical objects represented as holograms projected in a joint three-dimensional collaborative space in front of them, using intuitive hand gestures. This Interactive Tools and Demos paper discusses an AR/VR environment for collaboratively exploring geometry conjectures about shapes and solids. We show data of learners collaborating in our environment using speech, dynamic actions on virtual objects, and hand gestures. We discuss the potential of AR and VR environments for mathematics learning.
Keywords: Augmented Reality, Virtual Reality; dynamic geometry; embodied cognition

Introduction

Augmented Reality (AR) and Virtual Reality (VR) platforms now support shared, immersive experiences that enable people to directly physically and perceptually engage with mathematical objects, including shared objects. These new forms of AR/VR technology – which we call *shared holographic AR/VR* (shAR) - enable multiple learners to manipulate and reason about the same mathematical objects represented as holograms projected in a joint three-dimensional collaborative space in front of them, using intuitive hand gestures. This is significant because shared, immersive manipulation of 3D objects has not previously been possible, and can bring together the affordances of physical and virtual learning experiences (Bujak et al., 2013). ShAR has the powerful dynamicity and immediate feedback of virtual manipulatives and simulations, combined with the gestural interface and 3D nature of physical objects, leveraging knowledge that is gestural, perceptual, and action-based in nature (McNeil, 2008). By supporting collaborative holographic manipulation, 3D spatial displays like shAR “have the potential to do for gestures what writing did for speaking” and will “transform how people generate, disseminate, and interact with knowledge” (Dimmel & Bock, 2019, p. 2). These platforms also allow for new possibilities for collaboration because physical interplay between learners is increasingly likely, and learners can interact and embody concepts in a coordinated way (Lindgren & Johnson-Glenberg, 2013). In this paper, we discuss a prototype AR/VR system for collaborative manipulation of holograms of geometric shapes. In our prototype, students work together to transform geometric shapes (e.g., a cylinder, a triangle) in 3-dimensions by using their hands to resize, move, and transform the objects, while exploring geometric conjectures and properties.

Theoretical framework

Recent advances have demonstrated how mathematical thinking is ***embodied*** – tied to perception, action, spatial systems, and physical motions like gestures. Johnson-Glenberg et al.’s (2014) taxonomy givees three dimensions key to the effectiveness of embodied learning interventions in educational settings – motoric engagement, gestural congruency, and perception of immersion. ShAR technologies allow for these criteria to be met in ways that were not previously possible. Motoric engagement in shAR is high as learners can gesture to modify objects and walk around and within objects. Gestural congruency in shAR is high as advanced hand tracking allows for stretching gestures to enlarge shapes, rotation gestures to turn shapes, etc. Learners can become immersed using wireless headsets that layer mathematical representations onto their environment (AR), or which creates a new virtual environment around them (VR), all while seeing their peers and holograms made by their peers.

Gestures—the spontaneous arm and hand movements that speakers produce when communicating—have been the subject of both observational and intervention research because of their relationship with thinking, social cuing, and cognitive development (Goldin-Meadow, 2005). A review by Alibali and Nathan (2012) found converging evidence that representational gestures, those depictive of objects and processes, exhibited students’ mental simulations of actions, perception, and conceptual metaphors. Gesture studies offer an important link between individualized and social forms of embodiment. This is because, while gesture production has well-established cognitive benefits for the individual actor (e.g., Goldin-Meadow, 2005), gesture production is facilitated when speakers operate in a social context (e.g., Goodwin, 2000; Moll & Tomasello, 2007; Vygotsky, 1978), even when the speakers cannot see one another (Alibali, Heath & Myers, 2001). During collaboration, gestures operate synchronously with speech, acting as a mechanism to create cohesion and bind conversational elements (Enyedy, 2005; Koschmann & LaBarron, 2002).

ShAR Geometry Simulation Environment

Our shAR Geometry Simulation Environment (shAR GSE) functions in both AR and VR and works on a variety of goggles including the Oculus Quest 1 and 2, the Microsoft HoloLens 1 and 2 and the HTC Vive. Our ICLS demonstration can be given virtually, as we can stream from the goggles while sharing our screen.

