THE INTERPLAY OF FRUSTRATION AND JOY: ELEMENTARY STUDENTS' PRODUCTIVE STRUGGLE WHEN ENGAGED IN UNSOLVED PROBLEMS

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In this study, I investigated Grade 4 and 5 students' emotions while they engaged in the exploration of unsolved mathematics problems, including parts of the Graceful Tree Conjecture. Ten students from an after-school program in the Midwest participated in seven task-based interviews. The students exhibited a variety of emotions throughout the study, with frustration and joy displayed most frequently. I found the interplay of these emotions, joy and frustration, to describe the productive struggle that the students experienced while working on parts of the unsolved problems. A descriptive case of two students, Bernice and Alia, are included to describe how the interplay of frustration and joy characterize productive struggle.

Keywords: Affect, Emotion, Beliefs, and Attitudes; Elementary School Education; Problem Solving

The purpose of this paper is to document what productive struggle look like. To do this, I examined the emotions ten students displayed while they were engaged in problem-solving with unsolved mathematics problems. The use of the unsolved problems permitted the students the opportunity to display multiple emotions and engage in struggle. Hiebert and Grouws (2007) defined *struggle* as when "students expend effort to make sense of mathematics, to figure something out that is not immediately apparent" (p. 387). Allowing students, the opportunity to struggle is beneficial (e.g., Hiebert & Grouws, 2007; Kapur, 2010; Reinhart, 2000). When students have spent time learning mathematics while engaged in productive struggle, they significantly outperform similar ability students who were not given the opportunity to struggle on a task (Kapur, 2010). According to Kapur (2010), these same students who have engaged in productive struggle are also able to engaged and transfer their knowledge to work on challenging mathematics concepts they have yet to explore. While past research has documented that productive struggle is beneficial, there is no research that described what productive struggle actually looks like as children are engaged in mathematical problem solving (Warshauer, 2015; Zeybek, 2016).

When students are engaged in problem solving, they experience both positive and negative emotions (Goldin, 2000a; Hannula, 2015). However, the bulk of the previous research on emotions is limited to surveys and does not document students' emotions while they are engaged in problem solving (Hannula, 2015). In order to look forward in mathematics education, it is critical to document the different emotions students experience while engaged in problem solving and to describe what it looks like when students are experiencing productive struggle.

Research Questions

The following questions guided my research study:

1. What are the emotions students displayed when they were engaged in the exploration of unsolved problems?
2. How are the emotions of frustration and joy related to the struggle students experience while they are engaged in the exploration of unsolved mathematics problems?

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Theoretical Framework

Positioning theory has been used in mathematics education as a way to analyze social interactions (e.g., Turner, Dominquez, Maldonado, & Empson, 2013; Wood, 2013; Yamakawa, Forman, & Ansell, 2009). Daher (2015) and Evans, Morgan, and Tsataroni (2006) have identified that students' emotions are linked to their positioning. In this study, positioning theory (van Langenhove & Harré, 1999) was employed as an overarching theoretical framework for how students positioned themselves through their interactions with other students and the mathematics. Their positions were displayed through their dispositions or more specifically, their emotions.

Positioning theory is "the study of local moral orders as ever-shifting patterns of mutual and contestable rights and obligations of speaking and acting" (van Langenhove & Harré, 1999, p. 1). It is "the discursive construction of personal stories that make a person's actions intelligible and relatively determinate as social acts and within which the members of the conversation have specific locations" (van Langenhove & Harré, 1999, p. 16). People can position themselves or be positioned by others in many different ways, such as being good at mathematics or bad at mathematics. A person is positioned based on actions and conversations. The conversations create storylines. The storylines can document students emotions at the moment they are displayed through their dispositions.

Methods

In this qualitative study, I investigated ten Grade 4 and 5 students (Alia, Amanda, Becca, Bernice, Edward, Hector, Iris, Joella, Karly, and Trevor; all of the names have been changed) while they engaged in the exploration of unsolved problems. The study took place at an after-school program at a community center in the Midwestern United States. Students participated in seven semi-structured, task-based interviews (Goldin, 2000b), which I will refer to as problem-solving sessions. These seven problem-solving sessions took place over three weeks and each lasted between 35 and 45 minutes. For the first six Problem-Solving Sessions, students worked on the Graceful Tree Conjecture. During Session 7, students were introduced to Collatz Conjecture.

