

Implementation of GeoGebra a Dynamic Mathematical Software for Teaching and Learning of Calculus in Ethiopia

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Abstract

GeoGebra mathematical software is the best alternative way of teaching and learning calculus in 21st century by exploration and visualization. This software opposes the traditional teaching and learning method, or solution only approaches and make the teaching and learning process more active in the classroom which goes with the constructive approach of teaching and learning methods. The study aims to implement the GeoGebra mathematical software/technological tool/ oriented lesson plan in classroom using some developed steps and list out some criteria that the teacher may need to know to implement this dynamic software in his or her classroom. Finally, the study also put the recommendations of the study.

Keywords: GeoGebra, Zone of proximal development, Scaffolding, Classroom practices, Calculus, Cycle model of GeoGebra implementation

1. Introduction

Many educators demonstrated that technology in education is very important for learning and teaching for example (Pierce & Ball, 2009; Voogt, 2008). Technology in education is not only benefit researchers but also, teachers, governments and funding agency. In 21st century technology is well equipped both for students and teachers that may help in learning and teaching the intended subject. The integration of technology into education system vary from country to country. And some scholars revealed that the integration of technology into the education system hugely depend on the country policy. Some country has more initiation from their educational policy.

Some researchers point out that the integration of technology into their educational system depend on the availability of the software. It is known that in the 21st century, one can consider that technology available both freely and on the market. It is clearly true that the technology that are freely find on the internet are used more than the technology that find on market. And educators in this century hard work on this and develop the open source software/free software.

For all subject respective scholars in this century are trying to invent software and prepare for teaching, learning and research purpose. The inventor of the software some time make the software free and the other make not free. The software that is free of its license is called open software. This open software can be downloaded from its site. There is also many open source software in this century. Some of the open sources are GeoGebra, PhET, CK-12, Khan academy, CueThink, CanfigureItGeometry, Desmos, Mathalicious, Virtual Nerd and Brilliant. This are the best Mathematics tools for high school students. This idea is searched from google by inserting the ten top open sources for Mathematics. Here I recommend the reader to search this and use it for his or her classroom teaching and learning processes.

To this GeoGebra mathematical software is user friendly. From the above open source, the most used software in this century is GeoGebra mathematical software. I can say this software as full of freedom (FOF) when compared to all other software that are developed in year of 2000 to 2020. Therefore, in order to help educators to integrate technology in teaching and learning mathematics, teachers can use GeoGebra as one of the alternatives. And in my opinion open source software are essential for developing country more than developed country because of cost. Ethiopia may one of the beneficiaries if the government turns the eyes to this open software that are found on internet for free. In this Covid-19 pandemic time, the students can use the software to try out the exercises that are given for the students as the software enables us for e-learning (Charles-Ogan and Ibibo ,2018). This software can be downloaded from the official website of GeoGebra free of charge and is able to work across various platforms, including Windows, Macintosh, Linux, and Unix (www.geogebra.org). The most interesting aspect of GeoGebra is a virtual community of users who frequently contribute to the free teaching materials produced. The tutorials may cover the subject of an integer, distribution, linear equations, coordinate systems, angles, triangles, the formula space, transformation, symmetry, circles, Bezier curves, quadratic equations, matrix transformation, parametric functions, polynomial functions, derivatives, integrals, complex numbers, line and space, vector functions, and others. GeoGebra can be used to teach Geometry, Algebra, and Calculus (Antohe, 2009). GeoGebra effectively disseminates knowledge that includes planning, delivery, guidance, and evaluation that aims to spread the knowledge or skills to students.

GeoGebra software has the potential to help teachers implement teaching test conjecture on geometry, algebra, and calculus.

2. Learning Calculus with GeoGebra (GGB)

The usage of technology in teaching calculus enable students to participate highly and motivate in the classroom by relating between visualization and experimentation as cited in (Nobre et al,2016).

Other studies revealed that instruction supported with GeoGebra software in the classroom enhances students' learning calculus subjects meaningfully and conceptually (Ocal, 2017; Ayub, Sembok, & Luan, 2008).

2.1. Statement of the problem

Calculus is a basic of mathematics. Because of this the scholar Charles-Ogan and Ibibo (2018) stated calculus as the limp of Mathematics. Calculus is one of the concepts in mathematics that both teachers and students find difficult to teach and learn specially in this 21st century. The study done by the scholar Rochowicz (1996) revealed that calculus is the courses that filters many students to drop out from science and engineering students. According to this study the calculus curriculum is outdated and need to revise the curriculum which is appropriate with the days technological oriented educational curriculum. This study is studied in the last of 19th century by having calculator as tool in teaching calculus and arrived at the above results. Surprisingly in 21st century the rapid growth of technology has no end and the research that done with the combination of technology and calculus become many as the review literature revealed. The scholar Tall(2019) point out that the reason why more studies are going on with subject calculus with technology by stating that the subject by itself is applicable for the study with technology, and others researchers revealed that because of its difficulties and important of the course that is a back bone of Mathematics. The research done in Brazil by Meyer and Souza (2002) cited in (Nobre *et al.*, 2016) revealed that many failures and evasions of college students is because of Calculus teaching and the same is true for universities, of the lack of energies to

make the course interesting or useful. And according to the researchers Lasut (2015); Charles-Ogan and Ibibo (2017) calculus is a subject that most students feels it difficult to understand the courses and thus it needs methods and approaches for effective learning the formulas. This study revealed that technology-based learning and teaching model is important when implemented in learning and teaching calculus that leads to philosophy of constructivism learning which is more active process.

Several technological tools are available, but few are worthy of mention here. These include an interactive whiteboard, calculators, Geometers sketchpad and GeoGebra. In contrast, in Ethiopia no software is integrated for teaching and learning of calculus till now. This study intends to elaborate the dynamic mathematical software to government, teachers and students of Ethiopia which enables students to learn Calculus in simple ways.

With the implementation of the new mathematics curriculum, it is very important to use interactive methods in teaching mathematics by integrating technological tools like dynamic software GeoGebra. GeoGebra software provides a visual and effective learning environment for students to improve their technological know-how, hence the need for such a software in the teaching of calculus concepts for the better understanding of the subject by students.

According to some scholars bringing the technology into the classroom teaching and learning is not enough for the successful integration of technology in the education rather it needs further giving training on the knowledge of the technology itself to the users (Cuban, Kirkpatrick and Peck,2001; Ruthven and Hennessy,2002). Teacher professional developmental stages may be considered too into account to implement the technology into the classroom. Because of the above ideas the problem of this study is to elaborate the steps in which the teacher can implement a technology/GeoGebra into the classroom teaching and learning calculus.

