

## Mathematical Insights in the Pioneering Educational Project FSTF

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### Abstract

*The educational project “From Soccerene to Fullerene” was a pioneering interdisciplinary experience accomplished by six pupils who discussed it in the 2013 Italian school-leaving examination. The author illustrated the FSTF in the 2014 national conference of the Italian Physical Society (Pisa) eliciting a genuine interest. Here we highlight the FSTF’s mathematical aspects.*

**Key Words:** *Plane and solid geometry, Archimedean solids, Deformable solids, Geometry and physics*

**MSC(2010):** *97G40, 51P05, 74-02, 97M50*

### INTRODUCTION:

During their pre-university year, six brilliant and motivated students joined the educational project “From Soccerene to Fullerene” (acronym FSTF) proposed by five teachers from the Departments of *Physical Education* and of *Mathematics and Physics* at the Scientific High School “G.B. Grassi” in Latina (Italy). The pupils investigated the *fullerene* buckyball ( $C_{60}$ ) also known as *soccerene* for its football-shaped structure (truncated icosahedron). From plastic sheets they cut twelve pentagons and twenty hexagons, with a calculated edge, sewing them together on a polystyrene sphere as faces of a soccer ball; they repeated the same operation twice, making two almost identical soccer-like balls. That multidisciplinary experience started on March 2013 and ended on June 2013. It was satisfactorily discussed in the 2013 school-leaving examination and raised a great interest in the 100<sup>th</sup> national conference of the Italian Physical Society in Pisa, where it was presented the following year [1], [5]. Now we briefly illustrate the FSTF, focusing on its mathematical features.

### PREMISES OF THE FSTF:

The teachers submitted the following four questions, asking the students to seek for the answers by themselves:

- (1) Why do we play soccer with a sphere?
- (2) Why is the soccer ball a truncated icosahedron?
- (3) What is the soccerene or fullerene?
- (4) Why is the fullerene so important?

#### **Why do we play soccer with a sphere?**

The pupils verified that, as fully expounded by Ludwig and Guerrerio [14], the physics of rubber elastic bodies [20] requires a perfectly spherical ball in order to avoid unpredictable trajectories after the rebound. In this way, the trajectory after the impact is independent of the point of contact between the ball and the hit surface.

#### **Why is the soccer ball a truncated icosahedron?**

The students detected that the curricular *Platonic solids* (tetrahedron, cube, octahedron, dodecahedron, icosahedron) were not suitable for constructing a spherical ball. They discovered how Archimedes introduced thirteen semi-regular convex polyhedra whose faces are two or more types of regular polygons. Then the pupils

recognized that, as clarified by Ludwig and Guerrerio [14], only two *Archimedean solids* are closely approximated by a sphere:

(1) Rhombicosidodecahedrons: with volume 94% of the circumscribed sphere (12 pentagons, 30 squares and 20 triangles).

(2) Truncated icosahedrons: with volume 87% of the circumscribed sphere (12 pentagons, 20 hexagons).

The students understood that, between them, the best choice for building a soccer ball was the *truncated icosahedron*, with less edges (90 Vs. 120) by the Descartes-Euler polyhedral formula stating that the number of edges is equal to the number of faces plus the number of vertices minus two ( $E = F + V - 2$ ) for any convex polyhedron [6], [21].

#### What is the soccerene or fullerene?

The pupils studied that, for its hexagonal texture interspersed by pentagonal asymmetries, the soccer ball is a miniaturized version of the geodesic domes patented in 1954 by the architect *Richard Buckminster Fuller* [15]. They learned that, to emphasize the structural analogy, the molecule of 60 carbon atoms was named *buckminsterfullerene* [13] and that *soccerene* [12] is another definition of the same  $C_{60}$  molecule (IUPAC name ( $C_{60-I_h}$ )[5,6]fullerene) whose shape resembles a soccer ball; hence the title of our project [19]. We remark that the pupils' team was already accustomed with the hexagonal geodesic dome of the planetarium "Livio Gratton" ([2], [3], [4]) located in the Scientific High School "G.B. Grassi" of Latina (Figs. 1, 2 and 3).

#### Why is the fullerene so important?

The students comprehended that the fullerene is a topical scientific issue, as explained by Porter [18] and remarked by Giuliano [9, 11]. They found that also the related new materials and nanotechnologies [16] are surely worthy of investigation in a scientific high school.



