USING A DISCURSIVE FRAMEWORK TO ANALYZE GEOMETRIC LEARNING AND INSTRUCTION

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In this study we applied a discursive perspective of learning (Sfard, 2008) to a sequence of 21 geometry mini-lessons taught in a fourth grade classroom. From this perspective, learning is defined as changes in mathematical discourse. We first characterize and then compare discourse from the beginning and the end of the mini-lesson sequence. We identify shifts in the discourse that occurred during the sequence. We then discuss how the characteristics of the students' geometric discourse informed task design and instruction. This perspective provided a useful means for linking instruction to student learning in an operationalized manner.

Keywords: Classroom Discourse, Geometry and Geometrical and Spatial Thinking, Instructional Activities and Practices, Learning Theories

While geometry is an important branch of mathematics, U.S. students' geometry achievement lags behind achievement in other areas of mathematics (Mullis, et al., 2016). At the same time, scholars have observed that the geometry instruction students receive is often lacking (Sarama & Clements, 2009). Much of the early research on geometry learning (e.g., Burger & Shaughnessy, 1986; van Hiele, 1959) has stemmed from a cognitive perspective, while more recent research has shifted to a participationist perspective (Lave & Wenger, 1991; Sfard, 1998; Wenger, 1998), specifically, a discursive perspective (Sfard, 2007, 2008; Sfard & Lavie, 2005). This shift results in advantages, both analytic and practical (Sinclair, Cirillo, & De Villiers, 2017). From an analytic perspective, a discursive theory operationalizes learning and does not require researchers to make inferences about unseen cognitive processes. From a practical point of view, the theoretical framework can provide suggestions for task design and implementation to inform instruction.

This project aimed to analyze the discourse about geometric shapes and properties that developed in a fourth grade classroom across a series of 21 geometry mini-lessons. We use a discursive framework (Sfard, 2008) to (1) characterize learning and (2) identify ways instructional changes were made in response to students' discourse. We focus on the following questions:

- What is the impact of geometry mini-lessons on students' geometric learning?
- How does a discursive theory of learning inform instruction?

Discursive Theory of Learning

According to Sfard (2007), "the different types of communication that bring some people together while excluding some others are called discourses" (p. 571). *Communication* is the patterned activity where the action of one individual is followed by the action of another individual and "*thinking* is an individualized version of (interpersonal) communicating" (Sfard, 2008, p. 81). Used in this way, the notion of discourse embodies more than the communicative features of talk, but positions one in a larger community of practice. Sfard (2008) identifies two

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important types of discourses: colloquial and literate. Colloquial discourses are those that arise in one's everyday experience, while literate mathematical discourses are those that make use of standard mathematical terminology and symbolism recognized by the broader mathematical community. Sfard notes that *commognitive conflic*t "arises when communication occurs across incommensurable discourses" (p. 296). Much of schooling involves this intersection of discourses, with students moving from the colloquial discourse to the literate discourse. From this perspective, *learning* is defined as a lasting change in discourse.

In Sfard's framework, four features distinguish mathematical discourse: word use, visual mediators, narratives, and routines. *Word use* consists of the various terms and words that are unique to mathematical discourse. The meaning that is conveyed by these words, however, may differ depending on the discourse community that one is acting in. *Visual mediators* represent the symbolic and visual artifacts that are used as the basis for mathematical communication. As with words, the way that we attend to these visual mediators depend on the discourse. *Narratives* embody the ways that we describe, engage with, and identify relationships between mathematical objects. Narratives can be endorsed or rejected within the discourse. For instance, a possible narrative may be that all squares are rectangles. This narrative may be rejected by students in a discourse community based on the notion that a rectangle should have two long sides and two short sides. Finally, *routines* represent the patterns of activity found in a discourse. For instance, a routine for identifying a shape in one discourse community might involve matching the shape to a set of canonical shapes. In another, it might involve measuring side lengths and angles. Similarly a routine for comparing shapes in one discourse community could focus on their size and orientation, while in another, the routine might focus on their geometric properties.

