# DEVELOPING PRACTICAL MEASURES TO SUPPORT THE IMPROVEMENT OF GEOMETRY FOR TEACHERS' COURSES

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*This paper reports on an ongoing project aimed at developing an inter-institutional system of professional support for the improvement of the Geometry for Teachers (GeT) courses that mathematics departments teach to preservice secondary teachers. In alignment with the literature on improvement science (see Bryk et al., 2015; Lewis, 2015), it is essential to develop and deploy practical measurement tools to inform improvement. We describe three key forms of measurement our team has been using to drive this work as well as some preliminary findings.*

Keywords: Geometry and Geometrical and Spatial Thinking, Post-Secondary Education, Teacher Education-Preservice

## Introduction

This paper reports on the development and use of instruments to study instruction at the college level, specifically focused on the undergraduate geometry course offered by many university mathematics departments for pre-service teachers (Geometry for Teachers, or GeT hereafter). The teaching of geometry in schools has been identified as weak and resulting in unacceptable levels of student performance (Clements, 2003; NCES, 2012). High school leaders have often described it as hard to find high school teachers who want to, and can, teach geometry. Thus, we consider increasing instructional capacity for high school geometry a systemic problem. We pose that the university GeT course is a crucial lever in preparing pre-service teachers to teach high school geometry, particularly as one of the few college mathematics courses that directly connects to high school geometry content. Yet, many mathematics education scholars (Zazkis & Leikin, 2010) and mathematicians (Wu, 2011) have questioned whether the content of university courses is sufficiently connected to what secondary teachers need to do their work. This led us to investigate the connection between participation in GeT courses and increased mathematical knowledge for teaching geometry (MKT-G).

We utilize the networked improvement communities approach described by Bryk, Gomez, Grunow, and LeMahieu (2015) to increase the capacity for geometry instruction at the K-12 level. This approach uses an organizational learning perspective in which all stakeholders are involved in the articulation of common problems as well as the design, monitoring, and continuous improvement of strategies to solve those problems. Bryk et al. (2015) indicate that achieving improvement at scale requires that particular attention be paid to variation within the system, and that understanding the sources of that variation is critical to achieving the goal of improvement research projects.

Within the scope of our project, we are currently measuring variation by gathering and analyzing data in the following ways: 1) through an assessment instrument which tests GeT students' mathematical knowledge for teaching geometry (MKT-G); 2) with end-of-term questionnaires used to collect data on course content, student composition, and technology use; and 3) through the use of instructional logs, administered at three points throughout the term, aimed at gathering more nuanced information about instructional practices in GeT courses. In this paper, we describe these measures in more detail, and explore the potential use of the data gathered from these three instruments.

## Theoretical Framework

### Taking a Networked Improvement Communities Approach

Bryk and colleagues (2015) set out a strategy for educational research and development called networked improvement communities (NICs), meant to harness the creative power of networked communities within an improvement science framework. One of the core principles of this approach is to "see the system that produces the current outcomes" (p. 57). The authors claim that oftentimes the traditional approach to solving complex educational problems is to quickly look for solutions to the problems without fully understanding the complex systems producing such problems. Russell and colleagues' (2017) lay out a framework for the implementation of networked improvement communities and claim that education researchers and practitioners often struggle to find effective solutions as "[the] field is not organized to learn systematically, accumulate, and disseminate the practical knowledge needed for the improvement of teaching and learning" (p. 1). A second core principle of the NIC approach is to "focus on variation in performance" (Bryk et al., 2015, p. 13), with an acknowledgement that complex systems often result in considerable variation. This focus on variation reduces the tendency to oversimplify by looking for universal solutions to the problem, and supports more realistic consideration of which solutions work, for whom, and under what set of conditions.

To study variation in performance, NICs must make use of consistent and practical measurement tools to inform the improvement efforts made by those involved (Morris & Hiebert, 2011). Additionally, a network hub is responsible for aggregating the data from those measures, as well as key insights that emerge, and feeding it back to the network so that innovations can be tested and integrated into new contexts (Bryk et al., 2015). We took on the role of network hub by creating a community of stakeholders and developing measures that would inform the community about progress toward a solution. As part of our work in this role, we facilitate opportunities for the network to engage with each other as well as the data being generated.

### Variation in GeT Courses

Grover and Connor (2000) conducted a survey of 108 randomly selected U.S. colleges and universities to study the content and instructional practices of geometry courses through analysis of questionnaire responses and selected syllabi. They found considerable variation in content (e.g., geometries covered and axiomatic approaches employed), pedagogy (e.g., lecture, group work, and alignment with NCTM Professional Teaching Standards), and assessment (e.g., in-class examinations, homework, and forms of alternative assessment).

