SIMULATIONS OF PRACTICE FOR THE EDUCATION OF MATHEMATICS TEACHERS

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This working group addresses the challenge and opportunity presented by the use of simulations of teaching practice as an educative tool for preservice and practicing teachers. We focus particularly on the development of teachers' skills in enacting the content-intensive work of teaching, including how simulations may be able to cultivate or reveal teacher MKT, either as propositional or applied knowledge or as an essential component of a larger teaching practice in which MKT is activated.

Keywords: Mathematical Knowledge for Teaching, Teacher Knowledge, Teacher Education-Preservice, Teacher Education-Inservice/Professional development

History of Topic and Elaboration of the Problem Space

We situate this working group in two complementary strands of work, one focusing on teacher knowledge and the other on teaching practice. Since Shulman (1986) introduced the construct of pedagogical content knowledge, scholars have identified specialized mathematical knowledge in and for teaching (e.g., Ball & Bass, 2003; Krauss et al., 2008; Thompson & Thompson, 1996). With this scholarship have come efforts to focus teachers' education on learning this specialized knowledge (e.g., Ball, Sleep, Boerst, & Bass, 2009; Ghousseini, 2017; Silverman & Thompson, 2008). This knowledge, sometimes identified as mathematical knowledge for teaching (MKT), coordinates purposes and reasoning from both mathematics and teaching. Developing teacher MKT has increasingly become a core component of teacher preparation for mathematics teachers (e.g. Association of Mathematics Teacher Educators, 2017; Conference Board of the Mathematical Sciences, 2012) with assessment of MKT generating a body of scholarship (e.g., Gitomer, Phelps, Weren, Howell & Croft, 2014; Herbst & Kosko, 2012; Hill, Schilling & Ball, 2004; Krauss et al., 2008) and recently crossing over into mainstream licensure tests (see https://www.ets.org/praxis/about/ckt/).

MKT is, by definition, knowledge that is demanded in response to the content-intensive work of teaching (Ball, Thames & Phelps, 2008) and has been described by scholars as *practice-based*, or more closely grounded in teaching practice than other forms of mathematical knowledge (Ball & Bass, 2003). Other scholars have noted that the examination of MKT as it is learned by teachers or used in teaching benefits from more clearly articulated conceptualizations of how that knowledge is held, enacted, activated, or drawn on. (Heid, Wilson, & Blume, 2015; Rowland, 2013) The Knowledge Quartet work, for example, provides a framework designed "to guide attention to, and analysis of, mathematical knowledge-in-use within teaching", the purpose of which is to help teachers reflect and learn from their teaching (Rowland & Ruthven, 2011, p.85). Ghousseini (2017) similarly distinguished between propositional knowledge and knowing as a form of action, and Heid et al. (2015) described mathematical knowledge in terms of proficiency and activity. These lines of work arguably push toward understandings of MKT as it is used in teaching in ways that are even more closely aligned to the practice of teaching.

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A complementary line of practice-based research has emerged that seeks to conceptualize critical teaching practices and to understand how teachers learn to engage in them effectively. Recent years have seen widespread effort to identify and prioritize teaching practices (e.g., http://www.teachingworks.org/work-of-teaching/high-leverage-practices, https://www.corepracticeconsortium.com/), shifting the field toward the systematic description of the most critical competencies for teachers to develop during preparation. At the same time, theories such as the Grossman, Hammerness, and McDonald (2009) pedagogies of enactment help to describe learning opportunities likely to support teachers' development of those competencies. A number of studies utilizing simulations of teaching practice are grounded in this theory, which describes the key pedagogy of *approximation of practice* as "opportunities to rehearse and enact discrete components of complex practice in settings of reduced complexity" (p. 283).

We situate this working group proposal at the intersection of these two lines of work. While not all simulated teaching is content-intensive, the simulation of content-intensive teaching practices may provide us with opportunities to observe, develop, and measure MKT, as recommended across elementary (e.g., Ball & Cohen, 1999; Hill, Sleep, Lewis, & Ball, 2007; Stylianides & Stylianides, 2013 and secondary levels (e.g., Ticknor, 2012; Wasserman, Weber, Villanueva, & Mejia-Ramos, 2018). And increasing attention to how MKT is activated and used in the practice of teaching can help us to better understand the relationship between knowledge and practice and how a teacher's MKT may relate to the quality of his or her practice or opportunities to learn from practice.

Enacting this approach brings a number of challenges. When working with simulations, teachers' attention can emphasize pedagogical concerns, "eclipsing" the intended specialized knowledge (Creager, Jacobson, & Aydeniz, 2016, p. 3); and at other times teachers may attend to mathematical ideas in a way that sidelines pedagogical reasoning (e.g., Schilling & Hill, 2007; Suzuka et al., 2009). In enactments of these activities, whether teachers' attention slides away from mathematics or away from pedagogy, the opportunity for using mathematics and pedagogy to inform each other is lost. In other words, one challenge facing the simulation designer is that of focus. The proposed working group will provide opportunities for simulation designers to work toward clearly defining objectives and determining which simulation design characteristics are critical in creating circumstances likely to meet those objectives.

