BACKWARD TRANSFER EFFECTS ON ACTION AND PROCESS VIEWS OF FUNCTIONS

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This study was conducted to gain understanding about potential influences that learning about quadratic functions has on high school algebra students' action versus process views of linear functions. Pre/post linear functions tests were given to two classrooms of Algebra II students (N=57) immediately before and immediately after they participated in a multi-day unit on quadratic functions. The purpose was to identify ways that their views of linear functions had changed. Results showed that on some measures, students across both classes shifted their views of linear functions similarly. However, on other measures, the results were different across the classes. These findings suggest that learning about quadratic functions can influence students' action or process views of linear. Furthermore, the instructional differences between classes provide insights into how to promote those influences that are productive for students' views.

Keywords: Algebra and Algebraic Thinking, High School Education, Learning Theory

A well-established and widely-held idea in the mathematics education research community is the importance of the relationship between prior ways of reasoning and new learning (e.g., Bransford & Schwartz, 1999; Roschelle, 1995; Vosniadou & Brewer, 1987). However, most of this prior research focuses on the foundational role that prior knowledge plays in new learning. In other words, this research has primarily examined the influence that prior ways of reasoning can have on new learning. This is typically referred to as the *transfer of knowledge* (Lobato, 2008). What has yet to be well examined, especially in the context of mathematics education, is the influence that new learning can have on prior ways of reasoning.

We use the forward and backward direction to distinguish between the two kinds of influences mentioned above. Specifically, we use *forward* to describe influences that go from prior ways of reasoning to new learning and *backward* to describe influences that go from new learning to prior ways of reasoning. While, forward influences (also known as *forward transfer*) have been a well-researched construct in mathematics education, backward influences is a new idea for mathematics education research. Our research addresses this gap by examining backward influences in real algebra classrooms.

We know of only a handful of studies about backward influences in the context of mathematics education, including Hohensee's (2014, 2016) studies on middle school students reasoning about functions, Macgregor and Stacey's (1997) study on secondary students' reasoning about algebra symbols, Young's (2015) study on AP calculus students' reasoning about differentiation and integration, and Moore's (2012) study on undergraduates' reasoning about calculus concepts.

Despite the limited research on this topic, these studies have revealed that backward influences can be unproductive or productive. *Unproductive* backward influences are when students' prior ways of reasoning become muddled or shift to a lower level, because they learn something new (e.g., Macgregor & Stacey, 1997). More theory and research on backward influences are needed to find ways to prevent or at least mitigate unproductive backward influences. *Productive* backward influences are when students' prior ways of reasoning become refined or enhanced because they learn something new (e.g., Hohensee, 2014). More theory and research are needed to uncover ways to promote productive backward influences. The goal for this study was to contribute, as an early step, to developing more understanding about backward influences that occur in high school algebra classrooms, so as to inform ways to inhibit unproductive backward influences and promote productive backward influences.

Theoretical Framework

Our theoretical framework has two parts. The first part involves how we conceptualized backward influences by new learning on prior ways of reasoning. The second part involves the theoretical perspective that guided our study about ways to reason about functions.

Conceptualizing Backward Influences by New Learning on Prior Ways of Reasoning

Backward influences by new learning on prior ways of reasoning in the context of mathematics education have not been well-studied or theorized. However, in other domains, backward influences have been regularly referred to as a form of transfer of learning called *backward transfer*. For example, the effect that learning a second language has on individuals' ability to produce and comprehend their native language has been conceptualized as backward transfer (e.g., Cook, 2003). Therefore, we conceived of backward influences in the context of mathematics education as backward transfer.

Broadening the conceptualization of transfer to include backward influences is a new idea for mathematics education. A theoretical implication from this broad conceptualization is that perhaps one of the well-articulated mathematics education theories of transfer may be a suitable candidate to extend to include backward transfer. Among the theories we considered, Lobato's (2008) *actor-oriented transfer* (AOT) *perspective* is a suitable candidate because of the emphasis in the definition on transfer as an influence. In particular, according to AOT perspective, transfer is "the *influence* of a learner's prior activities on his or her activity in novel situations" (p. 169, emphasis added). An assumption underlying the AOT perspective is that transfer has occurred whenever a learner's prior activities influence their activities in a novel situation (i.e., forward influence), regardless of whether, from an outside-observer's perspective, the new activity involves normative or non-normative performance.

