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Abstract. Mental models are representations of students' minds concepts to explain a situation or an on-going process. The purpose of this study is to describe students' mental model in solving mathematical patterns of generalization problem. Subjects in this study were the VII grade students of junior high school in Situbondo, East Java, Indonesia. This study was conducted using the qualitative descriptive method to investigate students' mental model during the process of solving the problem of generalization patterns. Students were required to speak aloud what he or she was thinking about solving problems *(think-aloud).* Based on the data obtained, the students' mental models in solving the patterns of generalization problem can be classified into two kinds of mental models, namely: (1) direct mental model and (2) indirect mental model. Students classified using direct mental model were using the only generalization of algebraic in solving the problem. Meanwhile, students classified using indirect mental model were using a combination of both generalizations of algebraic and arithmetic. Based on the results of the study, teachers are required to take into account students' mental model in solving the generalization patterns problem to achieve better learning process.

1. Introduction

Learning is an active process of constructing knowledge which means that knowledge will be formed if the students do the construction process actively. A suitable technique for teaching courses needs to be adopted to improve the learning effectiveness of students [1]. The process of building knowledge in the context of learning Mathematics is done continuously in order to create complete knowledge for students. The knowledge that is formed can be used to build a new concept or used to solve problems encountered.

Harrison & Treagust [2] say that mental model represents the thinking of students that can be used to describe and explain a phenomenon during the learning process. Mental models are pictures of the concepts existing within students' minds to explain a situation or process that is going on ([3]: 106). Thus, the role of mental models are to explain the process of thinking and reasoning of the individual when trying to understand, predict or solve Mathematics problems.

Vosniadou and brewer [4] identify three worthwhile categories to explain the students' mental models of a concept: initial, synthetic, and formal. Students are included in the category who have the initial mental model as a framework for students in problem-solving which based on the concept of the initial knowledge of students [4]. Students are included in the category of having a synthetic mental model when they develop a framework to solve problems based on synthesizing their ideas. They have attempted to set the conceptual structure to assimilate new information while maintaining the current structure of their knowledge. Students included in the category of having a formal mental model as a framework to solve problems when they are able to reflect a good understanding in accordance with the rules of formal Mathematics.

Mental models are difficult to identify, elusive and difficult to describe because they are abstract, complex, unstable and varied [5]. However, one's mental model can be identified through interpretation of expressed mental models that is the mental model expressed using oral, written or images [5]. Chamizo [6] mentions that expressed mental model is a material model.

Because mental models are complex and varied, the characterization will require the interpretation of exquisite expressed mental model data collection from the examined subject (for instance in the form of pictures, writings, or verbal explanations). Identification of a mental model of learners in Mathematics and science generally uses qualitative data which are collected through interviews, questionnaire or diagnostic tool [7].

Several studies of mental models have been carried out. Bofferding [8] reported on the mental model of first grade in primary school regarding the understanding of the sequence, the magnitude, and direction of integers. In the study, Bofferding [8] showed that students develop a mental model of the order and the value of the integer into five types, namely: initial mental model, the mental model of the first transition, the mental model of the synthesis, the mental model of transition II, and formal mental model. Students develop mental models about the direction of the magnitude integers in three types: initial mental models, mental models of synthesis and formal mental models. In that study, it was also reported that students experiencing mental model changes after following the intervention of learning, for all groups of learning about the meaning of a negative sign (unari, binary, and combined). But even so Bofferding [8] did not indicate whether or not students always develop a mental model of the synthesis before having a formal mental model when solving Mathematical problems. Therefore, the research about how the mental model used by students in solving Mathematical problems are still open for further investigation.

According to the above-mentioned explanation, the mental model of an internal representation that involves recalling and processing information held in memory aims at solving mathematical problems.

One of the Mathematical topics that are deemed important to master by students is a topic of generalizing patterns. The concept of generalization pattern is constructed by a student within their mind during the learning process, thus it forms a mental model. In solving the pattern of generalization problem, [9] distinguishes the strategies used by the students to Na'ive induction strategy, the strategy is a generalization pattern based on trial and error and strategic generalization which consists of generalization of arithmetic and algebraic. Algebraic generalization strategy includes factual, contextual, and symbolic generalization.

