Can Teaching Fractions Improve Teachers' Fraction Understanding? Insights from a Causal-Comparative Study

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Abstract

The purpose of this causal-comparative study was to gain insight into whether teaching fractions improves teachers' understanding of fractions. University master's students (*n* = 25) conducted tutoring focused on fraction magnitude using number line representations across 39 sessions (3 per week) at grades 3-5. The contrast condition, 17 master's students drawn from the same pool of research assistant applicants, conducted tutoring designed to improve science and social studies text comprehension over 42 sessions (3 per week) at grades 3-5. In the fractions condition, students were at risk for mathematics difficulty; in the text comprehension condition, at-risk for reading difficulty. Across conditions, tutoring was similarly structured, with ongoing support to implement methods with fidelity while permitting flexibility to address individual student needs. When tutoring ended, accuracy of number line placement of single fractions and fraction sums as well as single whole numbers and whole number sums was assessed. Results indicated superior accuracy for fraction tutors over text comprehension tutors on fraction sums, with an effect size of 0.75. The effect size for placement of single fractions was 0.47. Implications are extrapolated from this study's sample of tutors to preservice and inservice teachers.

Can Teaching Fractions Improve Teachers' Fraction Understanding?

Insights from a Causal-Comparative Study

Competence with fractions, especially understanding of fraction magnitude, is linked to later success with algebra as well as other forms of more advanced mathematics and post-school employment (Booth & Newton, 2012; Booth et al., 2014; Brown & Quinn, 2007; Empson & Levi, 2011; Geary et al., 2012; National Mathematics Advisory Panel, 2008; Siegler et al., 2012). Fractions differ from whole numbers in fundamental ways. For example, fractions are expressed as a ratio of two integers; the value of fractions with the same numerator decreases as denominators increase; an infinite number of fractions exists on any segment of a number line; and the value of fraction products and quotients sometimes violate expectations based on whole-number operations.

It is not surprising, therefore, that many elementary-grade students, even those with adequate developmental trajectories on whole numbers, experience difficulty with fractions (Namkung et al., 2018). Confusion often persists in middle- and high-school (Durkin & Rittle-Johnson, 2015; Kallai & Tzelgov, 2009; Siegler & Lortie-Forgues, 2015; Siegler et al., 2012; Van Hoof et al., 2013) and through adulthood (Schneider & Siegler, 2010; Stigler et al., 2010).

As documented in numerous studies (e.g., Barbieri et al., 2011; Fuchs et al., 2013; Siegler et al., 2011), developing proficiency with fraction concepts relies at least in part on an understanding that, like whole numbers, fractions have magnitude and can be represented as points on the number line. Saxe et al. (2013) demonstrated the efficacy of fractions instruction that explicitly connects integers and fractions into one coherent framework, with number lines as the principal representational context. Students who received this instructional approach significantly outperformed peers across measures of fraction understanding, and an advantage was maintained at follow-up. In a line of randomized controlled studies, Fuchs et al. (2017) also demonstrated the benefits of fractions intervention focused primarily on number lines to build fraction magnitude understanding in students at-risk for mathematics difficulties.

Nevertheless, because the professionals charged with teaching students about fractions often experience poor fraction knowledge, questions persist about teachers' capacity to deliver such instruction (Ball, 1990; Ma, 1999; Post et al., 1991; Siegler & Lortie-Forgues, 2015). For example, Zhou et al. (2006) assessed teachers' fraction subject knowledge in China and the U.S. to shed light on reasons for the widely documented fraction-learning gap between students in these countries (e.g., Mullis et al., 2016). Zhou et al. found that American teachers performed significantly lower on fraction concepts, fraction word problems, and fraction computations, even when controlling for years of schooling and years of teaching. On the basis of interviews with American teachers, Zhou et al. (2006) concluded that teachers' difficulty with fractions generally and fraction magnitude understanding specifically contributes to their students' difficulty with fractions.

In fact, teachers' mathematics knowledge has been shown to predict student learning (Hill et al., 2005) even on simple whole-number content (Ma, 1999). For this reason, a pressing need exists to identify effective strategies for improving teachers' fraction understanding. This was the focus of the present study. In the remaining sections of this introduction, we provide an overview of the literature on the effects of teacher professional development (PD) on fraction outcomes; summarize the methods and results of two key studies designed to improve teachers' fraction knowledge, one focused on preservice PD and the other on inservice PD; and explain how the present study extends the literature by focusing on a third approach not previously evaluated for improving adults' understanding of fractions.

