The Impact of Whole School Professional Learning on Students' Multiplicative Thinking

Ann Downton
Monash University
<ann.downton@monash.edu >

Kerry Giumelli
CEDP
<kgiumelli@parra.catholic.edu.au >

Barbara McHugh
CEDP
< bmchugh@parra.catholic.edu >

Tammy Roosen
CEDP
<troosen@ parra.catholic.edu.au>

Nadine Meredith
CEDP
<nmeredith@ parra.catholic.edu.au>

Geraldine Caleta
CEDP
<gcaleta@ parra.catholic.edu.au>

Melissa King
CEDP
<mking4@ parra.catholic.edu.au>

Kathryn Salkeld
CEDP
<ksalkeld@parra.catholic.edu.au>

Paul Stenning
CEDP
<pstenning@parra.catholic.edu.au>

The complex nature of multiplicative thinking can be challenging for students and teachers to navigate. We report on the impact on student learning of school-based professional learning that targeted teachers' pedagogical content knowledge related to Multiplicative Thinking. Analysis of Year 4 students' longitudinal data indicated greater growth over time in schools involved in the learning than in non-participating schools. Six schools that received additional classroom support from teaching educators, showed the greatest growth over time. These findings suggest that school-based professional learning which includes a coaching component impacts on knowledge and practice, and subsequent student learning.

A key goal in mathematics teaching is to move students from counting-based strategies to multiplicative reasoning strategies, which is a basis for higher levels of mathematics, such as proportional reasoning. A critical issue for teachers is how to move students from having to model their thinking to abstracting. Many researchers (e.g. Fuson, 2003) reported that students' thinking moves from a reliance on materials through to a transition stage where they visualise or imagine, to an understanding of number properties that can be used to solve problems. Professional learning (PL) can assist teachers to understand the complexity of this shift, and the progression from concrete to visual to abstract thinking.

Within this project, a professional learning model was offered to schools that had identified stagnation in their student data in multiplication and division. This paper presents evidence of the impact of a PL program targeting multiplicative thinking and teachers' pedagogical content knowledge (PCK) on student learning. We report that increased PCK, and in-situ support that directly relates to classroom practice, resulted in a growth in student achievement as assessed by the Mathematics Assessment Interview (MAI; a refinement of the Early Numeracy Interview, Clarke et al., 2002) and the National Assessment Program - Literacy and Numeracy (NAPLAN).

Background Literature

To situate the study we briefly review the literature related to teacher professional learning models, and the complexities associated with the teaching and learning of multiplicative thinking.

2019. In G. Hine, S. Blackley, & A. Cooke (Eds.). Mathematics Education Research: Impacting Practice *(Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia)* pp. 228-235. Perth: MERGA.

Teacher Professional Learning Models

Much has been reported about models of PL (e.g., Clarke & Hollingsworth, 2002; Desimone, 2009; Guskey, 1986) and development opportunities for teachers that improve instructional practice and student outcomes. Some considered key components such as: Content focused on area of need, active participation, and conducted over an extended period of time (e.g., Desimone, 2009); teachers as learners in a community of practice (e.g., Clarke & Hollingsworth, 2002; Goos, 2014); is embedded in classroom experiences and practices (e.g., Bruce, Esmonde, Ross, Dookie, & Beatty, 2010); and includes opportunities for reflection (e.g., Clarke & Hollingsworth, 2002). Others suggested that quality PL that assists teachers to deepen their knowledge and change their pedagogical practices must incorporate three key areas of focus: subject matter knowledge for teaching, how students learn the subject matter, and how to convey the content in a meaningful way (e.g., Borko, 2004; Hill, Ball, & Schilling, 2008). Sowder (2007), on the other hand, indicated a key goal of PL was to understand how children think and learn mathematics. In addition, Timperley, Wilson, Barrar & Fung, (2007) argued that effective PL opportunities need to focus on student outcomes; promote deep teacher learning through integrating theory and practice; challenge teachers' beliefs and expectations; and provide multiple opportunities to apply new learning. In fact, Guskey maintained that change in teacher beliefs and attitudes are largely derived from classroom experiences, and occur after teachers see evidence of improved student learning.

