

SECONDARY MATHEMATICS STUDENT TEACHERS' TYPES OF NOTICING WHILE TEACHING

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The purpose of this paper is to describe secondary mathematics student teachers' types of noticing while teaching. We discuss the importance of focusing on the interrelatedness of the noticing skills rather than reporting on individual skills separately. We apply the types of noticing to videos of student teachers to identify their ability to elicit and interpret student mathematical thinking in-the-moment while teaching. Results suggest that our student teachers did elicit and attend to student mathematical thinking while teaching, but how they interpreted the elicited student thinking varied. We hypothesize three reasons for why student teachers may have interpreted student mathematical thinking at a general level.

Keywords: Teacher Education–Preservice, Teacher Knowledge, Mathematical Knowledge for Teaching

Teaching is a complex activity that requires teachers to make purposeful in-the-moment decisions to attend to some activities while disregarding others. Sherin and Star (2011) noted that teachers are “bombarded with a blooming, buzzing confusion of sensory data” (p. 69) that he/she must sift through to make in-the-moment decisions to support student learning. For preservice teachers (PTs), the cacophony of sensory data can be more difficult to sift through than experienced teachers resulting in PTs becoming overwhelmed, focusing on small tasks (e.g., one group of students, one solution strategy, one students' voice) and neglecting or missing other important aspects of teaching (e.g., multiple ways of student thinking, actions of all students in the class).

Researchers have used noticing to focus teachers' attention on important aspects of teaching; however, because teachers tend to notice a variety of information in a classroom (e.g., teacher actions, student actions, classroom management, posters on the wall) guidance on what to notice, especially for PTs, is necessary (Jacobs, Lamb, Philipp, & Schappelleel, 2011; Sherin & van Es, 2005; Star & Strickland, 2008; Star, Lynch, & Perova, 2011; Stockero, 2014). Jacobs, Lamb, and Philipp (2010) extended the construct of teacher noticing to —professional noticing of children's mathematical thinking, which is conceptualized “as a set of three interrelated skills: *attending* to children's strategies, *interpreting* children's understanding, and *deciding* how to respond on the bases of children's understanding” (Jacobs et al., 2010, p. 172, *emphasis added*).

Mathematics education researchers have used Jacobs et al.'s (2010) framework to learn what teachers attend to and how teachers interpret and respond to children's mathematical thinking (e.g., Schack, Fisher, & Wilhelm, 2017). As a field, we have learned more about professional noticing, specifically researchers often focus on one of the three interrelated skills (attending, interpreting, or responding) in a reflective setting, thus artificially compartmentalizing these interrelated skills. We argue in this paper that investigating *the interrelatedness* of the noticing skills is critical to understanding how teachers apply these skills when teaching. We also share how we, as researchers, utilized professional noticing to determine how our secondary mathematics student teachers elicited and acted on student mathematical thinking (SMT) in-the-moment *while teaching*.

In this paper, we report on the following research question: To what extent do secondary student teachers elicit SMT and interpret that thinking in-the-moment while student teaching?

Theoretical Framework

To analyze and interpret our data we frame our view of eliciting and acting on SMT through Piaget's (1955) construct of decentering and Teuscher, Leatham, and Peterson's (2017) types of noticing.

The construct of decentering provides a powerful lens for examining student-teacher interactions because it focuses on the teacher's interpretations of students' verbal and written explanations to make in-the-moment decisions while teaching. We seek to draw inferences about how student teachers' understanding of SMT relates to their actions in-the-moment of teaching. According to Piaget (1955) decentering characterizes how the actions of an observer (i.e., student teacher) attempting to understand how an individual's (i.e., student's) perspective differs from his/her own. Steffe and Thompson (2000) and Thompson (2000) distinguished two ways in which individuals interact with others, which are described by the type of model that an individual creates of other's thinking. Thompson (2000) described individuals as participating unreflectively or reflectively. Individuals who participate unreflectively were describe as creating first-order models. "The models an individual constructs to organize, comprehend and control *his or her experiences*, i.e., their own mathematical knowledge" (Steffe & Olive, 2010, p. 16 *emphasis added*). Whereas, individuals who participate reflectively were described as creating second-order models, attempting to elicit and interpret SMT, and assist students in furthering their thinking based on the students' perspectives, not their own (i.e., decentering).

