

EMBODIED MATHEMATICAL IMAGINATION AND COGNITION (EMIC) WORKING GROUP

<u>Erin R. Ottmar</u> Worcester Polytechnic Institute erottmar@wpi.edu	<u>Candace Walkington</u> Southern Methodist University email	<u>Dor Abrahamson</u> Univ. of California, Berkeley dor@berkeley.edu
<u>Mitchell J. Nathan</u> Univ. of Wisconsin-Madison mnathan@wisc.edu	<u>Avery Harrison</u> Worcester Polytechnic Institute aeharrison@wpi.edu	<u>Carmen Smith</u> University of Vermont carmen.smith@uvm.edu

Embodied cognition is growing in theoretical importance and as driving a set of design principles for curriculum activities and technology innovations for mathematics education. The central aim of the EMIC (Embodied Mathematical Imagination and Cognition) Working Group is to connect with inspired colleagues in this growing community of discourse around theoretical, technological, and methodological developments to advance the study of embodied cognition for mathematics education. Our thriving, informed, and interconnected community of scholars organized around embodied mathematical cognition will continue to broaden the range of activities, practices, and emerging technologies that contribute to mathematics teaching and learning as well as to research on these phenomena. This year's proposed EMIC working group builds upon our prior working groups with a specific focus on collaboratively creating embodied activities for mathematics learning that utilize different types of physicality, from full-body to gestural movements. In particular, we aim to develop and evaluate novel activities that apply principles of embodied cognition to foster mathematics learning through engaging in the enactment of carefully crafted movement. Our ongoing goal is to connect researchers and educators as we all create activities which can be implemented in mathematics classrooms.

Keywords: Learning theory, Cognition, Technology, Instructional activities and practices

Motivations for This Working Group

Empirical, theoretical, and methodological developments in embodied cognition and gesture studies provide a solid and generative foundation for the continuation of the established, **regularly held Embodied Mathematical Imagination and Cognition (EMIC) Working Group** for PME-NA. The central aim of EMIC is to attract engaged and inspired colleagues into a growing community of discourse around theoretical, technological, and methodological developments for advancing the study of embodied cognition for mathematics education, including, but not limited to, studies of mathematical reasoning, instruction, the design and use of technological innovations, and learning in and outside of formal educational settings.

The interplay of multiple perspectives and intellectual trajectories is vital for the study of embodied mathematical cognition to flourish. While there is significant convergence of theoretical, technological, and methodological developments in embodied cognition, there is also a trove of questions that must be addressed through formulating and implementing experimental design principles. As a group, we aim to (1) synthesize the work of leading scholars into a coherent theory of EMIC, (2) identify the most promising ideas for opportunities for methodological and technological integration, (3) curate and disseminate a set of evidence-based

design principles for enhancing mathematics education and broadening participation in STEM fields, and (4) articulate a future research agenda in the growing area of embodied design.

We aim to address basic theoretical questions such as, *What is grounding?* And practical ones such as, *How can we reliably engineer the grounding of specific mathematical ideas?* We want to understand how variations in actions and perceptions influence mathematical reasoning, including self-initiated vs. prescribed actions, and actions that take place in intrapersonal versus interpersonal interactions; how gestural point-of-view when enacting phenomena from a first-versus-third-person perspective, including how gestures move through space, influences reasoning and communication; how actions enacted by oneself, observed in others, or imagined influence cognition; how gestures connect with external visual representations, and how gestures are used to forge collaborative thinking (Abrahamson, 2018; Abrahamson & Bakker, 2016; Alibali & Nathan, 2012; in press; Walkington et al., in press).

From an applied level, we are also witnessing the emergence of a new genre of educational technologies and interventions for promoting STEM, rooted in theories of embodied cognition. These new uses of technology, in turn, offer novel opportunities for students and scientists to engage in math visualization, symbolization, intuition, and reasoning. In order for these designs to successfully scale up, they must be informed by research that demonstrates both ecological and internal validity. As technology becomes more affordable, more integrated in mathematics education spaces, and more common in classrooms, we need the proposed intellectually rigorous synthesis of theory and design principles to help shape approaches and activities that help make mathematics education accessible to a wider range of students.