Mathematics PD's Effects on Fractions Learning

We identified three syntheses centered on the effects of teacher PD largely focused on *student* achievement. Blank and de las Alas's (2009) meta-analysis included 12 randomized controlled trials or quasi-experiments. Mathematics PD significantly improved student learning with an ES of 0.21, but none of the studies focused exclusively on fraction learning. Yoon et al. (2007) identified 1,300 studies focused on PD's effects on student learning. Of the nine that met What Works Clearinghouse standards for evidence, two focused exclusively on mathematics PD, with a mean ES of 0.51. In the only study in the Yoon et al. meta-analysis centered on fraction knowledge, Saxe et al. (2001) examined PD to encourage deep conceptual explanations of fraction problems and flexibility in strategy use (i.e., the capacity to apply multiple strategies to solve the same problem). ESs for this study ranged from -0.53 for fraction computations to 2.39 for fraction concepts.

In the third synthesis, Gersten et al. (2014) reviewed 643 PD studies, 32 of which focused on mathematics. Among the 5 (of 32) studies meeting What Works Clearinghouse standards, two reported positive effects (ESs from 0.21 - 0.84), whereas the other three reported minimal or no effects. One of the two with positive effects focused on improving students' as well as teachers' fraction knowledge: Lewis and Perry (2017) randomly assigned 213 teachers to one of two PD treatment conditions or a control group. For treatment, teachers received a guided lesson study with a fractions resource kit (including course study materials, sample lesson plans and videos, research articles, a guide for digesting the materials) or the same guided lesson study without the resource kit. The resource kit had a strong focus on number lines, magnitude, and developing flexible strategies for approaching fraction problems. Reliance on this resource kit significantly improved *teachers'* fraction knowledge (*ES* = 0.19) as well as their *students'* fraction knowledge (*ES* = 0.50) over the other treatment group and control group combined.

In light of mixed findings across studies investigating the effects of PD, Gersten et al. (2014) raised caution about limited evidence to support traditional PD activities for improving student learning. Yet, although few PD studies focus on fractions, some provide support. Specifically, Saxe at al. (2013) found positive effects on *students'* fraction learning when PD centers on fraction number lines as the representation context. Moreover, Lewis and Perry's (2017) positive effects on *student* and *teacher* outcomes provides rationale for immersing adults in a magnitude-focused PD curriculum. In the present study, we took this approach, but we used a non-traditional mode for delivering that PD.

Preservice and Inservice Approaches to Improving Teachers' Fraction Knowledge

Before explaining the present study's approach, we further contextualize the present study by elaborating on two influential PD investigations that took contrasting but traditional approaches for improving *teachers*' understanding of fractions. In one study, the focus was preservice knowledge-building via formal coursework; in the other, the approach was inservice PD.

Preservice PD

Focusing at the preservice level, Newton (2008) assessed fraction knowledge before and after education majors (n = 99) completed a course designed to deepen knowledge of elementary school mathematics. Fractions were a major focus of the course, which integrated conceptual and procedural knowledge. The goal was to improve fraction computations, basic concepts, word-problem solving, use of alternative strategies (i.e., *flexibility*), and transfer (a problem that could not be solved with taught algorithms).

At pretest, although preservice teachers accurately answered approximately 75% of fraction computation problems, word problems, and basic concepts problems, their flexibility to choose alternative or more efficient solution methods for these problems was low (<30% on an 11-item measure). Thus, as described in the study report, this sample of preservice teachers possessed "limited and fragmented" (p. 1104) knowledge of fractions, with differentially low performance on division and flexibility in alternative solution strategies. Errors reflected prior fraction misconceptions and often involved procedural misapplication resulting from a focus on superficial problem features (e.g., using addition algorithms to solve multiplication problems with like denominators). This corroborates Braithwaite et al.'s (2017) computational modeling.

Posttest performance indicated improved computational skill for these preservice teachers, especially for division, with errors reflecting deepened understanding. Basic concepts and word-problem solving also reliably improved. Even so, flexibility did not, and participants found the transfer problem (administered only at posttest) to be challenging. Although the study did not include a control condition, empirical work provides no reason to think that knowledge would improve without a concerted effort to produce change. This study therefore illustrates the potential of preservice coursework to deepen fraction knowledge, although flexibility, which did not improve, may be important in helping teachers address their students' individualized needs (Hill et al., 2005). Additional research is required to assess whether the kinds of improvement realized with coursework translate to better teaching or student learning.

Inservice PD

In a large randomized controlled trial, Garet et al. (2011) instead focused on inservice teachers' rational number knowledge, while assessing effects of PD on *student* and *teacher* knowledge of fractions. Thirty-nine schools were randomly assigned, within 12 school districts, to participate in a control group or receive PD, which was delivered to ~100 seventh-grade math teachers in 12 districts the first year and ~50 in six districts in the second. PD included a summer institute, a series of one-day follow-up seminars during the school year, and in-school coaching visits by seminar facilitators (from commercial PD entities) conducted in connection with seminars. Teachers received 68% of the full PD dosage due to teachers changing schools as the 2-year study progressed.

