Studying Student Interactive Positioning in Collaborative Mathematics Problem Solving: The Case of Four Chinese Students

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This paper reports on an investigation of the student interaction and group development of a four-student group during collaborative mathematics problem solving. Video and written transcript of the group interaction were analysed. The group problem solving has been described by the flow of negotiative events in terms of topics discussed within the group. Positioning theory framed four interaction positions which described student interaction. A mutual relationship between the two has been identified in this case. Implications for collaborative problem solving are discussed.

Collaborative group work has been increasingly adopted as a daily classroom practice with a more constructivist view of learning (Esmonde, 2009). Research in collaborative group work has been linked to positive learning outcomes across various settings (Webb & Palincsar, 1996). Collaborative learning was suggested to be a main activity through which students are able to maintain motivation and engage in verbal communication in classrooms, especially in mathematics lessons (Roschelle & Teasley, 1995). Research in mathematics education has focused on the social interaction among students, which can lead to better problem solving in mathematical tasks (Barron, 2003), or improvements in students' mathematical reasoning and learning (DeJarnette, 2018). One line of research is the examination of students' positioning and identity within the group during collaborative discussions (Esmonde, 2009; DeJarnette, 2018). This line of research which aims to identify students' interactional patterns may be useful for understanding how groups coordinate and learn mathematics.

Theoretical Frameworks

Analysis in this study was informed by positioning theory which has been widely used in the area of social sciences. Harré and van Langenhove (1999) described positioning as "the discursive construction of personal stories that make a person's actions intelligible and relatively determinate as social acts and within which the members of the conversation have specific locations" (p.16). In different contexts, there might be various ways of positioning; for instance, individuals may physically position themselves or metaphorically position themselves in relation to a person's attributes as a speaker (Harré & van Langenhove, 1999; Wagner & Herbel-Eisenmann, 2009).

According to Harré and van Langenhove (1999), there are three orders of positioning which might happen in a conversation. The first order positioning refers to the ways people locate themselves and others within an essentially moral space by using several categories and storylines. When a person initiates an interaction, he or she positions himself or herself relative to the other interactants in current interaction (DeJarnette, 2018). A second order positioning occurs when the first order positioning is questioned and has to be negotiated, that is, the second order positioning occurs when the first order positioning is not taken for

granted by one of the others in the discussion. The second order positioning could have been acknowledged implicitly or explicitly. Besides, the third order positioning status of 'having a footing' is imminent, in a way that one can "enter a conversation, a game, a trial, someone else' private affairs, unchallenged, as of right" (Harré, Moghaddam, Cairnie, Rothbart & Sabat, 2009, p. 12). Someone in a group evaluates the content or quality of discussion with a 'footing' is listened to and taken notice of.

In the context of mathematics classrooms, students' academic and social status can influence their positioning in the interaction within the community (Bishop, 2012; Esmonde, 2009). There are different perspectives among researchers in terms of students positioning in collaborative problem solving. Some view positioning over time during which students hold stable roles or positions, such as expert, novice or facilitator (Esmonde, 2009) toward each other in the group, while some see it as a moment-by-moment behaviour from a dynamic perspective which stresses more on the power of students' instant actions in the group that shift the positioning among the group members (Wagner & Herbel-Eisenmann, 2009). Both perspectives on positioning help to capture the characteristics of students' group work, since a moment-by-moment base positioning may also extend to a relative stable pattern of positioning in a group (DeJarnette, 2018).

With regard to this study in the context of collaborative problem solving, student positioning provides a particular way for analysing the complex interactions that students engaging. Students' personal characteristics and peer interaction in the classroom both influence how students position themselves or are positioned within the group (Esmonde, 2009). This study analyses student positioning from both a 'moment-by-moment' and an 'over-time' perspective in terms of positioning orders. In this paper, the use of *interactive positioning,* as represented by *initiation, response, evaluation* and *non-interactive,* is defined over time as the inclination towards positioning orders in conversations. The research question addressed in this paper is: How do students interactively position themselves in the group so as to be involved in collaborative problem solving?

