SIXTH-GRADE STUDENTS' RETENTION OF EARLY ALGEBRA UNDERSTANDINGS AFTER AN ELEMENTARY GRADES INTERVENTION

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This research focuses on the retention of students' algebraic understandings one year following a Grades 3–5 early algebra intervention. Participants included 1455 Grade 6 students who had participated in a cluster randomized trial. Approximately half of these students received an early algebra intervention as part of their regular instruction in Grades 3–5, while the other half received only their regular mathematics instruction. Results show that, as was the case at the end of Grades 3, 4 and 5, treatment students outperformed control students at the end of Grade 6 on a measure of algebraic understanding. This was despite the fact that treatment students experienced a significant decline in performance and control students a significant increase in performance after the intervention. An item-by-item analysis performed within condition revealed the areas in which students in the two groups experienced a change in performance.

Keywords: Algebra and Algebraic Thinking

Mathematics education scholars have advocated for some time that students be provided long-term experiences, beginning in the elementary grades, that can support the development of their algebraic thinking. While this view is widely accepted, research is needed to document how the algebraic understandings of students who are provided such long-term, sustained algebra experiences in the elementary grades compare to those of students who have not had these experiences once students are in middle grades. This paper focuses on a retention study in which the algebraic understandings of students who participated in a Grades 3–5 early algebra intervention were assessed at the end of Grade 6, a year after the completion of the intervention.

Theoretical Perspectives

Our work aims to develop and test a model of early algebra instruction in order to produce research-vetted curricular frameworks and instructional materials from which we can better understand early algebra's impact on students' algebraic thinking. This work has involved small scale cross-sectional (Blanton et al., 2015) and longitudinal (Blanton et al., 2019) studies and more recently a large-scale, longitudinal, cluster randomized trial (Blanton et al., in press).

In a process fully described in Blanton et al. (2018), we built a curricular framework, instructional intervention, and associated assessments around the algebraic thinking practices of generalizing, representing generalizations, justifying generalizations, and reasoning with generalizations (see also Blanton, Levi, Crites, & Dougherty, 2011) and the big algebraic ideas in which these practices can occur, namely, generalized arithmetic; equivalence, expressions, equations, and inequalities; and functional thinking.

Previous Research: The Longitudinal Study

In order to provide context, we briefly describe the longitudinal study of the Grades 3–5 intervention that preceded the retention study that is the focus of this paper. The study used a cluster randomized trial design and is fully described in Blanton et al. (in press). Participants included approximately 3000 students and took place in 46 elementary schools across three school districts, with half of the schools randomly assigned to a either a treatment or control condition. Students in treatment schools were taught our early algebra intervention in Grades 3–5, which consisted of 18 lessons per year, as part of their regular mathematics instruction by their classroom teachers. Teachers took part in professional development that addressed the big algebraic ideas and algebraic thinking practices that were the focus of the intervention.

Students completed written assessments prior to the start of the intervention in Grade 3 and at the end of Grades 3, 4 and 5. Assessments consisted of 12-14 items designed to measure understanding of the big algebraic ideas and algebraic thinking practices that formed the basis of the intervention. We found that while there were no differences between groups in performance or structural strategy use (Kieran, 2007, 2018) prior to the intervention, treatment students improved at a significantly faster rate than control students in Grade 3. While students in the two conditions made parallel gains across Grades 4 and 5, the treatment students' initial gains were maintained so that they remained significantly ahead of control students in their understanding of big algebraic ideas and thinking practices as they entered middle school (Blanton et al., in press).

Given the results of our Grades 3–5 study, we were interested in revisiting these same students one year later to assess the intervention's longer-term impact. Our research question was thus the following: *How does the performance of students who took part in a Grades 3–5 early algebra intervention compare to that of students who experienced only their regular Grades 3–5 mathematics curriculum one year after the conclusion of the intervention?*

Method

Participants

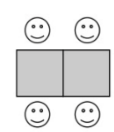
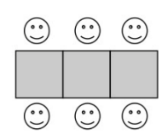
Sixth grade retention data was collected from 1455 students across 23 middle schools in the three school districts that took part in the Grades 3–5 longitudinal study. Of these students, 716 were in control schools and received only their regular instruction in Grades 3–5, while 739 were in treatment schools and were taught the intervention as part of their regular instruction.

