Helping preschoolers learn math: The impact of emphasizing the patterns in objects and numbers

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Abstract

Preschoolers' repeating patterning knowledge is predictive of their concurrent and later math and numeracy knowledge, but strong experimental evidence is needed to determine if these relations are causal. The purpose of the current study was to examine the causal effects of repeating patterning and numeracy tutoring on repeating patterning, numeracy, and general mathematics knowledge in the year prior to kindergarten (i.e., preK). Children in preK (N = 211) were randomly assigned to receive 5 sessions of researcher-delivered tutoring (a) on repeating patterns and numeracy or (b) on numeracy (and literacy as an active control), or received no tutoring and business as usual classroom instruction (control). Children who received tutoring in repeating patterning and numeracy improved in their repeating patterning knowledge the most. However, children's general math and numeracy knowledge improved similarly across conditions, and a specific aspect of numeracy emphasized during the tutoring did not improve. Children's repeating patterning knowledge is malleable, but this initial attempt to demonstrate causal links between repeating patterning and math knowledge was not successful. Results parallel mixed success in research training other skills, such as working memory or spatial skills, for improving mathematics knowledge. Findings are discussed in terms of the relations between patterning, numeracy, and general math knowledge in preschoolers.

Keywords: Patterning knowledge, general mathematics knowledge, numeracy knowledge, successor and predecessor principle knowledge, tutoring

Educational Impact and Implications Statement

We know that preschool children with better repeating patterning knowledge (i.e., understanding predictable sequences with a part that repeats, like objects arranged ABABAB) also tend to do better on math and numeracy tasks. How can we improve preschool children's patterning knowledge and does teaching preschoolers patterning along with numbers improve their math knowledge? Our 5 tutoring sessions improved children's patterning knowledge, but did not improve their math knowledge. The patterning instruction was designed to improve children's knowledge of and ability to duplicate, extend, and abstract (i.e., duplication with different materials) repeating patterns, and identify the unit of repeat in those patterns. Although it was not designed to be used for classroom instruction, key features of our tutoring protocol can be incorporated into preschool teachers' patterning instruction.

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Children's mathematics knowledge prior to formal schooling plays an important role in their future success. Specifically, children's math knowledge at the beginning of kindergarten varies substantially (Starkey et al., 2004) and strongly predicts their later math and reading skills (Duncan et al., 2007; Jordan et al., 2009; Nguyen et al., 2016; Watts et al., 2014). Additionally, individuals with better math knowledge often have higher incomes, attain more prestigious careers, and make better healthcare decisions (Lipkus & Peters, 2009; Ritchie & Bates, 2013; Shapka et al., 2006). While most research has explored the contributions of early numeracy knowledge to math development, some work has highlighted that repeating patterning knowledge (the ability to notice and use predictable sequences that have a part that repeats) also predicts children's math knowledge concurrently as well as months and years later (Fyfe et al., 2019; Rittle-Johnson et al., 2017, 2019; Zippert et al., 2019, 2020). Emerging research has further suggested that repeating patterning knowledge is associated with general and specific aspects of numeracy knowledge (Rittle-Johnson et al., 2019; Wijns et al., 2019). However, the majority of the evidence linking repeating patterning knowledge to math and numeracy knowledge has been correlational, and the two existing training studies have not used rigorous experimental designs. Thus, existing research neither reveals the specificity of the effects of repeating patterning instruction on math and numeracy knowledge, nor the directionality or causal nature of these relations. Further, repeating patterning knowledge appears to be malleable (Papic et al., 2011), but interventions for improving repeating patterning knowledge are not well developed nor rigorously tested (Burgoyne et al., 2017).

Thus, the current study had three aims. First, we examined the malleability of repeating patterning knowledge in response to moderately intensive 2-week tutoring with children in preK compared to both an active control group and regular classroom instruction. Second, we examined whether one pattern-intensive aspect of numeracy—successor and predecessor principle knowledge—could be improved through tutoring. Finally, we examined whether tutoring in repeating patterning knowledge together with numeracy knowledge would improve children's numeracy and general math knowledge. Theoretically, this research is needed to better understand the roles of non-numerical and numerical knowledge for young children's math development (see Figure 1 for the conceptual model guiding our research). Practically, this research is needed because while many early math standards and curricula emphasize the importance of patterning in math instruction (National Association for the Education of Young Children, 2014; National Council of Teachers of Mathematics, 2006), the Common Core State Standards (CCSS; 2010) did not include repeating patterning knowledge as a content standard in the early grades. Additional evidence is needed to inform efforts to revise the CCSS to include patterning in the early grades. In the following sections, we review evidence on repeating patterning knowledge and evidence of its relation to math and numeracy knowledge.

Repeating Patterning Knowledge

Patterning knowledge refers to the ability to notice and use predictable sequences (Rittle-Johnson et al., 2015). Preschool children are especially adept at learning repeating patterns, or linearly arranged sequences with a repeating unit (e.g., ABABAB). Around age 3, children begin to develop explicit knowledge of repeating patterns, recognizing repeating patterns around them with some accuracy, such as on a striped shirt (Clements & Sarama, 2014). Typical repeating patterning tasks include duplicating and extending model patterns using identical materials, and sometimes abstracting patterns by creating the same type of pattern using different materials (Papic et al., 2011; Rittle-Johnson et al., 2015; Sarama & Clements, 2004; Starkey et al., 2004).

Overall, duplicating patterns tends to be easier for preschoolers than extending them, and pattern extension items are easier than pattern abstraction items (Collins & Laski, 2015; Mulligan & Mitchelmore, 2009; Rittle-Johnson et al., 2015, 2019). Analyses of preschool children's patterning errors and strategies suggests that one of the main challenges with early patterning lies in their inability to recognize the role of the unit of repeat in representing a pattern's structure (Collins & Laski, 2015; Lüken & Sauzet, 2020). In addition to instruction in literacy and numeracy, children often work with repeating patterns in preschool and early elementary school (Economopoulos, 1998), although the emphasis on patterning may be changing in elementary school with the adoption of the Common Core State Standards (2010). Repeating patterns are most developmentally appropriate for this age as they rely on perceptual dimensions such as shape or color, making them less reliant on prior knowledge (e.g., numeracy) than other types of patterns, such as growing patterns (e.g., numbers or object sequences that increase by 2 each time, e.g., 5, 7, 9, 11; Sarama & Clements, 2004; Wijns et al., 2019). For example, 4- and 5-year-olds perform below chance even on growing pattern items that do not contain numerals, such as ∆□∆□□∆□□□ (Wijns et al., 2019). This is likely because growing patterning knowledge is theorized to develop in kindergarten and first grade (Sarama & Clements, 2009), and children are less likely to recognize examples of growing sequences as patterns as compared to repeating sequences (McGarvey, 2012).

Patterning knowledge appears to be malleable, although evidence is quite limited**.** In one study, preschool children received less than an hour's worth of tutoring across two consecutive days to detect the shared pattern unit in sequences with the same structure but different items (i.e., abstract patterns; Rittle-Johnson et al., 2015). Compared to their pretest scores, children improved in their performance on the abstract pattern items, although not the extend pattern items. Additionally, there was no control condition. Given the lack of a control condition, improvements in task performance may have been attributable to repeated exposure to task items at pretest and during tutoring.

In another study involving a year-long classroom-based intervention, children attending two Australian preschool centers either received 6-months of repeating and spatial pattern training and “patternized” curricula (that included copying, memorizing, finding missing items, extending, and generating new repeating and spatial patterns) or regular classroom instruction (Papic et al., 2011). Spatial pattern activities included copying dot arrays, grid patterns, subitizing, and creating spatial figures with dots. While repeating and spatial patterning knowledge in the two classrooms seemed comparable at pretest, at the end of the school year as well as the end of the following school year, children who received the patterning intervention and enhanced preschool curriculum tended to score higher on both task types. At the end of kindergarten, children from the intervention group also tended to show superior performance on repeating and growing spatial patterns (item sets increasing in quantity and arranged as triangles or squares). However, a number of methodological issues (i.e., no random assignment to condition, initial condition differences were not statistically confirmed, selective reporting of study results on specific items, assessor was not blind to the hypotheses at posttest, and training experiences and group size were neither controlled nor consistent) limit the generalizability of the findings.

Several studies by one team of researchers indicate that other, more complex types of patterning knowledge are malleable in older children who struggle with patterning (Hendricks et al., 2006; Kidd et al., 2013, 2014; Pasnak et al., 2015). First-grade students who were identified through screening as struggling with a variety of types of patterns were randomly assigned to receive intensive training (15-minutes 3 times per week for 6 months) in growing, rotating, and symmetrical patterns (e.g., numerical sequences, alphabet sequences, time on analog clocks) or instruction in other subjects (e.g., math, reading or social studies; Kidd et al., 2013, 2014). Students receiving patterning training scored significantly higher in patterning knowledge, including on new types of patterns at the end of the school year than those trained in reading and social studies (Hendricks et al., 2006; Kidd et al., 2014; Pasnak et al., 2015).

Overall, patterning knowledge appears to be malleable. However, a bulk of the evidence exists for knowledge of growing and more complex patterns in school-age children who struggled with patterning. Controlled, experimental studies promoting repeating patterning knowledge (the appropriate aspect of patterning knowledge for preschoolers) using random assignment to condition, a control condition, and clearly specified instruction have not been conducted in a single study with preschoolers. Thus, it is important to establish effective methods for promoting repeating patterning knowledge in preschool.

Relations Between Repeating Patterning, Math, and Numeracy Knowledge

Repeating patterning knowledge has been theorized to be an important aspect of mathematical thinking. First, math involves patterning (i.e., identifying, extending, and describing predictable sequences in objects and numbers; Charles, 2005; Sarama & Clements, 2004; Steen, 1988). Further, some theorists describe patterning as early algebraic thinking given its emphasis on awareness of regularities and structural relationships (Carraher et al., 2006; Mason et al., 2009; Sarama & Clements, 2009). Additionally, several research-based early childhood math curricula emphasize repeating patterning instruction (Clements & Sarama, 2007a; Greenes et al., 2004; Mulligan & Mitchelmore, 2009; Sarama & Clements, 2004; Starkey et al., 2004). One example includes a promising kindergarten-2nd grade curriculum supplement that adopts a much broader construct termed “pattern and structure.” This program encompasses a very wide range of topics within elementary school mathematics, including subitizing, equal spacing and partitioning, and multiplicative reasoning or grouping (Mulligan et al., 2020; Mulligan & Mitchelmore, 2009, 2018).