2.2. Aims and objectives of the study

The aim of the study is to develop a conducive landscape /lesson plan for applying GeoGebra in teaching basic topics of calculus. The specific objective of the study is

- i. lists some features of GeoGebra
- ii. Identify steps to prepare lesson plan for basic calculus that compatible for GeoGebra software

- iii. Indicate how to use the prepared lesson plan in classroom practices
- iv. Identify the steps of solving calculus by GeoGebra

3. Methodology

3.1. Features of GeoGebra

As it is discussed early, GGB is developed for educational purpose and the developer of the software has got many awards. Because of its dynamic in nature the developer of the software adds new things on its interface or features time to time. For instance, in the beginning the developer of the software did not develop GeoGebra Air, but through a time this application is developed. The GeoGebra has also international institutes.

The developer of the software developed the elements or components of GeoGebra with their application in interesting way for the users. Among this the researcher Ogwel(2009) try to list some of the components or elements of GeoGebra. This researcher introduced the elements of the GeoGebra software as Menus, Tools, Views, Input bar, Tool bar, Graphics window (DGSW), Algebra window (CASW), In put bar and Menu bar as cited in (Akanmu,2015). This all features are shown on the top of the screen if the setting is not changed.

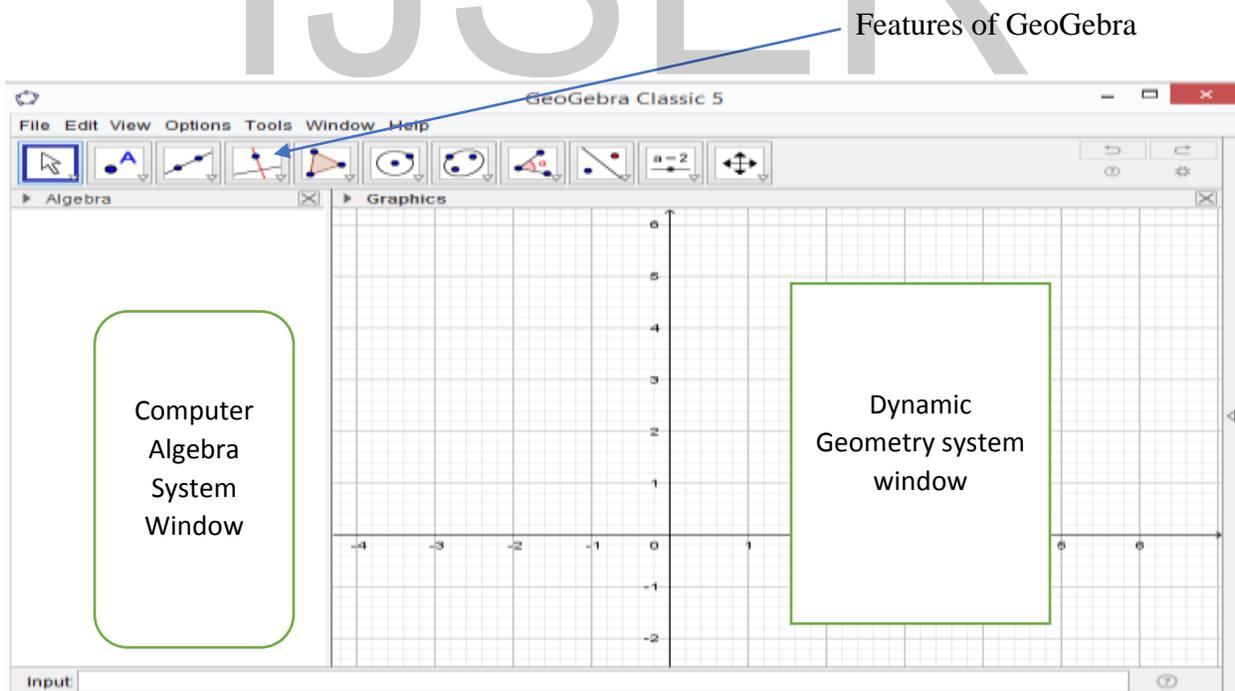


Figure 1. Features of GeoGebra

The following are some of the features of GeoGebra.

The main window is divided to views. By default, Algebra View is displayed on the left side and Graphics View (graphics, graphics2, 3D graphics) on the right as shown in the above figure. Above these views there is a Menu bar and Toolbar, underneath Navigation Bar can be placed. Many features of GeoGebra can be accessed via Keyboard Shortcuts. GeoGebra also includes accessibility features such as Virtual Keyboard. The main components of the GeoGebra are Menu bar, Tool bar, Context Menu Navigation Bar, Virtual Keyboard and Input Bar. The menu bar is file menu, edit menu, view menu, options menu, tools menu, window menu and help menu which was shown on the top of the window by default.

3.2. Conducive environment /landscape for applying GeoGebra

The researchers used the ideas of the scholar Ruthven (2009) to develop a lesson plan that goes with the GeoGebra (GGB) mathematical software. The researcher identifies five key structuring features of classroom resources in technology oriented/integrated which is to be practiced in the classroom. These are working environment/classroom, resource system (collection of mathematical tools and materials in classroom use), activity format (generic templates for action and interaction which frame the contributions of teacher and students to particular types of lesson segment), curriculum script (evolving teacher knowledge gained in the course of her own experience of learning and teaching a given topic) and time economy.

In order to apply this idea, the researchers used *working environment* as the laboratory in which the learning and teaching process is take place. In many schools, lessons must be relocated from the regular classroom to a computer laboratory for computers to be available in enough numbers for students to work with them (Ruthven ,2009).

Resource system the researchers used GeoGebra mathematical software, Laptop or PC, table, chair etc.

Activity format: - The activities that are given in the classroom/laboratory and the interaction of teacher and student was considered. Here when the interaction take place between the teacher and student the sociocultural theory principles can be applied. The researchers recommend the Ruthven on this practice the zone of proximal development should be considered. When a teacher interacts with the student it may leads to scaffolding and student

may again interact with each other. The combination of *resource system*, *working environment* and *activity format* can be observed from the following figure.

