USING INTERPERSONAL DISCOURSE IN SMALL GROUP DEVELOPMENT OF MATHEMATICAL ARGUMENTS

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The development of proofs and argumentation is one of the major standards for mathematical practices in K-12 education that researchers and practitioners alike are continuing to improve. Further, the use of discourse is considered essential in the learning of mathematical concepts at these levels. However, K-12 educators continue to confound how best to utilize student interpersonal discourse to advance the development of mathematical arguments. This qualitative study examines the nature of student discourse in small-group interactions as students create collective arguments based on mathematical evidence. In examining the patterns of discourse in small groups, the study concludes the effectiveness of various types of discourse in peer-to-peer interaction as students develop more analytical thoughts through the support of their discourse with one another in creating proofs and arguments.

Keywords: Argumentation, Reasoning and Proof, Classroom Discourse

The ability for students to develop their own mathematical arguments at the K-12 level remains a priority in mathematics education. Scholars agree that developing mathematical arguments ensures true understanding when one can convince oneself and others of one's mathematical explanation (Ellis, 2007, p. 195; Cáceres, Nussbaum, Marroquín, Gleisner, & Marquínez, 2017, p. 356, Stylianou, 2013, p. 23). Additionally, the eight mathematical practice standards in which students should consistently engage include the development of mathematical arguments and the critique of others' arguments (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). A major factor in creating the space for students to develop and critique mathematical arguments with one another is the dynamic of social interaction in the mathematics classroom. Student participation in a mathematical learning community in a classroom is dependent on the culture. Civil and Hunter (2015) found it was necessary to have an open atmosphere that allows social talk and humor, so that students feel comfortable to share, make mistake, and engage in dialogue about the mathematical learning. To further the mathematical learning, Rojas-Drummond and Zapata (2004) state that this opened the door for students to engage in exploratory talk, where they felt free to examine their own opinions, observations, and explanations of the mathematics. This interpersonal discourse allowed students to engage in healthy and open dialogue focused on the mathematics.

Purpose of the Study

The connection between student interpersonal dialogue and mathematical understanding remains the area that researchers have yet to fully explore. The mathematics education community seems to agree upon and have substantial research on the importance of mathematical argument development at the K-12 level (Brown, 2017; Byrne, 2013; Yee, Boyle, Ko, & Bleiler-Baxter, 2017). However, a dearth of research about the interpersonal dialogue and its relationship to the development of mathematical arguments at the K-12 level persists. This study analyzes student discourse and the mathematical arguments developed. Further, the project will not only discover research-based facts for the elements of and the reasoning for social discourse in student interactions with the mathematics, but also the pedagogical approaches that K-12 mathematics teachers can take towards implementation in regular practices in their classrooms. Finally, as a result of collaborative discourse, students will begin to grow in their autonomous thoughts about the mathematics and thus develop their own valid mathematical arguments.

Theoretical Framework

The situated learning lens is used in this study to understand how students learn. Using this perspective, we seek to understand learning in the context in which it happens. Particularly, this research study emphasizes social participation of students in the small group discussions, so they consider social norms and the culture of the classroom as they participate in their argument development. All of these factors impact their learning in the K-12 setting (Anderson, Greeno, Reder, & Simon, 2000). Learning mathematics in the context of the standards of mathematical practice is a collaborative process, and it should be studied within the contexts in which it is occurring, particularly for K-12 education. This project study of small group discourse in collective argumentation requires participation in the mathematical learning community. The community can influence the discourse that supports student learning.

Further, with the emphasis on discourse of the students, this research takes on a discursive framework, narrowing in on the elements of discussion that students use with one another in their collaborative small groups as they discuss, argue, and critique mathematical concepts with their own ideas of mathematical facts and evidence.

Finally, both the situated learning and discursive framework fall within a greater idea of social constructivism, in which student learning is formed through the interactions that students have with the content in the classroom settings. The foundational learning theory of constructivism provides the necessary lens to look at a deeper, more refined look of learning in this research in the overlapping ideas of the situated learning lens and discursive learning framework.

Review of the Literature

Student creation of mathematical arguments has been established as a primary focus in K-12 mathematics classrooms by national organizations and the recent creation of national standards for mathematical practices. The National Council of Teachers of Mathematics (1989) recommended that reasoning and proof should regularly be incorporated in K-12 classrooms, which was emphasized again in 2010 by the Common Core State Standards (CCSS) publication of eight mathematical practice standards in which students should consistently engage (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Although argumentation has been established as a recommendation for K-12 mathematics, classroom teachers still lack in fostering student development of valid mathematical arguments in a way that satisfies these standards.