Informed by the work of Guskey (1986), Clarke and Hollingsworth (2002) presented an interconnected model of professional growth that focused more broadly on the components of a teacher's world. They suggested that change occurs through "the mediating process of reflection and enactment within four domains that encompass the teacher's world" (p. 950). These domains include: the teacher's personal domain (knowledge, beliefs and attitude); the domain of practice (professional experimentation); the external domain (external sources of stimulus and support); and domain of consequence (salient outcomes). Each domain is connected through the mediating processes of *enaction* and *reflection* so that change in one domain translates to change in another domain. Enaction is the process of interpreting and acting on a set of beliefs and pedagogy, that is, putting new ideas or new beliefs into practice, and reflection works with enaction to ensure that the implemented action is actively considered over time. Bruce et al. (2010) concurred that the PL should be situated within the school and is characterised by a cycle of planning, practice, and reflecting. In contrast, Desimone (2009) proposed a conceptual framework that focuses on teacher and student characteristics, curriculum, school leadership, and policy environment. The purpose of the framework is to study the effects of PL on changing teacher practice and the extent to which such change impacts on student learning.

Complexities Associated with Developing Multiplicative Thinking

Multiplicative thinking is central to students' mathematical understanding; is complex; requires a different level of thinking than additive reasoning (e.g., Clark & Kamii, 1996); and is the foundation of proportional reasoning (e.g., Hilton, Hilton, Dole, Goos, & O'Brien, 2012). Indeed, several studies indicated that some students in upper primary grades rely on additive reasoning to solve problems that require multiplicative reasoning (e.g., Larsson, Pettersson & Andrews, 2017; Siemon, Breed, and Virgona, 2005). In fact, Siemon et al., found that 22% of Grades 5 to 9 students relied on additive strategies, such as count-all, or skip counting by twos to solve multiplicative problems involving large whole numbers, decimals, ratio and percent. Others indicated that student difficulties associated with proportional reasoning are directly related to their limited experience with different multiplicative situations, such as multiplicative comparison, rectangular arrays, rate, and Cartesian products (e.g., Downton & Sullivan, 2017; Greer, 1992). In addition to understanding the multiplicative structure, students need to understand the commutative, associative, and distributive properties, in order to develop efficient mental and written strategies and flexibility in their thinking (Larsson et al., 2017). For instance, Larsson et al. found that students who were reliant on equal groups and repeated addition were not fluent in their use of commutative or distributive properties.

Within the Australian context, Clarke et al. (2002) found that 51% of students in Grade 2 (aged 7 and 8) either could not correctly solve or did so by, using *a counting all by ones* strategy to solve: 'Fifteen teddies are sitting in rows at the movies. They are sitting in three equal rows. How many teddies are in each row?' These students were not yet able to use skip counting or a known fact, to solve the problem, which indicated they were unable to simultaneously coordinate two composite units mentally, without the use of perceptual models, known as *abstraction*. Sullivan, Clarke, Cheeseman, and Mulligan (2001) argued that teachers' reluctance to engage students in problems that gradually remove physical prompts and encourage students to form mental images of multiplicative situations is possibly the reason why students do not make the transition to abstracting. Given the complexity and importance associated with developing multiplicative thinking, teachers need support in enhancing their pedagogical content knowledge for developing multiplicative thinking in their students.

Informed by the research literature, the study reported in this paper was conducted within teachers' own school; was directly related to their pedagogical practice associated with the teaching and learning of multiplicative thinking; and was spaced across three school terms. Initial findings related to the impact of the professional learning (PL) on the teachers' content knowledge and practice was reported previously (Downton et al., 2018). The aspect of the study reported in this paper focuses on the impact of such PL on students' development of multiplicative thinking.

Methodology

In this paper we report on the cohort of schools involved in the first year of a longitudinal project from 2016 to 2018. In 2016, 14 of the 57 Catholic Education Diocese of Parramatta (CEDP) primary schools were involved in the project (Multiplicative Thinking (MT) Schools). The 14 schools had identified multiplicative thinking as a focus of their Action Plan for 2016 because 54% of Year 3 and 38% of Year 4 students had not progressed beyond modelling multiplication and division situations using objects. That is, they were still at Growth Points 0-2 (see Figure 1).

All 14 schools engaged in five 90-minute PL modules that were co-created by the first author and the CEDP Teaching Educators (TEs), and delivered by the TEs at spaced intervals throughout the year. The PL was aimed at Stage 2 but all teachers from the school, including leadership (n=230), participated in the learning. Six of the 14 schools received additional PL from the TEs in the form of classroom support for the classroom teachers and the Lead Numeracy Teacher while they implemented the multiplicative thinking problems. These schools had been assigned to the TEs prior to the study.

