STATISTICS EDUCATION: (RE)FRAMING PAST WORK FOR TAKING A HOLISTIC APPROACH IN THE FUTURE

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The goal of the proposed working group is to create a space for those interested in researching issues around the teaching and learning of statistics to meet, discuss, synthesize past research, and begin to strategize ways of leveraging multiple perspectives and expertise to identify and address current challenges in statistics education. The nature of statistics being a methodological discipline make it such that statistics education is made up of a diverse array of people from various backgrounds, disciplines, fields, interests and expertise. We propose this working group to create a space for dialogue among people with diverse perspectives to tackle important issues in the teaching and learning of statistics. Diverse perspective help to look at problems in new ways and come up with new solutions. However, we also want to pragmatically make progress towards a goal, which requires some common direction as well. To balance these tensions we frame our work in the notion of learning environments as a way of organizing past work as well as ideas for future projects into a meaningful structure. Additionally we are layering on the consideration of teachers and teaching in the design and enactment of learning environments.

Keywords: Data analysis and statistics

Introduction

Creating opportunities to develop statistical literacy in public schooling is crucial for education in the 21st century in data centric societies, and thus a top priority for statistics education (Franklin et al., 2007; Steen, 2001). The focal point of statistics education, an interdisciplinary field, is the teaching and learning of statistics and can trace its beginning to the early eighties (Zieffler, Garfield, & Fry, 2018). While statistics education has emerged as a discipline in its own right, it evolved from statistics and mathematics education drawing on the practice of statisticians, theories of learning and development, as well as methods of inquiry (Ben-Zvi & Grafield, 2008; Garfield & Ben-Zvi, 2008; Zieffler et al., 2018). In chronicling the emergence of statistics education, Ben-Zvi and Garfield (2008) identified core areas of importance in this emerging field: statistical literacy, challenges related to learning and teaching statistics, development of the field of statistics education, collaborations among statisticians and mathematics educators, and the relationship between statistics and mathematics. They argue that statistics can be perceived as a bridge connecting mathematics and science.

While much progress has been made to establish statistics education as a discipline, there are still many challenges that educators and researchers face in order for the field to evolve as a collective community of practice. Because of the interdisciplinary nature of statistics education and its roots in statistics, mathematics and science education, statistics educators come from a diverse array of disciplines, backgrounds, and expertise and, as such, have a wide variety of visions for the direction of statistics education. This is no surprise to anyone who agrees with Cobb and Moore's (1997) characterization of statistics:

> Statistics is a methodological discipline. It exists not for itself but rather to offer to other fields of study a coherent set of ideas and tools for dealing with data. The need for such a discipline arises from the omnipresence of variability (p. 801).

While having a diversity of perspectives is a strength, it is also a challenge. Another challenge for statistics educators is that the teaching and learning of statistics is often situated within the mathematics curriculum and in mathematics departments. Perhaps as a result, statistics is often still positioned as a branch of mathematics as opposed to a distinct discipline that draws heavily upon concepts and practices from mathematics (Cobb & Moore, 1997; Franklin et al., 2007; Groth, 2015).

The goal of the proposed working group is to create a space for those interested in researching issues around the teaching and learning of statistics to meet, discuss, synthesize past research, and begin to strategize ways of leveraging multiple perspectives and expertise to identify and address current challenges in statistics education. Organizing within the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA) acknowledges the roots of statistics education in mathematics education, and acknowledges the importance the authors place in drawing from educational theories, research, and practices in studying the teaching and learning of statistics. Additionally, PME-NA is also well known for promoting and stimulating interdisciplinary research; thus, it would be an ideal space for mathematics and statistics educators to work and collaborate, along with mathematicians, statisticians and scientists, on issues relating to the teaching and learning of statistics. Furthermore, in line with the conference theme, in a world where we are saturated with data in our daily lives, we view the new horizon as a place where we have the opportunity to prepare individuals to critically engage with data and make evidence-based decisions. This new horizon points to the critical need for fostering statistical literacy among all members of society to critically make sense of the world around them (e.g., Arnold, 2017). The new horizon also holds great potential for the future, with the explosion of interest in statistics and statistics education from the many fields it offers a coherent set of ideas and tools for dealing with data.

