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VISUAL IMAGES RELATED TO ANGLE CONCEPT IN THE 5TH GRADE MATHS COURSEBOOKS

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Abstract
There are different perspectives in the literature on the examination of visual images used in course books. In this study, illustrations from the visuals used in geometry in the lower learning area of school mathematics were examined and summarized first by researchers and then by teachers and students. For many students, the mathematics coursebook is the first material they encounter in understanding the course content and conveying the mathematics culture. In this study, the illustrations related to the concept of Angle used in mathematics coursebook, two books published abroad, two books published in Turkey, for 5th grade students were analyzed. The type of research is Document Analysis and the illustrations were examined in qualitative and quantitative terms. In this way, the study aimed to analyze the contribution of illustrations in mathematics education to the conceptualization ofgeometrical definitions through the coursebook and the contribution and effect of improving geometrical thinking. Results of the study showed that illustrations for the field of mathematics that are in coursebooks must include an algebraic representation.

Keywords: 5^{th} Grade Mathematics textbook, Angle Concept, Virtual images.

INTRODUCTION

Geometry deals with mental beings (geometric figures) with conceptual and shapely characters (Fischbein, 1993). Concepts and images are considered to be fundamentally different categories of mental being. Pieron (1957; as cited in Fischbein, 1993) defines a concept as a symbolic representation (almost always verbal) used in the process of abstract thinking and expresses that it has a general significance corresponding to a concrete collection of representations about what they have in common. Mental imagery is mental representations of visual/ spatial information involving the physical properties of represented events and objects, but not identical to them are exemplary representations (Shepard, 1978). For example, an angle is an abstract ideal concept, but it also has figural properties. In fact, although we find many different angle contexts, the absolute perfection of a geometric angle cannot actually be found.

Fujito, Jones, and Yamamoto (2004) say that geometric figures, as they cited from Fischbein, can be related to the harmony between formal and conceptual constraints of successful reasoning in geometry. They argue that in order to be a successful problem-solver in geometry, one must develop skills in the following subjects, based on the idea that mental structures with conceptual and formal characteristics can be represented.
* Creating and changing geometric figures in mind,
* Detecting geometric features,
* Associating images with concepts and theorems in geometry, and
* Deciding where and how to start when solving problems in geometry.

Mathematics coursebooks are one of the basic elements used in the development of research and inquiry skills. Paintings, photographs, and illustrations are used in secondary school coursebooks. This situation has left teachers with interpretations of these paintings, photographs, and illustrations, as well as graphical representations.

Visualizing angles has the potential to inform and build students and teachers about the concept. The angle is the second variable we use after the concept of distance in school mathematics. It is the concept that advances to see the length of the circle. If students use imagery to represent spatial and visual information, imagery should be closely related to educational content that contains concrete objects. In geometry, imagery can be applied by mental rotations. Illustrated images improve students' learning from texts (Carney and Levin, 2002).

The illustration is described as painting, that is, the painting that describes or decorates an article in a book. The iconography, which is 'illustration' in English, is the picture describing the subject with its shortest opening. A drawing that draws attention to the object rather than the shape is its art. Its purpose is to help explain and understand a subject, rather than art. It is to create a conceptual image on the subject.

For this purpose, , graphic representations of angle with photography and illustration were examined in four Mathematics coursebooks, two Mathematics coursebooks published in Singapore and Australia, two in Turkey used in secondary schools. In this regard, it was focused on what is the understanding of teachers and 5th-grade students about illustrations. Photographic and graphical representations were not dealt with.

In relation to the development of the concept of angle, OX and OY rays, whose starting point is common, divides the plane into two regions with respect to the process perspective, each of which is called an angle, and is an object of mathematics. Unless otherwise stated, the inner region between the rays is taken. The term representation refers to both the process and the concrete object or somehow expresses a mathematical concept or relationship and the act of understanding the form itself "(NCTM, 2000, p. 67). So representation is an important part of mathematical activity and is a tool for understanding mathematical concepts (Goldin & Shteingold, 2001). Different illustrations (eg representations can help students understand a function as a process; illustrative illustrations help students understand a function in concrete terms. Therefore, it is important to firmly understand these two perspectives (Clement and Battista, 1989) and use appropriate illustrations in different contexts (Cuoco, 2001; NCTM, 2000; 2009).

The data for this study were collected through 12 illustrations related to the angle in four 5th grade coursebooks, two from abroad, and two from our country. The books are the ones published by the Mone in Turkey and the countries that are listed in the ranking of successful countries in international examinations, one of which was published by the Mone in 2018 and the other which was used in 1974. Inductive content analysis was used to determine the frequency of observable features in notations..