Research on the generalization patterns in Indonesian context has been done. Several studies have been carried out [10], [11] and [12]. Sutarto, Toto Nusantara and Subanji [10] evaluate students' basis in establishing a pattern of generalizations, and Zayyadi and Kurniati [11] identify students' reasoning in generalizing the patterns that proved by generalizing the structural generalizations, whereas Inganah [12] examines the gesture in the algebraic thought process of students when generalizing patterns. Based on research that has been done, there is no research that explores the students' mental models in solving the problem of generalization based on the direction of student thinking patterns. Do the students' thinking in solving the patterns of generalization problem always use a strategy of arithmetic or strategy of algebraic only (Direct Mental Models) or even there is a change of direction to think of using a strategy of arithmetic into the strategy of algebraic generalization and vice versa (Indirect Mental Models). The aforementioned notion requires being examined further. This study, therefore, will examine "Students Direct and Indirect Mental Models in Solving the Pattern of Generalization Problems".

2. Methods

This study applied qualitative research methods. In qualitative studies, the goal is to provide an accurate description of a real situation. To this end, such studies attempt to directly present the opinions of individual participants and to collect data through detailed and in-depth methods. As the study investigates the way in which students use mental model, a case study with qualitative research techniques was produced

2.1 Participants

The research participants were 21 students of grade 8 of a junior high school in Situbondo, East Java, Indonesia. They were students who have learned the material of pattern generalization. The participants were determined based on their levels of mathematical ability as follows: students with good mathematics ability; students with medium mathematics ability; and students with less mathematics ability. The determination of the mathematical ability of the students is based on the score of the previous report, and the opinions of the mathematics teacher and homeroom teacher. Determination of research participants also considered the good communication of students in expressing their ideas and their willingness to voluntarily participate in the study

2.2 Data collection tool

The data retrieval was done by giving a simple problem related to the material of number pattern to be completed by students. The problems are used to examine the student's mental models when solving the pattern of numbers generalization problems. The given problem in this research is presented as follow.

Consider the following numbers pattern, with x and y
are integer number:
1, 3, 6, 11, 20, x, 70, y, ...
Find $x + y$!

2.3 Process and data analysis

The participants were told that their answers and that the interview notes would only be used to understand their thoughts during problem-solving. The students were then asked to think aloud when solving the problems so as to be abler to explain their thoughts. The researcher also asked them questions when necessary. In addition, questions such as "How did you think of this?" and "How did you draw this?" were asked in order to reveal the students' thought processes. The interviews were performed in a silent environment in the school library and recorded using video. The problem-solving process for each student lasted approximately 30 minutes. For the analysis of the study data, the video records were first transferred to the interview forms without any corrections being performed.

3. Results and Discussion

According to the analysis results on the students' interview, transcript and think-aloud, it was obtained two students' mental models based on thinking structure possessed by students research subject. The following is the description of two c□aracteristics of students' mental models based on the thinking structure.

3.1 Direct Mental Model Structure

The following figure is one of the examples of students' (S1) answer in solving a generalization of patterns problem which is included in the direct mental model.