Much of the research that has applied a discursive framework to geometry learning has focused on narrow instructional segments or small populations. Some of these studies have included students. For example, Sinclair and Moss (2012) analyzed one 30-minute lesson on creating and transforming triangles with a group of 11 kindergarten students. A pair of bilingual high school calculus students exploring in a dynamic geometry environment were the participants in a study by Ng (2015). Other studies have teacher participants. Pre- and post-tests were administered to 63 prospective elementary and middle school teachers in a study by Wang and Kinzel (2014). They report on two participants' geometric discourse but do not link to instruction. Sinclair and Yurita (2008) analyze the discourse of a high school geometry teacher, comparing static and dynamic environments. A goal of our project was to expand beyond the scope of these studies and explore the shifts in geometric discourse for a whole classroom over instructional episodes spanning several weeks. Further, we wanted to investigate how the discursive framework could also inform instruction.

Methodology

A teaching experiment methodology (Steffe & Thompson, 2000) was adopted for this study. As teacher-researchers in the project, we developed and implemented 21 geometry mini-lessons over the course of 11 weeks in a fourth grade classroom in a small, midwestern town. Approximately 22 students were in each class session. The mini-lessons occurred before students received geometry instruction in the regular fourth-grade math class. For each mini-lesson, one researcher acted as the instructor, while the other was an observer.

The geometry mini-lessons followed a format similar to that of *number talks* (e.g., Humphreys & Parker, 2015). Like number talks, these geometry mini-lessons were 15-20 minutes long and focused on geometric shapes and relationships (as opposed to number and computation). The teacher solicited ideas from several students and facilitated the subsequent discussion connecting these ideas. The mini-lesson tasks were designed to allow for multiple entry points and a range of student responses. We discuss three types of tasks:

- Quick Image (Wheatley, 2007). Students were shown a figure for 2-3 seconds and asked to draw it. Students described how they saw the figure and how they knew what to draw. This was repeated with three other related figures. The task concluded with a discussion of the similarities and differences of the figures.
- Which One is not Like the Others? (Danielson, 2016). Students were presented with four shapes and asked: Which one is not like the others? Any of the four shapes could be chosen as not like the others. Students discussed their reasons for their choice.
- Guess My Shape. The class was presented with a collection of shapes, one of which had been secretly selected by the teacher. The students asked yes/no questions that the teacher answered. The teacher led a discussion of the consequences of a yes or no answer to the questions before answering. The instructor strategically selected questions to answer based on the goals for the mini-lesson.

The data sources for this report come from video recordings, copies of students' written work, and field notes of each mini-lesson, along with notes from teacher-researcher post-lesson debriefing. Two video cameras captured the lesson from different angles in the classroom. When viewing video of the sessions in ATLAS.ti, each student utterance was identified and coded in two ways. First, codes were created to identify the geometric properties and shapes under consideration in the utterance. Then the utterance was coded for features of discourse, namely word use, routines, narratives, and visual mediators (Sfard, 2008). Any words related to the geometric properties and shapes, whether colloquial or literate, were coded. Instances where students compared or identified shapes were coded with the specific routines they used. Following Sfard, we coded students' descriptions of shapes as narratives.

We then looked at all the lessons and identified the main geometric topics addressed, e.g. angles, parallel lines, congruence. We examined each task and identified the potential geometric topics to be discussed by students. We looked for the emergence of these topics in the classroom discourse and linked together sessions that addressed the same topic. This gave us a mapping of the geometric topics that were addressed during the 21 mini-lessons.

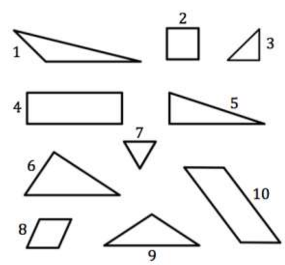
Changes in Discourse over Time

To identify learning that occurred during the sequence of mini-lessons, we characterize the discourse of two lessons, one occurring near the beginning (mini-lesson 4) and one near the end (mini-lesson 18) of the 21-lesson sequence. Sfard (2007) noted that, "because mathematical discourse learned in school is a modification of children's everyday discourses, learning mathematics may be seen as transforming these spontaneously learned colloquial discourses rather than building new ones from scratch" (p 573). As such, we focused on the characteristics of the students' use of words, visual mediators, narratives, and routines in both mini-lessons, and examined how these shifted. Both mini-lessons used the Guess My Shape task structure, so the nature of the mathematical activity was similar.

