JUMPING INTO MODELING: ELEMENTARY MATHEMATICAL MODELING WITH SCHOOL AND COMMUNITY CONTEXTS

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Mathematical modeling is a high-leverage topic, critical STEM education and civic engagement. This study investigates culturally responsive, school and community-based approaches that support mathematical modeling with elementary students. Specifically, we analyzed two modeling lessons in one fifth grade classroom, with a focus on how students drew upon their experiences and funds of knowledge as they engaged in the mathematical modeling process. Our findings illustrate different ways that students' experiences and situational knowledge informed and guided their modeling activity, including the quantities that they deemed relevant, and how they interpreted and refined their solutions. We also attend to the teacher's role in supporting such connections. Implications for mathematics teacher educators and research are included.

Keywords: Elementary School Education, Equity and justice, Modeling, Instructional activities and practices

Mathematical modeling is a high-leverage topic, critical for participation in STEM education and civic engagement (Aguirre, Anhalt, Cortez, Turner & Simic Muller, in press). Unlike typical textbook word problems that often require students to disregard realistic considerations (Greer, 1997; Verschaffel, De Corte & Borghart, 1997), modeling tasks invite students to consider real-world contexts, as well as real-world solutions. Students engage in a cyclical process of: (a) analyzing situations; (b) constructing models that represent the situation, based on information and assumptions; (c) using models to perform operations and reason about results; (d) validating or revising models; and (e) reporting conclusions (CCSSM, 2010; Lesh & Doerr, 2003).

Although mathematical modeling has a well-established research base in secondary and undergraduate education (Doerr & Tripp, 1999; Gainsburg, 2006), it has been underemphasized and under-supported at the elementary level, with a few notable exceptions (e.g., Carlson, Wickstrom, Burroughs & Fulton, 2018; English, 2006; Suh, Matson, Seshaiyar, 2017). This may be due to limited attention to modeling in elementary teacher preparation, and an absence of mathematical modeling tasks in elementary curricula (Burkhardt, 2006). However, research conducted with elementary grade students and teachers demonstrates that mathematical modeling *is accessible* to children in elementary grades (GAIME, 2016), including students with limited prior experience with modeling (Chan, 2009; English, 2006), and students from a diverse range of mathematical and cultural backgrounds (Turner et al., 2009).

Mathematical Modeling with Cultural and Community Contexts

Research suggests that culturally responsive, community-based approaches to teaching mathematics have added benefits, particularly for students from underrepresented groups

(Aguirre & Zavala, 2013; Civil, 2007; Ladson-Billings, 2009; Lipka et al., 2005; Turner et al., 2008). Grounding mathematics in meaningful contexts that connect to students' experiences can enhance student engagement and learning, and encourage students to draw upon situational knowledge and real-world considerations, instead of "cutting bonds with reality" (Bahmaei, 2011). Moreover, these connections help students understand how mathematics matters in personal and socially meaningful contexts (Anhalt, Cortez & Smith, 2017).

We build on this prior research to investigate culturally responsive, school and community-based approaches that support mathematical modeling with elementary students. Specifically, we collaborated with elementary teachers to develop modeling lessons that build on students' mathematical thinking (Carpenter et al., 1998) *and* their community-based knowledge and experiences (Civil, 2007). As teachers enact tasks in classrooms, a focus of our research has been on how students' contextual knowledge and experiences inform and support their modeling activity. Specifically, we focused on these research questions: How do students draw upon their experiences and funds of knowledge as they engage in the *mathematical modeling process*? How do teachers support these connections, and how do they shape students' modeling activity? We explored these foci via an analysis of modeling lessons in one fifth grade classroom (Mr. H).

The Mathematical Modeling Process

Figure 1 depicts the mathematical modeling process we have used with teachers and students. This model is informed by prior research (Anhalt, Cortez, & Bennett, 2018), and the modeling cycle included in the CCSSM's (2010) standard. The arrows represent what we have found to be frequent pathways through the modeling process, but others are possible.

