DISSECTING CURRICULAR REASONING: AN EXAMINATION OF MIDDLE GRADE TEACHERS' REASONING BEHIND THEIR INSTRUCTIONAL DECISIONS

Shannon W. Dingman
University of Arkansas
sdingman@uark.edu

Dawn Teuscher
Brigham Young University
dawn.teuscher@byu.edu

Lisa A. Kasmer
Grand Valley State University
kasmerl@gvsu.edu

Travis A. Olson
University of Nevada-Las Vegas
travis.olson@unlv.edu

Mathematics teachers are vital components in determining what mathematics students have the opportunity to learn. There are a vast number of factors and reasons that influence a teacher's instructional decisions. As such, teachers rely heavily on their curricular reasoning (CR) to make decisions about what content to teach, how that content is taught, and the tasks to use to facilitate student learning. In this paper, we outline five strands of CR gleaned from research with middle grades mathematics teachers as they plan and implement instruction with unfamiliar curricular resources. These strands lay the foundation for our Instructional Pyramid model of CR and provide a lens through which teacher decision-making can be further understood and enhanced.

Keywords: Instructional Activities and Practices; Curriculum Enactment; Instructional Vision

Mathematics teachers are critical constituents in creating learning environments that provide students with the opportunity to learn important mathematics and that assist in developing student mathematical knowledge. One of the most important tasks required of mathematics teachers is the planning and enactment of instruction—the "what" and "how" of mathematics teaching. Figure 1 (Mathis, 2019 adapted from Stein, Smith, & Remillard, 2007) highlights the many instructional decisions during the teaching process that shape the overall mathematics lesson. While there are many decisions teachers make during the teaching process, we focus on *mathematical decisions*, defined as those decisions that influence students' opportunity to learn mathematics, and teachers' reasoning for those decisions.

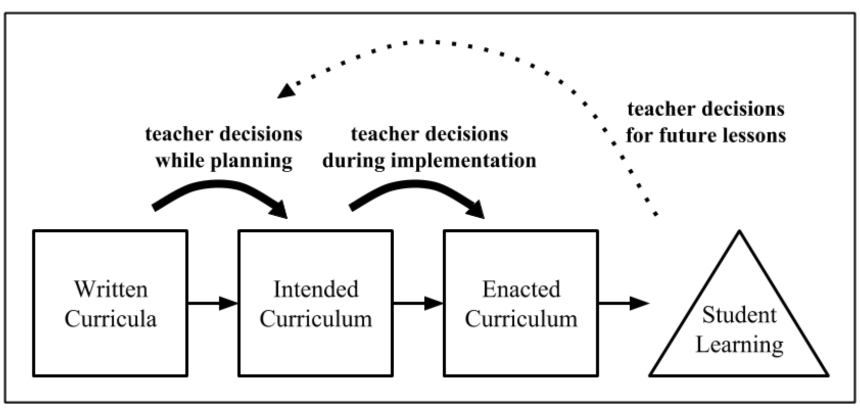

Figure 1: The Teaching Process and Teachers' Decisions That Affects Students' Opportunity to Learn (Mathis, 2019 adapted from Stein et al., 2007)

Researchers have sought to identify connections between teachers' implied decisions and possible factors influencing the teaching process (Bush, 1986; Graybeal, 2010; Nicol & Crespo, 2006; Remillard, 2000; Stein & Kaufman, 2010), but have rarely studied teachers' reasons for their decisions. These *factors* are not teachers' reasons for their decisions, but rather internal (e.g., mathematical knowledge, prior experiences) or external (e.g., professional development, textbooks) entities that influence or correlate with teachers' implied decisions. In contrast, *teacher reasons* are their own justifications for their decisions. Stein and Kaufman (2010) identified teachers' implied decisions during the teaching process based on learning opportunities afforded students. The authors investigated whether teachers' capacity (e.g., teacher experience, teacher education, professional development) or teachers' use of the curriculum correlated with the learning opportunities afforded students. Stein and Kaufman (2010) found that teachers' attention to the big mathematical ideas (i.e., implicit decision), which was related to the different curricula teachers used (i.e., factors), was highly correlated with students' learning opportunities. These data imply that the teachers' decisions about the big mathematical ideas within a lesson affected students' opportunity to learn mathematics. However, we do not know *why* some teachers choose to focus on the big mathematical ideas within a lesson and others did not, namely the teachers' reasons for their decisions that led to the differing student learning opportunities.

