EXPLORING THE NATURE OF MATHEMATICAL MODELING IN THE EARLY GRADES

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This working group will engage PMENA members to better understanding the nature of mathematical modeling in the early grades while considering the student perspective and recognizing the importance of teachers knowing their students and the contexts that are meaningful to their students. We will investigate how PK-6 teachers demonstrate the interdisciplinary nature of mathematical modeling, the diversity of mathematical approaches taken by student modelers, and the multiple pathways the teacher can use to elicit students' mathematical thinking. We will explore how mathematical modeling bridges equity and social community in teaching and learning mathematics for all students. Exemplar tasks that emphasized local contexts and tapped into students' funds of knowledge and student artifacts will be shared to illustrate the child's perspective and the developmental progression. These topics will facilitate group discussions exploring the learning progression for mathematical modeling thinking and habits of mind that can develop for emergent mathematical modelers from an early grade. Finally, based on the interests of the participants, we will devote work time to finding synergistic collaborative topics to pursue for future research and practice.

Keywords: Mathematical Modeling, Elementary Education, Teaching Practices, Professional Development, Learning Progressions, Knowledge of Content and Pedagogy

Overview of the Working Group

This working group began at the 2017 PMENA in Indianapolis, IN and continued its meeting in 2018 in Greenville, SC. We proposed that this working group have a special focus on early mathematical modeling and continue to build on PMENA's long tradition of working groups on

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Models and Modeling. Through the 2017 and 2018 working group meetings, we found that there are researchers and practitioners with a keen focus on broadening the access of mathematical modeling to diverse learners in the elementary grades and advancing the field's collective understanding of the interrelated processes of mathematical modeling in the elementary grades and beyond. Although there has been a long history of mathematical modeling at PME and PMENA, the focus was primarily middle, high school and university levels. We believe it is critically important to understand the learning progression of mathematical modeling from early elementary to secondary grades to ensure coherence and rigor in the mathematics curriculum.

In our first year, the working group leaders proposed an edited volume and a special issues journal venue for MM where participants interested in submitting manuscripts could work together to provide a comprehensive research trajectory documenting the progression of mathematical modeling from emergent levels to more sophisticated levels of modeling. We are excited to share that this working group was able to secure a contract with Springer to publish an edited volume on this very topic. In our second year of the working group, one of our lead facilitators announced an exciting networking meeting at the upcoming MSRI meeting in 2019 focused on modeling and the connection to community and cultural contexts. It is clear that this PMENA working group is facilitating ways to bring synergy among researchers across North America, and we hope to continue this working group so that we can invite more mathematics educators to take part in the important research of MM in the early grades.

Implementing MM in the elementary grades is not just going "light" with the high school math modeling curriculum. Instead we advocate integrating aspects of mathematical modeling in the early grades effectively to enhance student learning and to help build their competency in real-world problem solving using their current mathematical knowledge. The latter content knowledge is expected to develop and evolve as students progress towards high school and beyond. So what does mathematical modeling look like in the elementary grades? Why focus on early grades? In addition to the direct benefits of modeling, the elementary school environment affords many advantages that complement work in mathematical modeling. Elementary students often rely on concrete referents such as objects, drawings, diagrams, and actions that can support the conceptualization and construction of carefully formulated arguments to solve a problem. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades (CCSSO 2010). Young students have great potential to become fluent – native speakers, thinkers, and dreamers of mathematics. Thinking creatively may come more easily to children first learning and exploring mathematical concepts. Kindergarteners can use manipulatives to independently solve traditional multiplication or division problems they have never seen before, which is evidence that students come with knowledge--it is not necessary to wait to incorporate modeling activities until we have "shown them how" to do everything. Because early grade teachers are generalists, they can address several subjects simultaneously through modeling activities. Mathematical modeling is of interest and relevance to the mathematics education community especially because it connects to the need for professional development focused on MM in the elementary grades.

We are also interested in focusing on different research methodologies used in mathematical modeling research. Some of our researchers use Design-Based Implementation Research methodology, DBIR (Fishman, Penuel, Allen, Cheng, & Sabelli, 2013) to examine the design of the professional development and to study and enhance the design through feedback from iterative implementation cycles. Each year at our working group meetings, we meet researchers who are also using Networked Improvement Communities (NICs) collaborating with school

districts working with diverse populations to examine what works, for whom, and under what conditions, which helps us better understand the nature of MM in the elementary grades with diverse learners across geographic regions. For example, in one of our NSF-funded projects, each university site worked with the collaborating district's teacher leaders to co-plan the professional development. Teachers became co-designers of the MM curriculum for the elementary classrooms. In our project, we engaged elementary teachers in MM using real world tasks that contained several of the following attributes: (a) Openness; (b) Problem-posing; (c) Creativity and choices; d) Iteration and revisions.