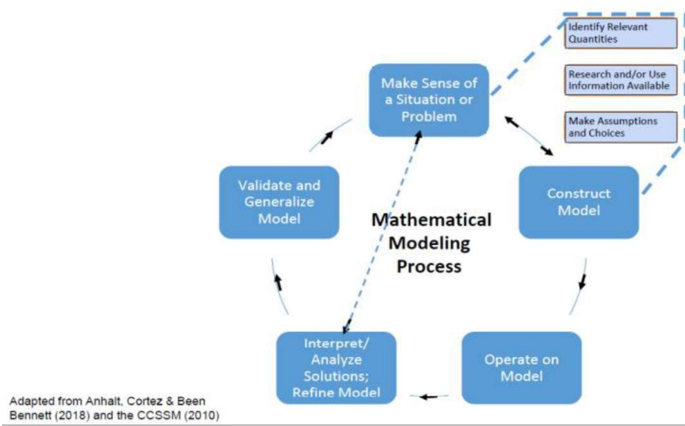

Figure 1: Mathematical Modeling Process for Elementary Students and Teachers

Phase 1, Make Sense of a Situation or Problem

Students begin the mathematical modeling process by making sense of a situation or problem. They consider questions such as: What do I know about this situation? What experiences have I had related to the situation? What additional information do I need?

Phase 2, Construct a Model

Next, students consider what quantities are relevant and important, and how those quantities relate to one another. They also consider what information is provided, what might need to be collected, and what they will need to assume or decide. These considerations are connected to the context, so as students construct a model, they continue to make sense of the situation.

Phase 3, Operate on Model

Next, students create a solution for the problem, perform computations, and check for precision both in their results and in their labels and/or explanation of their work.

Phase 4, Interpret/Analyze Solutions and Refine Model

Students then interpret their solutions in relation to the original situation. They ask whether their solution makes sense based on their experiences and knowledge about the context, draw conclusions about what the solution(s) imply, and refine and revise their model (if needed).

Phase 5, Validate and Generalize Model

This final phase in the mathematical modeling process involves validating and generalizing the model so that it is reusable and allows for applications to similar scenarios.

Study Context and Participants

As part of the broader project, approximately 30 teachers from two districts participated in 30-hour summer institutes focused on *teaching mathematical modeling in grades 3-5.* During the institutes, teachers engaged in modeling tasks developed by the research team, explored key phases of mathematical modeling, and discussed critical features of meaningful, relevant tasks.

During the academic year, teachers met on a monthly basis to plan, discuss, and reflect on modeling tasks enacted in their classrooms. Twice per year, researchers presented tasks and related lesson materials (photos, handouts, lesson launch slides) designed by members of the research team. These tasks addressed content-standards reflected in district quarterly curriculum maps, and connected to relevant contexts in schools and communities. All teachers were invited to enact these tasks, to facilitate analysis of a common lesson across different contexts.

Focal Teacher: Mr. H

We selected Mr. H's class because Mr. H enacted multiple mathematical modeling tasks across the school year, including tasks that connected to different school and family contexts. Mr. H is a White 5th grade teacher with 13 years of teaching experience. In his class of 27 students, 6 students were new learners of English. The racial/ethnic demographics mirrored the school with 30% Latinx, 19% White, 15% African American, 14% 2 or more races, 9% Asian, 3% Native Hawaiian/Pacific Islander, 1% American Indian/Alaskan Native. His classroom included a typical range of student backgrounds in mathematics.

Focal Modeling Lessons

We analyzed two mathematical modeling lessons in Mr. H's classroom, *Abuelo's Birthday* from the beginning of the year, and *Upcycling Jump Ropes* from the end of the year.

Abuelo's birthday. This task adapted from Aguirre and Zavala (2013) presented a realistic scenario about four grandchildren who want to share the cost of a birthday gift for their grandfather. Using information about each grandchild, students generate a "fair" plan for sharing the costs, and explain how their plan could be used in similar situations.