Repeating patterning knowledge is likely linked to math knowledge at least in part because of its relation to numeracy knowledge specifically (see Figure 1). Numeracy knowledge in early childhood encompasses knowledge of number (i.e., recognizing quantities through subitizing and counting, associating quantities with verbal and written labels), number relations (i.e., comparing magnitudes and representing them on a mental number line) and number operations (i.e., addition and subtraction with objects, in verbal story problems and with numerals; Jordan et al., 2009, 2010). Both repeating patterning and numeracy knowledge involve deducing underlying rules in sequences. An example for number and number operations knowledge is inferring the successor principle (i.e., the understanding that adding one means the next number in the count sequence), which is thought to involve noticing and generalizing a pattern in the associations between quantities and their verbal and written labels (Carey, 2004; Cheung et al., 2017; Rittle-Johnson et al., 2017; Zippert et al., 2019, 2020). Repeating patterning knowledge may also promote number knowledge by helping children notice other patterns in the count sequence, such as the repetition of the ones digits when counting to 100 (Papic et al., 2011; Zippert et al., 2020), and supporting counting by 2’s or 5’s (Clements & Sarama, 2014). Given that even preschool children can find underlying rules in patterns with objects and sounds, developing such skills at a young age may support their noticing and use of patterns in numbers as they acquire numeracy knowledge. This is a key idea of the conceptual model guiding the current study.

Correlational Evidence

Empirical evidence provides strong support for the correlational link between patterning and general math knowledge. Repeating patterning knowledge in preschool is predictive of general math knowledge concurrently and longitudinally, including years later (Fyfe et al., 2019; Nguyen et al., 2016; Rittle-Johnson et al., 2017, 2019; Zippert et al., 2019, 2020). For example, repeating patterning knowledge assessed at the beginning of preK predicted general math knowledge concurrently and 7 months later (Rittle-Johnson et al., 2019). In two studies using the same sample of children, end-of-the-year preK repeating patterning knowledge predicted math achievement on standardized math assessments in 5th grade and state math achievement tests in 4th- 6th grades (Fyfe et al., 2019; Rittle-Johnson et al., 2017).

Empirical evidence for the correlational link between repeating patterning, general numeracy knowledge, and specific aspects of numeracy knowledge is accumulating. For example, preschoolers' beginning-of-the-year repeating patterning knowledge predicted general numeracy knowledge concurrently and 7-months later (Rittle-Johnson et al., 2019; Wijns et al., 2019; Zippert et al., 2019). Evidence also suggests that repeating patterning knowledge relates to specific aspects of numeracy knowledge. For instance, repeating patterning knowledge correlated with concurrent magnitude comparison and number operations knowledge (Zippert et al., 2019). Similarly, early elementary school children's repeating patterning knowledge correlated with their concurrent number operations knowledge, but not their knowledge of two math concepts (i.e., math equivalence and inversion). The authors suggested that repeating patterning knowledge is most useful for learning predictable sequences over a long period of time, such as arithmetic facts, rather than insights about single math concepts (Fyfe et al., 2017; MacKay & De Smedt, 2019).

Further, numeracy knowledge may serve a mediating role in explaining the association between early repeating patterning and later math knowledge. Specifically, end-of-pre-K repeating patterning knowledge predicted symbolic mapping (e.g., linking symbolic to non-symbolic number representations and comparing the magnitude of symbolic numbers), number operations and repeating patterning knowledge in first grade, which in turn helped predict performance on standardized math measures at age 11, when most children were finishing 5th grade (Rittle-Johnson et al., 2017). Similar results were also found for predicting state achievement test scores in math (Fyfe et al., 2019). Overall, this suggests that numeracy knowledge may serve as a pathway through which repeating patterning knowledge supports mathematics knowledge more broadly, as depicted in Figure 1.

Causal Evidence

The causal nature of links between patterning, math, and numeracy knowledge is underexamined. Only a few studies have examined the impact of patterning training on children's math knowledge, and only one of those focused on repeating patterns.

One study focused on the impact of repeating patterning instruction on children's numeracy knowledge (Papic et al., 2011). Children who attended a preschool with special repeating and spatial patterning training showed greater gains in their numeracy knowledge (e.g., forward and backward counting, indicating the next and previous number, symbolic mapping, and use of advanced calculation strategies) at the end of the next school year (in kindergarten) than children who attended a preschool with only regular classroom instruction (Papic et al., 2011). However, children in the patterning training classroom also received some numeracy-relevant instruction and practice (e.g., labeling and generating small sets of objects, duplicating spatially arranged shapes made of dots). Thus, it is unclear whether the patterning or numeracy training alone, or the combination of the two contributed to children's greater numeracy knowledge. Isolating the effects of numeracy instruction with and without repeating patterning instruction is imperative to understanding the mechanism by which children's numeracy knowledge was improved. Beyond, this, the study had several methodological weaknesses, outlined above. Thus, no rigorous experimental evidence exists for the causal impact of repeating patterning instruction on general math or numeracy knowledge.

Promise for the causal impact of patterning instruction comes from the intervention studies focused on growing and other complex patterns described above. Across three studies, struggling first-graders in the patterning training condition outperformed children in the other conditions on some, although not all, standardized math measures (Kidd et al., 2013, 2014; Pasnak et al., 2015). For example, children who received patterning training had higher end-of-the-year performance on a general standardized math measure (i.e., counting, number identification, shapes, symbols, math terms and formulas, and filling in growing patterns), but not an applied math test of story problems, compared to children who received training in reading or social studies (Kidd et al., 2013). Additionally, they had better or comparable math performance compared to children who received general training in math (e.g., counting, adding, shapes, and basic fractions). In another study, patterning knowledge at posttest fully mediated the relation between condition and math knowledge at posttest (Kidd et al., 2014). This suggests that growth in patterning knowledge accounted for the effect of patterning training on improvement in math knowledge. Interestingly, in one study, children in the general math training condition had similar end-of-the-year patterning knowledge as children in the patterning training condition (Kidd et al., 2013). This suggests potential reciprocal effects of math and complex patterning training on patterning and math knowledge. The patterning training and measures in these studies were rarely focused on repeating patterns and often involved patterns created with numbers. Thus, there is no clear causal evidence that improving repeating patterning knowledge leads to improvements in math knowledge, including numeracy knowledge.

Current Study

The primary goal of the current study was to evaluate the causal impact of repeating patterning tutoring on preschool children's repeating patterning, numeracy, and general math knowledge. First, we worked to establish an effective short-term approach for promoting repeating patterning knowledge in preschool, to provide experimental evidence for the malleability of repeating patterning knowledge. Because learners, especially young learners, often need scaffolding to transfer their knowledge to new contexts (Vygotsky, 1978; Wood & Middleton, 1975), we did not anticipate that preschool children would spontaneously and immediately transfer knowledge gained in short-term repeating patterning tutoring to math knowledge. Rather, we anticipated that repeating patterning tutoring would better prepare preschoolers to learn from numeracy tutoring, with explicit prompting to build on repeating patterning knowledge. Indeed, prior patterning training studies attended to numeracy content as well (see, e.g., Kidd et al., 2014, 2014; Papic et al., 2011; Pasnak et al., 2015). In our numeracy tutoring, we chose to focus on one related aspect of numeracy knowledge – successor and predecessor principle knowledge. These pattern-intensive numeracy principles involve knowledge that the next or previous number name in the counting sequence signifies adding or subtracting one, respectively (Sarnecka & Carey, 2008; Sella & Lucangeli, 2020). Knowledge of these principles is thought to be important for children to be able to use their knowledge of the counting string to develop numerical magnitude and number operations knowledge (Carey, 2004; Sarnecka & Carey, 2008; Sella & Lucangeli, 2020). Instruction in these principles also provides opportunities for practicing other numeracy skills, such as counting and symbolic-quantity mapping.

Thus, one condition included repeating patterning plus numeracy tutoring in each of 5 tutoring sessions (patterning+numeracy condition). A comparison condition included literacy plus numeracy tutoring in each session (literacy+numeracy condition) to tease apart the effects of patterning+numeracy tutoring vs. numeracy tutoring with additional tutoring activities to control for effects of general attention and academic content. In particular, it allowed us to test whether repeating patterning tutoring better prepares children to learn from numeracy tutoring. A third condition involved business-as-usual instruction in their preschool classrooms. Note that instruction in all three aforementioned subjects occurred to some extent during the school day for all children. We worked with 4- to 5-year-old children in the U.S. enrolled in preK programs–the year of preschool prior to beginning kindergarten.

Research Goals and Hypotheses

The goals of this research and associated hypotheses were threefold.

1) First, we examined if providing tutoring on repeating patterning and numeracy (i.e., patterning+numeracy condition) would improve repeating patterning knowledge more than tutoring on numeracy knowledge without patterning tutoring or regular classroom instruction in preK. We hypothesized that it would.
2) Second, we examined if providing tutoring on specific aspects of numeracy knowledge (e.g., successor and predecessor principle knowledge) would improve math and numeracy knowledge more than regular classroom instruction. We hypothesized that tutoring in numeracy (in the two tutoring conditions) would improve general math and numeracy

knowledge and a specific aspect of numeracy knowledge (i.e. successor and predecessor principle knowledge) more so than regular classroom instruction.

3) Finally, we examined if providing patterning+numeracy tutoring would improve math and numeracy knowledge more than numeracy tutoring without patterning tutoring. We hypothesized that it would.

Method

Our method was preregistered on <https://aspredicted.org/d5ig7.pdf>. Additionally, data, code, and study measures and materials are available at https://osf.io/9hd6p/ (Zippert, Douglas, & Rittle-Johnson, 2020).

Participants

Participants were a convenience sample of 211 preschool children ages 4- to 5-years-old (M = 4.7 SD = .38), almost evenly divided by gender (56% male). Participants were recruited from 12 preschools (5 public (42%) and 7 private schools (58%)). Parent consent forms were distributed to families by preschool teachers at each school, and we exceeded our pre-registered target of 180 children because more parents returned consent forms at some preschools than anticipated. Twenty-one students were dropped from the study, 10 due to absences (7 at pretest, 3 from multiple tutoring sessions), 9 because of behavior issues that prevented full instruction delivered to the child and their partner, and 2 because the child did not give verbal assent). Parents reported that 113 children (54%) were White, 61 (29%) were African American, 13 (6%) were Biracial, 10 (5%) were Hispanic, 9 (4%) were Asian, and 2 (1%) were Somali or Saudi Arabian. Parents did not report the race or ethnicity of 3 (2%) children. Twenty-one children (10%) spoke a language other than English at home. Seventy-four participants (35%) received at least some financial assistance to attend preschool. Ten participants (6%) received special education services in school (although data were missing for 39 participants (19%)). Race, receiving financial assistance and school type were confounded; 71% of white children attended private preschools, without financial assistance, compared to 18% of Black children and 11% of other students of color. This reflects the segregated nature of schooling in our community and makes it inappropriate to try to disentangle the three factors in our study. Teachers (n = 41) across schools had an average of 13 years of experience teaching pre-k (SD = 12.40) and 3 years of experience teaching other grades (SD = 5.51). Institutional review board approval was obtained [Ref. 151356: Exploring the Roles of Pattern and Spatial Skills in Early Mathematics Development, Vanderbilt University].