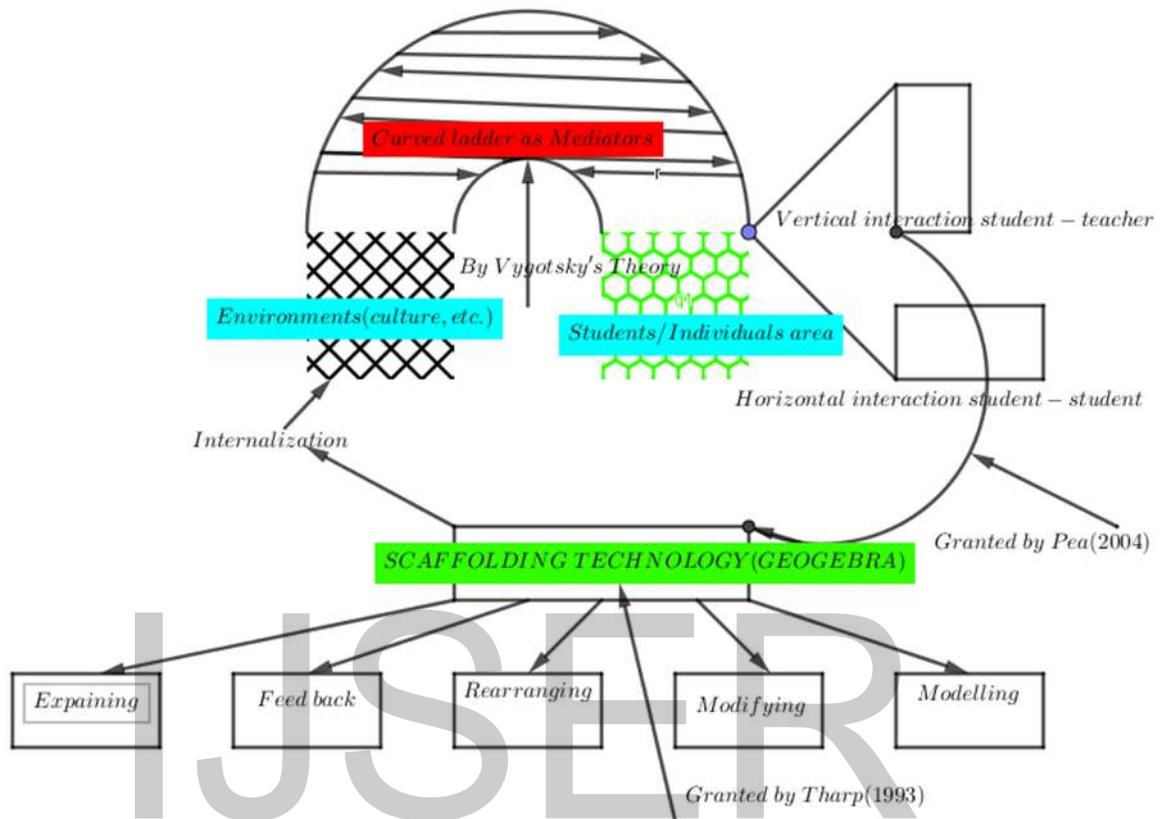


Figure 2: The interaction of environments (e.g. Mathematics), teacher and students with technology /tools (IEST) developed the hypothesized cycle model in teaching and learning mathematics (unpublished work developed by Tola Bekene)

In the figure the author Pea (2004) outlines the two axes of learning and Tharp (1993) outlines the activities within the scaffolding.

Curriculum script: The researchers took the harmonized Ethiopian higher Education curriculum on the subject calculus I.

Time economy. The researchers considered the time of Wachemo university, Ethiopia schedule and took it to develop the lesson plan of teaching and learning.

3.3. A way of using a prepared lesson plan with GeoGebra Mathematical software

First, the researchers should check whether the above section is fulfilled or not. In addition to the above discussed ideas the teacher should know how to manipulate the discussion into the classroom. This is to mean that the teacher development must be assured before the program is employed into the classroom. According to the scaffolding criteria the term uses the more knowledge others (MKO). MKO is defined as an essential component of the learning process and with more knowledge than the students (Vygotsky, 1978).

Many researchers point out that teacher progress in implementation of technology in the classroom is determined by considering Pedagogical Technology Knowledge (PTK). They also define how fluently the individual implement the technology and the techniques required to build situations incorporating it in teaching and learning.

In order to apply the prepared lesson plan, the teacher should consider another condition which is known as the Zone of proximal development. According to Vygotsky's idea a place at which good learning occurs is at the Zone of Proximal development. The zone of proximal(potential) development (ZPD) is the gap between what a student can do independently and what they can potentially do with support(guidance) and assistance (Daniels,2001).

The researcher Gallimore and Tharp (1990) revealed out that the ZPD has four stages. According their study these stages were stage1:in this stage modifying for transfer, assistance and task performance will be applied;stage2: this is a place where the performance is monitored by self/assisted by self and still the learner is not automatized;stage3: this is a place where the performance is automatized, fossilized and developed; and stage4: this is a place where the de atomization of performances leads to the recursion through ZPD.

In general, to apply the developed lesson plan with GeoGebra Mathematical software I consider the following steps.

1. Teachers professional development

The technology-oriented teachers professional development helps teacher to understand how the teacher can implement the technology in the classroom. Teacher professional development reduces the barriers (such as resources and others) that hinder a teacher to integrate the technology in the classroom (Pierce & Ball,2009). "The integration of technology into mathematics teachers' classroom practice is a complex undertaking regardless of whether

teachers find this an 'easy' transformation of practice or not''(Monaghan, 2004 p.352). The study also reveal that the government should understand the complexity of integrating the technology into the classroom.

Unlike the traditional "make and take" professional development experiences, the technology oriented teacher professional development give the opportunities to sharpen their mathematics skills, reinforcing both their understanding of mathematics and their ideas about what it means to learn and teach mathematics(Orrill and Galloway, 2001). In their study they stated that teachers are low in hand experience with computer unless they posed to the technology. Teacher behavior in a technology-rich environment is the instrumental approach to tool use, and the notion of instrumental orchestration (Drijvers *et al.*, 2010). The scholars Drijvers et al (2010) identified three elements within instrumental orchestration such as *didactic configuration*, *an exploitation mode* and *a didactical performance*. They discussed the elements as follows: -

A didactical configuration is an arrangement of artefacts in the environment (Ruthven,2009), or, in other words, a configuration of the teaching setting and the artefacts involved in it. In the musical metaphor of orchestration, setting up the didactical configuration can be compared with choosing musical instruments to be included in the band, and arranging them in space so that the different sounds result in polyphonic music, which in the mathematics classroom might come down to a sound and converging mathematical discourse. In this line this is up to the teacher that do this principle. So, the teacher development must be assured.

An exploitation mode is the way the teacher decides to exploit a didactical configuration for the benefit of his or her didactical intentions. This includes decisions on the way a task is introduced and worked through, on the possible roles of the artefacts to be played, and on the schemes and techniques to be developed and established by the students.

A didactical performance involves the ad hoc decisions taken while teaching on how to actually perform in the chosen *didactic configuration* and *exploitation mode*: what question to pose now, how to do justice to (or to set aside) any particular student input, how to deal with an unexpected aspect of the mathematical task or the technological tool, or other emerging goals.

