Presented by the George W. Bush Institute's Middle School Matters Program

How Math Instruction and Math Interventions Can Improve Student Outcomes

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Research-based principles and practices in math intervention research can be applied in the middle grades to improve student outcomes. These research-based strategies will enable middle grade students to develop important mathematics knowledge and skills to become algebra-ready.

THE GEORGE W. BUSH INSTITUTE SEEKS WITH ITS MIDDLE SCHOOL MATTERS INITIATIVE TO INCREASE THE NUMBER OF MIDDLE GRADE (GRADES 6–8) STUDENTS WHO ARE PREPARED TO COMPLETE HIGH SCHOOL AND EARN A DIPLOMA.

The Middle School Matters initiative at the George W. Bush Institute seeks to increase the number of middle grade students who are prepared for high school and postsecondary success. Middle School Matters' mission is to transform the middle grades by promoting high-quality research-based strategies and practices and to drive the development of policies that lead to better student outcomes. The two primary methods of achieving these goals are 1) to use solid research to develop practical tools and support for middle grade reform initiatives and 2) to promote the importance of research-based strategies and practices with individuals who make decisions for middle grade students.

The ability to problem-solve, reason, and think analytically—knowledge and skills at the heart of mathematics—is essential for students' success in the middle grades and beyond. This practice guide highlights several research-based practices to help middle grade students become more proficient at mathematical reasoning, computation, and problem solving, and ultimately, to become prepared for entry into algebra. School leaders and math teachers can use this information to compare and contrast the practices currently implemented at their school with those that have been validated by research and then identify ways to strengthen instruction and improve student success.

Success in math is important to all students; it provides them with college and career options in the fields of science, technology, engineering, and mathematics. In the middle grades, students build upon the critical skills taught in elementary school and prepare for entry into algebra, which is considered a gateway to later achievement in school and career. Research-based principles and practices in math intervention research can be applied in the middle grades to improve student outcomes. These research-based strategies will enable middle grade students to develop important mathematics knowledge and skills to become algebra-ready. What follows are a few questions school leaders might ask while providing instructional leadership, observing classrooms, or working with teachers on effective math instruction or intervention, and how Middle School Matters' math researchers would respond based on their knowledge of research in the area.

QUESTION:

Are there any school-wide practices we should adopt to enhance students' understanding and application of mathematics?

Yes! Mathematics is a foundational tool that can be used across many content areas (e.g., integrated science, social studies, health), and students often find solving problems that are situated in real-world examples to be motivating and relevant to their experiences. Content area teachers should consider the mathematics relevant to their disciplines and expect students to use their mathematical knowledge and skills to better understand and communicate what they are learning in other content areas.

Students can use their understanding of mathematics concepts and procedures to draw conclusions and propose solutions in content areas such as history, science, social studies, and economics. For example, data collected from observations of moving objects (e.g., the distance an object moves and the time it takes to move that distance) can be organized in tables and graphs, and conclusions can be drawn about the objects' features that result in differences in speed.

Students can also engage in quantitative reasoning by studying historical events and summarizing or analyzing these events using data. For example, students may struggle to understand why various spices were so valuable in the fourteenth century. However, if students are presented with data comparing the value of today's common spices with their value to Europeans in the 1300s, students will quickly understand why people risked their lives to search for new spice markets. Students can then use their quantitative reasoning to explore predictions for similar risks people take today.

QUESTION:

Research indicates that students' preparation for success in algebra is greatly influenced by their understanding of numeracy, or "number sense." How can math teachers help students enhance their understanding of number sense and prepare them for algebra?

Numeracy, or number sense, refers to how flexible and fluent a student is with numbers, including the understanding of what numbers mean, the ability to perform mental math, and the ability to look at the world and make comparisons. Numeracy also includes students' conceptual understanding of number systems and of the properties and operations that govern lawful application during problem solving. Students need to understand how these number systems relate to each other and be able to make generalizations from one number system to another (e.g., fractions to decimals). By generalizing, students can apply their knowledge of concrete numbers to algebraic reasoning.

