AN INTERNATIONAL STUDY OF PROSPECTIVE SECONDARY TEACHERS' NOTICING OF STUDENT THINKING

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Professional noticing of student thinking is considered a useful construct to investigate instructional decisions and activity made by teachers to support learning. This paper highlights the findings from one component of a multi-university (United States & Australia) research project focused on improving pre-service secondary mathematics teachers' (PSMTs) abilities to notice student thinking. The pre-post video component was specifically designed to document PSMTs' abilities to attend, interpret, and respond to student thinking. Results indicate that despite an improvement in ability to attend to and interpret student thinking, there was relatively no improvement in PSMTs' responding scores. This finding, which contradicts Monson et al.'s (2019) study, raises important questions about context and potential intervening factors to which researchers should attend when designing and implementing noticing interventions.

Keywords: Preservice teacher education, Instructional activities and practices, Teacher knowledge.

Introduction

In light of decades of research on how students learn mathematics (e.g., NRC, 2001), a more active vision of mathematics teaching, one that foregrounds student understanding and fosters student participation in the learning process, has emerged (NCTM, 2014). Teaching that uses student thinking as the basis for instructional decision-making, also known as responsive teaching, places significant demands on the teacher. Specifically, teachers need to attend to the substance of student ideas, recognize the disciplinary import of these ideas, and make intentional decisions about how to pursue those ideas (Robertson, Adkins, Levin, & Richards, 2016).

Skills such as interpreting and assessing the mathematical potential in student contributions, necessary prerequisites to responsive teaching, are especially difficult for prospective secondary mathematics teachers (PSMTs). PSMTs are understandably at a very different place than practicing teachers along the continuum of learning to teach (Feiman-Nemser, 2012) and often have had very limited opportunities to analyze student work in their teacher preparation programs (Jenset, Klette, & Hammerness, 2018; Simpson & Haltiwanger, 2017). However, research has demonstrated that PSMTs' abilities to attend to, interpret, and respond to student thinking can be developed through intentional, structured opportunities to engage in this work.

To that end, we sought to replicate a successful intervention, an Interview Module designed for secondary methods courses in the United States (Monson, Krupa, Lesseig & Casey, 2019), in an Australian context. The Australian context was deemed appropriate for two reasons: first, the topic of quadratics (the content central to the Interview Module) is taught to similar-aged students in both Australia and the United States; and second, the teacher education programs at the researchers' universities have similar structures and goals. Before detailing our research study, we briefly review the theoretical underpinnings of professional noticing of student thinking—the competency this intervention is designed to assess and develop—and discuss related empirical research. In this review, we highlight the goals and successes of intervention studies as well as the challenges revealed in research with PSMTs.

Theoretical Perspectives and Related Research

Noticing has proven to be a useful construct to investigate instructional moves and decisions teachers make in relation to student thinking (Schack, Fischer, & Wilhelm, 2017; Sherin, Jacobs, & Philipp, 2011). Teacher noticing builds on Goodwin's (1994) definition of professional vision as "socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (p. 606). Subsequently, teachers' professional vision, or noticing, refers to the ways in which teachers are attuned to 'notice' significant events in a teaching situation and engage in principled reasoning when interpreting and reacting to those events (Sherin et al., 2011). With roots in both cognitive and situative perspectives, the construct of teacher noticing has potential to bridge the research divide between teacher dispositions and teaching performance (Scheiner, 2016). In particular, noticing research, focused on situation-specific skills of perception, interpretation, and decision-making, helps reveal complex interactions among teacher knowledge and actions (Sánchez-Matamoros, Fernández, & Llinares, 2018; Thomas, Jong, Fisher, & Schack, 2017).

Our work is situated in a particular aspect of noticing referred to as *professional noticing of student thinking* (Jacobs, Lamb, & Philipp, 2010). Professional noticing of student thinking [hereafter professional noticing] consists of three interrelated practices of (1) *attending* to the mathematics evidenced in student work (writing, verbal communication, or actions); (2) *interpreting* what students understand and/or do not understand based on their mathematical work; and (3) *deciding* how to respond in light of this interpretation.