To gather information on normal classroom activities experienced by children during the timeframe of the tutoring, all teachers of participating students reported the number of school days they provided instruction on numeracy, patterning, and shapes in their classroom within the last month (out of 20 school days; see Table 1). Four teachers completed the survey twice as they had participating students during 2 semesters. Teachers reported providing numeracy and patterning instruction during about half of the school days each month (M = 11.67, SD = 4.28). The numeracy and patterning concepts they taught during the most school days per month were object counting and discussing patterns in days of the week, months in the year, or seasons. The least frequent concepts taught were number operations and abstracting patterns. Teachers reported providing numeracy instruction more often (M = 13.40 school days per month, SD = 3.66) than patterning instruction (M = 9.93, SD = 2.60), $t(43) = 4.57$, $p < .001$, d = 1.09. Finally, only 47% of teachers reported that they used published math curricula or commercial math activities and games in their math instruction. Rather, most teachers reported that they used activities in their math instruction that they made themselves (89% of teachers) or sourced online or from other teachers (78%).

Measures

General Math and Numeracy Knowledge

General math knowledge was measured using the REMA Short Form, which consisted of 19 items (Weiland et al., 2012) and took less than 15 minutes to administer. Thirteen items assessed numeracy knowledge (serving as our general numeracy measure), 5 items assessed shape knowledge, and 1 item assessed patterning knowledge. Numeracy items included easier items such as object and rote counting and subitizing small quantities, moderately difficult items such as matching numerals to their respective non-symbolic quantities and non-symbolic addition and subtraction of small quantities, as well as more challenging items such as magnitude comparison of large number words and non-symbolic arithmetic with large quantities. Shape knowledge items included identifying triangles and rhombuses and constructing shapes, identifying the number of sides on a shape, and determining the result of cutting a shape in two pieces. The patterning item required children to abstract an ABB pattern.

Items were ordered in each section by item response theory (IRT) difficulty estimates, and stop criteria was met when a child answered 3 consecutive items incorrectly in each section (recommended by Weiland et al., 2012). All but 4 items were scored as correct or incorrect. The 4 partial credit items were scored according to the criteria in Weiland et al. (2012), although scores on all partial credit items except for the shape construction item (item 15) had to be collapsed into fewer categories to accommodate the distribution of scores in our sample.

IRT ability estimates were generated using a Rasch model for items that were scored as correct or incorrect, and a partial credit model for items that were polytomous. IRT ability estimates were generated for the entire measure and for the numeracy and shape knowledge sections separately. The informative prior distribution on the item difficulty parameters and the sum-to-zero constraints on the item location and threshold parameters were chosen as per suggestions from Weiland et al. (2012). Internal consistency in our sample using IRT scores was good for general math knowledge ($\rho_{XX'}$ = .79 at pretest and $\rho_{XX'}$ = .80 at posttest) and for general numeracy ($\rho_{XX'}$ = .80 at pretest and posttest), but was weak for shape knowledge ($\rho_{XX'}$ = .56 at pretest and $\rho_{XX'}$ = .51 at posttest). Internal consistency previously reported by Weiland et al. (2012) was acceptable for general math knowledge (α = .71 and .79 in two samples) and the numeracy subscale (α = .79), but was weak for the shape knowledge subscale (α =.58).

Patterning Knowledge

Teacher-Based Patterning Assessment (TBP). One patterning knowledge measure, teacher-based patterning, is intended to measure children's repeating patterning ability in a manner similar to what might be used in a classroom setting and takes about 5 minutes to administer (Rittle-Johnson et al., 2019; Zippert et al., 2018). Children were presented with pictures of model patterns and given a set of small, laminated pictures to complete the patterning task. This assessment consisted of 8 items: identify the missing element (3 items with units AB, ABC, and ABB), extend by 1 element (2 items with units AB and ABC) and extend by 4 elements (3 items with units AB, AABB and ABC). Items were scored as correct or incorrect. Ability estimates were generated using a Rasch model with a Laplace approximation and empirical Bayesian prediction method, which is stable for sample sizes around 50 (Cho & Rabe-Hesketh, 2011). Laplace approximation was implemented in R (http://www.r-project.org), using the glmer function of the lme4 package (Bates et al., 2008). Internal consistency in our sample was good (α = .81 at pretest and α = .80 at posttest).

Research-Based Patterning Assessment (RBP). The other patterning measure, research-based patterning, is based on a learning trajectory and construct map, uses 3D materials and takes about 15 minutes to administer (Miller et al., 2016; Rittle-Johnson et al., 2015). The same learning trajectory and construct map was used to design the repeating-patterning tutoring, making it a more proximal measure of patterning knowledge gained from the tutoring. It also includes more difficult items than the teacher-based patterning measure. Three easier AB pattern items were added to the assessment because of floor effects in a previous study (Rittle-Johnson et al., 2019). It consisted of 12 items: identify the missing element (1 item with unit AB), copy (3 items with units AAB, ABAB, and AABB), extend by at least one unit (3 items with units AB, AAB, and AABB), abstract (3 items with units ABB and 2 AABB), create from memory (1 item with unit ABB), and identify units (1 item with unit AAB); a majority of items used tangram blocks. To reduce testing time and child frustration, stop criteria was implemented. Thus, if children answered all extend items or all abstract items incorrectly, the assessment was stopped. Ability estimates were generated using the previously described procedure utilized to generate TBP estimates. Internal consistency in our sample was good (α = .87 at posttest).

Children's incorrect responses were coded for the type of error that they made using the coding scheme described in Table 2. All incorrect responses were double coded by the third author and an undergraduate research assistant whose percentage agreement was 95.3%. The coders resolved disagreements via discussion and created a final set of codes. The coders were blind to hypotheses.

Successor and Predecessor Principle Knowledge

Children's knowledge of the successor principle was measured using an adapted version of the unit task (Cheung et al., 2017; Sarnecka & Carey, 2008), modified to use a fish pond context by David Barner and colleagues. We added new items to measure predecessor knowledge with small set sizes. Our measure consisted of 10 items and was designed to assess children's understanding that a set of objects increases or decreases by exactly one unit when you add or remove one object from a set. There were seven addition items (with starting quantities 6, 3, 7, 15, 12, 34, and 20) and three subtraction items (with starting quantities 2, 4, and 5), with items blocked by operation. Participants were presented with some fish (e.g. 3) which were then hidden under a lily pad. An additional fish was visibly placed next to the lily pad for addition items while one fish was removed from under the lily pad for subtraction items. Children were then asked how many fish there were and given two number options (e.g., "now are there 4 or 5 fish?"). Children received feedback on a practice item before the addition and subtraction blocks. Ability estimates were generated for successor and predecessor principle knowledge combined, and successor principle knowledge separately, using the previously described procedure utilized to generate TBP estimates. Perhaps because chance was 50%, internal consistency in our sample was poor (α = .51 at pretest and posttest); internal consistency has not been reported in past studies using successor principle measures.

Tutoring Materials and Topics

All tutoring materials, including scripts, are posted on the project website <https://peabody.vanderbilt.edu/departments/psych/research/research_labs/childrens_learning_lab/early-mathematics-development.php> and Open Science Framework <https://osf.io/9hd6p/> (Zippert, Douglas, & Rittle-Johnson, 2020).

Patterning+Numeracy Group. Tutoring consisted of instruction on repeating patterning and numeracy knowledge during each of five sessions as outlined in Table 3. Sample activities are included in the Appendix. The tasks were developed from an established math curriculum for preschool-aged children, Real Math Building Blocks, and the learning trajectory for repeating patterning that guided its development (Clements & Sarama, 2007b). The patterning instruction was designed to improve participants' knowledge of and ability to duplicate, extend, and abstract repeating patterns, and identify the unit of repeat in those patterns. A variety of materials were used to make the patterns (e.g. tangram blocks, drawn shapes, string and beads, unifix cubes, pipe cleaners, pompoms, and body movements). The numeracy instruction focused on successor and predecessor principle knowledge and aimed to help participants develop a conceptual understanding that numbers in the counting sequence follow a specific rule (e.g., the cardinality of a number is the cardinality of the preceding number in the number sequence plus one) just like repeating patterns follow rules. The numeracy instruction also aimed to improve participants' ability to utilize successor and predecessor principle knowledge to solve number problems (see Appendix for example). Materials for the numeracy tasks included unifix cubes, foam cubes, toy bugs, and songs. Across tasks, an experimenter provided brief direct instruction, modeling and feedback. Children were asked to answer prompts, make predictions, complete tasks (e.g., copy patterns) and explain their responses.

Literacy+Numeracy Group. Tutoring consisted of instruction on literacy and numeracy knowledge during each of the five sessions, as outlined in Table 3. The literacy instruction included oral language and early literacy skills taken from an established literacy curriculum, Opening the World of Learning (OWL) by Dickinson et al. (2014), and was matched in length to the patterning activities as much as possible. The numeracy instruction was the same as in the patterning+numeracy group.

Procedure

Children's general math and numeracy, successor and predecessor principle, and patterning knowledge were first assessed individually by one of six graduate-level research assistants in a 30-minute session. Only one measure of patterning knowledge (TBP) was given at pretest due to time constraints. Next, participants were randomly assigned to 1 of 3 conditions (i.e. control, patterning+numeracy, or literacy+numeracy). Mixed ability grouping was used to pair children based on their patterning knowledge at pretest. The final sample consisted of 71 children in the business-as-usual (BAU) control group, 72 in the patterning+numeracy condition, and 68 in the literacy+numeracy condition. Participants in each tutoring group received five 30-minute sessions of instruction in pairs from one of four graduate-level research assistants. Each 30-minute session consisted of approximately 20 minutes of instruction on patterning or literacy activities, followed by 10 minutes of instruction on numeracy activities. The tutoring sessions were completed over a span of two weeks, with no more than 3 sessions in a week. Following the completion of tutoring, all participants were individually post-tested by one of five research assistants, who were blind to the child's condition, in two 30-minute sessions. The first posttest session occurred the day after tutoring was completed and included the two measures most closely aligned to tutoring, the RBP and successor and predecessor principle knowledge measures. The second posttest session included our transfer measures of REMA and TBP and occurred 4.16 days after tutoring was completed (median = 3 days, range = 1 to 12 days, *SD* = 2.29); 91% of our sample was posttested for a second time within a week of tutoring completion). A subset of participants (n = 56) were also administered a measure of specific numeracy knowledge at the end of each posttest session (taken from Spaepen et al., 2018), but too few children completed the measures for us to report findings. The sessions were audiotaped or videotaped, based on the preference of parents. The teacher survey was distributed to teachers on the day posttesting began. A graphic of the procedure is included in Figure 2 to further illustrate the study components.

Twenty percent of each trainer's intervention groups (44 groups in total) was coded to determine how consistently trainers implemented the intervention (i.e., their adherence to the tutoring script and instructions). Trainers received a score of 1 for each expectation that they met (about 30 expectations per session). The overall fidelity was excellent (98%) for both conditions.