It is Sr. Aguirre's 70th Birthday. Four of his grandchildren want to buy him a gift. They found a photo printer on sale for $119.99. They want to buy him the printer to print family photos.

- Alex, a 9th grader, earns between $15 and $20 each week from babysitting jobs.
- Sam, a 6th grader, earns $10 each week taking care of a neighbor's pets.
- Elena, a 4th grader, earns about $5 each week doing odd jobs for an aunt.
- Jaden, a 1st grader, has no weekly job but has saved $8 in her piggy bank

One of the grandchildren says that they should split the cost of the printer among them and each pay the same amount. Another grandchild says that it is not fair and they should each pay different amounts. Help the children make a plan to share costs in a fair way to buy the gift.

- Your plan should work in other situations where family members want to share costs fairly.

Figure 2: Abuelo's Birthday Task

Upcycling jump ropes. This task focused on upcycling plastic bags to make jump ropes. This task was inspired by students' interest in environmental issues, including recycling and upcycling to reduce waste. Students designed a set of jump ropes for their school, calculated the number of plastic bags needed to make the jump ropes in the set, and then explained how their model could be used in other situations. The task materials included a video that showed how to braid plastic bags to make ropes, and the number of bags needed per foot of rope (3 bags/foot).

We selected these two modeling lessons because they were designed to facilitate different kinds of connections to students' experiences (i.e., to family practices in the Abuelo's birthday task, and to environmental concerns and school-based play practices in the Jump Rope task).

We want to make a jump rope set to be used in gym class. A set contains jump ropes of different lengths. How many plastic bags will be needed? Your plan for making the jump rope set must show:

- How you know that you will have enough jump ropes for gym class, without a lot of extra
- How many plastic bags you will need
- How others could use your plan to make jump rope sets for their school

Figure 3: Upcycling Jump Ropes task

Methods

Case Study Design

Using a qualitative case study design (Creswell, 2013; Stake, 1995; Yin, 2013), we considered lesson as a case, and the students who participated in the lesson as the primary focus of study. Case study research lends itself to "how and why questions" regarding social phenomena, especially questions that require "extensive and in-depth description" (Yin, 2013, p. 4). Correspondingly, we examined patterns within and across the two lessons (Stake, 2006).

Data Sources

Data sources included video-recorded observations of mathematics lessons, post-observation teacher interviews, teacher reflections on lesson enactments during a subsequent teach study group, student work, and other lesson artifacts (e.g., images of board work). The two lessons analyzed in this study ranged in length from 1.5 to 2 hours. When video-recording, we followed the teacher to capture instructional decisions and moves. All interviews were audio-recorded and transcribed. Videos were selectively transcribed with a focus on pivotal teacher questions and prompts, and examples of students' thinking and experiences that they brought to the task.

Data Analysis and Analytical Framework

Through multiple and iterative cycles of analysis, we developed preliminary categories based on themes identified in the literature related to modeling and connecting to students' experiences and funds of knowledge, and emerging themes evidenced in our data. These included: teacher moves to elicit and/or connect to students' experiences; ways students' experiences/knowledge connected with each phase of the modeling cycle; ways that connections to students' experiences supported sense-making. We engaged in iterative cycles of sorting data under these categories and writing analytic memos to identify and refine themes (Miles, Huberman, & Saldaña, 2013). To achieve *interpretive convergence*, multiple researchers developed and reviewed the memos. While viewing the data through the lens of the phases of the modeling cycle, we generated a *narrative compilation* (Creswell, 2013) of preliminary findings across the two lessons.

Findings

We structure our findings by the first four phases of the mathematical modeling process. (We address Phase 5 in the discussion.) In each section, we draw on examples from lessons analyzed.

Phase 1: Making Sense of the Situation or Problem

In both lessons, students drew upon understandings and experiences, from family and community, and school activities, to make sense of the task context. Mr. H also shared his own stories relevant to the context, as way to mirror the kinds of connections students might make.

