EXAMINING THE TENSIONS BETWEEN HIGH QUALITY DISCOURSE AND SHARING MATHEMATICAL AUTHORITY WITH STUDENTS

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Current reform efforts challenge teachers to create more student-centered classrooms focused on high quality classroom discourse (NCTM, 2014). There are difficulties, however, teachers face in bringing this vision to fruition. Over the past three years, we have worked with a group of 7-12 teachers supporting their efforts to implement high quality classroom discourse. Although their espoused beliefs aligned with our vision of high-quality discourse, their enacted practices did not align with those espoused beliefs. Further analysis suggested that many of these challenges are related to sharing mathematical authority with their students. We intend to share a set of "Look Fors" to build awareness of features of sharing mathematical authority that might be inhibiting the quality of teachers' classroom discourse. A set of questions will also be shared to help teachers decide upon a plan of action to overcome those challenges.

Keywords: Classroom Discourse, Teacher Education-Inservice/Professional Development, Mathematical Knowledge for Teaching

Purpose of the Study

Over the past three years practicing middle and secondary mathematics teachers (n = 16) from small, rural school districts (75% from high school needs school districts) participated in a grant project to improve the quality of classroom discourse in their classroom. During the project, teachers engaged in over 300 hours of professional development focused on Effective Mathematics Teaching Practices (NCTM, 2014) that supported high quality classroom discourse. Emphasized practices included: 1) implementing tasks that promote reasoning and problem solving, 2) facilitating meaningful mathematical discourse, and 3) posing purposeful questions. In spite of our best efforts to support their growth in these areas, teachers faced challenges in improving the quality of their classroom discourse. The quote from one teacher captures the emotional toll of these challenges:

> "I am really trying to change the way I teach, I really am, but I am so frustrated! I turn my students loose with a task, but our classroom discussions seem to get bogged down! The students are frustrated, I am frustrated! So, I end up going back to my old way of teaching: telling them how to do everything!"

We sought to better understand the nature of these challenges. Analyzing teaching episodes, we explored four questions: 1) Did teachers' espoused beliefs align with their practices, 2) Were there elements of their specific elements of their classroom discourse that reflected growth, 3) What role did sharing mathematical authority play in teachers' ability to implement high quality classroom discourse, 4) As facilitators of professional development, what are the additional tools that we need to provide teachers with so that they can navigate the challenges of implementing high quality classroom discourse?

Perspective

Teacher Beliefs

Guskey (2002) suggests a sequential model of teacher change moving from the professional development experiences to the teachers' classroom practices, to changes in student learning outcomes, and lastly, to changes in teachers' beliefs and attitudes (p. 383). Using Guskey's model, it was our view that if we were able to quantify changes in teachers' beliefs aligned with our desired instructional practices then we would expect to see those beliefs enacted in their instructional practices. The REU (Research Experience for Undergraduates) Beliefs Instrument (O'Hanlon et al., 2015) seemed to be an appropriate instrument to measure these changes because it assessed the three domains of beliefs (teaching, student learning, and personal learning) that we believed were central to teachers' decision-making about their instructional practices.

High Quality Discourse

The Instructional Quality Assessment (IQA) Classroom Observation Tool (Boston, 2012) was used as a tool to quantitatively measure elements of what we viewed as high quality classroom discourse. It also served as a shared lens for teachers to reflect upon the nature of their classroom discourse. The IQA consists of two components, academic rigor and accountable talk. Each component has five aspects each of which has an accompanying rubric. These elements include 1) implementing cognitively demanding tasks (AR1 and AR2); 2) holding students accountable for their thinking (AR3, AT4, AT5); 3) asking academically relevant questions (AR-Q); 4) linking mathematical contributions (AT2, AT3); and 5) whole-class engagement (AT1).

Table 1: Instructional Quality Assessment Rubrics

Academic Rigor	Accountable Talk
AR1: Potential of the Task	AT1: Participation
AR2: Implementation of the Task	AT2: Teacher's Linking
AR3: Student Discussion After Task	AT3: Students' Linking
AR-Q: Questioning	AT4: Asking (Teacher Press)
AR-X: Mathematical Residue	AT5: Providing (Student Response)

Mathematical Authority

It was our hypothesis that a necessary condition of high-quality classroom discourse, as described by the elements of the IQA, was teachers' capacity to share mathematical authority with their students. Sharing mathematical authority requires teachers to listen to students' thinking, process and act upon potentially unplanned, and sometimes unfamiliar, mathematical statements. It also requires teachers to press students for explanations, and ask questions, in the moment, to move students' thinking forward. Sharing mathematical authority is not about who is in charge of the classroom but about who gets to decide which tools to use to solve a problem and who determines the correctness of mathematical contributions (Gresalfi, Martin, Hand, & Greeno, 2009; Hiebert et al, 1997). It is also about the nature in which students' mathematical reasoning and sense-making is valued and affirmed by the teacher during classroom discourse. Sharing mathematical authority is about the opportunities teachers give students to share ideas, evaluate others' ideas (Harel & Sowder, 2007) , and provide justification for their reasoning and sense-making (Hufford-Ackles, Fuson, & Sherin, 2004).