Early Discourse

The fourth mini-lesson in the instructional sequence featured a collection of triangles and quadrilaterals (Fig. 1a). Notably, the shapes included right, acute, and obtuse triangles, as well as several shapes with different configurations of congruent sides. The students used these shapes as visual mediators to establish routines for categorizing or differentiating between shapes. The routines could be identified primarily through the questions that they suggested, as well as the accompanying discussion surrounding those questions. Figure 1b shows the questions generated by the students during the lesson.

Many of these questions demonstrated that students' routines for distinguishing between two shapes were tied primarily to the shapes' visual characteristics such as size and orientation. Only two questions focused on using geometric properties as a basis for distinguishing shapes, namely side congruence or number of sides. Even the use of the terms "rectangular" and "triangular" rather than "a rectangle" or "a triangle" highlight a focus on visual characteristics rather than identifying the shape as part of a class of shapes.



(a)

1. Is it slanted or up?
2. Is it upside down?
3. Is it long?
4. Is it short?
5. Is it on its side or flat?
6. Does it have four sides?
7. Is it rectangular?
8. Is it triangular?
9. Are the sides equal?

(b)

Figure 1: Early Discourse Guess My Shape Task and Student-generated Questions

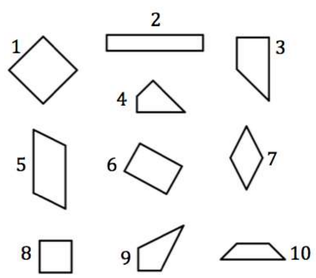
The questions and the discussion surrounding them heavily featured colloquial word use such as *slanted*, *up*, *flat*, *upright*, *upside down*, *laying*, and *corner*, words that are not well defined in the literate geometric discourse. Some words, which do have specific meaning in the literate discourse, were used in alternative ways. For example, a student referred to a shape being "straight" as a way to differentiate it from shapes she considered to be "slanted." Similarly, the sides were considered "equal" rather than "congruent." A rhombus (Fig. 1, shape 8) was referenced as a "little square" and a parallelogram (shape 10) was referred to as "rectangular."

Several competing narratives regarding the characteristics of the shapes under consideration emerged during the discussion. In the end, the students were left considering shapes 1, 5, and 6. The students were considering the question, "is it up"? When asked to show with their fingers how many of the shapes were "up," the responses were roughly equally distributed between 1, 2, 3, and 5. Some students only considered the isosceles triangle (shape 9) as being "up." Others included shapes that had vertical sides (e.g., shapes 1 and 5). Generally, the students did not clearly communicate their descriptions (narratives) of shapes that were "up" in a manner where it could be subjected to endorsement and agreement by the group. After the questions with geometric properties (e.g., number of sides and congruence) were answered, there was no clear resolution of the task due to the differences in the discourse. The words and narratives students were using made it difficult for them to determine which shapes to eliminate. In the end, the teacher had to reveal to the students which shape was selected.

Later Discourse

The eighteenth mini-lesson occurred approximately six weeks after the aforementioned mini-lesson. The quadrilaterals in the Guess My Shape task (Fig. 2a) were purposefully selected and oriented to serve as visual mediators for the students' geometric discourse. As with the previous mini-lesson, the shapes we selected gave students an opportunity to focus on both visual characteristics, such as size and orientation, as well as geometric properties, such as presence of right angles, parallel sides, and congruent sides.

The discursive routines students used to distinguish between shapes is evident in the questions they posed (Fig. 2b). Except for the final two, each question featured a distinguishing geometric property. The question, “Is the shape congruent?” was later clarified by the student to mean, “Does the shape have congruent sides?” The final two questions came at the end of the mini-lesson after all the shapes had been eliminated except for the two squares (shapes 1 & 8). Since both squares shared all salient geometric properties, the students’ routines naturally reverted to using orientation and size, both visual characteristics.



(a)

1. Does it have two sets of parallel sides?
2. Does it have a right angle?
3. Is the shape congruent?
4. Does the shape have congruent sides?
5. Is it a rectangle?
6. Does it have obtuse angles?
7. Are all four sides congruent?
8. Is it big?
9. Is it tilted?