Data Analysis

Data on successor and predecessor principle knowledge at pretest was missing for 2 children; thus, those 2 data points were imputed through multiple imputations in SPSS by generating 50 datasets and substituting the mean of the imputed scores for the missing values. Because children received tutoring in pairs, we calculated intraclass correlations to test for nonindependence in partner posttest scores (Kenny et al., 2006). Indeed, partners' posttest scores were often modestly related at the level of the dyad for the tutoring conditions (intraclass correlations ranged from .03 for the teacher-based patterning measure in the patterning+numeracy condition to .18 for teacher-based patterning measure in the literacy+numeracy condition). This violated the assumption of nonindependence in traditional analysis of variance models for some posttest measures. Following the recommendations of Kenny et al. (2006), we used multilevel modeling incorporating the actor-partner interdependence model (see http://davidakenny.net/dyad.htm for a tutorial and details on implementing this approach in SPSS). As indicated by Kenny et al. (2006), we specified the use of restricted maximum likelihood estimation and heterogeneous compound symmetry for the variance–covariance structure in the models, allowing the variances for the higher and lower ability dyad members to vary. The significance tests used the Satterthwaite (1946) approximation to estimate the degrees of freedom, which generally results in fractional degrees of freedom (see Kenny et al., 2006). Our model had two levels—the individual level and the dyad level.

As specified in our pre-registration, we conducted preliminary models to identify the most relevant covariates to include in our final models. In these preliminary models, we included the child's and their partner's pretest scores, meaning that both a child's own pretest scores and those of their partner were used as predictors of each individual's outcomes. We also included children's demographic characteristics (i.e., gender, age, race (coded as white (1) or student of color (0) because the distribution by race did not allow for a finer grain distinction), whether they received financial assistance to attend preschool). There were no significant effects of partner's scores or the child's gender, or receiving financial assistance on posttest measures (controlling for pretest and other assessment variables). Thus, we present analyses including variables for child's pretest scores, age, and race to present the most parsimonious models. We remind the reader that race and receiving financial assistance were highly related in the current sample.

To examine condition differences, we conducted 2 orthogonal planned comparisons with each dependent measure. In line with our first hypothesis, for analyses with our patterning measures as outcomes, we tested for an effect of patterning instruction versus no patterning instruction (literacy+numeracy and control), and then tested for differences in performance in the two non-patterning instruction conditions (literacy+numeracy vs. control conditions). In line with our second and third hypotheses, for our math measures, we tested for an effect of tutoring (regardless of type because both conditions included numeracy tutoring), and then tested for an effect of instruction type (patterning+numeracy vs. literacy+numeracy). Note that there was an error in our preregistration of how planned coding of conditions in our statistical models mapped to each of our hypotheses, which we needed to correct. Parallel exploratory Bayesian linear mixed effects analyses for all of our main models were conducted to quantify uncertainty in our estimates by producing 95% highest density interval credible values given the data and model (brms package in R; Bürkner, 2017). These analyses yielded comparable results (i.e., variables with significant effects in the frequentist analyses also yielded 95% credible interval estimates that did not include zero in Bayesian analyses). We also calculated Bayes Factors based on the Bayesian models to evaluate the plausibility of the alternative versus the null hypothesis and the strength of the evidence for our treatment effects (i.e., comparing evidence for models with and without condition codes in the model).

Results

We present results in line with our 3 hypotheses surrounding the effectiveness of combining patterning and numeracy tutoring in relation to numeracy tutoring without patterning tutoring or regular classroom instruction. Means on each of the measures, by condition, are presented in Table 4. At pretest, children performed comparably on all measures across conditions, *p*s > .56.

Research Goal 1

In line with our first research goal, we examined whether patterning+numeracy tutoring improved patterning knowledge more than literacy+numeracy tutoring or regular classroom instruction (see Table 5 for results from all models). In alignment with our hypothesis, children who received patterning+numeracy tutoring scored higher on our research-based patterning measure, Cohen's *d* = .43. Also, in line with hypothesis 1, there was no significant difference in performance between the literacy+numeracy and control groups (see Tables 4 and 5), *d* = .25. The Bayes factor comparing evidence for the model with and without our treatment effects codes (BF_{10} = 175.81) indicated extreme evidence in support of hypothesis 1, in accordance with the interpretation suggestions of Jeffreys (1961). This suggest that the data were over 175 times more likely under the alternative (with treatment effects codes) than the null hypothesis (control variables without treatment effects codes). As shown in Table 5, pretest patterning performance (measured via the teacher-based patterning pretest measure), general math knowledge, child age, and race were also significant positive predictors of research-based patterning performance at posttest.

Contrary to our hypothesis, we found no significant condition differences in performance on our teacher-based patterning posttest measure (Cohen's d = .02 and .10). The Bayes factor comparing evidence for the model with and without the treatment effect codes (BF_{10} = .05) indicated strong evidence for the null (e.g., that the patterning+numeracy condition would not outperform the literacy+numeracy and control conditions). As shown in Table 5, pretest patterning and general math knowledge were both significant predictors of teacher-based patterning posttest performance.

As an additional measure of learning, we conducted exploratory analyses to examine the quality of children's errors on the research-based patterning assessment. As shown in Table 2, the most frequent errors were creating at least one full unit of the model pattern, but having errors in the rest of the pattern (i.e. partial correct) or making a linear sequence that was not a repeating pattern (i.e. nonpattern). We hypothesized that children who received patterning+numeracy tutoring would make more sophisticated errors (i.e. more partially correct responses and less nonpattern responses). We used the frequency of these two error types as the dependent measures in generalized linear mixed-effects poisson models (parallel to the previously described frequentist models but accommodating the non-normal distribution of the count data) to test this hypothesis. Indeed, children who received patterning+numeracy tutoring gave more partially correct responses than children in the literacy+numeracy tutoring group and the control group, $F(1, 203) = 8.73$, $p < .01$, $d = .42$, with no significant difference in performance between the literacy+numeracy and control groups, $F(1, 203) = .11$, $p < .74$, $d = .06$. Additionally, children who received patterning+numeracy tutoring gave significantly less nonpattern responses than children in the literacy+numeracy tutoring group and the control group, $F(1, 203) = 15.61$, $p < .001$, $d = .56$, with no significant difference in performance between the literacy+numeracy and control groups, $F(1, 203) = .00$, $p < .97$, $d = .05$. We also explored whether there were significant differences in the number of wrong pattern errors by condition using similar models; however, no significant differences were found, *F*s < 2.59, *d*s < .27.

Research Goals 2 and 3

For our second and third research goals, we examined whether numeracy tutoring, provided in both the patterning+numeracy and literacy+numeracy conditions, improved math and numeracy knowledge, including specific aspects of numeracy knowledge (i.e., successor and predecessor principle knowledge), more so than regular classroom instruction, and more so in the patterning+numeracy than literacy+numeracy condition, respectively. In contrast to our hypotheses, the three conditions performed similarly on all 3 math measures (see Table 5). There were no significant differences in general math and numeracy knowledge between the two tutoring groups and the control condition (Cohen's *d*s = .02 and .09 respectively), nor between the patterning+numeracy and literacy+numeracy conditions (Cohen's *d*s = .02; see Table 5). The Bayes factors (BF_{10}) comparing evidence for the models predicting general math and numeracy knowledge at posttest with and without our treatment effect codes were .03 and .01, respectively. These values suggested that there was very strong evidence for the null hypotheses. In other words, the data were 33 to 100 times more likely under the null than the alternative hypotheses. Further, our successor and predecessor principle knowledge tutoring was not effective, as successor and predecessor principle knowledge did not differ between the two tutoring groups and the control condition (Cohen's *d* = .02), nor between the patterning+numeracy and literacy+numeracy conditions (Cohen's *d* = .05; see Table 5). The Bayes factor for comparing evidence for the model with and without our treatment effect codes (BF_{10} = .00) indicated extreme evidence for the null hypothesis. Additional analyses suggested that there were no tutoring effects when successor knowledge was examined separately from predecessor knowledge (Cohen's *d* = .05 and .09; see Table S1). Bayes factors (BF_{10}) comparing evidence for this model with and without treatment effects codes was .01, suggesting very strong evidence for the null. We did not examine predecessor knowledge separately given that there were only three items assessing it.

Rather than condition, pretest measures of math (and often patterning knowledge) were the primary predictors across posttest measures of math knowledge (see Table 5). Inspection of means suggested that general mathematics and numeracy knowledge improved from pretest to posttest, regardless of condition, but successor and predecessor principle knowledge did not (see Table 4). Results were confirmed by performing exploratory repeated-measures ANOVAs with test time as a within-subject factor, $F(1, 208) = 144.29$, $p < .001$, $F(1, 208) = 125.66$, $p < .001$, and $F(1, 208) = .02$, $p = .89$, respectively.

Discussion

The present study examined the effects of combining patterning and numeracy tutoring on preschoolers' repeating patterning, numeracy, and general mathematics knowledge in comparison to numeracy tutoring without patterning tutoring and to regular classroom instruction. We obtained causal evidence that preschoolers' repeating patterning knowledge can be improved through targeted instruction, in line with our first hypothesis. However, neither this tutoring nor our other numeracy tutoring condition led to significant improvements in specific or general measures of numeracy and mathematics knowledge compared to regular classroom instruction, counter to our second and third hypotheses. We discuss our findings in relation to our conceptual model on the role of repeating patterning and numeracy knowledge in early mathematics development (see Figure 1), including next steps to examine potential underlying causal mechanisms linking repeating patterning, numeracy, and mathematics knowledge.

Improving Repeating Patterning Knowledge in Preschoolers

One major contribution of the current study was that it utilized a well-controlled experimental design and clearly specified tutoring to provide causal evidence that preschoolers' repeating patterning knowledge can be improved through targeted instruction compared to both active and passive control experiences. Prior research had methodological issues that limited the generalizability of their findings. For example, research on preschoolers often did not randomly assign participants to condition, had limited or no measures of pretest knowledge, and did not standardize the tutoring experiences in terms of content taught nor group size (see, e.g., Papic et al., 2011). The current study also extends past patterning intervention research by Papic et al. (2011) by demonstrating that numeracy instruction without repeating patterning instruction is insufficient to improve repeating patterning knowledge, at least in the short-term.

Our repeating patterning instruction was carefully designed based on the existing literature, and multiple features likely contributed to its effectiveness. First, both tasks and pattern units were carefully sequenced to follow a learning trajectory, beginning with copying AB patterns and gradually increasing to more complex tasks and pattern units, such as abstracting ABC patterns and identifying the underlying rule (Clements & Sarama, 2014; Rittle-Johnson et al., 2015). Second, children were exposed to language describing the patterns in abstract terms and using quantitative language when labeling pattern units (e.g., describing an AABB pattern as "2 of one and 2 of another" with supporting gestures). Multiple patterns were labeled using the same language, (e.g., "The part that repeats is the same. They both have 2 of one and 2 of another"), as shared labels encourage comparison and promote learning of abstract concepts (Namy & Gentner, 2002). Recent experimental research confirms that using abstract, quantitative labels improves preschoolers' repeating patterning performance to a greater extent than using no labels or applying arbitrary symbolic labels (Flynn et al., 2020). Focusing children's attention on quantitative aspects of repeating patterns might redirect attention away from superficial perceptual features of items (such as shape and color) onto more easily duplicable and transferable properties, such as how many of a particular item is in the pattern unit (Collins & Laski, 2015; Fyfe et al., 2015). Third, children received frequent feedback when they completed patterning tasks, including process-level feedback to help them generate a correct response and understand why it was correct, as process-level feedback aids learning (Hattie & Timperley, 2007). Fourth, children were prompted to generate explanations because self-explanation prompts often improve learning, including about patterning (Rittle-Johnson et al., 2008). Fifth, they were asked to distinguish patterns from non-patterns, as contrasting exemplars and non-exemplars improves learning and transfer (see, e.g., Namy & Clepper, 2010).