Methods and Analysis

A mixed-method design was used to capture both quantitative and qualitative aspects of teachers' classroom discourse. At the beginning of the professional development experience, teachers were asked to complete the REU (Research Experience for Undergraduates) Beliefs Instrument (O'Hanlon et al., 2015). During the first year, teachers engaged in a series of professional development experiences (over 100 hours) to understand the Cognitive Demand Framework (Stein, Smith, Henningsen & Silver, 2000) and the rubrics of the Instructional Quality Assessment. During the first year, several teachers agreed to have teaching episodes video-recorded. With the support of our coaching and using the IQA, the teaching episodes were evaluated by their peers and feedback was provided. This process supported two goals: 1) to create a professional learning culture in which participants were comfortable sharing episodes of their teaching with peers and willing to receive constructive feedback from their peers, 2) to use these episodes to develop consistency in rubric ratings.

Each of the IQA rubrics are scaled from 0 to 4 with 0 being the lowest rating and 4 being the highest rating. The rubrics of the IQA focus on two dimensions of high-quality classroom discourse: academic rigor and accountable talk. For example, one of the rubrics of the IQA related to academic rigor *is Potential of the Task.* At the highest ratings, students are engaged in a task that involves complex non-algorithmic thinking or applying a broad general procedure that is closely connected to mathematical concepts. In order to be considered a 4-rating the task must explicitly prompt for evidence of student thinking. At the lower ratings students are either engaging in no mathematical activity or memorizing rules, formulae or definitions. The *Potential of the Task* is strictly about what the teacher puts in front of students to do. Likewise, one of the rubrics related to student accountability during classroom discourse is *Asking (Teachers' Press)*. The higher ratings of the rubric correspond with the teacher consistently asking students to provide evidence of their contributions (i.e. press for conceptual explanations) or to explain their reasoning. At the lower ratings there is no discussion or efforts to ask students to provide evidence for their contributions (Boston, 2012). It is important to note that during the first year, each teacher was visited at least once by a researcher, and their teaching episode was scored using the IQA.

During Year 2 and Year 3, each teacher was provided with a SWIVL robot and iPad to record lessons. Teachers were given the autonomy to record and share lessons that they believed best represented their growth in classroom discourse. These lessons were shared with researchers via the cloud and scored using the IQA rubrics. Each teacher who agreed to share lesson recordings was scored at least once. These episodes were rated by two or more facilitators who were trained in scoring the IQA. Due to scheduling conflicts, the facilitators were unable to develop inter-rater reliability. As a result, the ratings for the cohort (n = 16) on each rubric for the last two years were averaged. Each teacher was also provided with scoring on each rubric for the episodes that were shared. Written constructive feedback corresponding to each rating was also provided.

Teachers continued to participate in professional development experience (over 100 hours each year). These experiences involved deepening the content knowledge (i.e., geometry and data analysis) and continuing to improve the quality of classroom discourse. These experiences also involved small groups analyzing teaching episodes using the IQA and providing constructive feedback to their peers. Teacher were assigned a "teacher buddy" to share additional teaching episodes and to provide feedback aligned with the rubrics of the IQA.

At the end of the third-year professional development experiences, the REU (Research Experience for Undergraduates) Beliefs Instrument was given to teachers again. Teacher responses were averaged and Cohen-D effect sizes were computed to measure the significance of changes in teachers' beliefs from the beginning to the end of the professional development experiences.

In the post analysis of our data, after analyzing changes in teachers' beliefs and the IQA rubric ratings of teaching episodes, we took a Grounded Theory approach (Corbin & Strauss, 1996) to understand the role of mathematical authority. Teachers had strong initial beliefs related to elements of sharing mathematical authority with students, and the strengths of those beliefs over the three years of professional were either sustained (see table 2, questions 19 and 21) or advanced (see table 2, question 6 and 14). We wanted to understand whether those beliefs were present in teachers' practices and, if so, what were the challenges teachers faced in sharing mathematical authority with students?