(b)

Figure 2: Later Discourse Guess My Shape Task and Student-generated Questions

The narratives generated in this mini-lesson emphasized geometric properties, and were communicated in ways that could be endorsed by other students and the broader mathematical community. Late in the lesson, the instructor answered yes to the question, “Is it a rectangle?” When asked to consider which shapes should be eliminated from consideration, a few competing narratives surfaced. One student indicated a rectangle needed to have “even” sides, while another indicated that a rectangle needed to have four right angles. The latter narrative was endorsed when the instructor asked whether shape 9 (kite) should be eliminated from consideration. Several students agreed, with one stating that even though it had a right angle, it should be eliminated, “since it [didn’t] have four [right angles].” This discussion came to a head when students were asked if shape 1 (square) should be eliminated. After checking that the shape did, indeed, have four right angles, the students were reminded that the question answered was “Is it a rectangle?” At this point, some students proclaimed that the shape was a square, not a rectangle. Others, however, argued that it should remain since a rectangle has four right angles. In the end, the class was asked to raise their hands to indicate if they felt the shape should still be considered, and all but one student agreed. It is important to note that there is little evidence to suggest that this constitutes endorsing the narrative *all squares are rectangles*. In fact, some students’ hesitation based on naming the shape “square” rather than “rectangle” suggests that they have not. This episode does suggest that the students were comfortable endorsing the narrative *rectangles have four right angles*.

Comparing the Discourse

We see a stark contrast in the discourse between the two mini-lessons. This change is learning (Sfard, 2008). To begin, there is a difference in the questions the students asked to identify the shapes (Fig. 1b & 2b). Although the shapes were similar in each mini-lesson, students focused on orientation and visual appearance in the early mini-lesson; whereas, they were mostly attuned to properties of the shapes in the later mini-lesson. This suggests their routines for comparing shapes changed, i.e., students learned to focus on the properties of the shapes rather than the appearance of the shapes.

Word use also changed markedly from the early to the later mini-lesson. The early discourse was filled with colloquial and sometimes ambiguous words when referring to the shapes. The only words students used from the literate geometric discourse were the names of specific shapes (e.g., rectangle, rhombus, square) and these were not always used in conventional ways. For example, one student referred to a rhombus as a "slanted square." A hallmark of the students' later discourse was the frequent use of geometric words recognized by the mathematical community. Only a few utterances featured colloquial word use. The discussion was replete with geometric terms such as *parallel*, *congruent*, *obtuse angle*, and *right angle*.

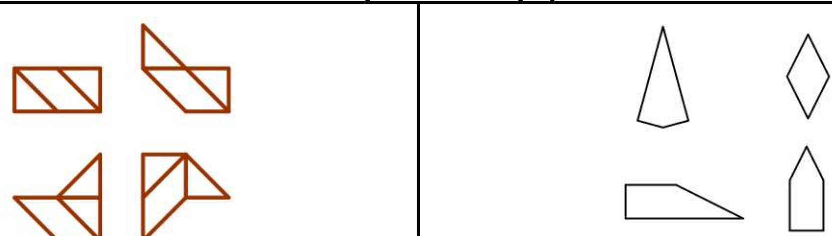
We can also see changes in the apparent student narratives for the geometric shapes. In the early discourse, for example, students described a rectangle as have 4 sides, two of them longer than the other. In the later discourse, students had come to describe rectangles as having 4 right angles. This change of narrative set the stage for future consideration of the narrative: *A square is a rectangle*. Class inclusion is the prevailing narrative in the broader mathematical community although it proves challenging for students (De Villiers, 1994).

Sfard's discursive framework proved useful in identifying students' geometric learning across the span of several mini-lessons. As students had opportunities to discuss geometric shapes, the words they used to describe shapes, the routines they used to compare shapes, and the descriptions (narratives) about shapes all changed. In sum, the students' everyday discourse was transformed to be closer to that of the literate discourse of geometric shapes. This did not happen in isolation, however, but was influenced by the instruction they received. As teacher-researchers, we were uniquely positioned having one foot in the students' discourse and the other in the discourse of the mathematics community.

Initiating Changes in Discourse through Instruction

We now turn our focus to show how the discursive framework allowed us to analyze the existing discourse and how it informed the instructional decisions for the subsequent mini-lesson. We consistently monitored the discourse in each mini-lesson and designed the subsequent mini-lesson to modify the students' discourse in specific ways to shift it closer to the discourse of the mathematics community.

The content goals for grade 4 geometry (CCSSI, 2010) include, among other things, identifying parallel lines in two-dimensional figures and classifying two-dimensional figures based on the presence or absence of parallel lines. After the first six mini-lessons, students had not yet noticed or discussed, either informally or formally, parallel lines.