In addition, we emphasized the importance of finding patterns in numbers. We noted that numbers follow rules just like repeating patterns follow rules, and that when we find a pattern, we know what comes next. By emphasizing number during patterning instruction, children may be primed to think about rules and regularities that are already apparent to them in the domain of number. This in turn may help them look for the rules and regularities in patterns (e.g., that repeating patterns have units that repeat, and this unit can be directly duplicated and abstracted using different materials), and more accurately solve repeating patterning problems. This argument would support the suggestion in our conceptual model (Figure 1) that numeracy knowledge influences repeating patterning knowledge. Future research is needed to identify which features of our tutoring were essential for improving patterning knowledge, as well as additional methods that might further improve learning about patterns. This work should likely consider repeating patterning knowledge as an early start to a much broader initiative that extends into early primary school, such as the Pattern and Structure Mathematics Awareness Program (Mulligan et al., 2020; Mulligan & Mitchelmore, 2009, 2018). Researchers should also be cautious in scaling up our brief tutoring session protocol, since the individualized attention could have accelerated learning to a greater extent than could be accomplished by whole class instruction.

Limitations to the effectiveness of our repeating patterning tutoring must be noted. Specifically, children in the patterning condition had higher performance on the research-based patterning measure, but not the teacher-based patterning measure. The two assessments were given on different days, with a longer delay between tutoring and the teacher-based patterning measure, so children's poorer performance on that measure may have been due to forgetting rather than a lack of transfer. The research-based patterning measure was also more closely aligned with our instructional materials and procedures than the teacher-based patterning measure. The two measures differ in several ways. For example, the teacher-based patterning measure does not include items involving abstracting or duplicating patterns. An informal examination of performance on the research-based patterning measure on these items suggested that students in the pattern+numeracy condition outperformed the other conditions on these two task types. The absence of these two task types on the teacher-based patterning measure may have contributed to it not being sensitive enough to detect condition differences. Additionally, differences in item response formatting may have contributed to poorer performance on the teacher-based versus the research-based patterning assessment. For example, it required a specified number of elements to be added to model patterns, which may have been awkward compared to a more open-ended, flexible format of the research-based patterning assessment. In addition, the patterning materials in the TBP measure included 2D materials that were images of complex objects (compared to the 3D, often simple materials used during tutoring and on the research-based patterning measure). Using images of complex objects on the TBP assessment may have distracted children from attending to underlying structure and impeded performance (Kaminski & Sloutsky, 2013).

Improving Prek Numeracy and Math Knowledge Through Targeted Numeracy Instruction

For our second research goal, we examined the extent to which targeted tutoring of a specific aspects of preschoolers' numeracy knowledge (i.e., successor and predecessor principle) would improve their understanding of this specific aspect of numeracy knowledge, as well as their numeracy and mathematics knowledge more broadly. However, our numeracy tutoring did not result in improved performance on any of our specific or general numeracy and math measures relative to the control condition. This contrasted with past work that has shown improvements in young children's numeracy knowledge from similarly brief trainings (Booth & Siegler, 2008; Ramani & Siegler, 2008; Scalise et al., 2018)

One issue was that there are no established methods for improving children's successor and predecessor principle knowledge in preschool, although success in this endeavor has been achieved with elementary-school children (Baroody et al., 2013). A previous training study with preschool children also failed to find a benefit of their successor principle training condition (Spaepen et al., 2018). Students in our study may have had limited or mixed perquisite prior knowledge, as recent research suggests that knowledge of cardinality and of number-after relations are important for understanding the successor principle (Baroody et al., 2013; Spaepen et al., 2018). Further, the response format may have not been sensitive enough to capture knowledge change on the successor and predecessor principle. Our measure used a forced choice format, while the tutoring elicited open-ended responses. Overall, there is increasing interest in the development of successor and predecessor principle knowledge (Sella & Lucangeli, 2020; Spaepen et al., 2018). Designing an instructional protocol that effectively promotes this knowledge is critical to understanding how it develops and how to support its development.

On the numeracy and general math measures, children's knowledge improved regardless of condition. Our five brief numeracy lessons in the tutoring conditions may have been too few to boost knowledge beyond their regular classroom math instruction. Teachers of children in the current study reported doing numeracy activities an average of about 13 of the past 20 school days. Although teachers' self-reports may be biased and do not capture the quality of the instruction, their self-reports of frequent numeracy input are in line with calls for sustained attention to mathematics instruction in early childhood settings (National Council of Teachers of Mathematics, 2006; National Mathematics Advisory Panel, 2008). Further, the frequency with which teachers reported doing numeracy activities shows that our experimental tutoring was limited in comparison to, and overlapped with children's regular classroom experiences.

Repeating Patterning Tutoring and Gains in Numeracy and Mathematics Knowledge

Our third research goal aimed to test if tutoring in repeating patterning knowledge in combination with numeracy knowledge (i.e., number and number operations) would transfer to improvements in specific and general aspects of numeracy and mathematics knowledge (see Figure 1). However, our tutoring in repeating patterning did not lead to improved numeracy or general mathematics knowledge for preschoolers. Comparing our design to past pattern training studies suggests potential features that might be needed for patterning instruction to improve numeracy knowledge (Hendricks et al., 2006; Kidd et al., 2013, 2014; Papic et al., 2011; Pasnak et al., 2015). First, our 2-week tutoring may not have been sufficiently intensive in comparison to past training studies using year-long interventions (see, e.g., Kidd et al., 2013, 2014; Papic et al., 2011) or extensive and unique enough from ongoing regular classroom instruction. Second, our study posttest may not have been delayed enough. Previous research with preschoolers testing the effects of repeating patterning training on numeracy knowledge did not conduct posttesting until the end of the following school year (Papic et al., 2011). Studies training growing and other complex patterning knowledge in elementary schoolers typically conducted posttesting at the end of the school year (see, e.g., Kidd et al., 2013, 2014). The delay in these studies potentially allowed children to fully benefit from the more explicit patterning instruction on their acquisition of more advanced math concepts introduced during the school year. Third, the children we recruited for our study may not have had as much to gain from our instruction. Past work with elementary-schoolers specifically targeted children struggling with patterning concepts (see, e.g., Kidd et al., 2013; 2014). Fourth, we may have focused on improving the wrong aspect of numeracy knowledge. More recent evidence has suggested that end-of-pre-K repeating patterning knowledge predicted other aspects of numeracy knowledge in kindergarten (i.e., rote counting to 100, associating quantities with verbal and written labels, comparing the magnitude of symbolic numbers, and addition and subtraction in verbal story problems; Rittle-Johnson et al., 2017; Zippert et al., 2020). Additional research suggests concurrent links between patterning knowledge and number relations and operations (i.e., comparing magnitudes and addition and subtraction in verbal story problems; Fyfe et al., 2017; MacKay & De Smedt, 2019; Zippert et al., 2019). Combining repeating patterning tutoring with instruction in other aspects of numeracy knowledge may thus be more effective.

In this study, we focused on promoting repeating patterning knowledge because prior research suggested it is closely tied to math and numerical knowledge development in preschool – and thus has potential to improve math knowledge. A growing body of work has affirmed concurrent and predictive relations between repeating patterning knowledge in preschool and numeracy knowledge months and years later (Rittle-Johnson et al., 2017, 2019; Zippert et al., 2019, 2020). However, we were unable to detect transfer effects from repeating patterning plus focused numeracy tutoring to numeracy and general mathematics knowledge.

This finding parallels mixed success in research promoting other skills (e.g., working memory and spatial skills) to improve mathematics knowledge. Instruction of a diverse group of preschoolers' in a spatial skill (i.e., spatial assembly) did not support overall transfer to gains in a general, conceptual measure of mathematics knowledge or a more symbolic-oriented math measure (Bower et al., 2020). Similarly, efforts to train working memory in preschoolers and kindergarteners have had mixed success in promoting gains in numeracy and math knowledge (Kroesbergen et al., 2014; Raghubar & Barnes, 2017; Ramani et al., 2019). Spatial and working memory skills are thought to be used when engaging in mathematics tasks, but improving these skills, and perhaps patterning knowledge as well, via instruction may not be an effective way to directly improve mathematics knowledge in young children. Further, past studies have not found benefits to improving numeracy and math knowledge when trained simultaneously with other cognitive abilities (Barnes et al., 2016; Kroesbergen et al., 2012; Kyttälä et al., 2015).

Conclusion

Preschoolers' repeating patterning knowledge predicts their later math and numeracy knowledge. The current study thus aimed to obtain causal evidence as to whether repeating patterning and numeracy tutoring impacted repeating patterning, numeracy, and general math knowledge in the pre-kindergarten year. Repeating patterning and numeracy tutoring together improved children's repeating patterning knowledge; however, there was no differential effects on improvements in children's general math and numeracy knowledge. Additionally, we were not successful in improving children's knowledge of the successor and predecessor principle. Thus, we demonstrated that repeating patterning in preschoolers can be improved through targeted tutoring, noting that the tutoring was not intended to be scaled up to a classroom intervention. However, additional research is needed to determine if improving repeating patterning knowledge, perhaps after a delay or over a longer period of time; using different outcomes (e.g., students' explanations), instructional approaches or integrated with whole-group instruction on other aspects of numeracy, can cause improvements in numeracy and mathematics knowledge.

References

Barnes, M. A., Klein, A., Swank, P., Starkey, P., McCandliss, B., Flynn, K., Zucker, T., Huang, C.-W., Fall, A.-M., & Roberts, G. (2016). Effects of tutorial interventions in mathematics and attention for low-performing preschool children. *Journal of Research on Educational Effectiveness*, *9*(4), 577–606. https://doi.org/10.1080/19345747.2016.1191575

Baroody, A. J., Eiland, M. D., Purpura, D. J., & Reid, E. E. (2013). Can computer-assisted discovery learning foster first graders' fluency with the most basic addition combinations? *American Educational Research Journal*, *50*(3), 533–573. https://doi.org/10.3102/0002831212473349

Bates, D., Maechler, M., & Dai, B. (2008). *Lme4: Linear mixed-effects models using s4 classes*. http://lme4.r-forge.rproject.org/

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, *79*(4), 1016–1031. JSTOR.

Bower, C., Zimmermann, L., Verdine, B., Toub, T. S., Islam, S., Foster, L., Evans, N., Odean, R., Cibischino, A., Pritulsky, C., Hirsh-Pasek, K., & Golinkoff, R. M. (2020). Piecing together the role of a spatial assembly intervention in preschoolers' spatial and mathematics learning: Influences of gesture, spatial language, and socioeconomic status. *Developmental Psychology*, *56*(4), 686–698. https://doi.org/10.1037/dev0000899

Burgoyne, K., Witteveen, K., Tolan, A., Malone, S., & Hulme, C. (2017). Pattern understanding: Relationships with arithmetic and reading development. *Child Development Perspectives*, *11*(4), 239–244. https://doi.org/10.1111/cdep.12240

Bürkner, P.-C. (2017). Brms: An R package for Bayesian multilevel models using Stan. *Journal of Statistical Software*, *80*(1), 1–28. https://doi.org/10.18637/jss.v080.i01

Carey, S. (2004). Bootstrapping & the origin of concepts. *Daedalus*, *133*(1), 59–68. https://doi.org/10.1162/001152604772746701

Carraher, D. W., Schliemann, A. D., Brizuela, B. M., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education*, *37*(2), 87–115.