While the characteristics of expert noticing are still evolving, there is some agreement on the nature of noticing necessary for responsive teaching. In particular, researchers have highlighted the importance of attending to salient mathematical details in a student's approach as well as the ability to connect those details to key mathematical understandings (Jacobs et al., 2010; Sánchez-Matamoros et al., 2018). On the contrary, more novice noticing is characterized by general statements, descriptions, or evaluations that are not tied to specific evidence from students (Jacobs et al., 2010; Mason, 2011). Research has demonstrated that professional noticing can be enhanced through collaborative, structured opportunities to interact with students' mathematical thinking through video (Ding & Domínquez, 2015; Schack et al., 2013), one-on-one student interviews (Monson et al., 2019; Fernandez, 2012) or samples of students' written work (Simpson & Haltiwanger, 2017; Son, 2013). While the majority of intervention studies report positive gains in the noticing abilities of prospective teachers, the results also reveal inconsistent improvements and highlight the complex interaction of noticing with other factors (e.g., teacher knowledge, the mathematical content of the task).

For example, prospective teachers have difficulty distinguishing between procedural and conceptual errors (Ding & Domínquez, 2015) or recognizing evidence of conceptual understanding (Bartell et al., 2013). Studies by Son (2013) and Son and Sinclair (2010) document difficulties prospective teachers had responding to students when conceptual errors were present, with most offering procedural explanations and other teacher-centered responses that did not aim to advance conceptual understanding. Content knowledge was insufficient. This was also evident in Son and Crespo's (2009) study in which PSMTs who were able to unpack the mathematics behind a student's non-traditional approach (i.e. those with strong content knowledge) were actually more likely to offer teacher-centered responses than their peers. In sum, studies consistently demonstrate that responding is the most difficult component of noticing to develop. The one exception is Monson and colleagues' (2019) study in which activities explicitly targeting PSMTs' ability to respond were added to an existing Interview Module.

Purpose of the Research

In this paper we present findings from a multi-university research project focused on improving PSMTs' abilities to notice student thinking within the context of secondary mathematics teaching and learning. The pre-post video component, on which this paper reports, was designed to specifically document PSMTs' abilities to attend, interpret and responding to student thinking. The overarching research question guiding the study was: *What, if any, improvements in attending to, interpreting, and responding to student thinking do PSMTs' demonstrate after completing the Interview Module?*

Research Design

The research is based on an Interview Module, which was developed by one of the authors and colleagues and has been used within the United States to demonstrate gains in PSMTs' noticing abilities. The Interview Module is comprised of a pre-post video assessment, prescribed readings, a one-on-one interview with a secondary student, and a sequence of responding assignments involving analysis of student work samples (Monson et al., 2019).

Participants and Context

Data for this paper come from the pre-post video assessment given to PSMTs enrolled in secondary mathematics teacher preparation programs in two countries. Nine PSMTs completed the assessment, with 6 PSMTs enrolled at one Australian university and 3 PSMTs at one university in the United States. The Australian participants were in their first or second year of their preparation program. Two of the US participants were in their second year of an undergraduate preparation program, though one was returning to university after working in a math-related field. The third had earned a BS in mathematics from a different university and was in her first semester of a 15-month Masters in Teaching program.

Pre-post video assessment. The researchers collected pre-post assessment data, which were based on PSMTs' written responses to short videos shown prior to and upon completion of the full Interview Module. These videos show a Mathematics Teacher Educator (MTE) conducting a task-based interview with a secondary student. In each video, the student is asked to solve two quadratic equations (one resulting in one real solution, the other with two imaginary solutions) which mirrors the student interview PSMTs conducted that focused on solving linear equations. The problems and prompts given to the student are provided in Table 1.

Table 1: Pre-post Video Assessment Problems and Prompts

Question 1:	Solve for x: $x^2-4x+4=0$
Probe:	Could you solve that another way?
Question 2:	Solve for x: $x^2-2x+3=0$
Probe:	Could you solve that another way?

