DESIGN PRINCIPLES FOR THE DEVELOPMENT OF PROFESSIONAL NOTICING OF STUDENTS' TECHNOLOGICAL MATHEMATICAL PRACTICES

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In support of standards for the learning and teaching of mathematics and statistics that advocate for the use of technology to promote reasoning and sense making, and to elicit student thinking, we draw on the use of authentic student work in the form of video case instruction to develop prospective secondary mathematics teachers' [PSMTs] knowledge of students' understanding, thinking, and learning with technology in mathematics. Specifically, we draw on the extant literature related to TPACK, video cases as learning objects, and noticing to propose a set of design principles intended to guide the development of materials to support PSMTs' acquisition of TPACK. Here we explicate six design principles situated in the literature, provide an example of a module designed based on these principles, and share findings from pilot studies utilizing the module.

Keywords: TPACK; Professional noticing; Preservice teacher education

The National Council of Teachers of Mathematics (NCTM, 2000) has long advocated that "technology is essential in teaching and learning mathematics; it influences what is taught and enhances students' learning" (p. 24). Given the impact that meaningful incorporation of technology tools can have on students' understanding of mathematics, it is important for teachers to develop a model of teaching and learning that goes beyond the specifics of a technology tool so they are able to make informed decisions about appropriate use of technology to develop mathematically proficient students. This was most recently articulated in the Association of Mathematics Teacher Educator's (AMTE) *Standards for Preparing Teachers of Mathematics* which states, "well-prepared beginning teachers of mathematics are proficient with tools and technology designed to support mathematical reasoning and sense making, both in doing mathematics themselves and in supporting student learning of mathematics" (2017, p. 11). This requires teachers to not only be proficient users of technologies, but also to understand how to use technology in meaningful ways to support students' thinking about mathematics. Whether or not the use of technology will enhance students' learning depends on teachers' decisions when using technology tools to design and implement meaningful tasks. These decisions are informed by teachers' knowledge of mathematics, technology, and pedagogy.

Consider the context of teaching trigonometric functions in high school. Teachers need to know how triangle and unit circle models of trigonometric functions are related and connected to each other (knowledge of content). They also need to know how to use technology to create connected representations of right triangle models, unit circle models, and representations of trigonometric functions (knowledge of technology specific to the content). Finally, teachers need to be able to design activities that align with the approaches that students might take when asked to make sense of the connections between right triangle and unit circle models of trigonometry (knowledge of pedagogy specific to the content). The intersection of these forms of knowledge has been identified as technological pedagogical content knowledge (TPACK).

Building off of the work of Grossman (1989), Niess (2005) articulated four components of TPACK: 1) an overarching conception of what it means to teach a particular subject while integrating technology in the learning; 2) knowledge of instructional strategies and representations for teaching particular topics with technology; 3) knowledge of students' understandings, thinking, and learning with technology in a specific subject; and 4) knowledge of curriculum and curriculum materials that integrate technology with learning in the subject area. It is the third component, knowledge of students' understandings, thinking, and learning with technology in a specific subject that is the focus of this paper. Specifically, drawing on the extant literature related to TPACK, video case instruction, and professional noticing we propose a set of design principles for the development of video-enhanced modules for PSMTs with an eye toward the development of their *knowledge of students' understandings, thinking, and learning with technology in mathematics,* an important aspect of TPACK.

Theoretical Foundations

Technological Pedagogical Mathematical and Statistical Knowledge

Within teacher education, many have built upon Shulman's (1986) idea of teachers' pedagogical content knowledge (PCK). Research and teacher education in mathematics education have been greatly influenced by Simon's hypothetical learning trajectory (1995), Even's (1990) work on the essential features of subject matter knowledge in mathematics (particularly for functions), and Ball and colleagues (e.g., Ball, Thames, & Phelps, 2008) work on the components of mathematical knowledge for teaching. However, none of this work considered the knowledge that comes with teaching with technology. The particular knowledge needed when technology is added has been identified as TPACK (e.g., Niess, 2005). This construct has been used by several researchers as a frame for their work to describe the development of PSMTs' abilities in using technology in mathematics teaching (e.g., Hollebrands, McCulloch, & Lee, 2016; Lee, Kersaint, Harper, Driskell, & Leatham, 2012).

