# MATHEMATICS AS A CONSCIOUS RAISING EXPERINCE: A BRIDGE TOWARDS SOCIAL TRANSFORMATION

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This study explores preservice teachers engaging in a ratio activity connecting the wages of men and women to the number of men and women in government positions. Mathematical learning expands the critical literacy framework reading and writing the word situating mathematics as the world itself. Furthermore, the bridging of mathematics, the self and the world, was constructed by incorporating the theory of conocimiento(s) in analyzing how critical consciousness is developed within a mathematics task. This study shows mathematical learning as a conscious raising experience in investigating social injustices towards acts of social justice.

Keywords: Equity and Diversity, Social Justice, Teacher Education-Preservice

#### Introduction

The purpose of this study is to learn how male pre-service teachers (PSTs) experience a mathematical task that explores wages earned by women and men in various occupations. To compare wages, an activity composed of five tasks using data taken from the 2016 U.S. Bureau of Labor Statistics was used in conjunction with the use of ratios and the creation of mathematical equations. The task was created with the intention of creating a learning space where PSTs would be able to reflect upon the content being learned and were provided with an opportunity to create a figure/diagram to share out any findings. Wherein the activity looked to situate learning in/as a form of revolutionary praxis, the union of action and reflection towards social transformation (Freire, 1976). More so, revolutionary praxis requires a critical consciousness, a socio-cultural-historical-political understanding of the world with the individual acknowledgement of being able to change the world. This study explored how a mathematics activity could transform learning into a conscious raising experience. To understand consciousness development during the learning mathematics, theories of conocimiento(s) (Anzaldua, 2003) were compared with how critical conscious is developed in reading and writing the word (Freire & Macedo, 2005), both described later. Wherein it is impossible to have a critical consciousness without multiple forms of conocimientos.

Without praxis in mathematics, mathematics becomes nothing more than a mechanical process. Any mechanical process can then become a dehumanizing force, shifting the learner from a position of subject to object (Freire, 1970; 2018). Teachers, students and researchers will be unable to address critical issues in mathematics without focusing on the entire experience of learning mathematics. Hence this study focuses on PSTs because "[p]rospective teachers ... need to begin to develop the political and ideological clarity that will guide [students] in denouncing discriminatory school and social conditions and practices." (Bartolome, 2004, p.119). The political and ideological clarity Bartolome spoke of is in line with critical consciousness and reflects the political power of teachers controlling learning in the classroom (Giroux, 1988). The activity created for this study uses ratios, algebraic generalization and re-representation of data to make meaning of the real world. The research question driving this study is, "how do pre-service

teachers develop critical consciousness through a ratio, algebraic generalization, and representation mathematics activity?"

#### Literature Review

## **Critical Consciousness**

A critical consciousness is defined as a socio-historical-political-cultural understanding of society with the self-awareness of being able to contribute to change in society (Freire, 1970). Critical consciousness is an elastic concept given society is in constantly flux (Freire, 1970; 2000; 2018). Prior to the development of a critical consciousness, is what Freire (1973) calls a transitional consciousness. Those with a sense of critical consciousness are always in between transitional and critical consciousness. Individuals not seeking critical consciousness are said to have a naïve consciousness (Freire, 1973) where they have been rendered objects due to dehumanization (Freire, 1970). Relative to developing critical consciousness, are ideas of praxis, critical literacy and conocimiento(s). Furthermore, Freire and Macedo (2005) provide a critical literacy approach in reading and writing the word (described later) which inherently allows for a socio-historical-political-cultural understanding of world. The praxis behind reading and writing the world is a revolutionary praxis (Freire, 1970) that requires a critical consciousness. Related to a transitional consciousness, Anzaldua (2004) offers conocimiento, seven stages of how consciousness changes (described later) that provide a more refined way of mapping naïve to transitional to critical consciousness development.

# Identity, Mathematics, and Dehumanization in Mathematics Education

Learning how to socialize is inherent in learning mathematics because teaching mathematics is a political act that impacts students understanding of the world (Gutierrez, 2017). Knowing that mathematics is part of the socialization, and identity development of learners helps educators identify factors that contribute to students' futures (Jackson, 2010). Insofar to say the connection between identity and learning is not an unknown relationship (Martin, 2010). Wherein utilizing the identities of students in the learning of mathematics has a growing body of research (Aguirre, 2013; Celedon & Ramirez, 2012; Civil, 1998; 2014; Martin, 2010; Wood, 2013). Specifically, Aguirre, Mayfield-Ingram and Martin (2013) "define a mathematics identity as the dispositions and deeply rooted held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the context of their lives" (p. 14). Although students' identities need our attention, we must also consider the role of the teacher and how their identities impact students (Macedo & Bartolome, 1997). Specifically, if we do not pay attention to students' identities then learning becomes a dehumanizing experience (Gutierrez, 2018) where students are never fully seen or heard and where teachers cannot fully meet the needs of students (Civil, 2014). Teachers (and researchers) need to focus on the development of young learners by paying attention to identity, which should be part of the daily life work of teachers (Aguirre, 2013).

