# WHAT INFLUENCES DO INSTRUCTORS OF THE GEOMETRY FOR TEACHERS COURSE NEED TO CONTEND WITH?

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*This paper reports on a project aimed at developing a system of professional support for the improvement of the Geometry for Teachers course that mathematics departments teach to preservice secondary teachers. We share data from interviews with 20 instructors to report on how they perceive their position of geometry instructors and the work they do in the course. To inspect this set of interviews, we use the framework of professional obligations to the discipline, to individual students, to the institution, and to the classroom community. We share how references to these professional obligations emerged in the interview data.*

Keywords: Geometry and Geometrical and Spatial Thinking, Post-Secondary Education, Teacher Education-Preservice

## Introduction

This paper reports on a study of instruction at the college level, specifically focused on the geometry course that many universities offer and is taken by prospective secondary teachers (Geometry for Teachers, or GeT hereafter). We report how GeT instructors perceive their position and the work they do in the GeT course in relation to institutional stakeholders. The literature on instruction at the college level is emerging (Mesa, Wladis, & Watkins, 2014) and in order to frame our focus, we can profit from considering as background the literature on K-12.

The study of mathematics instruction at the K-12 level has often considered the classroom as a container within which interactions among teacher, students, and content unfold. The influence of institutional context on instruction has not always been part of that consideration. Cohen et al.'s (2003) instructional triangle calls attention to the environments in which instruction is situated but most studies of instruction pay little attention to how those environments influence instruction. Some of that is justified on the received wisdom that instruction is "loosely coupled" with administration (Weick, 1976). Awareness in our field of the importance to look at the relationships between institutional and instructional issues has been brewing, particularly from research focused on systemic reform (e.g., Cobb, Jackson, Smith, Sorum, & Henrick, 2013) and on equity (e.g., Lubienski, 2002; Walker, 2007). The realization, particularly from the latter research, is that some of the phenomena that happen at the classroom level (e.g., little access to good mathematics) owes to issues that are structural (e.g., tracking, teacher placement, school climate). As the era of accountability starts to affect also higher education (Levine, 2017), there is reason to consider how environments affect instruction also at the college level. On this matter the K-12 literature can provides some theoretical resources.

Chazan, Herbst, and Clark (2016) describe the position of the teacher as one that connects the institution and its stakeholders with the roles and relationships at play in instruction. Attempts to improve instruction have often relied on the agency of teachers. Chazan et al. (2016) contend

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that such attention to agency needs to be complemented with attention to structural issues at play in educational institutions and in society at large if we are to understand how instruction can be improved. This paper argues that the GeT course is one that could benefit from instructional improvement and asks the question of how instructors of this course experience the influence of institutional issues that might support or hinder that improvement.

We inscribe the paper within an improvement effort that tries to follow the approach described Bryk, Gómez, Grunow, and Le Mahieu (2015), which challenges the usual improvement paradigm of diffusion of innovation. Rather than conceiving a solution to a problem and seeking to implement it with fidelity and evaluate its results, the approach described by Bryk et al. (2015) starts from creating a *networked improvement community* (NIC). Using an organizational learning perspective, this approach engages the whole network in designing, monitoring, and continuously developing solutions that might improve the system, particularly by attending to variation in performance. This methodology for improvement starts from understanding the system that produces the outcomes that need to be improved. Our attention to how instructors perceive their institutional positions, at the hinge between the institution and classroom instruction, is key in understanding the system in question.