Each module focused on a different multiplicative structure (rectangular arrays, times-as-many, ratio); included professional reading; reflection; analysis of student work samples; pedagogical practices; and challenging open-ended multiplicative problems that teachers would use in their classrooms. Much of the emphasis was on challenging teachers' current practice and their knowledge of multiplicative thinking; the multiplicative structures; observing and eliciting student thinking; and the importance of developing mental models.

Data Collection

There were two sources of data collection - MAI data and NAPLAN data. Each February CEDP collects MAI data from all students (Kindergarten to Year 6). The MAI data presented in this paper is from February 2016 (Year 4 students), which was prior to the intervention (Multiplicative PL for teachers) through to 2018 (Year 6). Year 4 data were selected because 14 of these schools were involved in the PL in 2016 and these students completed the NAPLAN test in 2015, when in Year 3. NAPLAN data were collected as an external measure to validate student growth in multiplicative (intervention) and non-multiplicative (non-intervention) schools from Year 3 (2015) to Year 5 (2017). The NAPLAN and the MAI data sets therefore represent the achievements of the same students, both prior to and post the intervention.

Data Analysis

The MAI data were analysed using the Growth Point Framework (Clarke et al., 2002) and the specific cohort data were extracted from the CEDP System MAI data to compare growth over time, and also to identify trends at all growth point levels. The Strategies for Multiplication and Division Growth Points (Figure 1) were used to code the students' strategies for the multiplicative domain of the interview and referred to in the results and discussion section. Direct comparisons of the percentage of students at lower growth points (Growth Points 0, 1, 2 and 3) were made between the participating (MT) and non-participating (Not-MT) schools. In a similar manner, comparisons between MT and Not-MT schools of the percentage of students at higher growth points (Growth Points 5, 6, 7 and 8) were made. A Watson U^2 test was conducted to determine if there was a statistical difference in the growth point distributions of participating (MT) and non-participating (Not-MT) schools both before the program and after program completion.

0. Not apparent.
1. Counting group items as ones.
2. Modelling multiplication and division (all objects perceived).
3. Partial modelling multiplication and division (some objects perceived).
4. Abstracting multiplication and division (no objects perceived).
5. Basic derived and intuitive strategies for multiplication.
6. Basic, derived and intuitive strategies for division.
7. Extending and applying multiplication and division.
8.Extending and applying multiplication and division to fractions and decimals.

Figure 1. Strategies for Multiplication and Division Growth Points.

All Year 3 (2015) students' NAPLAN scores were mapped to their Year 5 (2017) scores on the Number, Patterns and Algebra scales. The data were then grouped into three categories to compare growth. These categories were: schools participating with TE support (TE support); schools participating with no TE support (no TE support); and schools not participating (Not-MT). The first level of analysis involved calculating mean growth for each cohort, and the second level was to identify any levels of significance between the means.

Results and Discussion

In this section the results of the analysis of the MAI data are presented, followed by the results of the analysis of the NAPLAN data.

MAI Results

The data in Table 1 show the distribution of growth points for students as they progressed from Year 4, 2016 to Year 6, 2018 for the 14 schools involved in the Multiplicative Thinking project coded as (MT); and the schools not involved prior to 2018, coded as (Not-MT).

Table 1

Distribution of Growth Points by Numbers of Students 2016-2018 MAI Data

GP	2016 (Year 4)				2017 (Year 5)				2018 (Year 6)			
	MT		Not MT		MT		Not MT		MT		Not MT	
	(N)	(%)	(N)	(%)	(N)	(%)	(N)	(%)	(N)	(%)	(N)	(%)
0	8	(1.1)	16	(1.6)	6	(0.9)	5	(0.6)	1	(0.1)	8	(0.8)
1	41	(5.6)	52	(5.1)	15	(2.3)	19	(2.1)	5	(0.7)	6	(0.6)
2	281	(38.2)	375	(36.5)	121	(18.8)	195	(21.9)	72	(10.4)	110	(11.1)
3	172	(23.4)	227	(22.1)	109	(16.9)	151	(17.0)	85	(12.3)	146	(14.7)
4	159	(21.6)	264	(25.7)	230	(35.7)	327	(36.7)	241	(34.9)	346	(34.9)
5	55	(7.5)	67	(6.5)	102	(15.8)	123	(13.8)	138	(20.0)	200	(20.2)
6	13	(1.8)	20	(1.9)	39	(6.0)	57	(6.4)	76	(11.0)	115	(11.6)
7	4	(0.5)	1	(0.1)	14	(2.2)	5	(0.6)	40	(5.8)	35	(3.5)
8	3	(0.4)	4	(0.4)	9	(1.4)	8	(0.9)	33	(4.8)	24	(2.4)
Total	736		1026		645		890		691		990	

As indicated in Table 1 there was little difference between the cohorts in 2016, prior to the commencement of the project, as indicated by the similarities of the growth point distribution. However, in 2017 and 2018 there is a greater reduction of students in the lower growth points and greater increase of students in the higher growth points in the MT schools than the not MT schools.