Teuscher et al. (2017) extended Jacobs et al. (2010) professional noticing framework describing types of noticing. These types of noticing were based on student teachers' written journal entries completed during student teaching. Attending to SMT was divided into two categories: *general observation* or *student mathematical thinking*. Interpreting SMT was also divided into two categories: *general interpretation* or *root interpretation*. Four types of noticing specifically connect the two skills of *attending to* and *interpreting* SMT. They were: (1) general observation, no interpretation; (2) general observation, general interpretation; (3) student mathematical thinking, general interpretation; and (4) student mathematical thinking, root interpretation (see Teuscher et al., 2017 for specific definitions). We seek to apply these four types of noticing to videos of student teachers by identifying their ability to elicit and interpret SMT in-the-moment *while teaching* (e.g., decentering). Specifically, we are interested in identifying what SMT student teachers attend to while teaching and in what ways they interpreted the SMT that was available to them.

Methods

Pairs of student teachers where assigned a 16-week placement at a junior high or high school. The university supervisor, first author, observed the student teachers once a week and provided feedback during their student teaching experience. Data were collected on four pairs of student teachers (eight student teachers) during two different semesters. The researchers purposefully selected four pairs of student teachers to allow for variability in the data set. All eight student teachers' lessons observed by the university supervisor were videotaped. The researchers analyzed three lesson videos (beginning, middle, and end of student teaching) for each student teacher, 24 total, and coded each using the *Practice of Probing Student Thinking* Framework (Teuscher, Switzer, & Morwood, 2016) to identify instances where student teachers appeared to elicit and act on SMT. We chose probing student thinking because it allowed the researchers to

identify instances of SMT and possible interpretations of the student thinking by the student teachers.

To identify a subset of instances of probing student thinking to analyze, we compared the researchers' instances to the student teachers' instances of probing student thinking and found 27 overlapping instances, which indicated that student teachers seemed aware of SMT. Using the researchers' instances from the overlapping instances, we conducted our analysis to determine how the student teachers were eliciting and acting on SMT in-the-moment while teaching. Our unit of analysis was an individual video instance of probing student thinking, defined as a complete interaction between a teacher and student(s). The researchers used the types of noticing framework (Teuscher et al., 2017) to identify the elicited SMT in the video instances and inferred the student teachers' interpretations of the SMT based on their response to the SMT in the video.

In the following section, we provide illustrative examples from our data set to describe these different types of noticing. Video instance 1 is the transcript for one probing student thinking instance that occurred near the end of a high school mathematics class. The high school students spent the class period learning about graphing quadratic inequalities and using the graphs to find the solutions to the quadratic inequalities.

Student Teacher: So, what did we notice the difference between these two answers [one answer was $x < 5$ or $x > 11$ and the other answer was $-5 < x < 11$]? The one from our previous problem and this one? What's the difference? Student 1?

Student 1: Um, it's not like infinity on it so it doesn't go on forever, it stops.

Student Teacher: Yeah, and what did we notice about the equations at the top of number nine and ten? How are they different in terms of the signs? Yeah, Student 2?

Student 2: One [function] is showing like one is less than zero and one [function] is greater than zero so that changes when it's outside or inside.

Student Teacher: And when do we know if it's going outside or inside?

Student 3: If it's greater or less than zero. Wait, what?

Student Teacher: Okay, say, what happens when it's greater than zero?

Student 3: Then the arrows on the outside go inside.

Student Teacher: Um, is it, let's see, I think in this case we also had our, both our parabolas, which way are they opening?

Students: Up.

Student Teacher: Yeah, so we want to make sure and graph and make sure they were opening up, because if it was opening down we might have a different answer.

