PROFESSIONAL NOTICING IN COMPLEX MATHEMATICAL CONTEXTS: EXAMINING PRESERVICE TEACHERS' CHANGES IN PERFORMANCE

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This paper examines the implementation of an instructional module on Preservice Elementary Teachers' (PSETs) professional noticing of children's mathematical thinking as defined by Jacobs, Lamb, and Philipp (2010). The module focuses on professional noticing skills through the content focus of early algebraic reasoning and uses complex video vignettes from whole class instruction in authentic elementary mathematics classrooms. It was found that two of the three components of professional noticing (attending and interpreting) showed statistically significant increases in a treatment group that did not occur in a comparison group. The deciding component remains a challenge that warrants further research.

Keywords: Teacher Education-Preservice, Teacher Knowledge, Professional Noticing, Algebra and Algebraic Reasoning, Instructional Activities and Practices

Introduction

Developing responsive mathematics teaching practices is a persistent challenge for teacher educators. Over the past decade, teacher noticing and variants of such noticing (i.e., professional noticing of children's mathematical thinking, etc.) have risen in prominence as a construct of great interest among mathematics education researchers (Schack, Fisher, & Wilhelm, 2017; Sherin, Jacobs, & Philipp, 2011). Much has been learned about teachers' development and performance of noticing practices. However, vexing questions remain regarding the fundamental nature of such noticing and the extent to which the practice should be considered a net (capturing as much of the activity as possible) or a filter (distilling the activity to key moments to act upon) (Thomas, 2017). These questions are exacerbated in complex environments featuring multiple actors, converging mathematical topics, and varied avenues for sound instructional decision-making. While research suggests that productive experiences help teachers engage in more sophisticated forms of noticing, in more complex contexts, the impact of such experiences (and even the nature of noticing itself) becomes less clear (Castro-Superfine, Fisher, Bragelman, & Amador, 2017). For this study, we focus on the impact of a professional learning experience conducted with preservice elementary teachers (PSETs) aimed at developing their noticing capacities in a complex mathematical environment. Given our prior focus on noticing performance with respect to individual students' counting/arithmetic strategies (Schack, Fisher, Thomas, Eisenhardt, Yoder, 2013), we elected to contextualize noticing within mathematical activities of multiple children and their early-algebraic reasoning. The research question guiding this inquiry was: *To what extent can teacher educators facilitate the development of PSETs' professional noticing of children's mathematical thinking in the context of algebraic thinking in whole class settings?*

Hodges, T.E., Roy, G. J., & Tyminski, A. M. (Eds.). (2018). *Proceedings of the 40th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Greenville, SC: University of South Carolina & Clemson University.

Theoretical Framework

Teacher Noticing and Professional Noticing

Variants of teacher noticing have been the focus of research for some time (Mason, 2002; Sherin et al., 2011; Schack et al., 2017). While teacher noticing has been organized around two related components, "attending to particular events in an instructional setting" and "making sense of events in an instructional setting" (Sherin et al, p.5), Jacobs et al. (2010), posit a third related component, *deciding*, which refers to teachers' responses ostensibly built upon interpretations of children's activities. These interpretations are, themselves, "derived from events and behaviors to which teachers had attended" (Thomas, 2017, p. 508). The assemblage of attending, interpreting, and deciding as interrelated component skills has been referred to as professional noticing of children's mathematical thinking, hereinafter as simply professional noticing (Jacobs et al., 2010). Further, professional noticing is typically considered a complex and challenging practice to develop (van Es, 2011), and, as with most complex practices, there are varying perspectives on how such practices may be implemented. In some instances, professional noticing may "be used as a filter to identify only the most impactful moments" while from other perspectives, noticing may be "focused on capturing and interpreting as much of the instructional landscape as possible" (Thomas, 2017, p. 508). While the former perspective is represented by the Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) analytic framework (Leatham, Peterson, Stockero, & Van Zoest, 2015), the latter is typified by the research of Wells (2017) and Schack et al. (2013). It is this latter perspective of more inclusive professional noticing that we use for this study.