Graceful Tree Conjecture

During the first six problem-solving session students worked on the Graceful Tree Conjecture. The Graceful Tree Conjecture is an unsolved conjecture from graph theory that is accessible to young children. The problem has students exploring different types of tree graphs, graphs that are connected (one piece) with no cycles (see Figure 1). This means a tree graph is acyclic, which means if you follow a path from node to node along the edges, you will never cycle back to the same node without repeating an edge. This means that tree graphs always have one more edge than node.

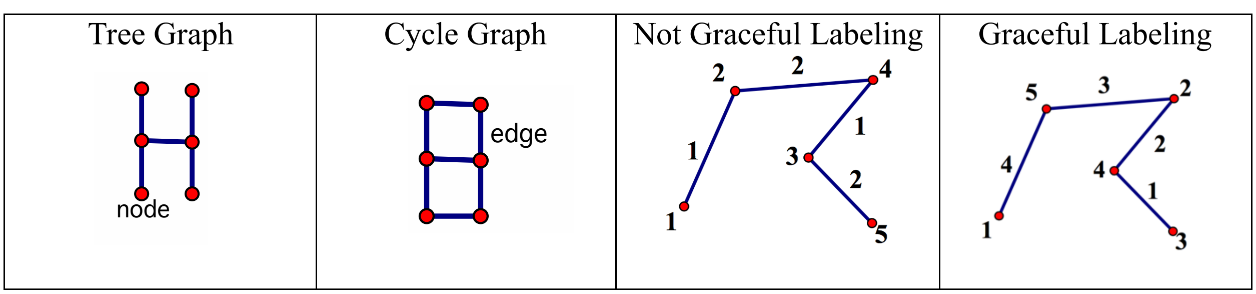

Figure 1. Concepts of the Graceful Tree Conjecture.

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Tree graphs are assigned numbers to the nodes that induce labeling of the edges. For a tree graph of order *n*, every node is labeled distinctly from 1 through *n* and the edges are labeled with the absolute value of the difference of the labels on their endpoints. For a tree graph to be labeled gracefully, the edges need to be labeled distinctly 1 through *n*-1 (see Figure 1).

Overview of Problem-Solving Sessions

During the first six sessions, students explore different categories of tree graphs in increasing sophistication (see Figure 2). I challenged the students to not only create a graceful labeling for each graph but to find a pattern or justification to show that any tree graph in the category could be labeled gracefully. While exploring the different tree graphs, students were given a page that contained four distinct tree graphs in a given category, enlarged copies of each of the graphs, and numbered circle and square chips (see Figure 3). This allowed students to try multiple solutions without having to erase.

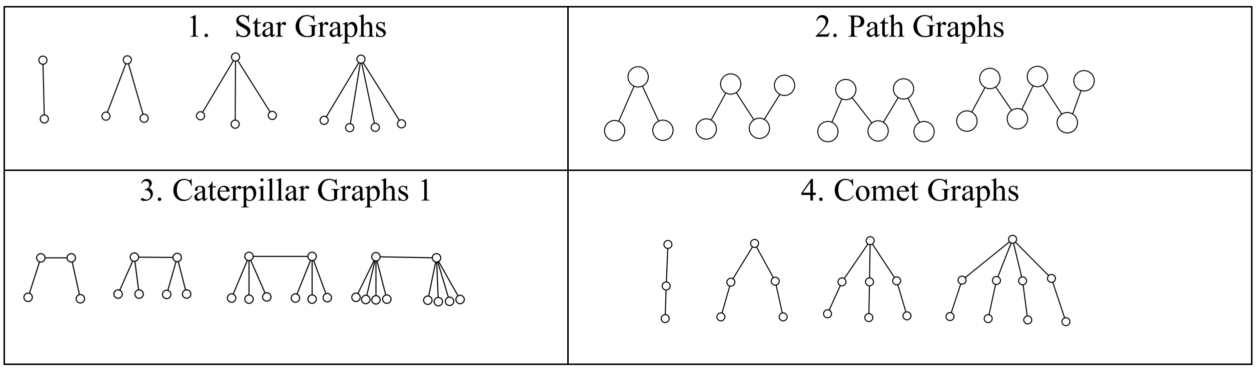

Figure 2. Tree Graphs in increasing sophistication.

