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Mathematical Education in the Italian Project FSTF

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Abstract

A modern school should encourage students to deepen the subjects autonomously and beyond the normal curricula. Such aim, requiring both well-disposed pupils and cooperative teachers, was reached by the pioneering interdisciplinary project "From Soccerene to Fullerene" in 2013. The scientific education was noticeably improved, especially for the mathematical aspect, through a constructivist activity never tried before in the Italian school system.

Keywords: Plane and solid geometry, Archimedean solids, Platonic solids, Fullerene.

Acknowledgment: I'm grateful to the Organizing and Scientific Committee of the IX EMS 2015 for having given me the chance to present a pioneering Italian educational experience in the marvelous venue of the ATINER in Greece. My thanks to the students Simone Ciotti and Valeria Volpe for their gift of a large amount of splendid pictures about, respectively, the planetarium and the ball, among which are the figures in the paper. I also wish to thank both the FSTF Manager Francesco Truppa and the Liceo Grassi's Headmaster Loretta Tufo for their confidence in my ability of divulging the project's characteristics and achievements through papers and talks around the world.

Introduction

Two years ago, six brilliant and motivated students of the Scientific High School "G.B. Grassi" in Latina (Italy) joined the educational project "From Soccerene to Fullerene" (acronym FSTF) proposed by five teachers from the Departments of *Physical Education* and of *Mathematics and Physics*. The pupils investigated the *fullerene* buckyball (C_{60}) also known as *soccerene* for its football-shaped structure (truncated icosahedron). After some calculations, they cut twelve black pentagons and twenty white hexagons and they sewed the plastic polygons together on a polystyrene sphere as faces of a soccer ball; the students repeated the same operation twice, making two almost identical soccer-like balls. The multidisciplinary experience (involving knowledge and skills of Mathematics, Physical Education, Physics and Science) started on March 2013 and ended on June 2013. It was satisfactorily discussed in the 2013 school-leaving examination and elicited a genuine interest during the 100^{th} national conference of the Italian Physical Society in 2014 [Ausili 2014, Bonacci 2014]. Now we briefly describe the educational results of the FSTF project, focusing on its mathematical features such as the path of gradual awareness about the difficulty to build a convex solid approximating a sphere and the importance of the thirteen *Archimedean solids*, often underestimated, with respect to the five *Platonic solids*, in the secondary school curriculum.

The FSTF Project

Crew of the FSTF
Students: Germana Cardarelli, Letizia Gambacorta, Marina Garrido Cutiño, Irene Olivadese, Marco Rosella, Kola Tytler.
Teachers: Enzo Bonacci, Roberta Calvitti, Aurora Curinga, Maria Grazia Saviano, Francesco Truppa.

Data of the FSTF
Project Title: "Dal Soccerene al Fullerene"
Project Manager: Francesco Truppa
Project Period: March 2013 – June 2013
Project Place: Liceo Scientifico Statale "G.B. Grassi", Latina (Italy)

Premises of the FSTF

The pupils learned the reasons why a soccer ball must be spherical [Ludwig and Guerrerio 2011]. They realized that the soccer ball is a miniaturized version of the geodesic domes patented in 1954 by the architect *Richard Buckminster Fuller* and that, for structural analogy, the molecule of 60 carbon atoms was named *buckminsterfullerene* or *soccerene*. The students understood that the fullerene is a topical scientific issue [Porter 1993] and that also the related new materials and nanotechnologies [Pedersen 2013] are

worthy of investigation in a scientific high school. The pupils' team was already accustomed with the hexagonal-only geodesic dome (Figure 1) of the planetarium "Livio Gratton" in Latina [Bonacci 2013].

