"GETTING BETTER AT STICKING WITH IT": EXAMINING PERSEVERANCE IMPROVEMENT IN SECONDARY MATHEMATICS STUDENTS

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Supporting students' in-the-moment perseverance is a vital objective in mathematics education because it promotes learning with understanding. Yet, little is written about if and how student perseverance can improve over time. I examine the specific ways in which secondary students improved their perseverance as they engaged with challenging mathematical tasks over six weeks. The results show that encouraging students to initially attend to their conceptual thinking can prolong productive effort upon impasse and explicitly improve selection of problem-solving strategies and affect regulation. These findings suggest learning environment designs that provide consistent opportunities for students to practice (and improve) their perseverance.

Keywords: Perseverance, Problem Solving, High School Education, Metacognition

In the context of problem-solving, perseverance is initiating and sustaining in-the-moment productive struggle in the face of mathematical obstacles, setbacks, or discouragements. The notions of tolerating uncertainty and overcoming obstacles have long been recognized as key processes supportive of learning with understanding (Dewey, 1910; Festinger, 1957; Polya, 1971). These ideas have been echoed for mathematics learning because students make meaning through productive struggle, or as they grapple with mathematical ideas that are within reach, but not yet well formed (Kapur, 2010, 2011; Hiebert, 2013; Hiebert & Grouws, 2007; Warshauer, 2014). Additionally, reconciling times of significant uncertainty (i.e., a perceived impasse) is critical for mathematics learning. The processes of struggle to approach, reach, and make continued progress despite a perceived impasse puts forth cognitive demands upon the learner that are conducive for development of conceptual ideas (Collins, Brown, & Newman, 1988; VanLehn et al., 2003; Zaslavsky, 2005). As such, supporting in-the-moment student perseverance has been made explicit as a way of improving teaching and learning in mathematics education, with the expectation that such support will nurture students' perseverance to improve over time (CCSS, 2010; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2014).

Supporting and Improving Student Perseverance with Mathematics Tasks

Several research efforts have sought to make explicit classroom practices that support student perseverance with challenging mathematics, yet little is known about how such practices can help improve student perseverance in specific ways over time. Studies aiming to unpack the nature of productive struggle (DiNapoli, 2019; DiNapoli & Marzocchi, 2017; Kapur, 2009, 2011; Sorto, McCabe, Warshauer, & Warshauer, 2009; Warshauer, 2014) generally have found that providing consistent opportunities for students to engage with unfamiliar mathematical tasks encouraged more variability in problem-solving strategies and greater learning gains, compared to providing consistent opportunities to engage with more procedural mathematics. Other researchers (Bass & Ball, 2015; DiNapoli, 2016, 2019; Kapur & Bielaczyc, 2012; Stein & Lane, 1996) have explored the nature of perseverance by investigating the effects of implementing classroom tasks with familiar entry points yet a complex structure. In general, there is empirical support for students leveraging opportunities within such low-floor/high-ceiling tasks to persevere in their efforts despite challenge and seemingly make mathematical progress. Additional scholarship has focused on the role of teacher feedback to encourage in-the-moment perseverance (Freeburn & Arbaugh, 2017; Housen, 2017; Kress, 2017; Sengupta-Irving & Agarwal, 2017). A synthesis of the findings from these works suggests non-leading teacher questioning encouraging student metacognition can facilitate more independent thinking and creative problem-solving during times of confusion.

Building from the previously mentioned literature, a recent study (DiNapoli, 2018) developed an operationalization of perseverance called the Three-Phase Perseverance Framework (3PP) (see Table 1), an analytical perspective by which perseverance can be qualitatively described and measured. The 3PP reflects perspectives of concept (Dolle, Gomez, Russell, & Bryk, 2013; Middleton, Tallman, Hatfield, & Davis, 2015), problem-solving actions (Pólya, 1971; Schoenfeld & Sloane, 2016; Silver, 2013;), self-regulation (Baumeister & Vohs, 2004; Carver & Scheier, 2001; Zimmerman & Schunk, 2011), and making and recognizing mathematical progress (Gresalfi & Barnes, 2015).

