WHAT ROLE DOES PROFESSIONAL NOTICING PLAY? EXPLORING CONNECTIONS TO AFFECT AND PEDAGOGICAL CONTENT KNOWLEDGE

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This research report describes a study that explored the intersections of professional noticing of children's mathematical thinking (PN), pedagogical content knowledge (PCK), and affective domains with preservice elementary teachers (PSETs). An instructional module on professional noticing, as defined by Jacobs, Lamb, and Philipp (2010), included three components: attending, interpreting, and deciding, was implemented with 170 PSETs. A comparison group of 121 PSETs were enrolled in mathematics methods courses but the PN module was not implemented. PSETs who participated in the module showed significant positive growth in attending, interpreting, and attitudes toward mathematics. While there was no significant change in dispositions toward teaching mathematics, a decrease was observed in PSET deciding and pedagogical content knowledge. However, findings showed promising connections among the constructs.

Keywords: Teacher Knowledge; Affect, Emotion, Beliefs, and Attitudes; Preservice Teachers

Introduction and Objectives

Research on teacher noticing has been prominent in the field of mathematics education over the past decade (Schack, Fisher, & Wilhelm, 2017; Sherin, Jacobs, & Philipp, 2011; Stahnke, Schueler, & Roesken-Winter, 2016). Much of this work has used proximal measures of teaching practices to examine teacher noticing. In addition to teacher noticing, mathematics education research has established strong connections between teacher knowledge and teaching practices (Kunter, Klusmann, Baumert, Richter, Voss, & Hachfeld, 2013). While affect (e.g. attitudes and beliefs) has also been studied extensively in the field, there have been fewer links made between affect and teaching practices in mathematics (Philipp, 2007; Schoenfeld, 2015; Swars, Smith, Smith, Carothers, & Myers, 2018). However, there is a dearth of literature that connects the constructs of teacher noticing, pedagogical content knowledge, and affect in mathematics among preservice teachers. As Philipp (2007) states, "Researchers studying teachers' knowledge, beliefs, and affect related to mathematics teaching and learning are still trying to tease out the relationships among these constructs and to determine how teachers' knowledge, belief, and affect relate to their teaching" (p. 257). In this study, we attempt to examine such intersecting constructs that have the potential to influence teaching practices. Specifically, we explored professional noticing, a specific branch of teacher noticing, with the following research questions:

1. *To what extent can the implementation of a PN module influence PSET professional noticing skills, pedagogical content knowledge, and affect toward teaching mathematics?*

2. *In what ways do PSET professional noticing skills relate to their pedagogical content knowledge and affect toward teaching mathematics?*

Theoretical Framework

Professional Noticing of Children's Mathematical Thinking

Professional noticing of children's mathematical thinking (PN) is a specific branch of teacher noticing defined by Jacobs, Lamb, and Philipp (2010) as the amalgam of three components, attending, interpreting and deciding. Jacobs et al.'s definition of PN is widely referenced in the mathematics education community. It is frequently shortened to PN often leading to confusion with the more general term, teacher noticing (Sherin, Jacobs, & Philipp, 2011), which usually includes only the attending and interpreting components. Schack, Fisher, Thomas, Eisenhardt, Tassell, & Yoder (2013) demonstrated that PN can be developed in preservice teachers with improved responses to prompts related to attending, interpreting, and deciding over the course of a mathematics methods course embedded with specific instruction on PN. Additionally, Fisher, Schack, Thomas, Jong, Eisenhardt, Tassell, & Yoder (2014) previously explored connections between PN and preservice teachers' attitudes toward mathematics. Using the Attitudes Toward Mathematics Inventory (ATMI) developed by Tapia and Marsh (2004) and adapted for preservice teachers by Schackow (2005), Fisher et al. (2014) found significant change from pre to post assessment on three of the four factors of the ATMI. There was, however, no significant correlation found between ATMI factors and PN. This study extends that work by engaging preservice teachers in more complex teaching situations and examining the interactions of pre-service teacher PN with affect, as measured by two domains of the Mathematics Experiences and Conceptions Survey (MECS) (Jong & Hodges, 2015), and pedagogical content knowledge, as measured by selected items of the TEDS-M (Brese & Tatto, 2012).