Statistics Education as a Field

In an effort to start a new working group focused on issues of teaching and learning statistics, we begin by briefly discussing the past history and current status of the field of statistics education in conjunction with mathematics education—particularly because of PME-NA's situatedness in mathematics education. We also use this section to make explicit some of the major challenges that statistics education currently faces as a field. This section serves as both a background and as a rationale for the work of the group.

Statistics Education and Mathematics Education

Mathematics education and statistics education have quite a long and, some might argue, complicated history. Mathematics education can trace its origins back to such writings as Thorndike (1922), Moore (1923), and *The First Yearbook* by the NCTM (Smith, 1926). In the US, although the National of Council of Teachers of Mathematics (NCTM) was established in

1920 (NCTM Board of Directors, 2017), the field of mathematics education was not formally recognized in the academe until 1962 when it was included as an option for doctoral graduates to choose as their primary field of study in the Survey of Earned Doctorates—a survey that has been used in the US to track earned doctorates since 1920 (Shih, Reys, & Engledowl, 2017). Historically, statistics has been included as part of the mathematics curriculum—dating back to at least 1923 in the US (National Committee on Mathematics Requirements, 1923).

It wasn't until the 1950s and 60s that statistics began to form its own identity apart from mathematics. For instance, although the American Statistical Association was founded in 1839 (ASA, 2019a)—49 years before the American Mathematical Society (AMS, 2019) and 76 years before the Mathematical Association of America were founded (MAA, 2019)—a separate department for statistics did not exist at Harvard until 1957, when founded by Frederick Mosteller (Powell, 2006). Further recognizing the distinct disciplines of mathematics and statistics, in 1968, the ASA and NCTM formed the *ASA/NCTM Joint Committee on the Curriculum in Statistics and Probability*, chaired by Frederick Mosteller (Scheaffer, 1982)—the outgoing ASA President. This first joint committee set out to produce a book that would "help the layman, and those in charge of secondary school mathematics curricula, understand the uses of statistics and probability" (Mosteller, 1971, p. 341). The committee went on to publish two curricula: *Statistics by Example* and *Statistics: A Guide to the Unknown* (Scheaffer, 1982)—the former of which was well received, with one book review stating that "from the point of view of interest and motivation, these problem sets are far superior to what has been used previously" (Greenhouse, 1975, p. 483). This relationship between NCTM and ASA would result in supporting roles in curriculum standards documents, such as the *Curriculum and Evaluation Standards* (NCTM, 1989), *Principles and Standards* (NCTM, 2000), and the *GAISE* report (Franklin, et al., 2007), and it played a role in the development of probability and statistics content in NSF-funded curricula, such as *Connected Mathematics Project* (Franklin, et al., 2015). Moreover, the *GAISE* report made a lasting impression on the field at large, and played a leading role in the probability and statistics standards included in the *Common Core State Standards for Mathematics* (NGA & CCSSO, 2010; Franklin et al., 2015). The joint committee is still in existence, and its charge still the same:

> To provide national leadership for the inclusion of statistics and probability in the Nation's mathematics curriculum; to promote awareness programs and quantitative literacy among teachers; and to support the development of appropriate curriculum materials. (ASA, 2019b, para. 1)

The current 6 members of the committee (3 from ASA, 3 from NCTM, Chair alternates between NCTM and ASA) are listed in Table 1 (taken from ASA, 2019b).

Table 1: Current ASA/NCTM Joint Committee Members

Name	Position/Affiliation	Term
Perrett, Jamis	Chair/ASA	2019–2019
Bargagliotti, Anna	NCTM	2016–2021

Gould, Robert	ASA	2018–2020
Maddox, Kathleen	NCTM	2014–2019
Miller, Stephen	NCTM	2015–2020
Tyson, Douglas	ASA	2016–2021

Through some of the efforts of ASA and NCTM described above, as well as others, there was a general recognition in the 1980s that statistics should be included in the school curriculum and improved at the tertiary level (Garfield & Ben-Zvi, 2008). In 1982, the first International Conference on Teaching Statistics (ICOTS) was held in order to improve statistics education at all levels, from elementary school to the training of professionals. Zieffler et al. (2018) suggest that statistics education can trace its roots as a discipline to the first ICOTS. ICOTS has continued to be held every four years ever since.