At the end of the second implementation year, there were no significant effects favoring PD over the control group (ES = 0.05) on *teacher* knowledge. This was the case for teachers' knowledge of rational numbers and on mathematics knowledge for teaching rational numbers. As might be expected, therefore, there was also no advantage for teachers who received PD over the group control on *student* performance with rational numbers (ES = –0.01). It is interesting to note that Garet et al.'s (2011) PD program did incorporate the features identified as effective by Blank and de Las Alas (2009), Desimone (2009), and Desimone and Garet (2015): subject content as well as pedagogical content, multiple activities to provide follow-up reinforcement of learning, and a duration of at least six months.

Insights

Based on Newton (2008), we conclude that coursework demonstrates promise for enhancing preservice teachers' fraction knowledge and that a controlled study to investigate a causal connection between teacher understanding and student learning is warranted. Also noteworthy, however, is that teachers' flexibility in problem solving did not improve with Newton's inservice approach. By contrast, the major study (Garet et al., 2011) assessing effects of well-designed PD for inservice teachers reveals less potential. Moreover, the absence of a concerted focus on fractions PD in reviews and meta-analyses likely overestimates meta-analytic conclusions about what can be expected for fractions PD, given sizeable deficits in teachers' fraction knowledge and given the greater challenge fractions present compared to many other mathematics and reading domains for teachers and students alike.

The Present Study's Approach: Immersion in a Tutoring Experience Centered on Fraction Magnitude and Number Line

In the present study, we tested a novel approach for enhancing adults' fraction knowledge: immersion in a tutoring experience, in which adults implemented a validated program for teaching students fraction-magnitude understanding, with number lines as the principal representational context. *Studies on tutor-learning effects provide the basis for hypothesizing that tutoring may enhance tutors' understanding of fractions*.

Tutoring is mutually beneficial for both the tutor and the tutee. Numerous studies report that more learning occurs for tutors than for comparable non-tutors in student-tutoring studies (Cohen et al., 1982; Mathes & Fuchs, 1994; Rohrbeck et al., 2003). For example, in Cohen et al.'s (1982) meta-analysis of 52 tutoring studies, they reported a pooled mean effect size of 0.40 among tutor dyads over similar students in comparison classrooms.

Roscoe and Chi (2007), who termed this the *tutoring learning effect*, demonstrated how this mutually beneficial learning opportunity occurs when tutors teaching material above their own knowledge level (Sprinthall & Scott, 1989); with sufficient tutoring duration (Topping & Bryce, 2004); with the autonomy and training to construct explanations and generate questions that are responsive to their students' needs (Fuchs et al., 1997; Rohrbeck et al., 2003); and with fidelity to the tutoring procedures (Dufrene et al., 2005). They attributed the tutoring learning effect to tutors "using and applying their subject matter knowledge … [to] transform their own knowledge in creative ways" (Roscoe & Chi, p. 540) and to a tutoring experience that is cognitively demanding for tutors (King, 1998). (These features were in place in the present study's fractions tutoring condition, as outlined in the method section below.)

In the context of the present study, it is important to note that the tutoring learning effect has been demonstrated among adults: when undergraduates teach classmates (e.g., Annis, 1993; Coleman et al., 1997) and when college students tutor younger children (e.g., Juel, 1996). Juel (1996) found that college students who were struggling readers significantly benefited when they tutored first-grade students who were also struggling readers. Tutored first-grade students made significantly greater gains from first to second grade compared to their peers. Codings of tutor-student interactions indicated that scaffolding and modeling specific content was particularly beneficial for both the tutor (college student) and the tutee (first-grade student).

The Present Study's Purpose

Identifying effective strategies for improving teachers' fraction-magnitude understanding is critical. Based on the data on preservice and inservice PD options, the most effective strategies for making these gains among teachers is unclear. Further, the paucity of research on the effects of improving teachers' rational number knowledge indicates the need for additional study. In the present study, we tested the effects of a novel approach for improving adult learning of fractions by combining the features of research-validated fractions intervention on student achievement with the notion that tutoring elementary-aged students has the potential to produce academic gains among the adults teaching the content.

Our research question was, Does immersing adult tutors in an effective fractions tutoring experience, centered on fraction-magnitude understanding via number lines representations, improve adult fraction-magnitude understanding? To establish an associative link between the fractions tutoring experience and the tutors' post-tutoring fractions performance, we contrasted the fraction tutors' outcomes against the post-tutoring fractions performance of a group of adults whose tutoring experience involved implementing a structured, effective reading comprehension intervention. Both groups of tutors received the same level of support and oversight for implementing their respective programs. Moreover, in both conditions, tutors were drawn from the same pool of adults who applied for tutoring research assistant positions. Assignments to various tutoring projects were based on tutors' scheduling availability, a variable not related to fraction knowledge or teaching experience.