The idea of humanization and dehumanization stems from the work of Paulo Freire regarding dialogue and the learning process known as a problem posing education, a humanizing approach and banking education, a dehumanizing education (Freire, 1970; 2000). A dehumanizing approach to education silences students, ignoring their identities and views students as mere objects. Shifting to mathematics education Gutierrez (2018) offers eight dimensions of humanizing practices for mathematics educators with ideas of participation/positioning, cultures/histories, creation and ownership, all having a direct connection to identity development (Nieto, 2018). Furthermore, a full discovery of identity and humanization allows for individuals

to discover or (re)discover their own agency and allows for the development of a critical consciousness (Shor & Freire, 1987). As previously mentioned, viewing the identities and even roles of teachers and students as interchangeable not only negotiates the power of the classroom (Shor, 2014) but also echoes Fasheh (1982) in that, "[t]here is a price for teaching math in a way that relates it to other aspects in society and culture which may result in raising the "critical consciousness" of the learners. (p. 288)."

# Critical Literacy, Mathematics, and Critical Consciousness

Mathematics as a form of critical literacy can best be summarized by Gutstein's adaptation of Freire and Macedo's (2005) *Literacy: Reading the word and the world* framework that Gutstein (2006) calls reading and writing the world with mathematics (RWWM). Freire and Macedo's framework is a way to understand the world by understanding the previous world through critical literacy (Freire, 1998). Critical literacy is the ability of communication and reflection inherent in being able to read and write (Freire, 2018). The ability to reflect and communicate is a necessity for revolutionary praxis. Gutstein (2006) developed RWWM and incorporated the role of mathematics to read the current and previous world. A "[c]ritical mathematics literacy enables the oppressed to use mathematics to accomplish their own ends and purpose" (Lenorard, 2010, p. 326). Being able to read and write the world within a critical literacy paradigm leads to empowerment and understanding of one's own agency (Freire, 1970; 2005). Knowing how to read the world and understanding one's agency directly relates to critical consciousness and (re)humanizing mathematics. Where a critical mathematics agency requires individuals to be see themselves as capable and powerful mathematical learners when participating in mathematics in personal and social ways (Aguirre, 2013).

## **Conceptual Framework**

In welcoming the connection between mathematics and the social world, the conceptual framework of reading and writing the word (Freire & Macedo, 2005) was utilized and paired with the theory of conocimiento(s) (Anzaldua, 2003; 2015), which also acted as the analytic framework for this study putting emphasis on mathematics as a way to discover both self and the world. The teaching of mathematics should reflect the social and physical environment when learning, with such an approach augmenting a deeper understanding and imagination (Joseph, 1987). The physical environment is constantly engaged when learning math directly or indirectly, as is the political nature of learning mathematics (Gutierrez, 2018). Furthermore, as Martin states "exposing the links between mathematics and social interests should not be a threat to mathematics" (1997, p.175).

In the understanding of social and structural problems, Freire and Macedo introduced the critical literacy framework, reading and writing the word (2005). Where the reading or understanding of the world requires the reading of the word and in order to read the word one must first read the previous reading of the world (historical awareness). The writing of the world which follows the reading of the world then becomes the new world (Freire, 2005). Where the learning of mathematics is the simultaneous act of learning more about mathematics, the world and self. Insofar to say the world cannot be understood without understanding how the individual fits in said world. The bridge between the conceptual and analytic framework can be seen in the reflection on how consciousness shifts towards the development of a critical consciousness.

#### Methods

To observe the process of mathematical learning of PSTs while identifying stages of conocimiento(s), a single holistic (Stake, 1995) qualitative case study bounded by the duration of the activity, was conducted. The study focused on understanding how PSTs engage in a contemporary issue that does not require the control of human behavior (Yin, 1998). In this study, the case is bounded by the space time of a classroom in a large Midwest university. The unique feature of this case is the looking at wage disparities between men and women in the U.S. using ratios. The facilitator of the activity was a participant observer and main researcher behind the study. The data includes written reflections from the PSTs after each task and a culminating reflection at the end of the entire activity. Written individual reflections were written prior to group discussions and used to triangulate findings. Group discussions along with whole class discussion were recorded.