Using the 3PP, this study (DiNapoli, 2018) investigated how prompting algebra students to create an artifact of their personal conceptualization of a mathematical task (Anghileri, 2006) could support perseverance at times of impasse. The results show how scaffolding tasks in this way encouraged making an additional attempt at solving via re-initiating and re-sustaining mathematically productive effort at impasse significantly more so than on tasks without such scaffolding. Participants articulated that the conceptual thinking recorded after engaging with the scaffold prompt acted as an organizational toolbox from which to draw a fresh mathematical idea, or a new connection between ideas, to use to re-engage with the task upon impasse and to continue to productively struggle to make sense of the mathematical situation. Participants were persevering in problem-solving cyclically, with each additional attempt as a new opportunity to productively struggle with a given task scaffolded by their own conceptual ideas (see Figure 1). Without recording their conceptual thinking on non-scaffolded tasks, participants felt frustrated after a setback and often gave up without making an additional attempt at solving.

Table 1: Three-Phase Perseverance Framework

Entrance Phase	
Clarity	Objectives were understood
Initial Obstacle	Solution pathway not immediately apparent
Initial Attempt Phase	
Initial Effort	Engaged with task
Sustained Effort	Used problem-solving heuristics to explore task
Outcome of Effort	Made mathematical progress
Additional Attempt Phase (after perceived impasse)	
Initial Effort	Engaged with task
Sustained Effort	Used problem-solving heuristics to explore task
Outcome of Effort	Made mathematical progress

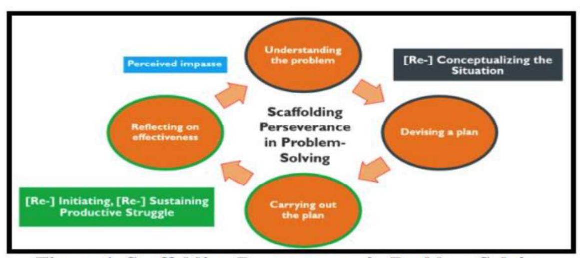


Figure 1. Scaffolding Perseverance in Problem-Solving

Despite the recent research on ways to support student perseverance in the moment, questions remain about whether these practices help nurture student perseverance to improve over time. There exists some work that shows evidence of student improvement in measures of grit (Polirstok, 2017) and time-on-task (Niemivirta & Tapola, 2007), however such work relies heavily on summative outcome variables that reveal little about the ways in which learners were challenged, overcame setbacks, and developed mathematical understanding, if they did at all (DiNapoli, 2018). Research on perseverance can produce insights into effective practices by which to learn mathematics with understanding, yet much of the empirical evidence of student perseverance have been situated in single points of time with little or no exploration of how those perseverance experiences may be related or demonstrate signs of specific improvement. Thus, the present study aimed to address these lingering concerns about whether and how student perseverance, when supported properly, can improve over time.

Methods

The participants for this qualitative study were 10 ninth-grade students from one suburban-area algebra class in a Mid-Atlantic state. They were purposely chosen to have demonstrated, via pretest, the prerequisite knowledge necessary to initially engage with each mathematical task included in the study. Each participant was observed engaging with five tasks across six weeks, approximately one per 7-10 days. These tasks were rated as analogous by the Mathematics Assessment Project because of their low-floor/high-ceiling structure, two objectives, and generalization requirements. Three tasks were randomly chosen to be scaffolded with conceptualization prompts (Anghileri, 2006), and two tasks were randomly chosen to be non-scaffolded. The conceptualization prompt embedded into the scaffolded tasks was "*Before you start, what mathematical ideas or steps do you think might be important for solving this problem? Write down your ideas in detail*." Each participant worked on these set of five tasks in a random task order. The results of this paper unpack participant perseverance across the three scaffolded tasks: Cross Totals, Sidewalk Stones, and Skeleton Tower. For context, Cross Totals asked students to generalize rules about how to arrange the integers 1-9 in a symmetric cross such that equal horizontal and vertical sums would be possible or not possible; Sidewalk Stones asked students to generalize rules about an evolving two-dimensional pattern of different types of stones; Skeleton Tower asked students to generalize rules about an evolving three-dimensional tower of cubes. All tasks are available for view at www.map.mathshell.org/.