Argumentation in the Development of Student Learning

Though argumentation has consistently been a recommended practice of K-12 mathematics, students are generally unable to produce valid arguments (Stylianou, Blanton, & Knuth, 2010), and current methods of teaching proofs and arguments are largely inaccessible to K-12 students (e.g. Karunakaran, Freeburn, Konuk, & Arbaugh, 2014). In several K-12 mathematics classrooms, students learn to create arguments simply by watching the teacher’s approach and attempting to recreate the same process. This type of instruction leads students to believe that the teacher is the supreme authority of an argument in terms of what is acceptable and what is not. Thus, students develop an authoritarian proof scheme (Harel & Sowder, 1998), wherein the process of developing a proof becomes a computational exercise to find the specific solution for which the teacher is searching.

In contrast to the traditional teaching approach to teaching mathematical arguments, encouraging collaboration and allowing students to engage in the process of proving can improve students' proof development (e.g. Brown, 2017; Byrne, 2013; Yee, Boyle, Ko, & Bleiler-Baxter, 2017). Collective argumentation, as one example, allows students to engage in a process where they collaborate to create arguments and come to agreement on the arguments that can be accepted within a community. Students create arguments within a group and present their work to the class, helping students gain authority (Stein, Engle, Smith, & Hughes, 2008) in their work and participate in authentic mathematical community.

The Role of Collaboration in the Mathematics Classroom

Research has shown that mathematical communication within a classroom community is crucial for the development of students' reasoning and mathematical understanding (Alrø & Skovsmose, 2003; Forman, 2003). Lampert and Cobb (2003) argue that by providing students the opportunity to discuss their ideas with others can develop their mathematic reasoning more readily. According to Howe and her colleagues, the most successful instances of collaboration occur when students propose and defend their ideas and when they explain their reasoning to each other (Howe et al, 2007). Further, Howe et al. (2007) discovered that collaboration was most productive when the teacher offered little intervention and allowed students to exercise their own authority in solving the mathematical tasks proposed to them.

Teacher Use of Mathematical Arguments

To discover more about the role of the teacher in fostering a community that encourage collaboration to develop student learning via mathematical arguments, Mercer (2008) builds upon the research of Howe et al. to explain that the teacher's role should be one where he or she guides the students in creating mathematical arguments. In this capacity, the teacher assists the students as they learn to collaborate effectively and utilize "exploratory talk" as a cultural and psychological tool to contribute to their development of reasoning (Mercer, 2008; Mercer, Daws, Wegerif, & Sams, 2004). Exploratory talk is the idea where partners engage critically but constructively with each other's ideas. Their statements and suggestions are considered jointly, as they challenge and counter-challenge, requiring justification and alternative hypotheses (Mercer, Daws, Wegerif, & Sams, 2004). Exploratory talk holds students accountable to reasoning.

To create an atmosphere where exploratory talk is commonplace in the mathematics classroom, Brown (2017) encourages teacher participation in a way that guides and pushes student thinking, as they listen and observe the activities of students in their small groups, as in the aforementioned collective argumentation model. The observation of activities can then inspire students by then challenging them to engage in different types of representations, explanations, and justifications (Brown, 2017) than what they had previously created. Brown (2017) continues that the teacher can do this by asking questions about representations, adding to the representations, or even by providing his or her own personal representation. The active role of the teacher can create an environment where students are not only accountable to developing viable mathematical arguments, but they also are inspired to actively engage in them as they are challenged to create new representations of the mathematics.

Discourse in Collaboration of Mathematical Arguments in a Mathematical Learning Community

Discourse in the classroom is dependent upon the social setting of the classroom and can have multiple meanings, involving more than language (Gee, 1996; Moschkovich, 2007). Moschkovic (2007) contends that discourse also involves representations and behaviors, which involves collaboration about arguments and proofs in the context of mathematics classrooms. The discourse of a mathematics classroom is important to note, then, because the language, representations, and behaviors in a class because the teacher and the students may have different interpretations to meanings and focus of attention.

