Young children's movement in outside spaces: could this reveal mathematical awareness?

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An awareness of pattern and structure (AMPS) is critical in young children's mathematical development (Mulligan & Mitchelmore, 2009). AMPS is formally assessed in the early years of school (K-3), in structured conditions using predetermined materials, requiring verbal and written responses. However, children also express mathematical thinking non-verbally. What is not understood, is if or how, pattern and structure may emerge informally through movement. For example, could AMPS be embodied through children's movement in outdoor play spaces? Theoretical perspectives on embodied mathematical cognition and mathematical pattern and structure will be presented to discuss possible relationships between AMPS and children's movement outdoors.

Studies on pattern and structure have identified an Awareness of Mathematical Pattern and Structure [AMPS] underlying the development of mathematical concepts in early childhood (Mulligan & Mitchelmore, 2009). However, AMPS is measured formally with a focus on children's verbal and recorded responses. A focus on children's verbal and written expression may limit the recognition of non-verbal mathematical ideas that could be embodied through children's actions (Kim, Roth & Thom, 2010).

Although a broad range of mathematical concepts are evident in young children's play and everyday experiences outdoors (Lee, 2012), the recognition of AMPS in these informal situations remains unnoticed. Young children's mathematical awareness may be evident in their movement outdoors, such moving up and down; round and round objects and boundaries; or aligning and realigning structures. However, this relationship is yet to be explored, as studies into AMPS were administered indoors with predetermined materials and tasks. This raises the question of whether, or not, there could be a relationship between young children's development AMPS and their movement in outside spaces.

In this paper, discussion will focus on how AMPS may be revealed informally, and the role of embodied cognition in illuminating young children's non-verbal expression of mathematical awareness. Limitations of current research will be identified to inform ongoing research into the role of movement in the formation of young children's mathematical ideas.

Background

Young children's mathematical ideas and the language of mathematics are developed formally and informally (Hunting, Mousley & Perry, 2012; Macmillan, 2009; Malaguzzi, 1993; Perry & Dockett, 2013). Formal learning involves the use of intentional activities and direct instruction by the educator to scaffold children's developing mathematical ideas in both preschool and school contexts (Hunting et al., 2012; Sullivan, 2011). Informal learning

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involves contexts children naturally engage with through their everyday experiences; for example, at home, school or preschool (Fleer & Raban, 2007; Macmillan, 2009; Sikder & Fleer, 2018). One context through which mathematical ideas may be developed informally is through children's engagement in outdoor spaces (Lee, 2012; Fleer & Raban, 2007).

Outdoor spaces

Outdoor spaces provide children with opportunity for movement that may be underutilised or restricted within indoor spaces (Department of Education, Employment and Workplace Relations [DEEWR], 2009; Early Childhood Australia, 2013). In preschool and school, outdoor spaces are defined as learning environments positioned outside the centre or classroom (Australian Curriculum and Reporting Authority, 2016; DEEWR, 2009; Moffett, 2011). They include natural settings such as trees, sandpits and gardens, the use of play equipment, and engagement with natural materials such as logs, rocks, sand, mud and water (DEEWR, 2009). Outdoor environments, are real-life contexts enabling children to internalise, transfer and apply mathematical ideas (Moffett, 2011). Children's use of positional and directional language, and broader engagement with spatial relationships, are predominate features in their play and movement in outdoor spaces (Lee, 2012). As children move, they may repeat actions indicating how they are engaging spatially within their environment (Athey, 2007) such as; going over, and under equipment; circling around trees; and running around boundaries. These physical actions could communicate awareness of mathematical ideas (Brock & Siraj-Blatchford, 2015).

Awareness of Mathematical Pattern and Structure (AMPS)

Young children's development of mathematical ideas involves their ability to form an awareness of patterns, structures and relationships (Mason, 1996; Mason, Stephens & Watson, 2009; Mulligan et al., 2009; Mulligan et al, 2013). A pattern is defined as "any predictable regularity, usually involving numerical, spatial or logical relationships [and its structure relates to] the way a pattern is organised" (Mulligan et al., 2009). In a suite of studies, AMPS has been found critical to the development of early mathematical understandings, supporting pre-algebraic reasoning, abstraction and generalisation of mathematical concepts (Mulligan et al., 2013). AMPS is based on "two interdependent components; one cognitive (knowledge of structure) and one meta-cognitive, i.e., 'spontaneous' (a tendency to seek and analyse patterns)" (Mulligan et al., 2013, p. 39). Both these aspects are considered to underlie how young children perceive and interact with their environment (Mulligan et al., 2013).