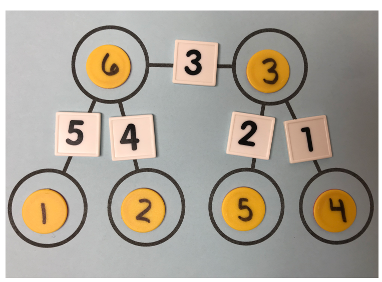

Figure 3. Enlarged Caterpillar Graph and circle and square numbered chips.

During Session 1, students were introduced to tree graphs, learned what a graceful labeling was, and explore a five node tree graph. Throughout Session 2, students worked to find a graceful labeling for Star Graphs. In Session 3, students finished Star Graphs and began to work on labeling Path Graphs. During Session 4, students continued to work on labeling Path Graphs. In Session 5, students finished labeling Path Graphs and began to work on Caterpillar Graphs. In Session 6, students worked on labeling Caterpillar Graphs. In Session 7, Collatz Conjecture was introduced but after twenty minutes of work the participants requested to continue working on the Graceful Tree Conjecture. The students then began to work on gracefully labeling Comet Graphs.

Analysis

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All seven problem-solving session were video recorded and student work was collected. All video was transcribed including non-verbal actions, such as facial expressions or arms raised in the air. Next, I constructed an analytic framework based on the ideas from Else-Quest, Hyde, and Hejmadi (2008) to characterize the emotions students displayed while they were engaged with the mathematics. During my first round of analysis, I modified Else-Quest et al.'s framework by combining emotions such as joy, pleasure, and pride because I was not able to distinguish a difference between these emotions without interviewing students to know how they were specifically feeling (see Table 1 for the emotion framework).

Table 1: Emotion Analytic Framework

Emotion/Code	**Definition**	**Example**
Anger/Disgust	Irritability, temper, rage	I hate you.
Tension	Worry, Tautness, rigidity	I don't want to work with my sister.
Sadness	Anguish, grief, misery	I am really sad.
Boredom/Apathy	Fatigue, lack of interest, indifference	I don't want to work on this today. I am tired.
Affection/Caring	Friendliness, warmth, encouragement	I can help you with this one.
Humor	Banter, joking, comedy	That looks like a tree.
Contempt	Snobbery, ridicule, mimicking	Bad job for you. Stop being scary.
Joy/Pleasure	Delight, excitement, happiness	I got it! Graceful!
Distress/Frustration	Complaining, impatience, upset, disappointment	I don't get it. I can't do it.

After creating my analytic framework, I examined all of transcripts from the problem-solving sessions and used the framework to document anytime an emotion was displayed by a student. If the student displayed several statements of frustration in a row, each individual statement was documented as its own sign of frustration. I did this process twice to check for consistency, similar interpretation, and to make sure nothing was missed. This process was completed using the qualitative software, Transana (Woods & Fassnacht, 2016). Next, I created reports through the use of Transana to account for each emotion that was displayed by a student.

Results

During analysis, I explored the emotions students displayed when they were engaged with the exploration of unsolved problems (see Table 2). Through my Transana reports, I found that the two most common emotions students displayed were joy and frustration.

Table 2: Emotions Displayed by Session

Emotions	1	2	3	4	5	6	7	Total
Anger/Disgust	0	0	0	0	0	0	0	0
Tension	0	0	0	0	0	0	0	0
Sadness	0	0	0	0	0	0	0	0
Boredom/Apathy	1	0	1	2	0	0	0	4
Affection/Caring	2	5	1	0	3	7	0	18

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Humor	3	9	0	3	1	2	3	21
Contempt	4	13	6	3	3	15	5	49
Joy/Pleasure	23	32	21	14	25	44	18	177
Distress/Frustration	22	26	56	21	23	38	37	223

Frustration

Documented a total of 223 times, frustration was displayed the most throughout the seven problem-solving sessions. Bernice, Trevor, and Joella displayed the emotion of frustration most commonly during the sessions (see Table 3). These emotions were most typically displayed while the students were working on finding a graceful labeling of a graph. Comments such as, “I don’t get this,” “Oh my gosh,” “This is hard,” and “It won’t work” were common statements students made that indicated the emotion of frustration. They also had actions like slamming their hand on the table or making moaning sounds.