Through our work, we are gaining a better sense of teaching practices and classroom routines that support modeling. We are contributing to the understanding of what is possible in early elementary grades and how these processes support the development of critical 21st century skills. As we continue in our research to consider what constitutes the practice of Mathematical Modeling (MM) and how it could be implemented in classrooms at different grain size, we invite the larger PMENA community to build on this knowledge. Over the past decades, working group leaders have individually, and in subgroups, been theorizing and collecting, analyzing, and reporting on data relating to mathematics modeling. This Working Group builds on and extends the work of previous Model and Modeling traditions by discussing current work from leading scholars from diverse perspectives.

Relevance to Psychology of Mathematics Education

In the spirit of exploring the theme of 2019 PMENA "Against a New Horizon," we will offer differing views of "expansion" and "growth" in relation to ways mathematical modeling can provide opportunities for learners, families, and their communities to engage in mathematics and supporting *all* students through a concerted focus on equitable teaching practices. In addition, this working group will attend to the interdisciplinary nature of mathematics and how it connects to social justice, STEM, and civic responsibility for our citizen in our country. Finally, we invite researchers from different countries in North America to broaden our understanding of how different national curricula attend to mathematical modeling as mathematical literacy, situational problem solving, and other curricular initiatives that develop critical thinking skills.

The purpose of this working group is to invite individuals across the research community interested in synthesizing the literature and collaborating on research focused on mathematical modeling along the developmental continuum. Our goal of mapping a learning progression of mathematical modeling from K-12 education, particularly starting from elementary to middle grades, is critically important to provide coherence in the mathematics curriculum.

The primary focus for this working group will be centered on the following three goals:

1. Examine current research and discuss the nature of mathematics modeling and detailing the development of teachers' content knowledge, teaching practices, and students' modeling competencies.
2. Map the learning pathways for mathematical modeling and task design for K-6 mathematics education and explore how mathematical modeling can bridge equity and social community in teaching and learning mathematics for all students.
3. Engage in dialogue and collaboration among individuals and groups conducting research on student- and teacher-related outcomes related to implementing mathematical modeling, ways mathematical modeling promotes social justice, 21st century skills, and ways in which early modeling can develop interdisciplinary skills in STEM.

Related Research

Mathematical proficiency, in today's world, moves beyond computational ability. It includes the development of 21st century skills (i.e., critical thinking, creativity, communication, and collaboration), conceptual understanding of mathematics (NCTM, 2014), and mathematics that has practical relevance outside of the classroom (Gravemeijer, Stephan, Julie, Lin, & Ohtani, 2017). Mathematical modeling (MM) is a powerful tool for developing students' 21st century skills (Suh, Matson, & Seshaiyer, 2017), advancing their conceptual understanding of mathematics, and developing their appreciation of mathematics as a tool for analyzing critical issues in the world outside the mathematics classroom (Greer & Mukhopadhyay, 2012). It provides the opportunity for students to solve genuine problems and to construct significant mathematical ideas and processes instead of simply executing previously taught procedures and is important in helping students understand the real world (English, 2010).

There is broad agreement among mathematics educators on the relevance of MM in schools, but the field has yet to come to a consensus on the definition of mathematical modeling or on how it might be taught and learned in schools (Kaiser, 2017). Although mathematical modeling has traditionally been reserved for secondary and college students, its enactment in schools contributes to broad educational goals that are relevant to learners of all ages (Ferri, 2018). In addition, scholars have argued that engaging in mathematical modeling is important for elementary school students (Carlson, Wickstrom, Burroughs, Fulton, 2016).