Connections to family practices sharing costs in Abuelo's birthday task. During the launch of the Abuelo's birthday task, Mr. H began by narrating a personal story about eating out at a restaurant with a friend, and determining a fair way to split the cost.

> Mr. H: This happened to me last night actually. I went out to dinner with a friend, and then we ordered all this food, and we had to think about how we were going to pay. … Is it fair that one person pays for all it? I make more than my friend, so should I pay for all of it, because I make more?
> Students: No!
> Mr. H: She ordered more, so she should pay for all the things that she got, and I pay for my things? [Students call out both Yes! And No!]
> Mr. H: So I want you guys to think, what situations with your family, friends or siblings, have you been involved in where you had to share costs for something? … Can you talk about that for a minute?

Students then contributed their own family stories related to sharing costs such as splitting the cost of a new video game system evenly among siblings, with one sibling contributing the extra funds to cover the tax, or collaborating with cousins and an uncle to pool the money needed to purchase a family television set, with everyone contributing as much as they could.

Connections to play activities in the Jump Rope task. Mr. H also used personal stories to invite students to share experiences in the Upcycling Jump Ropes task. For example, he described his own attempts at specific jump rope styles.

> Mr. H: Yesterday I tried to use this [pointing to a jump rope] in some ways I was successful and some ways I was not as successful. ... Maybe it's [the jump rope] not a good fit for me. … If I had the longer one I would have been able to do the cross over thing [gesturing by crossing his hands in front of him to cross the rope] and that would have been kind of cool.

This prompted students to discuss different jump rope activities (i.e., "double dutch" versus "normal jumping"), and their ideas about height and rope length, noting that jump ropes need to be "different lengths, because … there are people of different sizes [in our class]."

Across the two lessons we found that Mr. H used his own stories, coupled with targeted probes to elicit students' experiences related to the task context, and to connect those experiences to key features of the task, such as quantities that would be important in building a model.

Phases 2 and 3: Constructing a Model and Operating on a Model

As students continued to make sense of the situations, they identified relevant quantities, and discussed information that was known, or that they needed to decide or assume. While we found similar patterns in both lessons, we focus here on *Abuelo's Birthday*, highlighting students' reasoning about quantities and relationships that were central to their model-building. Given that students' often iterated between constructing and relating quantities (Phase 2) and operating on those quantities (Phase 3), we describe both phases in this section.

Reasoning about the role of tax. As students in Mr. H's class discussed how the cost of the printer should be shared among the grandchildren, they considered the role of sales tax in the total price. Several groups assumed that tax would be approximately 5 dollars, because, as one student explained, "usually if you buy something from the grocery store, it's [tax] usually underneath $5." Other students drew on experiences shopping at dollar stores to reason about the tax. In the excerpt below Students 1 and 3 suggest that Jaden (the youngest sibling) pay the tax using his savings, because tax "is never over 5 dollars." Student 2 disagrees, drawing on experiences at the dollar store to explain that tax is charged at a rate of 10 cents per dollar.

Student 1: The littlest one [Jaden] can just pay tax.
Student 2: Eight dollars wouldn't be enough for tax [Jaden has $8 saved.]
Student 3: The tax is never gonna be like over 5 dollars.
Student 2: Tax is always over 5 dollars! Tax is ten cents, ten cents on a dollar. That means it would be 10 dollars [estimating the tax on the printer].
Student 1: But if you are [only] buying one thing-
Student 2: Listen. 1 dollar is 1 dollar and 10 cents, 2 dollars is 2 dollars and 20 cents.
Tax is 10 cents [per dollar]! Do you guys ever like go to the dollar store? You buy [one] thing and have to give them one dollar and 10 cents. …
Student 1: Tax is usually like under 5 dollars. Like at the grocery store. …
Student 4: We can add the tax after we do everything else. That's what people usually do.