Mini-Lesson 7
(a)

Mini-Lesson 8
(b)

Figure 3: Four Tasks Used in Mini-Lessons

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The task for the mini-lesson 7 provided opportunities for students to talk about parallel lines due to the figures chosen for the task. The Quick Image task (Fig. 3a) featured four different arrangements and orientations of parallelograms and right triangles. The teacher projected the first image (upper left) for the students to draw. Then, the teacher asked the students to describe what they drew. Students described the figure as having a sideways slanted square, a rectangle, a slanted rectangle, and a square. Some students commented on the diagonal lines but after reviewing their work, we noticed many students had drawn two diagonal lines while some had drawn three.

As we progressed sequentially through the images, a student noticed a parallelogram in each figure. The teacher asked the class what made that shape a parallelogram. The only verbal response was that it had four sides, although one student gestured the shape, indicating a routine of identifying a shape by matching it to a visual prototype. The teacher continued by mentioning "parallel" was part of the word *parallelogram*. "What does parallel mean?" the teacher asked. Students responses included, "the two shorter sides and the two longer sides are the same," "same size," and "side by side."

By the end of the discussion, it was clear that students did not share a narrative for parallel lines. Moreover, the students did not spontaneously use the word parallel, as this exchange was initiated by the teacher. The student-used words (side by side, same size) are not part of the literate mathematical discourse for parallel. Also, students' routines for classifying the parallelogram focused on congruence rather than parallel sides. This, then, informed their narratives about the meaning of parallel, conflating it with that of congruence.

In an attempt to change the students' discourse (i.e., learning) about parallel lines, we used the discursive framework to design and implement mini-lesson 8. We used a task where students would decide Which One is not Like the Others? We selected shapes that had the potential to encourage students to identify and discuss parallel lines. In particular, we selected three shapes that had parallel sides and one that did not (Fig. 3b). Other properties that we included were symmetry, right angles, and number of sides. Furthermore, we wanted to introduce a narrative: *parallel lines go in the same direction*. To this end, we had two long, thin wood dowels to position over the sides of the projected shapes to further illustrate that the sides go in the same direction. We specifically chose to focus on direction rather than the alternative narrative that *parallel lines do not intersect*. We wanted to avoid the possibility of introducing a narrative that conflated segments with lines. Two segments that do not intersect are not necessarily parallel. For example the left and right sides of the right trapezoid do not intersect and they are not parallel.

When we implemented this task in the classroom, students described ways in which the right trapezoid ("The only one pointing to the side."), the pentagon ("The only one with 5 sides."), and the rhombus ("It has two spikes [acute angles].") were different from the other shapes. At this point in the mini-lesson, the students have not provided any reasons for why the kite was different from the other shapes. The teacher had students recall the parallelogram in the previous mini-lesson. He introduced the dowels as a way to focus on the direction the sides were going and introduced the description of parallel lines as going in the same direction. Together the class revisited each shape and checked to see if any had parallel sides. Since the dowels extended the sides, it was clear that parallel sides did not intersect, but, more important, they had the same slope (go in the same direction). After checking the shapes, the mini-lesson ended with a student noting that the kite was the "different one because all of the other ones have parallel sides," to which no one disagreed.

Although new words, visual mediators (dowels), narratives, and routines for determining parallel lines were introduced by the teacher, they became part of the students' discourse. As we saw in the discussion of the later mini-lesson, the students' routine for comparing shapes included noticing the presence or absence of parallel lines.

Discussion

As we have shown, Sfard's discursive framework was useful for analyzing the impact of the mini-lesson sequence on students' geometric learning, defined here as a change in discourse. By operationalizing learning in this way, we can see "the expansion of the existing discourse, attained through extending a vocabulary, constructing new routines, and producing new endorsed narratives" (Sfard, 2007, p. 573). Taken together, these characteristics are pieces of a larger puzzle. Using the words rectangle and parallelogram was not always indicative that students could identify distinguishing properties (e.g., right or acute angles) when comparing them. Likewise, students were not necessarily willing to call a square a rectangle even though they acknowledge they both had four right angles.