2. Conducive environment/Classroom practices

After the teacher developmental stage were accomplished it is up to the teachers that brings the technology into the classroom even if the integration of the technology is a complex process. If the teacher becomes more knowledgeable others in the definition of Vygotsky theory it is up to him/her to implement the technology into the classroom by identifying the above listed classroom practices (Ruthven ,2009).

3. Zone of proximal development

This idea should be considered in order to implement the technology/GeoGebra into the classroom. In order to elaborate this step, we used the following depicted figure.

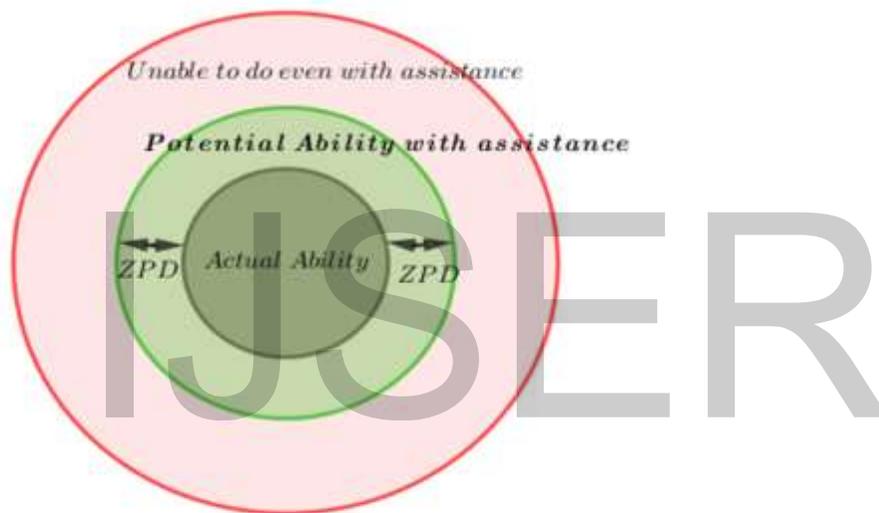


Figure summary of the ZPD

Figure 3: summary of the ZPD

To implement the technology in the classroom the teacher should know that students has the actual ability. The assistance should be given depending on the ability of the students they have before. A way of scaffolding must be measured. The term scaffolding was introduced first by(Wood, Bruner and Ross, 1976). Wood et al. (1976) defined scaffolding as a form of adult assistance "that enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts" (p. 90). The scaffolding should be slowly with drawn according to the stages in ZPD (Gallimore & Tharp ,1990).

3.4. The steps of solving calculus and Discussions

We open the GeoGebra mathematical software on our PC, Desktop, or Smart phones as we want. Then we should consider the features of the software on the screen of our device. We see that at the bottom we see the in-put bar and we write the function we want and click insert/Enter key. Then finally we see the calculated and its visual on the Algebra system on the left side and Geometry system on the right side respectively. For more understanding features of GeoGebra we recommend the reader of this material to see the official GeoGebra manual document.

In this material I wish to discuss the input bar. This bar permits you to precisely generate and re describe mathematical objects/functions in the Algebra View by inserting or adjusting their algebraic statements (e.g. Values, coordinates, equations). After writing function you want in input bar, we always hit the enter on our keyboard computer or devices. To implement the developed lesson plan, we may see our developed lesson to teach our students in the classroom.

Lesson plan

The designed teaching-learning scenario allows students and teachers to focus on specific mathematics learning and teaching and to make sense to the mathematics with foreseeable results for the full range of students in classroom. In a time of technological change and economic globalization, we need to use every resource in our power to improve the ways we organize our lives. Lesson study viewed within a framework of long-term growth of mathematical thinking is a powerful tool to improve how we learn to think mathematically (Isoda & Tall, 2007).

To show the steps in solving the calculus I directly take activities from our lesson plan which was prepared by the teacher in this case the researchers.

Example 1. Sketch the graph of the function $f(x)=x^2$.

To draw the graph of this function $f(x)=x^2$, we follow the following steps.

Step1. Write $f(x)=x^2$ at input bar

Step2. Hit the Enter on the keyboard

Step3. See the output functions both on algebra view and Geometry view

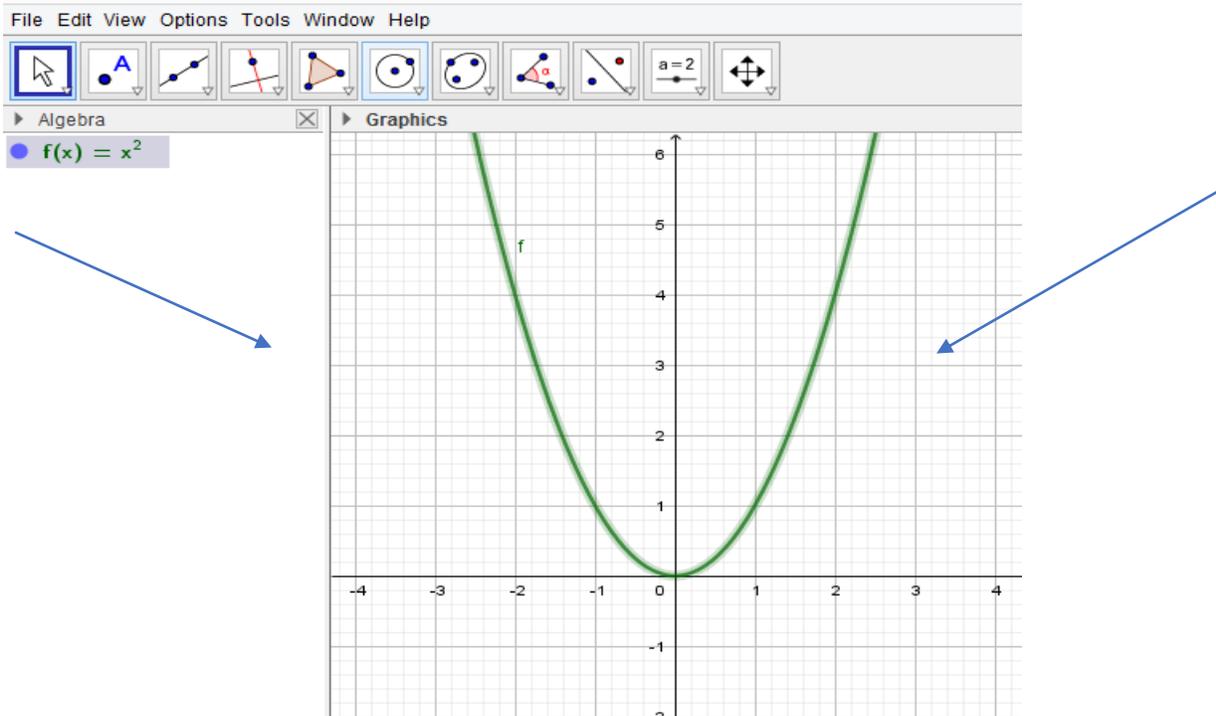


Figure 4: The graph of $f(x)=x^2$

Then after the above graph is done, we can find the derivatives of the function by simply writing derivative[$f(x)$] on the in-put bar and see the result both on algebra view and Geometry view as shown below. To do this we repeat the above steps repeatedly.