Ultimately, this group decided to follow Student 4's suggestion to focus first on how the four grandchildren should split the cost of the printer (without tax) and "add the tax after." Yet their conversation evidences that they leveraged their experiences with tax charged at different stores to reason and make decisions about key quantities in their model.

Deciding how to share costs "fairly". Most students decided that the youngest sibling should only pay the tax; yet they made different decisions about "fair" plans for the other siblings. Some assumed that each sibling should pay the same amount, "that wouldn't be fair if 1 kid paid more." Others reasoned that the older siblings should pay different amounts, proportional to their earnings. Figure 4 displays one group's solution - the older three siblings each contribute their weekly earnings for four weeks, which added to Jaden's savings ($8), gives $148 total.

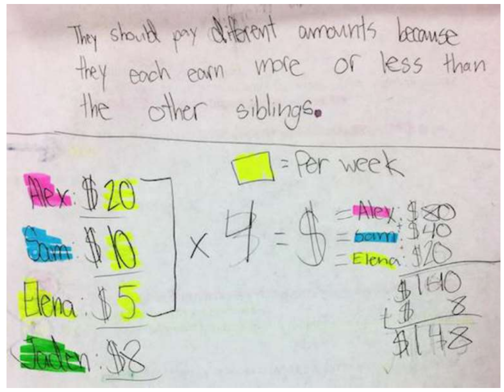

Figure 4: Equal Time Model for Sharing Costs. Each Sibling Contributes Earnings for 4 Weeks

In the discussion that follows, students explain both the quantities they deemed important (in this case, each sibling's weekly earnings) and the way that they related and operated on those quantities in their model (multiplying each weekly earnings by a set number of weeks).

> Student 1: we got what they all made in a week, except for Jaden. And we multiplied it by 4 weeks. And we got these [total earned per sibling in 4 weeks] and we added them …
> Mr. H: Why did you pick 4 weeks to multiply by?) Do you remember?
> Student 2: Because when we did 3 weeks we had less than what we needed, so then we added another week. … Alex paid $80, Sam paid $40, and Elena paid $20 and Jaden paid $8.
> Mr. H: Why did you think that was fair? What was your thinking? … Was there a reason, though, that you let them pay different amounts?
> Student 3: Well because each person, well, Alex made the most. So he would pay like most of it. Well, like Elena, and Sam would pay some of it too. ..
> Student 4: A question I have is … since the printer was $120, but when I look at your paper, it said they paid $148 if you add everything together.
> Student 3: We will just have extra.
> Student 5: They could use the extra money they had to pay tax.

Students drew on shopping experiences, including understandings about tax, as well as perspectives on "fair" ways to share costs to relate and operate on quantities in their models. While some students proposed models based on equally dividing the cost among the three older siblings, other students drew on sensibilities about fairness and sharing in families to reason that fair does not always mean equal. Yet all models were based on the assumption that all four siblings should contribute to the gift, something that students reasoned was important.

Phases 4: Interpret/Analyze Solution, and Refine Model

After students operated on their models to determine the amount paid by each sibling (in Abuelo's Birthday) or the number of bags needed to make the jump ropes in their set (in Jump Ropes), they analyzed their solutions, and in some instances, refinements were needed. While we found similar patterns in both lessons, we focus here on *Upcycling Jump Ropes*, with attention to how students' experiences informed this phase of their modeling activity.

In some instances, students' analysis of their emerging models led to revisions in how they defined an appropriate set of jump ropes for their target audience. For example, several groups of students proposed models based on grouping students by height, and then providing each group of students with a set of appropriate length ropes. As one student explained, “We came up with the idea that we could split people into different categories based on their height because some people are different sizes.” Often students also included one or more long jump ropes in the set, to allow for group jumping activities. Figure 5 below display such a solution.