Returning to the notion of professional noticing as a complex practice, studies have demonstrated that preservice teachers at different levels of practice (i.e., elementary, middle level, secondary) can engage in productive professional noticing (Floro & Bostic, 2017; Krupa, Huey, Leisseg, Casey, & Monson, 2017). However, there is some emerging conjecture that professional noticing considerations and development may vary somewhat depending upon grade level (Krupa et al., 2017). Germane to this study, though, are inquiries of noticing performance within elementary grades. In a previous study, we found evidence that PSETs may advance their practice of professional noticing as measured via growth in the component skills of attending, interpreting, and deciding (Schack et al., 2013). Using a video-based instrument, we found statistically significant performance gains in each of the component skills (attending, interpreting, and deciding) with the largest gains occurring in the deciding component. Note, though, that this study was conducted in the context of a single child's mathematical activity along a mathematical progression of counting and arithmetic reasoning (Steffe, von Glaserfeld, Richards, & Cobb, 1983; Thomas and Tabor, 2012). There has been some study of preservice teachers' professional noticing performance that have grounded such noticing in complex pedagogical domains such as the enactment of specific mathematical practices (Floro & Bostic, 2017). However, this study is unique in that we have adopted an inclusive professional noticing perspective to examine PSET performance within a complex mathematical domain – namely early algebraic reasoning.

Early Algebraic Reasoning as a Complex Domain

Mentioned earlier, our previous research focused on the professional noticing of an individual child's mathematical thinking in the area of counting and arithmetic strategies. We relied upon a highly descriptive framework detailing children's early arithmetic strategies and changing conceptions of unit (Steffe et al., 1983; Wright, Martland, & Stafford, 2006). While this context proved fruitful for the development of foundational noticing capacities (Schack et

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al., 2013), we concluded that such contexts did not accurately represent the "blooming, buzzing confusion of sensory data" that occurs when multiple children are acting and interacting upon a convergence of mathematical topics (Sherin & Star, 2011, p. 69).

Representative of such mathematical convergence, early-algebraic reasoning comprises and intertwines several mathematical domains including properties of operations, equality, patterning, symbolic representation, and functions (Kaput, Carraher, & Blanton, 2007; Russell, Schifter, & Bastable, 2011; Warren, 2005). As with other domains such as counting and arithmetic reasoning (Steffe, 1992; Thomas and Tabor, 2012), each of the converging domains of early-algebraic reasoning may be thought to have some manner of developmental progression. Further, these progressions have been defined via empirical study for some of these domains (Clements & Sarama, 2009). However, the convergence of these individual progressions and resultant *conglomerate trajectory* may be rightly considered a significantly complex area of mathematical content rife with possibilities and pitfalls regarding the myriad aspects to which one might attend, interpret, and respond instructionally. While there are many other domain convergences and conglomerate trajectories within the mathematics education landscape, early-algebraic reasoning should be one of the first encountered by children and enacted by their teachers. As such, this represents a rich mathematical context for the study of professional noticing.

Methodology

Participants

Participants in the study included 296 preservice elementary teachers (PSETs) enrolled in an elementary mathematics methods course at one of five universities in the south central United States. Among the total participants, 171 completed the measures as part of an intervention group and the remaining 125 were in the comparison group. Participants with any missing scores on the professional noticing measure were removed, thus resulting in an intervention sample size of 147 and comparison sample size of 121.

Instructional Module

PSETs in the intervention group experienced an instructional module focusing on professional noticing and early algebraic thinking. The in-class module was taught by three professors at two of the institutions in the study. The three-session module gradually introduces each of the three components of professional noticing. The first session of the module, which focuses on just the attending and interpreting components, begins with a discussion of a prerequisite reading on the early understanding of equality and a review of the three components of professional noticing. Then, the PSETs analyze two videos with class discussion, match equality tasks to the Common Core State Standards for Mathematics, use their professional noticing skills to analyze children's written work, and complete a play-by-play storyboard from a video of whole class completing an algebraic task. The second session adds deciding and includes video analysis and discussions of authentic classroom experiences as well as analysis of children's written work. The final session includes all three components but focuses more deeply on productive decision making through the video analyses and discussion. All three sessions contain homework consisting of readings, mathematical tasks, or video analyses focused on algebraic thinking.