Context

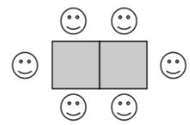
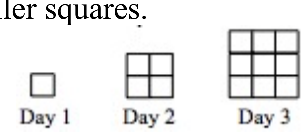
While participating students were consistently in a control or treatment elementary school throughout Grades 3–5, all students moved to new (middle) schools in Grade 6 and were intermixed. The students who participated in the retention study experienced their schools' regular mathematics curriculum during Grade 6 and no instructional intervention from our project. Grade 6 teachers cited use of a variety of mathematics curricula but in all cases were expected to align their instruction with the *Common Core Standards for School Mathematics* (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center & CCSSO], 2010).

Data Collection

Students completed a written assessment at the end of Grade 6 consisting of 11 open-ended items, most of which contained multiple parts. Nine of these 11 items also appeared on the Grade 5 assessment and will be the focus of the results we share in this paper (see Figure 1).

<table>
<tr><th></th><th>Big Idea(s)</th><th></th></tr>
<tr><td>1</td><td>Equivalence, expressions, equations</td><td>Fill in the blank with the value that makes the number sentence true.
7 + 3 = ____ + 4
Explain how you got your answer.</td></tr>
<tr><td>3</td><td>Generalized arithmetic</td><td>Marcy’s teacher asks her to solve “23 + 15.” She adds the two numbers and gets 38. The teacher then asks her to solve “15 + 23.” Marcy already knows the answer is 38 because the numbers are just “turned around.”
a) Do you think Marcy’s idea will work for any two numbers? Why or why not?
b) Write an equation using variables (letters) to represent the idea that you can add two numbers in any order and get the same result.</td></tr>
<tr><td>4</td><td>Generalized Arithmetic</td><td>Brian knows that if you add any three odd numbers, you will get an odd number. Explain why this is true.</td></tr>
<tr><td>5</td><td>Equivalence, expressions, equations</td><td>Tim and Angela each have a piggy bank. They know that their piggy banks each contain the same number of pennies, but they don’t know how many. Angela also has 8 pennies in her hand.
a) How would you represent the number of pennies Tim has?
b) How would you represent the total number of pennies Angela has?
c) Angela and Tim combine all of their pennies. How would you represent the number of pennies they have all together?
Suppose Angela and Tim now count their pennies and find they have 16 all together. Write an equation with a variable (letter) that represents the relationship between this total and the expression you wrote above.</td></tr>
<tr><td>9</td><td>Functional thinking</td><td>Brady is celebrating his birthday at school. He wants to make sure he has a seat for everyone. He has square desks.
He can seat 2 people at one desk in the following way: 
If he joins another desk to the first one, he can seat 4 people: 
If he joins another desk to the second one, he can seat 6 people: 
a) Fill in the table below to show how many people Brady can seat at different numbers of desks.
<table><tr><th>Number of desks</th><th>Number of people</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td></td></tr><tr><td>5</td><td></td></tr><tr><td>6</td><td></td></tr><tr><td>7</td><td></td></tr></table></td></tr>
</table>

Otten, S., Candela, A. G., de Araujo, Z., Haines, C., & Munter, C. (2019). *Proceedings of the forty-first annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. St Louis, MO: University of Missouri.

<table>
<tr>
<td></td>
<td></td>
<td>b) Do you see any patterns in the table from part a? If so, describe them.
c) Think about the relationship between the number of desks and the number of people.
Use words to write the rule that describes this relationship.
Use variables (letters) to write the rule that describes this relationship.
d) If Brady has 100 desks, how many people can he seat? Show how you got your answer.
e) Brady figured out he could seat more people if two people sat on the ends of the row of desks. For example, if Brady had 2 desks, he could seat 6 people.

How does this new information affect the rule you wrote in part c?
Use words to write your new rule.
Use variables (letters) to write your new rule.</td>
</tr>
<tr>
<td>10</td>
<td>Functional thinking; Equivalence, expressions, equations</td>
<td>The table below shows the relationship between two variables, k and p. The rule $p = 2 \times k + 1$ describes their relationship.
a) Some numbers in the table are missing. Use this rule to fill in the missing numbers.
<table>
<tr><th>k</th><th>p</th></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td>9</td></tr>
</table>
b) What is the value of p when $k = 21$? Show how you got your answer.
c) What is the value of k when $p = 61$? Show how you got your answer.</td>
</tr>
<tr>
<td>14</td>
<td>Functional thinking</td>
<td>The following magic square is growing so that each day it is made up of more and more smaller squares.