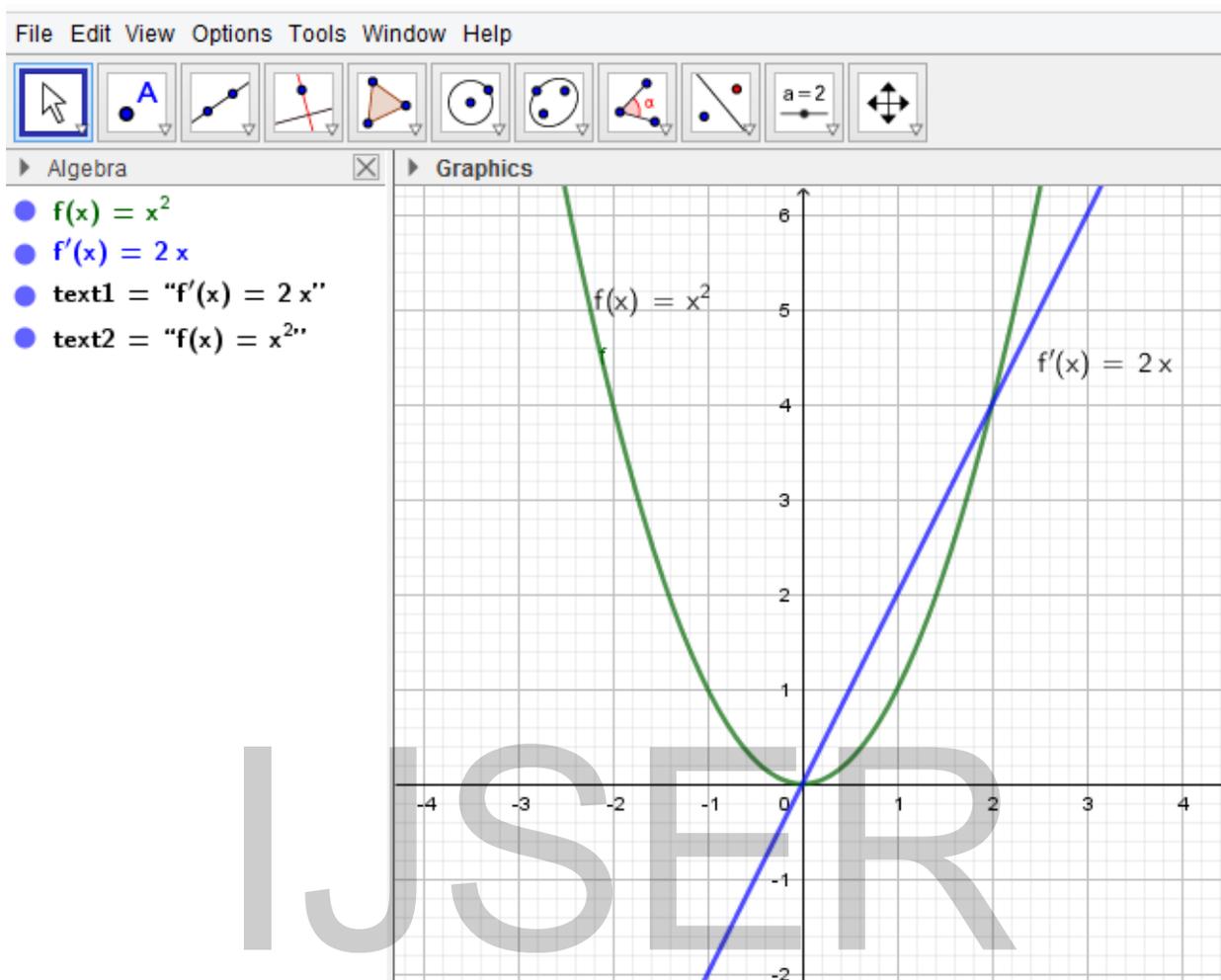


Figure 5: The graph of the function $f(x)=x^2$ and its derivatives.

As we see here that we can draw the function and its derivative on the same window. At this case students can escape from solution only approach to solution visualization approach of solving calculus problem.

Example 2 sketch the graph of the function $g(x)=x^x$ and find the first and the second derivatives.

In the same procedures we can sketch the graph of $g(x)=x^x$. The sketched graph by GeoGebra looks like the following.

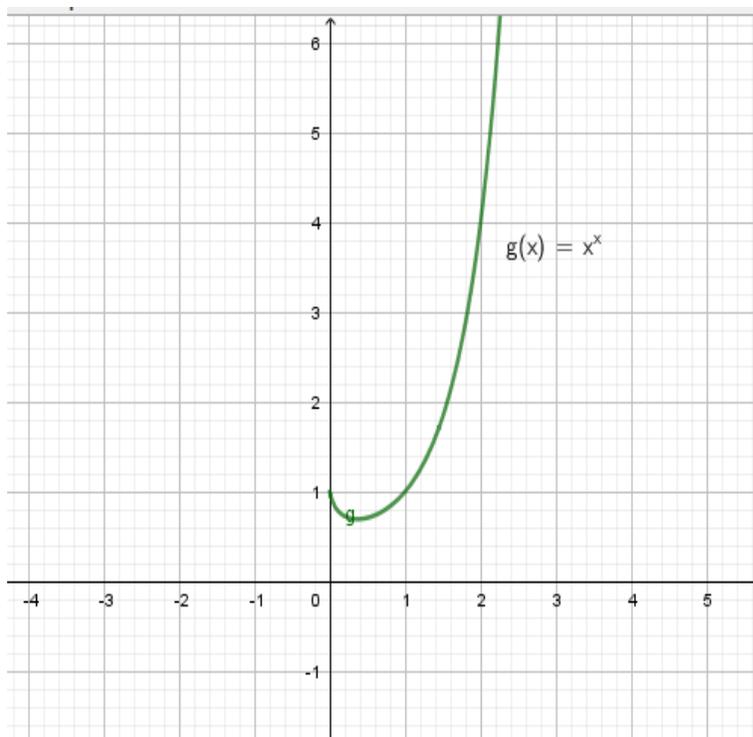


Figure 6: The graph of $g(x) = x^x$

In order to sketch the graph of this $g(x)=x^x$ by hand may takes time and may difficult for low ability students. By solution only approach the derivative of $g(x)=x^x$ can be done by writing this in terms of the combination of the natural exponential and natural logarithm as $x^x = e^{x \ln(x)}$. This needs the knowledge of recalling how to find the derivative of both natural exponential and natural logarithm. But the derivative of $g(x)=x^x$ by using GeoGebra mathematical software simplify the life of both the teacher and students as it computes the derivatives of the function by itself. Here is the derivative of the function $g(x)=x^x$ as it is shown below.