# **Analytic Framework: Conocimientos**

Conocimiento(s) consists of seven stages and represents a change in consciousness wherein to say an individual that has experience a conocimiento(s) is no longer the same person (Anzaldua, 2015; Keating 2008). The result of no longer being the same person changes an individual's identity because gaining conocimiento(s) results in action that would be impossible without, the aforementioned, change in consciousness (Anzaldua 1999; Keating, 2016; Lara, 2005). Furthermore, conocimientos is a pivotal theory in the epistemological understanding of the interconnectedness of all people (Keating, 2006) which leads to self-discovery of one's own freewill upon the world (Anzaldua, 2015). This interconnectedness allows for the individual to be interconnected with not only others but with mathematics itself. The seven stages of conocimiento(s)—a non-linear, non-sequential and cyclical (both a cycle and/or repeating) process of consciousness (Anzaldúa, 2003)—described in Table 1, will act as the analytic framework in analyzing the data.

Table 1: Stages of Conocimiento(s) (Anzaldúa's, 2003)

Stage	Description	
el arrebato	This is the stage that kicks you out of your comfort zone. This is	
	the stage that pushes you to learn/change. It can be a positive o	
	negative experience.	
nepantla	This is the clash of worlds and or ideas both the friction and	
	overlap of two or more thoughts. Since we are always learning	
	we are always in nepantla but necessary not aware of it.	
coatlicue	This is the pain of knowing and the realization that you need to	
	change. The coatlicue stage is also known as deconocimientos	
	representing that learning and growing never stops.	
compromison	This is the realization that nothing in this world is fixed, which	
	implies change is possible.	
putting	This is where you reinvent yourself as a new person. The	
Coyolxuhqui	transformation that results from fragmentation of self, both	
together	positive and negative experiences that you learn from.	
the blow up	This is the realization that change has happened and that you are	
-	a new person. It is where/when it is first internalized that change	
	has occurred.	

shifting realties	This is an internal/external spiritual transformation of being	
	conscious of others, a shift from how this affects the individual to	
	how this affects the world. The act of changing.	

# **Participants**

All four of the participants were White male PSTs from Iowa, taking a secondary mathematics methods course in 2017 at a large predominantly White mid-west university. All participants are traditional undergraduate mathematics majors that were one semester away from student teaching. All participants were under the age of 22. The course they were taking did not focus on the development of critical consciousness nor did any other mathematics methods course specifically focus on critical consciousness in the 4-year university-based teacher preparation program. Three of the four PSTs have previously taken one course on multicultural education that is part of the institution's teacher preparation program. The methods course where data was collected consisted of 12 students, six male and six females.

### **Task**

The task was developed to be part of an algebra methods course with the intent to raise critical consciousness of the learner, where "critical consciousness is brought about not through an intellectual effort alone, but through praxis – through the authentic union of action and reflection" (Freire, 1973). The role of groups was designed to allow students to interact with each other because students need to support each other in meeting their goals (Zacko-Simth, 2013). The understanding of ratios, generalization, and re-representation were the topics covered throughout the algebra methods course. Ratios were selected because the concept is frequently connected to the real world and proportional reasoning is a difficult topic to learn/teach (Lamon, 2007). Generalizations were selected to provide PSTs with the opportunity to make conclusions based on the difference of data over time (Driscoll, 1999). Generalizations allow for the extrapolation of data. Re-representation was incorporated into the activity because of its potential in being the action component of the activity. Re-representation allows learners to reword and reflect on what they have learned using visual representation. The study consisted of an activity created by the researcher comprised of five parts described in Table 2.

**Table 2: Task Overview** 

Part	Student Engagement/Action/Reflection	
1	View raw data from 2016 U.S. Bureau of Labor Statistics of weekly median incomes.	
2	Construct ratios of weekly women median income to weekly men median income and	
	find the value of ratios. Participants reflected individually then as a group on real-world context of the ratios.	
3	The generalization task uses the value of the ratio(s) and creates an equation to predict what happens after a year.	
4	Ratios of the numbers of men to the number of women in the US Senate and House of Representatives along with the relationships in state governorships were constructed to the value of each ratio. Participants reflected individually then as a group on real-world context of the ratios.	
5	Re-representation of the entire activity.	