Mathematical modeling has received increased attention in the United States since the release of the Common Core Standards in Mathematics (the Common Core hereafter) in 2010. Modeling is incorporated as a specific area of expertise that teachers should cultivate in students across Grades K–12. The Common Core's Standards for Mathematical Practice, SMP4 is called *Model with Mathematics*. Although SMP4, as a mathematical practice, cuts across Grades K–12, mathematical-modeling opportunities are not highlighted in connection with the K–8 content standards, presenting an implementation challenge for teachers (Cirillo, Pelesko, Felton-Koestler, & Rubel, 2016). Modeling with mathematics, the topic of SMP4, refers to both modeling mathematics and mathematical modeling. The distinction between modeling mathematics and mathematical modeling is not clear to many teachers (Meyer, 2015), nor is it clear in Common Core documents or in mathematics education literature (Cirillo et al., 2016). The key difference between mathematical modeling and modeling mathematics is where the mathematical activity begins. Modeling mathematics begins in the mathematical world (Van de Walle, Karp, & Bay-Williams, 2016), whereas mathematical modeling begins in the unedited real world (Pollak, 2007). The explicit focus on getting a problem outside of mathematics into a mathematical formulation and explicitly translating the mathematical solution back into the real world is what differentiates mathematical modeling from modeling mathematics. The real-world focus also distinguishes mathematical modeling from problem solving and application problems (Lesh & Caylor, 2007; Schukajlow et al., 2012).

One of the ways the researchers in this working group have approached MM in the elementary grades was to immerse students in a relatable and personally-meaningful real-world situation within their local contexts. In bringing mathematics closer to social community spaces, mathematical modeling became a vehicle that brought teaching and learning mathematics closer to all students. Reforms in mathematics have advocated for mathematics to be more related to students' lives by building on community and cultural knowledge and practices with issues that

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matter to them, which then helps students view mathematics as a vehicle through which they learn to be active change agents for social justice (Bartell et al., 2017; Civil, 2007).

To keep the initial problem open, students were encouraged to develop the habit of mind of being problem posers by identifying the many questions around the real phenomenon, then defining a mathematical problem that can be solved by way of mathematics. After the identification process of the problem, the modeler makes assumptions, eliminates unnecessary information, and identifies important quantities to develop a solution. The mathematical solution focuses on the usefulness of mathematics to solve a real-world problem. It should be noted that there can be several mathematical solutions for a given real-world situation. After solving the problem, the results are translated back to the real-world and interpreted in the original context. The problem-solver then validates the solution by checking whether it is appropriate or reasonable for the purpose. This process of making assumptions, identifying variables, formulating a solution, interpreting the result, and validating the usefulness of the solution is iterative in nature and modified and repeated until a satisfactory solution is obtained and communicated (Blum, 2002).

It is important to note that teachers play a crucial role in MM and must be able to: (1) find appropriate questions to move students through the modeling cycle, (2) handle discussions in nondirective but supportive ways, (3) allow students time for productive struggle, and (4) provide scaffolding without directing the problem or its solution (Burkhardt, 2006). Teachers also need to develop problem-posing expertise (Suh et al., 2017) and to base their instructional decisions on responses to students' work (Bleiler-Baxter et al., 2016). Thus, learning to teach MM requires teachers develop multiple knowledge bases. For example, teachers must understand modeling processes and tasks, including the potential mathematical content embedded within tasks; learn about students' mathematical and personal experiences to predict the strategies they might use when responding to modeling tasks; know what content is on the mathematical horizon to anticipate what mathematical ideas students might construct; and learn to engage individual and groups of students in the modeling process (Blum, 2011; Ferri, 2018).

Previous work with elementary school children demonstrated it is feasible for them to develop a disposition towards realistic mathematical modeling (Verschaffel & De Corte, 1997). One of the issues in implementing MM at the elementary level is that MM can be difficult for both teachers and students to implement (Blum & Ferri, 2009). MM can be difficult for teachers to implement as they must be able to merge mathematical content and real-world applications while teaching in a more open and less predictable way (Blum & Ferri, 2009). Mathematical modeling can be a challenge for students because each step of the modeling process presents a possible cognitive barrier (Blum & Ferri, 2009). As stated in the Common Core Standards for Mathematical Modeling,

"Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. These real-world problems tend to be messy and require multiple math concepts, a creative approach to math, and involves a cyclical process of revising and analyzing the model" (Carter et. al., 2009).

To model, students need support to develop mathematical modeling competency -- i.e., the ability to independently carry out the various phases of the modeling process (Vorhölter & Kaiser, 2016) and its related sub-competencies (Schukajlow et al., 2015). Based on the Standards for Mathematical Practice 4 in the Common Core: Model with Mathematics, Bleiler-Baxter, Barlow, and Stephens (2016) identified the mathematical modeling sub-competencies needed by students as simplification (e.g., making assumptions), relationship mapping (e.g., identifying

important quantities and their relationships), and situation analysis (e.g., interpreting results in the context of the situation). In addition, students need to develop a metacognitive modeling competency because it is "indispensable in order to enable students to solve complex modeling problems independently, which is an indispensable part of true modeling activities" (Vorhölter & Kaiser, 2016, p. 279).