In our fine-grained analysis of a few teaching episodes, IQA ratings (3 or higher) in the rubrics related to task potential (AR1) and questioning (AR-Q), we noticed that sharing mathematical authority was not as prevalent in the actual teaching episodes as teacher beliefs would suggest. Also, we noticed that, in the instances in which teachers did attempt to share mathematical authority with students, there were different challenges that they faced in continuing to share the mathematical authority with students. These instances were analyzed to categorize the nature the classroom activities that either had the potential to or resulted in the sharing of mathematical authority with students. Themes that emerged were: 1) a teacher pressed a student to further explain their reasoning, 2) a teacher asked a question to advance students' reasoning (Bill & Smith, 2008), 3) a student generated a conjecture, 4) a student asked a question, or 5) the validity of mathematical contribution by a student needed to be established.

Results

Teacher Beliefs

Items from the REU (Research Experience for Undergraduates) Beliefs Instrument (O'Hanlon et al., 2015) related to elements of classroom discourse are shared. Bolded item numbers are reverse scored and those with large effect sizes (Cohen's D > 0.60) are starred.

Table 2: Pre and Post Significant Changes in Beliefs (|ES| > 0.60)

#	Question	Pre	Post	Effect Size
6	During class discussions, students should analyze and critique another students' work.	$\bar{x} = 3.80$ $s = 1.23$	$\bar{x} = 4.40$ $s = 0.60$	0.68*
11	The teacher should demonstrate how to solve mathematical problems before the students are allowed to solve problems.	$\bar{x} = 3.00$ $s = 0.82$	$\bar{x} = 4.05$ $s = 0.51$	1.84*
14	During class discussions, the teacher should be the authority in terms of whether a student's mathematical conjecture or justification is correct.	$\bar{x} = 2.60$ $s = 1.07$	$\bar{x} = 3.50$ $s = 0.89$	0.91*

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19	Teachers should provide opportunities for students to critique mathematical arguments, and discuss their own conjectures.	$\bar{x} = \mathbf{4.50}$ $s = \mathbf{0.52}$	$\bar{x} = \mathbf{4.50}$ $s = \mathbf{0.53}$	0.00
21	During class discussions, students should play a role in determining whether mathematical justifications are valid.	$\bar{x} = \mathbf{4.50}$ $s = \mathbf{0.52}$	$\bar{x} = \mathbf{4.50}$ $s = \mathbf{0.52}$	0.00
31	Struggling with mathematical concepts is detrimental to understanding.	$\bar{x} = \mathbf{2.80}$ $s = \mathbf{1.55}$	$\bar{x} = \mathbf{3.94}$ $s = \mathbf{1.12}$	0.84*
32	Group discussions often lead to tangents, or incorrect mathematics, and should be limited in their use.	$\bar{x} = \mathbf{3.70}$ $s = 0.82$	$\bar{x} = \mathbf{4.25}$ $s = 0.45$	0.83*
49	The teacher should provide verification for mathematical arguments, rather than expecting students to do so.	$\bar{x} = \mathbf{3.50}$ $s = \mathbf{0.97}$	$\bar{x} = \mathbf{3.81}$ $s = \mathbf{0.75}$	0.36

Quality of Classroom Discourse

These results suggest that the cohort's beliefs about classroom discourse either moved, or were already consistent, in the direction we espoused. The average ratings for Years 2 and 3 are shown below. A two-sample unequal variances t-test was conducted to determine whether there was a statistically significant change in average cohort ratings (n = 16) on the rubrics of the IQA.

Table 3: Comparing Year 2 and Year 3 IQA Cohort Ratings

	Year 2	Year 3	P-Value
Academic Rigor			
R1: Task Potential	2.63	2.97	0.25
R2: Task Implementation	1.91	2.22	0.32
R3: Student Discussion	1.69	1.91	0.45
AR-Q: Questioning	1.84	1.78	0.84
AR-X: Mathematical Residue	1.85	2.07	0.46
Accountable Talk			
AT1: Participation	2.02	2.56	0.049*
AT2: Teachers' Linking	1.68	1.94	0.32
AT3: Students' Linking	1.63	1.27	0.19
AT4: Asking (Teacher Press)	1.82	1.94	0.68
AT5: Providing (Student Response)	1.66	1.81	0.61

These results indicate that the only statistically significant ($\alpha < 0.05$) change in the ratings occurred in Participation (AT1) rubric, the percentage of students participating in teacher-facilitated discussion. It is important to note that the numerical rating indicate an average of between 50% and 75% of students participating in class discussions.