For each task and participant, I conducted think-aloud interviews while they worked on a task and video-reflection interviews immediately after they finished working. Additionally, once a participant had engaged with all five tasks (and thus all five think-aloud interviews and video-reflection interviews), I conducted an exit interview to elicit reflections on the overall experience working on the five tasks. In all, I conducted 11 interviews with each participant, or 110 interviews in total for this study. I adopted an inductive coding process (Strauss & Corbin, 1990) using the 3PP to capture the ways in which students were persevering, or not, across the five tasks (DiNapoli, 2018, Table 1). The 3PP considered if the task at hand warranted perseverance for a participant (the Entrance Phase), the ways in which a participant initiated and sustained productive struggle (the Initial Attempt Phase), and the ways in which a participants re-initiated and re-sustained productive struggle, if they reached an impasse as a result of their initial attempt (the Additional Attempt Phase). A participant was determined to have reached a perceived impasse if they affirmed they were unsure how to continue (VenLehn et al., 2003). Mathematical productivity was determined based on if the participant perceived themselves as better understanding the mathematical situation as a result of their efforts (Gresalfi & Barnes, 2015).

I used a point-based analysis with the 3PP to help inform deeper investigation of the ways in which participants persevered. Each participants' experiences with each task were analyzed using the framework, and each component in the Initial Attempt and Additional Attempt Phases were coded as 1 or 0, as affirming evidence or otherwise, respectively. Since each task had two objectives and six components per objective, there were 12 framework components to consider, per participant, per task. Thus, 3PP scores ranged from 12 to 0, depicting optimal to minimal demonstrated perseverance in this context, respectively. I conducted regression analyses to compare the ways in which 3PP scores were changing over time for participant work on scaffolded tasks and on non-scaffolded tasks. I also inductively coded interviews to uncover from the participant perspective how and why their perseverance may have been changing over time. I enlisted help from two independent coders to analyze participant perseverance and their reasons for doing so. Our inter-rater reliability was 93%.

Results

Overall, participants' perseverance across mathematical tasks improved over time, more so on scaffolded tasks than on non-scaffolded tasks. Participants' demonstrated perseverance on the three scaffolded tasks improved in quality over time as evidenced by increasing mean 3PP scores (see Figure 2). Participants' demonstrated perseverance on the two non-scaffolded tasks also improved in quality over time. However, the average rate at which scores on non-scaffolded tasks improved was three times less than the rate of improvement of scores on scaffolded tasks.

A simple linear regression was calculated to predict participants' 3PP scores on their second scaffolded task based on their first scaffolded task (see Table 2). A significant regression equation was found ($F(1, 8) = 58.593$, $p < .001$), with an R^2 of .880, indicating participants' 3PP scores on their first scaffolded task explained 88% of the variance in their 3PP scores on their second scaffolded task. Also, participants' 3PP scores on their first scaffolded task was a significant predictor of their 3PP scores on their second scaffolded task, with a one-point increase on their first scaffolded task predicting a .727-point increase on their second scaffolded task. A multiple linear regression was calculated to predict participants' 3PP scores on their third scaffolded task based on their first and second scaffolded tasks (see Table 2). A significant regression equation was found ($F(2, 7) = 10.741$, $p = .007$), with an adjusted R^2 of .684, indicating a participants' 3PP scores on their first and second scaffolded tasks, together, conservatively explained 68.4% of the variance in their 3PP scores on their third scaffolded task.