After watching the pre- and post-videos, the PSMTs were asked to independently respond in writing to three prompts, each corresponding to one component in the noticing framework: (1) What do you notice? (Attending) (2) How would you describe what this student understands? (Interpreting) and (3) Describe some ways you might respond to this student and explain why you chose those responses (Responding).

Analysis and Coding

Both researchers used a coding scheme adapted from the module developers (Krupa, Huey, Lesseig, Casey & Monson, 2017), and independently coded PSMTs' ability to attend to, interpret, and respond to student thinking on the pre- and post-assessments. Each question was coded as demonstrating either no evidence (0), limited ability (1), or emerging ability (2). Initially, each researcher coded responses (pre- & post-) from three participants before meeting virtually to discuss any discrepancies in coding. After resolving all coding differences and reaching consensus, the researchers coded the responses from the remaining six participants before meeting virtually again to agree on a consensus score for each response. Reliability in coding was enhanced by the creation and maintenance of an operative codebook with examples and non-examples of responses at each level (Miles, Huberman, & Saldaña, 2013), and any discrepancy was discussed in reference to the codebook until consensus was reached.

Findings

The pre-post video assessment data provided the researchers with an opportunity to look for growth in PSMTs' attending, interpreting and responding skills. While in the larger study these data were compared with other components of the Interview Module, this paper outlines changes in PSMTs' skills as evidenced in the video assessment only. The scores from pre-post video assessment data are presented in Table 2 (all participant names are pseudonyms). The scores represent the following: 0 = No evidence, 1 = Limited Ability, 2 = Emerging Ability. Tables 3, 4 and 5 outline verbatim participant responses according to *attending*, *interpreting* and *responding* skills, respectively, together with the video phase (pre-/post-) and the consensus score (0, 1, 2) given to the response by the researchers. While not necessarily meant as exemplars, these responses have been included to indicate a qualitative range and illustrate our coding levels.

Table 2: Pre-post Video Scores for Attending, Interpreting and Responding

	Attending		Interpreting		Responding	
	Pre	Post	Pre	Post	Pre	Post
Andi (US)	2	2	2	2	2	2
Brook (US)	1	2	1	1	2	2
Rachel (US)	1	2	1	2	2	2
Benedict (AUS)	0	2	0	1	0	1
Georgia (AUS)	1	1	1	1	0	0
Jai (AUS)	1	2	1	2	1	1
Emmeline (AUS)	1	2	2	2	2	2
Steven (AUS)	0	1	0	0	1	0*
Owen (AUS)	1	2	1	2	1	2

Key: * = score decreased from pre- to post; score with gray shading (e.g. 2) = increased; no shading = no change.

Changes in PSMTs' Noticing in Pre-Post Video Assessment

Attending. With the exception of two participants (Andi & Georgia, who remained at the *limited ability* level for pre- and post-), all participants improved their pre-video attending score by at least 1 point in the post-video assessment. Participant Benedict demonstrated the greatest growth in the attending component, scoring at the *no evidence* level in the pre-video assessment and *emerging ability* in the post-video assessment. Benedict attended to the student's affective dispositions in the pre-video assessment (see Table 3), yet documented in detail a number of mathematical terms used by the student in the post-video assessment (e.g., "...knew of two ways to solve a quadratic, factor and formula. Showed/said her understanding of graphing").

Table 3: Examples of Participants' Attending Responses

Participant	Response
Benedict (Pre-video, 0)	Girl always waited slightly to get confirmation about her process when she was uncertain; tried to logically solve it and talked through the process.
Georgia (Post-video, 1)	The student was checking that her factorization was correct. [She] simplified the answer as much as possible without a calculator. The student could think about other ways to answer the question.
Brook (Post-video, 2)	The student correctly factored and solved the first problem. She was also able to sketch a general graph of the equation. Moreover, she realized graphing was an alternative method of solving the equation when asked for another method. The second equation she factored, but in checking herself realized her factoring was not correct. She knew she needed to use the quadratic formula to solve the equation. However, she could not graph the equation. Additionally, the interviewer prompted her to correct a negative sign.