## The Need for Improving the Geometry Course for Teachers

A geometry course (HSG, hereafter) has been part of the US high school curriculum for more than 100 years (Sinclair, 2008) and it has traditionally been key in inducting students into mathematical practices such as conjecturing and proving. The changes proposed by the Common Core (Common Core State Standards Initiative, 2010), and the corresponding state assessments have substantial impacts on the HSG curriculum (Wu, 2014). These changes have been accompanied by an increase in the use of students' achievement for individual teachers' accountability (Roth McDuffie et al., 2015). Likewise, the description of "highly qualified teachers" ushered in by the No Child Left Behind [NCLB] legislation (Bush, 2002) suggests the need for teachers to have substantial content preparation in the disciplines they will teach. This suggests an institutional pressure on those who prepare teachers, to align what they teach preservice teachers with what the latter will need to teach their own students. This pressure can be seen in the CAEP standards adopted by 33 states through partnership with NCATE which state that teacher education provider programs must "ensure that candidates demonstrate skills and commitment that afford all P-12 students access to rigorous college- and career-ready standards (e.g., […] Common Core State Standards)." The pressure can also be understood by examining the MET II documents that call secondary teacher preparation programs to offer courses specifically designed to focus on mathematics at the high school level from an advanced perspective, including "address[ing] the CCSS approach to Euclidean geometry based upon translations, rotations, reflections and dilations" (CBMS, 2012, p. 7). But, are GeT courses providing teacher candidates with the knowledge they need in order to teach HSG? Grover and Connor (2000) surveyed the content and instructional practices of geometry courses at 108 randomly selected U.S. colleges and universities and found that GeT course content varies greatly: From a review of middle and high school topics to the development of elementary axioms or a study of non-Euclidean and projective geometries using alternative transformational and analytic approaches. A comparison of the textbooks used for GeT courses produced similar differences. Grover and Connor (2000) concluded that there is no typical curriculum for GeT. This variability in GeT courses prompts questioning their usefulness for teachers (Wu, 2011).

It seems that institutions should have reason to be interested in improving the GeT course by better aligning what is taught to future teachers with the knowledge they need to teach HSG.

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However, because Euclidean geometry is no longer an area of active mathematical research, the geometry content high school students study has few stewards in university mathematics departments (Steen, 1988; see also Atiyah, 2001). With the GeT course being a service course for fewer students than other service courses (such as calculus or linear algebra), it is hard for mathematics departments to create local communities to steward the GeT course. Improvement is needed but it may need more than local attention; it could use a network approach.

We are interested in improving the GeT course using the approach described by Bryk et al. (2015), which requires us to start from understanding the system in need of improvement. The GeT course is one where environmental influences (e.g., instructors need to prepare PSTs to teach geometry) could connect with the outcomes of the course (viz., better mathematical knowledge for teaching geometry of preservice teachers). As some research has shown connections between MKT and the mathematical quality of instruction as well as K-12 student outcomes (Hill, Rowan, & Ball, 2005; Hill et al., 2008), increasing MKT would be desirable. Based on performance data, Clements (2003) suggested that students' knowledge of geometry could use improvement. Increasing MKT in geometry might be one lever. Given that the need for improvement in HS geometry instruction points to the possibility to improve the GeT course, a question that can be asked is whether a process of continuous improvement based on principles of organizational learning can be used productively to improve geometry for teachers.

Our project has started developing a networked improvement community by bringing individuals together. We began by locating institutions with large teacher preparation programs—as we are interested in the undergraduate geometry course serving future teachers, rather than geometry courses in general. Within those institutions, we looked at mathematics departments for geometry courses serving secondary mathematics pre-service teachers and identified instructors of those courses as the natural candidates to be members of this community. We are also incorporating other stakeholders, including high school geometry teachers and mathematics supervisors who influence certification policies at state levels. Our first step has been to do a set of initial interviews of instructors of the GeT course. The interviews help us describe members of this group in terms of their professional position as instructors of college students.

## Theoretical Framework: Practical Rationality of GeT Instructors

As we consider the effort involved in improving GeT, we are keen to note that like the case in K-12 instructional improvement, the improvement of curriculum and instruction for GeT is likely to need more than resources and networks: It needs know-how, anchored in an understanding of what instructional practice is like in its institutional context (Halverson, 2003). In a review of the research literature on collegiate mathematics, Speer, Smith, and Horvath (2010) reported that there exists "very little research [that] has focused directly on teaching practice—what teachers do and think daily, in class and out, as they perform their teaching work" (p. 99). Further, they argue that "the community's efforts to support instructors as they learn to teach college mathematics is often not informed by data and research on what is involved in teaching college mathematics" and recommend conducting research in this area to guide the professional development efforts designed to improve collegiate mathematics (p. 111). Specifically, in considering what is involved in the improvement of GeT, it would be helpful to understand how GeT instructors negotiate the multiple demands of their role preparing HSG teachers. How do faculty relate to the dual expectation that GeT be a university mathematics class and prepare students to teach high school geometry?