Young children's development of patterns and structures are described as five interrelated structural groupings:

- *Sequences* such as patterns;
- *Structured Counting and grouping* involving subitising and equal groups*;*
- *Shape and Alignment* includes concepts such as co-linearity, similarity and congruence;
- *Equal spacing*, for example using units of measure; and,
- *Partitioning* shapes and objects into equal parts (Mulligan et al., 2018, p. 22).

Strengthening one structural grouping will impact the development of other structures across mathematical concepts (Mulligan et al., 2018).

Could AMPS be noticed through observing children's movement?

Children's AMPS, and consequent awareness of underlying structures, is assessed formally in the early years of school (K-3). However, it is possible that these underlying structures could be observed through children's movement in outdoor spaces. For example, children may:

- engage in a movement sequence, such as going up and down a tree (*Patterned Sequence*);
- gather and rearrange a collection of found objects with similar geometric features (*Shape and Alignment*); and,
- construct cubbies through comparing lengths of branches to form sides, adjusting the arrangement of branches to create space inside the cubby (*Shape and Alignment, Equal Spacing,* and *Partitioning*).

Although mathematical studies of movement suggest that mathematical concepts are embodied through gesturing, vocalising, and bodily orientation (Kim et al., 2010; Bautista, Roth & Thom, 2012), no studies to date have investigated children's embodied mathematical cognition in outdoor contexts, nor the possibility of underlying structures.

Thus, it is not known how, or if, children's movement may be related to AMPS; that is, if movement patterns contribute to the development of underlying mathematical structures. Furthermore, it is possible a different relationship might form between children's movement and their structural awareness, that may not relate to AMPS.

Interdisciplinary perspectives

Two theoretical perspectives—embodied mathematical cognition; and mathematical pattern and structure are presented to investigate possible interdisciplinary relationships between young children's movement and their developing mathematical awareness.

Embodied mathematical cognition

Embodied cognition is described as a bodily sense of knowing, expressed through physical movement and sensory exploration with environments (Kim et al., 2010; Merleau Ponty, 2002; Smith & Gasser, 2005; Varela et al., 1991). There is complexity in the processes that may be involved in the development of embodied cognition as "knowledge depends on being in a world that is inseparable from our bodies, our language, our social history" (Varela et al., 1991, p. 173). Smith et al. (2005) also describe an interplay between numerous factors in the expression of embodied cognition in babies; such as the importance of language, social interaction, engagement of multimodal senses, and opportunity to physically explore environments. Additionally, contentious perspectives such as 'mathematics in the flesh', propose that movement reveals mathematical insight; however, this contrasts with views of insight learning as a purely cognitive process (Bautista, et al., 2012).

Research into embodied mathematical cognition has investigated children's use of gesture (Bautista et al., 2012; Kim et al., 2010; McNeill, 1992); movement of objects (Kim et al., 2010; Thom, 2016); changes in bodily orientation (Bautista et al., 2012; Kim et al., 2010) and expression of sounds and rhythmic patterns (Bautista et al., 2012; Bautista & Roth, 2012a; Bautista & Roth, 2012b). Additionally, these actions may emerge alongside, and be observable when children visualise, draw and discuss concepts (Elia, Evangelou & Gagatsis 2016; Thom, 2016; Thom & McGarvey, 2015).

In a study of 23 Canadian second graders, Kim et al. (2010) analysed children's embodiment of geometric properties through their use of gesturing, along with changes in bodily orientation, such as moving up, down and around. Findings indicated that movement was often accompanied by speech, however in new, unfamiliar situations the children's bodies became the prime source for exploring and investigating problems (Kim et al., 2010). In group situations, co-emerging gestures enabled concepts to be collectively expressed and understood, revealing the interactive, social nature of embodied cognition (Kim et al., 2010). However, what was not clearly understood in this study were the broader cognitive processes involved (Kim et al., 2010) nor the connection to other mathematical concepts. Additional research could systematically examine these relationships along with changes in children's embodied cognition over time.