Other researchers have explicitly researched teachers' decisions and their reasoning during the teaching process; however, they hypothesized about specific factors or teacher reasons they think affect the teaching process rather than considering all potential factors or teacher reasons to give insight into teachers' decisions. Choppin (2011) hypothesized that teachers' *understanding of resources* and *attention to student thinking* would impact teachers' decisions about how they used curriculum materials; however, it may be the case that there were other factors or teacher reasons that were more prominent for why teachers used the curriculum materials in the way they did. Researchers have identified many different factors or teacher reasons that suggests we may not have a full understanding of teachers' mathematical decisions made during the teaching process if we do not consider all factors or teacher reasons.

With such a wide array of factors and teacher reasons identified that influence teachers' decisions throughout the teaching process, we suggest that teacher reasons are connected to their curricular reasoning (CR) – defined as *the thinking processes that teachers engage in and employ as they plan and enact the mathematics curriculum*. The purpose of this paper is to present a framework to characterize mathematics teachers' CR. We do this by defining and presenting five teacher-thinking processes that we refer to as CR strands. We then argue for the need to modify the Instructional Triangle (Cohen, Raudenbush & Ball, 2003) by adding a fourth dimension resulting in the Instructional Pyramid. Finally, we present the relationship of the five CR strands to the Instructional Pyramid and the interrelatedness of the CR strands in regards to teachers' mathematical decisions.

Context

As part of our NSF-funded project (#1561542, 1561554, 1561569, 1561617) that examines teachers' mathematical decisions and their reasoning as they navigate the teaching process, we developed our framework based on a sample of grade 8 mathematics teachers who taught a unit on geometric transformations (reflections, translations, rotations, and sequence of transformations). The topic of geometric transformations has historically been included in high school geometry courses; however, with the widespread adoption of the Common Core State Standards for Mathematics (CCSSM) this content was moved to grade 8 (Tran, Reys, Teuscher, Dingman & Kasmer, 2016). In addition, the authors of CCSSM use geometric transformations to build the definition of congruence, an approach rarely used in past standards nor in middle grades textbooks.

Teachers in our project were given the *University of Chicago School Mathematics Project (UCSMP)* geometry curriculum (Benson et al., 2009) to serve as the foundation for their instructional decision-making. This curriculum was chosen for two reasons: (1) its alignment with the approach to geometric transformations as found in Grade 8 CCSSM; and (2) its unfamiliarity to teachers in our study. Therefore, we aimed to identify teachers' mathematical decisions during the teaching process and their reasoning when planning with these unfamiliar curricular materials (*UCSMP*) and enacting this geometric transformation unit that was new to their grade level.

CR Strands

The teacher reasons for their decisions and reflections fell into five CR strands. In other words, we identified five key *thinking processes* that teachers used when making mathematical decisions during the teaching process. These strands are *Viewing Mathematics from the Learner's Perspective, Mapping Learning Trajectories, Considering Mathematical Meanings, Analyzing Curriculum Materials,* and *Revising Curriculum Materials.* Three of these strands—*Viewing Mathematics from the Learner's Perspective, Mapping Learning Trajectories*, and *Revising Curriculum Materials*—build from previous research on CR (Roth McDuffie & Mather, 2009; Breyfogle, Roth McDuffie & Wohlhuter, 2010), while the remaining two strands were identified through the open coding of our data. Below we define each CR strand and provide an example to demonstrate teachers' decisions and their reasoning for the particular CR strands. The interview excerpts below use pseudonyms for the participating teachers in the study.

Viewing Mathematics from the Learner's Perspective

As teachers make decisions or reflections, teachers reason about how their students will perceive and view the mathematics of the lesson. This thinking process is *Viewing Mathematics from the Learner's Perspective*, and defined as the teacher discussing the mathematics content of the lesson through the lens of student interpretations. Specific indicators of this CR strand are when teachers reasoned with specific details of the mathematics within the lesson and one of the following: (a) predicted (actual) student interpretations of the mathematics; (b) predicted (actual) areas of what students will do (did) with the mathematics or assessed student understanding; or (c) predicted (actual) student misconceptions. This reasoning allowed the teacher to articulate the perceived mathematical needs or mathematical knowledge held by the students. From a practical standpoint, this CR strand was typically utilized by the teacher when considering the students' prior knowledge, the students' responses to other students' thinking, the students' needs or struggles, the students' access points to the mathematics in the lesson, or students' anticipated thinking about the mathematics in the lesson.

To illustrate this CR strand, we use the following excerpt where Helen provides her reasoning for how the definition of reflection assisted her students:

> Helen: I think it helped them to recognize the pattern and recognize why those patterns were there. It also helped them, going back to that word "orientation" … and how that effects the ordered pair.