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In their account of the practical rationality of mathematics teaching, Herbst and Chazan (2012; see also Chazan, Herbst, & Clark, 2016) identify sources of justification that instructors might use to make their actions reasonable or sensible. Many actions in teaching mathematics go without saying, they don't call for justification but rather are habitual or normative. But quite often instructors deviate from the norm—e.g., instead of correcting a mistaken response, an instructor might ask his or her class to consider whether the response makes sense. Herbst and Chazan (2012) propose that such departures from the norm might be perceived as justifiable by a professional if they help meet one or more of four professional obligations: An obligation to the discipline of mathematics, an obligation to students as individuals, an obligation to the class as a community, and an obligation to the institutions that make room for instruction.

The obligations framework has important implications for the ways we think about supporting GeT instructors in improving their instructional practice. In particular, we can use this framework to unpack the tensions that they perceive as undergirding GeT instruction. For example, the institutional obligation (to the Teacher Education program and State certification agency calling for mathematics departments to offer GeT courses) might compel the GeT instructor to cover the content that his or her students would need to teach in schools, while the instructor's obligation to the discipline of mathematics might compel them to bring in considerations that are more general (e.g., that Euclidean geometry is only one geometry, but different choices of postulates might give rise to different geometries) in order to better represent the discipline. Because time is limited, instructors can't just avoid these tensions, they need to manage them. We see the many tensions that might exist among the professional obligations of undergraduate instructors as fertile places where to start an inquiry toward the systemic improvement of the GeT course. In this paper, we begin that work by sharing what we've learned from GeT instructors through a set of initial interviews. Our research questions are: (1) How does the framework of professional obligations help us understand the professional position of geometry instructors? and (2) How does such understanding help us move toward alignment between the GeT course and the HSG course?

## Methods

We recruited 20 participants (8 men and 12 women) from 17 universities across the U.S., all of whom were identified as mathematics department faculty, hold doctoral degrees in mathematics or mathematics education, and have recently taught a geometry course aimed at serving pre-service teachers in large teacher preparation programs. We developed a semi-structured interview protocol to learn about the instructor, their context, and their dispositions toward improving the GeT course. The protocol had three sections: 16 questions about the GeT course and the challenges that come with teaching it, 3 questions (with follow-ups) about the various forms of content covered in the course, and 3 questions about the background of the participant. In this paper we focus responses to the first section, and questions such as: "What experiences do you aim for your students to have in the geometry course for teachers?" and "What are the expectations that shape the geometry course for teachers that you teach?"

We piloted the interview with two non-participants who are experienced instructors of the GeT course before administering it to the rest of the participants. We conducted the interviews using online video-conferencing software that allowed us to capture audio and video records of the interaction. The analysis of the interview data was a multi-step process. We began by taking field notes during the interview and then improving those notes with one or two reviews of the video after the fact. Using the professional obligations framework, we individually coded participants' contributions across the first section's interview questions, meeting together to

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compare, discuss, and reconcile our understanding of each category as well as identify emerging themes.

## Results

In this section, we use the professional obligations of mathematics teaching to organize initial gleanings from the interviews. We share the various ways that each of the professional obligations emerged in the data, illustrating each theme with examples from the data. Lastly, because part of what we aim to do is to understand how well the obligations framework helps us account for instructors' perceptions of their position as instructors of the GeT course, we also share data that did not fall neatly within any of the four professional obligations framework.

### Instructors Dispositions toward the Disciplinary Obligation

We had hypothesized that that GeT instructors would relate to their professional position by recognizing an obligation to the discipline of mathematics. Unsurprisingly, we observed a plethora of responses that illustrated various dispositions toward the disciplinary obligation. Three themes emerged from that analysis; a disposition to attend to (1) geometry as a body of confirmed and correct knowledge that GeT students need to know; (2) mathematics as a practice of inquiry, discovery, invention, or knowledge development that people need to engage in; (3) geometry as a set of models which are useful for solving problems. In Figure 1 we illustrate these themes with participants' responses:

| Mathematics as ... | Participant Response |
|---|---|
| A body of confirmed and correct knowledge | "I do think it's important for mathematicians to teach this course… it's a surprisingly mathematically sophisticated course. I think that it's mathematically sophisticated enough that mathematicians should be teaching it" (MV). |
| A practice of inquiry, discovery, invention, or knowledge development | "High school teachers were definitely as good as I was in recognizing patterns or solving problems. But when they found an answer, they were happy to move on. They didn't seem to me to need to have a rigorous argument about why this pattern worked or why things were the way they were...that's something that I want to convey to my students" (RR). |
| Providing models useful for solving problems | "[Community members such as businesses] want their people to be problem solvers and have the ability to deal with an unfamiliar situation. ... I think that that's the main value of the class to the society as a whole" (SA). |

**Figure 1.** Examples of data coded as evidence of GeT instructors' disciplinary obligation. Note: Two letters are used to refer to each of the different participants.