During the act of collaboration in a mathematics class, collective mathematical understanding may take place when students work together on one mathematical task (Martin, Towers, & Pirie, 2006). This collective understanding requires the social context of the learning environment, and it cannot be described by looking at the actions of the individual learners. Through the process of working jointly on a problem, problem-solving leads students to share ideas and their ways of solving, so individual understanding becomes shared. Teachers should establish a classroom environment that encourages this type of discourse where students will jointly partake in a discourse that transforms individual student thinking about mathematics due to the collective understanding that takes place via student language, patterns, behavior, and interactions with the mathematics as well as each other. This study pursues the nature of the small group discussions and the class-wide, whole-group consensus in collective argumentation, hoping to clarify the elements of these discussions that most encourage student learning through the development of mathematical arguments.

Modes of Inquiry

Research Design

The research is qualitative by nature, as audio recorded conversations of the individual small groups for each day of the instructional sequence were transcribed verbatim to allow for coding to occur. We use open and a priori coding to discover patterns concerning student discourse, particularly the types of discourse used that led to mathematical breakthroughs or insights into developing their mathematical arguments. By gaining insight into the patterns of discourse, we see the necessity of interpersonal discourse of the mathematics so that students can be able to defend and justify their critical thinking as others critique their reasoning, as encouraged by the third standard for mathematical practices. Because of the theoretical grounding in a situated learning lens via social constructivism, a qualitative study of the discourse in the small groups will open doors to understanding different factors influencing valid argument development. The findings will lead to discovery of what elements of the discussion best support autonomous student formation of mathematically sounds arguments, as well the practices that teachers can use to best support this environment in a K-12 mathematics classroom.

Methods

Our study took place during the 2018-2019 school year at a private middle school in the Southeast. The participants included 44 eighth-grade students enrolled in one of four different Pre-Algebra classes. The students were all taught by the same teacher, and they were accustomed to collaborating with one another while working on mathematical tasks. For the purposes of our study, we co-taught each of the four class periods for three days of instruction (55-minute class periods), which made up the entire teaching series. On each day of instruction, students worked on one task (see Figure 1) where they were to create a collective mathematical argument. Each class discussed the criteria to create a valid mathematical argument, and the criteria was written on a whiteboard to ensure that it was visible to each member of the class.

Within each class, students were placed into groups of three or four (14 groups in total), and we used Brown's (2017) key-word format to organize our time. Students first spent four minutes creating an argument on their own, then they spent twenty minutes creating a collective argument within their groups. After creating collective arguments, groups shared their arguments with the class to be validated based on the criteria from the beginning of the teaching series.

Task 1

The sum of any three consecutive integers is divisible by three. Is this conjecture true? Write an argument for why or why not.

Task 2

Conjecture: *The sum of two odd numbers is even.* Is this conjecture true? Write an argument for why or why not.

Task 3

How many triplet primes of the form *p, p+2, p+4* are there? Write an argument for how you know.

Figure 1: Tasks

Data Collection

Three types of data were collected: student written work (both in class as collective argumentation groups and homework assignments), audio recordings of the group discussions, and field notes written. Individual written arguments (during the four minutes of individual think time) were not collected, as they were not included in the analysis of the data. The arguments created by each group on each task are the primary source of data collection. Groups were tracked to allow comparisons to be made between each task. Observational field notes were taken during the instructional sequence to record any relevant remarks made by students in both small group and whole group discussions.

Data Analysis

To determine how students' arguments developed as a result of the discourse in collective argumentation, a coding system was developed. Students' collective arguments as evidenced by the audio recordings and written work were coded according to Stylianides and Stylianides (2009) framework. Their framework consists of five hierarchical levels for judging the sophistication of a mathematical argument. Students create *non-genuine arguments* (Level 1) when they commit little effort. *Empirical arguments* (Level 2) are arguments that rely on examples as warrants for a mathematical claim. An *unsuccessful attempt at a valid general argument* (Level 3) is an argument that uses general warrants but contains flaws. A *valid general argument but not a proof* (Level 4) is an argument that uses a deductive chain to argue a claim but uses warrants that are not accepted by the mathematical community. A *proof* (Level 5) is a deductive argument that uses warrants accepted by the mathematical community.