Measurement and Scoring

All PSETs in the comparison and intervention sites completed pre- and post-assessments to measure their professional noticing skills. All pre-assessments were administered near the beginning of the semester and the post-assessments were administered at the end of the

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instructional module for the intervention sites and near the end of the semester for the comparison group. The professional noticing measure consisted of a 74 second video involving a group of second grade students grappling with the task "10 + 10 = ___ + 5" where they were discussing the missing number in the mathematical sentence. In the video, four children provided different answers and explanations for the task. The explanations for each child are provided in Table 1.

Table 1. Transcripts of Individual Student Responses

Student	Answer	Transcript of Student Explanation
1	25	"I added 10 and….I added both 10's and the 5. Ten plus 10 is 20 and add a 5 equals 25."
2	20	*<No explanation - blurts answer out of turn>*
3	4	"Cause if you add a 5…and then you count up by 5's up to 4, it'd be 20."
4	15	"Because I counted 15, 16, 17, 18, 19, 20" (*using his right hand to track the count*). Teacher (in background) asks: "Why did you start with 15?" Student stated: "Because I thought to myself 5 plus 10 equals 15, but then I thought of adding 15 with the 5 and I counted up and it was 20."

PSETs are asked to view the video and describe what they would do next if they were the teacher (deciding), what mathematical thinking and actions they observed (attending), and what the children understood about mathematics that influenced their answers (interpreting). Decision trees were used to aid in the scoring process. All responses were scored on a scale from 0 to 3 with a score of 3 representing the most advanced responses (See Table 2 for examples). All responses were scored using a double-blind process and any scores not in agreement between the two scorers were discussed and negotiated. In general, PSETs were assigned higher scores when they thoughtfully addressed the diversity in responses instead of focusing on one response or their own assumption of what the intended outcome should be in this situation.

Table 2. Sample PSET Responses

Component	Score	Sample Response
Attending	0	The children were explaining how they got the answer to the problem.
	2	Some of the kids added the first side of the problem, then added 5 and thought that was the answer. Other students were able to understand that the equal sign meant both sides had to be the same, and they were able to move numbers around to make them equal.
	3	The children were given the equation 10+10=___+5. The first child to respond was adding all the numbers together, he said the answer was 25. He was under the impression that the blank should be the sum of all the numbers. Another student said he thought the number should be 4 because it would take 4 5's to make 20 which is what 10 and 10 made. The last student came to the conclusion that it should be 15. He explained that 15 and 5 added together made 20 which is what 10 and 10 made.
Interpreting	0	I would make sure that the students understood what the equal sign meant. I would use manipulatives to show this.

	1	I was surprised to find such an interesting method for finding the answer. It was obvious that he understood the relation of counting by 10s and 5s.
	3	Some students were getting the correct answer (15) but he did not know where he got it from, or he could not explain it. Another student was adding all the numbers together to get the blank number so he needed to break down the number sentence into smaller parts to better understand it.
Deciding	0	I would have the children come up to the board and list a few different ways and ideas that work and don't work.
	1	Next I might show a true expression that looks different on both sides. I might ask, "Are both of these sides equal to each other?" Then I would use student responses to guide the discussion. Some sample questions I may ask along the way would include: How did you get that answer? Are both sides the same? Are both sides different? How are both sides the same? How are both sides different?
	2	I would continue to ask students for more strategies. I would write those strategies on the board. Then I would try some of the strategies on the board to show which ones were correct and which ones were incorrect. I would emphasize that there are multiple ways to solve this problem. I would explain that many of the students had the right answer, but got to it in different ways. I would encourage the students who solved it incorrectly towards the correct answer. It seems like they are thinking in the correct way, but that they had a few miscalculations. I would address those, so that the student still feels successful.
	3	I would have other students see if they can explain the last students thinking since he is on the right track. This would get all the students involved without saying that the other students who shared their opinions were wrong.

Results

When comparing the mean scores for the three professional noticing components (See Table 3), we found that the attending and interpreting scores increased from pre- to post-assessment in the intervention group, but slightly decreased, but not significantly, in the deciding component. The comparison participants decreased in all three components. Not all intervention participants improved, however, and Table 4 outlines the number of participants in each group that increased, stayed the same, and decreased from pre- to post-assessment.