The shAR GSE is designed in the Unity engine and combines Azure spatial anchors and photon multiplayer servers to connect a user's headset to these virtual shared spaces. The system uses a client-server architecture where the client is the Geometry Simulation application and teachers and students use the client. The server is photon and is responsible for communication between students and teachers, synchronizing object position, rotation, and simulation state. The client is designed using an object-oriented architecture and uses the C# language. The simulation is accessed as an app installed on AR/VR goggles. Users enter their name and select from a list of classes to join (opened by instructors). Each classroom holds up to 6 people. When students join a classroom, if they are joining in VR, they appear as a generic head and torso with their name hovering overhead; the head and torso moves around the environment as they move. When they move their hands within view of the goggles, their hands are tracked and displayed in real time to the other users in the simulation through hand-tracking. Users on an Oculus Quest device must designate a play space of 6.5ft by 6.5ft to ensure they can move around in their virtual environment; all devices need adequate lighting for proper positional and hand tracking. All devices require Wi-Fi and learners can hear each other's voices through the headsets.

When shAR GSE launches, there are three shapes for learners to work with – two triangles and a cylinder. Learners can touch the shapes with their hands to select them, turning them green. Once shapes are selected, learners can grasp them and move them around. They can also be rotated or turned in the same manner. The triangle's three vertices have small cubes attached, which learners can select and drag with their hands in order to transform the triangle's side lengths and angles. The cylinder has similar manipulation points which allow its height and radius to be modified. The triangle's angle, side, and area measurements and the cylinder's radius, height, volume, and surface area are all displayed and updated automatically. All users see and interact with the same holograms, with the holograms updating based on collaborators' actions in real time.

Learners move to the second stage of the simulation by placing the shapes in puzzle outlines – to do this, they must resize them appropriately. In the second stage of the simulation, learners can collaboratively manipulate a cube, a square pyramid, a sphere, a hexagonal prism, and a torus. Although there are a wide variety of mathematical tasks that could be facilitated in the environment, Table 1 shows tasks that were given to learners during our pilot study. The design of our environment was tightly linked to our theoretical framework. We wanted learners to be able to manipulate the objects using gesture and to be able to see each others' manipulations and hand gestures in real time to promote embodied communication and reasoning. We wanted gestures, actions on objects, and speech to operate synchronously to allow learners to jointly embody geometric principles.

Table 1: Tasks given to learners in shAR GSE pilot study

Stage	Tasks
Stage 1	1. One of your students conjectures that the volume of a cylinder changes by the same amount whether you increase the radius by 1 cm or increase the height by 1 cm. Do you think this conjecture is true or false? Why? Try it out with the cylinder in front of you. 2. Can you make it so the cylinder looks like a circle from both your viewpoint and your partner(s) viewpoint(s), at the same time? 3. One of your students conjectures that for all triangles, the largest side is always opposite from the biggest angle. Do you think this conjecture is true or false? Why? 4. Can you make the two triangles into a square, with one person controlling each triangle?

Stage 2	5. Can you and your partner(s) place your hands on as many faces of the cube as possible? Point your fingers to as many vertices of the cube as possible? Use your index finger and thumbs to cover as many edges as possible? How many vertices, faces, and edges does a cube have? 6. Choose two of the solids. Size them such that you have created two solids that you believe would have the same volume. How do you know they have approximately the same volume?

Demonstration of learners using environment

We conducted a pilot study where 9 in-service teachers used shAR GSE in groups of 2-3, with an instructor present in their room. They explored the tasks in Table 1, and learners did the simulation in VR as they were enrolled in a synchronous virtual course for math teachers at a university in the Southern United States. The teachers checked out Oculus Rift goggles to participate in the simulation and joined from their individual homes where they were instructed to have a clear area to play. Teachers had an average of 3.9 years of teaching experience, with 3 teachers teaching grades 5-6 mathematics, 5 teaching grades 7-10 mathematics, and 1 working as a grades 4-6 technology lead. Figure 1 shows a group of three learners exploring Task 6 in Table 1, trying to create a torus and sphere with the same volume. They discuss which shape has the larger volume, by manipulating the shapes, gesturing around the shapes using their virtual hands, and moving their virtual bodies around the shapes to change their perspective.