Just as learning is a change in discourse, instruction can be viewed as catalyzing changes in the discourse. Sfard (2008) argued, "proactive participation of the expert interlocutors is critical to the success of learning" (p. 605). One of the most important roles of the teacher is to serve as the source of commognitive conflict where the encounter between the teacher's and the students' discourses necessitates a change in discourse. Commognitive conflict arises in the interaction between two discourses that do not operate under the same discursive rules. We can observe this commognitive conflict taking shape in the above episodes surrounding the norms that govern how shapes are described or differentiated. In the instructor's discourse, the means for discussing shapes was through identifying the geometric properties of the shapes, while the students allowed for the use of visual characterizations such as size and orientation. It was not enough to just introduce the properties into the students' discourse, the goal was for the students to similarly focus on geometric properties when examining shapes. In order for this shift in discourse to occur, it is important that the students agree that this shift is both necessary and advantageous.

The tasks outlined above were chosen because they had specific objectives (e.g., guess the mystery shape, find the shape that does not belong) that were more effectively resolved through the use of geometric properties than visual characteristics. As we saw in mini-lessons 4 and 8, the students' existing discursive routines acted as a hindrance to accomplishing this objective. In lesson four, the teacher had to eventually tell the students the mystery shape since their questions could not adequately distinguish the shapes. Similarly, the students could not tell why the kite did not belong without attending to parallel sides. The introduction of the word parallel, the routine of checking the direction of the sides with the dowels, and the narrative of parallel lines going in the same direction served as important tools to successfully resolve the task.

As teacher-researchers investigating elementary school students' geometry learning, we found great utility for Sfard's discursive theory in both unpacking the nature of the learning that took place, as well as being explicit about the instructional actions to support learning. By viewing learning as changing discourse, and attending to the characteristics of the students' discourse, we believe that teachers could be supported to become more explicit about their instructional goals.

References

Burger, W.F., & Shaughnessy, J.M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education*, *17*, 31–48.

Common Core State Standards Initiative (CCSSI). (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Danielson, C. (2016). *Which one doesn't belong?* Portland, ME: Stenhouse.

Humphreys, C., & Parker, R. (2015). *Making number talks matter*. Portland, ME: Stenhouse.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 international results in mathematics*. Retrieved from Boston College, TIMSS & PIRLS International Study Center website: http://timssandpirls.bc.edu/timss2015/international-results/

Ng., O. (2016). The interplay between language, gestures, dragging and diagrams in bilingual learners' mathematical communications. *Educational Studies in Mathematics*, *91*, 307–326.

Sarama, J., & Clements, D. H. (2009). Early childhood mathematics education research: Learning trajectories for young children. New York: Routledge.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher*, *27*(2), 4–13.

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from commognitive standpoint. *Journal of the Learning Sciences*, *16*(4), 565–613.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. New York: Cambridge University Press.

Sfard, A., & Lavie, I. (2005). Why cannot children see as the same what grown-ups cannot see as different?: Early numerical thinking revisited. *Cognition and Instruction*, *23*(2), 237–309.

Sinclair, N., Cirillo, M, & de Villiers, M. (2017). The teaching and learning of geometry. In J. Cai (ed.), *Compendium for Research in Mathematics Education* (pp. 457–489). Reston, VA: National Council of Teachers of Mathematics.

Sinclair, N., & Moss, J. (2012). The more it changes, the more it becomes the same: The development of the routine of shape identification in dynamic geometry environment. *International Journal of Educational Research*, *51-52*, 28–44.

Sinclair, N., & Yurita, V. (2008). To be or to become: How dynamic geometry changes discourse. *Research in Mathematics Education*, *10*(2), 135–150.

Steffe, L. P., and Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In A. E. Kelly & R. A. Lesh (eds.), *Handbook of research design in mathematics and science education* (pp. 267–306). Mahwah, NJ: Lawrence Erlbaum.

Van Hiele, P. M. (1959/2004). The child's thought and geometry. In T. P. Carpenter, J. A. Dossey, and J. L. Koehler (eds.), *Classics in mathematics education research* (pp. 61–66). Reston, VA: NCTM.

Wang, S., & Kinzel, M. (2014). How do they know it is a parallelogram? Analyzing geometric discourse at van Hiele level 3. *Research in Mathematics Education*, *16*(3), 288–305.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, UK: Cambridge University Press.

Wheatley, G. H. (2007). *Quick draw: Developing spatial sense in mathematics*, 2nd edition. Bethany Beach, DE: Mathematics Learning.