Bautista et al. (2012), identify aspects of children's movement and expression that suggests awareness of geometrical concepts, involving:

- *bodily orientation of movement* towards objects of interest;
- *thinking in movement* before words are expressed (evidenced through gestures);
- *verbal awareness* emphasising *realisations* discovered through embodied actions; and,
- *non-verbal* expressions, including glances, smiles and eye contact that communicate awareness of concepts (pp. 378-380).

Earlier studies, identified that sounds also emerge alongside children's rhythmic patterns reflecting awareness of geometric properties (Bautista & Roth, 2012a). A sense of rhythm can be identified through children's use of *beat gestures*, and changes in *body positions* and *object orientation* (Bautista et al., 2012, p. 368). Findings suggest that rhythm may reveal *patterns of regularity* and the emergence of generalised embodied mathematical consciousness (Radford, Bardini & Sabena, 2007, cited in Bautista & Roth, 2012b, p. 44).

Children's gesturing alongside other semiotic forms of representation such as drawings, words, and models have been found to also support the construction of geometric understandings (Elia et al., 2016). In a case study of one child in Kindergarten, Elia et al. (2016) found that in most occurrences there was synchronicity and congruency between the changes in different modes of expression. Differences between verbal and non-verbal expressions were related to the complexity of concepts under investigation (Elia, et al., 2016). Gesturing was found to be integral in identifying changes in the development of the child's geometric awareness of concepts. Thus, without access to the embodied expression essential information regarding how the child constructed and deconstructed images, along with their development of geometric ideas is left unknown. Analysis of the child's movement beyond the use of gesturing with hands and arms, was limited. Thus, research into changes in orientation of the whole body through differing geometric planes of movement could reveal a greater breadth of embodied geometric thinking and reasoning. Additionally, further research is needed to see if this relationship is generalisable across a larger study of children.

Mathematical pattern and structure

Patterns have predictable elements that repeat and structural understanding involves reasoning about the relationships between patterns, recognising and engaging with similarities across mathematical concepts (Mason et al., 2009; Mulligan et al., 2009). Longitudinal research indicates that AMPS enables emergent forms of generalisation, supporting young children in developing pre-algebraic reasoning (Mulligan, English & Mitchelmore, 2013). Integral to this research is the development of an interview-based *Pattern and Structure Awareness Assessment (PASA)* that measures children's levels of AMPS through stages ranging from *pre-structural, emergent, partial structural, structural,* to *advanced structural* (Mulligan, Mitchelmore & Stephanou, 2015). An evaluation study of a Pattern and Structure Mathematical Awareness Program (PASMAP) revealed that children who accessed intervention strengthened their level of structural awareness and early algebraic reasoning compared to children who were not involved in the intervention program (Mulligan et al, 2013). "This evaluation study involving 316 Kindergarten students from 4 schools … [found] highly significant differences on PASA scores for [the intervention] PASMAP students" after the second year of formal schooling (Mulligan et al., 2013, p. 337). Further, findings from this research indicate that children identified with low level AMPS are at risk of not performing successfully with school mathematics. Implications for ongoing research is to further explore avenues for identifying children at risk of low AMPS and implement early intervention, and or professional learning programs (Mulligan et al. 2013). However, limitations of this research were that the development of AMPS was not systematically evaluated beyond grade 1, and transition from preschool to formal school was not analysed. Furthermore, children in the pattern and structure studies were assessed through clinical interview and observation in controlled, structured learning environments with specified materials. There may have been some critical aspects of AMPS evident in children's broader everyday contexts.

Studies into young children's sense of patterning prior to school indicate their developing awareness of structural features of repeating patterns (Luken, 2011; Papic et al. 2011; Papic, 2015). Although these studies were implemented formally, in interview conditions, they could provide insight into structural stages of patterning that may be observable in more informal situations.

In the *Patterns and Early Algebra Program (PEAP)*, two hundred and fifty-five predominately Australian Indigenous preschool children were assessed prior to the project using the *Early Mathematical Patterning Assessment [EMPA]* (Papic, 2015). Children's representation of patterning was also categorised into levels of increasing structural awareness, that is; *Pre-structural, Emergent, Structural*, and *Advanced Structural 1* (Papic, 2015, p. 528). Findings indicated a range of abilities in children's development in patterning prior to school, and the effectiveness of the intervention program in strengthening practitioner analysis of individual children's mathematical patterning. These findings support the positioning of assessment tools, alongside observational methods, in the development of young children's mathematics (Papic, 2015).