This causal-comparative design, in which relations between independent and dependent variables are observed after an event (here tutoring) has occurred (Salkind, 2010), provides insight into whether a structured experience teaching fraction magnitude with number lines improves adults' fraction understanding. Based on the literatures just described concerning the benefits of tutoring on tutor performance and concerning effects of PD centered on fraction magnitudes and number lines on teachers' fraction knowledge, we hypothesized that this association would be demonstrated. If so, it provides the basis for investment in a large-scale randomized controlled trial.

Method

Participants

We selected 42 adult tutors to participate based on the following criteria: availability during school days, a car to travel to schools, experience working with children when possible, and a successful interview. All participants were university master's level students in the College of Education at a top-tier university. The College of Education includes 18 master's of education or public policy degree programs. From the pool of tutors who met criteria, research staff assigned master's students to six tutoring research projects based on scheduling availability.

(Random assignment was not feasible due to tutors' scheduling constraints; hence, the causal-comparative design.) Tutors from two of these six projects, those focused on tutoring fractions *or* text comprehension, comprised this study's sample. Few had teaching experience; none had former experience teaching fractions. (We did not include tutors on the other four projects because they were conducted at other grade levels.)

We therefore assumed tutors in both conditions began the study with gaps in their own fraction understanding, parallel to previous studies on adult knowledge of fractions (e.g., Zhou et al., 2006). Yet, because fraction tutoring was conducted with third- through fifth-grade students at risk for mathematics difficulties, whose fraction knowledge was minimal, we assumed tutor-tutee gaps in fraction knowledge were universally present in the fraction tutoring condition, in line with the tutoring learning effect literature. In Table 1, we show tutors' demographics and mean Graduate Record Exam (Educational Testing Service, n.d.) verbal reasoning and quantitative reasoning scores by condition. There were no statistically significant differences between conditions. (Note that this adult tutor study took place in the context of randomized controlled trials, one in fractions and the other in reading comprehension, testing the efficacy of intervention on third through fifth graders identified as at-risk for developing mathematics or reading difficulties. The present article does not discuss those randomized controlled trials, except to provide essential information for understanding the methods of the present study.)

Measures: Four Number Line Estimation Tasks

We followed Braithwaite et al. (2018) to assess fraction and whole-number magnitude understanding using four number line estimation tasks. Two assessments relied on 0-1 fraction number lines; two on 0-1000 whole number lines. One task required participants to place 18 single fractions on a 0-1 number line and another to place 18 single whole numbers on a 0-1000 number line. The magnitudes for these fraction and whole-number tasks paralleled each other, such that the location of a single fraction on the 0-1 number line had an equivalent estimate of a whole number on the 0-1000 number line (i.e., multiplying each fraction decimal value by 1000). For example, the fraction 1/8 (decimal value = 0.125) had a parallel item of 125 on the 0-1000 number line.

Magnitudes were evenly distributed across quartiles of the number line: Six items were in the lowest quartile (< or = 0.25 or 0-250); four items in the second (.251-.400 or 251-500), third (.501-.75 or 501-750), and fourth quartiles (0.75-1 or 751-1000). The other two tasks involved placement of 16 fraction addition problems all with unlike denominators on a 0-1 number line and 16 whole-number addition problems on a 0-1000 number line. The value of the answer of each fraction addition problem paralleled the whole-number addition problem answers. Also, some numbers used for the placement of single fractions were used in the fraction sum task, and some whole numbers used for placement of whole numbers were used in the whole number sum task. See Table 2 for a list of items on the number line estimation tasks. Due to programming challenges, we diverged from Braithwaite et al. (2018) in three ways. We did not adjust whole numbers to avoid zero digits. We did not randomize the order of addends within problems. We permitted correct estimates to fall in the same half of the numeric range on three consecutive items.

Participants estimated fraction magnitudes before whole number magnitudes and items appeared randomly within each task and for each participant. The tasks were administered on a laptop using a set of standard directions, which instructed participants not to calculate answers, but instead to estimate placements as quickly as possible, within a few seconds. Participants did not have access to scratch paper to solve addition problems and were not permitted to execute strategies involving benchmarks or to count spaces on the number line. Each fraction and each fraction sum appeared above a number line labeled with the endpoints 0 and 1. Each whole number and each whole number sum appeared above a number line labeled with the endpoints 0 and 1000. The participant used the cursor to place the number on the line. Then that number disappeared, and a new screen with a new number above the line appeared.

