TACIT, TRICK, OR "TEACH": WHAT IS GINA'S MATHEMATICAL REALITY?

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We present study findings that depict the natural and fractional number knowledge of one third grade student with learning disabilities (LDs) in seven experimental sessions. We utilize qualitative analysis methods to illustrate how this student evidenced her knowledge of natural number and fractions through her interactions with varied learning situations. We argue the child reverted to pseudo-empirical abstractions and tricks to make sense of the situations as opposed her own reasoning. Providing children interventions that promote procedures and actions may be preventing children from adapting their thinking structures.

Natural and rational number understandings are among the most relentless areas of difficulty in school mathematics, especially for children who have varying exceptionalities, such as learning disabilities (LDs) (Mazzocco & Devlin, 2008). Difficulties in understanding natural numbers and fractions impact these students from the early elementary years through their adult life (e.g., Lewis, 2014; Mazzocco, Myers, Lewis, Hanich, & Murphy, 2013) and effect problem solving, computational procedures, and development of reasoning and sense making.

In this paper, we present findings from a design experiment that depicts the natural and fractional number knowledge of one third grade student with learning disabilities (LDs) in seven experimental sessions. We utilize qualitative analysis methods to illustrate how this student evidenced her knowledge of natural number and fractions through her interactions with varied learning situations. Through presenting this data, we raise questions about the child's reality and what the child's apparent knowing and learning was relying upon. The research questions are (1) What is one child identified with a LD's mathematical reality specific to her number and fractional knowledge as modeled across varying task types and (2) What persistent difficulties did the child experience? What seemed to be the function of these difficulties?

Theoretical Framework: The Mathematical Reality of Children with LDs

Historical perspectives regarding the mathematical realities of children with LDs arose from a medical depiction of LD as either innate, neurological impairment (Lewandowsky & Steelman, 1908) or an acquired 'disabling' of calculation centers in the brain despite normal language and speaking skills (Henshan, 1928). Over time, children not affected by injury and thought to have otherwise average or above average intelligence showed similar 'impairments' in mathematics performance (U.S. Office of Education, 1968). Researchers began to define ways to address the perceived impairments in mathematical knowledge (e.g., Hudson & Miller, 2006). In recent years, specific factors (e.g., working memory, processing, spatial reasoning, retrieve basic facts, identifying and/or compare number magnitudes and symbols) were tested alongside instruction (or before and after instruction) in a predictive manner to both explain and remediate the child's mathematical reality (e.g., Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012; Jordan, 2007; Mazzocco & Devlin, 2008; Murphy, Mazzocco, Hanich, & Early, 2007; Vukovic, 2012). By and large, the field continues to define LDs in children's mathematical realities as innate, neurological differences even today.

Yet, this definition of LD is incomplete and at times misleading for those who do not equate *biological difference* as a *disabler* or a *remediation* with *learning*. We argue that biological variations in the brain are far more dissimilar than they are similar (Compton et

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al., 2012) and that regardless of variations from a norm, these children still hold knowledge (DiSessa, 1988). If this is true, then we also have to accept that even if children labeled LD may evidence knowing or engage with situations in unexpected ways that their reasoning cannot be conceptualized as "impaired" or even "different". It must be conceptualized as *their knowing*: a complex, unique organism truly known only to the child. In this way, we draw from Piaget (1972/1980), who viewed *learning as adaptation*, one that any child makes to her internal cognitive schemes, as she navigates her environment. In this way, Piaget (1972) links adaptation with instruction.

Adaptations children make in their reasoning, or not, become negotiated by *the environment and the larger system*. These negotiations *do* begin in the child's mind, yet the child's mind and the goal-driven activity that produces learning is variant and system dependent. Depending on the system, a child with "LDs" might very well be considered disabled in one classroom or learning environment and not another (McDermott, 1993). In a similar way, this child could be disabled in one kind of mathematical instruction (Lambert, 2014) but not in another. These factors can work to enable or disable aspects of the child's unique knowing and learning.