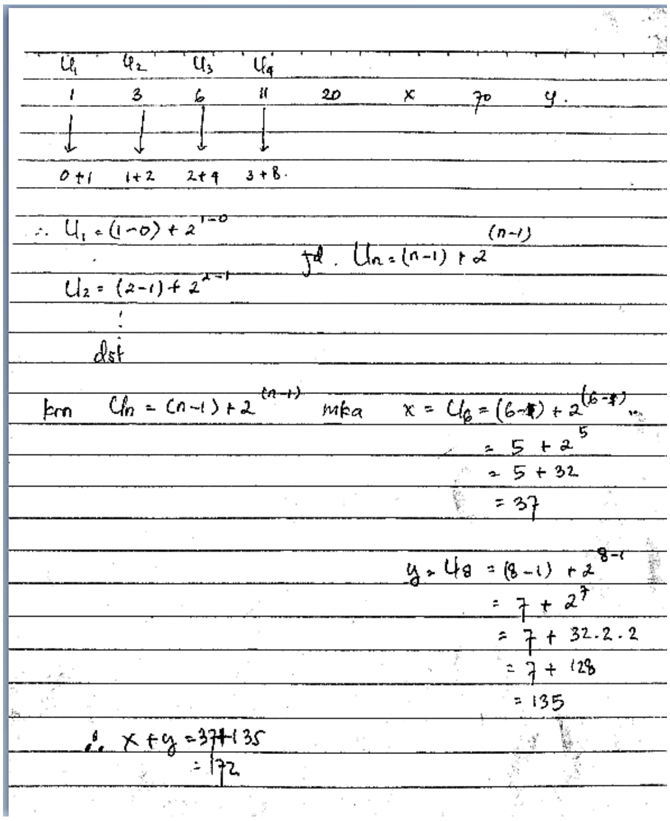

Figure 1. Example of Student's (S1) Answer which Reflect Direct Mental Model

According to the Fig 1, S1's Think Aloud and interview transcript, student's thinking structure in solving a generalization of patterns problem given by teacher could be depicted in the thinking process scheme as in Fig 2.

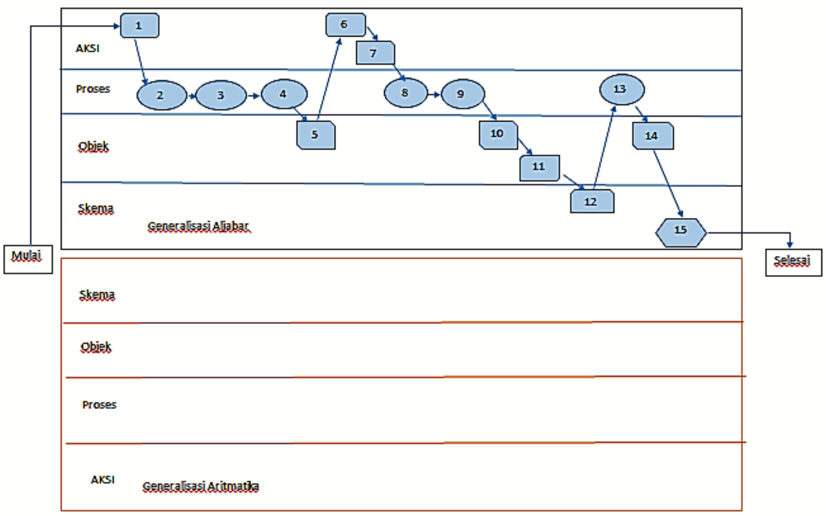

Figure 2. S1's Thinking Structure with *Direct Mental Model.*

Remarks:
1. Observing the pattern sequence of numbers: 1, 3, 6, 11, 20, *x*, 70, *y*, ...
2. Writing the variable which declares terms by using symbols, e.g, *U1, U2, U3,* and so forth.
3. Generating relationship between variables *U1* and 1, *U2* and 3 , *U3* and 6 , and so forth until *U6* and *x*, *U8* and *y*
4. Manipulating numbers in rows 1, 3, 6, 11, 20, ... becomes multiplication and/or summation of two integers
5. Declaring *U1 = 1 = 0 + 1; U2 = 3 = 1+2; U3= 6=2+4; U4=11=3+8; U5=20=4+16;*
6. Observing the pattern sequence of numbers *U1 = 1 = 0 + 1; U2 = 3 = 1+2; U3= 6=2+4; U4=11=3+8; U5=20=4+16; ....*
7. Writing line/circle/signs showing regularity in the number patterns result of manipulation
8. Generating relationship between *n* index expressing the sequence of the discovered pattern
9. Predicting rules of order of sequence number from the discovered regularity i.e. *(n-1) +2(n-1)*
10. Expressing the general formula for the terms to n of line numbers 1, 3, 6, 11, 20, *x*, 70, *y*,... is *Un=(n-1)+2(n-1)* for *n* as the natural number
11. Demonstrating through calculation which *Un=(n-1)+2(n-1)* true for some *n*
12. Stating the belief that *Un=(n-1)+2(n-1)* true for the entire *n* natural numbers.
13. Calculating *x* and *y* using general formula of *Un=(n-1)+2(n-1)*
14. Writing and expressing *U6= x =37*, dan *U8=y=135*
15. Providing an explanation to convince that *x=37* and *y=135* thus n *x+y =172*