Pedagogical Content Knowledge

Pedagogical content knowledge refers to that knowledge about content that is specifically needed to teach the content. Shulman (1986) drew upon Ong's 1958 observation that the medieval university made no distinction between content and pedagogy. Shifts in teacher preparation that emphasized pedagogy over content led Shulman to examine the need for content/subject knowledge in one's pedagogical approach. He defines pedagogical content knowledge as "the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). Teachers with strong pedagogical content knowledge also are knowledgeable of the potential misconceptions (Shulman, 1986) and speed bumps students will encounter in their learning, preparing instructional environments that allow students to grapple with the content, and ultimately overcome the speed bump. Pedagogical content knowledge has been the subject of much research over the past three decades (Ball, Thames, & Phelps, 2008; Hill et al., 2008; Depaepe, Verschaffel, & Kelchtermans, 2013) following on the premise that Shulman presented his theory as a heuristic method for studying the types and processes of knowledge needed by teachers. Ball, Thames, and Phelps (2008) attempted to refine Shulman's knowledge types, including pedagogical content knowledge, but acknowledged that further research was needed to bring clarification to the definition.

Depaepe et al. (2013) performed a systematic review of research on pedagogical content knowledge in mathematics education seeking to determine how PCK is conceptualized and investigated in empirical studies. The results demonstrated much alignment in researchers' definitions with Shulman's but there were also variations that drew upon other conceptualizations of pedagogical content knowledge (Ball et al., 2008; Hill et al., 2008).

Depaepe et al.'s (2013) conclusions illustrate that some researchers study a cognitive conceptualization of PCK, one in which PCK is within the mind of the teacher and often assessed through a test, and others study a situated conceptualization of PCK, which is related to "knowing-to act", one of Mason and Spence's (1999) four types of knowing.

Affect in Mathematics Education

Affect is used as a broad term that most commonly includes attitudes and dispositions, and are also closely linked to beliefs and self-efficacy, which vary in where they fall along the spectrum from emotion to cognition (Philipp, 2007). Schoenfeld (2015) points out the complexities of understanding affect in different contexts and while asserting that underlying beliefs influence practices, he also states that actions are what ultimately matter. He also urges researchers to examine changes in affective factors over time as he notes that since it takes years to develop beliefs and practices, then one would expect the same for substantive changes accompanied by support over time. Although there are affordances and drawback to various methods that examine affect in mathematics teaching, scholars agree that beliefs and attitudes matter (Aguirre & Speer, 1999; Jacobson & Kilpatrick, 2015; Wilkins, 2008).

Connections: Professional Noticing, Pedagogical Content Knowledge, and Affect

Thomas, Jong, Fisher, & Schack (2017) examined potential theoretical connections between two frameworks: mathematical knowledge for teaching and PN of children's mathematical thinking. They theorized that the skills of PN mediate mathematical knowledge for teaching and teacher responsiveness to children's mathematics. They developed a hypothesized trajectory of effective PN skills as an outcome of a well-developed MKT and responsiveness to student thinking and this research will continue to investigate these relationships. Similarly, Stahnke et al (2016) argue that such "skills display the missing link between mathematics teachers' dispositions (professional knowledge, affective motivational features) and their performance (observable behavior)" (p. 24). They are essentially making a case that PN skills have the potential to bridge affect, pedagogical content knowledge (referred to as "professional knowledge"), and teaching practices. Swars et al. (2018) recently conducted a study on the preparation of elementary mathematics specialists. They examined connections among pedagogical beliefs, teaching self-efficacy beliefs, specialized content knowledge, and observed teaching practices at the end of the program. The only significant relationship they found was between pedagogical beliefs and teaching self-efficacy; however, it might have been partly due to the self-reported nature of the instruments. Newton, Leonard, Evans, and Eastburn (2012) examined preservice teachers' self-efficacy, pedagogical content knowledge, and outcome expectancy before and after a mathematics methods course. Results from their study showed a consistent moderate positive correlation between self-efficacy and pedagogical content knowledge, but no relationship with outcome expectancy.