Past History of Working Group

While PME-NA 41 will be the inaugural meeting of this working group, past working groups have focused on a closely related topic, the teaching and learning of probability. Although probability plays an important role in statistics education, past working groups have placed more emphasis on probability rather than statistics. The first group's work centered on learning to reason probabilistically (Maher, Speiser, Friel, & Konold, 1998) and has convened at ten annual conferences up to 2006 (Lee, Tarr, & Powell, 2005; Maher & Speiser, 1999; 2001; 2002; Powell & Wilkins, 2006; Speiser, 2000; Stohl & Tarr, 2003; Tarr & Stohl, 2004). In 2009, Lee, Lee, Wilkins, and Angotti extended the focus of the working group to include statistics by exploring ideas related to learning to reason probabilistically and statistically through experiments and simulations. The following year, the focal point of the working group returned to using technology to teach and learn probability (Radakovic, Karadag, & McDougall, 2010).

Challenges of the Field

Mathematics and statistics. Though both mathematics and statistics are part of the mathematical sciences, statistics is its own distinct discipline—not a sub-discipline or branch of mathematics (Cobb & Moore, 1997; Franklin et al., 2007; Gattuso & Ottaviani, 2011; Groth, 2013). As Steen (2001) points out, "Although each of these subjects shares with mathematics many foundational tools, each has its own distinctive character, methodologies, standards, and accomplishments" (p. 4). Statistics relies heavily on mathematics, but there are distinct practices and habits of mind in statistics which are non-mathematical as well (Groth, 2007, 2013). Part of this reliance is through probability, which is necessary for statistical inference and firmly a part of mathematics (Fienberg, 1992).

There is a strong literature base that discusses important differences that should be considered between the discipline of mathematics and statistics in undergraduate and school settings (Cobb & Moore, 1997; Franklin et al., 2007, 2015; Gattuso & Ottaviani, 2011; Groth, 2007, 2015; Scheaffer, 2006; Usiskin, 2014). The most prominently discussed differences include the treatment of context, variability, inductive versus deductive reasoning, and uncertainty. Scholars generally support the location of the teaching of statistics in the mathematics curriculum (Gattuso & Ottaviani, 2011; Scheaffer, 2006), and some scholars also point out it should also be distributed cross the teaching of all disciplines (Usiskin, 2014). This situatedness can also be viewed in the earlier discussion of the relationship between mathematics and statistics education.

While there are interrelations between mathematics and statistics, differences in the disciplines mean there are some important differences in the thinking and reasoning encouraged in both disciplines, which relates to teaching and learning. For example, mathematics primarily relies on deductive reasoning using definitions, axioms, and theorems, in a logical chain of reasoning, to come to a conclusion. A student could use Euclid's definition of a circle and his first and third postulates to construct an equilateral triangle. At the same time, Euclidean geometry is based on certain unprovable assumptions such as the parallel postulate, which if changed creates an entirely new type of geometry and way of viewing the world (Katz, 2009).

Statistics, on the other hand, begins with a question for which data is collected to answer (Franklin et al., 2007; Wild & Pfannkuch, 1999). It is from the data that information is empirically derived. This can lead to issues in teaching statistics, as teachers who have had few experiences with statistics may attempt to deduce solutions from rules and assumptions rather than inducing them from the data. Such differences in teaching result in the need for the investigation of issues relevant to the teaching and learning of statistics, which may differ in some regards to the teaching and learning of mathematics. It is these differences that have fueled the need for the field of statistics education. However, As Groth (2015) cautions,

Although the evolution of a statistics education community of practice can be viewed as a positive development for research on the teaching and learning of statistics, there is also a danger that the disciplines of statistics education and mathematics education may become increasingly insular and non-communicative with each other. Sustained boundary interactions are vital to preventing insularity from contributing to the stagnation of interrelated communities of practice. (p.5)

This caution is why a working group such as the one being proposed is so important—to create a community of practice that focuses on sustained boundary interactions between mathematics and statistics education. The need for such a space is discussed in the section that follows.

Collaborating and communicating across fields and disciplines. Statistics education as a discipline has established its own organizations, conferences and journals. At the international level, the International Association of Statistics Education (IASE) serves as an organizing body, which also houses the Statistics Education Research Journal (SERJ) to help disseminate the work of its members. The IASE also helps to organize a number of events and research conferences, both on a yearly basis in conjunction with the International Statistics Institute, and every four years for the main research conference (i.e., ICOTS). In the US, the ASA disseminates scholarly work through the *Journal of Statistics Education* (JSE).