Number Sequences and Composite Units

Within the child's mind lies the potential construction of a meaningful mathematical reality. Steffe and Cobb (1988) describe four distinct types of number sequences children may evidence and adapt to understand other mathematics: (1) Initial Number Sequence, (2) Tacitly-Nested Number Sequence, (3) Explicitly-Nested Number Sequence, and (4) Generalized Number Sequence. Each number sequence can be related to children's composite unit coordination.

Initial Number Sequence. Steffe (1992) posits that children who "count on" think with an *Initial Number Sequence (INS).* A child who uses an INS is characterized by her counting of single units and then segmenting of a numerical sequence. Through this segmenting, the child unitizes numerical sequences and interiorizes the rules of patterns (e.g., develop one composite unit and count on from this composite unit to a second composite unit, 4…5-6-7-8).

Tacitly-Nested Number Sequence. Once a child has developed and coordinated composite units through an INS, she can use composite units both as a unit in which to count on from and one to keep track of when counting. This is a *Tacitly-Nested Number Sequence (TNS).* The activity, while similar to an INS, supports the child to hold a start and stop value AND count on additional items from the stop value (e.g., three more items were added to the total of 8…9-10-11). Essentially, a child who thinks with a TNS take the result of her counting as a composite unit in which to use for a new problem. Ulrich (2015) explains this child is aware of her counting acts as both (a) a segment of a numerical sequence (i.e., the difference between 4 and 11 is 5, 6, 7, 8, 9, 10, 11) and (b) the numerical sequence created through this segmenting (i.e., 5 is 1, 6 is 2, 7 is 3, 8 is 4, 9, is 5, 10 is 6, 11 is 7). The awareness of one number sequence contained inside another, or *double-counting,* is an indication of TNS, as is a skip count to solve early multiplicative kinds of problems, such as how many 3's are contained in 39.

Ulrich and Wilkins (2017) further distinguish between two types of TNS, early TNS (eTNS) and advanced TNS (aTNS). The two main distinctions between these subsets of TNS are described as the degree of abstraction a child relies on when understanding one composite unit while developing a second composite unit. For instance, when asked to find the parts of a whole in a fraction task (with a continuous model), a child who thinks with an eTNS will engage in *equi-segmenting* where she develops a composite unit that she needs to adjust in relation to the whole (Steffe, 2002; Ulrich & Wilkins, 2017). A child who thinks with an aTNS anticipates using

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a composite unit prior to the activity in a task. The common aspect of each way of reasoning is the propensity to be aware of one number sequence in relation to another number sequence.

Explicitly-Nested Number Sequence. Children who use composite units and the whole simultaneously disembed parts from wholes and develop iterable units of one. These children are thinking with an *Explicitly-Nested Number Sequence* (ENS) (Olive, 1999; Ulrich & Wilkins, 2017). An ENS supports children to multiplicatively understand a single unit and a composite unit without disrupting the whole because they see the summation of the parts and the whole as interchangeable (2017). Olive explains that each nested unit needs to be understood abstractly, allowing two levels of units as a singular abstract composite unit. This nesting of abstract units provides children number structures that are necessary for multiplicative reasoning. For example, a child at an aTNS stage versus an ENS stage would solve the following task in very different ways: 1 + 1 + 1 + 1 + 1. A child at an aTNS stage would need to consider each 1 as its own unit in which to understand the whole (e.g., 1+1=2, 1+2=3, 1+3=4; 1+4=5). A child at an ENS stage would be able to consider the five 1s and the whole 5 simultaneously.

Other sequences: Doubles and halves. Doubling and halving are described as representing initial forms of partitioning and iterating development that children use when transitioning from additive structures towards multiplicative structures. Research studies focused on doubling and halving are minimal in mathematics education (i.e., Confrey, 1994; Kieran, 1994; Empson & Turner, 2006; Steffe, 2002). Findings from these studies suggest that children do not need multiplicative structures to initialize engagement with halving because they can develop doubling operations through symmetry and halving operations through activity such as considering parts of parts (Author, Confrey, 1994).