Focal Issues in the Psychology of Mathematics Education

Emerging, yet influential, views of thinking and learning as embodied experiences have grown from several major intellectual developments in philosophy, psychology, anthropology, education, and the learning sciences that frame human communication as multimodal interaction, and human thinking as multi-modal simulation of sensory-motor activity (Clark, 2008; Hostetter & Alibali, 2008; Hutto, Kirchoff, & Abrahamson, 2015; Lave, 1988; Nathan, 2014; Newen, Bruin, & Gallagher, 2018; Varela et al., 1992; Wilson, 2002). As Stevens (2012, p. 346) argues in his introduction to the *JLS* special issue on embodiment of mathematical reasoning, “it will be hard to consign the body to the sidelines of mathematical cognition ever again if our goal is to make sense of how people make sense and take action with mathematical ideas, tools, and forms.”

Four major ideas exemplify the plurality of ways that embodied cognition perspectives are relevant for the study of mathematical understanding: (1) *Grounding of abstraction in perceptuo-motor activity as one alternative to representing concepts as purely amodal, abstract, arbitrary, and self-referential symbol systems.* This conception shifts the locus of “thinking” from a central processor to a distributed web of perceptuo-motor activity situated within a physical and social setting. (2) *Cognition emerges from perceptually guided action* (Varela, Thompson, & Rosch, 1991). This tenet implies that things, including mathematical symbols and representations, are understood by the actions and practices we can perform with them, and by mentally simulating and imagining the actions and practices that underlie or constitute them. (3) *Mathematics learning is always affective:* There are no purely procedural or “neutral” forms of reasoning detached from the circulation of bodily-based feelings and interpretations surrounding our encounters with them. (4) *Mathematical ideas are conveyed using rich, multimodal forms of communication, including gestures and tangible objects in the world.*

In addition to theoretical and empirical advances, new technical advances in multi-modal and spatial analysis have allowed scholars to collect new sources of evidence and subject them to powerful analytic procedures, from which they may propose new theories of embodied mathematical cognition and learning. Growth of interest in multi-modal aspects of communication have been enabled by high quality video recording of human activity (e.g., Alibali et al., 2014; Levine & Scollon, 2004), motion capture technology (Hall, Ma, & Nemirovsky, 2014; Sinclair, 2014), eye-tracking instruments (e.g., Abrahamson, Shayan, Bakker, & van der Schaaf, 2016), developments in brain imaging (e.g., Barsalou, 2008; Gallese & Lakoff, 2005), multimodal learning analytics (Worsley & Blikstein, 2014), and data logs generated from embodied math learning technologies that interacts with touch and mouse-based interfaces (Manzo, Ottmar, & Landy, 2016).

Past Meetings and Achievements of the EMIC Working Group

The first PME-NA meeting of the EMIC working group, “Mathematics Learning and Embodied Cognition,” took place in East Lansing, MI in 2015. Our group has been growing ever since. In addition to the PME-NA meeting each year, there are a number of ongoing activities that our members engage in. We have built an active website which connects members, provides updates on projects, and hosts resources. We have also created a space for members to share information about their research activities – particularly for videos of the complex gesture and action-based interactions that are difficult to express in text format. In addition, we have a common publications repository to share files or links (including to ResearchGate or Academia.edu publication profiles, so members don’t have to upload their files in multiple places). Our members collaborate on ongoing projects and have presented at other conferences and workshop events annually. Several research programs have formed to investigate the embodied nature of mathematics (e.g., Abrahamson 2014; Alibali & Nathan, 2012; Arzarello et al., 2009; De Freitas & Sinclair, 2014; Edwards, Ferrara, & Moore-Russo, 2014; Lakoff & Núñez, 2000; Melcer & Isbister, 2016; Ottmar & Landy, 2016; Radford 2009; Nathan, Walkington, Boncoddio, Pier, Williams, & Alibali, 2014; Soto-Johnson & Troup, 2014; Soto-Johnson, Hancock, & Oehrtman, 2016; Walkington et al., in press), demonstrating a “critical mass” of projects, findings, senior and junior investigators, and conceptual frameworks to support an ongoing community of like-minded scholars within the mathematics education research community.