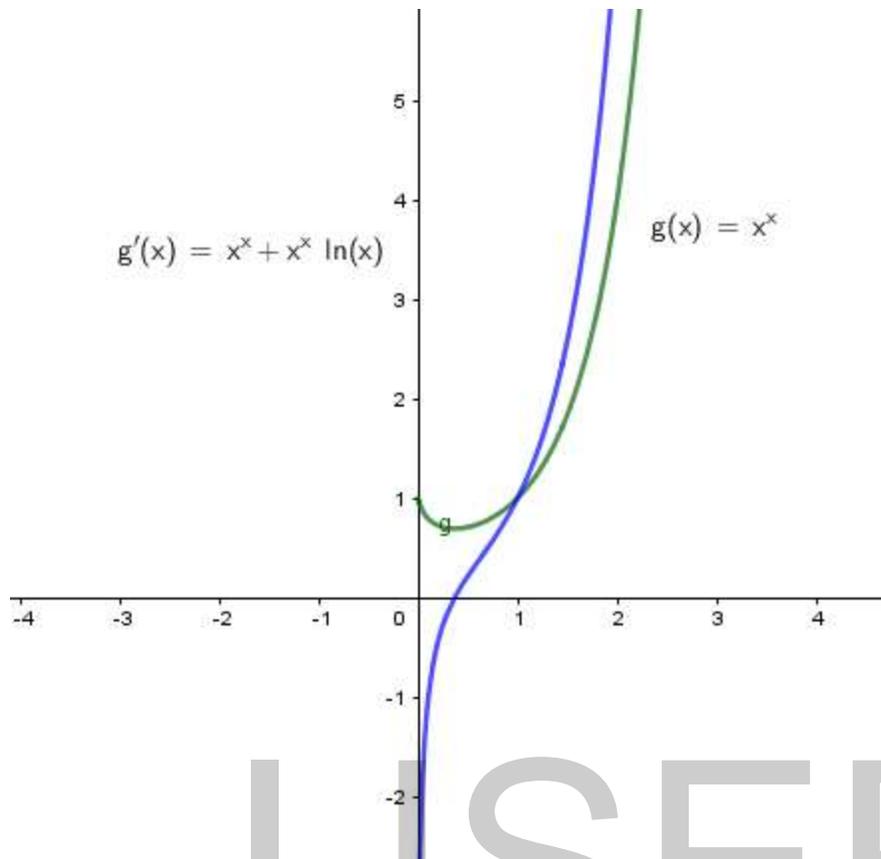


Figure 7: The graph of $g(x)=x^x$ and both result of its derivatives and graphs

Example 3: Find equation of the tangent line of the function $f(x)=x^2+2$ at the point (2,6).

By using hand process it needs some recalling of the formula of equation of the tangent line to the graph ,for instance, at a point(a,b) the equation of tangent line to the graph is given by $g : y = f'(a)(x - a) + b$.

By using GeoGebra Mathematical software it is to ease to find the result and its graph and visualize both the graph and its results. To find this we follow the following steps.

Step1: write the function $f(x)=x^2+2$ on input bar and hit the Enter key (Note for all do this on the first steps)

Step2: write tangent [2, f(x)] on input bar and hit the Enter key.

Step3: Read the result on algebra view and click it to see the graph on Geometry view to see the graph of the function if you draw more than this function. And you can drag the result to its

graph carefully.

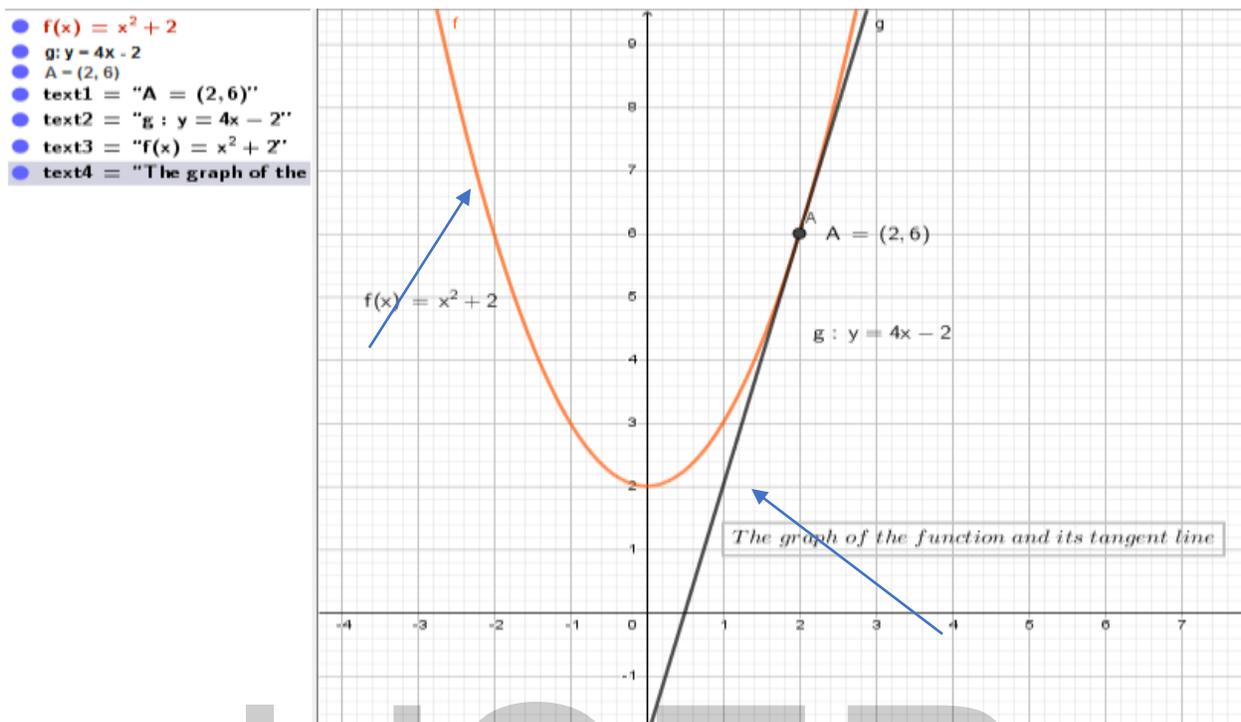


Figure 8: The equation of tangent line to the function $f(x)=x^2+2$ at the point $(2,6)$ and its graph

Example 4: Consider the function $f(x)=4x^3+9x^2-12x+3$

- Sketch the graph of $f(x)=4x^3+9x^2-12x+3$
- Show and find the extreme value(s) and root(s) of the function $f(x)=4x^3+9x^2-12x+3$
- Construct the tangent of the function $f(x)=4x^3+9x^2-12x+3$ at the point $x=-2$

To do these question(s) we need to follow the following steps.