Based on the AOT perspective, we defined backward transfer as the "influence that learning something new has on a learners' prior ways of reasoning about a different or related concept" (Hohensee, 2014, p. 136). Consistent with the AOT perspective, we consider any backward influences by new learning on prior ways of reasoning, regardless of whether they lead to more- or less-normative performance, as cases of backward transfer. A primary reason to study backward transfer in mathematics education contexts is because of the potential that backward transfer unintentionally undermines or weakens learners' prior ways of reasoning (i.e., leads to less-normative performance). Understanding more about backward transfer could enable mathematics educators to develop instructional approaches that minimize unproductive effects.

Note that Young (2015) and Moore (2012), cited previously, adopted the same extension of the AOT perspective to conceptualize backward influences, while Macgregor and Stacey (1997) conceived of backward influences differently (i.e., as interference of learning).

Theoretical Perspective on Ways to Reason about Functions

Within the field of mathematics education research, a number of perspectives on ways of reasoning about functions have been put forth. The perspective we used for this study was the APOS perspective on ways of reasoning about functions (Breidenbach, Dubinsky, Hawks, &

Nichols, 1992). APOS stands for *action*, *process*, *object* and *schema*, and represents four ways to view functions (as well as other mathematical concepts). For our study, action- and process-views of functions were most relevant. An action view is described as "any repeatable physical or mental manipulation that transforms objects (e.g., numbers, geometric figures, sets) to obtain objects" (p. 249), and as "a static conception in that the subject will tend to think about it one step at a time (e.g., one evaluation of an expression)" (p. 251). In contrast, a process view is described as "[the] total action can take place entirely in the mind of the subject, or just be imagined as taking place, without necessarily running through all of the specific steps" (p. 249) and as "a dynamic transformation of objects according to some repeatable means…a complete activity beginning with objects of some kind, doing something to these objects, and obtaining new objects as a result of what was done" (p. 251). According to APOS Theory, an action view is a necessary precursor to a process view in the development of conceptions of functions.

One reason the distinction between action- and process-views of functions was important for our study was because, as Breidenbach et al. (1992) point out, "many individuals will be in transition from action to process…the progress is never in a single direction" (p. 251). In other words, an individual can change between action- and process-views of functions. For our study, we wondered if students' views of linear functions would change because of influences by new learning about quadratic functions. Our research question was the following: *In what ways do algebra students' prior ways of reasoning about linear functions change, if at all, along the dimension of an action versus process view, after they participate in an instructional unit on quadratic functions?* Next, we describe the methods we used to address this question.

Methods

Participants and Setting

The participants were Algebra II students from two classrooms at two high schools in the Mid-Atlantic region of the US. Both schools were ethnically diverse and drew from an urban population. All students in both classes volunteered to participate in the study (24 in Class 1; 33 in Class 2; N=57). The students reflected the ethnic diversity of their respective schools. The study took place from March to May of the school year. Each class had an experienced teacher: Ms. Henry (Class 1) had 8-years of teaching experience; Mr. Anderson (Class 2) had 17-years of teaching experience.

Procedure

The study procedures followed a pre/post format, in which pre- and post-tests on linear functions bookended an instructional unit on quadratic functions. Before the instructional unit, all students took a 45-minute paper-and-pencil linear functions pre-test. Also, four students from each class participated in one-on-one pre-interviews about their responses on the pre-test.

Students then participated in a multi-day instructional unit on quadratic functions taught by their regular algebra teacher. All lessons were observed and video recorded.

After the instructional unit, all students took a 45-minute paper-and-pencil linear functions post-test. Finally, the same four students from each class who participated in the pre-interviews, participated in one-on-one post-interviews about their responses on the post-test.

Linear Functions Pre- and Post-Tests

We used two versions of the linear functions test. Students were randomly sorted into two equal groups and assigned to either take Version A as the pre-test and Version B as the post-test, or vice versa. Each problem on Version A had a structurally similar problem on Version B. The purpose of having two versions and varying the order was to control for the possibility that students would do better on the post-test if they took the same test pre and post, and for the possibility that items on the versions were not comparable. Each version had one problem based on a graph, one problem based on a table and one problem based on a picture. Results from the picture problem are presented in this report (see Figure 1 for the picture).