When considering TPACK within the context of preparing prospective secondary mathematics teachers [PSMTs], we believe it is *most* essential to focus on the intersections of content (mathematical and statistical) knowledge with technological and pedagogical knowledge. So although we acknowledge the importance of general knowledge of technology and pedagogy, we focus on thinking about mathematics and statistics content, and the use of technology tools specific to teaching mathematics and statistics, as well as pedagogical and technological knowledge that is central to teaching and learning mathematics and statistics (Figure 1). This means developing specific types of knowledge for teaching secondary mathematics/statistics with technology: 1) Mathematics/Statistics Knowledge (Content Knowledge of mathematics and statistics); 2) Technological Mathematical/Statistical Knowledge (Technological Content Knowledge with appropriate tools used in mathematics and statistics); 3) Pedagogical Mathematical and Statistical Knowledge (Pedagogical Content Knowledge for teaching mathematics and statistics); and 4) Technological Pedagogical Mathematical and Statistical Knowledge (as a specific type of Technological Pedagogical Content Knowledge). With a focus on content, notice that the largest circle represents our approach in that mathematics/statistics knowledge is foundational to developing the other three knowledge types. In designing modules to focus on examining students' mathematical practices with technology, we also necessarily concentrate on increasing undergraduate PSMTs' Mathematical and Statistical Knowledge (MSK) and Technological Pedagogical Mathematics and Statistics Knowledge (TPMSK).

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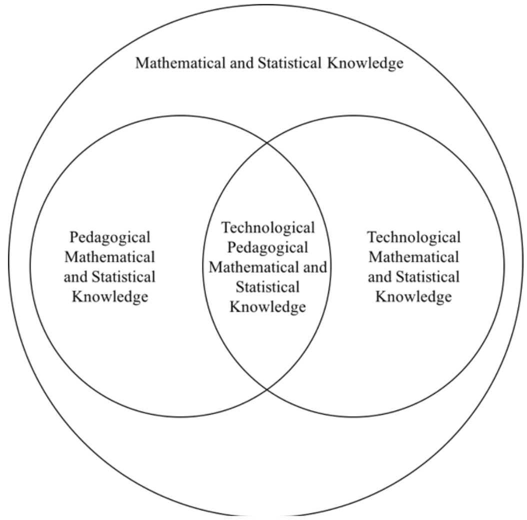

Figure 1. Framing Technological Pedagogical Mathematical and Statistical Knowledge

Analyzing Student Work

Research has pointed to the important role that students' mathematical thinking plays in high-quality instruction (e.g., Jacobs & Spangler, 2017). This points to the need for PSMTs to have opportunities to grapple with and make sense of how students think about mathematics. NCTM's publication, *Principles to Action*, identified "elicit and use evidence of student thinking" as one of the eight mathematics teaching practices (2014, p. 10). For PSMTs this skill must be purposefully developed via teaching practice. One method that has been shown to help PSMTs develop an understanding of student thinking is analysis of authentic student work (e.g., Jansen & Spitzer, 2009; Philipp, 2008).

Authentic student work can come in the form of written artifacts or video cases. Here we focus on video cases and their corresponding written artifacts as together they provide insight to student thinking as they are engaged in mathematical work. Video cases have been shown to improve PSMTs' ability to critically observe classroom practice, attending to teacher choices and student thinking rather than merely content delivery (e.g., Star & Strickland, 2008; Sherin & van Es, 2005). Additionally, a focus on student thinking through video case analysis has been shown to improve PSMTs' abilities to draw attention to and describe teachers' instructional moves to make student thinking visible, to reason about impact of teacher's decisions on student learning, and to propose alternatives to what was observed in the video (Santagata & Guarino, 2011).

While video case instruction has been shown to be very beneficial for PSMTs, researchers caution that the *selection* of video clips (e.g., Kurz, Llama, & Savenye, 2005; Sherin, Linsenmeier, & van Es, 2009) and *how* video cases are used is critical to promoting teacher learning (e.g., Brophy, 2004). To this end, Sherin et al. (2009) articulated a framework for selecting video clips to develop cases that attend to the extent to which a clip provides a *window* into student thinking, the *depth* of student thinking, and *clarity* of student thinking. To ensure the best video clip possible they suggest that all three criteria are considered. It is also suggested that cases be designed so that they focus on aspects of student work in which there are elements of confusion or surprise (Shulman, 1996; Sherin et al. 2009). Once video clips are selected, the activities that surround their use must be carefully designed, articulating clear goals to focus the analysis of the video (Borko, Jacobs, Eiteljorg, & Pittmann, 2008). A method often used to guide PSMTs' analysis of student work in video cases is the *professional noticing* construct developed by Jacobs, Lamb and Philipp (2010). The three components of the professional noticing

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construct are attending to students' strategies, interpreting students' mathematical thinking, and deciding how to respond on the basis of students' understandings.

Much of the research on PSMTs analyzing student work has been completed through the lens of professional noticing. Within professional noticing work more attention has been paid to professional noticing of whole class video (e.g., Krupa, Huey, Lesseig, Casey & Monson, 2017; McDuffie et. al, 2013), with less on prospective teachers' noticing of student written work (e.g., Dick, 2017; Goldsmith & Seago, 2011). In terms of PSMTs' professional noticing of students' understanding, thinking, and learning mathematical tasks in technological environments, very few studies have been conducted (e.g., Wilson, Lee & Hollebrands, 2011).