Research indicates that number lines can be used as a central representational tool to help students recognize number systems and expand their understanding beyond whole numbers to integers and rational numbers. Teachers can do this in a number of ways, including the following:

- Ask students to calculate the sum or difference of positive and negative integers by counting on a number line
- Provide opportunities for students to measure objects using a common number line with fractions
- Ask students to connect fractions to a number line and use a number line to identify equivalent fractions
- Use number lines to illustrate equivalence between fractions and decimals, such as *0.40 = 4/10 = 2/5 or 8/10 = 0.80 = 80%*

QUESTION:

What is the best way to develop students' conceptual understanding in mathematics?

Research evidence suggests that using visual representations during instruction can improve students' conceptual understanding and lead to improved procedural fluency in problem solving. Using visual representations is especially important for students who are struggling with math. Instruction should be carefully sequenced. Teachers should first introduce mathematical ideas and concepts through a concrete model (e.g., math manipulatives, cups, sticks, paper plates). Students then transition to using a visual representation (e.g., drawing, picture, or chart) to develop a strategy for translating word problems into abstract numerical statements.

The purpose of this type of instruction, which is known as the Concrete-Representational-Abstract (CRA) sequence, is to ensure that students have a thorough understanding of the math concept or skills being learned. When students are allowed to first develop a concrete understanding of the concept, they are more likely to perform that math skill accurately and understand concepts at an abstract level.

When observing math instruction, look for the following:

- Concrete (e.g., manipulatives) and visual (i.e., semi-concrete) representations of mathematical concepts as teachers develop students' foundational knowledge
- Teachers making an explicit link between the visual representation or model and the abstract mathematical symbols or concept
- Consistent and precise language being used across similar representations so that students see the link between the concrete, visual, and abstract representations

QUESTION:

Are there any research-based practices associated with helping students solve word problems?

Math instruction at all grade levels should include strategies for solving word problems. First, students should be taught an organizational strategy for setting up and solving problems. Teachers may provide either a list of prompts to help students monitor and reflect during the problem-solving process or a list of important steps or tasks to use when solving problems (e.g., identify what the problem is asking, identify the problem type, use a visual to help solve the problem, solve the problem, and then check the problem). Whichever method is chosen, it is important that the teacher models the strategy for the students and that the students use the strategy consistently.

Second, students should be taught common problem types and their structures, and they should be taught how to categorize and select appropriate solution methods for each problem type. Common word-problem structures have growing support in research literature and have been used in mathematics instruction for some time. For example, typical word-problem structures for additive word problems include combine, compare, and change. In the middle grades, a common problem structure relates to ratios where two problems are being compared and the student is asked to use that comparison to find a missing number. Students who receive instruction in finding the underlying structure of a problem are able to solve problems more efficiently because they have a schema for them and do not have to relearn how to set up each new problem they encounter.

Proficiency with problem solving depends on the student's ability to see common problem types and connect them to viable solutions. Struggling students need to receive explicit instruction on organizing information presented in word problems, on common problem types, and on appropriate solutions.

QUESTION:

How can teachers help students who experience significant difficulties with math?

The instruction that teachers provide during core instruction may be insufficient to help struggling students make progress toward expected outcomes. Students having difficulty in math require explicit and systematic instruction (and supplemental intervention) that includes modeling of concepts, procedures, and proficient problem solving processes; verbalizing thought processes (i.e., modeling, think alouds); guided practice; corrective feedback; and frequent cumulative review.

When observing math instruction and intervention, look for the following:

- Clear modeling with think alouds, concise language, and planned examples
- Guided practice in which students and teachers practice together
- A mixture of high-level (why and how) and low-level (what and when) questioning by the teacher
- Frequent opportunities for students to respond in a variety of ways (e.g., whole class, turn and talks, partners, written)
- Immediate and specific feedback provided to students
- Use of concrete models (3D objects) and visual representations (2D objects) to support the understanding of abstract mathematical numerals, symbols, and algorithms
- Ongoing practice and review of basic math facts to support computational and procedural fluency
- Consistent use of a problem-solving attack strategy and problem types (i.e., schema) when solving word problems

SUMMARY

Extensive research and numerous syntheses have been conducted in the areas of mathematics instruction and intervention for middle grade students. The question-and-answer segment summarizes key information from extant research for improving math achievement for all students as well as those who are experiencing math difficulties. Providing opportunities for students to develop strong conceptual understanding, apply effective problem-solving strategies, and acquire strong numeracy skills will enhance the development of the math skills they need to become algebra-ready and prepared for math courses in high school and beyond.