**Figure 1.** The planetarium "Livio Gratton" at the Scientific High School "G.B. Grassi" in Latina (Italy).



**Figure 2.** The geodesic dome of the planetarium “Livio Gratton” at the Scientific High School “G.B. Grassi”.



**Figure 3.** Detail of the hexagonal-only geodesic dome of the Latina’s planetarium “Livio Gratton” (Italy).

### THE FSTF EDUCATIONAL PROJECT:

Stimulated by five teachers (Department of Physical Education: M.G. Saviano, F. Truppa; Dept. of Mathematics and Physics: A. Curinga, R. Calvitti, the author), in march 2013 six Italian pupils aged 18-19 (G. Cardarelli, M. Garrido Cutiño, L. Gambacorta, I. Olivadese, M. Rosella, K. Tytler) explored the footballs' structure, familiarly known as *bucky-ball* [6, 13] still after *R. Buckminster Fuller* and his famous geodesic domes. According to the geometric construction of regular polygons, the students drew hexagons and pentagons on differently coloured plastic sheets (following the soccer balls' pattern: *white* hexagons and *black* pentagons) and then they cut them off. The cut polygons were then applied on a polystyrene ball with a diameter ( $d = 25\text{ cm}$ ) chosen a bit larger than the official football "size 5" ( $d = 22 \div 23\text{ cm}$ ) for practical purposes. The pupils used pins and black thread to emulate the seams of traditional footballs (Figs. 4, 5, 7 and 8). The students made two almost identical soccer-like balls, describing them in their State Exam (July 2013). As reported on the artefacts' book-shaped bases (Figs. 4, 5 and 8), this cross-sectional project involved knowledge and skills from four subjects: *Mathematics, Physical Education, Physics and Science*.



Figure 4. Front view of the 2013 FSTF artefact.



Figure 5. Back view of the 2013 FSTF artefact.

### THE MATHEMATICS OF THE FSTF:

Once assimilated the geometry of the Carbon-60 molecule, the pupils were given two identical polystyrene spheres (each of a diameter of  $25\text{ cm}$ ), some plastic sheets (black and white), a  $50\text{ m}$  long black thread and around 2000 pins. Let us show the four computational steps autonomously taken by the pupils' team.

#### Step 1. The unknown common edge

The students considered that each of their two polystyrene spheres had the radius:

$$(1) \quad r = 12.5\text{ cm}$$

and the spherical surface was:

$$(2) \quad S_s = 4\pi r^2 = 4\pi \left(\frac{25}{2}\right)^2 = 625\pi \text{ cm}^2$$

The pupils realized that, since 20 hexagons and 12 pentagons must cover the whole spherical surface (Eq. (2)) they had to calculate the unknown common  $x$ -edge (or  $x$ -side, see Fig. 6) between the polygons.

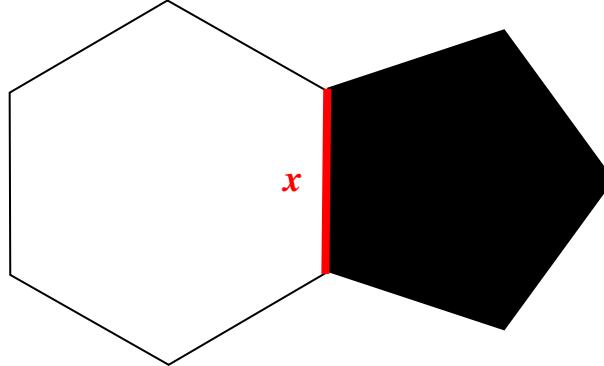


Figure 6. The common  $x$ -side of hexagons and pentagons.

### Step 2. The hexagons' side length

The students' team started from the hexagon's area, linked to the  $x$ -side by:

$$(3) \quad A_h = \frac{3\sqrt{3}}{2} x^2$$

The part of sphere covered by 20 hexagons was calculated as:

$$(4) \quad S_h = 20A_h = 30\sqrt{3}x^2$$

### Step 3. The pentagons' side length

The students' team continued with the pentagon's area, linked to the  $x$ -side by:

$$(5) \quad A_p = \frac{5}{4} x^2 \tan 54^\circ = \frac{5}{4} x^2 \frac{1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}$$

The part of sphere covered by 12 pentagons was calculated as:

$$(6) \quad S_p = 12A_p = 15 \frac{1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}} x^2$$

### Step 4. Value of the $x$ -side

Since the sum of the hexagons' total surface (Eq. (4)) and of the pentagons' total surface (Eq. (6)) must cover the whole spherical surface (Eq. (2)), the pupils calculated the  $x$ -value as follows:

$$(7) \quad S_s = S_h + S_p$$

$$(8) \quad 625\pi = 30\sqrt{3}x^2 + 15 \frac{1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}} x^2$$

$$(9) \quad x^2 \approx 27 \text{ cm}^2$$

$$(10) \quad x \approx 5.2 \text{ cm}$$

The students were ready to cover their two polystyrene spheres by sewing together the white hexagons and black pentagons shown in Fig. 7, whose identical side was approximately 5.2 cm long.