While these organizations, conferences and journals make important contributions to the knowledge base of scholarly work and best teaching practices, we conjecture that work may not be as widely accessed by mathematics educators and researchers, or others, who do not identify as statistics educators and scholars. In turn, statistics educators and scholars, especially those who identify more broadly as mathematics educators, science educators, learning scientists, mathematicians, or statisticians likely publish their work outside of journals such as SERJ and JSE. Thus, the interdisciplinary nature of statistics education contributes to challenges in disseminating work through organizational efforts, conferences and scholarly publications. We argue that while it is a challenge to access research across different fields and disciplines, it is necessary to build on all contributions to move statistics education forward.

An additional challenge to collaborating and communicating about important issues in statistics education is the multitude of stakeholders with vested interests in improving the teaching and learning of statistics: K-12 mathematics teachers, university and community college mathematics and statistics instructors, statisticians, mathematics education researchers, statistics education researchers, mathematics teacher educators, learning scientists, as well as others. When statistics educators and scholars disseminate their work, they are faced with the challenge of appealing to such a wide audience.

Lastly, another challenge in working across disciplines is the relationship between science education and statistics education. Statistics has very strong roots in science, and as a methodological discipline is drawn upon heavily by all the sciences, which has implications for standards. For example, the new Next Generation Science Standards (NGSS Lead States, 2013) includes science and engineering practices that are deeply connected with the practice of statistics, such as analyzing and interpreting data and engaging in argument from evidence. While there are strong disciplinary boundaries between mathematics and science at the K-12 level in the US, we argue that there should be stronger ties between these communities. The work of Lehrer and Schauble (e.g., Lehrer, Kim, & Schauble, 2007; Lehrer & Schauble, 2000, 2004; Petrosino, Lehrer, & Schauble, 2003) around data modeling is an exemplar of how drawing on multiple fields can contribute to improving the teaching and learning of statistics. Work along these boundaries holds much promise for the future, particularly given Ben-Zvi and Garfield's (2008) call for statistics to serve as a bridge between mathematics and science. It is our hope that the proposed working group will foster a space for such boundary work to occur, and to consider how best to disseminate that work to the various stakeholders and audiences of interest.

Researching the Teaching and Learning of Statistics

The challenges of the field we describe in the previous section are part of the reason why it is so important to form spaces to bring together people from various backgrounds, disciplines, fields, interests and expertise to tackle important topics in the teaching and learning of statistics. We propose this working group for that very purpose, to create a space for dialogue among people with diverse perspectives. Diverse perspective help to look at problems in new ways and come up with new solutions. However, we also want to pragmatically make progress towards a goal, which requires some common direction as well. To balance between acknowledging and drawing from diverse perspectives and working towards a common goal we have decided to take the approach of framing our work as a way of organizing past work as well as ideas for future projects into a meaningful structure.

To frame the activities of the working group, we draw on Ben-Zvi, Gravemeijer, and Ainley's (2018) recommendations in taking a holistic approach to investigating the teaching and learning of statistics by focusing on learning environments and drawing from a multitude of theories from various levels of generality. They call for more researchers to take up a holistic perspective of investigating many or all of the factors of a learning environment, and furthermore ask researchers to consider which supports for teachers and teacher education are necessary for creating and facilitating such learning environments. Their work highlights the importance of understanding how various dimensions of learning environments are interrelated.

Responding to the call of Ben-Zvi et al. (2018), the initial goal of the working group is to review what the field knows about each of the components of a learning environment described, and to also conceptualize the role of teaching and teacher education in relation to learning environments and synthesize the literature related to that as well. The rationale for this approach is to synthesize the literature through this new lens to identify areas of need and have a basis for identifying specific problems for the working group to brainstorm ways of investigating and form collaborations for future work around. To frame the work of the working group, we briefly describe the factors in learning environments described by Ben-Zvi et al. (2018) below and then begin to conceptualize the role of teaching and teacher education in relation to learning environments.