To see if this variation in GeT courses persists, we conducted an analysis of GeT course artifacts from 17 initial participants in this study (i.e., syllabi supplemented with course catalogs, interviews, and poster sessions). The results of our study of GeT courses were very well aligned with Grover and Connor's (2000) results, indicating that there still exists wide variation in GeT courses.

## Methods

As the project is intended to help provide support for the instructors of university GeT courses, it is important to work collaboratively with them in order to help define the problem space. We began by locating institutions with teacher preparation programs—as we are interested in the undergraduate geometry course serving teachers, rather than geometry courses in general. Within those institutions, we looked in mathematics departments for geometry courses serving secondary mathematics pre-service teachers and identified instructors of those courses as the natural members of this community. We conducted a set of 19 initial interviews to gather their professional perspectives as instructors of GeT courses. We organized the interview data according to common challenges that many of the instructors noted as embedded in the work of teaching GeT courses. These common problems, or *tensions*, that many instructors identify in their own work we identify as inherent in the work of teaching college geometry, which are perhaps distinct from the set of problems or defects that an outsider might identify in the work of a GeT instructor (Herbst, Milewski, Ion, & Bleecker, 2018; Milewski et al., 2019).

In June 2018, we held a two-day conference which gathered more than fifty various stakeholders involved in the teaching of the GeT course (GeT instructors, high school district leaders, high school teacher leaders, and education researchers). Using what we had learned from the interviews with GeT instructors about the tensions that come with teaching the GeT course, we worked with conference attendees with a common problem in mind: to improve capacity for teaching K-12 geometry. Two of the tensions most salient in the conversations of the conference working groups were:

- *Knowledge tension:* This tension arises in contexts where GeT instructors need to consider the question "What is the *knowledge* students need to learn in the GeT course?" On one side of the tension, the course exists to provide novice teachers with the knowledge needed to teach the secondary geometry course. This includes knowledge like, "What are the common ways that students think about parallelograms?" or "How does one design good proof problems for high school geometry students?" On the other side, GeT is a university mathematics course that needs to be comparable in terms of rigor with other advanced mathematical coursework that students are expected engage with. Ideally, the course could include both kinds of mathematical knowledge, however, time is a limited resource and GeT instructors need to decide what to prioritize.
- *Experiences tension:* This tension arises in contexts where GeT instructors need to consider the question "What *experiences* can support students' learning in the GeT course?" On one side of the tension, as GeT is a mathematics course, it would be reasonable to assume that students in the course are learning to think and act like mathematicians. Thus, students should experience opportunities to engage in reasoning, problem-solving, and other mathematical practices, which could serve them as future teachers as it would engage them in work similar to what they would have their eventual students engage. On the other side, the course acts a service course for pre-service teachers who are apprenticing into the work of a teacher, and this would suggest that they should experience opportunities to practice the work of teaching, like explaining a mathematical concept to the whole class or to each other, or interpreting fellow students' work.

Rallying around these tensions, the conference participants collaborated to establish a set of projects to engage with over the course of the next four years that would address these tensions. Through the use of an online collaboration platform, teams of GeT instructors unite to work on a common project. An example of one of these projects is the *Geometry Knowledge Needed for Teaching* group, where 17 GeT instructors are currently working together to answer the questions: *What is the mathematical content knowledge needed for teaching high school geometry? What do we need to teach our college geometry students so they begin their teaching careers with the appropriate knowledge?* To answer these questions and discuss the tensions that come up in the work of teaching the GeT course as highlighted above, this group meets over video conference once a month, as well as throughout the month asynchronously, on an online forum.

In addition to collaborating on these projects, GeT instructors also volunteer to take and administer various measurement instruments throughout the year. The details of these instruments are outlined in the following section. We understand the potential pitfalls of large variability in those courses at the same time that we honor the academic freedom and professional judgment of instructors. The project forums, like the one described above, are meant to promote improvement by creating community. To inform the improvement efforts made by instructors who participate in the community, we track variation in course offerings and variation in outcomes (in terms of MKT-G), with the hope to ascertain whether changes in the former are related to changes in the latter. If reduction in course variability could be correlated with increases in MKT-G gains, this would lend credibility to the notion that reducing variability in the course offerings would be desirable.