Table 2: Summary of Regression Analyses of Perseverance Scores

Task Type	Comparison	R^2	Adjusted R^2	B	SE(B)	Sig. (p)
Scaffolded	1st → 2nd	.880	.865	.727	.095	< .001
	1st/2nd → 3rd	.754	.684	.385/.234	.348/.449	.007 (.305/.618)
Non-Scaffolded	1st → 2nd	.553	.497	.680	.216	.014

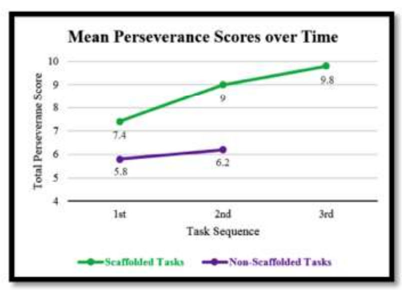


Figure 2. Mean Perseverance Scores Over Time

A simple linear regression was also calculated to predict participants' 3PP scores on their second non-scaffolded task based on their first non-scaffolded task (see Table 2). A significant regression equation was found ($F(1, 8) = 9.879$, $p = .014$), with an R^2 of .553, indicating participants' 3PP scores on their first non-scaffolded task explained 55.3% of the variance in their 3PP scores on their second non-scaffolded task. Also, participants' 3PP scores on their first non-scaffolded task was a significant predictor of their 3PP scores on their second non-scaffolded task, with a one-point increase on their first non-scaffolded task predicting a .680-point increase on their second non-scaffolded task. Thus, participants' 3PP scores were significantly improving over time, more so on scaffolded tasks than on non-scaffolded tasks.

During their exit interviews, participants revealed they noticed improvements, over time, in their engagement with the challenging mathematical tasks. Most participants (8 out of 10) mentioned they thought they were getting better, somehow, as they had more practice with these types of problems. Several participants (6 out of 10) mentioned their improved work on tasks specifically prompting them to conceptualize the situation prior to starting, i.e., the scaffolded tasks. Arguably the most noticeable improvement, from the participants' point of view, was affective in nature. Many participants (7 out of 10) explained in their exit interview that they felt like they were getting better at handling the stress of the situation as they reached impasses within their work on mathematical tasks they did not know how to solve. Some participants (4 out of 10) reported cognitive gains, believing the way they were thinking about the mathematics was changing for the better, over time, and that their problem-solving skills were improving.

Illustrative Case: Sandra's Perseverance Improvement across Scaffolded Tasks

Unpacking representative participant Sandra's experiences across her scaffolded tasks generally illustrates how perseverance improved over time, mostly by supporting participants to make a more quality additional attempt at solving. Sandra encountered Cross Totals first (first overall), then Sidewalk Stones (second overall), and then Skeleton Tower (fourth overall). Importantly, Sandra passed through the 3PP Entrance Phase on all three tasks by affirming she understood all of the objectives, but was not immediately sure how to achieve them. She earned a 3PP score of 6 for her perseverance on Cross Totals, a score of 9 on Sidewalk Stones, and a score of 12 on Skeleton Tower (see Table 3). Like her peers, Sandra made no additional attempt at solving while working on her first task, but progressively improved her perseverance after a setback in the Additional Attempt Phase as she had more experiences with tasks necessitating productive struggle. For comparison, Sandra earned 3PP scores of 6 for her work on both non-scaffolded tasks. She encountered non-scaffolded tasks third and fifth overall.

Table 3: Sandra's 3PP scores for Cross Totals, Sidewalk Stones, and Skeleton Tower

INITIAL ATTEMPT PHASE	Cross Totals (S, 1^{st})		Sidewalk Stones (S, 2^{nd})		Skeleton Tower (S, 4^{th})	
Evidence of Perseverance	Obj. 1	Obj. 2	Obj. 1	Obj. 2	Obj. 1	Obj. 2
Initial Effort	✓	✓	✓	✓	✓	✓
Sustained Effort	✓	✓	✓	✓	✓	✓
Outcome of Effort	✓	✓	✓	✗	✓	✓
ADDITIONAL ATTEMPT PHASE	Cross Totals (S, 1^{st})		Sidewalk Stones (S, 2^{nd})		Skeleton Tower (S, 4^{th})	
Evidence of Perseverance	Obj. 1	Obj. 2	Obj. 1	Obj. 2	Obj. 1	Obj. 2
Initial Effort	✗	✗	✓	✓	✓	✓
Sustained Effort	✗	✗	✓	✓	✓	✓
Outcome of Effort	✗	✗	✗	✗	✓	✓
Total Perseverance Score	**6**		**9**		**12**	