Mathematical Authority

Although teachers expressed strong beliefs about sharing mathematical authority with students (see table 2), we identified very few teaching episodes in which teachers shared mathematical authority with their students. Analyzing the specific instances in which teachers did share mathematical authority with students, we attempted to categorize teacher actions that supported those efforts, or hypothesized about the nature of the challenges they faced in as they shared mathematical authority with their students. From this analysis, a set of guiding questions were generated to enable us to further reflect upon teacher actions that supported the sharing of mathematical authority with students.

1. Are students given an opportunity to choose the tools, or understandings, they want to use to make sense of the problem or task?
2. Are students deciding the mathematical correctness of an answer or a students' contribution to the discussion?
3. Are students given an opportunity to share their reasoning and sense-making with the class?
    a. If so, in what forum? (e.g. group discussions, whole class, etc.)
    b. Are these contributions valued by other members of the class? By the teacher?
4. Is the teacher asking pressing/assessing questions to better understand their students' reasoning and sense-making?
5. Is the teacher asking questions to advance students' understanding of their ideas?
6. Is the teacher giving their students opportunities to explore their own conjectures?
    a. If so, do my students have access to the necessary tools to do so?
    b. If so, does the teacher value these experiences? How is the teacher making this evident to students?
    c. If so, what demands is this placing on the teachers' knowledge (e.g. mathematical, pedagogical content knowledge, etc.) to meaningfully orchestrate this experience?

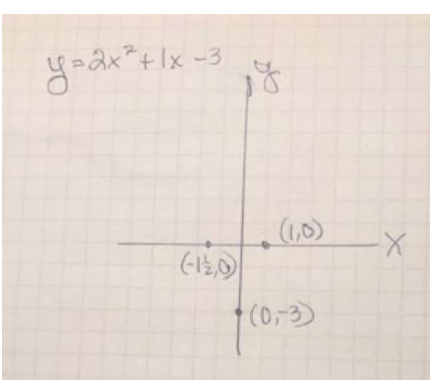
Discussion

It seems important to share a few, brief teaching episodes that exemplify the role of sharing mathematical authority with students in high-quality discourse. The episodes also illuminate a few of the demands sharing mathematical authority with students places on teachers.

Mrs. Barnes Classroom

Mrs. Barnes (pseudo-name) gave the following task to her 10th grade students, "Graph $y = 2x^2 + 1x - 3$." Working in groups, students were provided graph paper, no technology, and no guidance. Most groups were able to locate the zeros and y-intercept, but most struggled to find the location of the vertex.

After a few minutes, one of the groups *shared a mathematical contribution* about the relationship between the zeros of the function and the location of the axis of symmetry (see fig. 1, lines 50-53). The exchange between Mrs. Barnes and the group is shown in Figure 1.



S1: Okay so we found the zeros, but we really don't know where to go from there. I mean we kind of guessed on the vertex but we don't really know.
T: You guessed the vertex…
S2: The axis of symmetry is right here.
T: Axis of symmetry is at the y-axis. Do you think it is right there at x equals zero? Why?
S1: No, because it is uneven. You have one at one and one at negative one and a half.
T: Could you find the middle between those two points?
S1: Yeah
T: Could you find the middle between those two points.
S1: We did, it's negative one-fourth. That is where we tried to put our point. We just don't really know what we are doing.
S2: Okay, so we used the negative three and went down three and then did that negative one-fourth, we think.
T: Okay, so do you think the negative one-fourth should be higher or lower than that negative three?

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Figure 1: Small Group Discussion of the Location of the Axis of Symmetry

During the small group discussion there are multiple ways in which Mrs. Barnes shared the mathematical authority with her students. She asked the group to *clarify a statement*, "we kind of guessed on the vertex" to better understand the meaning of <u>their</u> words (see fig. 1, lines 46-48). She also asked students to *provide a justification for a conjecture* she inferred from the location of their finger on the graph, "Axis of symmetry is at the y-axis. Do you think it is right there at *x* equals zero? Why?" (lines 50-51). Also, instead of giving the students the answer, she *asked a question to advance students' understanding of their ideas* related to the possible location of the vertex (lines 60-61), "So do you think the negative one-fourth should be higher or lower than the negative three? "[referring to the location of the vertex].

However, during the whole class discussion Mrs. Barnes struggled to continuing sharing the mathematical authority with her students (see fig. 2).

T: What if I can't immediately look at that and know it? Is there a way for me to find the axis of symmetry just by doing some math?
S1: Can't you just add both of them and divide by two?
T: You mean your intercepts? Yeah, we could average them. We could average the intercepts. We could also take negative b over 2a. Do you remember we did the quadratic formula and we started with negative b plus or minus you got that big long square root and then it was 2a. Did you know that the number on the outside of the radical, not the discriminant, the number to the outside, actually procedures the axis of symmetry?