Kieran (1994) first noted that young children were aware of multiplicative patterns through paper folding; Empson and Turner (2006) further explored this development with elementary school age students. Empson and Turner found that engagement with empirical material resulted in four types of development (non-recursive activity, emergent recursive activity, recursive activity, and functional multiplication). Through this development, Empson and Turner noted that young children began by not being able to relate their folds to the number of parts and to use this solution to solve problems (non-recursive activity) and that first-grade students did not transition beyond this activity. After transitioning from non-recursive activity, students could build towards early recursive activity (emergent recursive) and develop recursive activity (recursive activity) and multiplicative structures where solutions were anticipated (2006). These stages in students' development suggest another means in which to make sense of how students transition from additive, natural number structure structures, towards other mathematics, such as multiplicative or fractional knowledge.

Methods

"Gina"

"Gina" (age = 10 years) attended elementary school in the Northwestern United States. She was identified by her school system as having a learning disability that affected her mathematics performance. Gina's performance on the mandated standardized state measure of math performance was at a low level. Her reading scores were at average levels. Gina received additional support in mathematics concepts and operations that included explicitly modeled strategies and procedures for operations. Gina's individual learning differences included significant difficulties with working memory.

Initial Interview and Design Experiment

Data collection was collected in two semi-structured clinical interviews (Ginsburg, 1997) and seven sessions of a design experiment (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). All sessions were used in data analysis. Sessions took place during school hours and were in addition to the child's regular math class time. The first author was the interviewer and the researcher-teacher and attended all sessions. The second author collaborated throughout the retrospective analysis of sessions. The first author utilized explanations and indicators of Gina's possible number (Olive, 2001), multiplicative (Hackenberg, 2013; Tzur et al., 2013), and fraction (Olive, 2001; Steffe & Olive, 2010; Tzur, 1999, 2000, 2007) conceptions to build a theoretical model of Gina's initial ways of operating. Then, we worked with Gina individually in sessions that lasted about 40 minutes. Although sessions were planned for 40-min time increments, not all sessions lasted 40 min (range of time = 30 to 50 min; average time per session ≈ 40 min). Researchers collected three sources of data: video recordings, written work, and field notes.

Tasks to promote Gina's number knowledge. Consulting the available research and resources (e.g., Tzur & Lambert, 2011), we designed several learning situations to support Gina to adapt her schemes for number toward abstract composite units. Our goal was to promote Gina to "know" that fluent addition strategies use number relationships and the structure of the number system- that numbers can be decomposed and added on in parts, not just by ones. One platform task involved rolling a dice and moving a marker across a linear number game board (Tzur & Lambert, 2011). The object of the game was for the child to tell "How Far from the Start" she was after combining two addends. Another platform task involved counters and covers. In some cases, counters were quantified by the child; the researcher then added to or subtracted from the total and asked about the new total. Constraints on the platform tasks (i.e., covering of an addend or both addends, larger numbers, moving the location of the unknown questions to promote anticipation) were planned such that Gina would hold a start value in her head, count on using the structure of numbers through ten as parts and wholes mentally to add two numbers- an early ENS with small composites. A third task involved small and large units presented as bakery pans. Gina was asked to convert back and forth between the larger (e.g., 3) and the smaller (i.e., 1) to "fill orders".

Fraction knowledge. We also included a second set of learning situations to assess and support Gina's unit fraction concept (Tzur, 2007). Unit fraction concepts can be initially accessed with a tacit number sequence. One platform task was to equally share one item among two people and then share among three and four people. Constraints (no folding for 3 sharers; make the size in fewest number of attempts for 4+ sharers) were designed such that Gina would use iteration, or repeating, of a unit she estimated to be the correct length across the whole and make adjustments opposite the number of pieces.

Data Analysis

Ongoing analysis of critical events (Powell, Francisco, & Maher, 2003) in the child's thinking and learning were noted and discussed before and after each session. The focus was on generating (and documenting) initial hypotheses as to what conceptions could underlie the child's apparent problem-solving strategies during these critical events. These hypotheses led to planning the following teaching episode.