The following table shows a given day and the number of small squares on that day:
<table>
<tr><th>Day</th><th>Number of small squares</th></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>9</td></tr>
<tr><td>4</td><td>16</td></tr>
<tr><td>5</td><td>25</td></tr>
<tr><td>6</td><td>36</td></tr>
</table>
a) Think about the relationship between the number of days and the number of small squares.</td>
</tr>
</table>

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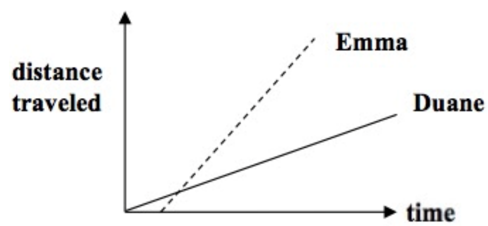
		Use words to write the rule that describes this relationship. Use variables (letters) to write the rule that describes this relationship. b) Use your rule to predict how many small squares will be inside the big square on day 100. Show how you got your answer.
21	Functional thinking	Duane and Emma each went for a bike ride. The graphs below represent the relationship between time spent riding and distance traveled for each rider. a) Who started riding first? How can you tell? b) Who rode faster? How can you tell? c) Is Emma riding the same speed on her whole trip, or is she speeding up or slowing down? How can you tell? 
23	Equivalence, expressions, equations	Do the following two equations have the same solution? Explain. $2 \times n + 15 = 31$ $2 \times n + 15 - 9 = 31 - 9$

Figure 1: Items Common to the Grade 5 and Grade 6 Assessments

Data Analysis

Coding schemes developed in previous work (see Blanton et al. [2015] for a full description of this process) were used to categorize student responses to assessment items both in terms of performance and in terms of strategy use. (In the results shared here, we focus only on performance.) Items were coded by trained coders unaware of students' treatment condition.

Changes in overall correctness over time were assessed using a repeated measures ANOVA, with treatment condition as a between-subjects factor. McNemar's test was then used to examine item-by-item changes in performance from Grade 5 to Grade 6 within each condition.

Results

The repeated measures ANOVA showed no significant impact of testing time across all students, but did reveal a significant interaction between testing time and treatment condition, $F(1, 1453) = 160.73$, $p < .001$. Overall, treatment students (M = 47.51% correct, SD = 21.54%) maintained their advantage over control students (M = 37.93% correct, SD = 19.74%) at the end of Grade 6, $F(1,1453) = 78.13$, $p < .001$. However, the gap between the two groups narrowed from Grade 5 to Grade 6, with control students experiencing an overall increase in performance from Grade 5 (M = 33.38%, SD = 19.43%) to Grade 6 (M = 37.93%, SD = 19.74%) and treatment students experiencing an overall decrease in performance from Grade 5 (M = 53.40%, SD = 24.17%) to Grade 6 (M = 47.51%, SD = 21.54%).

McNemar's test was used to compare performance (percent correct) across the two conditions on the nine individual assessment items (with a total of 24 individual item parts) common to Grades 5 and 6. The items are given in Table 1.

Table 1: Percent Correct on Assessment Items Across Conditions and Testing Times

Item	Control	Treatment

	Grade 5	Grade 6	Grade 5	Grade 6
1	66.76%	71.23%*	84.84%	82.68%
3a	54.05%	62.99%*	67.66%	70.09%
3b	21.65%	35.20%*	67.25%	59.40%**
4	10.34%	7.54%**	23.41%	11.91%**
5a	19.13%	36.73%*	59.95%	55.89%
5b	7.68%	10.89%*	29.50%	19.62%**
5c1	11.17%	9.36%	51.42%	16.51%**
5c2	8.52%	4.89%**	38.16%	8.93%**
9a	94.27%	96.79%*	94.86%	95.81%
9c1	21.79%	16.48%**	35.32%	23.55%**
9c2	27.23%	31.70%*	60.76%	48.17%**
9d	78.91%	79.47%	84.17%	82.68%
9e1	9.22%	6.98%	22.73%	13.80%**
9e2	16.48%	16.62%	42.49%	31.80%**
10a	56.01%	59.22%	73.61%	71.72%
10b	35.47%	48.32%*	58.46%	60.62%
10c	31.28%	38.55%*	45.33%	47.90%
14a1	8.38%	15.64%*	26.39%	24.90%
14a2	13.27%	20.25%*	49.53%	45.33%**
14b	35.89%	42.60%*	46.96%	50.34%
21a	61.59%	74.86%*	72.80%	79.70%*
21b	60.89%	70.11%*	72.67%	74.42%
21c	27.65%	34.22%*	45.60%	38.97%**
23	23.46%	19.69%	27.74%	25.58%
TOTAL	**33.38%**	**37.93%**	**53.40%**	**47.51%**