Step1. Write the function $f(x)$ on input bar and note to write power of x use shift+6 then write the value of powers

Step2. Write $\text{extremum}[f(x)]$ on input bar and see the results with its graph on CAS and DGS by hitting the Enter key.

Step3. Write $\text{root}[f(x)]$ on input bar and press Enter key and observe the results on the screen.

Step4. Write $\text{tangent}[f(x)]$ at the point $x=-2$ and see the constructed on the geometry view. At the same time this software enables us to see the equation of the tangent line at the point of tangency.

After you did these three steps you see the following result.

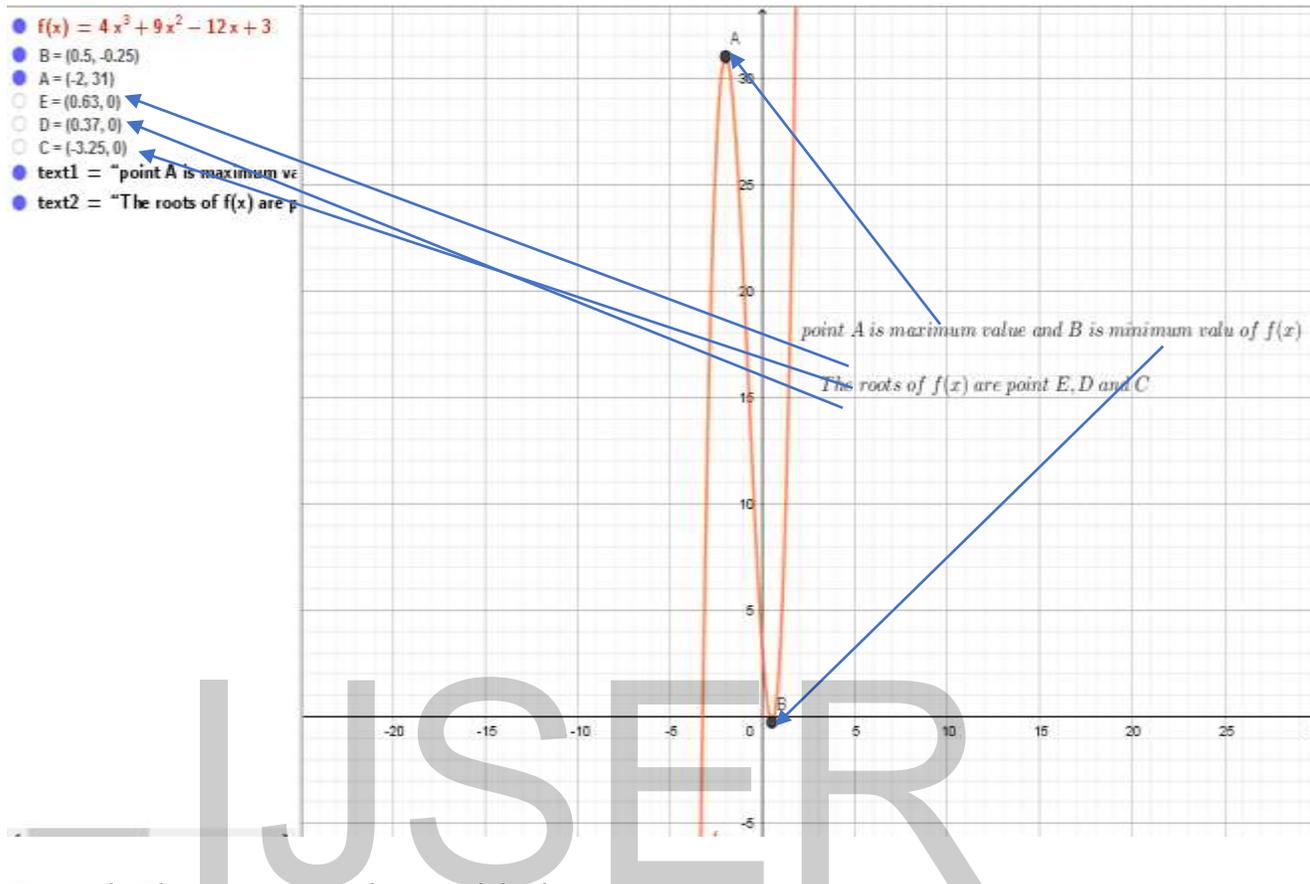


Figure 9: The extremum and roots of the function

For questions (a) and (b) we can see the answers from figure 9 above. As we see that the maximum and minimum value of the function $f(x)$ are 31 and -0.25 that occurs at -2 and 0.5 respectively.

And the roots of the function $f(x)$ can be done by hand process which take times by using either rational root test method or other methods. In a short time, the software is able us to compute the roots of the function by simply writing step3 above. Thus, the roots of the function as it is shown in the CAS view are 0.63,0.37 and -3.25. This software saves the time of both teachers and students in calculation. Complex calculation in the calculus can be simplified by using GeoGebra as we see by using this example. This concept line with the finding of (Ayub, Sembok, & Luan, 2008).

The solution to (c) can be constructed at the point of tangency. Since we are given with the point of tangency and we can find equation of tangent line by hand process by remembering

formula of equation of tangent line and construct it. Thanks to this software we can construct the tangent line to the given function and simply observe its equation on DGS and CAS views respectively which is shown in the following figure.