The seminal work on PSMTs' professional noticing of students' understanding, thinking, and learning of mathematics with technology comes from Wilson et al. (2011). They not only indicated that engaging preservice teachers in analyzing video cases of students' technological mathematical work resulted in identifying different ways of constructing models of student thinking, but also made a call for the need of research to more fully understand the role of these models in the development of PSMTs' TPACK. Even so, a broad search of the literature (including unpublished dissertations) indicates there has been very little continued work in this direction. This might be due to the complex nature of designing such materials for PSMTs. To address this, and support others who are aiming to support PSMTs in their development of TPMSK, we draw on the literature described here to propose a set of design principles for engaging PSMTs in professional noticing of students' mathematical technological practices.

Design Principles for Supporting Professional Noticing of Students' Technological Mathematical Practices

The design principles we propose draw on the integration of the literature on developing TPACK, video case pedagogies, and the construct of professional noticing. Specifically, we propose that by beginning with the philosophy of an integrated approach to develop skills in a specific content area, pedagogy and technology, the development of MSK and TPMSK can be done through the use of video cases of student practices on technology-based mathematics and statistics tasks. Guiding PSMTs' analysis of video cases is the use of the professional noticing construct. Specifically, we propose the following design principles.

1. PSMTs need to observe secondary students engaged in technology-based tasks of high-cognitive demand. As such, the selected tasks must be of high cognitive demand (Smith & Stein, 1998) and position the use of technology to develop mathematical or statistical understanding (Dick & Hollebrands, 2011).
2. Video clips (and associated written artifacts) should focus on aspects of student work in which there are elements of confusion or surprise, as is suggested by Sherin et al. (2009) and Shulman (1996).
3. Final clips should be selected based on Sherin et. al's (2009) recommendations for dimensions of video clips that support teacher discussion of students' mathematical thinking (i.e., *window* into student thinking, *depth* of student thinking, and *clarity* of student thinking). This includes use of picture-in-picture so that students' technological work is visible as well as any gestures they are making in relation to their technological work (e.g., pointing).
4. To support PSMTs' development of TMSK they must engage with the technology-based task first as learners (Lee, Hollebrands, & McCulloch, 2015).

5. To support PSMTs' development of TPMSK through the analysis of video clips, guiding questions should be designed based on Jacobs et al. (2010) framework for professional noticing. This means specifically asking PSMTs to attend, interpret, predict, and make pedagogical decisions based on their analysis of the video cases.

We conjecture that careful selection of tasks, technology, video case clips, and the questions included in a video case will work together to provide PSMTs an opportunity to develop their knowledge of students' understandings, thinking, and learning with technology in mathematics.

An Example: The Function Concept - Functions and Non-functions

To illustrate our vision for video cases that can promote PSMTs' professional noticing of students' technological mathematical practices we provide an example. We begin by having PSMTs engage with a preconstructed GeoGebra applet designed to provoke a dilemma in relation to their understanding of function (Design Principle 4) PSMTs are asked to engage with this applet and answer questions related to their own understanding of function, representations of function, and consider how they might use the applet with students. (Figure 2). This applet has been designed using a vending machine metaphor so that PSMTs grapple with making sense of function, domain, and range in a context that does not use traditional algebraic representations (Design Principle 1) (see McCulloch, Lovett, & Edgington (2017) for a full discussion of the design of this applet and one study of its use with undergraduate students).