Figure 2: Excerpt of Mrs. Barnes Whole Class Discussion

In reflecting upon the teaching episode, Mrs. Barnes expressed frustration that she was unable to do more with the conjecture shared by students, "the axis of symmetry of symmetry is halfway between the two x-intercepts" (see fig. 1, lines 50-53). She indicated that the conjecture was unanticipated, and that it challenged her because she did not know, in the moment, what to do to further advance students' reasoning and sense-making." She recognized that she dismissed the group's contribution by stating "Is there a way for me to find the axis of symmetry just by doing some math?" (see fig. 2, lines 242-243). She also stated that this group was actually "doing mathematics" based on their engagement in multiple Standards of Mathematical Practice (CCSSI, 2010). It is interesting to note that even in the midst of Mrs. Barnes explanation, the student, desiring to retake the mathematical authority exclaimed, "Can't you just add both of them and divide by two?" (line 244).

Mr. Pho's Classroom

Unlike Mrs. Barnes' classroom episode, in which there were productive moments of sharing the mathematical authority with students, Mr. Pho's classroom episode did not have those same moments. Mr. Pho (a pseudo-name) gave students the following task, *A virus is doubling every 30 minutes, if the initial number of virus present is 2000, how long will it take before there are 3 million virus? 10 million virus? Write an equation to model the situation.* After a lack of meaningful small group discourse, Mr. Pho started a class discussion in which the following dialogue occurred.

T: What unit are we on?
S1: Exponents
T: So you know it has something to do with exponents.
S2: B squared…B squared. I don't know.
T: We are looking at exponential functions. [T goes to back of class and grabs a student-made poster that involves characteristics of exponential functions.]
T: We are looking at exponential functions.
S3: Okay, prime factors.
T: Exponential functions
S4 Hey, can I see that again!
S5: a times b, whatever that is supposed to mean.
T: Yes
S6: Didn't you explain this one once like it was time times growth?
T: I am giving you a hint. [Holding poster in front of the student.]
S3: It is how many times it duplicates.
S7: Multiply it.
T: Exponential functions
S3: Two-thousand
S4: What's the multiplier
S2: Three

Figure 3: Excerpt of Mr. Pho's Whole-Class Discussion

In reflecting upon the teaching episode with Mr. Pho, we hypothesized as to the nature of the challenges he faced in sharing the mathematical authority with students, and potential strategies to overcome those challenges. Mr. Pho recognized that he lowered the cognitive demand of the task, and reclaimed mathematical authority when he stated, "what unit are we on? (Line 46)". He stated, however, that, in the moment, he could not think of a question to advance students' reasoning and sense-making without imposing his mathematical authority. It was our hypothesis that students did not seem to be able to access the necessary tools to reason and make sense of the task. By *asking an advancing question*, such as "Could you make a table to make sense of the situation?" or "How could you find the number of virus after 30 minutes, 1 hour, etc.?" , Mr. Pho may have been able to continue to share the mathematical authority with students. Thus, letting students' reasoning and sense-making drive the discussion instead of Mr. Pho's mathematical topic hint. We also hypothesized that, asking students, as part of the initial prompt, to "write an equation", instead of giving students an opportunity to make sense of the task may have hindered their ability to assume the mathematical authority. Giving students an opportunity to read the stem of the initial prompt and ask him clarifying questions, such as "What does it mean to double every 30 minutes?", might have been enough for students to feel that they had the permission, and necessary reasoning tools, to assume the mathematical authority.

Conclusion

As the teaching episodes of Mrs. Barnes and Mr. Pho illustrate, teachers' capacity to implement high quality discourse is closely connected to their ability to share mathematical authority with their students. While teachers may have strong beliefs that support sharing mathematical authority with students, there are significant demands placed on teachers in enacting instructional practices that support those beliefs. Sharing mathematical authority with students requires the teacher to not only listen and make sense of student contributions, but to, in the moment, act upon what was heard and understood. Effectively acting upon student contributions requires teachers to determine the nature and validity of student reasoning, and ask questions to advance their current understanding. In other instances, sharing mathematical authority with students challenges teachers to identify instructional barriers (e.g., unfamiliarity with the task, lack of reasoning tools) that are hindering students ability to assume the mathematical authority. Future teacher training sessions that involve improving the quality of classroom discourse needs to include: 1) opportunities to develop an awareness of the nature of the challenges associated with sharing mathematical authority with students, and 2) sharing of specific strategies that support teachers efforts to share the mathematical authority with students.

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