In this reflection of her lesson, Helen reasons about how the mathematics of the lesson, namely the definition of a reflection and the concept of orientation, helped her students to recognize and generalize patterns when reflecting across the different axes on the coordinate plane. To this point, she is reasoning about the lesson from the students' perspective regarding what helped them to be successful in the lesson.

Mapping Learning Trajectories

Another CR strand is related to teachers' reasoning about how a teacher maps out a lesson, either within the immediate lesson, or across units within a school year or across courses. We termed this thinking process as *Mapping Learning Trajectories*, and defined it as a teacher considering either how the mathematical concepts will unfold within a lesson, or discussing how the mathematics topics of a current lesson connect to either past or future mathematics topics students learn. Specific indicators for this CR strand are teachers' reasoning about: (1) how the mathematical concepts or skills in a lesson connected (did not connect) to past or future mathematics content; or (2) how the concepts or skills unfolded within a lesson or a unit. Therefore, the idea of a learning trajectory can take on either a short-term nature, where the teacher reasons about how the day's lesson will progress or how the given unit of lessons for geometric transformations are sequenced, or a longer-term outlook, where the teacher envisions how the lessons will connect with the mathematics content taught either in a given grade-level or across multiple grade-levels.

To illustrate this CR strand, in the following excerpt, Jill shares her reasoning for the success of her translation lesson.

> Interviewer: Do you feel like your task overall promoted student learning in the way you had hoped? How so?
> Jill: Yes, I feel that it did. It allowed the kids to explore the composition [of] reflections to determine what a translation is, and then we talked about the translation properties and which ones are preserved, and then they actually practiced. So, I feel like it did go, do what I was hoping it would do.

In her response, Jill reasons with the short-term nature of mapping learning trajectories, examining how the sequence of the lesson helped to support student learning by focusing on translations and their properties first before the students attempted to translate figures.

Considering Mathematical Meanings

The role of mathematical knowledge in the art of teaching is fundamental, and that knowledge plays a critical role of planning and enacting instruction (Ball, Thames, & Phelps, 2008). The CR strand *Considering Mathematical Meanings* is defined as the teacher's mathematical meanings of the mathematics within the lesson, or articulation of the anticipated student mathematical meanings that will be developed as a result of the lesson. Specific indicators for this CR strand are: (1) the teacher expresses his/her own mathematical meaning, which could be correct or incorrect, of the mathematics related to the lesson; or (2) the teacher expresses the mathematics students should learn during the lesson. This CR strand differs from the *Viewing Mathematics from the Learner's Perspective* in that the *Considering Mathematical Meaning* strand is from the viewpoint of the teacher and focuses on what the teacher thinks students should know, while the *Viewing Mathematics from the Learner's Perspective* strand stems from the viewpoint of the students—their misconceptions, their interpretations of the task, and their potential ways of thinking.

In the following excerpt, Judy reasons with her mathematical meaning about the relationship between reflections and rotations:

> Interviewer: Do you see a connection between reflections and rotations, besides the equal distance idea?
> Judy: …So it's really just that equal distance, but the congruent shapes are still there. And I mean, as I was talking in 2nd period, that's things they still said. But I don't know—besides that, besides them being equal and have that equal distance, I'm not sure that there's anything else I would compare those.

In this response, Judy reasons with her meaning of the properties that connect reflections and rotations. In the *UCSMP* textbook, rotations are seen as a composite of two reflections across intersecting lines (e.g., a pre-image in the first quadrant of the Cartesian coordinate system that is reflected over the x-axis and then over the y-axis results in the same image that is rotated 180° around the origin). In addition, the connection between reflections and rotations serves to highlight the properties shared by these two transformations. As this is not an approach traditionally taken in many textbooks, Judy shares her mathematical meaning of the distance preservation property, but does not discuss other properties shared by these two transformations (e.g., preservation of angle measures, collinearity of points).