Joy

Joy was the second most common emotion displayed. It was documented 177 times. Becca, Alia, and Bernice displayed the emotion of joy most frequently throughout the seven sessions (see Table 3). Joy was displayed most frequently when students found a graceful labeling for a graph. Comments marked as joy included statements such as, “Got it!” “I am done,” “I did it, I finished it!” “Finished,” and Graceful!” Many times these comments were also made with students throwing their arms in the air in celebration or with clapping.

Productive Struggle

Because of the large number of times students displayed joy and frustration, I wanted to examine frustration and joy in more detail. I broke down frustration and joy by student and session (see Table 3). When I did this, I found that every student displayed both frustration and joy multiple times throughout the problem-solving sessions. Upon a closer look at when the students were displaying the frustration and joy, I found that students would display multiple signs of frustration followed by a moment of joy. Most of the time the joy was displayed when a student found a graceful labeling of a graph. I will share two students’ stories throughout a session to document the interplay of frustration and joy.

Table 3: Frustration and Joy By Student and Session

	Alia		Amanda		Becca		Bernice		Edward		Hector		Iris		Joella		Karly		Trevor	
Session #	F	J	F	J	F	J	F	J	F	J	F	J	F	J	F	J	F	J	F	J
1	2	4	3	1	-	-	6	6	2	5	0	1	0	0	3	3	1	0	5	3
2	0	12	1	2	3	3	8	0	2	2	-	-	0	3	3	2	-	-	9	8
3	8	5	1	2	5	2	15	2	1	2	4	0	1	3	10	3	5	2	6	0
4	-	-	1	1	7	1	9	5	1	4	-	-	1	1	2	1	-	-	0	1
5	3	9	0	1	0	5	5	4	1	1	2	0	2	2	5	2	0	1	5	0
6	4	5	2	7	2	10	6	6	1	3	7	4	1	2	11	1	1	2	3	4
7	2	6	1	0	9	3	9	5	1	0	-	-	0	2	1	1	-	-	14	1
Total	19	41	9	14	26	24	58	28	9	17	13	5	5	13	35	13	7	5	42	17

Note. – means a student was absent for the problem-solving session. F stands for frustration and J stands for joy.

Bernice. Bernice was a Grade 4 student. Her story was chosen to be documented because she was vocal in her feelings and displayed the most frustration and the second most amount of joy.

During the Session 6, Bernice began with attempting to gracefully label the fourth graph in the Caterpillar class. This graph contained 10 nodes (previously she had only done graphs that contained eight nodes). Bernice began her attempts at labeling the graph. Several minutes into working she said, "Oh, dang it. I need a nine," showing frustration. She continued to work and changed her frustration into excitement with the statement, "I am doing it, I am doing it." Only two minutes later, while she was still trying to label the graph gracefully she said, "One, two. What happened? Oh no, I missed one." (A labeling of a node.) Again shifting from joy back to frustration. This frustration continued as she worked and the following conversation took place with myself, the teacher.

Bernice: Seven minus three equals four (has her yellow page almost filled). Oh my god, Ms. Teacher, look how close I was. But this last one doesn't make sense.

Teacher: Well are you looking at your sheet for a pattern?

Bernice: No.

Then Bernice continued to struggle while displaying signs of frustration with the following statements: "This makes, oh my god this doesn't make sense" and "It does not make sense."

Several minutes later, Bernice completed a graceful label and shouted with her arms in the air, "I did it. I am done. I need a pen. Ms. Teacher, I did it." At this time her frustration oscillated to joy (see Figure 4 of her work on the first set of Caterpillar graphs).