Indirect Mental Model Structure
The following figure is one of the examples of students' answer in solving a generalization of patterns problem which is included in the indirect mental model.

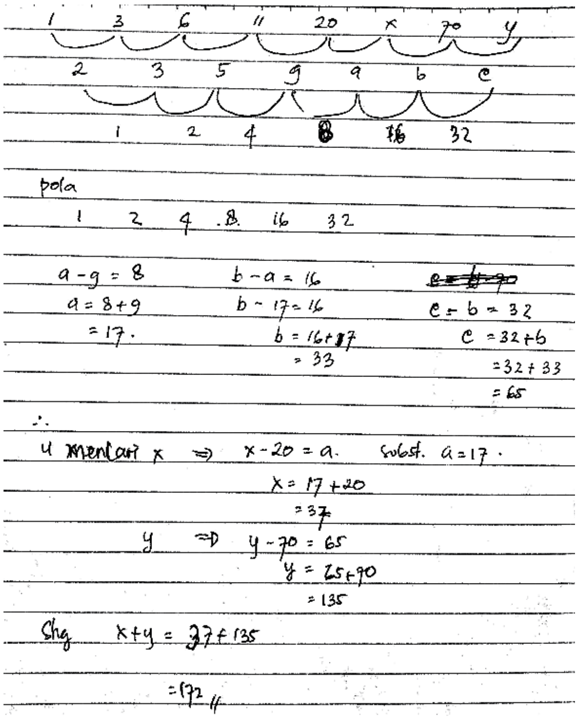

Figure 3. Example of Student's (S6) Answer which Included in Indirect Mental Model

According to the Fig 3, S6's Think Aloud and interview transcript, S6 thinking structure in solving a generalization of patterns problem given by teacher could be depicted in the thinking process scheme as in Fig 4.