Interpreting. Five of nine participants demonstrated growth in the interpreting component, with the remaining four participants maintaining the same score from pre-video to post-video assessments. Only two participants scored at the *emerging ability* level during the pre-video stage, with this number increasing to five after the post-video assessment. In the pre-video assessment, Rachel only wrote a list of things the student did incorrectly, (e.g. "...the student cancelled a 2 wrong from Problem 3 and she didn't know how to simplify or solve for a negative square root...", scoring a 1 for this response. Rachel's post-video interpreting response was scored as a 2 due to the list of correct and incorrect things the student did, coupled with various statements about what the student understands supported with evidence, (e.g. "...she knows the graphs for parabolas with real solutions, but she needs some help with understanding imaginary solutions and their graphs").

Table 4: Examples of Participants' Interpreting Responses

Participant	Response
Steven (Pre-video, 0)	Jumps straight into the first question. Understood how to solve the 1st question but got the graph wrong when asked what it was (sic) looked like.
Brook (Post-video, 1)	She has a very good grasp of the material. When graphing the first problem, she states the graph would contain two zeros. However, when she sketches the graph, she draws it correctly with only one zero. She solved the 2nd problem correctly using the quadratic equation. She seemed to understand how to graph it but was not immediately able to estimate $2\pm\sqrt{2}$ to pick values for the x-axis.
Emmeline (Pre-video, 2)	Understands factors of factorisation. Perfect square (notion of x^2 and $\sqrt{x}$). Can solve for x, understands inverse relationship of operations. Has an understanding that different method(s) can be used. Knows the quadratic formula. Understands $\sqrt{-x}$, that it isn't possible, therefore that 'no solutions' are possible.

Responding. Eight of nine participants either improved or maintained their pre-video responding score in the post-video assessment (Steven's score decreased by 1 point). Four participants scored at the *emerging ability* level during the pre-video stage, with this number increasing to five on the post-video assessment. To illustrate, Owen scored a 1 in the pre-video assessment after providing a limited rationale of how he intended to respond to the student, (e.g. "Would encourage her to understand a visual representation of the graphs. Might suggest that she considers what's the best approach at the beginning, in case there are short-cuts"). The post-video response from Owen was:

> Can you explain to me your working in solving/using the QF? (help her find the mistake). Is your QF correct? (Direct her to mistake if she doesn't find herself). What does the root of a negative number give you? (Must give understanding as to why it's special).

This response was scored as a 2 by both researchers, as they felt Owen anchored his response and rationale, which he included in parenthesis, to his observations of the student's error in working with the quadratic formula and the result of obtaining a negative discriminant.

Table 4: Examples of Participants' Responding Responses

Participant	Response
Georgia (Pre-video, 0)	Directive questioning -- open questioning to allow her to explain her thought process (e.g., rather than to assume her thought process)?
Jai (Post-video, 1)	I would ask her to check her work to avoid follow-through errors...ask her why she thinks eq. 1 has 2 roots. Might explain why she draws a y-int @ 2.
Andi (Pre-video, 2)	Show her some problems regarding reducing fractions with addition on the top to remind her of why reducing too early gives incorrect answers. To help her graph and aid her in understanding how the first graph looks, I would have her plot points from 0 to 4 to help her visualize the graph. I would also have her plot out points for the graphs x^2=y and $\sqrt{x}$=y to help her understand where she got the idea for her first graph from.

Discussion

As demonstrated in Table 1, the largest improvements in professional noticing were evidenced in PSMTs' ability to attend to student thinking, with seven of nine PSMTs scoring at the *emerging level* on the post-video assessment. Further analysis revealed that while PSMTs mainly attended to mathematical aspects in both the pre- and post-video assessments, they did so with even greater focus after the intervention. Five of nine PSMTs made comments about the interviewer and/or student disposition in the pre-assessment. In particular, PSMTs noted that the 'teacher' did not correct or tell the student if she was correct; but instead prompted her with questions. In the post-video assessment there were no comments about interviewer actions and only one comment that the student in this second video was "quite confident". These promising findings from Australia add to the US literature confirming that noticing skills can be developed through targeted interventions.