We identified video instance 1 as a *General Observation* and *General Interpretation*. The general observation is because we could not infer the SMT. We have some idea that students are discussing whether a quadratic function is greater than zero or less than zero, but we were unable to determine what students meant when they said, "*that changes when it's outside or inside*" (Student 2) or "*then the arrows on the outside go inside*" (Student 3). The general interpretation was based on the student teacher's response to Students 2 and 3 because she seemed to carry the discussion on without seeking clarification of what students meant by "*that changes when it's outside or inside*" (Student 2) or "*then the arrows on the outside go inside*" (Student 3).

Video instance 2 is the transcript for another probing student thinking instance that occurred at the beginning of a middle school mathematics class. Students were learning about unit rates, constant growth rates, and linear equations. Students were shown a graph of a linear function (number of hands as a function of number of people) and asked to answer the following three questions: (1) What is the unit rate and what does it represent? (2) If there are 6 hands, how many people are present? (3) How many hands are there if there are 52 people? The transcript is from the class discussion about the first question.

Student 1: I was looking at this one right here [points to the point] this dot right here (1, 2), and since it is the lowest one that is not (0, 0) that is the unit rate because all you need to do is multiply it by two and it will keep going on forever.

Student Teacher: Multiple what by two?

Student 1: Multiply people and hands by two.

Student Teacher: People and hands? I am not sure I understand what you mean.

Student 1: One person times two is two people, well (pause). You add one person here then you add two hands. Whenever you go up one dot it goes add one person then it goes add two hands.

Student Teacher: Ok, so you are saying...

Student 2: There is a constant growth pattern.

Student Teacher: [revoicing] There is a constant growth pattern that you add one here (pointing to the person axis) and you add two here (pointing to the hands axis).

Student 1: Yes

Student Teacher: And then continue on?

Student 1: Yes

Student Teacher: Okay. Okay. Awesome, so your unit rate is what?

Student 1: One to two.

Student Teacher: [revoicing and writes on the board: 1 to 2], One to two, and what are your units?

Student 1: Units?

Student Teacher: What does this 1 represent [pointing to the 1 on the board]

Student 1: People. People. People to hands

We identified video instance 2 as Student Mathematical Thinking and Root Interpretation. The SMT was “I was looking at this one right here [points to the point] this dot right here (1, 2), and since it is the lowest one that is not (0, 0) that is the unit rate because all you need to do is multiply it by two and it will keep going on forever” (Student 1). In other words, the student is saying that you can use the point on the line closest to (0, 0) to determine the unit rate, which is the point (2, 1) to which he points. We inferred that the student teacher made a root interpretation – a more in-depth analysis of what the students might have meant by their utterance, or what that thinking means with respect to student understanding – because the teacher was unsure what Student 1 referred to when he states, “multiple it by 2.” Therefore, the student teacher asks clarifying questions to which the student responds with “One person times two is two people, well (pause). You add one person here then you add two hands. Whenever you go up one dot it goes add one person then it goes add two hands” (Student 1). The student teacher seemed aware of the difficulty that middle school students have when beginning to work with multiplicative relationships. Following up on the statement, “you multiply it by 2” (Student 1), allowed Student

1's thinking to be made public. In responding, Student 1 begins to think about the multiplicative relationship between the number of hands and people. As he is explaining his thinking he realizes he is not correct and reverts to an additive relationship for which he is familiar.

Results

Table 1 displays the percentage of video instances based on the types of noticing. We found that merging the separate noticing skills captured the interrelatedness of attending and interpreting that reveals important and distinguishing aspects of how the student teachers acted on SMT. Of the 27 video instances the researchers coded 23 (85.2%) provide evidence of the student teachers eliciting SMT and 4 video instances (14.8%) provided evidence of the student teachers making a general observation of the SMT. However, of the 23 video instances that include evidence of SMT, 14 video instances (51.9%) include evidence of a root interpretation and 9 video instances (33.3%) include evidence of a general interpretation. Therefore, the results indicate that the student teachers were highly successful in eliciting SMT, but often did not generate a root interpretation of the elicited SMT.