Learning Environments

Taking a learning environment perspective acknowledges the inherent complexity in a classroom, which is at the intersection of social, cultural, temporal, and spatial dimensions and represents an interactional space that includes the influence of many different stakeholders including students, parents, teachers, administrators, politicians, etc. Such complexity requires design efforts that do not just take a narrow focus on a single element of a learning environment, such as the written curriculum used, but instead focuses on how to take into account many interrelated dimensions of the environment. Ben-Avi et al. (2018) identify key interrelated dimensions that are critical to designing learning environments to support students in developing productive statistical thinking: focus on central ideas, well designed tasks, real or realistic data, technology tools, classroom culture, and assessment to monitor and evaluate. Ben-Zvi et al.'s proposed interrelation of these elements can be seen in Figure 1.

Researchers taking up this perspective often take a design research approach (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). Our goal in drawing from this framework is not to subscribe to particular orienting or background theories or specific methodologies. Instead our initial goal is to use this framework as a way of organizing past research in a manner that will serve as a starting point for designing research projects that leverage what is known by the field and target areas of need in a holistic manner attending to the learning system as a whole versus only its parts. In framing our review of the literature, we feel it is important to explicitly incorporate the role of teachers, teaching and teacher education in learning environments. We elaborate on these dimensions in the section that follows.

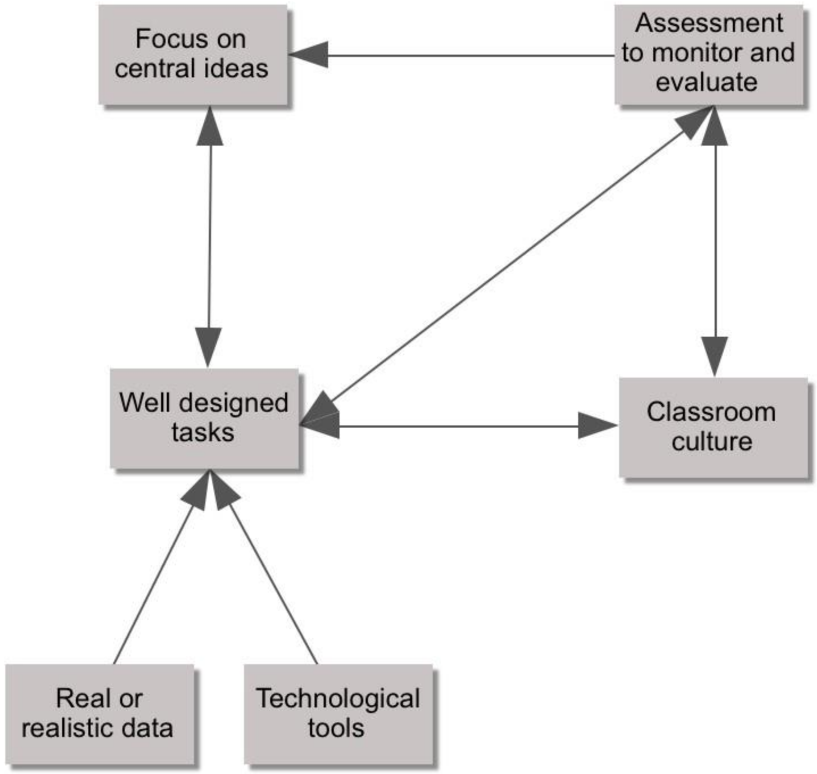

Figure 1: Ben-Zvi et al.'s (2018) Interrelated Dimensions of a Learning Environment

Teaching and Teacher Education

There is a wide variety of models or notions of teaching. In teachers' enactment of their curriculum, they must work as both diagnosticians and river guides (Russ, Sherin, & Sherin, 2011) by both probing and making sense of student learning, and being flexible to navigate the complexities and dynamics of the practice of teaching in the moment. This includes navigating the complex interactions between teacher and student, teacher and content, student and content, and between students situated in learning environments (Cohen, Raudenbush, & Ball, 2003). These interactions are influential in shaping what is taught as mathematics/statistics, how to do mathematics/statistics, and students' identification in relation to mathematics/statistics (Boaler, 2002; Boaler & Greeno, 2000). To be able to navigate these interactions requires that teachers have a strong content knowledge—be familiar with the content they are teaching—as well as have a strong pedagogical content knowledge—be familiar with how to create effective learning experiences for others to learn content knowledge (Ball, Thames, & Phelps, 2008; Baumert et al., 2010; Hill et al., 2008; Shulman, 1986). Furthermore, content knowledge and pedagogical content knowledge are deeply intertwined and are inseparable in the practice of expert teachers (Baumert et al., 2010). As a result it is important for mathematics teachers to not only have a strong understanding of the mathematics they teach, but to also have powerful pedagogical practices for effectively teaching mathematics content to students from diverse backgrounds (Chao, Murray, & Gutiérrez, 2014).