Methodology

Data sources

The data reported in this paper came from the Australian government-funded research project, the Social Essentials of Learning (Chan, Clarke, & Cao, 2018). As part of the project, classes of Year 7 students in Australia and in China were given separate open-ended tasks to complete individually, in pairs, and in small groups (four to six students). The problem-solving process of each group was video recorded and their written work collected for analysis. For this paper, one group of four students, two boys (S1 and S2) and two girls (S3 and S4), was chosen as a case study. The selected group worked on the task for 15 minutes and 42 seconds. The video recording was transcribed for analysis. The group was chosen for analysis because there appeared to be a lot of verbal exchanges between the group members during the problem solving activity. The instruction of the group task and the group's solution are translated and shown below (Figure 1).

Figure 1. Provided task instruction and group solution of one student group.

Analytical approach

The analysis was conducted by first employing negotiative events as the unit of analysis. The definition of a *negotiative event* is an utterance sequence constituting a social interaction with a single identifiable purpose (Chan & Clarke, 2017). Tracing the flow of negotiative events helps to identify how a group operates as whole, and the contribution of individual members, such as noting who initiated an idea within a negotiative event; who responded and evaluated the initial idea; and, who were not involved in the discussion within the negotiative event. In terms of implementation of the analysis, a discussion topic was identified for every negotiative event. The researcher (first author) first partitioned the transcript and identified the point in the transcript where there was a change in topic in the discussion. The researcher then viewed the video recording of the students' discussion to ensure that the change of topic aligned with what was identified in the transcript. The following example shows an example of a negotiative event.

Table 1
An example of a negotiative event (NE)

Negotiative Event 4: Whether 2 m^2 is big enough for a toilet?	
S4 (4.1)	A 2 m^2 bathroom is OK.
S2 (4.1)	That's impossible. I tell you.
S3 (4.1)	Possibly? No. 2 m^2 is impossible. If 2 m^2, even a toilet bowl is not enough.
S1 (4.1)	How is it impossible ?
S2 (4.2)	Firstly, you should have a…
S4 (4.2)	Then 5 m^2.
S2 (4.3)	Firstly, it must have a place to wash. In a toilet, you must have a place to wash. A toilet should be as big as the teacher's platform.
S1 (4.2)	No. No. No. You don't know, some hotels have a place to wash. It is just like, It is direct there…
Negotiative Event 5: Whether 5 m^2 is big enough for a toilet?	
S4 (5.1)	5 m^2 is actually around that area.
...	…

Then a closer look was taken the student interaction, drawing on positioning theory, to investigate how students interact with each other. Four types of interactive positions (initiation, response, evaluation and non-interactive) have been conceptualised partly drawing on first order positioning, second order positioning and third order positioning from positioning theory. The non-interactive positioning has also been noted since it is common that students do not interact with others within the group. It is also worthy to note that these four categories are not scales; rather, these are descriptive categories.

Table 2 takes Negotiative Event 4 (see Table 1) as an example to illustrate how interactive positioning is corresponding to the group. As can be seen, S4 initiates a discussion which has been addressed by the group, she positions herself in a position of *initiation*. S4's initiation has been responded firstly by S2 who challenges the idea [S2 (4.1)] and provides explanation for his standing [S2 (4.2)]. S2 then occupies the response position. S3 follows S4's initiation and S2's challenge with an evaluation on S4's initiation. S1 asks for explanation and evaluation from both S4 and S2, his question arises following discussion on the topic which gives him a position of footing in evaluation as well. Everybody is engaged in the discussion, therefore, no speaker occupies the non-interactive position.

Table 2
Coding of students' interactive positioning in Negotiative Event 4 (in Table 1)

Positioning Theory	Interactive Positioning	Speaker
First order positioning	Initiation	S4
Second order positioning	Response	S2
Third order positioning	Evaluation	S1, S3
N/A	Non-interactive	None

Analysis

Tracing the flow of negotiative events. First of all, since the transition from one negotiative event to the next is determined by the change of topic, the following table summarises how the group moved from one topic to another and who proposes the specific topic, which gives a view of the operation within the group.