Notes: S = Scaffolded Task; Obj. = Objective; 1^{st} = First task overall; 2^{nd} = Second task overall; 4^{th} = Fourth task overall

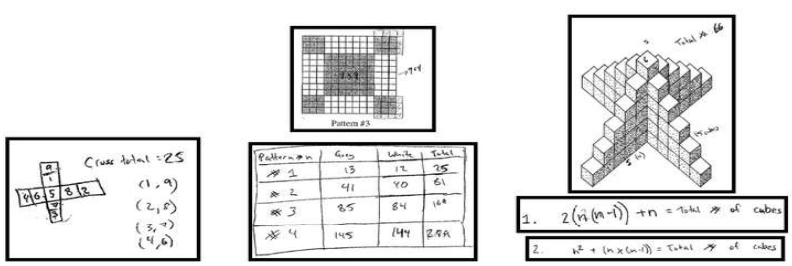
Cross Totals. Sandra began Cross Totals (her first scaffolded/overall task) by responding to the scaffold prompt and initiated her effort toward both objectives by stating her plan to reason about the magnitude of integers. She sustained her effort toward both objectives by exploring the logical ramifications of distributing different integers within the cross. Sandra perceived she was making mathematical progress in better understanding the situation by which cross totals may be possible or impossible. She found one example of a possible cross total (see Figure 3a), but soon after reported she had reached an impasse. During her think-aloud she said, "I know this one works, but it's just one example. These things aren't for all cross totals. It wants me to get it in

Otten, S., Candela, A. G., de Araujo, Z., Haines, C., & Munter, C. (2019). *Proceedings of the forty-first annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. St Louis, MO: University of Missouri.

general. I don't know." Although she recognized she had more work to do to better generalize the situation, Sandra decided to record two admittedly incorrect rules based on what she had found and not to make an additional attempt at exploring the problem. Sandra clarified this decision during her video-reflection interview: "In this moment I was just overwhelmed. I felt any other rules, general rules, would be more complicated to get and I just ran out of ideas."

Sidewalk Stones. Sandra's work on Sidewalk Stones (her second scaffolded/overall task) showed specific evidence of perseverance improvement compared to her work on Cross Totals. She similarly began by recording her ideas under the scaffold prompt about the mathematical relationships at play and ways to explore them. Importantly, Sandra revealed during her think-aloud that she was planning ahead when she said, "I was still stuck, or couldn't find a general rule, I could make up examples and check them." Sandra initiated her effort toward both objectives of the task by stating her plan to make a table of values representative of the different kinds of stones. She sustained her effort toward both objectives by searching for patterns within the table that could help her discern how the gray and white stones were changing. Despite some perceived progress, Sandra reached a perceived impasse about the generalization objectives. During her think-aloud she said, "How do I know about Pattern #*n*? This general stuff is hard." She clarified this moment during her video-reflection interview: "I was starting to see the gray pattern, but I had no clue about Pattern #*n*. I was thinking about quitting. I got farther than I thought, but general stuff, it's hard, like finding #*n*. I was stumped."