Table 3: Mean Scores of Professional Noticing Components

	Treatment		Control	
	Pre	Post	Pre	Post
Attending	1.020	1.340	0.959	0.826
Interpreting	0.769	1.218	0.876	0.661
Deciding	1.361	1.340	1.554	1.446

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Table 4: Noticing Score Changes

Component	Change	Treatment (%; N = 147)	Control (%; N = 121)
Attending	Increase	38	23
	Same	41	47
	Decrease	20	30
Interpreting	Increase	46	18
	Same	37	55
	Decrease	16	26
Deciding	Increase	29	28
	Same	39	32
	Decrease	31	40

In order to determine whether the increases from pre- to post-assessment in attending and interpreting and the decrease in deciding were statistically significant, Wilcoxon signed rank tests were conducted. The test was significant for attending ($Z = -3.219$, $p = .001$) and interpreting ($Z = -3.961$, $p < .001$), but not for deciding ($Z = -.384$, $p = .701$). Noting that an effective intervention should raise scores to above the level of the control group (which scored higher on interpreting and deciding at pre-test than the treatment group did), Mann-Whitney U tests were conducted to determine whether the treatment group's post-assessment scores were higher than the maximum (i.e., pre-) scores from the control group. The test was significant for attending ($U = 6913$, $p = .001$) and interpreting ($U = 7172$, $p = .004$). The test was also significant for deciding, but in the other direction; the control group received higher deciding scores than the treatment group ($U = 7654$, $p = .039$).

Discussion and Conclusion

Previous research by this team revealed that professional noticing was a teachable skill when taught in a one-on-one teacher to student setting (Schack et al., 2013). In a similar study using an intervention and control group, but within a one-to-one teacher to student setting, PSETs in the intervention group showed statistically significant growth in all three areas of professional noticing (Fisher, Thomas, Schack, Jong, & Tassell, 2017). However, in that same study, the comparison group did not show statistically significant increases in attending and interpreting, but they did show a statistically significant increase in the deciding component. This opens the discussion to the pedagogy taught in the mathematics methods course and how it could potentially impact the deciding component scores.

To further advance the research, this video vignette of a whole-class discussion focused on an algebraic concept was used to determine if the professional noticing skills were still teachable when expanded to a more complex setting. It is interesting to note that attending and interpreting skills both still revealed a statistically significant increase in the intervention group, but both components decreased in the comparison group. These results indicate that attending and interpreting are both responsive to intervention in the varying settings (one-on-one vs. whole class) but deciding needs further research and discussion.

Many questions still remain within the deciding aspect of professional noticing; However, we believe there is still much to learn from the results. Perhaps more research should be conducted on what constitutes effective deciding. While defined mathematical practices provide some organization for considering such decisions, there are myriad decisions a teacher might make in each moment and the productivity of such decisions likely varies greatly according to many different factors (e.g., instructional goals, students' mathematical needs, availability of resources,

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etc.). This variability is further compounded when considering potential decisions within complex mathematical domains such as the early-algebraic convergence.

While we recognize that many limitations exist with this study, such as the short timeline for learning and assessing these skills (4-6 weeks for implementation sites) or the less-than-diverse sampling that can be very difficult to overcome in a study of elementary education majors, perhaps the most troublesome limitation is the quantification of qualitative responses. For example, scorers credited more highly PSET responses that formally addressed each of the student's thinking in the video vignette, while realizing the difficulty of doing so in an authentic setting.

However, many times, responding to each child may have diminished the robustness of the decision. There remains a delicate balance in how to best quantify two different, yet both important, aspects in this situation: teaching to all and productive decision-making. We believe there should be a place for both, but these results suggest we may not have found the best way to measure their intersection as productive decision-making can be quite circumstantial and adequate measurements for those circumstances do not exist.

In summary, we remain optimistic that teacher educators can facilitate the development of PSETs' professional noticing skills within the context of algebraic thinking in whole class settings. We are most hopeful that this facilitation exists at least within the attending and interpreting components. More difficult to demonstrate, however, is the deciding component, thus we encourage the development of measurement tools that consider the complexities of classroom contexts when measuring productive decision-making. Tools that could potentially be used for both research purposes and teaching more productive decision-making for preservice and inservice teachers would be most desirable.

Acknowledgment

This research was supported by a collaborative grant from the National Science Foundation (I-USE: grant #'s 1431133, 1431631, 1431098). The opinions expressed herein are those of the authors and do not necessarily reflect views of the National Science Foundation.

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