A second challenge is that of common vocabulary. As Grossman, Compton et al. (2009) noted, simulations such as approximations of practice "can vary significantly, both in terms of comprehensiveness and authenticity" (p. 2065), ranging from responding to written cases to role-playing in mixed reality settings. In addition, the simulations under consideration vary significantly with respect to factors that matter in simulation design such as the level of interactivity or the types of records in which the practice is made visible. This variability in turn may amplify the potential for confusion across lines of work. Lesson planning, for example, might be considered a teaching practice to be simulated by one project, or as a part of the preparatory cycle for a simulation of interactive teaching by another, and it is likely to be conceptualized, supported, standardized, recorded, related to teacher knowledge, and attended to in very different ways as a result. The proposed working group will provide opportunities to work toward a common vocabulary or way of describing simulations of teaching practice

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Rationale for the Working Group

This working group is timely and critical. The field is still at the early stages of developing a robust understanding of the ways in which practice-based professional preparation occurs within teacher education, especially in terms of how to leverage simulated environments to productively develop teachers' competencies (Sykes & Wilson, 2015). Recent years have seen multiple nationally-funded efforts to improve the mathematical preparation of teachers by developing materials with simulations of teaching practice for use in mathematics and methods courses. Most of these efforts build on the success of small-scale pilot studies (e.g., Lischka, Strayer, Watson, & Quinn, 2017; Straub, Dieker, Hynes, and Hughes, 2015; Wasserman, Fukawa-Connelly, Villanueva, Mejia-Ramos, & Weber, 2017).

While approximations of practice such as role playing or peer teaching are common practices in teacher preparation, more systematic approaches such as those using trained actors to provide standardization of opportunity (e.g., Dotger, Masingila, Bearkland, & Dotger, 2014; Shaugnessy & Boerst, 2018) are relatively rare, and investigators have noted the novelty and rarity of the very idea of incorporating technologically-supported simulations into mathematics courses (e.g., Ensley & Fiorini, 1998; Lai & Patterson, 2017; Wasserman et al., 2017). As noted in the 2018 PME-NA call, such technologies create opportunities to "explore how technology can be used in the service of mathematics education and research," (Hodges, Roy, & Tyminski, 2018) both by allowing us to explore best practices around the use of such technologies and because of the windows of opportunity they create to observe teacher learning in more systematic ways.

There is thus an emerging community of mathematician and teacher educators who are interested in attending to teachers' mathematical development through simulations of practice. For example, one of the NSF-funded projects listed previously supported a recent conference on the topic of simulation use in teacher education attended by one of the working session co-leads, who reported that major learning from sessions included (1) a significant variation across models of implementation, structure of cycles of enactment, and purposes and (2) a need expressed among expert panel members for the field to describe the parameters of simulation task design in a way that helps to build mappings between those parameters and task design.

Focus of Work

This working group seeks to explore the following questions:

1) How can we conceptualize the theories of action by which teacher learning is expected to result from engagement in simulation activities in contexts such as mathematics and methods coursework?

Theories of how teacher learning occurs, characteristics of simulation activities, the role of MKT, and how the activities are evaluated differ in substantial ways across projects. For instance the ULTRA project (Wasserman et al., 2018) is based on a learning theory of transfer and has revised its simulations in tandem with revising decompositions of teaching practice, whereas the MODULE(S^2) project (Lai, Strayer, & Lischka, 2018) is based on a learning theory of decentering and has revised its simulations in tandem with revising an adaptation of the Knowledge Quartet framework for use with simulations. This working group will invite comparison of theories of action and explore the variability in how researchers are conceptualizing simulation.

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2) How can we articulate design principles rising out of or grounded in the theories of action identified in question 1?

Many simulation projects utilize iterative cycles of design to revise and refine simulation tasks. That is, simulations are designed based on a provisional theory of action, the simulations are then enacted in instruction, the enactments and instruction are studied in relation to intended outcomes, and action is taken to revise the simulations and underlying theory based on studying enactments.

This revision process is the foundation for a growing understanding of how characteristics of simulations provide opportunities for teachers to develop their teaching practice. But we lack a common language for what these characteristics are, and how they differ by context. Without a common language, it is difficult for the field to build systematically on this emerging base and capitalize on interest. So now is the time to contextualize what different educators are doing, how they draw on theory, and the rationale for choosing these theories. As Kennedy (2016) argued, the rationale matters more than the particular actions, because it's the reasoning that can change practice.

3) How can we measure the development of MKT, teaching practice, or other valued outcomes through the use of simulations?