Next, the researchers engaged in a blend of retrospective analyses (Cobb et al., 2003) and fine-grained analysis (Siegler, 2007) to consider Gina's adaptations and interactions with the learning situations. To begin, we gathered the video data for all sessions and transcribed these sessions verbatim. We also examined corresponding written student work and anecdotal notes taken during these sessions. Then, we closely inspected the data for evidence of Gina's

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understandings and/or shifts in understandings. That is, we conducted a line by line examination of the transcribed data with a side by side of video data to consider how Gina's thinking progressed, or not, across the sessions. We then chunked the data into smaller, more meaningful parts. For each identified segment, we examined very closely each child's spoken and gestured actions along with her utterances and representations, consistently and systematically searching for confirming or disconfirming evidence (Strauss & Corbin, 1994) to ensure credibility. This process allowed us to elaborate on the model of Gina's mathematics as she interacted with the learning environment over time (Steffe, 1995) to gain a clearer picture of adaptations made to her thinking.

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Gina's Initial Mathematics: Clinical Interview and Session One

In the clinical interview, Gina solved several tasks involving whole number operations the revealed a seemingly tacit nature of her thinking. For instance, she skip-counted by threes to 18 yet had to resort to the use of ones after 15 (e.g., "3, 6, 9, 12, 15…16-17-**18**; 19-20-**21**…."). When asked to solve 8 + __ = 14 in a contextualized story problem, Gina stated, "bottom number bigger, break apart a ten", first counting back with her fingers from 14, keeping a double count, and wrote a number sentence (algorithm). Interestingly, she writes "108" as the answer. When asked if that matched what she did with her fingers, Gina stated that "you have to put a ten there". When asked different ways to make 17, Gina stated there were not many. When given a start number of six, Gina counted up from five to 11 with her fingers, then counted up another six verbally to 17. She did not put the five and six together as a solution. We were unsure of Gina's number sequence.

We also asked questions around Gina' fraction knowledge. When asked to say for which of two cookies she would get a bigger share (one is cut into two parts vertically; the other diagonally), Gina stated that they were the same because "they each are a split of one cookie". In another problem, Gina worked to share a thin rectangular paper called a French fry (Tzur & Hunt, 2014) between two friends. Gina suggested partitioning the bar "into halves", partitioning the bar in the center. When asked if there was a way to know the parts are equal, the child made three parts on each side of the partition line and seemed to explain "equal" as the same number on each side. When asked to share the fry among three people and constrained not to use cubes or fold, the child drew three relatively equal parts that did not take up the entire rectangle. To address the space not used by her three parts, Gina suggested we cut off the rest of the rectangle.

To confirm the participatory nature of Gina's tacit number knowledge, we began Session 1 with the "How Far from the Start Are You?" game. *Excerpt a* begins with Gina's response to "How Far from the Start" she was after rolling 9 then 11 (the first nine spaces were covered with a paper and a "9" was written on the cover):

Excerpt a: Move nine spaces, then move 11 spaces. How far from start are you?

R: Are you thinking about how to make nine?
G: Yeah let me show you [counts up five, places a finger, then counts another four and places a finger, then counts up one and places a finger].
R: Hmm. So, you did five- 1-2-3-4-5 – and then you figured out that the other part of nine was four [child nods]. Did you figure that out in your head before you did anything?
G: Yeah, I was about to do three plus one is four [rolls dice again and gets 11]. I'm trying to figure out how to get to the 11; I haven't done that one before [child uses game board to count ten more by ones, then another for 11 spaces].