Despite the urge to quit, Sandra decided to keep working by amending her current plan and changing strategies. She said during her think-aloud, "Well, I guess I can make another example like I said I would." Sandra was referring to her scaffold work, in which she stated that if she "was still stuck" with the objective of generalization, she could "make up examples and check them." Thus, drawing from her earlier conceptualization work prompted by the scaffold, Sandra decided to make an additional attempt at solving Sidewalk Stones. She re-initiated her effort by choosing a different problem-solving heuristic, drawing a partial diagram of a new pattern of stones, and re-sustained her effort by amending her table of values and searching for a pattern amongst all available data (see Figure 3b). Sandra ultimately did not successfully find two general rules, and, by her own admission, she did not believe she better understood the mathematical situation as a result of her additional efforts. Yet, compared to her engagement with Cross Totals, Sandra's perseverance with Sidewalk Stones was much improved primarily because she included in her initial conceptualization work a plan for if she got "stuck." Sandra's backup plan helped her make an additional attempt at solving the problem upon a perceived impasse, an additional attempt she did not make one week earlier with Cross Totals.



(a) (b) (c)

Figure 3. Samples of Work from Sandra's Engagement on Scaffolded Tasks

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Skeleton Tower. Sandra's work on Skeleton Tower (her third scaffolded/fourth overall task) showed even further evidence of perseverance improvement compared to prior tasks. Under the scaffold prompt, she first recorded her mathematical ideas and problem-solving plans, which included "adding up all the cubes" and "knowing the area of a square, s^2, to maybe get an equation." During her video-reflection interview, Sandra clarified that she was preparing for the two generalization objectives: "I thought maybe the s^2 would help me get a general answer with an equation. Something I could just plug height into. The general stuff has been hard in all these problems so I thought using variables might help." Sandra initiated her effort toward finding one general rule by stating she would look for patterns in the diagram, and sustained this effort by counting the cubes in various parts of the tower and reasoning about how to algebraically represent these parts if the height of the tower was n. Sandra visualized piecing two legs of the tower together, stating in her think-aloud, "it's like two squares put together if you flip two of the legs." Eventually, she posited, "I think $(n\text{-}1)^2$ x 2 could be my rule for the total blocks because that's like two squares made up of the legs." Then, Sandra tested her equation and realized it was incorrect. Despite her ample perceived mathematical progress on this task, Sandra shared that she was "annoyed that it wasn't working" and was "stuck."

Despite the perceived impasse as a result of her first attempt to find one general rule, Sandra eventually decided to make an additional attempt by revisiting her past idea about the area of a square. During her video-reflection interview, she clarified, "Well I had that plan to use s^2 and I saw it here and really thought it was a good idea, so I decided to try it again, to maybe think about it another way." Sandra's earlier conceptualization work of Skeleton Tower helped her rethink about deconstructing the tower into squares and algebraically modeling the situation by considering the expression s^2. During this time when she was most "annoyed", it was revisiting her scaffold work that encouraged her to make an additional attempt at solving. From here, Sandra re-initiated her effort by deciding to draw parts of the tower separately, and re-sustained her effort by thinking-aloud about how to piece together the deconstructed parts of the tower. After diligent exploration she exclaimed, "Oh! It's not a square, it's a rectangle!" During her video-reflection interview, Sandra clarified these moments: "I wasn't sure that I was doing it right, but I kept going here. I knew I had a good plan and then, boom, it happened. I figured it out." Sandra went on to use algebraic expressions to model her discovery and ultimately wrote a correct rule that generalized the situation (see Figure 3c). While recapping her success during her video-reflection interview, Sandra cited the importance of incorporating a general equation in her initial conceptualization. She said, "Having an equation (Area = s^2) in mind from the start was a big help. It helped me keep going and helped me with the general part."

Unlike her work on Cross Totals and Sidewalk Stones, on which she worked toward both task objectives simultaneously, Sandra worked toward one objective at a time on Skeleton Tower. This meant that after her breakthrough above, she essentially started over to try to generalize the situation in a different way. Sandra used new strategies and a different point of view to persevere a second time on Skeleton Tower. As she did before, Sandra admitted she had reached another impasse during her first attempt toward the second objective, but ultimately changed her point of view to overcome the obstacle and make an additional, successful attempt (see Figure 3c). Compared to her work with Sidewalk Stones and Cross Totals, Sandra's perseverance with Skeleton Tower was much improved mainly because she planned to work with variables from the outset, during her scaffold work, to generalize the situation. Sandra's more-refined initial planning helped her make an additional attempt at times when she was most frustrated and go on to make progress and solve the problem.