Research Problem

1. How are the angle-related illustrations in class coursebooks in our country and abroad in terms of accuracy, connectivity (attribution), clarity, and contextuality according to Kim (2018)'s classification?
2. What are the teachers ' views about the illustrations in 5th-grade coursebooks in our country and abroad?
3. How do the 5th grade students view the illustrations in the coursebooks mentioned?
4. Play roles to help students understand the concept of angle. For example, algebraic?

METHOD

Research Model

The conceptual framework of the research's data is stated in Table 1 and Table 2. According to this framework, each illustration used in geometry and sub-areas of learning measurement in the 5th-grade mathematics coursebooks, 2 from Turkey and 2 from abroad (Australia - Singapore), was collected from the teachers as "should be corrected, can be used, insufficient" and asking for an explanation for its reason. Likewise, according to each of the "Accuracy, Attribution, Clarity, Contextuality" analysis, it was evaluated (0; not suitable, 1; acceptable, 2; suitable) separately (according to the criteria given in Table 2) and coded. Data were collected by a document review technique. The document review technique involves the analysis of written texts containing the fact or facts targeted in the research (Yıldırım & Şimşek, 2008).

Data Collection Tools and Data Analysis

Table 1: Criteria Regarding Teachers' Opinions About the Angle in 5th Grade Coursebook

Illustrations About the Angle	Point	Criteria	Explanation
Rating	2	Should be corrected	
	1	Can be used	
	0	Insufficient	

Table 2: Criteria for Analysis of Illustrations Related to Angle in 5th Grade Coursebook (Kim, 2012).

Status of Visual Items	Point	Criteria
Accuracy	**2**	A non-text element accurately describes the definition of a concept or is correct to show a concept based on its definition.
	1	A non-text element is meaningful in terms of the definition or meaning of a concept. However, it does not show every required mathematical situation (eg some necessary notations are missing or misleading) or some features are not suitable for explaining a concept.
	0	A non-text element is wrong for the definition of a concept (for example, it has a pronounced error). Or an inappropriate realistic object is used to present a concept. There is a big mistake or concern for using the realistic object or context for a concept. There is no mathematical concept for a non-text element.
Connectivity	**2**	A non-text element is clearly and exactly related to the mathematical content in the text. It indicates a direct concept or problem.
	1	A non-text partly relates to mathematical content in the text. A non-text item has some missing or irrelevant information. It shows the content, but not clear how it depends on the content.
	0	A non-text element has nothing to do with mathematical content. It can give some clues about the context in the texts (for example, the river when the problem is related to the length of the river).
Clarity	**2**	A non-textual element is simple to illustrate a concept or problem without any distractions or other factors.
	1	A non-text element is simple to illustrate a concept or problem with some other factors that may be useful for the concept.
	0	A non-text element has distractions or other factors that are useless in addition to the factors required to represent a concept or problem.
Contextualities	**2**	A non-text element uses a realistic object or context with a mathematical link.
	1	There are no mathematical ideas or concepts, but there is some realistic contextual information (this is used to provide context or objects to problems or to facilitate related activities).
	0	There is no real object or realistic content in the non-text item.

This assessment was made independently by a mathematics educator and a specialist teacher who went on to a doctorate. These evaluations were later calculated as the percentage of agreement suggested by Miles and Huberman (1994), namely reliability (Reliability = Consensus / (Consensus +

Disagreement). This percentage of agreement among the researchers was first calculated at 91.6%. Disagreeable points were identified and discussed and the consensus was reached.

Table 3: Criteria for Analysis of Illustrations Related to Angle in 5th Grade Coursebook (Kim, 2012).

Analysis of the visual element	Illustrations
Accuracy	**1.08**
Connectivity	**1.66**
Clarity	**1.66**
Contextualities	**1.91**

RESULTS

The illustrations in the books are 12. And 11 (eleven) are qualitative, except 1 (one). In one, only the measurement of the angle was used. 19 secondary school math teachers participated in the study. The feedback from the teachers has been in written form. They used the scores for the illustrations but did not make much of an explanation for the illustration in the desired descriptions column.