In an explorative study of six, three to five-year-old kindergarten children, Luken (2018) categorised structural strategies used during patterning tasks. Children's use of strategies involved:

- *no reference to pattern*: random arrangement of elements;
- *use of pattern elements*: engaging with a common aspect such as colour, shape and size;
- *comparison*: elements of regularity may emerge, such as identifying 'sameness';
- *focus on the sequence*: awareness of the order of elements is evident; and,
- *view of unit of repeat*: demonstrating knowledge of the structure of the pattern through identifying and using unit of repeat (Luken, 2018, pp. 41-42).

Findings were consistent with other related studies, indicating the breadth of preschool children's patterning competencies (Papic et al., 2011; Rittle-Johnston, Fife, Loehr & Miller, 2015, cited in Luken, 2018, p. 49). Limited engagement with repeating patterns was noted in the three-year-old children, although the overall impression of the pattern (gestalt), such as the linear arrangement was apparent (Luken, 2018). The use of strategies, such as c*omparison* and *focus on the sequence* of patterns, were evident in four and five-year-old children's patterning, this was observed through children's attention to sameness, and alternate features of patterns (Luken, 2018). Although five-year-old children exhibited greater awareness of strategies, and engaged with more complex patterns, the notion of the *unit of repeat* was still not evident in the results (Luken, 2018). However, the children's engagement with patterns beyond the formal interview was not considered. There may be examples of the children's patterning through movement that could relate to stages of structural awareness. For example, children may engage with basic movement patterns, such as up/down, or on/off indicating *use of pattern elements*. Comparison may be noticed when the children identify and compare a similar aspect, such as 'jump off here too'. Engaging with order, such as 'first here, then there, next …' could indicate a *focus on sequence*. Awareness of structure of a movement sequence, such as 'let's do it again, start here' could suggest notion of a *unit of repeat*. Hence, young children's movement may reveal structural stages of patterning that could inform research into their developing mathematical awareness.

Conclusion and recommendations

A review of the literature reveals a general consensus among educators that mathematical concepts are developed through young children's informal play and everyday experiences, as well as through children's movement in outdoor spaces (Lee, 2012; Fleer & Raben, 2007). Research into the development of young children's awareness of patterns and structures has focussed on observing and categorising mathematical processes children use while engaged in pre-determined mathematical tasks indoors, usually in the formal school setting. Key patterns and structures have been found critical and salient to mathematical development, and support pre-algebraic reasoning; for example, making simple repetitions, and using shape and alignment, equal spacing and partitioning. However, there are few studies investigating children's mathematical development through movement in outdoor spaces that focus on children's awareness of patterns and structures. The early development of these structures must begin long before formal schooling and in informal environments including a range of outdoor experiences. We know that as children move, they may engage in repeating actions indicating how they are engaging spatially within their environment (Athey, 2007).

The field of embodied cognition recognises the role of children's gestures and specific movements in communicating and developing mathematical thought. The review of the literature has indicated that analyses of children's use of gestures and bodily orientation provides much insight into their mathematical development. However, the field needs to investigate the relationships between children's broader range of movements and their awareness of mathematical concepts.

Further studies could be designed to observe children's movement patterns when engaged in free play in outdoor spaces, both alone and with peers. This could involve the use of playground equipment and when they are engaged freely such as climbing or skipping. Documenting changes in the children's movement over time, could provide insight into the role of the body in the development of mathematical concepts. These movement patterns may develop their awareness of pattern and structure; for example, through repeated spatial actions and navigating pathways. The pace, rhythm and changes in orientation (Bautista et al., 2012; Bautista & Roth, 2012b) of children's movement patterns could be systematically analysed longitudinally for evidence of children's developing mathematical structures such as skip counting, equal spacing and transformation skills. Children's self-awareness of their engagement with mathematical patterns could be studied through implementing *Video Stimulated Recall* (VSR) (refer Morgan, 2007; Rowe & Claire, 2009) and a *Drawing Telling* approach (Wright, 2007). A drawing-telling process, used effectively in early childhood mathematics education research (MacDonald, 2010; MacDonald & Lowrie, 2011), provides opportunity for children to represent and express meaning verbally and non-verbally (Wright, 2007, cited in MacDonald & Lowrie, 2011).

How, or why the body may be involved in the emergence of cognition is not fully understood. Therefore, further research is required to illuminate a broader, more holistic perspective into the role of movement in the development of young children's mathematical awareness.

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