The computer automatically derived percent of absolute error (PAE) by calculating the absolute deviation between the correct placement and the respondent's cursor placement. The onscreen number lines have 512 pixels between endpoints; PAE is the pixel deviations from exact placement. With PAE, lower scores indicate greater accuracy as the respondent's deviation from exact placement approaches 0. To put the 0-1000 number line scores on the same scale as the 0-1 items, we divided by 1000. Then, within each task, we calculated the mean PAE by multiplying each by 100 and dividing by the number of items for that task. Average response time was 4.99 sec (*SD* = 1.55) for fractions; 5.12 sec (*SD* = 1.54) for fraction sums; 3.20 sec (*SD* = 0.94) for whole numbers; and 4.01 sec (*SD* = 1.58) for whole number sums. As in Table 2, not all fraction and whole number values are exact matches. Across items, the mean pixelated difference between fraction and whole-number matched pairs is of 0.82 (mismatched by 0.2%), consistent with Braithwaite et al.'s (2018) items. Test-retest reliability on similar computer number line assessments (e.g., Fuchs et al., 2013) is .82.

We chose this measure due to the difficulty of its fraction items and because few items were used in the tutoring program (in which denominators are restricted to 1, 2, 3, 4, 5, 6, 8, 10, and 12). Further, none of the fraction sums was easily calculated using the strategies taught in program, and the intervention program did not include any computerized number line tasks. In these ways, the measure should reflect participants' overall estimation ability and conceptual understanding, rather than application of the specific strategies taught in the program or fluency with particular items.

Training and Support Procedures across Conditions

Methods for training and supporting tutors, which were similar across the fraction tutoring and the text comprehension tutoring conditions, were in line with principles derived from the tutoring learning effect literature. Tutors participated in a 2-day workshop (16 hours) to learn the content and demonstrated 95% accuracy role-playing lessons with trainers before they began tutoring students in schools. Training focused on effectively implementing the tutoring program's prescribed activities with the prescribed methods. Research staff did not spend training time teaching tutors about fractions. Research staff also conducted weekly meetings to solve ongoing problems, answer questions, and preview upcoming content. Periodic live observations (approximately 200 min of live observations per tutor) and review of a random sample of 20% of audiotaped tutoring sessions across the 39 lessons provided the basis for ongoing corrective feedback. Corrective feedback typically focused on developing more efficient and effective correction procedures for student misconceptions, in line with the language used in the tutoring program.

This support system was supplemented with detailed lesson guides and structured activity and help cards to ensure tutors relied on the program's methods. The support system also encouraged tutors to generatively supplement lessons guides and help cards to address student needs responsively. At weekly meetings, tutors described student struggles. Sometimes, this identified misapplication of tutoring methods, which were resolved. Other times, tutors participated in discussion with fellow tutors and the research staff to generate and fine-tune strategies for addressing individual student needs. Occasionally, the whole group of tutors added these methods generatively to their toolkit.

Fraction Tutoring

Fraction tutoring relied on the 13-week (39 lessons) research-validated *Super Solvers* program (e.g., Fuchs et al., 2019; Fuchs et al., 2020). The program focuses on developing students' fraction-magnitude understanding with conceptually-based explanations and efficient strategies for assessing magnitude. Number lines connect students' understanding of whole numbers and fractions and serve as the primary representational context for assessing magnitude. Students learn efficient estimation strategies centered on benchmarking to 1 and ½. Activities include comparing fractions, ordering fractions, placing fractions on the number line, fraction word problems, fraction calculations, developing proportional reasoning, and developing automaticity with fraction equivalencies.

Tutors explicitly model and explain new concepts and strategies with manipulatives and worked examples designed to reduce cognitive load. Tutors introduce and gradually fade problem-solving cards to support student application of strategies for thinking through comparison, ordering, and paper number line activities. (No computerized number line activities, as in the present study's assessment tasks, are included in the program.) At third grade, the focus on fraction magnitudes is contextualized and extended with conceptual explanations and efficient strategies for solving word problems; at fourth and fifth grades, for understanding and executing fraction operations. Across grades, tutors require students to apply taught strategies to help students consider the reasonableness of fraction magnitude assessment, word-problem, or computational answers. Pacing and skills differed for grade 3 versus grades 4 and 5. Fidelity to the tutoring procedures was strong, with 91% of intended activities and explanations implemented based on coding of audiotaped fidelity recordings. Across a series of randomized controlled trials, iterations of this program have significantly improved student understandings of fractions (e.g., Fuchs et al., 2017; Fuchs et al., 2020). Contact lynn.a.davies@vanderbilt.edu for information on how to obtain a tutoring manual.

Text Comprehension Tutoring

Text comprehension tutoring relied on the research-validated *Reading PI* program (D. Fuchs et al, 2018), comprising 42 sessions over 14 school weeks. A comprehensive array of strategies is used to teach text comprehension, centering on previewing the text, reading and engaging with the text, and answering questions about the text. Instruction focuses on vocabulary, identifying text features and text structure, clarifying and making connections, summarizing, and inferencing. Tutors explicitly model and provide feedback to guide students toward independence. All three grade levels focused on the same content, but pacing and texts were customized to support developmental differences at grade 3 versus grades 4 and 5. Fidelity to the tutoring procedures was strong, with 94% of activities implemented as intended based on coding of live observations and audiotaped recordings. Across a series of randomized controlled trials, iterations of this program have significantly improved student performance on near- and mid-transfer text comprehension tasks (e.g., D. Fuchs et al., 2018). Contact lynn.a.davies@vanderbilt.edu for a tutoring manual.