Instructional Unit

Each teacher taught their instructional unit on quadratic functions using the Integrated Mathematic II curriculum and were given no direction by the researchers about how to teach the unit. Both teachers typically lectured for part of the class, gave students a seatwork assignment and held a whole-class discussion about the solutions. Ms. Henry's instructional unit was comprised of 16 lessons, and each lesson was 70 min. Mr. Anderson's instructional unit was comprised of 11 lessons, and each lesson was 45-80 min., due to a rotating block schedule. Both teachers seated students in groups. Both teachers were well liked by their students and their lesson were conducted in an orderly manner.

A difference in teaching styles was that Ms. Henry typically used an online platform that students accessed with laptops (i.e., Class Lab), that allowed her to monitor student responses in real time. She would then go over students' responses with the class and provide feedback. In contrast, Mr. Anderson typically had students go the board and write down their solutions to problems. Then, he would go over the responses with the class and provide feedback.

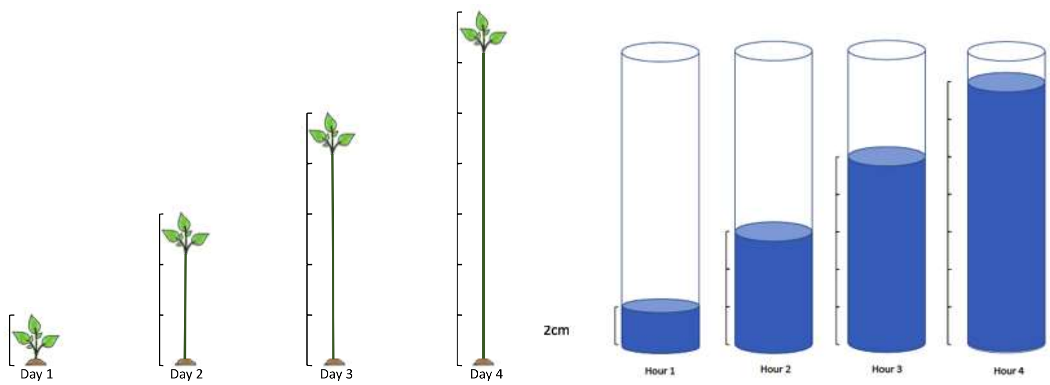

Figure 1: Growing Plant (Version A) and Container Filling with Rain Water (Version B)

A second difference between teachers was the time spent on specific quadratic functions topics. For example, Ms. Henry spent the most lessons on factoring quadratics expressions, real-world problems modelled with quadratic functions and graphing quadratic functions (see Table 1). In contrast, Mr. Anderson spent the most lessons on solving quadratic equations.

Table 1: Breakdown of Lessons Devoted to Each Quadratic Functions Topic

Teacher	Lesson Topic	# of Lessons
Ms. Henry	Lessons on factoring (factoring diamonds, difference of squares, leading coefficient≠1)	4
	Lessons on real-world problems	4
	Lessons on graphing (x/y-intercept, max/min, axis of symmetry)	3
	Lessons on solving by graphing	2

Otten, S., Candela, A. G., de Araujo, Z., Haines, C., & Munter, C. (2019). *Proceedings of the forty-first annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. St Louis, MO: University of Missouri.

	Lessons on solving by factoring	2
	Lesson on standard, vertex and x-intercept equation forms	1
Mr. Anderson	Lessons on solving (square root both sides, complete the square, quadratic formula)	5
	Lessons on graphing (x/y-intercepts, max/min, axis of symmetry, translations/dilations)	2
	Lesson on solving by factoring	1
	Lesson on factoring	1
	Lesson on real-world problems	1
	Lesson on standard, vertex and x-intercept equation forms	1

Data Set

Our data set was comprised of the pre-/post-tests, video-recorded pre-/post-interviews (4 students per class), video-recorded classroom observations and observation field notes.

Data Analysis

Analysis of the data was conducted in three stages: (a) analysis of the pre-/post-test responses and interviews for students who participated in interviews, (b) analysis of the pre-/post-test responses for remaining students, and (c) analysis of fieldnotes from the classroom observations.

Analysis of pre-/post-test responses and interviews for interviewed students. For stage 1 of the analysis, the first author compared the pre-/post-test responses and the pre-/post-interviews for the 4 students from each class that were interviewed. The unit of analysis was each sub-part of each question. From these comparisons, the first author developed an initial set of codes with *grounded theory* (Strauss & Corbin, 1994). Initial codes and supporting evidence were presented to the second and third authors for feedback on the validity of the codes. Once agreement was reached about the interviewed students, the authors proceeded to the next step in the analysis.