Methodology

Participants and Context

Participants were preservice elementary teachers enrolled in mathematics methods courses during one of five semesters at five universities (two urban, three rural) in the south-central United States. Treatment group PSETs ($N = 170$) participated in a module designed by the researchers to develop PN in the context of early algebraic thinking in a whole class setting. The *Examining Essential Expressions in Algebra* (E^3A) module included three 60-minute sessions focused on PN skills through the content of early algebraic reasoning and used complex video vignettes from whole class instruction in elementary mathematics classrooms to prompt discussion about the three components of PN: attending, interpreting, and deciding. Comparison group participants (*N* = 126) completed their mathematics methods course "business as usual".

Data Collection

Participants completed pre and post assessments consisting of three instruments, although not all participants completed all assessments, thus the varying *N*s. PN was assessed using a video-based assessment (*N* = 268). Affect was measured using the MECS (*N* = 149). We focused on the MECS' two-subscales of attitudes and dispositions to capture affect (Philipp, 2007). Additionally, preservice teachers responded to selected TEDS-M items (*N* = 196) chosen for their relation to pedagogical content knowledge in that some items represented student (mis)conception and required participants to identify such and discuss how they might respond instructionally. Selected TEDS-M items also matched the module's algebraic thinking content.

Video-based assessment. We assessed preservice teacher PN through a video clip of an authentic classroom in which children are engaging in the meaning of the equal sign in a whole group setting. The teacher has presented a number sentence, 10+10=__+5 asking the children to determine what number to put in the blank to make the number sentence true. The full class setting requires preservice teachers to attend to multiple understandings as displayed by the various children's responses in close temporal proximity to each other, make sense of the responses, and make an instructional decision based on the responses. The assessment prompts are aligned with the three components of PN in the order of deciding, attending, and interpreting: 1) *Pretend that you are the classroom teacher. What might you do next? Provide a rationale.*, 2) *What mathematical thinking and actions did you observe?*, and 3) *What did you learn about the children's mathematical thinking that influenced your decision in question 1?*. We intentionally prompted participants to decide first in an attempt to capture their in-the-moment thinking, more closely. PSET responses were scored by research team members on a four-point scale using decision trees programmed in JavaScript for automated scoring (see Schack, Dueber, Jong, Thomas, & Fisher, 2019 for details).

Mathematics Experiences and Conceptions Surveys (MECS). MECS is a set of instruments consisting primarily of six-point Likert-scale items (ranging from *strongly agree* to *strongly disagree*) designed to measure various affective factors related to teaching mathematics, such as attitudes, beliefs, dispositions, and self-efficacy over time (Jong & Hodges, 2015). MECS utilized the Philipp (2007) definitions of attitudes as "manners of acting, feeling, or thinking… Attitudes, like emotions, may involve positive or negative feelings. …Attitudes are more cognitive than emotions, but less cognitive than beliefs" (p. 259). It was also informed by his broader description of affect as "a disposition or tendency or an emotion or feeling attached to an idea or object", the main distinction between attitudes and dispositions being a feeling versus a tendency that might be more directly linked to an action. The two subscales used in the study were attitudes (MECSA) and dispositions (MECSD) toward mathematics. MECSA consists of six items, such as: "Mathematics is one of my favorite subjects." and "I look forward to teaching mathematics." MECSD consists of 10 items, such as "I plan to engage students in mathematics discussions." and "I plan to encourage students to share their thinking."

Teacher Education and Development Study in Mathematics (TEDS-M). TEDS-M was a study, funded by the International Association for the Evaluation of Educational Achievement, that examined how preservice teachers were prepared to teach mathematics at both the elementary and secondary levels in teacher education programs across 17 countries (Brese & Tatto, 2012). A major aim of TEDS-M was to capture preservice teachers' knowledge about mathematics. To assess pedagogical content knowledge in our study, we used a subset of eight

TEDS-M released items from the number and algebra content domains. Responses were then scored according to the TEDS-M scoring guide with varying scores of 0-2 based on the problem. Three open-response questions were added to the eight items to assist the researchers in further understanding PSETs' thinking. We did this by simply asking, "Please explain your response." We had specifically selected items within the number and algebra content domains because the TEDS-M post-test was administered immediately after the E^3A module was implemented in an attempt to capture PCK that was closely related to the content goals.