## Measures

### Mathematical Knowledge for Teaching Geometry (MKT-G)

Herbst and Kosko (2014) have developed and validated an instrument used to measure MKT-G that targets four of the six domains of content knowledge for teaching identified by Ball, Thames, and Phelps (2008). These items are closed-ended and graded as either correct or incorrect. In a prior study, the instrument was administered to 387 inservice teachers from 47 states, as well as 195 preservice teachers from 7 institutions. Results from these tests showed that preservice teachers' scores on the MKT-G after taking a GeT course were .23 standard deviations lower than inservice teachers' MKT-G scores. While this result was unsurprising, it raised the question of whether the test could detect differences in MKT-G gained over the course of a semester of GeT instruction.

Over the course of the project, we will be collecting data from GeT students who will take this instrument at the beginning and end of their GeT course. Progress toward readiness to teach high school geometry will be measured in terms of average change in scores between pre- and post- assessments. Throughout the term, we share three sets of reports with participating GeT instructors: 1) individualized reports showing how their students performed on the pre-test compared to the national sample of practicing teachers; 2) individualized reports showing how their students performed on both the pre- and post- tests, with change in scores over the term reported in relation to the national sample of inservice teachers; and 3) aggregated reports showing each instructor how students from their institution performed on average compared to the overall average scores of all GeT students in this study. Our hope is that the instructors will use the individualized and aggregate results from these reports to reflect on and inform their course design and teaching practices, as well as spark conversations within the network about rationales, costs, and benefits of any potential changes in practice with respect to the knowledge and experiences tensions identified in their work.

**Course Questionnaire and Instructional Logs**

Inspired by the work of Grover and Connor (2000), we have developed a course questionnaire to be taken by GeT instructors at the end of each term in which the course is offered. In the questionnaire, instructors are asked about the course sequence at their institution, student composition, textbook use, dynamic geometry software use, as well as types of geometry taught in relation to various axiomatic approaches. Some examples of questions include:

- *Approximately what percentage of the students enrolled in the geometry course are prospective high school or middle school teachers?*
- *[What] courses are students required to take before taking this section of geometry?*
- *List the most important Euclidean geometry theorem(s) you cover.*

To better understand the experiences students are having in GeT courses, we have developed a set of instructional logs to be completed by GeT instructors three times throughout the term in which the course is offered. In each of these logs, instructors are asked to report on the planning and implementation of two lessons. As part of each log, instructors are asked to share the set of expectations for the lesson, as well as the amount of class time they devoted to each expectation. Following this, they are asked about the instructional approaches employed throughout the lesson, reporting how much class time was spent on each (e.g., whole class lecture, student presentations). These set of logs serve as one way of describing the experiences students are having in the GeT courses.

We expect these results to be useful in gauging variability across GeT courses, with the hypothesis that variability will be reduced as instructors participate in the networked improvement community. One reason that variability may be reduced over time is that GeT instructors will have access to these aggregated reports, may see evidence of other instructional practices being employed within the community, and will participate in discussions of the data all together.

## Preliminary Results and Discussion

**Fall 2018 MKT-G Results**

In Fall 2018, 94 students from six GeT courses completed the pre- and post- MKT-G assessment. The level of MKT-G possessed by each student was provided as IRT scores derived from the IRT (item-response theory) model which takes account of the difficulty and discrimination of items (how well each item can discriminate participants' ability). In Table 1, we report on the pre- and post-IRT scores of the 94 GeT students, as well as the IRT gains from this sample. An average score of 0 for the sample of GeT students would indicate a level of knowledge equivalent to the average amount of knowledge of a national sample of in-service teachers (N=605) previously collected. The negative IRT scores reported indicate that GeT students' knowledge is below the average amount of MKT-G of the in-service teacher sample (Ayala, 2009; Crocker & Algina, 2006). The positive gain in IRT scores (0.15) between the pre- and post-test indicates that GeT students made progress over the course of the term, on average, with respect to MKT-G, and the gain was significant ($t(93)=-2.85$, $p=0.0027$). The use of the data from practicing teachers allows us to understand the GeT students' relative standing to the practicing teachers.

**Table 1: Pre- and Post- test Score Comparison of GeT Students (N=94) and ISTs (N=605)**

| | IRT θ | | |
|---|---|---|---|
| | **PRE_IRT** | **POST_IRT** | **GAIN_IRT** |
| **GeT students (N=94)** | **-1.29** | **-1.13** | **0.15** |
| **ISTs (N=605)** | **0.00** | **0.00** | **N/A** |

Despite overall positive gains by students on average, only four of the six GeT instructors showed positive gains in their students' IRT scores. The graph below (Figure 1) represents the standing of the GeT students' IRT scores in pre-test and post-test within the distribution of IRT scores from the national sample of in-service teachers. The GeT students' post-test scores are shifted slightly to the right from their pre-test scores, indicating improvement in MKT-G over the semester of GeT instruction. Compared to the ISTs, most of the 94 students have lower than average level of MKT-G in both pre- and post- test scores and this result is consistent with the prior study.