In order to sustain collaboration and connect emerging and established scholars, eight of our members will also host an NSF funded Synthesis and Design workshop at the University of Wisconsin–Madison (UW–Madison) in May 2019. This workshop will bring together leading scholars in mathematical reasoning, teaching, and learning who work on embodied design with the goal to form a ten-year research agenda that will provide a coherent set of evidence-based design principles for enhancing mathematics education and broadening participation in all STEM fields. The organizers will seek to attract an interdisciplinary set of 30 scholars from education research, cognitive science, the learning sciences, developmental psychology, movement science, computer science, and mathematics, as well as 6 teachers. The research presented will span K–16 topics in content areas such as arithmetic and algebra, proportional reasoning and fractions, geometry, complex numbers and functions, statistics, and calculus. It will focus on design of systems for classroom learning settings, with attention to equity and access for underrepresented groups, while examining evidence of learning both in *and* outside of school. The reach of such an endeavor can extend to studies of mathematical intuition and reasoning, learning in and

outside of formal educational settings, professional development, classroom instruction and assessment, and STEM integration. The hope is that the 2019 PME workshop can serve as a time to follow up, disseminate, and extend what was learned from this workshop.

Current Working Group Organizers

As the Working Group has matured and expanded, we have a broadening set of organizers that represent a range of institutions and theoretical perspectives (and is beyond the limit of six authors in the submission system). This, we believe, enriches the Working Group experience and the long-term viability of the scholarly community. The current organizers for 2019 are (alphabetical by first name):

- Candace Walkington, Southern Methodist University
- Carmen J. Petrick Smith, University of Vermont
- Caro Williams-Pierce, University at Albany, SUNY
- David Landy, Indiana University
- Dor Abrahamson, University of California, Berkeley
- Erin Ottmar, Worcester Polytechnic Institute
- Hortensia Soto-Johnson, University of Northern Colorado
- Ivon Arroyo, Worcester Polytechnic Institute
- Martha W. Alibali, University of Wisconsin-Madison
- Mitchell J. Nathan, University of Wisconsin-Madison

Some of **our collaborative accomplishments** since last year's PME-NA working group include:

1. A successful proposal to host a three-day NSF Synthesis and Design workshop for researchers and teachers on "The Future of Embodied Design for Mathematical Imagination and Cognition" with 8 members serving as organizers <https://www.embodiedmathematics.com/>
2. Invitation by Springer to write a book on our collective work on Embodied Cognition in Mathematics for the "Research in Mathematics Education" Series
3. Submission of an additional NSF Workshop Proposal to host a 3 day Workshop in 2020 on Embodied Cognition for K-16 math educators)
4. Several members creating and teaching Embodied Cognition and Gesture seminars at their institutions.
5. Several grants awarded by IES CASL program to study embodied cognition, including the role of action in pre-college proof performance in geometry (Funded 2016-2020 for Nathan & Walkington) as well as the use of perceptual learning technology to study algebra learning (Ottmar & Landy, 2018)
6. Expanding a group website using the Google Sites platform to connect scholars, support ongoing interactions throughout the year, and regularly adding additional resources/activities <https://sites.google.com/site/emicpmena/home>
7. Extending the embodied-design agenda into special education in dialogue with Universal Design for Learning (Abrahamson, Flood, Miele, & Siu, in press)
8. Some senior members joining junior members' grant proposals as Co-PIs and advisors

EMIC 2019: Creating Embodied Instructional Activities for Mathematics

Last year, the EMIC working group focused on extending theoretical frameworks of embodied cognition (Melcer & Isbister, 2016) as well as exploring the role of technology in assessing and assisting mathematics learning. This year, we will continue this discourse as we effectively transition from theory-driven discussion and research to application in mathematics education. Specifically, we will continue to focus on theoretical frameworks which tie various perspectives on embodiment to different forms of physicality in educational technology (Melcer & Isbister, 2016; see Figure 1 below) as foundations to collaboratively design novel activities for mathematics education which utilize action, objects, and the surrounding environment in distinct ways. We will follow up on several of the discussions from the workshop held at UW-M, plan the proposed 2020 workshop, and explore the ten-year research agenda that aims to provide a coherent set of evidence-based design principles for enhancing mathematics education and broadening participation in all STEM fields.