### Instructors Dispositions toward the Individual Obligation

We hypothesized that GeT instructors would relate to their professional position through a recognition of their obligation to the individual student. While not as prevalent as the evidence for instructors' disciplinary obligation, we observed many participant responses that we agreed would be categorized under the individual obligation. We share two themes that emerged from that analysis: the obligation to attend to the individual as: (1) a cognitive being (who can think,

process mentally, etc.); (2) an emotional being (who can feel anger, joy, fear, disgust, sadness, etc.). Figure 2 illustrates these themes with participants' responses:

| Individual as | Participant Response |
|---|---|
| A cognitive being | "[The mathematical experience is] a good thing for a person's spirit…to be challenged and succeed" (IL). |
| An emotional being | "So I've definitely have gotten some good feedback about people who were scared about, or nervous about teaching geometry at the high school level and after they took the college geometry course at our university they felt like they were ready, or maybe even excited about teaching geometry" (RL). |

**Figure 2.** Examples of data coded as evidence of GeT instructors' individual obligation.

**Instructors Dispositions toward the Institutional Obligation**

We hypothesized that GeT instructors would relate to their professional position through a recognition of their obligation to the institution. Three themes emerged from that analysis: the obligation to attend to the institution as: (1) a place that provides service to young members of society; (2) a place that has external accountability (teacher preparation programs and certification agencies); (3) an organization that has rules, policies, etc. We illustrate these themes (see Figure 3) with participants' responses to various interview questions:

| Institution as ... | Participant Response |
|---|---|
| A place that provides service to young members of society | "[T]he content in the course and the student work in the course is related to how well they do on things like Praxis tests as well as how it relates to how they actually teach in the classroom two years later" (IF). |
| A place that has external accountability | "We redesigned it based on the MET II document, actually based on the MET and then revised again with the MET II, and also focusing on the um Common Core Stand—State Standards as to what geometry the teachers are going to have to teach. And so we have totally revamped the course so the focus is on those aspects of geometry" (IF). |
| An organization that has rules, budgets, etc | "The catalogue affects me - the description talks about axiomatics and finite geometries. While finite geometry is a nice playground, because they can build the models and see everything, I wouldn't have to do that. That course description constrains me" (SL). |

**Figure 3.** Examples of data coded as evidence of GeT instructors' institutional obligation.

**Instructors' Dispositions toward the Interpersonal Obligation**

We hypothesized that GeT instructors would relate to their professional position through a recognition of their obligation to the interpersonal collective of the classroom. Unlike the previous three obligations, we found few of the instructors' responses that could be filed under the interpersonal obligation. Thus, here we have just one theme that emerged from our analysis:

the obligation to attend to the group of students as a discourse community, that needs to partake of communicative exchanges (Figure 4).

| Group of students as ... | Participant Response |
|---|---|
| A discourse community, that needs to partake of communicative exchanges | "So I want them to collaborate mathematically in and outside of class, I want them to get experience communicating mathematics both in person, we do a lot of presentations at the board, a lot" (MV). |

**Figure 4.** Examples of data coded as evidence of GeT instructors' institutional obligation.

**Those Responses Falling Outside the Four Professional Obligations**

Prior to closing this section, we take a moment to review some of the instructor responses that felt important to describe how GeT instructors relate to their professional position but were not captured by the professional obligations. These exceptions are rare in the data, as we have only found two instances. These instances can be attributed to instructor's personal resources, including knowledge, experience, beliefs, and identity. One of the responses came in the context of discussing whether or not GeT instructors held any responsibility for teaching students how to engage in work that was specific to the work of teaching, like creating questions for a geometry exam or understanding students' difficulties with geometric proof. RR's response serves as an example of how an individual's knowledge or experiences can elevate the tensions that one experiences in teaching the GeT course:

> I was trained as a mathematician, I was not given any formal training on student teaching/learning - anything I know about student teaching/learning is something that I've read or picked up from [a mathematics education colleague] but there are other [mathematicians] who don't have [mathematics education colleagues] who come at these courses - geometry, or upper level algebra/analysis course who don't have any of that experience. (RR)

## Conclusion

The professional obligations framework is useful to understand the professional position of GeT instructors, positions toward the GeT course, though the data speaks also about the importance to attend to personal resources. The interviews suggest that efforts to improve the course by developing a networked improvement community may need collective awareness of the variety of ways in which individuals relate to their positions as instructors.