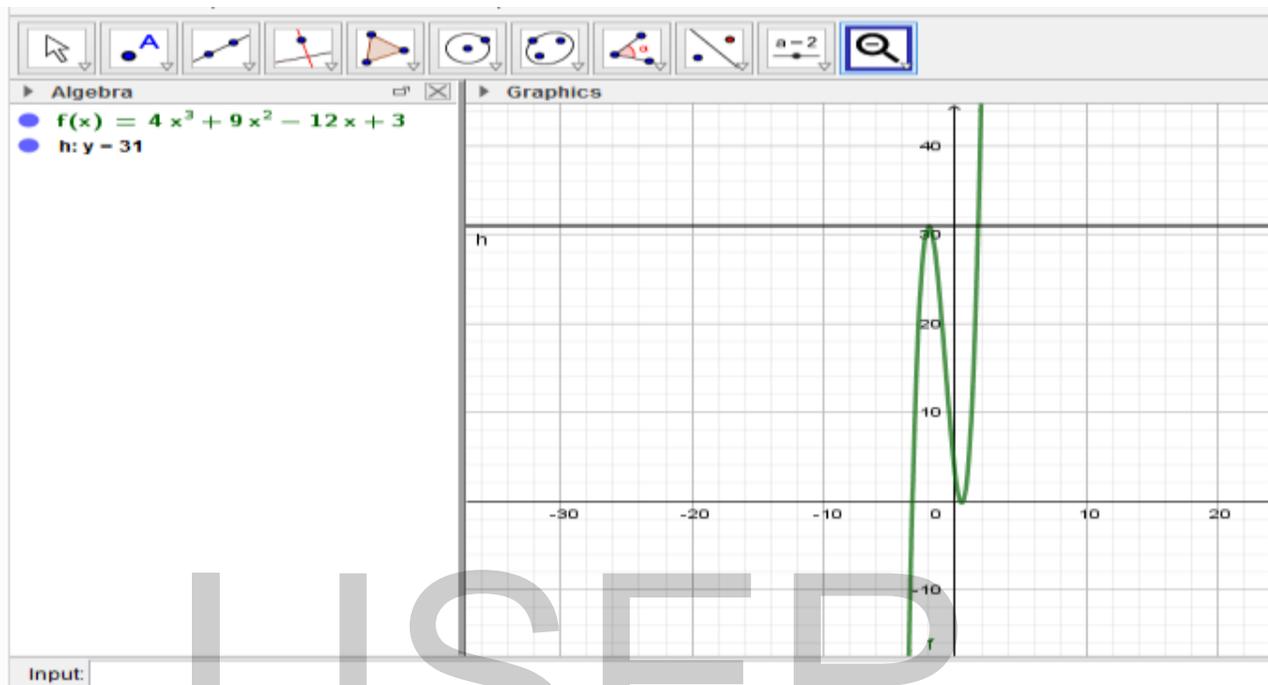
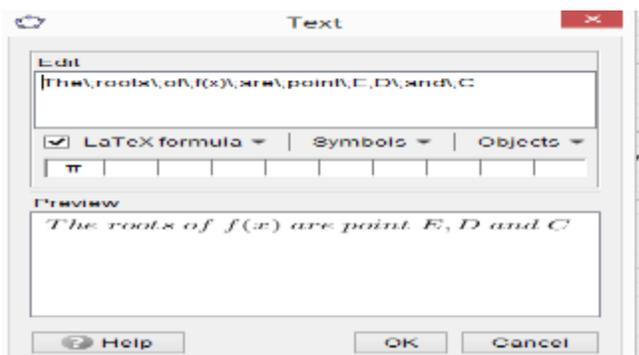


Figure 10: Equation of the tangent line to the function at $x=-2$ and its graph

As we can observe from the above graph, we see the blue and white circle on algebra view. By clicking these circles, we can hide and unhide object the texts from Dynamics Geometric view or graphics view.

Note: The text in the above figure is written by using Latex formula which is shown as in the following figure.



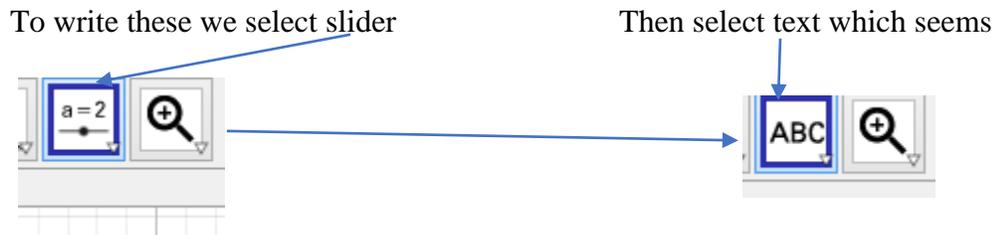


Figure 11. places of slider ,text and Latex

Finally you can write by clicking twice on the window.

Note:-In all above examples, on the dynamic Geometry view the graph can be simulated which can be shown in the classroom by the teacher of the subject. In this case we should care of the ability of the students/prior understandings see also Saxe's Model as cited in Monaghan(2004) of the lesson as *adding water on stone may take time to penetrate into the stone*. And the activities in the hypothesized cycle model can be implemented by the teacher in the classroom while the teaching and learning process with the help of GeoGebra is implemented. The cycle imply that the teacher teaches students and the students become a teacher after some times or stages then they also teaches their students if they internalized in their environment. This cycle continues in the society.

Conclusion

GeoGebra mathematical software is the best alternative way of teaching and learning calculus in 21st century by exploration and visualization. This software opposes the traditional teaching and learning method, or solution only approaches and make the teaching and learning process more active in the classroom which goes with the constructive approach of teaching and learning. A teacher can use the GeoGebra mathematical software to attract students in his/her classroom in the Zone of proximal development. This encouraged a more interactive teacher-student, student-student interpersonal environment where everyone worked as a group to guide (should be in the ideas of scaffolding), help and assist one another to reach the required goals in ZPD.

To implement the GeoGebra mathematical software in the classroom the teacher may need to identify whether h/she trained/ or equipped with the technology/GeoGebra (Teacher developmental), identify whether the five classroom practices is there or not and lastly identify what are the stages in the Zone of proximal development. After implementing the program in the

classroom, the teacher should check whether the output of his/her work is fruit full or not by using the theory of Vygotsky. To check this, student assessment may be implemented at the last of the intervention and Pedagogical Technology Knowledge (PTK) may be also checked.

This study established a new teaching and learning method using GeoGebra software as a tool in their instructional activities. Students can download GeoGebra freely from its sites and can use it both in the classroom and at home. The software provided teachers and students with a free new technological tool, a new way of using technology with visual aids to help students to interact with the mathematical concepts individually or in groups in the classroom, or at home.

Generally, GeoGebra mathematical software provides a visual and effective learning environment for students to improve their technological know-how as it is simple to learn the intended lesson. This software may help the user to search for another software by bearing in mind the advantage they obtained from the GeoGebra software. I arrive at this statement starting from my experiences.

Recommendations

1. Mathematics teacher should be encouraged and trained to use GeoGebra
2. The government should provide Mathematics laboratory and the concerned body should download the software and install it on the computer and use it for teaching and learning purpose
3. The university in Ethiopia should have a GeoGebra international institute so that both the teachers and the students know, know-how of the software and use it for their educational purpose
4. The teacher professional development program of every country should have training with this software/integrate in line with post- Vygotsky theory of learning.
5. Untrained teacher my use the developed GeoGebra oriented lesson plan and follow the steps in this study to implement in their classroom to teach calculus.
6. The developer of the software should add some steps of the manipulation of the results in mathematics.

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