The Bush Institute's Middle School Matters program worked with Drs. David Chard, Leanne Ketterlin Gellar, and Sarah Powell to locate the best available research evidence to assist schools in reaching their goals. Areas of research have been compiled in the user-friendly Middle School Matters Field Guide (www.GreatMiddleSchools.org/field-guide), which translates important research findings into overarching principles and practices for middle school math instruction. Additionally, toolkits related to these practices are available at www.GreatMiddleSchools.org/toolkits. Here, teachers can locate professional development resources, videos, and instructional materials needed to implement practices.

Dr. David Chard is President of Wheelock College and a member of the Middle School Matters Advisory Board. He was formerly the Dean of the Annette Caldwell Simmons School of Education and Human Development at Southern Methodist University. In 2011, Dr. Chard was nominated by President Barack Obama to serve on the Board of Directors of the National Board for Education Sciences. He was confirmed in 2012 by the U.S. Senate and elected chair of the board in 2013. President Obama reappointed Dr. Chard to his second term in 2016.

Dr. Leanne Ketterlin Gellar is a professor, the Texas Instruments Endowed Chair in Education, the Director of Research in Mathematics Education, and the Director of STEM Initiatives at the Caruth Institute for Engineering Education at Southern Methodist University.

Dr. Sarah Powell is an assistant professor in the Department of Special Education at the University of Texas at Austin. She was a National Academy of Education/Spencer Postdoctoral Fellow from 2014 to 2016, and she is currently a Faculty Fellow with the Greater Texas Foundation.

RESOURCES

To learn more about these recommendations, see the below resources. A complete list of resources can be found in the Middle School Matters Field Guide.

Ball, D. L., Ferrini-Mundy, J., Kilpatrick, J., Milgram, R. J., Schmid, W., & Schaar, R. (2005). Reaching for common ground in K-12 mathematics education. *Notices of the AMS, 52*(9), 1055–58.

DeWolf, M., Bassok, M., & Holyoak, K. J. (2015). From rational numbers to algebra: Separable contributions of decimal magnitude and relational understanding of fractions. *Journal of Experimental Child Psychology, 133*, 72–84. doi:10.1016/j.jecp.2015.01.013

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance. Retrieved from http://ies.ed.gov/ncee/wwc/practiceguide.aspx?sid=2

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *Journal of Special Education, 33*(1), 18–28. doi:10.1177/002246699903300102

Jitendra, A. K., Petersen-Brown, S., Lein, A. E., Zaslofsky, A. F., Kunkel, A. K., Jung, P. G., & Egan, A. M. (2015). Teaching mathematical word problem solving: The quality of evidence for strategy instruction priming the problem structure. *Journal of Learning Disabilities, 48*(1), 51–72. doi:10.1177/0022219413487408

Krawec, J., Huang, J., Montague, M., Kressler, B., & De Alba, A. M. (2012). The effects of cognitive strategy instruction on knowledge of math problem-solving processes of middle school students with learning disabilities. *Learning Disability Quarterly, 36*(2), 80–92. doi:10.1177/0731948712463368

Montague, M., Krawec, J., Enders, C., & Dietz, S. (2014). The effects of cognitive strategy instruction on math problem solving of middle-school students of varying ability. *Journal of Educational Psychology, 106*(2), 469–481. doi:10.1037/a0035176

National Mathematics Advisory Panel. (2008). Foundations for success: *The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education. Retrieved from http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Rakes, C. R., Valentine, J. C., McGatha, M. B., & Ronau, R. N. (2010). Methods of instructional improvement in algebra: A systematic review and meta-analysis. *Review of Educational Research, 80*(3), 372–400. doi:10.3102/0034654310374880

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade: A practice guide* (NCEE 2010-4039). Washington, DC: National Center for Education Evaluation and Regional Assistance. Retrieved from http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=15

Torbeyns, J., Schneider, M., Xin, Z., & Siegler, R. S. (2015). Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different countries. *Learning and Instruction, 37*, 5–13. doi:10.1016/j.learninstruc.2014.03.002

Van Garderen, D., Scheuermann, A., & Jackson, C. (2012). Examining how students with diverse abilities use diagrams to solve mathematics word problems. *Learning Disability Quarterly, 36*(3), 145–160. doi:10.1177/0731948712438558

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