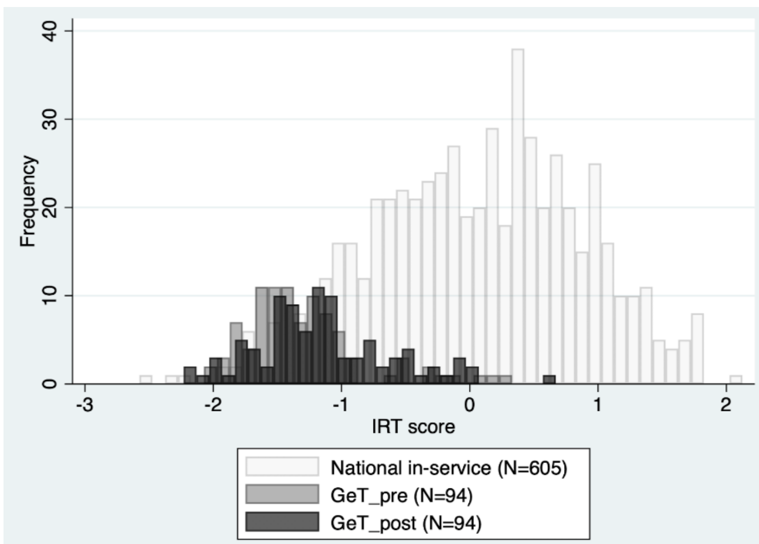


**Figure 1: Histogram of the MKT-G Scores of GeT Students (N=94) and ISTs (N=605).**

## Course Questionnaire and Logs Results

In our first distribution of the course questionnaire, while we have a small sample of instructors (N=7), we are able to make similar observations as Grover and Connor (2000) with respect to large variability in the GeT courses. Some sample responses are shown in Table 2.

**Table 2: Descriptive Results from Two Course Questionnaire Prompts (N=7)**

| | <=50% | 51% - 80% | >81% |
|---|---|---|---|

| | | | |
|---|---|---|---|
| Q9: *Approximately what percentage of the students enrolled in the geometry course are prospective high school or middle school teachers? (N=7)* | n=2 | n=2 | n=3 |
| Q31: *Indicate what percentage of the course is devoted to Euclidean geometry. (N=7)* | n=2 | n=2 | n=3 |

Also, in the Fall 2018 term, we collected a total of 20 instructional logs from five GeT instructors. An example of results from one such question is shown in Figure 2.

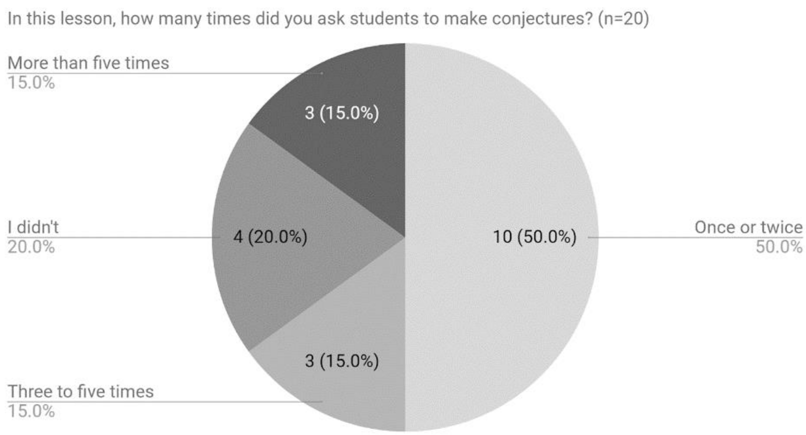


**Figure 2: Aggregated Responses (N=20) to One of the Instructional Log Questions**

The results from the course questionnaire and instructional logs have the potential to provide evidence of ways in which the knowledge and experiences tensions are present in the work of teaching the GeT course. We may find that GeT instructors with student populations that are less than 50% PSTs are less likely to spend considerable time on Euclidean Geometry, for example. Other uses for the data generated by these instruments is to measure variability across courses over time. The results shared here are from the first instructional term of this study, but we expect to look at how courses change over time when that data becomes available. For the time being, the instruments described here seem promising to allow us to describe the variability present across offerings of the GeT course.

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