Overall experience. Sandra's experiences with her scaffolded tasks helped illuminate the ways in which participants were improving in their perseverance. Sandra's perseverance gains were most noticeable in the improved quality of her additional attempt at solving a task after reaching an impasse. Sandra's scaffold work – recording her conceptualization of the mathematical situation at hand – played a role in why her perseverance changed because the types of ideas she recorded changed over time as well. During her exit interview, Sandra explained how she thought she was improving in the way she engaged with scaffolded tasks:

> I got better at writing out my ideas in those problems. When you keep doing it you get better at the planning stuff. All the problems ask for a general rule, and I started to learn about how to do that. I was ***getting better at sticking with it*** (emphasis added), too. Like after getting stuck or making a mistake. You just have to get into the problem and maybe even make some mistakes to figure it out. That got easier for me, not getting too annoyed after mistakes.

This perspective is indicative of how cognitive and affective changes impacted participants' perseverance improving over time for scaffolded tasks. Sandra was "getting better at sticking with it" because she better prepared strategies for generalization objectives and better regulated her frustration at key moments during problem-solving. All participants encountered impasses with the scaffolded tasks, yet, as they had more experience in situations requiring perseverance and requiring planning, they persevered more and noticed cognitive and affective improvement.

Sandra earned 3PP scores of 6 on both of her non-scaffolded tasks. She did not demonstrate any evidence of specifically preparing for mathematical generalization, nor did she report in any of her interviews specific ways she was changing how she prepared. Also, Sandra did not mention if she felt better about handling the stress during work on the non-scaffolded tasks, even though they were the third and fifth task with which she worked. Sandra's experiences with non-scaffolded tasks was illustrative of most participants in this study. This suggests that exposure to such tasks warranting struggle did not alone improve participants' perseverance in specific ways, but responding to the initial scaffold prompt, in which participants attended explicitly to conceptualizing the situation, played an influential role in perseverance improvement.

Discussion and Conclusion

Several effective practices for supporting student perseverance have been made apparent in recent research, yet there is little evidence to show if and how student perseverance can improve over time – a vital objective of most reform efforts. This study extends previous research by explaining how and why student perseverance improved over time, and by unpacking this process from the student point of view. These results suggest that developing students' perseverance for solving challenging mathematics tasks may be possible through the process of deliberate practice, a systematic effort to improve performance in a specific domain (Ericsson, 2016). Through this study's design, participants essentially deliberately practiced to improve their perseverance. They worked toward specific objectives of challenge, demonstrated appropriate prior knowledge, invested their full effort and attention to make progress on these tasks (relying heavily on self-control to not give up at moments of impasse and continue to persevere), were a self-source of feedback when recognizing a setback and modified their efforts accordingly, and had opportunities to repeat and practice working through these processes every week. With the scaffolded tasks, this repeated opportunity helped students refine their strategies over that time to learn to persevere in more effective ways. Although students also had a chance to repeat the processes of deliberate practice with non-scaffolded tasks, the data showed that they did not refine their strategies in the same high-quality ways as they did with scaffolded tasks.

Interpreting the apparent perseverance improvement in this study through a lens of deliberate practice makes clearer the process by which perseverance can be nurtured and developed, over time, in students. In mathematics education, deliberate practice has been studied primarily in the context of helping students learn specific skills and developing competencies in particular mathematical domains. Yet, findings in this study suggest mathematical practices like perseverance, in addition to domain-competencies, are malleable and able to improve through processes of deliberate practice, especially with support systems in place that encourage initial conceptual thinking. More work is needed to replicate these findings in different contexts. Still, teachers should take away from this study the key tenets of a learning environment conducive of developing more perseverant learners. No students are always perseverant, but regularly providing learning opportunities that encourage conceptual thinking and prize their productive struggle can support them "getting better at sticking with it" to make meaning of mathematics.

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