Procedure

In August, September, and October, tutors completed initial training, separately for fractions and for text comprehension depending on the study to which they had been assigned. In April, adult tutors who participated in the present study provided background information and completed the number line tasks in a single session in a private space. Each participant received a $25 gift card for completing this session. Data were collected such that they are not traceable to participant identities.

In terms of the parent studies (D. Fuchs et al., 2018; L.S. Fuchs et al., 2019; L.S. Fuchs et al., 2020), which focused on student learning outcomes, third, fourth, and fifth graders were identified as qualifying for tutoring due to low academic performance in the relevant domain, via a systematic screening process involving researcher-administered mathematics or reading tests (depending on student condition). Low performers who entered each study were pretested in the relevant academic domain and were randomly assigned to receive tutoring or participate in a control group. Tutoring began in October and concluded in February-March, with weekly tutor meetings held throughout this period. Each tutor worked with one to five groups; each group had two students. At the end of tutoring, children were posttested on measures relevant to their study's academic domain.

Results

Means and standard deviations on the four number line tasks are shown in Table 3. Analysis of variance revealed a significant difference favoring the fraction over the text comprehension tutoring condition on fraction sums, $F(1, 40) = 5.29$, $p = .027$, with an ES of 0.75. On the other three measures, differences between conditions were not statistically significant; however, on each measure, they favored the fraction tutoring condition: for placing fractions, $F(1, 40) = 2.25$, $p = .142$, ES = 0.47; for placing whole numbers, $F(1, 40) = 0.98$, $p = .327$, ES = 0.31; and for placing whole number sums, $F(1, 40) = 0.70$, $p = .408$, ES = 0.26.

Discussion

On the fraction number line task involving placement of fraction sums, the advantage for the fraction tutors over the comparable group of text comprehension tutors was significant and large, with an ES of 0.75 *SD*. This fraction sums task is more difficult than placement of single fractions by requiring respondents to conceptualize the first addend's magnitude, the second addend's magnitude, and estimate the combined quantity's number line placements as quickly as possible, within a few seconds. Alternatively, the respondent may first estimate placement of the first addend and then extend the distance closer to the number line endpoint by a distance approximating the magnitude of the second addend. Either way, it is not surprising that the ES for the placement of individual fractions was smaller, approximately two-thirds the size of the ES for fraction sums. Although not significant, the ES for placement of individual fractions was of moderate size (0.47).

These results lend support to the hypothesis that immersing adult tutors into a research-validated fractions curriculum (e.g., Fuchs et al., 2019; Fuchs et al., 2020), centered on fraction-magnitude understanding and number lines, improves adults' ability to understand fraction magnitude. We judge the Braithewaite et al. (2018) fraction sums number line tasks to be reflective of participants' overall fraction magnitude understanding, rather than application of the specific strategies taught in the program or fluency with particular items, for three reasons. First, few addends on the measure appeared in the tutoring program. Second, none of the fraction sums was easily calculated via the program's taught strategies. Third, the computerized number line task was novel (the intervention program did not include any computerized number line activities and did not include estimation of fraction sums on or off the number line).

Although we did not code participants' strategy use during the assessment, the response times for fraction sums (5.12 sec, *SD* = 1.54) was nearly the same as for single fractions (4.99 sec, *SD* = 1.55). Thus, it seems unlikely that participants quickly calculated common denominators before estimating the magnitude of sums. It also seems unlikely that they relied on a mental analogue of the tutoring program's strategic steps they children taught to think through and systematically execute to derive paper-pencil number line placements. More plausible is that participants' consistent use of benchmarking to 1 and ½ as they taught the fractions program developed deep understanding of magnitude and flexibility in high-level strategic behavior, which produced more accurate estimates with low response times on the difficult test problems.

In contrast to these large and moderate ESs for the fraction placement tasks, the ESs contrasting the accuracy of whole number placement between the tutor conditions fell in the small range: 0.31 for single whole numbers and 0.26 for whole number sums. The mean for whole number placement across the two relevant tasks reflected twice the accuracy than the mean fraction placement: 2.85 PAE as opposed to 6.10. The small advantage in ES for fraction tutors over text comprehension tutors on the whole number tasks probably reflects fraction tutors' ongoing immersion with number line thinking, while few opportunities were likely available in everyday life to the text comprehension tutors.