### **Analysis**

In Vivo coding was conducted (Creswell, 2016) due to its usefulness in thematic analysis (Saldana, 2016), wherein short phases were analyzed. Thematic coding has emerged as a rigorous approach in analyzing themes (Fereday & Muir-Cochrane, 2006) where Braun and Clarke (2006) provided six phases for thematic analysis: (1) Familiarization with the data, (2) Coding, (3) Searching for themes, (4) Reviewing themes, (5) Defining and naming themes and (6) Writing up. After Coding was completed in Reviewing themes an analytic memo was written for each code. The analytic memos were part of Searching for themes, specifically a generative theme emerged from each analytic memo. In Vivo coding was then conducted and accompanied with/by analytic memos for each code. The memos were then coded using the analytic framework. The reflection written on worksheets were used to triangulate and in the reviewing of coding. Next, the codebook was created, see the sample in Table 3.

**Table 3: Sample of Codebook** 

Stage	Code Description	Example invivo codes(s)
C1 – el arrebato	Related to discomfort or excitement caused either by the mathematics or by the context or connection to the real world	"a lot of numbers" "You are right holy crap I don't know what to tell her that sucks"
C2 – nepantla	Related to the comparisons of mathematics, professions, genders, history and other structures	"so we write ratio as a fraction or what" "because women are dominantly known as teachers, have always been, way back in the day they make less than we do"

## **Findings**

#### Part 1

The start of the activity provided multiple *el arrebato* 's due to the context of the problem. Mathematical shake ups range from being overwhelmed by the data, "a lot of numbers" to being overwhelmed by the difference in wages (the context) "they should be on the same pay scale" to discovering things by doing mathematics "Men make 700 more dollars ... oh wow." This shows that viewing and discussing real world data provides an entry (*el arrebato*) and an environment (*nepantla*) to explore the world. What we found was that the data allowed the PSTs to find contradictions to the view that men always earn more than women by finding cases where women make more. For example, one student said "I got one I got one where women make more" being surprised and excited (*el arrebato*) to find a contradiction (*compromision*) to the fixed idea that men always earn more.

We then began to see a comparison (*nepantla*) not only of wages but of number of employees. The comparison of wages and number of employees can be seen in one PST exploring elementary/middle school teachers where "like 80% women" and another PST asking, "what's the wage" only to hear the wages and say, "wow that is a big jump." The mathematical discrepancy directly relates to the real-world disparity in wages between women and men and to the results in of PSTs trying to justify the differences. At this stage of the activity the mathematics was limited to additive comparisons of not only individual professions but across, as can be seen when one PST said, "weird it is so small when women make more [yet] when men make more it's like a thousand dollars more."

## Part 2

In the second part of the activity a shift from direct additive comparisons to the use of ratios, which was expected as a direct result of the task. The shift to using ratios from direct comparisons while exploring different professions among women and men is nepantla on multiple levels. On one level when a PST said "so we write ratio as a fraction" ratio and fraction represent the clash of two mathematical sets or worlds. Specifically, the use of ratios was naturally *nepantla* as a ratio represents the relationship between two quantities and even how ratios are represented can be seen as nepantla within nepantla as one PST said "there are 17 females to 100 males or .17 to every one male." On other levels we see; the comparison of professions in comparing police officers to engineers to teachers; or the comparison of expertise in questions posed by the PSTs like "do you think that has to do with levels of expertise?"; or historical comparisons "because women are dominantly known as teachers, have always been, [since] way back in the day;" or by breaking the border between the mathematical world of the activity and the ones individual world as can be seen in comments like "they make less than we do." Additional el arrebato's occurred during task two of the activity. The first was related to limitations of being unable to generalize claims with one PST saying, "from the data we cannot make any conclusions" and another saying, "doesn't matter I am not an expert." The second manifests when unexpected outcomes occur in the real-world data, as one PST said "so why is that not the case for education" in being shocked at male teachers making more even though there are more women in the profession. In this part of the task el arrebato and nepantla leads to coatlicue or the pain of knowing and can be seen when it was said "that the problem [is] you only have four tenths of a teacher." Whereby directly stating a problem hints there needs to be a solution implying the realization of change.

#### Part 3

Part 3 of the activity began with a shake up as the group selects a profession and one individual saying, "ours is going to be awkward" where a PST's ability of estimation led to *el arrebato*. The group was already aware of the difference in the weekly median income along with the unit rate relationship between men and women but in creating a formula for "n" many weeks they discovered "after a year, men have made 10,920, nearly eleven thousand [dollars] more" encountering *el arrebato* once more. Furthermore, in *nepantla*, in the comparison of positive and negative values the group discussed when it would be appropriate to use a positive difference (men – women) versus a negative difference (women – men). One PST said they would always subtract the numerically larger value from the numerically smaller value to ensure a positive number with another replying that means you will always use women subtracted from men only to hear "that was a low blow." Calling it a low blow is a direct result of *coatlicue* and the pain knowing that leads to a *shift in reality* as the conversation connects negative values with disadvantages in saying "it depends on how you want to see the data."