Figure 1. *The Planetarium "Livio Gratton" and its Geodesic Dome*

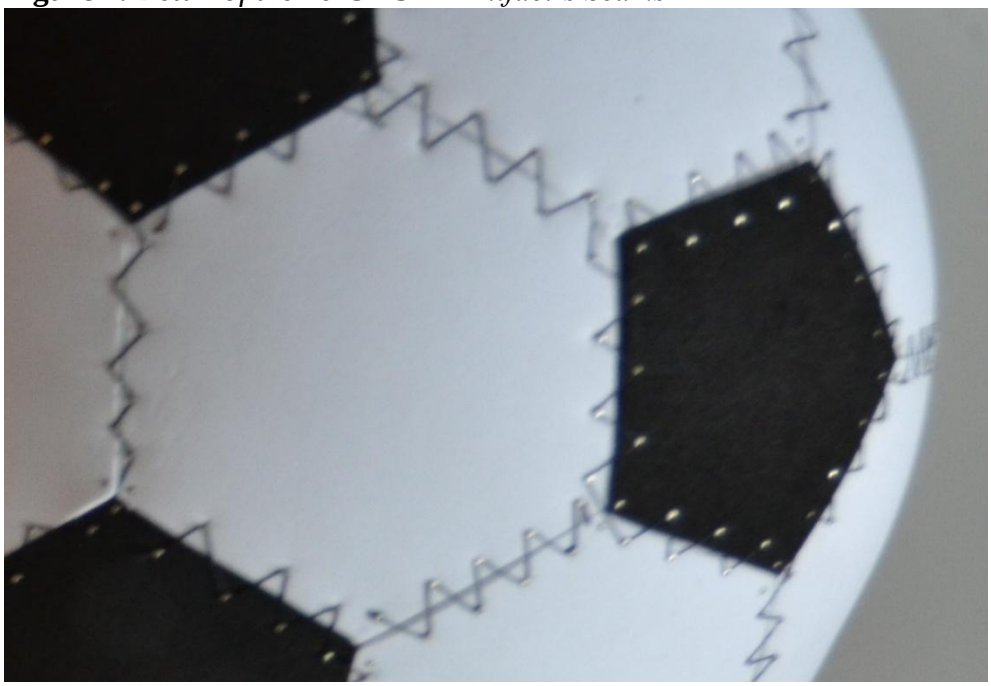
Mathematical Education in the FSTF Project

Once assimilated the geometry of the Carbon-60 molecule, the pupils were given two identical polystyrene spheres (each of a diameter of 25 cm), some plastic sheets (black and white), a 50 m long black thread and around 2000 pins. The students inferred soon that the curricular *Platonic solids* (tetrahedron, cube, octahedron, dodecahedron, icosahedron) were not suitable for constructing a spherical ball. They discovered how Archimedes introduced thirteen semi-regular convex polyhedra, whose faces are two or more types of regular polygons, and that only two types of *Archimedean solids* are closely approximated by a sphere: *rhombicosidodecahedrons* (12 pentagons, 30 squares and 20 triangles) and *truncated icosahedrons* (12 pentagons, 20 hexagons). The students used the Descartes-Euler polyhedral formula to establish a well-known result: the best choice for building a soccer ball is the *truncated icosahedron* [Chung and Sternberg 1993]. The pupils found that, since 20 hexagons and 12 pentagons must cover the whole spherical surface they had to calculate the unknown common x-edge between the polygons. Then, the pupils' team took four computational steps autonomously:

1) Since the radius was $r = 12.5\ cm$, the spherical surface of each polystyrene sphere had to be $S_s = \mathbf{625}\pi \approx \mathbf{1963\ cm^2}$.
2) The surface covered by 20 hexagons had to be $S_h = 30\sqrt{3}x^2$.
3) The surface covered by 12 pentagons had to be $S_p = 15\frac{1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}x^2$.
4) Since $S_s = S_h + S_p$ then $625\pi = 30\sqrt{3}x^2 + 15\frac{1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}x^2$.
5)

The solution $x \approx 5.2\ cm$ was the length of the polygons' side (Figure 2).

Figure 2. *Detail of the 2013 FSTF Artifact's Seams*



The FSTF Artifacts

The students drew hexagons and pentagons (of side approximately 5.2 cm) on, respectively, white and black plastic sheets. Then they cut them off and applied the polygons on the polystyrene balls, each with a diameter (d=25 cm) a bit larger than the official football "size 5" (d=22÷23 cm). The students used pins and thread to reproduce the football's seams (Figure 3).

Figure 3. *Top Front View of the 2013 FSTF Artifact*

Further FSTF Achievements

Beyond an incisive mathematical education and the realization of peculiar artifacts (Figure 3), the 2013 FSTF project had other educational results, such as: the collaboration between traditionally distant Departments; an application of the *learning by doing* and *constructivism* in Science [Giuliano 2013]; a discussion of cross-sectional issues.

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