Charles, R. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Educational Leadership*, *73*(3), 9–24.

Cheung, P., Rubenson, M., & Barner, D. (2017). To infinity and beyond: Children generalize the successor function to all possible numbers years after learning to count. *Cognitive Psychology*, *92*, 22–36. https://doi.org/10.1016/j.cogpsych.2016.11.002

Cho, S.-J., & Rabe-Hesketh, S. (2011). Alternating imputation posterior estimation of models with crossed random effects. *Computational Statistics & Data Analysis*, *55*(1), 12–25. https://doi.org/10.1016/j.csda.2010.04.015

Clements, D. H., & Sarama, J. (2007a). Effects of a preschool mathematics curriculum: Summative research on the Building Blocks Project. *Journal for Research in Mathematics Education*, *38*(2), 136–163. https://doi.org/10.2307/30034954

Clements, D. H., & Sarama, J. (2007b). *Real math building blocks*. McGraw-Hill Education.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). Routledge.

Collins, M. A., & Laski, E. V. (2015). Preschoolers' strategies for solving visual pattern tasks. *Early Childhood Research Quarterly*, *32*(3), 204–214. https://doi.org/10.1016/j.ecresq.2015.04.004

Common Core State Standards. (2010). Mathematics standards. http://www.corestandards.org/Math/

Dickinson, D. K., Copley, J. V., Izquierdo, E., Lederman, J. S., Schickedanz, J. A., & Wright, L. (2014). *Opening the world of learning planning and assessment*. Pearson Learning Solutions.

Duncan, G. J., Claessens, A., Magnuson, K., Klebanov, P., Pagani, L. S., Feinstein, L., Engel, M., Brooks-gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446. https://doi.org/10.1037/0012-1649.43.6.1428

Economopoulos, K. (1998). What comes next?: The mathematics of pattern in kindergarten. *Teaching Children Mathematics*, *5*(4), 230–233.

Flynn, M. E., Guba, T. P., & Fyfe, E. R. (2020). ABBABB or 1212: Abstract language facilitates children's early patterning skills. *Journal of Experimental Child Psychology*, *193*, 104791. https://doi.org/10.1016/j.jecp.2019.104791

Fyfe, E. R., Evans, J. L., Matz, L. E., Hunt, K. M., & Alibali, M. W. (2017). Relations between patterning skill and differing aspects of early mathematics knowledge. *Cognitive Development*, *44*, 1–11. https://doi.org/10.1016/j.cogdev.2017.07.003

Fyfe, E. R., McNeil, N. M., & Rittle-Johnson, B. (2015). Easy as ABCABC: Abstract language facilitates performance on a concrete patterning task. *Child Development*, *86*(3), 927–935. https://doi.org/10.1111/cdev.12331

Fyfe, E. R., Rittle-Johnson, B., & Farran, D. C. (2019). Predicting success on high-stakes math tests from preschool math measures among children from low-income homes. *Journal of Educational Psychology*, *111*(3), 402–413. https://doi.org/10.1037/edu0000298

Greenes, C., Ginsburg, H. P., & Balfanz, R. (2004). Big math for little kids. *Early Childhood Research Quarterly*, *19*(1), 159–166. https://doi.org/10.1016/j.ecresq.2004.01.010

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81–112. https://doi.org/10.3102/003465430298487

Hendricks, C., Trueblood, L., & Pasnak, R. (2006). Effects of teaching patterning to 1st-graders. *Journal of Research in Childhood Education, 21*(1), 79–89. https://doi.org/10.1080/02568540609594580

Jeffreys, H. (1961). *Theory of probability* (Third Edition). Oxford University Press.

Jordan, N. C., Glutting, J., Ramineni, C., & Watkins, M. W. (2010). Validating a number sense screening tool for use in kindergarten and first grade: Prediction of mathematics proficiency in third grade. *School Psychology Review*, *39*(2), 16.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*(3), 850–867. https://doi.org/10.1037/a0014939

Kaminski, J. A., & Sloutsky, V. M. (2013). Extraneous perceptual information interferes with children's acquisition of mathematical knowledge. *Journal of Educational Psychology, 105*(2), 351–363. https://doi.org/10.1037/a0031040

Kenny, D. A., Kashy, D. A., & Cook, W. L. (2006). *Dyadic data analysis*. Guilford Press.

Kidd, J. K., Carlson, A. G., Gadzichowski, K. M., Boyer, C. E., Gallington, D. A., & Pasnak, R. (2013). Effects of patterning instruction on the academic achievement of 1st-grade children. *Journal of Research in Childhood Education*, *27*(2), 224–238. https://doi.org/10.1080/02568543.2013.766664

Kidd, J. K., Pasnak, R., Gadzichowski, K. M., Gallington, D. A., McKnight, P., Boyer, C. E., & Carlson, A. (2014). Instructing first-grade children on patterning improves reading and mathematics. *Early Education and Development*, *25*(1), 134–151. https://doi.org/10.1080/10409289.2013.794448

Kroesbergen, E. H., Van ’t Noordende, J., & Kolkman, M. E. (2014). Training working memory in kindergarten children: Effects on working memory and early numeracy. *Child Neuropsychology*, *20*(1), 23–37. https://doi.org/10.1080/09297049.2012.736483

Kroesbergen, E. H., Van’t Noordende, J. E., & Kolkman, M. E. (2012). Number sense in low-performing kindergarten children: Effects of a working memory and an early math training. In Z. Breznitz, O. Rubinsten, V. J. Molfese, & D. L. Molfese (Eds.), *Reading, Writing, Mathematics and the Developing Brain: Listening to Many Voices* (pp. 295–313). Springer Netherlands. https://doi.org/10.1007/978-94-007-4086-0_16

Kyttälä, M., Kanerva, K., & Kroesbergen, E. (2015). Training counting skills and working memory in preschool. *Scandinavian Journal of Psychology*, *56*(4), 363–370. https://doi.org/10.1111/sjop.12221

Lipkus, I. M., & Peters, E. (2009). Understanding the role of numeracy in health: Proposed theoretical framework and practical insights. *Health Education & Behavior*, *36*(6), 1065–1081. https://doi.org/10.1177/1090198109341533

Lüken, M. M., & Sauzet, O. (2020). Patterning strategies in early childhood: A mixed methods study examining 3- to 5-year-old children’s patterning competencies. *Mathematical Thinking and Learning*, Advance Online Publication. https://doi.org/10.1080/10986065.2020.1719452

MacKay, K. J., & De Smedt, B. (2019). Patterning counts: Individual differences in children's calculation are uniquely predicted by sequence patterning. *Journal of Experimental Child Psychology*, *177*, 152–165. https://doi.org/10.1016/j.jecp.2018.07.016

Mason, J., Stephens, M., & Watson, A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal*, *21*(2), 10–32. https://doi.org/10.1007/BF03217543

McGarvey, L. M. (2012). What Is a pattern? Criteria used by teachers and young children. *Mathematical Thinking and Learning*, *14*(4), 310–337. https://doi.org/10.1080/10986065.2012.717380

Miller, M. R., Rittle-Johnson, B., Loehr, A. M., & Fyfe, E. R. (2016). The influence of relational knowledge and executive function on preschoolers' repeating pattern knowledge. *Journal of Cognition and Development*, *17*(1), 85–104. https://doi.org/10.1080/15248372.2015.1023307

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, *21*(1), 33–49. https://doi.org/10.1007/BF03217544

Mulligan, J., & Mitchelmore, M. (2018). Promoting early mathematical structural development through an integrated assessment and pedagogical program. In I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank, & C. Benz (Eds.), *Contemporary Research and Perspectives on Early Childhood Mathematics Education* (pp. 17–33). Springer International Publishing. https://doi.org/10.1007/978-3-319-73432-3_2

Mulligan, J., Oslington, G., & English, L. (2020). Supporting early mathematical development through a 'pattern and structure' intervention program. *ZDM*, *52*(4), 663–676. https://doi.org/10.1007/s11858-020-01147-9

Namy, L. L., & Clepper, L. E. (2010). The differing roles of comparison and contrast in children's categorization. *Journal of Experimental Child Psychology*, *107*(3), 291–305. https://doi.org/10.1016/j.jecp.2010.05.013

Namy, L. L., & Gentner, D. (2002). Making a silk purse out of two sow's ears: Young children's use of comparison in category learning. *Journal of Experimental Psychology: General*, *131*(1), 5–15. https://doi.org/10.3102/003465430298487

National Association for the Education of Young Children. (2002). *Early childhood mathematics: Promoting good beginnings*. http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Early+Childhood+Mathematics+:+Promoting+Good+Beginnings#4

National Association for the Education of Young Children. (2014). *NAEYC early childhood program standards and accreditation criteria*. National Association for the Education of Young Children.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics*. National Council of Teachers of Mathematics.

National Mathematics Advisory Panel. (2008). The final report of the national mathematics advisory panel. *Foundations*, *37*, 595–601.

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly*, *36*(3), 550–560. https://doi.org/10.1016/j.ecresq.2016.02.003

Papic, M. M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education*, *42*(3), 237–269. https://doi.org/10.5951/jresematheduc.42.3.0237

Pasnak, R., Kidd, J. K., Gadzichowski, K. M., Gallington, D. A., Schmerold, K. L., & West, H. (2015). Abstracting sequences: Reasoning that is a key to academic achievement. *The Journal of Genetic Psychology*, *176*(3), 171–193. https://doi.org/10.1080/00221325.2015.1024198

Raghubar, K. P., & Barnes, M. A. (2017). Early numeracy skills in preschool-aged children: A review of neurocognitive findings and implications for assessment and intervention. *The Clinical Neuropsychologist*, *31*(2), 329–351. https://doi.org/10.1080/13854046.2016.1259387

Ramani, G. B., Daubert, E. N., Lin, G. C., Kamarsu, S., Wodzinski, A., & Jaeggi, S. M. (2019). Racing dragons and remembering aliens: Benefits of playing number and working memory games on kindergartners' numerical knowledge. *Developmental Science*, *23*(4) https://doi.org/10.1111/desc.12908

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development*, *79*(2), 375–394. https://doi.org/10.1111/j.1467-8624.2007.01131.x

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science*, *24*(7), 1301–1308. https://doi.org/10.1177/0956797612466268

Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development*, *88*(5), 1727–1742. https://doi.org/10.1111/cdev.12662

Rittle-Johnson, B., Fyfe, E. R., Loehr, A. M., & Miller, M. R. (2015). Beyond numeracy in preschool: Adding patterns to the equation. *Early Childhood Research Quarterly*, *31*, 101–112. https://doi.org/10.1016/j.ecresq.2015.01.005