In relation to the lower growth points (0-3), the percentage of students in MT and not-MT schools that were at, or below GP3 from 2016 (Year 4) to 2018 (Year 6) were approximately: 68%, 39%, and 24% (MT schools), compared to 65%, 42% and 27% (Not-MT schools). This showed overall reduction of students at these lower growth points for the MT schools of 45%, compared to 38% for the Not-MT schools.

A similar pattern was evident in the higher growth points (5-8). The percentage of students that were at, or above GP5 from 2016 to 2018 in MT schools were approximately: 10%, 25%, and 42%, compared to 9%, 22% and 38% in Not-MT schools. The overall growth from 2016 to 2018 was 31% and 29% respectively, which suggests that the PL may have had a subsequent impact on student learning.

A Watson U^2 test was carried out to investigate any changes in the distribution of growth points for the two groups of schools (intervention and non-intervention) over the period of 2016 to 2018. The results of the statistical analysis are shown in Table 2.

Table 2
Pre and Post Intervention Watson U^2 Test of MAI Multiplicative Data for School Groups

Year	Watson U^2 Statistic	P-Value
2016	0.0330642	0.935978
2018	0.0632716	0.587371

The value of the Watson U^2 test for the 2016 distributions (MT and Not-MT schools) represents a 0.94 probability that both distributions were sampled from the same population. This finding confirms that there was no difference between the two school populations before the PL program. The 2018 distributions (MT and Not-MT schools) showed a reduced probability (0.59) that the distribution of growth points between the two school groups were the same. The null hypothesis that the datasets have the same distribution is not rejected at the 5% level based on the Watson U^2 test. While the growth point distributions are not significantly different, the reduced p-value does highlight the movement in the distributions, particularly the reduction in lower growth points (0-3) and the increase in higher growth points (5-8), when comparing MT and Not-MT schools.

NAPLAN Data Analysis

Individual student NAPLAN data were matched across their Year 3 (2015) and Year 5 (2017) scores for the Number, Patterns and Algebra scale to determine a raw growth figure. The data were grouped by school into three categories: schools participating in the PL with in classroom coaching support from a Teaching Educator (TE support), schools participating in the PL but with no in class coaching TE support (no TE support) and schools that were not involved in the professional learning (Not-MT). Differences in mean growth between these groups were determined; the analysis of which is shown in Table 3.

Table 3
NAPLAN Mean Growth of Schools from 2015 to 2017

Groups	Number	Mean of growth between scores	Variance
TE support	278	94.07	4316
No TE support	361	88.25	3196
Not MT	2159	84.28	4424

The cohort with TE support had the largest mean growth of 94.07 over the two years from 2015 to 2017, followed by the schools involved in the PL with no TE classroom coaching support had a mean growth of 88.25, whereas the mean growth of school with no involvement was 84.28. Further investigation was carried out to determine whether there were significant differences between the groups using an Analysis of Variance (ANOVA). The analysis as identified in Table 4 showed that there was a difference between the means of the growth of the groups ($p = 0.046$).

Table 4
Analysis of Variance

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	26147	2	13073	3.0725	0.04645	2.998
Within Groups	11892615	2795	4254			
Total	11918762	2797				

Students' t-tests were used to examine the difference in mean growth taking the school groupings in pairs (e.g., TE support and no TE support; TE support and Not MT; no TE support and Not-MT). The statistically significant difference 'Between Groups' at p = 0.02 was between the TE support ($\bar{x} = 94.04$) and the not MT group ($\bar{x} = 84.29$). The effect size for the TE support group was calculated at 0.70 compared to 0.64 for the Not-MT group. Both groups demonstrated an effect greater than 0.4, which is generally considered to be the demarcation between normal and accelerated learning (Hattie, 1992).

The results of the analysis of MAI and NAPLAN student data indicate greater growth in student learning of multiplicative thinking in the schools that participated in the PL combined with the additional TE in class coaching support.