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R: So, we went nine [grabs cover and writes "9"; covers first nine spaces] and then we went 11. How far from the start are we?
G: [*pauses for 6 seconds; the grabs pencil and begins to write algorithm*]. R: Can you figure it out without writing it down?
G: [*pauses for 9 seconds*] 20. R: How'd you know it was 20?
G: Whenever I do math problems, it's like a paper and pen to the wall showing the math problem and the numbers magically appear. It's just my brain.
R: Tell me what you were thinking in your brain. G: [writes down 9 + 11 = 20 vertically]
R: Tell me what that means.
G: This [points to 9 + 11 that she wrote, write 9 + 1 = 10] is how much this adds up. It's sort of like we're grouping.
R: So, you took the nine and the one from the 11 and you did ten?
G: [Smiles] Or - you could do 9 [lifts fingers one at a time]…10-11-12-13-14-15-16-17-18-19-20.

In the first session, Gina evidences what could be viewed as tacit knowledge of number through a count on. That is, she seems to start from the nine and count up 11 to stop at 20. Yet, because she already solved the problem visually by "showing the math problem on the wall", we asked her another problem involving 2 + __ = 8. Gina's thinking in this problem told a different story.

Namely, Gina began to count up from five and stopped. She then stated that four and four is eight, raised two fingers, began to count up and stopped at seven. The child then began to verbally recite number facts and, after several attempts, eventually stated that two and six would make eight. We also used the fraction tasks from the first session, yet this time we asked Gina to test a share size to see if it was the correct size for one of three equal shares. To do so, Gina repeated the share three times to show a length, yet called the part "one-half". She then described her activity as "two times", further explaining the activity as 2 + 1. Looking across the tasks, Gina's activity suggested that she was operating with a participatory tacit number sequence. Yet, procedural residue made it difficult to say this with certainty.

Sessions Two and Six

In session two, Gina continues to display what we called tacit knowledge in both number and in fractional knowledge. Excerpt b begins with Gina's response to a missing addends task where she was shown six counters, then the counters were covered. The researcher then said, "I placed more counters under the cover; altogether there are 13. How many counters in my hand?":

<u>Excerpt b: 6 counters covered, 7 in the researcher's hand</u>

G: [mumbling]. Six… [pauses 12 seconds] Trick question. R: How so?
G: Well, I already know that six plus four is ten. The trick I was taught was to add another three [shows three fingers to R]. So, it would have been six plus…like…13 [shugs]. Because there's possibly no way to get to 13 because we've already got six [shows six fingers, stares at her fingers. Then begins to raise two more fingers]. Eight…. [pauses 7 seconds] it's 12. I have to get to….
R: Well I had six and I added some more and now I have 13. You were trying to think about… G: Is this some sort of trick question or what [smiles]?
R: [smiles at child] Let's back up to something you were saying. Did you say that you had six, and then you put four more onto that and got to ten?
G: [nods]
R: Then how many more did you say to 13? G: Three more.
R: So [shows three on one hand and hour on the other]?
G: 30…34. No, seven. Seven?
R: Do you think I put seven under there?
G: Yes [lifts covers and counts to check, smiles].

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In the second session, Gina continues to show a kind of tacit knowledge of number through a count on. That is, she seems to start from the six and counts up to 10. Interestingly, she then seems to no longer rely on her own knowledge. Instead, she reverts to a "trick" to get from 10 to 13. Good evidence for this claims rests in the notion that Gina could not put the four [6… 7-8-9- 10] with the 3 to arrive at a missing addend of seven. Instead, Gina sees a "trick question".

Similar activity seemed to continually show and then fade in Gina's reasoning throughout the remainder of the sessions, and became exacerbated when the location of the session was changed from the small environment used in the first four sessions to the room where Gina was used to coming for small group math intervention time during the school day. In the seventh session where Gina's response to task where she was shown 4 red units (each red unit measured 2 white units). The researcher asked Gina to think about how many white units would be needed to equal the same length. Gina aligned the white units long ways, reasoning that three white units aligned with the red unit as opposed to two and muttered, "3, 6, 9, 12, 15, 18" and smiled. Excerpt c begins with a request from the teacher to consider the total amount of white units if two more red units were added. Gina answered incorrectly and was asked to explain:

Excerpt c: Add two more red units (3s)- how many total white units (1s)?