Beyond the arguments themselves, we transcribed and studied the small group discussions to account for different types of discourse that impacted student thinking, particularly as they moved to more advanced levels of arguments. We discovered which patterns of explanation, interjection, questioning and defense, for example, best led students to understanding mathematics and advancement of mathematical arguments. Further, we begin to understand what the classroom environment requires for this collective argumentation and collaborative discussion to foster advanced mathematical proofs and arguments. This will develop findings in pedagogy to inspire further research and practitioner models to encourage small group discourse

Data Sample

When split into groups of 3 students (or less depending on number of students in the class), students were audio-recorded by group. One specific group developed a mathematical induction proof orally in explaining his reasoning to his group:

> So, you start with 0,1,2 as your base one. And then, what you do to bring it up to 1,2,3, is you add one to each of the integers. So, when you're adding one to each of the integers, you're really just adding three. So, that's why it's divisible by three. And then, that's why it's divisible by three every single time.

This oral argument aligns with a proof by mathematical induction. The speaker explains that the base case (0+1+2) is divisible by three. Then, he explains adding one to each integer will gives the next case (e.g. 1+2+3). He explains why this new case is divisibly by three by stating "you're really just adding three" appealing to the property that adding three to a number already divisible by three creates a new number that is also divisible by three. His group asked for further clarification, to which the speaker further explained his reasoning.

- Speaker 2: But I do feel like in some instances it won't be right.
- Speaker 1: No, it will be right every time because you're just adding three every time.
- Speaker 2: But, why? We need facts and evidence.
- Speaker 1: Because if you have three consecutive numbers, if you go up one for each of them, you just add three. And if the base one does—is divisible by three that means all the other ones will be divisible by three. And in this instance the base one is divisible by three.

In being able to provide a rationale to a problem, particularly in response to a question about the validity of the response, the speaker thoughtfully engages in personal discourse to promote a valid argument and its reasoning.

Results

Table 1 shows the frequency of the types of arguments that were created by each group across the three tasks. The data suggest an overall increase in the level of argument created by the groups over the three tasks. There was a steady decrease in the use of empirical arguments; however, students had less success in creating a proof on task 3 than on task 2. This is likely due to the difficulty of task 3 in comparison to task 2, and the expectation of creating a conjecture before attempting to write an argument. Even so, the data suggests that by task 3, students understood that an argument that uses examples (empirical argument) was not viable in their mathematical setting.

To understand how each group progressed across the tasks, we tracked their level of argument from task 1 to task 3. Table 2 shows the level of argument for each of the fourteen groups (listed A-N) on task 1 and task 3 and the difference in level of argument. Most groups increased in their level of argument while few groups decreased or remained the same. To determine whether there was a statistically significant difference in level of argument from task 1 to task 3, a Wilcoxon-signed rank test was used. There was no statistically significant difference in the level of argument the groups created from task 1 to task 3 (T=47.5, p=0.178). The sample size, lack of variability in the levels of argument, and ceiling effects of not including baseline data likely contributed to the non-significant test statistic. Still, the descriptives illustrate that groups generally made progress in the sophistication of their collective arguments from task 1 to task 3.

Table 1: Level of Argument by Task

Task	Level 1: Non-genuine Argument	Level 2: Empirical Argument	Level 3: Unsuccessful Attempt at Valid General Argument	Level 4: Valid General Argument but Not a Proof	Level 5: Proof
Task 1	2	8	2	0	2
Task 2	1	5	2	2	4
Task 3	3	1	5	4	1

Table 2: Difference in Level of Argument from Task 1 to Task 3

	Groups													
Tasks	*A*	*B*	*C*	*D*	*E*	F	G	H	I	J	K	L	M	N
Task 1	1	2	5	5	1	3	2	2	3	2	2	2	2	2
Task 3	1	3	3	4	1	4	1	4	4	3	3	2	5	3
Difference	0	-1	2	1	0	-1	1	-2	-1	-1	-1	0	-3	-1

Discussion

Preliminary quantitative results of this study suggest that the eighth-grade students benefited from engaging in interpersonal discourse via collective argumentation. Communication among peers provided access to the standards for a viable mathematical argument, thus making it more likely that they will have success in creating their own arguments. Further, engaging in collective argumentation gives students opportunities to critique and validate others' work which places them in a position to learn directly from their peers, as supported by the review of the literature. Our study was unique in its application of the aforementioned supports for middle grade students, and it agrees with findings from previous research that students benefit from interpersonal discourse in creating mathematical arguments (Rojas-Drummond & Zapata, 2004). However, more research should be devoted to how students use interpersonal discourse to create increasingly sophisticated arguments. Furthermore, scholars should test the supports mentioned in this paper in other settings such as primary and secondary schools. The preliminary results of this paper is promising in providing further promotion of and support for K-12 mathematical argument development.

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