Data Analysis

Psychometric properties (e.g. model fit, factor analyses) of the aforementioned instruments were completed using WINSTEPS (Linacre, 2018) to inform subsequent analyses. Preliminary analyses of MECS' affect subscales indicated that the attitudes and dispositions scales were the most reliable, given the sample size. A Rasch rating scale model was used to examine reliability and model fit of the scales. Both subscales (at both pre and post) had reliability estimates ranging from 0.70 to 0.88. Principal components analysis of the residuals from the Rasch model also indicated strong unidimensionality for both scales, but a few items had poor fit according to INFIT and OUTFIT (Bond & Fox, 2007) and thus were eliminated from the analyses. Preliminary analyses of the TEDS-M items revealed excellent item fit according to INFIT and OUTFIT; however, the TEDS-M showed poor reliability (0.65 for pre and 0.55 for post) due to the limited number of items. This low reliability can reduce power in univariate significance testing such as *t* tests (Kanyongo, Brook, Kyei-Blankson, & Gocmen, 2007), and make the results of multivariable statistical analyses such as multiple regression untrustworthy (Cole & Preacher, 2014). Thus, we present findings including TEDS-M scores with a great deal of caution. While there are major limitations of the claims we can make with the TEDS-M due to the low reliability, we thought it would be beneficial to present results as a way of being transparent.

As PN video-based assessment scores are not interval level data, Wilcoxon signed rank tests were performed to determine whether PN scores significantly changed from the pre to the post assessment. As TEDS-M, MECSA, and MECSD scores are interval level data, a mixed design ANOVA was conducted on pre and post data for these scales to explore changes given the treatment effect and repeated measures aspects of the design. In the event of a significant interaction term, indicating a greater score change for the treatment group than the control group, a paired *t* test was performed to provide a significance test and Cohen's *d* effect size for the score change of the treatment group.

Next, in order to determine whether variables other than the treatment condition contributed to post-test scores, a multiple regression model was employed for each outcome of interest in which all measured variables (e.g. sum scores for TEDS-M and MECS factors) at pre-test were used as predictors. Models were tested with and without interaction terms (e.g., PreAttending × Treatment), but none of the interaction terms had significant regression coefficients and these interaction models did not explain significantly more of the variability in the outcome than the main effects models. Therefore, only main effects models are reported. As not all measures were completed by all participants, missing data were handled using full information maximum likelihood, an approach which utilizes all available data and makes fewer assumptions about the nature of missingness than listwise or pairwise deletion techniques (Enders, 2010).

Because of the way measurement error attenuates correlations, measurement error in the predictors of a multiple regression model can change the strength and pattern of effects. On the other hand, measurement error in the outcomes of a multiple regression model attenuate the strength of effects. Therefore, correcting for measurement error can give a more accurate description of the true underlying relationships (Cole & Preacher, 2014). Due to substantial measurement error in the TEDS-M data, we performed a sensitivity analysis in which results using observed variables were compared to results using a single indicator latent variable (SILV; Hayduk, 1987) technique for correcting for measurement error in the MECSA, MECSD, and TEDS-M variables.

Results

Changes in Constructs

Table 1 provides average pre and post scores of the various constructs examined among PSETs from both the comparison group and those who participated in the E^3A module. For the PN component scores of the treatment group, Wilcoxon signed-rank tests indicated that significant positive changes were observed in attending (Z = -3.219, p = .001) and interpreting (Z = -3.961, p < .001), but not in deciding (Z = -.384, p = .701). Furthermore, since scores for all three components decreased for the comparison group, there is evidence to suggest that the treatment was effective in the areas of attending and interpreting; however, the deciding component warrants further investigation.