Analysis of pre-/post-test responses for the remaining students. Each author took one-third of the remaining tests and again compared sub-parts on one test for each student to the associated sub-part on the other test, noting whenever a change in reasoning fit one of the existing codes. During this analysis, we blinded ourselves to which tests were pre-tests and which were post-tests. The initial codes were in some cases insufficient to capture changes in students' views of functions. In those cases, the coder either refined an existing code or created a new code. Once, each author had coded their dataset, we met in pairs to discuss changes in reasoning that had been identified, to check for agreement on whether the evidence that a change had occurred was compelling and the right code had been applied. Whenever a pair of coders failed to reach consensus on a response, it was flagged and discussed by all three authors until consensus was reached. As such, we continued to refine our grounded theory with a *constant comparison approach* (Strauss & Corbin, 1994). Once all coded responses had been discussed in pairs or by all three authors, and the codes had stabilized, we recoded all the coded responses to ensure that the refined codes fit the entire data set.

Analysis of classroom observations. Analysis of the classroom observations is ongoing. Thus far, the first author has summarized the field notes on a spreadsheet, and identified differences between the two classes in terms of numbers of lessons devoted to specific quadratic functions topic (see Table 1). In future analyses efforts, the three authors will divide up the recordings of the lessons and identify particular episodes in which potential connections to the changes in reasoning observed in the pre-/post-tests exist. Episodes will be transcribed and analyzed to identify the interactions, visual representations, gestures, etc., that could account for to the observed changes in reasoning identified in the pre-/post-test responses.

Results

Analysis of the pre- and post-tests revealed that, in a number of cases, students' views of functions had changed in terms of an action versus a process view. In some cases, the changes in views were similar across the two classes, whereas in other cases, they differed. Next, we show how students' views of functions changed on each of the three sub-parts for the picture problem.

Reasoning with a Build-up Process or a Repeated Build-up Process

Our first finding was that several students changed the way they reasoned on the first sub-part of the pictorial problem, which asked "*Explain in words how to find the height of the plant on day 17*" (Version A) or "*Explain in words how to find the total amount of rainfall if the storm lasts for 11 hours*" (Version B). One reasoning strategy was to repeatedly add the amount of growth, one day or one hour at a time, while simultaneously keeping track of the days or hours, until the height on the desired day or hour was attained. Kaput and West (1994) called reasoning about linear functions in this way a *build-up process*. The second strategy was to multiply the rate of growth by the number of days or hours. Kaput and West (1994) called this an *abbreviated build-up process*. In other words, the abbreviated build-up process required one step, while the build-up strategy required multiple steps. A third strategy was to add a combination of *given* heights and times to find a desired height, such as finding the height on day 11 by doubling the given height on day 4, and adding the given height on day 3. Since this third strategy also required multiple steps, we included this strategy as a build-up process.

Frequency counts supporting this claim. A change from *build-up* to *abbreviated build-up* process was observed for 6 of Ms. Henry's 24 students and 7 of Mr. Anderson's 33 students (see Table 1). A change in the other direction was less common: only 3 of Ms. Henry's students and 2 of Mr. Anderson's students. The remaining students maintained one strategy on both tests or did not provide sufficient responses for us to determine if their reasoning had changed. Note that the more common change from build-up to abbreviated build-up process was similar across classes.

Table 2: Change Involving Build-up Process and Abbreviated Build-up Process

Teacher	Students per class	From build-up to abbreviated build-up	From abbreviated build-up to build-up	Maintained build-up: Maintained abbreviated build-up
Henry	24	6	3	12:2
Anderson	33	7	2	8:3

Change interpreted in terms of action versus process view of functions. We interpreted a change from reasoning with a build-up process to reasoning with an abbreviated build-up process as a possible shift away from an action view towards a process view of functions, because with a shift to an abbreviated build-up process, students reasoned as if they knew, without calculating each separate change in height, that all the changes in height would be the same and that they could simply multiply by how many changes in height there were. This aligns with Asiala et al. (1997), who described the process view as "it is not necessary to perform the operations, but to only think about them being performed" (p. 8).