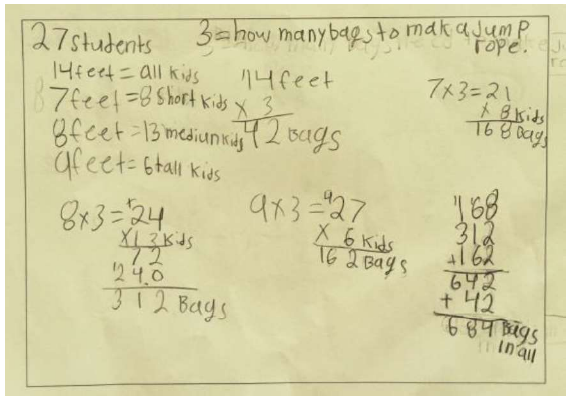

Figure 5: Mr. H’s Students Operate on a Model Based on Grouping Students by Height

As this group explained their model to the class, they highlighted the ropes of different lengths, and how they operated on those quantities to find the total number of bags needed (i.e., “we thought if one foot is 3 bags, we did 3 times 7 is 21 [bags for one 7-foot rope], 21 times 8 is 168 bags [for eight 7-foot ropes], etc.” Yet because they included only one 14-foot rope in their set, a classmate (Student 1) asked them to consider whether that configuration would allow for double-dutch style jumping (which requires two 14 foot jump ropes).

Student 1: For double-dutch you have to have two [jump ropes].
Mr. H: Oh, for double-dutch you have to have two? (to group of students who shared their model) Did you mean actual double-dutch or big jumping?
Student 2: [We meant] double dutch.
Student 3: I just want to jump rope.
Mr. H: If actual double-dutch was a requirement of our class, would that change our outcome? What would we need [to change]? … So if you are doing double-dutch what do we have to do for the bags for the 14 foot [rope] if we are doing double-dutch?
Student 2: We have to double it [double the number of bags]
Mr. H: We could double it to get a double dutch rope [to have enough for two ropes].

This exchange highlights how students’ prior experience and understandings related to jumping rope informed their analysis and interpretation of their own solutions (and those of peers), as well as the teacher’s roles in prompting students to consider refinements. We found that as students analyzed, interpreted and refined quantities (e.g., the number of jump ropes of a given length in a set), they often refined ideas generated in *Phase 2*, and these refinements were informed by the knowledge and experiences students brought to the task.

Discussion

Across the two lessons, we found that students' (and in some instances the teacher's) experiences and understandings were pivotal to their engagement in each phase of mathematical modeling. Students leveraged their experiences to identify important quantities and relationships, to make assumptions, to analyze and interpret the reasonableness of their solutions, and to revise their models when needed. Students' reliance on their own experiences and sense making highlights the potential of relevant mathematical modeling lessons to support students' empowerment as mathematical learners, as well as their real-world reasoning (Bahmaei, 2011). Perhaps the salience of student experience was a reflection of the relevance of the tasks. A well-chosen context supports students in developing "informal, highly context-specific models and solving strategies," (Doorman & Gravemeijer, 2009). Yet we also suspect that Mr. H's frequent invitations for students to share their experiences played a key role. Consistent with other research that has noted the positive impact of honoring students' ideas (Ladson-Billings, 2009), we found that students responded to teachers' invitations with interest, and that these discussions provided opportunities for students to consider and respond to the perspectives of others.

The most challenging phase of the modeling process was generalizing. This is in part due to time constraints; after an extended period of work students were left with minimal time to explore how their models could be used by others. Though not taken up, we see Mr. H's invitation for students to consider how their models could work in other similar situations as an initial step. Future research should investigate how to support generalization in elementary mathematical modeling, both through task development and lesson implementation.

Implications and Conclusion

Our findings have important implications for mathematics modeling instruction and research. Given the salience of children's funds of knowledge across all phases of the modelling process, teachers should explicitly elicit students' experiences and perspectives, and position these experiences as resources to support meaningful engagement in mathematical modeling. A detailed analysis of how specific teacher moves support connections to students' experiences during modeling lessons would be a productive focus for future research.

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