Analyzing Curriculum Materials

As teachers make decisions regarding the mathematics lesson, they often use textbooks, online resources, and/or other supplementary materials at their disposal. These curriculum materials, defined as the "printed or electronic, often published, materials designed for use by teachers and students before, during, or after mathematics instruction" (Stein, Remillard, & Smith, 2007, p. 232), help to organize and structure the learning opportunities created for students. *Analyzing Curriculum Materials* is defined as when the teacher reasons about the curriculum materials by comparing the curriculum materials to other materials, providing analyses of potential strengths and weaknesses and detailing differing approaches. Specific indicators of teachers reasoning with this CR strand are (1) an analysis of a curriculum, pointing out appealing features or components that were unfavorable or that would be changed; or (2) a comparison of two or more curricula with respect to how these materials provide coverage of topics, how topics are sequenced, or for activities the teacher favored when enacting the lesson. In our analysis, we included standards documents, namely CCSSM, and state assessments in our definition of curriculum materials. Given CCSSM's important role in determining what is taught at given grade-levels as well as the prominence given to state-mandated assessments, these forms of curricula influence the decisions teachers make and play a critical role in a teacher's CR.

To illustrate this CR strand, we share a response from Ava who used the *Connected Mathematics Project* (*CMP3*) (Lappan, Phillips, Fey, & Friel, 2014) as the district adopted textbook. Given her preference for the *CMP3* curriculum, Ava was asked to compare *CMP3* to the *UCSMP* materials she was given to plan the unit on geometric transformations.

> Ava: I like the launch of them [*UCSMP*] creating their own [figure]; *CMP* just gives them a flag and tells them to reflect it. So I like that idea of starting with a white piece of paper and doing their own thing.

Ava reasons with the *Analyzing Curriculum Materials* strand as she compares the two curriculum materials, discussing features of both curriculum she liked. In subsequent data collections, Ava continued to contrast the *UCSMP* materials with what she normally taught in *CMP*, making it apparent she held a favorable disposition towards the curriculum in which she was more familiar.

Revising Curriculum Materials

The final CR strand concerns the iterative process of reflecting upon one's practice and changing parts of the lesson in order to improve the implementation or modify parts of the lesson that did not go according to plan. The *Revising Curriculum Materials* CR strand is defined as a teacher considering modifications and changes to a lesson based upon past teaching experiences. This reasoning, however, suggests a dynamic relationship between the teacher and the curriculum materials, one in which teachers reason with their CR to alter the curriculum based on experience.

In the following excerpt, Tracy is asked about her upcoming lesson that she planned, but based upon how the same lesson went in a prior class period, she is second-guessing her approach.

> Interviewer: Do you expect to get through all of the questions?
> Tracy: [Laughs] Sure, but now I'm like, NO! I mean I…no. So now I have to decide...
> Interviewer: So if you had to decide which ones you would skip or leave out, what would you decide?
> Tracy: I would probably skip the overlapping one. I did a grid one in their video of overlapping, and so I probably would skip the overlapping one. The other thing I might do is, say out of these three [questions], do two with a MIRA and do one with a protractor. So that they [students] get through the three things, but they're picking... they don't have to do it each twice. Because the protractor is going to take a little more time than the MIRA is, so if they just do one with the protractor. I mean, because I'm going to have to walk them through one, I'm going to have to go through one. I guess I need to decide which one I'm going to go through with them, because they don't know how to use that stuff. And then whichever two, they have to do one with a MIRA and one with a protractor. That probably, honestly, would be time management-wise, OK.

Based upon Tracy's reflection of the previous class period's lesson, she decided to skip some of the problems she had developed for her students. This revision of her lesson stems from student confusion during the previous class period's lesson as well as the fact that some of the problems in the planned lesson were repetitive from problems Tracy had already worked with her students.

Interplay of CR Strands

One of the predominant models regarding mathematics instruction is the Instructional Triangle (Cohen, Raudenbush, & Ball, 2003), which connects teachers, mathematics, and the students as vertices of the triangular model and where the edges of the triangle signify the interactions among these three critical components in the classroom environment (Nipper & Sztajn, 2008). This model highlights how various factors and resources influence teachers' instruction and subsequently student achievement. However, given our lens in focusing on teacher's CR and the role curriculum materials play in teachers' mathematical decisions during planning and enactment of lessons, we argue that a fourth component—the curriculum—should be added to include the various interactions that occur as teachers engage in the teaching process. Figure 2 illustrates our model that reflects the four main components of the Instructional Pyramid that influence teacher decisions during the teaching process.