Table 4: Teachers' Views on Illustrations Used in Mathematics Coursebooks Geometry and Measurement Sub-Learning Area

Country	Visual Item	Can be used (2)		Should be corrected (1)		Insufficient (0)	
		f	%	f	%	f	%
Australia	İ1	15	78,9	4	21,1	-	-
	İ2	13	68,4	6	31,6	-	-
	İ3	13	68,4	5	26,3	1	5,3
Singapour	İ4	11	57,9	6	31,6	2	10,5
	İ5	15	78,9	3	15,8	1	5,3
	İ6	15	78,9	3	15,8	1	5,3
	İ7	13	68,4	5	26,3	1	5,3
2018 MONE Turkey	İ8	7	36,8	7	36,8	5	26,4
	İ9	17	89,5	2	10,5	-	-
	İ10	7	36,8	8	42,1	4	21,1
1974 MONE Turkey	İ11	14	73,7	3	15,8	2	10,5
	İ12	10	52,6	8	42,1	1	5,3

I1 coded illustration is the top left and top view of a table. Teachers (78.9%) said the I1-coded illustration in the book published in Australia could be used. It is teachers (21.1%) who say it should be corrected. Although the picture shows the angle between the two Rays in terms of accuracy and the counterclockwise direction as the rotation, no algebraic representation representing the angle was used. T3 (see. Table 4) interpreted the angle at the corner of the table as an acute angle according to its appearance.

Table 5 Students' Views on I1 Coded Illustration Used in Geometry and Measurement Sub-Learning Field in Mathematics Coursebooks

S1	It would be better if the right angle put a point here, and if it made the table like this, it would be better to give a bird's eye view. Since it is a beam, it is formed by combining two rays.
S2	There's a right angle here. It looks like a crooked table, not looking upright. But if some don't understand, they might call it an acute angle.
S3	They tried to show an angle, acute angle. There is an angle sign, but some students may not understand it. They could draw the table with the arrows to make it more effective.

S4	I understand this as a right angle to the corner of the table. It is among the things we use in daily life

In the I2 coded representation, the hour and over seconds representation, the angles between the hour and minute hands have been used to measure the angle in the opposite direction of the clock. When using the watch in angle notation, we can think of moving (dynamic) representation. All four books were used as illustrations. The percentage of teachers who say it should be corrected is 31.6%. It is seen that all students (See table 5) adopt the clock as a prototype in the concept of angle.

Table 6 Students' Views on I2 Code Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub-Learning Area

S1	It is easier to explain the obtuse angle here, the beautiful angles here with hours. Our primary school teacher used to tell it like that. It is logical to tell with time, with a scorpion and minute hand.
S2	There is an obtuse angle in the picture. I think it may seem good. It is easy to understand that it is an obtuse angle.
S3	it's like an obtuse angle, I can't understand it because I can't measure it right now, but it should be an obtuse angle. It shows the angle and its arms are clear. So, the picture of the clock is a proper one.
S4	The second picture looks like an obtuse angle to me. They could have given it more clearly. In a way that we can understand even more. Like at the table.

For the angle coded between the ceiling and roof of a house (here, the slope can be considered) in the i3 coded illustration, teachers used the correction with 26.3%, and there was 1 teacher (5.3%) who said it was inadequate. Teachers did not provide explanations and justification for their statements that should only be corrected, sufficient, and inadequate. They stated that they did not use the coursebooks much. The students found the house illustration complicated. They wanted the shape to be a little simpler (see table 6).

Table 7 Students' Opinions about İ3 Code Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub Learning Area

S1	Acute angle. It could have been a simpler visual. There are other examples of this part of the house. We need to delete unnecessary things.
S2	It may be appropriate for an acute angle.
S3	I beg your sorry. It seemed different to me. Why does this show the angle that a better picture could have been put in its place? Maybe, the kids can understand the angle between the two.
S4	there is an acute angle, so the room is fine because it's illustrated.

In the coursebook published in Singapore, teachers (57.9%) said that they can be used for I4 coded illustrations. It is the teachers (31.6%) who say it should be corrected. The teachers who say "not enough" are (10,5). In the illustration, a curved tablet was used on a ground and there was an indication that it can be considered as an angle, there was an angle indicator, but the arrow indicating the direction of rotation of the angle was not used. No algebraic representation representing the angle was used in this illustration. An intuition for measuring the angle wasn't developed. The students also described this illustration as complex (See table 7).

Table 8 Students' Opinions about I4 Coded Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub Learning Area

S1	Acute angle. This is an obtuse angle, and this is an acute angle. It is confusing. There is a straight angle, too.
S2	There is an acute angle on the right and an obtuse angle on the left. So it looks like what's put on the canvases. It is not very good, just medium.