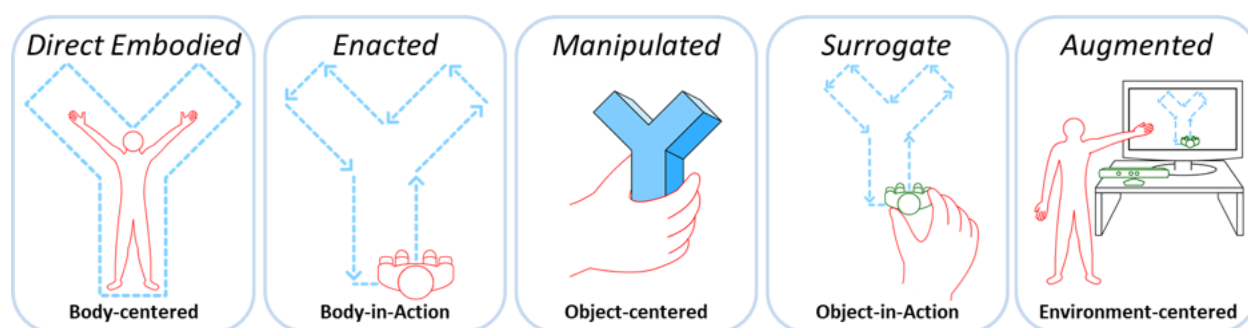


Figure 1: Five Distinct Approaches to Facilitating Embodiment through Bodily Action, Objects, and the Surrounding Environment in Educational Technology

In previous years, participants experienced theory-driven embodied instructional activities for mathematics learning. Two years ago, the EMIC working group focused on embodied instructional activities in geometry and most recently, the group focused on instructional activities which emphasize gestures through novel technologies. Examples include exploring mathematical transformations while using a dynamic technology tool (Ottmar & Landy, 2016), playing and creating embodied technology games to teach mathematics and computational thinking (Arroyo et al., 2017; Melcer & Isbister, 2018; Nathan & Walkington, 2017); using dual eye tracking (Shvarts & Abrahamson, 2018), and a teacher guiding the movements of a learner exploring ratios (Abrahamson & Sánchez-García, 2016). From experiencing these embodied activities, we explored questions such as: what role does technology play on supporting connections between the brain, body, and action? Even as we continue to bear these formative questions in mind, this year, participants in our EMIC workshop will shift from experiencing such activities and reflecting on the roles of physicality, technology, and collaboration to applying their perspective of embodied cognition to the creation of future novel activities for research and/or learning contexts.



Figure 2: A Small Selection of Embodied Activities Created by EMIC Organizers and Experienced by EMIC Participants

Plan for Active Engagement of Participants

In the past years of PME-NA working groups, we successfully engaged participants in open-ended math activities at the beginning of each session that steered our discussions towards elements of mathematics that the group found most provocative. This year, we intend to engage participants by facilitating one central open-ended activity that places participants in the driver seat for their working group experience—collaborating in small groups to design novel embodied instructional activities for mathematics (Figure 3).

On **Day 1**, we will focus on introductions and goals for the three sessions. After introductions to one another as well as an overview of the EMIC working group, we will discuss the goals for PME-NA 2019. The overarching goal will be to design and demonstrate a small collection of embodied activities for mathematics learning that utilize the enactment of goal-oriented movement in unique ways. This will give way to a discussion of the theoretical framework of physicality in embodied activities (Melcer & Isbister, 2016), which will drive the structures of our instructional activities. We will divide into small groups which will work together over the course of three days to create an instructional activity. The groups will mix educators, researchers, and students to create synergistic groups with different interests and knowledge of mathematics education. After a brainstorming session in small groups, the EMIC working group will close with a general discussion of ideas for instructional activities. On **Day 2**, we will primarily work in our small groups to continue designing embodied instructional activities. Towards the end of the session, groups will share their progress on activity design and participate in a guided discussion about the role of physicality, technology, and collaboration in the designed activities to reflect on how the structure of the activity contributes to connections between the mind, body, and action. **Day 3** will be focused on finalizing the created activities among small groups as well as planning for continued engagement and the dissemination of these activities.