Rittle-Johnson, B., Saylor, M., & Swygert, K. E. (2008). Learning from explaining: Does it matter if mom is listening? *Journal of Experimental Child Psychology*, *100*(3), 215–224. https://doi.org/10.1016/j.jecp.2007.10.002

Rittle-Johnson, B., Zippert, E., & Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly*, *46*, 166–178. https://doi.org/10.1016/j.ecresq.2018.03.006

Sarama, J., & Clements, D. H. (2004). Building blocks for early childhood mathematics. *Early Childhood Research Quarterly*, *19*(1), 181–189. https://doi.org/10.1016/j.ecresq.2004.01.014

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition*, *108*(3), 662–674. https://doi.org/10.1016/j.cognition.2008.05.007

Satterthwaite, F. E. (1946). An approximate distribution of estimates of variance components. *Biometrics Bulletin*, *2*(6), 110–114. JSTOR. https://doi.org/10.2307/3002019

Scalise, N. R., Daubert, E. N., & Ramani, G. B. (2018). Narrowing the early mathematics gap: A play-based intervention to promote low-income preschoolers' number skills. *Journal of Numerical Cognition*, *3*(3), 559–581. https://doi.org/10.5964/jnc.v3i3.72

Sella, F., & Lucangeli, D. (2020). The knowledge of the preceding number reveals a mature understanding of the number sequence. *Cognition*, *194*, 104104. https://doi.org/10.1016/j.cognition.2019.104104

Shapka, J. D., Domene, J. F., & Keating, D. P. (2006). Trajectories of career aspirations through adolescence and young adulthood: Early math achievement as a critical filter. *Educational Research and Evaluation*, *12*(4), 347–358. https://doi.org/10.1080/13803610600765752

Spaepen, E., Gunderson, E. A., Gibson, D., Goldin-Meadow, S., & Levine, S. C. (2018). Meaning before order: Cardinal principle knowledge predicts improvement in understanding the successor principle and exact ordering. *Cognition*, *180*, 59–81. https://doi.org/10.1016/j.cognition.2018.06.012

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly*, *19*(1), 99–120. https://doi.org/10.1016/j.ecresq.2004.01.002

Steen, L. A. (1988). The science of patterns. *Science*, *240*(4852), 611–616. https://doi.org/10.1126/science.240.4852.611

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds.). Harvard University Press.

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher*, *43*(7), 352–360. https://doi.org/10.3102/0013189X14553660

Weiland, C., Wolfe, C. B., Hurwitz, M. D., Clements, D. H., Sarama, J. H., & Yoshikawa, H. (2012). Early mathematics assessment: Validation of the short form of a prekindergarten and kindergarten mathematics measure. *Educational Psychology*, *32*(3), 311–333. https://doi.org/10.1080/01443410.2011.654190

Wijns, N., Torbeyns, J., Bakker, M., De Smedt, B., & Verschaffel, L. (2019). Four-year olds' understanding of repeating and growing patterns and its association with early numerical ability. *Early Childhood Research Quarterly, 49,* 152–163. https://doi.org/10.1016/j.ecresq.2019.06.004

Wood, D., & Middleton, D. (1975). A study of assisted problem-solving. *British Journal of Psychology*, *66*(2), 181–191. https://doi.org/10.1111/j.2044-8295.1975.tb01454.x

Zippert, E., Clayback, K., & Rittle-Johnson, B. (2019). Not just IQ: Patterning predicts preschoolers' math knowledge beyond fluid reasoning. *Journal of Cognition and Development*, *20*(5), 752–771. https://doi.org/10.1080/15248372.2019.1658587

Zippert, E., Douglas, A., & Rittle-Johnson, B. (2020). Finding patterns in objects and numbers: Repeating patterning in pre-K predicts kindergarten mathematics knowledge. *Journal of Experimental Child Psychology*, *200*, 104965. https://doi.org/10.1016/j.jecp.2020.103965.

Zippert, E., Loehr, A. M., & Rittle-Johnson, B. (2018, February). *A new teacher-based assessment of preschoolers' patterning skills* [Poster]. Society for Research on Educational Effectiveness, Washington, D.C.

Table 1

Frequency of Teacher-Reported Numeracy and Patterning Activities Per Month

Math Concept	***M* (*SD*)**
Numeracy[a]	13.40 (3.66)
Count items	19.39 (1.90)
Count out loud above 10	17.16 (5.39)
Match number names to appropriate set of objects	13.70 (7.12)
Talk about what number comes before or after another	13.42 (7.42)
Compare quantities	12.93 (6.34)
Name written numerals	12.74 (7.72)
Simple adding & subtracting with objects	11.97 (6.99)
Count backwards	10.07 (7.82)
Add simple sums or talk about number facts without objects	8.21 (8.44)
Patterning[b]	9.93 (2.60)
Discuss patterns in days of week, months in year, or seasons	15.18 (7.38)
Make or copy pattern with objects or sounds	10.80 (7.63)
Figure out what comes next in pattern	10.02 (7.11)
Describe patterns in words	9.36 (7.34)
Copy a pattern with different materials	6.79 (7.54)
Identify the part that repeats in patterns	8.46 (7.58)

Note. [a]Average of teachers' reports (*n* = 44) of including nine numeracy activities in math instruction. [b]Average of teachers' reports (*n* = 45) of including six patterning activities in math instruction.

Table 2

Descriptive Statistics on Responses Types on the Research-based Patterning Measure

Response Type	**Description**	**Proportion across all responses *M*(*SD*)**			
		Overall	**Patterning**	**Literacy**	**Control**
Correct	Correct. At least one full unit model pattern.	0.55(0.25)	0.62(0.22)	0.49(0.26)	0.54(0.27)
Incorrect					
Partial Correct	At least 1 full unit (2 for AB patterns) at beginning or end. Errors present.	0.11(0.10)	0.13(0.10)	0.09(0.09)	0.09(0.10)
Wrong Pattern AB	At least 3 consecutive units AB (or BA). Errors may be present before and/or after.	0.04(0.08)	0.06(0.03)	0.05(0.10)	0.05(0.08)
Wrong Pattern Other	At least 2 consecutive units of a proper pattern. Errors may be present before and/or after.	0.03(0.05)	0.03(0.06)	0.03(0.05)	0.02(0.04)
Non-Pattern	Linear sequence with no discernible repeating patterns (including sequences sorted by color or shape).	0.26(0.27)	0.17(0.21)	0.31(0.29)	0.29(0.30)
Off Task	Not a linear sequence.	0.02(0.09)	0.02(0.10)	0.03(0.08)	0.01(0.09)

Note. Frequencies exclude item 1 which required students to identify the missing item in an AB pattern. It was coded using a different coding scheme (i.e., whether children responded correctly (*M* = .69), chose the incorrect item in the pattern (*M* = .26), or chose a random item to complete the pattern (*M* = .05)).

Table 3

Description of Tutoring Activities by Session

	Session 1	Session 2	Session 3	Session 4	Session 5
Pattern	• Duplicate and extend patterns using pattern strips • Make patterns with dance and body movements	• Duplicate and extend patterns using string beads • Make patterns with dance and body movements	• Identify core unit of patterns using cubes and pattern strips • Make patterns with dance and body movements	• Abstract patterns using pattern strips • Make patterns with dance and body movements	• Review core unit and abstract patterns using pattern strips • Make patterns with dance and body movements
Numeracy	• Bird Book, adding 1 bird each page • What's different activity – hiding x number of items under a cloth and adding 1 (with small numbers, 1-5)	• Create stairs with Unifix cubes – adding 1 cube each time with small numbers 1-5	• 5 little monkeys song • What's different activity – hiding x number of items under a cloth and adding 1 (with small numbers, 1-5)	• Create stairs with Unifix cubes – adding 1 cube each time with large numbers 6-10	• What's different activity – hiding x number of items under a cloth and adding 1 (with small & large numbers, 1-10)

Table 4

Descriptive Statistics of Study Measures

Variables	Accuracy by Condition *M (SD)*					
	Control[a]		Literacy+Numeracy[b]		Patterning+Numeracy[c]	
	Pretest	**Posttest**	**Pretest**	**Posttest**	**Pretest**	**Posttest**
IRT Scores						
Research-based Patterning	--	-0.05 (2.33)	--	-0.61 (2.23)	--	0.63 (2.07)
Teacher-based Patterning	0.04 (1.41)	0.07 (1.37)	-0.10 (1.45)	-0.07 (1.46)	0.17 (1.30)	0.02 (1.30)
General Math	-0.93 (0.93)	-0.43 (0.96)	-0.92 (0.93)	-0.42 (0.82)	-0.93 (0.92)	-0.40 (1.00)
General Numeracy	-0.72 (1.70)	0.12 (1.55)	-0.72 (1.60)	0.00 (1.46)	-0.84 (1.57)	-0.03 (1.55)
Successor and Predecessor Principle	0.04 (0.46)	0.00 (0.57)	-0.03 (0.46)	-0.01 (0.42)	0.00 (0.48)	0.02 (0.56)
Raw Scores						
Research-based Patterning	--	5.80(3.38)	--	5.00(3.27)	--	6.83(3.03)
Teacher-based Patterning	2.93(2.55)	3.80(2.54)	2.66(2.62)	3.54(2.67)	3.13(2.36)	3.75(2.41)
General Math	10.90(4.20)	11.99(4.48)	10.99(4.16)	12.04(3.93)	10.97(4.22)	12.13(4.72)
General Numeracy	7.62(2.79)	8.55(2.79)	7.65(2.64)	8.37(2.62)	7.46(2.70)	8.26(2.87)
Successor and Predecessor Principle	6.77(1.89)	6.73(2.11)	6.49(1.88)	6.77(1.59)	7.62(1.96)	6.82(2.07)

[a] *n* = 71. [b] *n* = 68. [c] *n* = 72.