S3	I think there are two angles. I think it's beautiful like the watch. I see one acute angle and one obtuse angle.
S4	He showed the acute angle over there, and the obtuse angle over there. It seemed a little more complicated to me. It could point directly at a single angle. it is not clear. We also don't understand what this object is.

Teachers (78.9%) said that they could be used in the I5 coded illustrations. It is the teachers (15.8%) who say it should be corrected. The teachers who said it was inadequate was (5,83). The angle between the hour and minute hands on the watch was shown using the concept of a beam, but again without the concept of direction. The line is drawn at 12 o'clock and 6 o'clock were presented with three different angles that made up the perigon, and again the measurement of the angle and the variable role of this measurement were neglected. The inner and outer region of the angle, the direction to take the inner region, and the relationship that would reveal the dynamic direction of the angle, unless stated otherwise, were expressed in three unknowns instead of the emphasis on creating a reflexive with the inner and outer region. The students agreed to use watches as prototypes (see table 8).

Table 9 Students' Views on I5 Coded Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub-Learning Area

S1	Here, a straight angle, an acute angle, and an obtuse angle. It can actually be difficult to describe three angles in one place, so it could be easier if it were two or more. It would be wrong to show them all at once.
S2	There is one obtuse angle, then there are acute angles at twelve completed hours. There is one right angle.
S3	I think it was aimed to show a right angle on the watch, but they drew one line because, besides the arrows, there is one angle of 180 degrees and even three angles. I think there is an angle here. What if the line was a little more clear, which is second.
S4	There is an obtuse angle. There's one straight angle there. There's a round angle over there. It showed different angles. I think it's beautiful.

In the I6 coded representation, for scissors used daily, teachers (78.9%) said it could be used, (15.8) should be corrected, and (5.83) insufficient. In this demonstration, no determinations were made to show the quantitative direction of the angle. With the open state of the scissors and two intersecting lines on it, it is desired to create an angle concept in the mind of the student. We avoid such determinations that we would call analogy in mathematics lessons. The students found the scissors sample effective (See table 9).

Table 10: Students' Views on I6 Code Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub-Learning Area

S1	There are two acute angels if I am not wrong. I think there is the right angle from the crosses. To use the scissors is something useful.
S2	Acute angle. There is one on the back and one in the end. There are two obtuse angles here. The obtuse angle from right, an acute angle from left is not very suitable. The line does not appear, but it can be seen if they put it.
S3	They asked both the angle here and the angle there. Four angles
S4	There is an acute angle of scissors over there. That's an obtuse angle. The scissors attracted my attention more.

In the I7 coded illustration, the feet of a table is presented as an angle representation. In this illustration, teachers (68.4%) said that it could be used while teachers (26.34%) said it should be corrected. The teachers who said it was insufficient were (5,83%). It is an illustration on a table foot with a slope that presents angles that produce a correct angle that does not use algebraic notation, such as in the case of scissors. In this image, teachers should only be corrected, they did not bring explanation and justification to their statements as it should just be corrected, it is sufficient and it is insufficient. Students wanted to give simpler examples in this illustration, which presents the concept of wholes tried to be explained on the feet of the table (see table 10).

Table 11: Students' Views on I7 Code Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub-Learning Area

S1	Actually, it may be difficult for us. It is hard to grasp the table, to understand that there is a line here, so what is there, they can put something a little simpler. They gave some difficult examples, it would be better if they gave simpler examples from daily life.
S2	The obtuse angle from the right and acute angle from left are not very suitable.
S3	They did not put an angle indicator either. It would be more descriptive if they put an angle sign.
S4	This showed that it was an obtuse angle. If we close here, this acute angle becomes obtuse angle. it sounded a little complicated.

In the coursebook published in Turkey, teachers (36.8%) said that i8 coded illustration can be used. It is teachers (36.8%) who say it should be corrected. It is teachers (26,4) who say it is insufficient. In the illustration, cars standing in a parking lot and the way cars park in the area upright and oblique, and the angle between them emphasizes the quantitative direction 45^ ^ 0. The angle is statically there. There is a measurement of the angle, but the direction of rotation of the angle was not used. The algebraic notation representing the angle was not used in this illustration either. An intuition for the measurement of the angle was developed. Students interpreted this picture as an acute angle and 45 degrees in a quantitative way. They tried to relate it to everyday life (see. Table 11).