Table 1: Descriptive Statistics

	Treatment		Comparison	
	Pre	Post	Pre	Post
	M (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Attending	1.03 (0.98)	1.34 (0.92)	0.96 (1.01)	0.83 (0.92)
Interpreting	0.77 (1.11)	1.22 (1.11)	0.88 (1.09)	0.66 (0.94)
Deciding	1.36 (0.81)	1.34 (0.88)	1.55 (0.98)	1.45 (0.99)
MECSA	21.85 (7.55)	24.09 (6.20)	22.73 (7.05)	22.49 (6.31)
MECSD	43.64 (3.95)	44.27 (3.35)	44.73 (2.91)	43.48 (4.47)
TEDSM	9.70 (2.75)	9.45 (2.13)	10.67 (2.33)	9.41 (2.05)

The mixed-effect ANOVAs of the TEDS-M and MECS results are reported in Table 2. On the left half of the table are the results of omnibus tests of interactions; all three interactions are significant, indicating that treatment group scores improved relative to control group scores. The results of post-hoc paired t tests on the treatment group can be found on the right half of Table 2. Affect, consisting of the MECS scales, yielded a significant positive change in attitudes (MECSA), and a non-significant positive change in dispositions (MECSD). Pedagogical Content Knowledge (PCK) included scores from the TEDS-M, in which average scores were lower, but not significantly lower than pre-treatment scores. Note that scores for MECSD and TEDS-M exhibited a treatment effect according to the ANOVA interaction term, but no significant improvement in scores for the treatment group. The significance of the treatment effect was therefore at least partially due to score decreases in the control group.

Table 2: Mixed-Design ANOVA and Post Hoc Tests

	Omnibus test of interaction effect				Treatment group paired *t* test				
	F	df	*p*	$\widehat{\eta_p^2}$	Diff.	t	df	*p*	*d*

MECSA	11.38	1, 146	.001	0.072	2.235	6.172	114	<.001	0.296
MECSD	6.322	1, 146	.013	0.042	0.635	1.762	114	.081	0.161
TEDSM	6.124	1, 162	.014	0.036	-0.254	-1.271	117	.206	0.092

Note. Cohen's d computed using pre-score standard deviation

Connections Among Constructs

Results of observed multiple regression models for all outcome variables are presented in Table 3. When the same models were estimated including SILV corrections for measurement error, all statistical decisions about significance were retained. Furthermore, parameter bias was slight except for the coefficients for the TEDS-M predictor, which were all smaller in the observed model than the SILV model. Accordingly, only the results of the more conservative observed model are reported and interpreted. All models had a significant R^2, however, for the TEDSM and MECSD variables, the only significant predictor was the pre-score of the same variable. The E^3A module, or treatment condition, was seen to be a significant positive predictor of Post-Attending (PostATT), Post-Interpreting (PostINT), and PostMECSA, consistent with earlier reporting of changes in constructs. Furthermore, PreINT significantly predicted PostATT, while PreTEDSM significantly predicted PostDEC. Also, PreMECSD significantly predicted PostMECSA.

Table 3: Standardized Regression Coefficients for Observed Analysis

	Outcome					
	PostATT	PostINT	PostDEC	PostTEDSM	PostMECSA	PostMECSD
	β	β	β	β	β	β
Treatment	.27***	.29***	-.00	.05	.21***	.15
PreATT	.27***	.01	.06	.01	.00	-.09
PreINT	.16**	.35***	-.01	-.06	.05	-.04
PreDEC	-.00	.06	.06	.01	.03	-.07
PreTEDSM	.15	.06	.24*	.58***	.02	.11
PreMECSA	-.05	-.04	.15	.11	.86***	-.01
PreMECSD	-.09	.03	.02	.03	.11*	.46***
R^2	.21***	.21***	.11*	.27***	.77***	.25***

Note. The coefficient for the treatment variable only standardizes the outcome, in order to provide a standardized difference between the two treatment and control groups while controlling for the other predictors. *$p < .05$. **$p < .01$. ***$p < .001$.

Discussion

In this study we endeavored to study the connections of PN, with PCK and affect. Multiple regression analyses confirmed ANOVA results, indicating that participation in the E^3A module was a significant positive predictor of PSETs' attending, interpreting, and affect. In regard to the connections among constructs, we found that pre-interpreting significantly predicted post-attitudes, demonstrating a connection between PN and affect. Pre-dispositions significantly predicted post-attitudes, which makes sense but needs to be further investigated. In addition, pre-PCK significantly predicted post-deciding. While tenuous, this shows a connection between PN and PCK. This was an interesting finding, given that significant differences were not observed from pre to post in either deciding or PCK. Post scores of every component skill, except deciding, were significantly predicted by the pre-score of that component skill. It is also worth noting that results yielded no connections between PCK and affect.