Despite improvement in both attending and interpreting, there was relatively no improvement in PSMTs' responding scores. This result is understandable in the US case, given that all three participants received scores of 2 on the pre-video assessment. However, the lack of improvement in scores for the Australian participants is in stark contrast to Monson et al.'s (2019) study in which participants showed the greatest gains in responding. In this regard, the findings from the Australian cohort more closely align with results from an earlier implementation of the Interview Module (Krupa et al., 2017), which prompted the addition of the responding assignments. In that initial study, with 36 PSMTs across three US universities, 38% of participants showed gains in attending, 25% gained in interpreting, but there was no notable change in responding after the intervention. While we are naturally hesitant to draw too many inferences based on the limited sample, we wonder about contextual factors that may have played a role. For example, in the US case, the three participants were the only students enrolled in the methods course, and as a result received more targeted instruction and feedback during the in-class responding assignment. In contrast, 39 PSMTs were enrolled in the course in Australia and thus may have relied more on their peer-group and received less 'expert' feedback when crafting responses to the student work presented. Another possible intervening factor, also potentially related to the disparate class sizes, is the fact that on both the pre- and post-video assessments, the US participants simply wrote more when addressing all three of the prompts. Finally, we wonder whether the in-class responding component of the module was sufficiently outlined in the Interview Module materials to support implementation by an MTE from outside the group. In other words, the different instructors may have enacted parts of the assignment very differently. Given that a key purpose of this study was to investigate the transferability of the Interview Module to a new context, this explanation is a cause for concern that warrants further investigation.

Finally, we note that although there were no significant changes in responding scores (due in part to the ceiling effect with the US participants), we did see qualitative differences in the ways PSMTs chose to respond. Most apparent was the shift toward more student-centered responses. On the pre-video assessment 4 PSMT responses were coded as teacher-centered (2 from the US participants), 3 were coded as student-centered, and 2 as mixed. In contrast there were no responses on the post-video assessment that were coded as strictly teacher-centered, with responses more focused on eliciting additional student thinking or building the student's understanding rather than providing further instruction. For example, there was a shift in language from "I would *show her*…" or "I would *go over* quadratic graphs and what they look like" to "*I would ask* the student…"

This finding is consistent with previous work in which "leaving room for student thinking" was the characteristic PSMTs were best able to meet when crafting responses on the take-home assignment associated with the responding components that were added to the Interview Module (Casey, Lesseig Monson & Krupa, 2018). We speculate that this aspect–allowing students to do their own thinking, rather than jumping in to 'tell'–is not only highly supported throughout the module (e.g., in the course readings as well as the in-class responding assignment), but also is less reliant on subject matter, or pedagogical content knowledge. Son and Crespo's (2009) study with elementary preservice teacher provides some support for this claim.

Limitations

In addition to the limitations alluded to above (e.g., small number of participants, differences in US and Australia class size) the study is subject to methodological limitations that plague noticing research more generally. Specifically, we acknowledge that there may well be differences between what is attended to explicitly versus implicitly (Scheiner, 2016). We accounted for this to some degree by encouraging students to take notes while watching the video, prior to addressing the first prompt. These notes were collected and included in our analysis and scoring. Second, video of a clinical interview (the format employed in the pre-post assessment) provides limited access to information pertinent to the teaching and learning episode (e.g., students' prior experiences or attitude toward mathematics, specific learning goals or associated curricular materials). Without this full context, PSMTs are left to "fill in the gaps", often relying on their own experiences with this topic as high school students. This not only impedes the situational awareness of the PSMTs, but also our abilities as researchers to infer meaning from PSMTs' responses (Nickerson et al., 2017). In future work, we intend to attend more carefully to PSMTs' background experiences and explore potential relationships among PSMTs' knowledge, skills, and dispositions; we encourage other researchers to do the same.

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