Table 1: Student Teachers' Video Instances by Types of Noticing

Attending	Interpreting	Instance Count	Percent
Student Mathematical Thinking	Root Interpretation	14	51.9%
	General Interpretation	9	33.3%
General Observation	General Interpretation	4	14.8%
Total		27	100.0%

For the remainder of this report, we focus our analysis on the nine video instances where the student teachers elicited SMT but made a general interpretation. In these instances, there was no evidence that the student teachers generated a root interpretation of the elicited SMT. We were interested in identifying potential factors that may have led to the student teachers not generating a root interpretation. We share two transcripts that are representative of the SMT with general interpretation type.

Video instance 3 occurs at the beginning of a high school lesson. Students were working on a task to determine if different situations were fair or unfair to assist them in generating a mathematical definition of fair. The following transcript provides the conversation between the student teacher and a student.

Student: I think it is fair, but I am trying to think how it might not be fair?

Student Teacher: What is your definition of fair?

Student: It would be more fair if everyone got a prize or something, but this isn't the type of thing where everyone could get, I don't know, I'm thinking realistic.

Student Teacher: So, are you reconsidering the definition then of fair?

Student: Kind of, literally fair is everyone gets it.

Student Teacher: Gotcha, so like equality. Like everyone would get one?

Student: But this scenario is confusing me; it wouldn't be fair if they said that the tallest person gets the prize if they are short.

Student Teacher: Gotcha, so then that would be bias against the short people, right?

Student: Yeah, so this is, I don't know, this is like everyone has the same opportunity.

Student Teacher: (revoicing) Okay, so everyone has the same opportunity. So, does this one fit that definition of fair, this scenario?

Student: Yeah, everyone got a ticket and everyone had the, like, gets the opportunity to be picked, it's just not everyone gets picked.

Student Teacher: Right so we all have, so you and I might be in the drawing and we might not both get picked, but we have equal chances?

Student: Yeah

Student Teacher: Which is what you were saying? As long as you explain I am ok with whatever you come up with.

Our analysis of video instance 3 found that the student teacher attended to SMT multiple times. The student teacher asked the student if she was reconsidering her definition of fair. The student suggested that "*fair means everyone gets a prize,*" which is a common definition of fair but not the mathematical definition. Then the student seems confused because she thinks "*fair means everyone has the same opportunity.*" While the student teacher seemed to identify that the student had two definitions of fair, how the student teacher acted on this SMT was revealing. The student teacher indicates to the student that it does not matter which way she thinks about it as long as she can explain her thinking. In other words, both of the student's definitions were acceptable. Throughout the lesson multiple students continued to bring up and discuss these two definitions of fair and the student teacher never distinguished the difference between the definitions. In concluding the lesson, the student teacher stated, "as long as we can explain why whatever method we use was fair then it should be fair, right?" We inferred that the student teacher's mathematical meaning for fair (Byerley & Thompson, 2017) may have resulted in her interpreting the SMT at a general level.

Video instance 4 occurs in a high school lesson where students were learning about probability. The students were given the following problem to work on in groups.

Two students, Lee and Rory, find a box containing 100 baseball cards. To determine who should get the cards, they decide to play a game with the following rules:

- One of the students repeatedly flips a coin.
- When the coin lands heads up, Lee gets a point.
- When the coin lands tails up, Rory gets a point.
- The first student to reach 20 points wins the game and gets the baseball cards.

As Lee and Rory are playing the game they are interrupted and are unable to continue. How should the 100 baseball cards be divided between the students given that the game was interrupted at the described moment? When they are interrupted Lee has 19 points and Rory has 17 points.

A student (Bill) shares with the class that Rory should get $\frac{17}{36}$ of the cards and Lee should get $\frac{19}{36}$ of the cards. Other students in the class respond with "*what?*" and "*where did the 36 come from?*" The following transcript captures the ensuing conversation between the student teacher and the students in the class.