The interaction of PN, pedagogical content knowledge, and affect offers the opportunity to examine the conceptualization of each construct. Our results indicate that connections between PN and affect in mathematics teaching can be detected in a brief period. While it makes sense that dispositions might serve to predict attitudes as related constructs, it is less clear why interpreting might serve as a predictor for attitudes. We suspect that PSETs who have a stronger grasp on interpreting children's mathematical thinking at the onset might be more enthused about teaching mathematics. If the TEDS-M instrument were more reliable, we would be able to say with more confidence that pre-PCK influenced post-deciding. However, both resulted in lower scores and it might be the case that the regression analysis picked up on similar slopes.

We are aware that there are limitations in the context and the instruments of this study. The context is limited by the short duration of the treatment embedded in a semester-long mathematics methods course. While significant changes for the treatment group were realized, this may have been associated with the course in which the treatment was embedded and not the treatment alone. The use of the comparison data is useful in debunking this particular limitation, we recognize that all instructors used in the study teach in diverse ways. The TEDS-M instrument is limited by the reliability due to the low number of items selected. The limitations of the TEDS-M also underscore the need for an instrument that is more reliable in assessing PCK and/or MKT of preservice teachers. Fisher, Thomas, Schack, Jong, & Tassel (2018) previously attempted to assess preservice teachers using the LMT (Hill, Schilling, & Ball, 2004), however, because this assessment was designed for inservice teachers, our results were limited. Given these limitations, we believe reporting these results is important for the advancement of research exploring the interaction of these constructs: PN, affect, and PCK. Our results indicate there are connections amongst the three constructs though not clearly among all three constructs and not among all components of the various constructs. Though limited, this is an encouraging step in the research that should be further explored with an improved assessment instrument for PCK and potentially better measurement strategies for PN.

As noted previously, the literature illustrates that many researchers are exploring such connections. Mired within this research are multiple challenges, not the least of which is the invisible nature of what occurs in a teacher's head. Jacobs, Sherin, and Philipp (2013) referred to teacher noticing as "the hidden skill of teaching" (p. 723). One might argue that affect and PCK are similarly hidden.

An additional challenge to researchers is in sorting through the language used to describe the various constructs. Questions arise. What is simply a language difference and what constitutes a different construct? In what ways is PN alike or different from the situated conceptualization of PCK (Depaepe et al., 2013)? In what ways is PN alike or different from the integrative perspective of teacher knowledge conceptualized by Gess-Newsome (1999) as the integration of subject matter, pedagogy, and context? Thomas et al. (2017) found connections in the theoretical space between PN and MKT. "[W]e contend that one productive lens for considering the practical outcomes of professional noticing is from the perspective of responsiveness. In this instance, responsiveness may be considered a broad manifestation of the coordinated component skills of professional noticing. . .[and] effective professional noticing occurs at the intersection of developed MKT and a high level of responsiveness to the mathematical activities of students" (pp. 13-14).

Ultimately, the goal is to support teachers to enact responsive teaching that benefits the learning of the students. To be responsive, teachers must constantly use converse and diverse thinking. Attending to an individual student's mathematical response requires converse, or very focused seeing (Broudy, 1984; Polanyi, 1966), that considers that specific student's response and the specific mathematics of the moment. However, the teacher must also think diversely of the needs of the whole class of students as well as the mathematics of the moment in context of a larger mathematical concept.

Are these descriptions of teacher practice describing the same or different constructs? Is it that many attempted to understand teacher thinking and practice from different perspectives, using different language? Does it benefit research on teacher preparation to study essentially the same construct under different names? If constructs are different paths to the same end, understanding "the hidden skill [or skills] of teaching" (Jacobs et al., 2013, p. 723), research such as this, that attempts to find the intersections is crucial to deepening our understanding of effective teacher thinking and with that, how to better prepare teachers.

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