Emerging Research on Early Mathematical Modeling

Developing modeling competencies in young mathematicians. English (In press) argues for the importance of incorporating mathematical modelling into the years of early education, where young children's learning potential often remains untapped. She states MM is ideally suited for early learning and should not be reserved for the later school years; Early MM facilitates interdisciplinary learning, in particular, linking the STEM disciplines as well as humanities (e.g., literature). She elaborates on (a) "pre-modelling" experiences (e.g., working with simple data and their representations), (b) mathematical modelling within different disciplinary contexts (e.g., engineering in the production of confectionary for a modelling problem that targeted this topic; using literature as introductory and supporting contexts), (c) the inclusion of supplementary science content, and (d) modelling experiences posed by children.

Anhalt, Cortez, and Aguirre (In press) share a construct for developing competency across grades and propose key modeling competencies that can be targeted for development early on, and thus, classroom activities can be designed with the goal of developing those competencies in students through modeling tasks and through other activities that do not necessarily engage the entire modeling process. They refer to it as *mathematical modeling thinking (MMT)*. In order to define MMT and determine what to include in it, they address the questions: (1) what are key competencies to be successful in mathematical modeling? And (2) what activities are effective at laying the foundation for the development of those key competencies?

Osana and Foster (In press) report on the genesis of modeling in kindergarten and unpack the key ingredients for professional development. The MEA that was orchestrated in the teachers' classrooms invited the children to design their ideal Kindergarten classroom. The activity encouraged the children to answer the questions they themselves generated through modeling (English, 2010), such as "How many tables and chairs do we need for all the kids in the class?" "How big should the tables be and where should we put them?" "How many kids should be at each table and why?" Manipulatives and other tools were made available to the children as they worked through the modeling cycles, and children worked collaboratively on the construction and revision of their models. The results yield valuable insights on the elements that are necessary for professional development in mathematical modeling with young children, particularly in classrooms with at-risk students.

Turner, McDuffie, Aguirre, Foote, Chappelle, Bennett, Granillo, and Ponnuru (In press) provide a description of the Upcycling Jump Rope task and state several important implications for mathematics education researchers to consider. They recommend that given the salience of children's funds of knowledge across all phases of the modelling process, teachers should explicitly elicit students' experiences and perspectives, and position these experiences as resources to support meaningful engagement in mathematical modeling. Second, their findings highlighted pedagogical tensions in mathematical modeling lessons that demand further investigation. For example, while students readily shared experiences related to the broader task context (plastic consumption, recycling, and pollution), teachers had to determine when to encourage this sharing and when to redirect the conversation to key features of the specific modeling task. A third implication is related to more effective engagement with the final phase

of the modeling process – generalizing. While generalization of models remained elusive, emerging evidence suggests that teachers can reframe tasks to facilitate shareable and re-usable models for similar situations.

Wickstrom and Yates (In press) analyze students' notions of mathematics as they consider how elementary students define mathematics and view themselves as learners while doing mathematical modeling and also during traditional instruction. Findings suggest that the third-grade students conceptualized math as computations and often compared themselves to peers to determine success in mathematics. In contrast, during mathematical modeling, students discussed that the task was more difficult than traditional mathematics, but also more rewarding and inclusive.

Developing core teaching practices for early mathematical modeling. Suh, Matson, Birkhead, Green, Rossbach, Seshaiyer, and Jamieson (In press) report ways in which researchers are collaborating with teacher designers to develop personally relevant and rigorous MM tasks for elementary students. The essential design skills include : 1) Leveraging problem posing routines to develop questioning skills: When posing an MM problem, teacher-designers adopted instructional routines for problem posing and worked on developing teacher and student questioning competence; 2) Connecting familiar context that engages students: Teachers, as designers, looked for situational features that warranted mathematizing and searched for contexts that were relevant and important to support students' engagement in modeling. In addition, teachers elicited students to think about how their solution was shareable, reuseable, or generalizable in order to evaluate whether a systematic model was created; 3) Connecting context with content: Teachers connected the need for mathematics in a modeling task with the curricular objectives of their grade level; 4) Considering categories of MM tasks: The modeling tasks tended to fall into four general categories (described below) where a mathematical solution or model could be used to describe, predict, optimize, and make decisions about real world situations.