Table 12: Students' Views on I8 Code Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub-Learning Area

S1	From daily life, the car is 45 degrees when leaving the park.
S2	Cars are at an acute angle. I think it might be appropriate because the shape they created seems to be so. They understand easily.
S3	There is one acute angle. Is that just to make it stand out? I wonder if it means that maybe 45 degrees make it easier to park.
S4	There is one acute angle here. They made it according to the positions of the cars. The existence of roads and vehicles is also different and I am interested in daily life.

Teachers (89.5%) said that they can be used in the illustrations with code I9. It is the teachers (10.5%) who say it should be corrected. The angle between the hour and minute hands on the watch wants to express obtuse, acute, and right angles with two half-lines without showing the direction and measurement without using the concept of the beam. The inner region and outer region of the angle tried to intuitively express the relationship that would reveal the inner side of the angle and the dynamic direction of the angle unless otherwise stated. The perigon angle was not highlighted. In this picture, which explains the angle types over 3 hours, the students explained the ideas in accordance with the definitions in their minds as the first prototype for understanding the angle types of the students (see table 12).

Table 13: Students' Views on the I9 Code Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub-Learning Area

S1	Obtuse angle acute angle, right angle. Right angle and they always draw like this. It would be better if the dot inside the square is at the right angles. One suspects quite a lot if they put it so.
S2	Obtuse angle, acute angle, right angle about clocks. Hours are easy to understand. They may be appropriate. It would be better understood if it went as a right angle.

S3	The clocks are suitable for me. There is an acute angle, an obtuse angle, and a right angle.
S4	Here is an acute angle like this. Same way over here.

It wants to develop the concept of angle based on intuition in a park environment in the I10 coded illustration. Teachers (36.8%) said that it can be used, (42.1) should be corrected, and (21.1) insufficient. Instead of intersecting lines, an intuition that can lead to the definition of curvature and curvature can be developed with this illustration. The students stated that they did not understand the illustration trying to explain the angles on the roads in a park (See table 13).

Table 14: Students' Views on I10 Coded Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub-Learning Area

S1	I did not understand anything. In fact, this example was wrong. I do not understand.
S2	It is a bit complicated. They can not understand. it is difficult to understand.
S3	There may be an angle here or there can be an angle there. it is not understood. You don't know what it wants.
S4	Here is an acute angle like this. Same way over here. It is nice but not a good example from daily life.

Two illustrations were used in a mathematics coursebook approved by the ministry of education and published in Turkey in 1974. The illustration, code I11, is the angle created between the arms of a broken meter. Teachers (73.7%) said it could be used, (15,8) it needed to be corrected, and (10,5) inadequate. Angle was marked in the notation, direction and measurement were not specified. (See fig. table 14).

Table 15: Students' Views on I11 Coded Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub-Learning Area

S1	Here, it explains the acute angle. Nobody is using those meters in daily life right now. Now there are more technological things. It was necessary to use the meter as t was broken very often.
S2	By using a ruler, an acute angel was formed. It is good.
S3	Acute angle. what is this ruler for?
S4	I did not understand much if a ruler was put. it is not clear.

The illustration, which has the code I12, attempts to intuitively describe the angle on the clock, as in the other three books. Although the region between the two rays is treated as thinking, the region is not marked and the measurement is not emphasized. (See fig. table 15).

Table 16: Students' Views on I12 Code Illustration Used in Mathematics Coursebooks Geometry and Measurement Sub-Learning Area

S1	Angles are right angle, acute angle obtuse angle, straight angle, and perigon. They give the exercises in the coursebook, Eighty percent of the exercises have answers right next to or on the other page. They answer whatever they say. There are very few exercises in the coursebook. So all teachers use smart notebooks or test books.
S2	It has also given some examples of clocks. There are right angles, acute angles, obtuse angles, perigon. It has given five angles. There are two arrows. In a row, perigon.
S3	They did not use the angle symbol here. Right angle, acute angle, straight angle. It can at 180 degrees or a perigon. It is related to illustrating. I couldn't understand where the other line is? Or is there just such a line? They are in a row. It can be 0 degrees.
S4	When describing fort he the first, it should be more understandable. There are right and acute, obtuse angle. This is a perigon. I couldn't understand that. The situation where the hour and minute hands overlap.

DISCUSSION AND CONCLUSION

The concept-oriented acquisition of the mathematics education program that was put into practice in 2018 in Turkey, is as follows: "M.5.2.1.4. It creates acute, right, and obtuse angles, referring to an angle of 90 °; determines whether a created angle is acute, right, or obtuse. a) Studies are done on checkered, dot paper, etc. b) When determining or creating angles, it may be required to use the corner of a paper, square or protractor as a reference. c) Angles are expressed by the name".