G: Because this and that [taps table]...it's like a multistep...so...what we do in my classroom. It's like, um, it's supposed to be hard. It used to be hard for me but...it's easy [writes down the word multistep on the paper]. So that would be considered multistep. Six red and you need two more red. And you already have this over here down [references red units she modeled earlier]. And so, six plus two equals eight [points to where she wrote "multistep"]. It's a multistep. Then 18 [writes 18] plus two, I mean eight.
R: Why eight?
G: [looks over at other teacher in room] Because six plus two equals eight.
R: That is true. How many white units equal one red unit? G: Three.
R: How many reds are you adding? G: Ah ha! 11! This all equals 11! R: Tell me how.
G: Because this right here... [draws a circle in the air over her earlier representation of the red and white units]. Because this [swipes where she wrote "multistep"] and three equals ... 11.

This excerpt provides clear evidence that what may have disabled Gina was not her innate, cognitive differences but instead prior "tricks" and teaching "strategies" that prohibited her from negotiating her own reasoning in the learning situation. Moreover, the mere presence of the teacher in the room seemed to change Gina's goals away from reasoning and sense making and toward displaying learning "strategies". While these strategies were arguable now part of her reality, we question whether this thinking was really "hers" in that she could not call upon it to explain and justify her words. We will expound upon further examples to illustrate this point further in session.

Discussion

We investigated how one student, Gina, evidenced her tacit use of composite units and early fractional reasoning. We begin to describe the difficulties this child experienced - namely, the use of tricks and previously taught strategies to engage with the mathematics as opposed her own reasoning. Toward this end, we add a unique contribution to the literature. Namely, we found that when Gina engaged in natural number tasks, she did appear to be relying on tacit knowledge, at least in part. However, when engaging with other tasks, she reverted to pseudo-empirical abstractions and tricks to make sense of the situations. Gina seemed to take up procedures that prevented her from reflecting on her actions upon units such that she might adapt

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her thinking structures. For instance, in session 2, Gina segmented the number sequence between six and 13 into four and three. However, we posit that these actions were not organized in such a way that Gina could make sense of the potentially powerful way of operations (i.e., a through ten thinking, or a breaking of seven into four and three). Instead, strategies and tricks seemed to be taught arbitrarily to Gina to afford her success in counting and performance in adding and subtracting.

In other sessions, Gina seemed to have a goal of mimicking her intervention teacher. This is evidence that learning is contributed to by both children and teachers and are negotiated by factors such as interactions, school curriculum, classroom culture, and so forth. For example, a teacher's reaction to a child with an exceptionality who has supplied a solution that they did not expect or time taken to answer may be, "That's not right" followed by explicitly taught teacher thinking. These responses likely reinforce the child as possessing the "problem." Yet, another response (e.g., "Say more") might change the context, opening up space for the child to access the mathematics from his or her own conceptions and develop practices that align with academic success. The interactional context of, "Did they say what I was expecting?", becomes replaced by, "How are they thinking about it?". The problem, removed from the child, becomes explorable as the narrative of the child as disabled begins to unravel.

We argue that instruction for Gina would have better served her mathematical needs if she had been given more opportunities for adapting *her own thinking* constructed within her own mathematical reality. When well-intentioned educators provide children interventions that promote procedures and actions, not only are they not serving their children's mathematics learning needs, they may be preventing them from engaging in learning situations that support the children to adapt their thinking structures and advance their learning. In this sense, from working with Gina, we have lingering questions regarding her development. For instance, over the course of the sessions, Gina evidence an affinity towards doubles and symmetry both within natural number development and early fractional reasoning. Other studies (e.g., Empson & Turner, 2006) have found students rely on doubles and halves in varying tasks. Although not fully reported here, Gina seems to rely heavily on these units across learning situations and across natural and fractional number reasoning. Could this have been a pathway forward which Gina could utilize to overcome her reliance on tricks and make sense her own thinking? More research is necessary to determine real causes of students' difficulties with number alongside research that conforms models of teaching that facilitate these children's understanding, reasoning, and sense-making.

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