Student Teacher: Okay, [student 1] has a question about where the 36 came from?

Bill: You take the $19+17$ to get the 36.

Student Teacher: So how many cards do you give each person than?

Bill: Um, still trying to figure that out.

Student Teacher: (Writes $\frac{17}{36} * \frac{x}{100}$ on the board) and how many cards do they each have?

Student 2: The first one is 52

Student Teacher: (revoices) 52 for Rory

Student 3: 52 for the other one 47 for that one

Student Teacher: (writes on the board $x=52$)

Student 4: 52.7 and 47.2

Student Teacher: (revoices) 52.7 and 47.2 and writes them on the board.

Student 3: That should actually be 0.8.

Student Teacher: Which one?

Student 3: That one (points to the Rory column)

Student Teacher: (Erases the 52.7 and writes 52.8), Kay, student, what do you think we should do with the 0.8 and 0.2 of a card?

Our analysis of video instance 4 found that the student teacher attended to SMT multiple times. The student teacher asked the Bill to explain where the 36 came from in response to another student's question. Students provide the number of cards that they calculated Rory and Lee should receive. The student teacher seems to identify that Bill has based the number of cards that Rory and Lee should receive on their respective theoretical probabilities of winning. The student teacher accepts Bill's strategy and immediately moves to asking the class what they think they "should do with the 0.2 and 0.8 of a card," assuming the rest of the students, those who initially did not understand Bill's reasoning, now understood his thinking. We inferred that the student teacher's not decentering (Piaget, 1955; Teuscher, Moore, & Carlson, 2016) may have resulted in her interpreting the other students' mathematical thinking at a general level.

Table 2 displays the three categories for the nine video instances where student teachers elicited SMT but made a general interpretation of the elicited SMT. The majority (66.7%) of video instances fell in the category of not decentering. As in video instance 4, the student teacher only seemed concerned with making sure that everyone had the correct answer, rather than helping students understand how Bill (the student) had come up with the answer and why that made sense. In two video instances (22.2%) we determined that the general interpretation was appropriate. This was because the student teacher was launching an activity where students later explored the ideas that they had shared. The goal during a launch of a task is to make student thinking public so it can be discussed; therefore, we felt that at that point in the lesson the general interpretation was appropriate. The last category was the influence of the student teacher's mathematical meaning as described in video instance 3.

Table 2: Possible Reasons Secondary Student Teachers' May Not Generate a Root Interpretation

Inferences for why student teachers make a general interpretation	Count	Percent
Not Decentering	6	66.7%
Appropriate	2	22.2%
Mathematical Meaning	1	11.1%

Implications

The purpose of this paper was to describe secondary mathematics student teachers' types of noticing while teaching. We demonstrated the importance of the interrelatedness of the attending and interpreting noticing skills because the types of noticing revealed important and distinguishing aspects of how the student teachers were acting on SMT. We found that our student teachers were highly effective with eliciting SMT in-the-moment of teaching. However, we found that our student teachers were less successful in interpreting the elicited SMT in-the-

moment of teaching with a root interpretation. While there were two video instances where it was appropriate for the student teachers to interpret at the general level, most video instances were student teachers interpreting at the general level that were because the student teachers were not decentering.

While our analysis is a small subset of video instances from eight student teachers, we believe that future research can focus on identifying differences among student teachers to identify what practices PTs need to focus on during their course work that will prepare them to attend to and interpret SMT in such a way that will build on all students mathematical thinking and improve student learning in their classrooms. We recommend that teacher educators need to assist PTs in learning how to interpret SMT from the perspective of the students (e.g., decentering). While we would agree that this is a challenging skill for PTs to develop, we had student teachers who identify a root interpretation of the SMT in-the-moment while teaching, which allowed the student teacher to orchestrate a productive discussion in their mathematics classroom with all students.

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