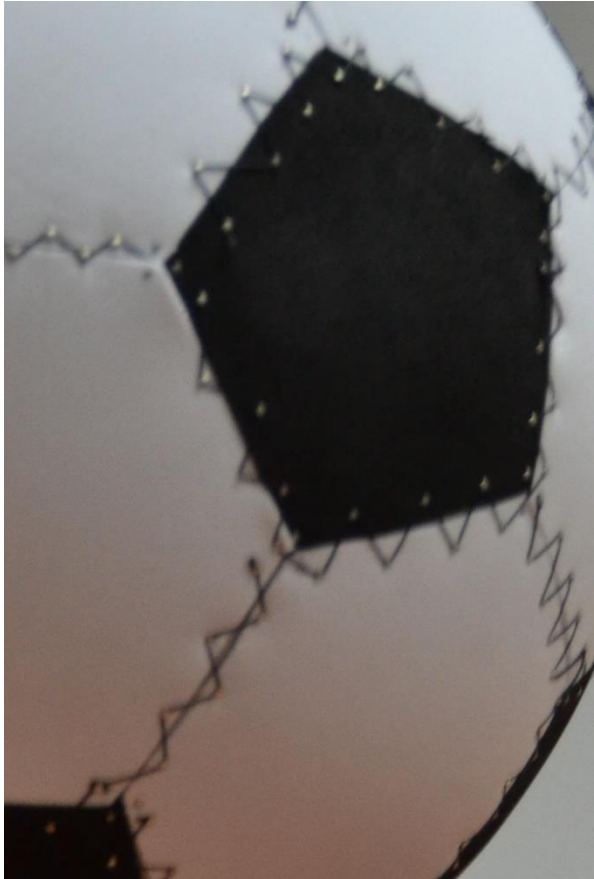


Figure 7. Detail of the 2013 FSTF artefact's sewings.

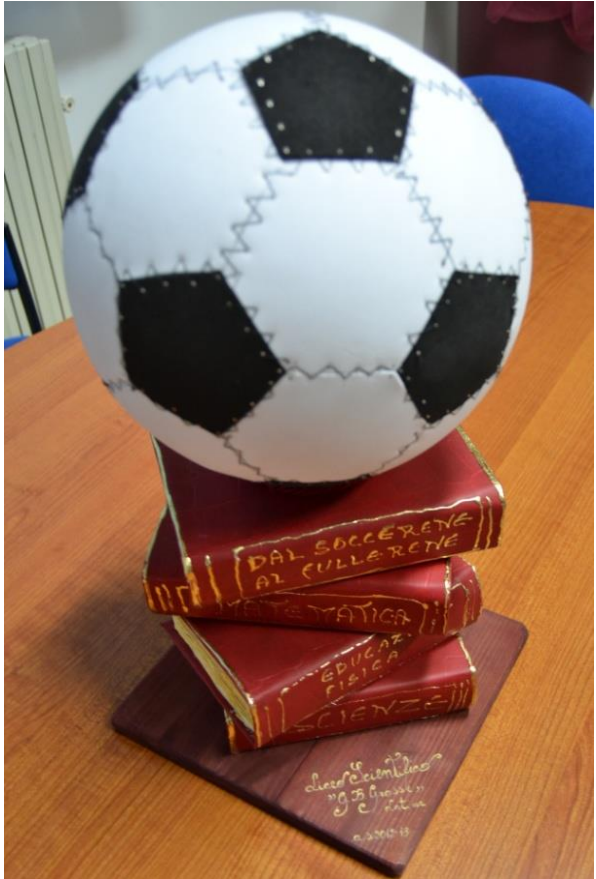


Figure 8. Top view of the 2013 FSTF artefact.

### CONCLUSIONS:

Beyond the realization of a nice artefact (Fig. 8), the 2013 FSTF project's results were educational, cognitive, epistemological and heuristic. Among them, we recall:

- (1) An innovative collaboration between traditionally distant Departments fixing an Italian school's weak point, i.e., the lack of enough interdisciplinary dialogue and activity.
- (2) An application of the widely auspicated *learning by doing* [7] and *constructivism* [17] in Science [8], [10].
- (3) The discussion of cross-sectional issues ([9], [11], [16], [18]) useful to improve the students' preparation for their state exam.
- (4) A consideration on how even apparently easy sport questions, like the choice to play with a *ball*, can induce non-trivial reflections of physical order [14], [20].
- (5) A path of gradual awareness about the difficulty to build, with flat surfaces, a convex solid approximating a sphere.
- (6) The suggestion to extend the Italian pre-university curriculum to the *Archimedean solids*, besides the five classic (Platonic) polyhedra.

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## Mathematical Insights in the Pioneering Educational Project FSTF

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