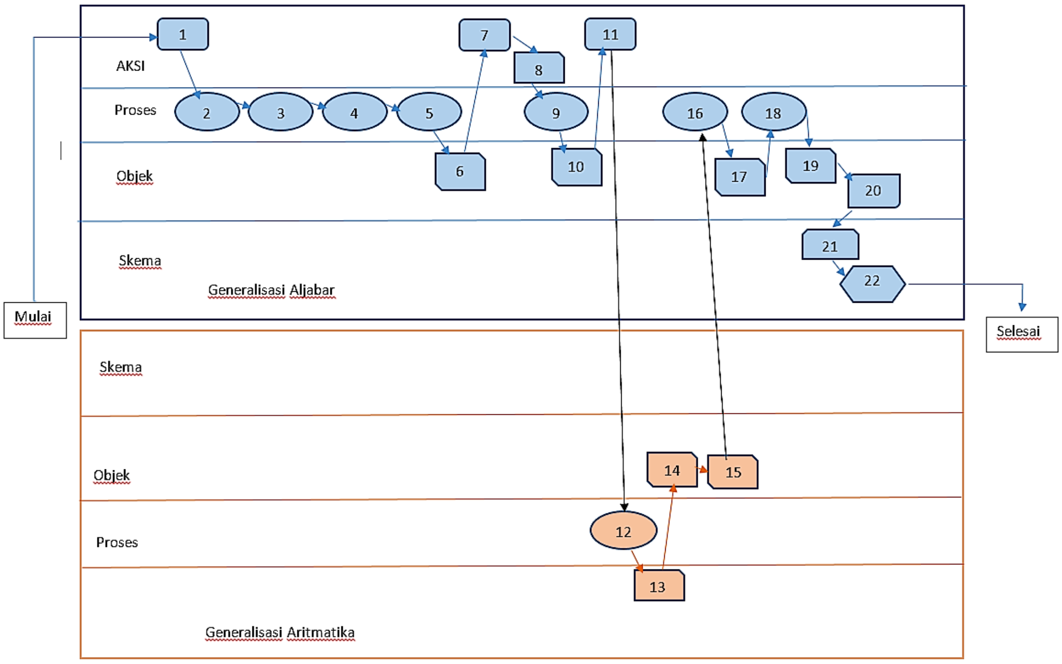

Figure 4. S6 Thinking Structure in *Indirect Mental Model*.\

Remarks:
1. Observing the pattern sequence of numbers: 1, 3, 6, 11, 20, *x*, 70, *y*
2. Writing the variable which declares terms by- .... by using symbol, e.g. *U1*, *U2*, *U3*, and so forth.
3. Generating relationship between variables *U1* and 1, *U2* and 3, *U3* and 6, and so forth until *U6* and *x*, *U8* and *y*
4. Calculating the difference of sequence *U2-U1; U3-U2; U4-U3; U5-U4; U6-U5; U7-U6; U8-U7.*
5. Generating relationship between *U6-U5=x-20* with the variable of *a*; *U7-U6=70-x* with the variable *b; U8-U7= y-70* with the variable of *c*
6. Expressing the difference in the sequence of numbers *1, 3, 6, 11, 20, x, 70, y*,...unidentical, getting higher, irregular, and formed a new sequence of numbers of *2, 3, 5, 9, a, b, c.*
7. Observing the pattern of the sequence of numbers which is a sequence difference of two numbers in the sequence of *1, 3,*
6, 11, 20, x, 70, y, ... i.e*: 2, 3, 5, 9, a, b, c, ...*
8. Writing curved line indicating a pattern difference between the two numbers in the sequence number of *2, 3, 5, 9, a, b, c, ...*
9. Calculating a difference between two adjacent numbers in sequence numbers of *2, 3, 5, 9, a, b, c, ...*
10. Expressing a difference on the sequence of numbers of *2, 3, 5, 9, a, b, c, ...* unidentical, getting higher, irregular and formed a new sequence of numbers of 1*, 2, 4, a-9, b-a, c-b, ...*
11. Observing the pattern of the sequence of numbers which is a sequence difference of two numbers in the sequence of *2, 3, 5, 9, a, b, c, ... i.e: 1, 2, 4, a-9, b-a, c-b, ...*
12. Manipulating numbers in the sequence of 1, 2, 4,... becomes multiplication or the summation of two integers
13. Writing line/signs showing regularity in the number patterns result of manipulation 1, 2, 4 that have been manipulated
14. Expressing a sequence of numbers 1, 2, 4, ... regularly follow the rule stating that the next number is twice higher

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from the previous number
15. Writing the next numbers from a sequence of numbers of 1, 2, 4, is 8, 16, 32, ...
16. Calculating *a: 8= a – 9* ; calculatng *b: 16= b-a*; calculating *c: 32=c-b*
17. Writing and expressing that *a=17, b = 33* and *c=65*
18. Calculating x*: x-20 = a* ; calculating *y: y-70=b*;
19. Writing and expressing that *U6= x =37*, and *U8=y=135*
20. Showing through calculation that *70= x + b* is true if *x=37* and *b=33*
21. Expressing that *x=27* and *y=135* according to the discovered rules
22. Providing an explanation to convince that *x=37* and *y=135* thus *x+y= 172*

The results of the study having been explained before indicated that the thinking structure of students in solving a generalization of patterns given by teacher is diverse and varied because it depends on the strategy used by students in solving the problem.