- Descriptive Modeling - Using math to describe, represent, and analyze a situation or a phenomenon.
- Optimization Modeling - Using data to find the "best" by optimizing or in some cases minimizing some variable (i.e., cost, space) in a situation.
- Rating and Ranking - Using a criterion where one assigns weights or mathematical measures as a way to rate and rank options to make decisions.
- Predictive Modeling - Using trends and data analysis to predict an outcome or using patterns (data analysis and algebra) to predict a situation and make decisions. In some tasks, probability and statistical modeling is used to search for patterns in data to explain a phenomenon (i.e., scientific phenomenon used in STEM contexts).

In addition, researchers are examining core practices that are essential in supporting student learning through modeling. Suh and Matson (In press) found four main categories of core teaching practices that emerged as being central to the success of enacting mathematical modeling in the elementary classroom: a) Questioning practices: Developing competence in asking productive questions of students; b) Data Practices: Connecting relevant data by formulating the problem and eliciting student thinking about important variables and assumption in a problem situation; c) Modeling Practices: Building solutions/models that can be

communicated and are useable to others through records of student work, concrete tools, written and verbal explanations, number sentences, and pictorial representations; d) Analytic and Interpretive Practices: Facilitating productive analysis of a model for the purpose of refining it.

Carlson (In press) report on teacher knowledge bases for engaging young children in MM. She explores the mathematical and pedagogical knowledge teachers need in order to engage young children in mathematical modeling. Analysis of data collected suggests that teachers developing and implementing modeling tasks draw on three knowledge bases: (1) knowledge of real world contexts around which students might pose and investigate mathematical problems, (2) knowledge of students' mathematical and local knowledge resources, and (3) knowledge of curricular mathematics – including the mathematical tools children may have at their disposal and ideas that are on the "mathematical horizon" and might be constructed during a modeling activity.

Formal and informal learning of modeling across disciplines and settings. Mathematical modelling is central to understanding different disciplinary contexts. Elementary teachers are generalists who teach multiple disciplines. Early caregivers, parents, and preschool teachers also have opportunities to leverage real-world situations to teach mathematics. Gallagher and Jones (In press) describe how elementary teacher candidates were introduced to mathematical modeling (MM) in their math methods course and how one of those candidates implemented a MM task related to economics. Students were asked to make economic decisions on the supplies needed for their classroom based on a list of choices and a budget prepared by their principal. The students chose items they found most necessary within the budget. They discussed how economics, as a field based on MM, provides a natural way to integrate MM into the elementary curriculum.

Yanisko and Minicucci (In press) share design features of a course designed for K-5 prospective teachers and aligned with two curricular goals – that students feel empowered by learning mathematics and that teachers recognize the assets of their students and leverage those assets to improve the effectiveness of mathematics instruction. This acquired knowledge is unpacked through an asset-focused lens and leveraged to build students' capacity for geometric modeling that is aligned with math standards. Additionally, teachers examined how to foster students' personal sense of power by investing them to use acquired math knowledge to positively impact themselves or their community.

Gilbert and Suh (In press) highlight how modeling principles across mathematics, science, and engineering converge toward a construct of integrated STEM modeling. These processes are framed as a disciplined inquiry approach that embrace cross-disciplinary connections to solve problems or better understand real-world phenomena. In particular, this instrumental case study investigated preservice teachers enrolled in a graduate-level integrated STEM course where activities were steeped in modeling tasks surrounding content and pedagogy involved in meaningful integrative processes. Findings suggest that preservice teachers could both prepare and enact integrated STEM approaches, and after teaching children in elementary contexts, recognized the creative freedom and motivation it brought to the children they taught as well as themselves. These researchers propose a model for the convergence of STEM practices and articulate the value for STEM modeling in elementary contexts.

Civil, Bennett, and Salazar (In press) look at ways to encourage MM in informal settings with professional development with families and caregivers. They report on their learning from Modeling with Mothers. They maintain that parents are intellectual resources and have a wealth of knowledge about various topics that interest their children. In alignment with the larger

project's overarching goal to bring together parents and teachers, the co-development exercise intertwined the roles of those inside and outside of the classroom, fusing researchers' knowledge of mathematical modeling with parents' funds of knowledge about the school community. Additionally, the mothers' insistence to involve their children signaled to the researchers the importance of collaboration between parents and children in curriculum development.