EMIC Working Group Agenda 2019		
Day 1	Introductions	<ul style="list-style-type: none"> • Introductions & Goal Setting • Discussion of Theory • Instructional Activity Design
Day 2	Creating Activities	<ul style="list-style-type: none"> • Collaborative Design of Embodied Instructional Activities
Day 3	Demonstrations	<ul style="list-style-type: none"> • Sharing Embodied Activities • Plans for Dissemination & Continued Collaboration

Figure 3: Agenda Outline for the 2019 Embodied Mathematical Imagination and Cognition Working Group

Building on the diverse work in embodied cognition and among our group members, possible topics for these activities may include:

1. Grounding Abstractions
 1. Conceptual blending (Tunmer & Fauconnier, 1995) and metaphor (Lakoff & Núñez, 2000)
 2. Perceptuo-motor grounding of abstractions (Barsalou, 2008; Glenberg, 1997; Ottmar & Landy, 2016; Landy, Allen, & Zednik, 2014)
 3. Progressive formalization (Nathan, 2012; Romberg, 2001) and concreteness fading (Fyfe, McNeil, Son, & Goldstone, 2014)
 4. Use of manipulatives (Martin & Schwartz, 2005)
2. Cognition emerges from perceptually guided action: Designing interactive learning environments for EMIC
 1. Development of spatial reasoning (Uttal et al., 2009)
 2. Mathematical cognition through action (Abrahamson, 2014; Nathan et al., 2014)
 3. Perceptual boundedness (Bieda & Nathan, 2009)
 4. Perceptuomotor integration (Ottmar, Landy, Goldstone, & Weitnauer, 2015; Nemirovsky, Kelton, & Rhodehamel, 2013)
 5. Attentional anchors and the emergence of mathematical objects (Abrahamson & Bakker, 2016; Abrahamson & Sánchez-García, 2016; Abrahamson et al., 2016; Duijzer et al., 2017)
 6. Mathematical imagination (Nemirovsky, Kelton, & Rhodehamel, 2012)
 7. Students' integer arithmetic learning depends on their actions (Nurnberger-Haag, 2015).
3. Affective Mathematics
 1. Modal engagements (Hall & Nemirovsky, 2012; Nathan et al., 2013)
 2. Sensuous cognition (Radford, 2009)

4. Gesture and Multimodality
 1. Gesture & multimodal instruction (Alibali & Nathan 2012; Cook et al., 2008; Edwards, 2009)
 2. Bodily activity of professional mathematicians (Nemirovsky & Smith, 2013; Soto-Johnson, Hancock, & Oehrtman, 2016)
 3. Simulation of sensory-motor activity (Hostetter & Alibali, 2008; Nemirovsky & Ferrara, 2009)
5. Universal Design for Learning in Special Education (Abrahamson et al., in press)

Follow-up Activities

Unique to this year, the EMIC working group intends to focus on collaboratively creating instructional activities for mathematics education. By the end of the third session, we aim to have a collection of novel activities for mathematics classrooms that utilize action and movement in various ways. After the conclusion of the working group, participants will be invited to assist the organizers with the dissemination of these activities on the EMIC website, in journal articles, and other math education forums.

Beyond dissemination of our instructional activities, we envision an emergent process for the specific follow-up activities based on participant input and our multi-day discussions. As in previous years, we will continue to develop a list of interested participants and grant them all access to our common discussion forum and literature compilation. Additionally, the EMIC organizers will continue to plan workshops that continue this line of work. All EMIC participants will be invited to attend these events.

In the past several years, we have seen a great deal of progress. This is perhaps best exemplified by the development and continuation of the EMIC community, the NSF workshops, the website, the ongoing collaborations between members, and the annual PME-NA workshops that draw participants from across the country. We will strive to explore ways to continue to reach farther outside of our young group to continually make our work relevant, while also seeking to bolster and refine the theoretical underpinnings of an embodied view of mathematical thinking and teaching.

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