Mental is "reverting to the mind" which means something to do with thinking or thought itself [13]. Thinking is a process of generating a new mental representation, through the transformation of information involving a complex interaction between mental attributes [14]. Mental attributes may include assessment, abstraction, and problem-solving. Further, it is said that the purpose of the thought process leads to a conclusion to generate solutions to problems. Thinking, according to Santrock [15], is to manipulate or manage and transform the information into memory. Therefore, thinking is a mental activity that involves recalling and processing information held in memory aiming at solving the problem.

Mental models of each individual are different, and mental models constructed by each individual is influenced by several factors. Lin and Chiu [16] state that the factors that can affect the mental model's of the students can be classified into five factors, namely: a) the teacher's explanations, b) language and words, c) the experience of everyday life, d) social environment and e) the causal relationship and intuition.

Based on 2 answers obtained in this study and further analysis, there are two kinds of mental models that were identified and revealed. Both of these mental models are Direct Mental Models and Indirect Mental Model.

Generalization of the pattern is considered particularly important because the mathematical structure can be observed by looking for patterns and relationships [17]. Associated with the pattern, National Council Of Teachers Of Mathematics (NCTM) [18] recommends students to participate the activities of a pattern from an early age with expectation that they will be able to (1) make generalizations regarding geometric and numerical patterns, (2) provide justification for their conjecture, and (3) represent patterns and functions in words, tables, and graphics. Patternbased methodological approach challenges students to use higher order thinking skills, emphasizing exploration, investigation, suspecting, and generalizing. Finding patterns is part of the problem solving that requires a strategy.

According to Radford [9], students can solve problems related to the generalization pattern using naïve strategy or generalization. Generalization is divided into a generalization of arithmetic and algebraic. Furthermore, Radford [9] describes the difference between na'ive induction (guessing by trial and error) and generalization. Na'ive induction is generalizations patterns based on trial and error. Furthermore, generalization consists of generalization of Arithmetics and algebraic. Generalization of algebraic consists of factual, contextual and symbolic generalizations.

Direct mental model shown by the students during the process of solving the patterns of generalization problem where the strategies used are a consistent strategy in one zone generalization strategy according to students' mental model [9]. While the indirect mental model shown during the process of solving the pattern of generalization problem where the strategy used is a combination of strategy and generalization of arithmetic algebraic by Radford [9].

The direct mental model shown by the students during the process of solving the given problem students encountered no confusion which attracted them to use other strategies. While the indirect mental model occurs when students were confused or encountered obstacles in solving the problem of generalization pattern. In this study, it was found that students, using indirect mental model, began to solve the given problems by using a generalization of algebraic. However, during the process of solving the given problems, they encountered a confusion and decided, according to the process of reflection done by students, to use arithmetic strategy to make a generalization.

The findings of this study indicate that students can only solve the problem of generalizing patterns by using different strategies. Different strategies may be either algebraic or arithmetic generalization pattern as described by Radford [9]. But even so, the students can just do a generalization strategy pattern which is a combination of generalization strategy pattern described by Radford [9]. The findings in this study confirm that students can move away from using generalization of algebraic to generalization of arithmetic and vice versa.

4. Conclusions

After all the data were collected, then the data were analyzed. Apparently, from the data collected, the structure of thinking which was used by students when solving the problems can be classified into two different mental models: Direct and Indirect Mental Models.

Considering the type of response groups of students which is included in the direct mental model, it appears that the students, when solving problems of generalizing patterns, always use a generalization of algebraic strategy. They do the activities of thinking when trying to find the rules of the settlement of a given problem. Mental model was directly identified when the entire activities of the solving problem is in a zone of generalization strategy, both algebraic and arithmetic.

On the other hand, the type of response group of students which is included in the indirect mental model, it appears that the students when solving problems of generalizing patterns are not always in one zone of generalization strategy. During the process of solving problems of generalizing patterns, they tend to alter their strategy from generalization of algebraic to generalization of arithmetic and vice versa. The process of altering can occur repeatedly.

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