Plan for Active Engagement of Participants

The working group will meet three times during the conference and virtually during the course of one year. In each session, PMENA members will engage in mathematical modeling while sharing their perspectives in teaching and learning mathematics, considering synergistic areas fruitful for future research and practice, and finding collaborators within our group. At each session, we will have facilitators share a research theme and provide participants time to get into smaller research groups so that participants can join one or visit with three smaller groups to network and find synergistic research interests with others at our working group.

WG-Research Group 1: Student Development Focused: Tapping into Students' Funds of Knowledge and Assessing Student MM Competencies and the Developmental Trajectories

Research Group 1 will focus on better understanding the nature of mathematical modeling in the elementary grades while considering the student perspective and recognizing the importance of teachers knowing their students and the contexts that are meaningful to them. We will investigate how K-6 teachers can assess math modeling in the elementary grades while appreciating the diversity of mathematical approaches taken by student modelers and the multiple pathways the teacher can use to elicit students' mathematical thinking. We will explore how mathematical modeling bridges equity and social community in teaching and learning mathematics for all students. Exemplar tasks that emphasized local contexts and tapped into students' funds of knowledge and student artifacts will be shared to illustrate the child's perspective and developmental progressions. These topics will facilitate group discussions exploring the learning progression for mathematical modeling thinking and habits of mind that can develop for emergent mathematical modelers from an early grade. We will map out productive learning pathways for mathematical modeling and task design for K-6 mathematics education and beyond.

WG-Research Group 2: Teacher Development Focused: Identifying Core Teaching Practices for Early Math Modeling and Unpacking the Knowledge of Content and Pedagogy Needed for Mathematical Modeling in the Elementary Grades

Research Group 2, we will focus on clearly defining modeling teaching practices and competencies needed for mathematical modeling and outlining research goals and objectives to monitor the enactment of these practices. We will detail classroom routines, such as the "organize - monitor - regroup" cycle (Carlson et al., 2017), and the Core Practices for Mathematical Modeling (Suh & Matson, in press) as we share designed activities and lesson vignettes to solicit more ideas around high leverage MM teaching practices. We will explore what mathematical knowledge is needed to "successfully" facilitate mathematical modeling tasks in elementary grades. As we synthesize the current research on early modeling, we will define the nature of mathematics modeling and detail the development of teachers' content knowledge, teaching practices, and students' modeling competencies.

WG-Research Group 3: **Professional Development Focused: Detailing PD Research and Development of Tools and Modules for PD & Finding Convergence between PD on Mathematical Modeling and Interdisciplinary Areas**

Research Group 3 will focus on detailing components of effective mathematical modeling professional development for educators, examining relevant research methodology and instruments for studying the nature of MM in the early grades, and outlining several 21st century skill frameworks and teaching approaches for mathematics educators, researchers, and practitioners. We will share PD modules designed for elementary teachers that engage learners to use mathematical modeling through problem-based tasks, STEM, and teaching social justice through MM. Connecting interdisciplinary topics across subjects afford modeling opportunities that will help educators value the complementary connections between subjects and common classroom practices that support MM. We will engage in dialogue and collaboration among individuals and groups conducting research on student- and teacher-related outcomes related to implementing mathematical modeling, ways mathematical modeling promotes 21st century skills, and ways in which early modeling can develop interdisciplinary learning.

Anticipated Follow-up Activities and Goals of Working Group

Each session will engage participants to share their research interests related to mathematical modeling and form groups that might pursue research collaboratively based on the interests of the participants. Some of the questions that we will engage in include:

- What defines successful mathematical modeling at different grade levels?
- How does mathematical modeling support each and every learner?
- How does mathematical modeling connect to issues of social justice, STEM, andcivic responsibility of citizens?
- What can we learn from teachers who implement MM regularly in their classrooms?
- How is mathematical modeling ambitious teaching and how can we support teachers enacting MM through lesson plans and other resources?
- How can we map out the learning pathways of MM across grade levels?
- How and what can we learn about models elicited from student artifacts from MM tasks?
- What do "successful" modeling practices look like in our elementary mathematics classrooms? How are they similar or different from practices in secondary classrooms?
- What does it mean to "see the math" in the components of mathematical modeling?
- How do teachers select and/or develop modeling problems? How can Professional Learning Communities or Teacher Study Groups help teachers anticipate how students will answer the MM questions?

Our goal is for the working group leaders to propose an edited handbook or a special issues journal venue for mathematical modeling where participants interested in submitting manuscripts can work together to provide a comprehensive research trajectory documenting the progression of mathematical modeling from emergent levels to more sophisticated levels of modeling.

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