* Items on which students showed significant gain from Grade 5 to Grade 6 ($p < .05$)
** Items on which students showed significant decline from Grade 5 to Grade 6 ($p < .05$)

As shown in Table 1, control students made gains in performance on individual assessment items much more so than did experimental students. Specifically, of the 24 individual item parts, control students improved on 15 of those parts and declined on three of those parts from Grade 5 to Grade 6. Treatment students, on the other hand, improved on just one of those parts and declined on 11 of those parts from Grade 5 to Grade 6.

Of particular interest in the control condition is the identification of the areas of algebra in which students made their gains. That is, in which areas did their regular middle school instruction help to close the gap between their performance and that of their treatment counterparts? Of particular interest in the treatment condition is the identification of the areas of algebra in which students a) maintained or gained in their Grade 5 performance levels and b) declined in performance after a year without a focused early algebra education. Answers to these questions will shed light on the algebraic content and thinking practices that “stuck” with students one year after our three-year early algebra intervention—whether because the learning was supported by their “business as usual” middle school instruction or was maintained over the course of the year despite this instruction—and the areas in which this was not the case.

For control students, significant gains occurred across a range of big algebraic ideas and thinking practices. Students showed an increased understanding of equality and equations on some items (Items 1, 10b, 10c). They were increasingly able to identify one-step function rules in words (Items 9c1, 14a1) and in one of these cases make a far prediction (Item 14b). They also made gains identifying an arithmetic property (Items 3a) and representing unknowns along with arithmetic and one-step functional relationships with variables (Items 3b, 5a, 5b, 9c2, 14a2). They did not experience gains on items involving advanced understanding of equality (Item 23), two-part function rules (Items 9e1 and 9e2) or using algebraic expressions of more than one term to represent problem situations (Items 5c1, 5c2).

Treatment students maintained their Grade 5 performance levels on items addressing their understanding of the equal sign (Items 1 and 23) and equations (Items 10a, 10b, 10c). They also maintained their ability to identify an arithmetic property (Item 3a), make far predictions when working with functional relationships (Items 9d, 14b) and represent an unknown with a variable (Item 5a). Treatment students showed a decline in performance from Grade 5 to Grade 6 where they needed to identify one- or two-step functional relationships in words (Items 9c1, 9e1), provide a general argument to justify a statement about the sum of odd numbers (Item 4), and provide a variable representation of more than one term to represent unknown quantities (Items 5b, 5c1, 5c2), an arithmetic property (Item 3b), and functional relationships (Items 9c2, 9e2).

Discussion

Results from our longitudinal Grades 3–5 study (Blanton et al. [in press]) offered evidence that providing students with sustained early algebra experiences across a range of big algebraic ideas and algebraic thinking practices can in fact place them at an advantage in terms of algebraic understanding relative to students who experience a more arithmetic-focused approach to elementary mathematics as they enter the middle grades. The study reported in this paper assessed the algebraic understanding of these students one year after the conclusion of the Grades 3–5 intervention, at the end of Grade 6. The most important result we have to report is that one year post-intervention, treatment students still significantly outperformed control students on an assessment measuring their understanding of big algebraic ideas and thinking practices. That is, early algebra (i.e., the Grades 3–5 intervention) *did* make a difference in terms of helping students be better prepared for algebra in the middle grades.

Digging deeper into the item-by-item results revealed the areas in which treatment students were able to maintain their Grade 5 performance levels and the areas in which their performance dropped. Likewise, we were able to identify areas in which control students made significant gains, likely signaling areas of relative strength in their middle school mathematics instruction.