Scholars argue that merely discussing teaching in terms of the interactions Cohen et al. (2003) describe, does not give enough credit to the historical, cultural, societal, and political environments that learning environments are situated in and influenced by (Chazan, Herbst, &

Clark, 2016). Classrooms are situated at the intersections of many different contexts, such as the specific social norms, policies, and physical layout of the classroom and school within which such interactions are occurring, as well as larger community and societal environments within which the school is situated (Davis & Sumara, 1997; de Freitas & Sinclair, 2014).

The importance of statistics education has gained traction in K-12 school settings in the US with the CCSSM (NGA Center & CCSSO, 2010). However, a common issue related to this is the teaching of statistics and the statistical education of teachers (Franklin et al., 2015). As Shaughnessy (2007) pointed out over a decade ago, and Horizon has found is still the case from two separate nationally representative surveys of K-12 mathematics teachers (Banilower et al., 2012; Banilower et al., 2018), many mathematics teachers have had little to no prior experiences with statistics. This is an issue because, as Cobb and Moore (1997) describe, the teaching of statistics has some differences from that of mathematics and this position has been echoed by a number of others since (Franklin et al., 2007; Gattuso & Ottaviani, 2011; Groth, 2007). More recently, the Horizon surveys found teachers across K-12 are not confident in teaching statistics in general (Banilower et al., 2012; Banilower et al., 2018), and, regarding specific statistics content, Lovett and Lee (2017) found that secondary teachers are not confident in their ability to teach a number of the new statistics and probability standards they are expected to teach. The consideration of teachers and teaching is crucial to developing statistics education in school settings. Teaching is however a complex social practice at the intersection of many different communities of practices and influencing factors.

To incorporate the consideration of teachers and teaching the working group will include considering the knowledge and pedagogy necessary to create and enact learning environments as well as how teacher education may need to be modified to help prepare teachers for such roles. These layers will be considered in addition to the learning environment framework from Ben-Zvi et al. (2018).

Plan for Active Engagement of Participants

The working group meetings at the PME-NA 2019 conference will be organized to introduce members to one another, explore frameworks for synthesizing statistics education research, synthesize findings from statistics education research, and make connections with researchers that have a similar focus within statistics education research. To make the most of our time together during the conferences the meetings will be organized as described below.

Session 1

- Participants introduce themselves to the group and share research focus in statistics education
- Working Group leaders introduce the focus of the group and share initial framework from Ben-Zvi and elaborate on components
- Participants break out into small groups to discuss and engage with the framework considering guiding questions provided by the working group leaders
- Participants share summary of their small group discussion and the group as a whole refines the framework and goals of the group

Session 2

- Participants brainstorm in small groups dimensions that are not in the Ben-Zvi framework that would be important to consider
- Groups share out and Working Group leaders facilitate discussion to build shared framework

- Participants identify areas of the framework that align with their expertise and/or interest

Session 3

- Participants work in groups to identify sources for literature within each dimension of the framework and brainstorm a plan for systematically reviewing the literature
- Participants share out sources of literature and thoughts on a plan for reviewing the literature
- Group refines plan for reviewing the literature
- Outline of draft of publication created
- Participants plan meeting times and shared work for after conference
- Possible venues for publication of work are discussed

After conference

- Small groups follow the plan for searching for literature along different dimensions of the framework
- Small groups add literature to google sheet repository
- Working Group leaders curate literature and write synthesis for each area
- Teams work on adding to the outlined draft
- Working group leaders plan 2020 conference time

It is our hope that this working group will serve as a basis for a sustainable working group that can continue on in later PME-NA conferences and serve not only to connect scholars currently in the field, but to also provide an environment that welcomes in early career scholars and graduate students interested in investigating issues around the teaching and learning of statistics.

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