Concluding Comments

This study sought to investigate the impact of a structured school based PL program on students' development of multiplicative thinking. As indicated in previous studies this can be challenging due to students' reliance on additive thinking (Siemon et al., 2005), and teachers' pedagogical content knowledge (Sullivan et al., 2001). In addition to the key features and models of professional learning identified in the literature, these findings suggest that well-structured school-based PL programs should include two key components. First, that the PL is spaced over an extended period of time; is school-based to enable collective participation, professional discourse and interaction; includes input of new learning on an identified area of need; focuses on enhancing teachers' pedagogical content knowledge; includes tasks that teachers can explore with their students; and promotes opportunities for enaction and reflection. Second, that a coaching component be embedded in the program that allows teachers to receive ongoing feedback and advice as they implement the new learning about content and practice in their classroom, and reflect on the impact on student learning. Such support is characterised by a cycle of planning, practice and reflection (Bruce et al., 2010), and the acknowledgment that sustained change in teacher practice and subsequent student learning takes time, and requires a whole school commitment.

References

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-5.

Bruce, C. D., Esmonde, I., Ross, J., Dookie, L., & & Beatty, R. (2010). The effects of sustained classroom-embedded teacher professional learning on teacher efficacy and related student achievement. *Teacher and Teacher Education: An International Journal of Research and Studies, 26*(8), 1598-1608.

Clark, F. B., & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1 - 5. *Journal for Research in Mathematics Education, 27(*1), 41-51.

Clarke, D. M., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., McDonough, A.,…, Rowley, G. (2002). *Early numeracy research project: Final report*. Melbourne: Australian Catholic University.

Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education, 18*(8), 947-967.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualisations and measures. *Educational Researcher, 38*(3), 181-199.

Downton, A., Giumelli, K., McHugh, B., Roosen, T., Meredith, N., Caleta, G., & Stenning, P. (2018). Pilot study, on the impact of in-situ spaced professional learning on teachers' mathematical knowledge of multiplicative thinking. In Hunter, J., Perger, P., & Darragh, L. (Eds.). *Making waves, opening spaces* (Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia) pp. 274-281. Auckland: MERGA.

Downton, A., & Sullivan, P. (2017). Posing complex problems requiring multiplicative thinking prompts students to use sophisticated strategies and build mathematical connections. *Educational Studies in Mathematics, 95*(3), 303–328.

Fuson, K. (2003). Developing mathematical power in whole number operations. In J. Kilpatrick, G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 68–94). Reston, VA: National Council of Teachers of Mathematics.

Goos, M. (2014). Researcher-teacher relationships and models for teaching developments in mathematics education. *ZDM Mathematics Education, 46*,189-200. DOI 10.1007/s11858-013-0556-9

Greer, B. (1992). Multiplication and division as models of situations. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 276-295). NY: Macmillan.

Guskey, T. R. (1986). Staff development and the process of teacher change. *Educational Researcher, 15*(5), 5-12.

Hattie, J. (1992). Measuring the effects of schooling. *Australian Journal of Education, 36*(1), 5-13.

Hill, H., Ball, D., & Schilling, S. (2008), Unpacking pedagogical content knowledge: conceptualising and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, *39*(4), 372–400.

Hilton, A., Hilton, G., Dole, S., Goos, M., & O'Brien (2012). Evaluating middle years students' proportional reasoning. In J. Dindyal, L. P. Cheng & S. F. Ng (Eds*.), Mathematics education: Expanding horizons* (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia, pp. 330-337). Singapore: MERGA.

Larsson, K., Petterssen, K., & Andrews, P. (2017). Students' conceptualisations of multiplication as repeated addition or equal groups in relation to multi-digit and decimal number. *Journal of Mathematical Behavior, 48*, 1-13.

Siemon, D., Breed, M., & Virgona, J. (2005). From additive to multiplicative thinking: The big challenge of the middle years. In J. Mousley, L. Bragg, & C. Campbell (Eds.), *Mathematics: Celebrating achievement* (Proceedings of the 42nd conference of the Mathematical Association of Victoria, pp. 278-286). Brunswick, Victoria: MAV.

Sowder, J. (2007). The mathematics education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157–223). Charlotte, NC: Information Age Publishing & National Council of Teachers of Mathematics.

Sullivan, P., Clarke, D., Cheeseman, J., & Mulligan, J. (2001). Moving beyond physical models in learning multiplicative reasoning. In M. van den Heuvel-Panhuizen (Ed*.), Proceedings of the 25th conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 233-240). Utrecht – The Netherlands: PME.

Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2007). *Teacher professional learning and development: Best evidence synthesis iteration.* Wellington: Ministry of Education. Retrieved 26 February 2017 from http://www.oecd.org/edu/school/48727127.pdf