Results lend support to the idea that an immersive fraction tutoring experience improves *adult tutors'* fraction magnitude understanding, at least when the fractions tutoring program centers on developing children's fraction magnitude understanding via flexibility in benchmarking and via reliance on number line as the principal representational context. The productivity of such an instructional focus for supporting the development of fraction understanding echoes Saxe et al. (2013) and Fuchs et al. (2017), who demonstrated the efficacy of fractions instruction that explicitly connects integers and fractions into a single framework, using number lines. Those studies focused on *children's* fraction outcomes. Lewis and Perry (2017), by contrast, demonstrated added value on *teacher* as well student understanding when teachers implemented guided fraction lesson study supplemented with a resource kit that incorporated a strong focus on magnitude and number lines.

The present study extends this prior work by demonstrating enhancement of adults' fraction knowledge via their implementation of an immersive tutoring experience. Similarly, Juel (1996) found that college students who were struggling readers significantly benefited from tutoring first-grade students who were also struggling readers. Codings of tutor-student interactions revealed that scaffolding and modeling were particularly beneficial for both the tutor (college student) and first-grade student. Consistent with Juel, master's students' in the present study, who relied heavily on scaffolding and modeling, demonstrated significant advantage from tutoring elementary-aged students struggling with mathematics. Unlike Juel, in which college student tutors also struggled academically, tutors in the present study were master's students in a top tier graduate degree program. Even so, their knowledge also benefited.

This finding is in line with the literature showing stronger learning for tutors than for comparable non-tutors (Cohen et al., 1982; Mathes & Fuchs, 1994; Rohrbeck et al., 2003) on a wide array of academic domains, including other forms of mathematics (Fuchs et al., 1997) and including college age tutors (Annis, 1993; Coleman et al., 1997). We identified no previous study investigating this idea for fractions.

As Roscoe and Chi (2007) clarified, such a tutoring learning effect occurs when the tutoring content is cognitively demanding, when tutoring duration is adequate, when tutors have the training and authority to extend explanations and generate questions that are responsive to their students' needs, and when fidelity to the tutoring procedures is strong. Because the present study's immersive fraction tutoring experience reflected these principles, it appears to have permitted the adult fraction tutors to use their existing fraction knowledge in generative ways not only to improve their tutees' fraction knowledge, but also to transform their own knowledge (Roscoe & Chi, 2007).

Implications for Research and Practice

This study therefore provides evidence supporting a link between fraction tutoring and adults' fraction understanding, even as it illustrates the potential for using an immersive fraction tutoring experience, focused on magnitudes and number lines, to improve preservice or inservice teachers' fraction understanding. Given the present study's causal-comparative design, it provides the basis for investment in a randomized controlled trial assessing the effects of an immersive fraction tutoring experience on preserve or inservice teachers' understanding of fractions.

Such a study should explicitly code strategies during adult participant's pre- and posttesting, even as it assesses the flexibility of their fraction thinking. Although we did not measure tutor outcomes in terms of flexibility, such effects seem plausible because flexibility in thinking about fractions is required when tutoring students who struggle with mathematics. A recursive feedback loop, in which tutors systematically vary the nature of their explanations to provide support for students with varying needs, should in turn improve tutor understanding of fraction concepts. This likely contributed to the strong ES on the fraction sums task reported in the present study.

The present study's findings may help guide future practicum experiences for preservice teachers. Such a fraction tutoring experience may be provided as a supplement to the kind of preservice coursework Newton (2008) assessed. Alternatively, for inservice teachers, such an experience may occur in schools that rely on multilevel systems of support to conduct small-group intervention to struggling learners. Given evidence that inservice professional development initiatives fail to improve teachers' or students' rational number knowledge (Garet et al., 2011), a structured fraction tutoring experience – implemented as a stand-alone experience as in the present study or in conjunction with a PD fractions program – may provide a stronger vehicle for improving inservice teacher knowledge. A beneficial side effect is that students simultaneously profit (Fuchs et al., 2017; Fuchs et al., 2020).

This prompts us to call attention to the fact, in the present study, fraction tutors not only improved their own fraction knowledge but also succeeded in helping their struggling third through fifth graders achieve dramatically stronger fraction outcomes than the contrast groups of struggling peers who did not receive this intervention. We know this because, as mentioned, the present study's fraction tutors implemented the fractions program as the independent variable in randomized controlled trials at third grade (Fuchs et al., 2020) and at grades 4 and 5 (Fuchs et al., 2019). Outcomes favoring students who received fraction tutoring over control were significant, with moderate to large ESs on a range of fraction tasks, including released fraction items from the National Assessment of Education Progress. Consequently, an important advantage of an immersive fraction tutoring experience for improving teacher fraction knowledge is the potential to simultaneously enhance the fraction knowledge of adults and students alike.

Limitations

Findings must, nevertheless, be interpreted within the constraints of the study's limitations. First and foremost, because the study relied on causal-comparative design, causal inference is only suggestive. Additionally, sample size was small, such that an ES of 0.47 on the single fraction task did not achieve statistical significance. Further, without a future student corroborating effects in preservice or inservice teacher, we cannot assume that effects would generalize to teachers' fraction knowledge. Also, as in many intervention studies, we cannot isolate which components of the intervention were accounted for the observed effects.