Reasoning about Independent and Dependent Variables

Our second finding was that several students changed their way of reasoning on the second sub-part of the pictorial problem, which asked "*Can you find the day* [independent variable] *the plant was measured if you were given the height* [dependent variable]*? If yes, explain how. If no, explain why not*" (Version A), or "*Can you find the hour* [independent variable] *the rain water was measured if given the height* [dependent variable]*? If yes, explain how. If no, explain why not*" (Version B). Students who exhibited this change, either reasoned on the pre-test *it is not possible* to use the dependent variable to find the corresponding value of the independent variable and on the post-test reasoned *it is possible*, or vice versa. Note that *it is possible* is correct because the independent variable in a linear function can be found (i.e., $x = (y - b) / m$).

Frequency counts supporting this claim. This change was more common for Ms. Henry's students than for Mr. Anderson's students, by a ratio of 9:2 (see Table 2). Additionally, Ms. Henry's students who exhibited this change were almost evenly split on changing from *it is* to *it is not possible* (4 students) versus changing from *it is not* to *it is possible* (5 students). In contrast, all of Mr. Anderson's students who exhibited this change, went from *it is not* to *it is possible* (2 students). The remaining students maintained the same reasoning on both tests or did not provide sufficient responses for us to determine if their reasoning had changed.

Table 3: Change Involving Finding Independent Variable from Dependent Variable

Teacher	Students per class	From *not possible* to *possible*	From *possible* to *not possible*	Maintained *possible*: Maintained *not possible*
Henry	24	5	4	5:1
Anderson	33	2	0	15:1

Change interpreted in terms of action versus process view of functions. We interpreted a change in reasoning from *it is not* to *it is possible*, as a possible shift toward a process view of functions and a change in the other direction as a possible shift toward an action view of functions. This interpretation is based on Asiala et al.'s (1997) characterization of a process view of functions as when a person can "reverse the steps of the transformation" (p. 7).

Reasoning about Specific or General Intervals of Change

Our third finding was that several students changed their reasoning on the third sub-part of the pictorial problem. Version A asked:

> You have to leave the plant in your office over the weekend. You did not measure the plant for 2.5 days. The plant grows at the same rate the whole time. How much did the plant grow in the 2.5 days you were gone? Show any work that helped you decide.

Version B asked:

> You fall asleep while watching TV. You did not measure the rain water for 3.5 hours. It rained the whole time at the same rate. How much rainwater was collected during the 3.5 hours that you were sleeping? Show any work that helped you decide.

Students either reasoned about *specific* or *general* intervals on this problem. When they reasoned about specific intervals, they either found the height of the plant or rainwater on day 2.5 or at hour 3.5 or the height 2.5 days or 3.5 hours after the last height depicted in the picture. When students reasoned about a general interval, they found the amount the plant would grow or the rainwater would rise over *any* 2.5-day or 3.5-hour interval in general. Notice that reasoning about general intervals is correct because, for each version of the problem, the amount of change in the independent variable is for a general interval, not a specific interval.

Frequency counts supporting this claim. Ms. Henry's and Mr. Anderson's classes showed opposite (and mixed) trends in changes in reasoning (see Table 4). A greater number of Ms. Henry's students went from reasoning about a *specific interval* to reasoning about a *general interval* (7 students), than vice versa (3 students). In contrast, a smaller number of Mr. Anderson's students went from reasoning about a specific interval to reasoning about a general interval (3 students), than vice versa (8 students).

Change interpreted in terms of action versus process view of functions. We interpreted reasoning with *general intervals* as more consistent with a *process view* and reasoning with *specific intervals* as more consistent with an *action view*. Our rationale was that to reason about any general 2.5-day or 3.5-hour interval, an individual would need to think about changes in height across all 2.5-day or 3.5-hour intervals in general, without individually calculating changes in heights for all those intervals. This aligns with Asiala et al.'s (1997) description of the process view as not needing to perform all the operations to think about the results of operations.

Table 4: Change Involving Reasoning about Specific or General Intervals

Teacher	Students per class	From *specific* to *general*	From *general* to *specific*	Maintained *specific*: Maintained *general*
Henry	24	7	3	2:1
Anderson	33	5	8	5:3

Discussion

To summarize our results, we saw one type of change in reasoning on each of the three sub-parts for the pictorial problem. This suggests that backward transfer effects by new learning about quadratic functions on prior ways of reasoning about linear functions may be a fairly frequent occurrence. However, the effects were more varied than we anticipated: they occurred in both classes or in one class only, and in the same direction or in opposite directions. We think these results suggest that backward transfer effects, in real classrooms and with teachers who are not purposefully trying to produce these effects, may be somewhat messy.