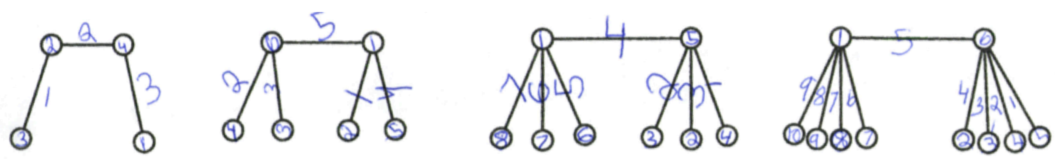

Figure 4. Bernice’s work on Caterpillar Graphs.

Alia. Alia was a Grade 4 student. She had a very positive and cheery demeanor. She also displayed her productive struggle through an oscillation of frustration and joy. Her story was chosen be documented because she displayed the most total accounts of joy throughout the sessions.

During Session 6, Alia was working on gracefully labeling Caterpillar graphs. After a few minutes of working, Alia said, "I am lost" and erased her page, displaying frustration. After another thirty seconds, Alia stated aloud, "Okay, I got it" and continued to work for several more minutes. Alia then noticed that she was missing an eight on one of her nodes and shouted, "No! I need an eight." This showed her frustration. Alia continued to work on finding a graceful labeling and just twenty-seven seconds following her last shout of frustration, she clapped her hands and then threw her arms in the air and shouted, "I did it! Woo!” changing her frustration to joy.

Alia began to work on labeling her next caterpillar graph gracefully. Two minutes into working, Alia took her hand and slammed it on the table and stated, "Oh, I messed up." Only twenty-four seconds later, Alia clapped her hands together and said, "I was right. I was right." This again showed her oscillation from frustration to joy. Alia continued to work. After six more minutes of work, Alia shouted with her arms in the air, "Oh, I am good at this" and continued to work. Several seconds later, Alia said, "No" displaying frustration and a solution that was not a graceful label. After six more minutes of working, Alia shouted, "Yes" with her arms in the air. After recording her solution, Alia turned to the instructor and said, "Okay, I am ready for the next one." Alia continued working throughout the rest of the time with the similar pattern of joy and frustration. She displayed frustration when she was struggling to find a solution followed by joy and excitement when she made progress on her work.

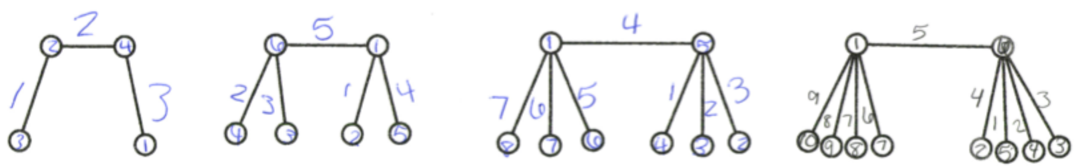

Figure 5. Alia's work on Caterpillar Graphs

Similar to Bernice, Alia also displayed a range of emotions and oscillated between being frustrated and then displaying signs of joy or pride when she produced a graceful labeling. I found this oscillation between joy and frustration a sign of productive struggle. This struggle was similar to the productive struggle Alia, Bernice, and the other students demonstrated during all the problem-solving sessions.

Discussion and Conclusions

The most common emotions that students displayed were frustration and joy. These results are similar to Else-Quest et al. (2008) who found the most common negative emotion students displayed were frustration and distress and the most common positive emotions students displayed were joy and pride. Students in this study also showed emotions of contempt, sadness, boredom, humor, and acts of caring.

Past research has documented different ways to help students during productive struggle (Warshauer, 2015; Zeybek, 2016), and that productive struggle is beneficial (Kapur, 2010). But, there is limited research in what productive struggle actually looks like as children are problem solving (Warshauer, 2015; Zeybek, 2016). Through this research study, I have documented that while students are engaged in productive struggle, they display the emotions of frustration and joy. Students display frustration while struggling through a portion of a problem. Frustration was displayed multiple times throughout their work. Once a student found some success or made progress towards the solution, they enacted joy. This process was repeated throughout the mathematical problem solving.

Overall, I believe a significant finding of this study pertains to the oscillation of joy and frustration which I have interpreted as the construct of productive struggle. All of the students in this study were able to persist through the struggle and were able to find joy and pride in their work. Unsolved mathematics problems are not typically part of elementary school education but I found that these types of problems allowed students the opportunity to engaged in productive struggle and experience mathematics more similar to how a mathematician might experience mathematics.

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