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## References

Atiyah, M. (2001). Mathematics in the 20th Century. *The American Mathematical Monthly, 108*(7), 654-666.

Bryk, A. S., Gomez, L. M., Grunow, A., & LeMahieu, P. G. (2015). *Learning to Improve: How America's Schools Can Get Better at Getting Better.* Cambridge, MA: Harvard Ed. Press.

Bush, G. W. (2002). No Child Left Behind, Act of 2001, Pub. L. No. 107-110, § 115. Stat, 1425, 107-110.

Chazan, D., Herbst, P., & Clark, L. (2016). Research on the teaching of mathematics: A call to theorize the role of society and schooling in mathematics instruction. In D. Gitomer and C. Bell (Eds.), *Handbook of research on teaching (5th edition, pp.* 1039-1097). Washington, DC: AERA.

Clements, D. H. (2003). Teaching and learning geometry. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 151–178). Reston, VA: NCTM.

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Conference Board of the Mathematical Sciences (CBMS). (2012). *The mathematical education of teachers II*. Providence, RI: American Mathematical Society.

Cobb, P., Jackson, K., Smith, T., Sorum, M., & Henrick, E. (2013). Design research with educational systems: Investigating and supporting improvements in the quality of mathematics teaching and learning at scale. *National Society for the Study of Education Yearbook*, *112*(2), 320-349.

Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis, 25*(2), 119-142.

Grover, B. W., & Connor, J. (2000). Characteristics of the college geometry course for preservice secondary teachers. *Journal of Mathematics Teacher Education, 3*(1), 47-67.

Halverson, R. (2003). Systems of practice: How leaders use artifacts to create professional community in schools. *Education Policy Analysis Archives, 11*(37).

Herbst, P., & Chazan, D. (2012). On the instructional triangle and sources of justification for actions in mathematics teaching. *ZDM, 44*(5), 601-612.

Hill, H. et al. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction. *Cognition and Instruction, 26*(4), 430-511.

Hill, H., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *Amer. Ed. Research Journal, 42*, 371-406.

Levine, A. (2017). How the academic profession is changing. In S. Steinberg (Ed.), *The American academic profession* (pp. 42-53). New York: Routledge.

Lubienski, S. T. (2002). Research, reform, and equity in U.S. mathematics education. *Mathematical Thinking and Learning, 4*, 103-125.

Mesa, V., Wladis, C., & Watkins, L. (2014). Research problems in community college mathematics education: Testing the boundaries of K—12 research. *Journal for Research in Mathematics Education, 45*(2), 173-192.

Roth McDuffie, A., Drake, C., Choppin, J., Davis, J. D., Magaña, M. V., & Carson, C. (2017). Middle school mathematics teachers' perceptions of the Common Core State Standards for Mathematics and related assessment and teacher evaluation systems. *Educational Policy*, *31*(2), 139-179.

Sinclair, N. (2008). *The history of the geometry curriculum in the United States*. Charlotte, NC: Information Age.

Speer, N. M., Smith III, J. P., & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. *The Journal of Mathematical Behavior, 29*(2), 99-114.

Steen, L. A. (1988). The science of patterns. *Science, 240*(4852), 611-616.

Walker, E. N. (2007). Why aren't more minorities taking advanced math? *Educational Leadership, 65*(3), 48.

Weick, K. E. (1976). Educational organizations as loosely coupled systems. *Administrative Science Quarterly, 21*(1), 1-19.

Wu, H. H. (2011). The mis-education of mathematics teachers. *Notices of the AMS*, *58*(3), 372-384.

Wu, H. H. (2014). Potential impact of the common core mathematics standards on the American curriculum. In Y. Li & G. Lappan (Eds.), *Mathematics curriculum in school education* (pp. 119-142). Dordrecht, The Netherlands: Springer.