The final limitation concerns the measures used in the present study. We identified no prior PD study that relied on computerized fraction number line tasks, including fraction or whole-number sums and individual numbers, as in the present study. Rather, the teacher fraction knowledge indexed in previous investigations typically involves paper-pencil measures on a variety of fraction problem types mirroring the knowledge forms (usually basic concepts, calculations, and problem solving) required of students at or near the grade levels at which participating teachers work (e.g., Garet et al., 2011; Lewis & Perry, 2017; Newton, 2008; Zhou et al., 2004). Sometimes these measures are supplemented with tasks tapping teacher flexibility with fraction content (e.g., Newton, 2008). Sometimes pedagogical knowledge relevant to fraction content is assessed (e.g., Garet et al., 2011; Zhou et al., 2004). Extensions to the present study should include a greater range of assessments, reflecting tutor fraction knowledge more proximal to the demands students face and relevant pedagogical knowledge (as in prior studies) as well as more generalized fraction understanding (as in the present study).

Pending future corroborating evidence from studies with random assignment of tutors to fraction tutoring and contrast tutoring conditions, with a larger number of tutor participants, with preservice or inservice teacher participants, and with a variety of measures, the present study strengthens the hypothesis that an immersive fraction tutoring experience, designed in line with prior research on the tutor learning effect (Roscoe & Chi, 2007) and relying on effective methods for improving student understanding of fractions (Saxe et al., 2013), is an effective approach to enhance teachers' fraction knowledge.

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Table 1
Demographics and GRE Scores (Verbal and Quantitative) by Condition

		Condition						
		Fractions Tutor (*n* = 25)				Reading Comprehension Tutor (*n* = 17)		
Variable		*M*	*SD*	%		*M*	*SD*	%
Age		27.48	(4.23)			25.59	(3.37)	
Race/Ethnicity								
African American				4.2%				-
White				83.3%				94.1%
Hispanic				4.2%				-
Other				8.3%				5.9%
GRE[a] Scores								
Verbal		157.32	(5.62)			156.73	(4.33)	
Quantitative		154.08	(6.37)			151.53	(6.68)	

[a]GRE is Graduate Record Exam (Educational Testing Service, https://www.ets.org/gre; Verbal is verbal reasoning; Quantitative is quantitative reasoning).

Table 2
Items on the Number Line Estimation Tasks

Fraction Measures (0-1 Number Line)		Whole Number Measures (0-1000 Number Line)	
Fractions Items (Decimal Equivalent)		*Whole Numbers* Items	
1/18	(0.056)	56	
1/14	(0.071)	71	
1/10	(0.010)	98	
1/9	(0.111)	111	
2/17	(0.118)	118	
1/8	(0.125)	125	
2/7	(0.286)	286	
4/13	(0.308)	308	
14/15	(0.933)	933	
Fraction Sums Items (Decimal Sum)		*Whole Numbers* Items (Sum)	
1/14+1/18	(0.127)	71+56	(127)
2/17+1/14	(0.189)	118+71	(189)
1/8+1/10	(0.225)	125+98	(223)
1/8+1/9	(0.236)	125+111	(236)
2/7+1/10	(0.386)	286+98	(384)
4/13+2/17	(0.425)	308+118	(426)
1/3+1/10	(0.433)	333+98	(431)
2/7+1/10	(0.386)	286+98	(384)
4/13+2/17	(0.425)	308+118	(426)
1/3+1/10	(0.433)	333+98	(431)
7/16+1/18	(0.493)	438+56	(494)
1/3+2/7	(0.619)	333+286	(619)
11/19+2/17	(0.697)	579+118	(697)
3/5+1/8	(0.725)	597+125	(722)
7/16+4/13	(0.745)	438+308	(746)
3/4+1/9	(0.861)	752+111	(863)
3/5+2/7	(0.886)	597+286	(883)
11/12+1/18	(0.972)	917+56	(973)
6/11+7/16	(0.983)	545+438	(983)

Note: Items are from Braithwaite et al. (2018). We present items in ascending order to enhance readability. Note that subjects saw items in random order, and ensured sequential items spanned the four quartiles.

Table 3
Outcome Means and Standard Deviations by Condition

	Condition			
Variable	Fractions Tutor (n = 25)		Reading Comprehension Tutor (n = 17)	
Outcomes				
Fractions	4.33	(1.89)	6.21	(5.86)
Fraction Sums	11.7	(3.94)	16.12	(8.45)
Whole Numbers	3.48	(1.21)	3.80	(1.39)
Whole Number Sums	5.25	(1.38)	5.65	(1.76)

The outcome scores are percentage absolute error on the four number line estimation tasks following Braithwaite et al. (2018).