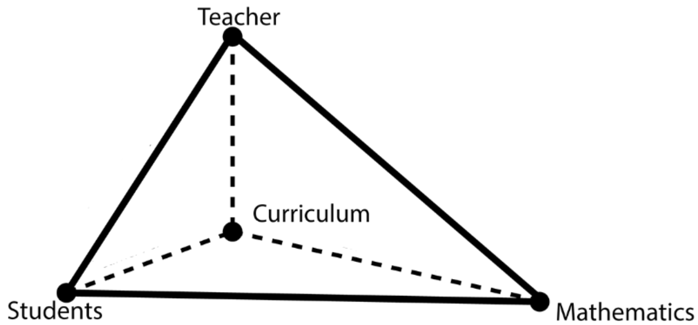

Figure 2: Instructional Pyramid for Curricular Reasoning

In the Instructional Pyramid, the edges represent CR strands, teachers' reasoning behind their mathematical decisions during the teaching process. The faces of the pyramid illustrate the use of multiple strands that interplay with one another as teachers plan and enact instruction. Figure 3 depicts the edges of the pyramid correlated to the CR strands used by teachers.

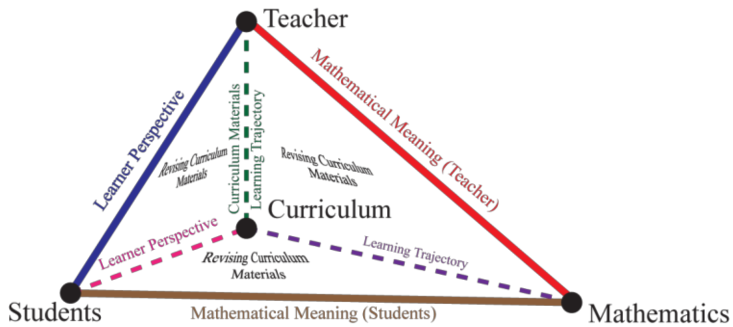

Figure 3: Model of CR Interplay

Our data set illustrates instances where teachers reason with one CR strand when making a decision, therefore existing along an edge of the pyramid. In other instances, teachers may coordinate multiple CR strands, thereby existing on multiple edges or faces of the pyramid. To highlight the interplay of these CR strands as teachers make and enact mathematical decisions, we use an interview excerpt from Judy, who planned to teach an overview lesson that would introduce her students to the language and vocabulary used with rigid transformations. Students were given a stack of geometric shapes that were translated, reflected, and rotated, and students were asked to describe what had happened to each figure.

> Interviewer: Are you planning to define any of the transformations, or are you just going to leave them in the vague terms?
> Judy: I might stay more vague today. My guess is that the words will come up, because they've heard them before. So I'm sure they'll come up. But I'm hoping that, so after this,

> I'm hoping that we'll get into days of, OK, here's a reflection. Tell me what it is. And that's when we're going to define it more. But today I think I'm going to stay a little more vague on it, and then later we'll get into more details.

In this exchange, Judy reasons with two CR strands: *Mapping Learning Trajectories* and *Considering Mathematical Meanings*. As Judy decides to "stay more vague with the definitions of each transformation" she employs the *Mapping Learning Trajectories* strand as she reasons about how the language of transformations can be more formalized over the coming days and weeks of the unit. With this reasoning, she is focusing on the Curriculum-Mathematics edge of the Instructional Pyramid. Building from this reasoning, Judy reasons with the *Considering Mathematical Meanings* strand as she delves into her thinking about staying more imprecise in this lesson. She sees this approach as fine for an introductory lesson, as she knows there will be time later in the unit to solidify the definitions of reflections, translations, and rotations. In this excerpt, Judy is imparting what she wants students to understand and gain from the day's lesson. With this reasoning, Judy is focusing on the Students-Mathematics edge of the pyramid. Taken together, the interplay of the *Mapping Learning Trajectories* and the *Considering Mathematical Meanings* strands suggests Judy's reasoning lies on the Students-Mathematics-Curriculum face of the Instructional Pyramid, as she reasons about the mathematical content of the lesson, what her students should know and understand about that content, and how that content will develop and grow throughout the unit.

Our data analysis is ongoing, but our hypothesis contends that teachers who reason with multiple CR strands when making mathematical decisions, and thus whose reasoning exists on multiple faces of the Instructional Pyramid, can provide different opportunities for students to learn mathematics. While teachers who reason with single CR strands may miss important support features in the teaching process that can improve their ability to assist student learning. The goal of the Instructional Pyramid is to provide a framework by which we can examine how teachers reason with the CR strands outlined above. This will allow researchers to examine the factors and reasons that drive teachers' decisions as they plan and enact mathematics lessons. By focusing on the CR strands teachers reason with when making mathematical decisions, teacher educators can work to support teachers' ongoing development of their use of these CR strands, thereby allowing teachers to flexibly move from the edges to the faces of the Instructional Pyramid. This will allow teachers to put into practice multidimensional CR and thus better support their abilities to make mathematical decisions that assist in promoting student learning.

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