Simulation use is additionally diverse in the varying degree to which it focuses on assessment and with respect to the targets of that assessment. Shaughnessy and Boerst (2018), for example, describe a program designed to measure preservice teachers' skill in eliciting student thinking through the use of a standardized on-demand interactive simulation. While the context of their work is indisputably content-intensive and they note that skill in eliciting may interact with MKT, the primary focus of their study is the teaching moves involved in elicitation. One could easily imagine, however, analyzing such data for evidence of MKT in use, similar to the approach taken by Lai et al. (2018). In contrast, Mikeska, Howell, & Straub (2017) acknowledge and propose to study the potential relationship between MKT and the targeted teaching practice of leading small group discussions in mathematics, but provide preservice teachers with preparatory materials in advance of the interactive simulated teaching to support their ability to make sense of the student work samples they will be discussing. Of note across these examples is that despite similar conceptualizations of MKT and a common understanding that it relates to the teaching practices simulated, MKT holds a very different role in what each project seeks to measure.

Organization and Plan for Active Engagement

The overall goal of this new working group is to create a community in which researchers and practitioners can explore how simulations of practice can be optimized to provide opportunities for teacher learning. Prior to convening in Missouri, we see value in collecting information from participants through a short survey on participants' conceptualizations of simulations of practice and how they use them in their teaching and/or research. This information will help create a canvas in which to begin to co-create our community.

The working group will consist of three sessions during the conference followed by virtual meetings through the following year. Across the three sessions participants will engage with facilitators to examine the learning theories that ground current work in simulations of practice, identify key design principles of the simulation tasks, and analyze our methods of assessing the impact of this work. In each session, participants will engage with simulation tasks of their own

or from the facilitators, make explicit the areas they want to dive into deeper, and link with others that want to dive in with them.

Session 1: Exploring our Simulations of Practice

For the first session we will use information from our short survey to ensure that we are anticipating how different researchers and practitioners are conceptualizing simulations of practice. We will introduce our conceptions of simulations of practices within our own work and expand to include those from the survey. Next, small groups will be formed based on common interpretations, and group members will participate in an activity to map features of simulations to the issues of teacher learning that they are attempting to address. These simulations of practice will be provided by attendees' own work or examples from co-leaders' projects.

This mapping activity will result in a working document of simulation characteristics linked to the objectives they are intended to meet. This mapping will also allow outlets for us to problematize the simulation space. These activities are intended to surface commonalities and differentiations in our theories of action and our intended use of simulations.

Session 2: Analysis of Design Elements of Simulation Tasks

For the second session we will continue utilizing the working document from the first session and delineate the characteristics or design elements of the simulations that provide opportunities for teachers to enact MKT and teaching practices. Participants will begin by placing examples of simulations on a continuum of authenticity and completeness in relation to their approximation of teaching (Grossman, Hammerness, et al., 2009). This activity will lead to group efforts to identify key components of simulations that afford or hinder teachers' opportunities to learn. From this we will identify common themes in how simulations are designed for particular contexts. This record of design characteristics will be the beginnings of our efforts to create a collection of design principles for simulations that will inform the field and help link theory to practice. Session participants will apply this preliminary collection of design principles to revise or write simulations of practice to incorporate these characteristics. Through this process we will also identify issues of congruence and contrast within the design process.

Session 3: Assessment of the Impact of Simulation Tasks

In the last session of the working group we will again employ the working document to dive deeper into how learning objectives are measured active projects represented by attendees. In our efforts to identify key components of simulations that afford learning we need to be transparent in what outcomes we are measuring as learning and how they align with the underlying theories guiding the simulation development. Outcomes and metrics used within projects by co-facilitators will be presented and examined as a starting point for this session. Next, participants will identify their outcomes and metrics within their own work and consider any possible revisions. The power of utilizing a consistent working document is that we will have a record of the revision processes and analyses that will further inform future simulations and underlying theory.

In the closing segment of this session we will collect contact information from participants, document subgroups and their future research interests, and share next steps.

Follow-up Activities

This working group is a key step in establishing a network of teacher educators and researchers that are engaged in utilizing simulations of practice to develop teaching knowledge and practice. Subgroups formed during the sessions will be encouraged to continue collaborations on co-constructed areas of inquiry. The facilitators will support the continuation

of collaboration by hosting virtual meetings through the year to check-in. We anticipate subgroups pursuing design revisions, creating simulations, exploring learning theories, examining the impact of simulations across varying contexts, and other emerging themes.

One of the future priorities of this working group is to channel the productivity of our sessions into a proposal for a special issue in the *Journal of Mathematics Teacher Educator*. In 2007 there were a series of special issues in *JMTE* that described the field's then-current understanding of mathematics-related tasks for the education of mathematics teachers. Those special issues highlighted the development process around task and as a result the editors saw "how the focus on tasks leads to 'meta-tasks' through which knowledge in and of teaching grows in practice through a research process" (Jaworski, 2007, p. 201). We see simulations as a form of "meta-task" and envision this special issue as a platform to describe our current understanding of simulations of practice for the education of mathematics teachers. Subgroups formed during the working group will be well positioned to contribute and advance this work.

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