PSTs' reflection on this activity provides an example of a shift moving from *coatlicue* to the *blow up* to the *el arrebato* stages of conocimientos. When asked how the activity makes you feel, one PST instantly said "this makes me feel like shit" this pain shows the realization of change (*the blow up*) because the next statement said "it's like [am] back at my therapist. Here the individual is connecting what was learned in this task to internal transformation. This cause *el arrebato* in others as they default to say, "I don't have feelings" and "men don't have feelings."

#### Part 4

The beginning of this activity was *el arrebato* as the worksheet for task 4 had 19 women senators and 71 non-women senators which does not equate to 100 senators. Knowing there should be 100 senators shows that the PSTs do have a political understanding of society.

Furthermore, a *shift in realties* can be seen as PSTs look up how many governors are women only to discover "8% of governors are women" as they declare it a "fun fact." One again this reflects the *blow up* of being in *nepantla* in discovering how women are represented in the government because "for every four men there is one woman." As the PSTs travel outside the task in *shifting realties* they discover currently there are "six women governors so the number is going up" a *compomison*, a reminder that the world is not fixed.

At the end of this activity a PST repeated the statement "I can neither deny or confirm this because I am no expert." The first time it was said it represented only *neplantla* because he was in between denial. The PST initially made the claim of not being an expert then became silent. The second time the same PST said "I can neither deny or confirm this because I am no expert" shows a changing by the PST not staying silent and offering his "guesstimation" on how he sees the percentages of women. The act of "guesstimating" represents a *blow up* and *shift in reality*, while the change from silence to speaking reveals a new person, *putting Coyolxauhqui together*.

## **Final Reflection**

When asked how all parts are related, one PST said "it takes money to run for government, right, if all these dudes are out here making more money than there in government more" followed by another PST stating "guys are making more money, so they are donating more and who are they donating to probably guys." This brief exchange shows a *blow up* for both individuals in better understanding why more men may run for office. The following exchange occurred when PSTs were asked "would you use this activity in your future classroom?":

PST A: I would adapt it to talk about what they, the students want to be in the future"

PST D: It would be discouraging if a girl comes up and says I want to be an engineer.

Researcher: what do you tell the girl?

PST D: I say ... um ... Oh I don't know

PST B: You are right holy crap I don't know what to tell her that sucks

PST C: life is not fair

PST A: She could be the outlier

PST A engages *nepantla* by saying the activity would be useful because of its real-world application then shifts to a *blow up* in wanting to adapt the problem specifically to what learners want to be in the future. PST D then confronts a potential *el arrebato* in the potential pain of knowing (*coatlicue*) for young women engaging in the activity. A further shake up comes when the group was asked what they would say to the young women who discovers she will earn less compared to a man. PST D does not know what to say as he experiences *el arrbato*, while PST B and P PST C are in *coatlicue* a realization that something needs to change. PST A has a shift *in reality* in offering to tell the young women she is not in a fixed world (*compromiosn*) because she can be an outlier.

#### **Discussion**

The findings show that stages of conocimientos can be experiences throughout this mathematical activity. The activity provided PSTs with one way to understand the current world by understanding how wages and government forces interact. The tasks within the activity provide a way to read the world as PSTs write the world by engaging in the problems. In going through the task, learners experienced opportunities to gain conocimiento(s) and shifted their consciousness. In bridging ratios of wages and ratios of governmental representatives the PSTs

discovered injustices. Discussion on injustices with mathematical discoveries of ratios represents a shift towards developing a critical consciousness. Where "men make more in all my categories" (nepantla) is both reading and writing the world in looking at men and women while doing the mathematics to show they are not making equal wages. *Nepantla* alone only represents a naïve consciousness. To say "life is not fair" (el arrebato) is one way to read the world, saying "you would adapt it to talk about what is ... the future" (putting Coyolxuhqui together) enters a transitional consciousness in writing the world. The coatlicue and the blow up stages of conocimiento(s) are respectively engaged in reading the world where the activity "makes me [PST] feel like shit" while discovering "it is like the rich get richer." Still in transitional consciousness the PSTs wrote the world by saying "it depends on how you use the data" (shifting realties) and telling future young women that they "could be an outlier" (compromison). Once out of *nepantla* the other six stages only represent a transitional level of consciousness and an imperfect understanding of world. The moment all stages of conocimiento(s) are lived then critical consciousness emerges as a socio-political-historical-cultural understanding in changing reality. Wherein bridging mathematics and conocimientos provides a path to see how mathematics is inherently part of self and world towards the transformion of mathematics itself.

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