Messy results for backward transfer effects are significant because they suggest backward transfer may be difficult for researchers to detect unless deliberately tuned to them. Our messy results may also help explain why teachers may be unaware of backward transfer effects in their students.

To add to the messiness, our findings suggest backward transfer effects can be either productive or unproductive. The evidence of productive backward transfer effects is significant because it suggests there are aspects of learning about quadratic functions instruction that could be emphasized to further enhance *productive* influences on students' views of linear functions. For example, the finding that a number of students changed productively from a build-up strategy to an abbreviated build-up strategy suggests that, with further emphasis, even more students could be supported to change along this dimension (e.g., such as by exploring how students could engage in a kind of abbreviated build-up strategy in a quadratic context).

The findings of unproductive backward transfer effects are also significant because they suggest there are aspects of learning about quadratic functions that could be emphasized to inhibit *unproductive* influences on students' views of linear functions. For example, the finding that a number of Mr. Anderson's students went from reasoning with a general interval to reasoning with a specific interval suggests that researchers and educators should look for ways to emphasize reasoning with general intervals to inhibit or even eliminate this unproductive backward transfer effect. Our future research on the topic of backward transfer in the context of linear functions conceptions and quadratic functions instruction will test some of these ideas.

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References

Asiala, M., Brown, A., DeVries, D., Dubinsky, E., Mathews, D., & Thomas, K. (1996). A framework for research and curriculum development in undergraduate mathematics education. In A. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), Research in collegiate mathematics education II, CBMS issues in mathematics education (pp. 1-32). American Mathematical Society.

Bransford, J. D., & Schwartz, D. L. (1999). Rethinking transfer: A simple proposal with multiple implications. In A. Iran-Nejad & P. D. Pearson (Eds.), *Review of Research in Education 24* (Vol. 24, pp. 61-100). Washington, DC: American Educational Research Association.

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational Studies in Mathematics, 23*(3), 247-285. doi: 10.1007/BF02309532

Cook, V. (2003). Introduction: The changing L1 in the L2 user's mind. In V. Cook (Ed.), *Effects of the second language on the first* (pp. 1-18). Clevedon, United Kingdom: Multilingual Matters.

Hohensee, C. (2014). Backward transfer: An investigation of the influence of quadratic functions instruction on students' prior ways of reasoning about linear functions. *Mathematical Thinking and Learning, 16*(2), 135-174. doi:10.1080/10986065.2014.889503

Hohensee, C. (2016). Teachers' awareness of the relationship between prior knowledge and new learning. *Journal for Research in Mathematics Education, 47*(1), 17-27. doi: 10.5951/jresematheduc.47.1.0017

Kaput, J. J., & West, M. M. (1994). Missing-value proportional reasoning problems: Factors affecting informal reasoning patterns. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 235-287). Albany: State University of New York Press.

Lobato, J. (2008). Research methods for alternate approaches to transfer: Implications for design experiments. In A. E. Kelly, R. A. Lesh, & J. Y. Baek (Eds.), *Handbook of Design Research Methods in Education: Innovations in Science, Technology, Engineering, and Mathematics Learning and Teaching* (pp. 167-194). New York: Routledge.

Macgregor, M. & Stacey, K. (1997). Students' understanding of algebraic notation: 11-15. *Educational Studies in Mathematics, 33*, 1-19. doi: 10.1023/A:1002970913563

Moore, T. (2012). *What calculus do students learn after calculus?* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI No. 3526217) Retrieved from http:// krex.k-state.edu/dspace/handle/2097/14090

Roschelle, J. (1995). Learning in interactive environments: Prior knowledge and new experience. In J. H. Falk, & L. D. Dierking (Eds.), *Public institutions for personal learning: Establishing a research agenda* (pp. 37-51). Washington, DC: American Association of Museums.

Strauss, A., & Corbin, J. (1994). Grounded theory methodology: An overview. In N. K. Denzin and Y. S. Lincoln (Eds.), *Handbook of Qualitative Research* (pp. 273-285). Thousand Oaks: Sage Publications.

Vosniadou, S., & Brewer, W. F. (1987). Theories of knowledge restructuring in development. *Review of Educational Research, 57*(1), 51-67. doi: 10.3102/00346543057001051

Young, S. (2015, February). *Investigating backward transfer effects in calculus students*. Poster session presented at the 18th Annual Conference of the Special Interest Group of the Mathematics Association of America on Research in Undergraduate Mathematics Education, Pittsburgh, PA.