1	Cathy:	((moves to the left to around to see through the torus))
2	Facilitator:	So which one
		(4.2)
3	Melinda:	Actually if I don't know. Maybe the sphere.
4	Cathy:	((walks away from the torus and sphere towards Facilitator, looks back at the sphere, then turns around to get new angle on the shapes – a profile view))
5	Facilitator:	Why are you thinking maybe the sph::ere?
6	Melinda:	We…The donut…I mean not only are we missing like the space in the middle, but it's also flatter.
7	Cathy:	((Nodding head))
		(1.8)
8	Facilitator:	Ohhhh.
		(2.3)
9	Jill:	((makes an open flat hand gesture roughly perpendicular to the torus, moving away and closer to show changes in width))
10	Jill:	Like we lo::se like width. Like measurements off the sides as well as in the middle.
		(2.1)
11	Facilitator:	What do y'all think… (.5) of that?
12	Melinda:	Well kind of the same reason you're obviously missing a whole space ((pointing with index finger indicating hole in center of torus))
13	Cathy:	((tilting head left and moving body to left))
14	Melinda:	In the middle, um, like even though like technically the length of the diameter is bigger or the radius…ummm it's still missing a hole in the middle. So, I think volume wise I think that the red sphere is…bigger.

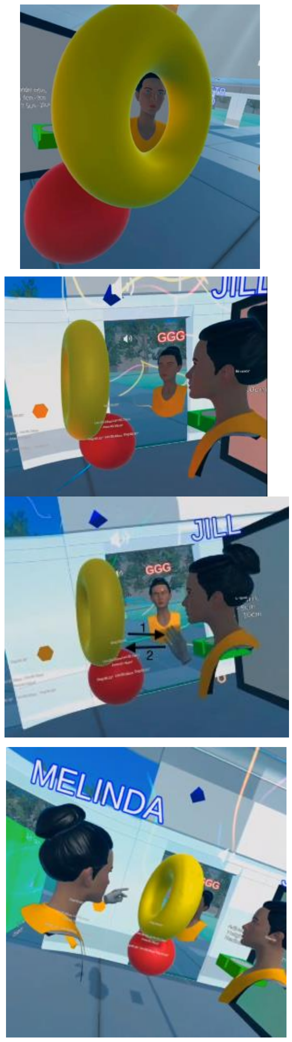

Figure 1. Multimodal analysis of a group of 3 learners using shAR GSE to explore Task 6 in Table 1. Transcription system developed by Gail Jefferson and described in Sacks, Schegloff, and Jefferson (1974)

The teachers were overwhelmingly positive about the VR experience on a post-lesson reflection. One teacher stated "It is the best technology I have seen for learning. Changing the dimensions of shapes is even better than manipulatives. Collaboration is easier because it is more like being in person." while another described how "It was awesome to interact with many different 3D shapes. I liked being able to 'touch' and manipulate them. I also loved the teamwork aspect of the activity and how it allowed for multiple group members to interact. The exposure to this type of advanced technology was also fascinating to me." The teachers acknowledged challenges to using shAR, including strain from extended use of the goggles, finding space for the simulation, the price of the goggles ($300-$400 per pair), and the technological expertise needed to troubleshoot the VR setup.

Challenges and future directions

We had originally intended for our simulation to be AR; however, the COVID-19 pandemic required us to allow for flexibility for people joining from different physical spaces. The switch to VR introduced technical and conceptual challenges to our design. First, unlike in AR, in VR collaborators' gestures are only visible if their goggles can see and successfully track their hands; as a result, gestures seemed to occur more rarely then in our previous studies. Second, in VR there is not access to collaborators' facial expressions, and it can even be challenging to determine who is talking. This presented challenges for leveraging our theory of embodied collaboration. And finally, we found that the hand-tracking and hand gestures to manipulate the shapes, while greatly improved from previous iterations, still had a learning curve associated with getting it to work smoothly.