On the other hand, in 2018, the 4th grade curriculum, M.4.2.3.4. It determines the angles as acute, right, obtuse, and straight by measuring them with standard angle measuring tools. a) comparison is made by taking the right angle reference. b) when examining angle models, it is considered that they are not larger than the straight angle. For these reasons, the students also showed a straight angle in the illustrations. They were even able to speak of zero degrees and perigon. In fact, the students are from a high-economic secondary school in a major city. But even so, the consistency in the curriculum still needs to continue. The use of clock examples in four books is well thought out in terms of revealing the dynamic structure of the angle. In addition, it was determined that in the 2009 and 2013 educational programs published by MONE, the types of angles and contextual studies for obtaining the angles dynamically were not mentioned ln 5th grades and the straight angle and the perigon were not given in the program.

Schunk (2009) states that the most important concern in research on first sample theory is that they take up more space in long-term memory than rules as a result of storing thousands of first samples. The other concern is that individuals may create false first examples if they store non-identifying features, rather than useful ones. For this reason, the visuals in the book should also be discussed by the teacher and students.

In the study, students and teachers accepted high rates of illustrations in the concept of angle overclock. The clock appears to be a prototype adopted by students. It should not be overlooked in the studies that the watch forms a perigon as this prototype, for example, between 1-2 and 300 on the clock. It should be added to the acquisitions.

The clock in 12/360 is an important prototype. According to the classical theory approach (Gagne, 1985; Smith & Medin, 1981), it is stated that different examples of a concept should be remembered at the same speed because each example makes judgments about examples based on their distinctive qualities. However, this is not always the case. Many people find it more difficult to confirm some examples of a category (for example, the angle measured in degrees) than others (for example, the angle is also measured in radians in trigonometry). This highlights the problem that many concepts simply cannot be identified with a set of distinguishing features. The importance of teachers discussing illustrations with students is clear.

Illustrations in coursebooks can be used to improve students' learning. Verbal narrators and two-dimensional illustrations are used, but real models of shapes are much more useful in terms of efficiency in learning. In I12, it may be easier for students to understand the concept of angle with collapsible meters, for example, to allow them to make shapes in their hands. This example is manipulatively among the mathematics lesson tools in our schools. Highly qualified contextual problems can be created from the angle-related positions of the apparatus attached behind a tablet to stand on the table or a telephone apparatus standing on the table (example I4 in the Singapore mathematics coursebook). In this way the angle switches from static to dynamic. This is true within the hour used in I2, I5, i9, I12. Examples of manual skills, audiovisual devices, and educational tools such as computer graphics make learning easier. While concrete tools are arguably more important to younger children than they lack the cognitive capacity to think over abstract concepts, students of all ages benefit from information represented by multi-modes.

Today's cognitive computing theories offer valuable information to use in classrooms. One of them is mental imagery. While mental imagery is used in the representation of spatial knowledge, this imagery must be clearly understood in long-term memory. There are also individual differences in terms of vision and developmental periods. Long-term memory, according to Paivio, stores two types of information in the form of a verbal system that combines information expressed in a word and an imaging system that records visual-spatial information, and these are related to each other. There are verbal descriptions of video code in every 4 coursebooks, but contextually these cases are different from the conversion of verbal code into video Code. The visual code is used to represent concrete objects or events, while the verbal system is suited to abstract information. Verbal code in the use of representations from process skills in mathematics teaching should also be supported as algebraic notation. Binary code theory (functioning memory - long-term memory) means that concrete words can be explained by verbal and visual codes, while abstract words can only be explained verbally. During recall, people run both memory systems for concrete words but only the verbal system can be used for abstract words. The verbal system related to the concept of angle should be used in 5th class coursebooks.

That the algebraic expression of the angle (AOB) is supported by the use of (AOB) = 〖30〗^0=c as an unknown is perhaps a state that should be in geometry after algebraic expressions in 6th class but not used.

For this reason, the use of the angle related to the angle concept in the first unknown form should also be considered in illustrations. The keywords in the student-teacher interaction of these illustrations are name, draw, and label due to the structure of geometry. Similar research should be done in terms of other concepts related to geometry in coursebooks. Geometry is as much about algebra as it is about numbers and number sense. Research should also be continued in terms of these linked areas.

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Abbreviations

MONE : Ministry of National Education
S1 : Student 1
S2 : Student 2
S3 : Student 3
S4 : Student 4