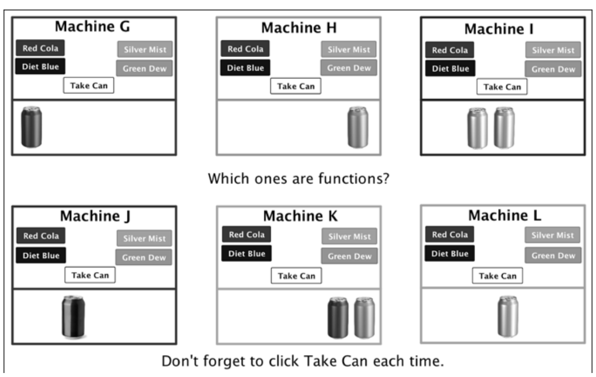

Figure 2. Vending machine applet

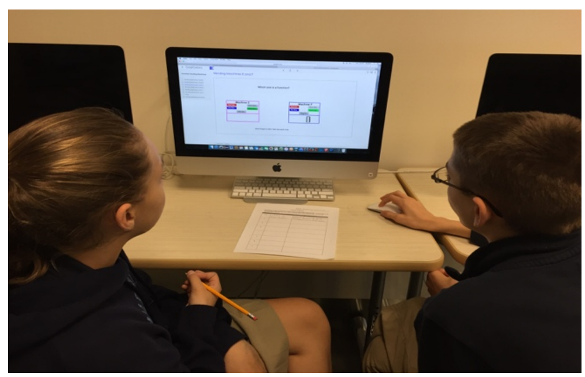

Figure 3. Students working on the applet

A second version of the Vending Machine task was designed specifically for secondary students (Design Principle 1). It was designed to be used as an introduction to function with students who had no previous experience with the term *function*. The goal of this version of the task is that students develop a definition of function based on their exploration of the machines in the applet. The applet was then implemented in an 8th grade math class with pairs of students working together and their work on the task was screen captured. Next, the students' videos were analyzed for the purpose of selecting examples of work. This included identifying video clips for episodes of confusion or surprise, followed by the analysis of the window, depth of student thinking, and clarity of student thinking (Design Principle 2 & 3). Once video episodes were selected, they were packaged as a case with questions designed based on Jacobs et al. (2010) professional noticing construct (Design Principle 5). For example, PSMTs were asked to attend to and interpret the coordination of student thinking and technological actions when analyzing the video cases. Further, they were asked to make predictions about student thinking and technological action based on their attending and interpreting. Using these materials, the video case was implemented with PSMTs as follows:

- PSMTs were asked to discuss revisions they would make to the original applet if they intended to use it with secondary students to develop the definition of function.
- After exploring the version of the applet that was used with secondary students, PSMTs anticipate how secondary students would engage with the task - considering actions with the technology and students' verbal and written responses to task prompts.
- PSMTs then analyze carefully selected screen capture clips of multiple pairs of students engaging with the applet (and their associated written work). These video clips show the pairs of students' engagement with some of the machines in the applet, but not all. PSMTs then predict how each student pair might identify each machine as a function or non-function and how they might engage with the remaining machines given their analysis of the first few.
- Given written definitions of function from five pairs of secondary students, PSMTs select which they would use to start a class discussion of the definition of function, why they selected that sample, and which machines in the applet they would draw upon for the discussion given their selection.

As is evident, the video case provided ample opportunities for the PSMTs to consider their own and students' technological mathematical work through engaging with a carefully selected task and analyze carefully selected video cases of students' technological mathematical work by engaging in professional noticing.

Discussion

Our proposed design principles for the development of professional noticing of students' technological mathematical practices are grounded in the literature and have been successfully used to frame the design of a module for PSMTs in the context of the use of a vending machine applet to build an understanding of the function concept. The vending machine task module has been piloted with 98 PSMTs in secondary mathematics education methods courses at six different universities. Data included PSMTs' pre and post function definitions, screen casts of their own mathematical work with the vending machine task, and written artifacts from the analyzing student work assignments. These studies indicate that this module was successful in eliciting PSMTs' MK, TMK, and TPMK (Lovett et al., Under Review). Specifically, within the realm of TPMK, not only were PSMTs able to show an understanding of *students' understandings, thinking, and learning with technology in mathematics*, but also as they stated their predictions of students' technological mathematical work they showed evidence of being able to conceive of how technology can support mathematical thinking (Lovett et al., Under Review), both important aspects of TPACK (Niess, 2005).

Pilot study results also indicate the use of professional noticing to frame the analysis of the video cases was important in that it elicited the specific ways in which prospective teachers drew upon different aspects of student work evident in the video cases. We found that some drew only upon students' spoken and written words, others drew upon only students' actions with the technology. However, those that coordinated both aspects of the students' work were better prepared to predict students' responses on related tasks and to make decisions to support student learning (Lovett, Dick, McCulloch, Sherman, & Martin, 2018). This coordination was especially important as they were making sense of a particular video clip in which students were confused.

Finally, it was found that PSMTs' knowledge of the mathematics deepened through their own engagement with the vending machine task as a learner, and for many this knowledge was expanded even more through their analysis of the video cases (Lovett et al., 2017). Thus, we have substantial evidence that these proposed design principles for developing video cases for examining students' practices with technology are promising.

Conclusion

Strong preparation of mathematics teachers must include opportunities to engage with technology-based mathematics tasks as learners as well as opportunities to develop an understanding of how to support students' learning in mathematical technological environments. As we consider the conference theme, *looking back* and understanding theories and methods that have been successful in supporting prospective teachers as they learn to make sense of student thinking can help us *look ahead* and move forward by drawing on this work to propose new theory about continuing this development in technological contexts. In this paper we have articulated a set of 6 principles to frame the design of video case materials to support PSMT development of TPACK. Results from pilot studies provide empirical support for the promise of these design principles. We now challenge ourselves and others in the field to keep these principles in mind as we work to ensure PSMTs are well-prepared to teach with technology in ways that support students' mathematical reasoning.

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