In future work, we are collaborating with GeoGebra (Hohenwater & Fuchs, 2004), a company which produces a 3D geometry AR system for mobile devices. GeoGebra's system will allow learners to collaboratively interact with a wide variety of geometric shapes in 3D using a gestural interface. We will be imbedding tasks like those in Table 1 into a game for learning geometry based on the *Flatland* novella. We will use this platform to test hypotheses about how embodied learning experiences can be effectively designed for collaboration, and the affordances of different modalities for math learning. These technologies are an important area of study because they allow learners to have experiences that are impossible in the real world, and only recently possible with technology – i.e., dynamic manipulation of 3D shapes in a collaborative manner. Such advances may allow for new, groundbreaking discoveries about the nature of mathematical cognition to be uncovered.

References

Alibali, M. W., Heath, D. C., & Myers, H. J. (2001). Effects of visibility between speaker and listener on gesture production: Some gestures are meant to be seen. *Journal of Memory and Language*, *44*(2), 169-188.

Alibali, M. W., & Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: Evidence from learners' and teachers' gestures. *Journal of the Learning Sciences*, *21*(2), 247–286.

Bujak, K. R., Radu, I., Catrambone, R., Macintyre, B., Zheng, R., & Golubski, G. (2013). A psychological perspective on augmented reality in the mathematics classroom. *Computers & Education*, *68*, 536-544.

Dimmel J., Bock C. (2019) Dynamic Mathematical Figures with Immersive Spatial Displays: The Case of Handwaver. In: Aldon G., Trgalová J. (eds) *Technology in Mathematics Teaching. Mathematics Education in the Digital Era* (pp. 99-122), vol 13. Springer, Cham.

Enyedy, N. (2005). Inventing mapping: Creating cultural forms to solve collective problems. *Cognition and Instruction*, *23*(4), 427-466.

Goldin-Meadow, S. (2005). *Hearing gesture: How our hands help us think*. Harvard University Press.

Goodwin, C. (2000). Gesture, aphasia, and interaction. *Language and Gesture*, *2*, 84-98.

Hohenwater, M., & Fuchs, K. (2004). Combination of dynamic geometry, algebra and calculus in the software system GeoGebra. University of Salzburg.

Johnson-Glenberg, M. C., Birchfield, D. A., Tolentino, L., & Koziupa, T. (2014). Collaborative embodied learning in mixed reality motion-capture environments: Two science studies. *Journal of Educational Psychology*, *106*(1), 86-104.

Koschmann, T., & LeBaron, C. (2002). Learner articulation as interactional achievement: Studying the conversation of gesture. *Cognition and Instruction*, *20*(2), 249-282.

Lindgren, R., & Johnson-Glenberg, M. (2013). Emboldened by embodiment: Six precepts for research on embodied learning and mixed reality. *Educational Researcher*, *42*(8), 445-452.

McNeil, N. M. (2008). Limitations to teaching children 2+ 2= 4: Typical arithmetic problems can hinder learning of mathematical equivalence. *Child Development*, *79*(5), 1524-1537.

Moll, H., & Tomasello, M. (2007). Cooperation and human cognition: the Vygotskian intelligence hypothesis. *Philosophical Transactions of the Royal Society B: Biological Sciences*, *362*(1480), 639-648.

Sacks, H., Schegloff, E. A., & Jefferson, G. (1978). A simplest systematics for the organization of turn taking for conversation. In *Studies in the organization of conversational interaction* (pp. 7-55). Academic Press.

International Society of the Learning Sciences

Acknowledgments

The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R305A200401 to Southern Methodist University. The opinions expressed are those of the authors and do not represent views of the Institute or the U.S. Department of Education.