One important area in which treatment students maintained their performance was the concept of equality and the meaning of the equal sign. This was the core concept with which we started our Grade 3 lessons, and it was consistently revisited throughout the three years of the early algebra intervention. While control students made some gains in this area as well, their gains were not consistent across items and did not include growth on our most advanced item involving the recognition of equivalent equations. While the *Common Core* (NGA Center & CCSSO, 2010) does address students' understandings of the equal sign to some extent, our findings suggest a need to strengthen and maintain this focus over multiple years.

An important Big Idea in our Grades 3–5 intervention in which treatment students' learning was not robustly maintained from Grade 5 to 6 is functional thinking. While treatment students maintained their performance identifying a simple exponential function in words, they experienced a decline on all other items requesting the identification of a functional relationship, whether in words or variable notation. Control students showed some gains on these items, yet they still fell short of the performance of their treatment counterparts. This is perhaps not surprising giving the lack of a deep treatment of functional thinking in the *Common Core* (NGA Center & CCSSO, 2010) prior to Grade 8, calling into question the choice to de-emphasize the role of functional thinking in elementary grades in these standards. Our work with students across Grades 3–5 as well as the work of Blanton and colleagues (Blanton, Brizuela, Gardiner, Sawrey, Newman-Owens, 2015) with much younger students has illustrated that elementary students are capable of engaging in such thinking, beginning with very simple relationships that increase in complexity over time. The decline treatment students demonstrated in performance from Grade 5 to Grade 6 suggests not that they are unable to engage in such thinking, but that opportunities to engage in such thinking need to be sustained over time.

Only one item (Item 4) on our assessment explicitly asked students to build an argument to justify an arithmetic generalization. This was among the most difficult of tasks for Grades 3–5 students in both conditions. By the end of Grade 5, 23.41% of treatment students in the Grades 5-6 retention study were able to produce a general argument to justify that the sum of three odd numbers is an odd number. By the end of Grade 6, only half of these students were able to do so. Control students likewise showed no improvement in their ability to produce such an argument. Like functional thinking, this is an area that does not receive a great deal of attention in elementary curricula. Also, like functional thinking, argumentation is an area in which we know elementary students are capable of engaging. Our previous work (Blanton et al., 2015; Blanton et al., in press), along with the work of others (Carpenter, Franke, & Levi, 2003; Carpenter & Levi, 2000; Russell, Schifter, & Bastable, 2011a, 2011b) illustrates that while building more general arguments is challenging, students are capable of using the context of arithmetic to engage in justifying mathematical claims. In our current work, we are successfully engaging K– Grade 2 students in generalizing and justifying generalizations about even and odd numbers. We thus argue that our findings in this area suggest not a lack of ability on the part of students, but rather a lack of access to important algebraic practices.

Finally, we point to students' performance producing variable representations across a variety of big algebraic ideas to argue that this, too, is an area in need of sustained focus. Treatment students showed a decline from Grade 5 to Grade 6 in their abilities to represent an arithmetic property, functional relationships, and related unknown quantities using variable notation. While control students showed some gains in this area, the performance levels they reached were nowhere near that of their treatment counterparts. We have evidence from our Grades 3–5 work as well as the work of others with much younger students (Brizuela, Blanton, Sawrey, Newman-Owens, & Gardiner, 2015) that students are capable of representing varying quantities with algebraic notation and in fact often prefer such representations over verbal ones. We thus argue again that what is needed is a sustained focus on such experiences over time.

Conclusion

Over twenty years ago, Kaput (1998) called for an end to "the most pernicious curricular element of today's school mathematics—late, abrupt, isolated, and superficial high school algebra courses" (p. 25). He argued that we could do so by viewing algebra as a K–12 experience, integrating algebraic thinking and reasoning throughout the mathematics curriculum. Our work developing and testing a Grades 3-5 early algebra intervention coupled with our Grades 5–6 retention study lends support to Kaput's argument. First, we found that, over time, treatment students retained a significant advantage over their control peers in their understandings of worthwhile big algebraic ideas and thinking practices. Second, we believe our findings lend support to the argument that algebra must be treated as a continuous K–12 strand of thinking, not as a subject that can be infused into students' elementary curricula for a few years only to be cast aside until its more formal treatment.

Acknowledgments

The research reported here was supported in part by the US Dept of Ed under IES Award #R305A140092. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the US Dept of Ed.

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