Table 5

Multilevel Modeling Frequentist and Bayesian Analysis Results for Posttest Measures

Parameter	Frequentist Analyses				Bayesian Analyses			
	Coefficient	*SE*	*df*	*t*	Estimate	*SE*	95% CI Lower	95% CI Upper
Research-based Patterning Post H1								
Intercept	-3.68	1.51	189.79	-2.44*	-3.23	1.56	-6.30	-0.19
Patterning vs. Literacy & Control	0.27	0.07	100.78	3.69**	0.27	0.08	0.12	0.42
Literacy vs. Control	-0.22	0.13	104.02	-1.70	-0.25	0.13	-0.53	0.01
Patterning Pre	0.35	0.09	199.40	3.87**	0.32	0.09	0.14	0.51
General Math Pre	1.12	0.15	184.95	7.34**	1.13	0.16	0.82	1.44
Successor & Predecessor Principle Pre	0.20	0.27	195.48	0.71	0.31	0.28	-0.24	0.84
Age	0.93	0.31	189.67	3.02**	0.85	0.32	0.22	1.48
Race	0.59	0.21	179.70	2.82**	0.49	0.22	0.05	0.93
Teacher-based Patterning Post H1								
Intercept	-0.16	0.98	195.37	-0.16	-0.40	1.02	-2.41	1.59
Patterning vs. Literacy & Control	-0.02	0.05	99.35	-0.41	-0.04	0.05	-0.14	0.05
Literacy vs. Control	-0.02	0.09	102.39	-0.18	-0.03	0.09	-0.20	0.15
Patterning Pre	0.56	0.06	198.38	9.61**	0.56	0.06	0.44	0.68
General Math Pre	0.28	0.10	182.36	2.90**	0.26	0.11	0.06	0.46
Successor & Predecessor Principle Pre	0.10	0.18	192.41	0.57	0.09	0.18	-0.25	0.45
Age	0.06	0.20	195.10	0.29	0.11	0.21	-0.29	0.51
Race	0.24	0.14	186.86	1.77	0.25	0.14	-0.02	0.51
General Math Knowledge Post H2, H3								
Intercept	0.30	0.58	191.68	0.52	0.16	0.60	-1.03	1.36
Patterning & Literacy vs Control	-0.00	0.03	101.96	0.23	0.01	0.03	-0.05	0.06
Patterning vs Literacy	0.01	0.05	99.20	-0.16	-0.02	0.05	-0.12	0.09
General Math Pre	0.60	0.06	185.27	10.28**	0.61	0.06	0.49	0.72
Patterning Pre	0.11	0.03	197.91	3.19**	0.11	0.04	0.04	0.18
Successor & Predecessor Principle Pre	0.28	0.10	194.64	2.69**	0.25	0.10	0.05	0.46
Age	-0.06	0.12	190.64	-0.49	-0.02	0.12	-0.26	0.22
Race	0.19	0.08	182.19	2.36*	0.17	0.08	0.01	0.33
General Numeracy Post H2, H3								
Intercept	-1.06	0.84	168.01	-1.26	-0.93	0.87	-2.64	0.78
Patterning & Literacy vs Control	-0.03	0.04	99.38	-0.71	-0.03	0.04	-0.12	0.06
Patterning vs Literacy	-0.03	0.07	96.76	-0.39	-0.03	0.08	-0.18	0.13
General Numeracy Pre	0.58	0.05	199.14	11.70**	0.57	0.05	0.47	0.67
Patterning Pre	0.08	0.05	195.54	1.48	0.09	0.06	-0.01	0.20
Successor & Predecessor Principle Pre	0.54	0.16	191.96	3.40**	0.60	0.16	0.29	0.91
Age	0.29	0.17	164.61	1.67	0.26	0.18	-0.10	0.62
Race	0.31	0.12	162.56	2.55*	0.28	0.13	0.03	0.53
Successor & Predecessor Principle Post H2, H3								
Intercept	0.03	0.39	189.03	0.10	0.07	0.39	-0.70	0.80
Patterning & Literacy vs Control	0.01	0.02	99.73	0.37	0.01	0.02	-0.03	0.04
Patterning vs Literacy	-0.01	0.03	96.81	-0.30	-0.01	0.03	-0.07	0.06
Successor & Predecessor Principle Pre	0.37	0.07	192.37	5.31**	0.38	0.07	0.24	0.52
General Math Pre	0.17	0.04	192.56	4.42**	0.18	0.04	0.10	0.26
Patterning Pre	0.06	0.02	197.31	2.69**	0.06	0.02	0.02	0.10
Age	0.02	0.08	184.34	0.25	0.01	0.08	-0.14	0.17
Race	0.06	0.05	180.76	1.03	0.06	0.05	-0.05	0.16

Note. CI = credible interval. Race is coded as 1 = white, 0 = student of color. * $p < .05$. ** $p < .01$

Figure 1.

Conceptual Model Depicting Pathways Through Which Each Training Type is Theorized to Influence Prek Early Numeracy and Mathematics Knowledge.

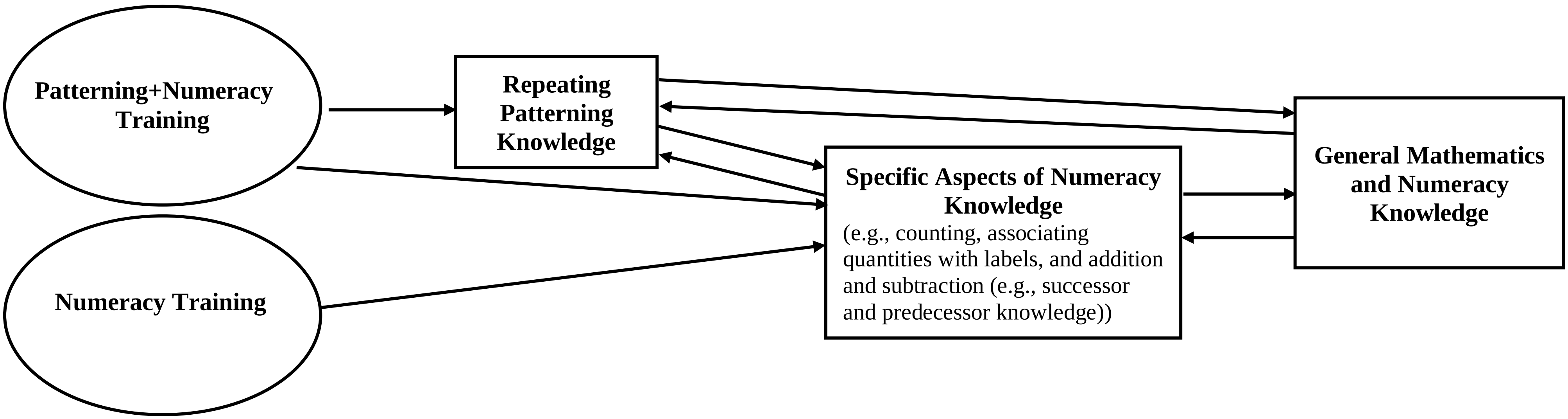
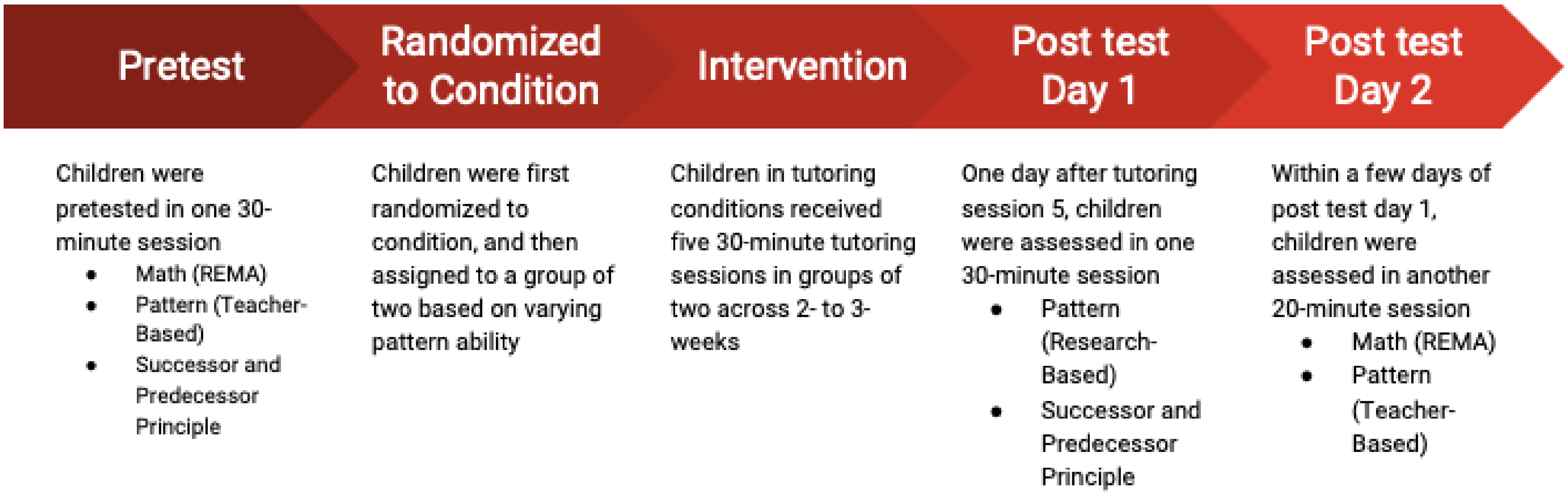


Figure 2.

Study Design.



Appendix

Sample Tutoring Activities

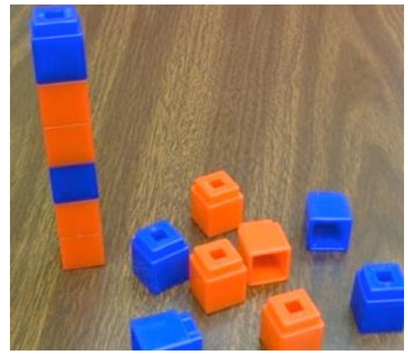
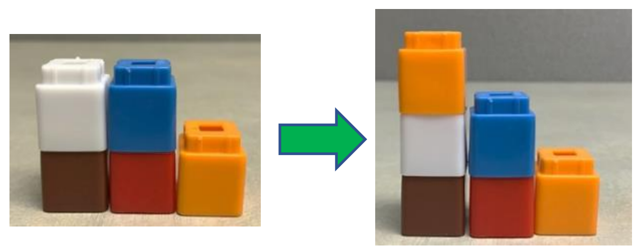
Tutoring Activity Type	Description	Example
Patterning Activity	Create AAB Core Unit: “In repeating patterns, the part that repeats over and over again is called the core unit. This is our core unit: Orange-Orange-Blue.” “Can you make the same core unit as me?” “Let’s put all of our core units together. Look, now we have a pattern!”	
Numeracy Activity	Build Cube Stairs: “We will make stairs out of these cubes, starting at 1 and going up to 5.” “The first stair has 1 cube, and the second stairs has 2. How many cubes do we need to make the next step?” “Right! Because the number that comes right after 2 is 3. Ok, I have 2 cubes but we need 3. What do we need to do?” “Right, add 1 more! 3 is 1 more than 2”	

Table S1

Multilevel Modeling Results for Successor Principle Measure

Parameter	Frequentist Analyses				Bayesian Analyses			
	Coefficient	*SE*	*df*	*t*	Estimate	*SE*	95% CI Lower	95% CI Upper
Successor Principle Knowledge Post H2, H3								
Intercept	0.17	0.35	190.24	0.48	0.13	0.35	-0.53	0.81
Patterning & Literacy vs Control	0.00	0.02	104.41	0.20	0.01	0.02	-0.03	0.04
Patterning vs Literacy	-0.03	0.03	99.86	-0.86	-0.03	0.03	-0.09	0.03
Successor Principle Pretest	0.53	0.08	187.85	6.63**	0.53	0.08	0.38	0.69
General Math Pretest	0.12	0.03	193.39	3.70**	0.12	0.03	0.05	0.19
Patterning Pretest	0.06	0.02	197.35	2.87**	0.06	0.02	0.02	0.10
Age	-0.01	0.07	187.48	-0.18	-0.01	0.07	-0.14	0.13
Race	0.02	0.05	183.09	0.45	0.02	0.05	-0.07	0.11

Note. Race is coded as 1 = White, 0 = Other. **$p < .01$