Table 3
The flow of negotiative events

Event Number	Topic Proposer	Duration (m:ss.ms)	Topic of Negotiative Event
NE1	S3	0:23.92	How big is 60m^2? How many rooms in the apartment?
NE2	S2	1:00.15	What kind of rooms could be in the apartment?
NE3	N/A	0:16.24	[*Casual talk without a specific topic*]
NE4	S4	0:24.48	Whether 2m^2 is big enough for a toilet?
NE5	S4	0:54.87	Whether 5m^2 is big enough for a toilet? How big is four bricks?
NE6	S3	1:00.18	How long is a side of a brick?
NE7	S3	1:55.05	How big is one brick and how big is four bricks?
NE8	S4	0:10.91	Whether 5m^2 is big enough for a toilet?
NE9	S2	0:31.24	If a draft paper is needed now for drawing the plan?
NE10	S3	0:59.51	Where to place the bathroom and toilet?

NE11	S4	1:10.19	How big should other rooms (bedroom and study room) be? How many square meters are left?
NE12	S1	0:22.71	Should the group start to draw down the work?
NE13	S4	0:22.31	Whether a rectangle of 6×10 is suitable?
NE14	N/A	0:23.08	[*Passing ruler*]
NE15	N/A	0:16.36	[*Students passing rulers and erasers*.]
NE16	S1	0:57.43	Is a scale needed? How big is the living room?
NE17	S3	0:16.23	If rectangle is suitable for the shape of an apartment?
NE18	S4	1:00.11	How many square meters are left? For a study room, is it ok? Are doors needed for drawing the work?
NE19	S4	0:43.93	Is 16m^2 big enough for a bedroom?
NE20	S3	0:38.66	What furniture should be in the living room? How many square meters are left for the kitchen?
NE21	S2	1:44.98	If the plan is possible in practice? [*Labelling rooms and doors*.]

Note: Description in [*bracket*] is a summary by researchers on group action or behaviour if needed or if no negotiative topics have been identified.

Interactive positioning. Student positioning has been viewed firstly from an individual perspective. Table 4 summarises how each of the students interacted with other group members for the duration of the task.

Table 4
Occurrence of interactive positions for each group member

	S1	S2	S3	S4
Non-interactive	2	8	2	3
Initiation	2	6	9	4
Response	6	6	7	5
Evaluation	9	0	1	8

For example, S1 tended to evaluate information for the group, S2 kept his place among initiation, response and non-interactive and never evaluated information. S3 initiated more often than other members. S4 seemed to position in various ways and evaluated more often. The following figure corresponds to student interactive positioning and visually represents changes across them, along with the flow of negotiative events.
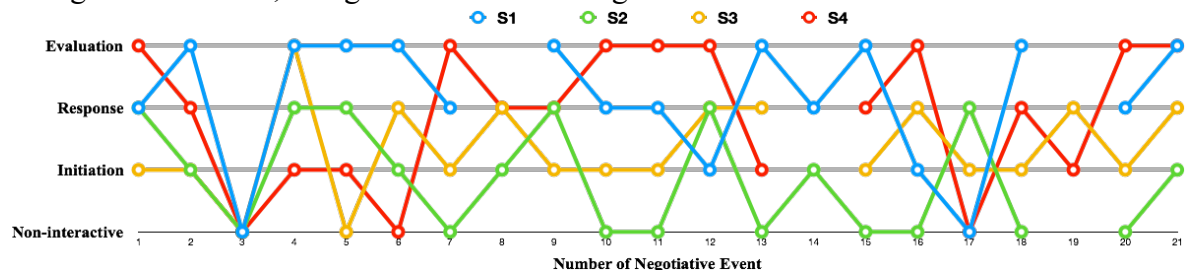

Figure 2. Student interactive positioning.

Results

Process of collaborative problem solving. The flow of negotiative events describes the development of a group's goals, and makes it easier to see who proposes a topic and which topic follows which. A simple summary of the negotiative events in terms of what purposes they serve for solving the problem is shown in Figure 3. The group discussion evolved in the following manner:

1. Clarifying the problem (NE1, NE2);
2. Discussion about the basic plan of one room and its area (NE4, NE5, NE8);
3. Negotiation on scale for estimating (NE5, NE6, NE7);
4. Discussion on physical objects, such as draft paper, worksheet, ruler, eraser (NE9, NE12, NE14, NE15);
5. Discussion on a rough plan of the apartment (NE10, NE11, NE12);
6. Identifying the shape of the apartment (NE13, NE17); and,
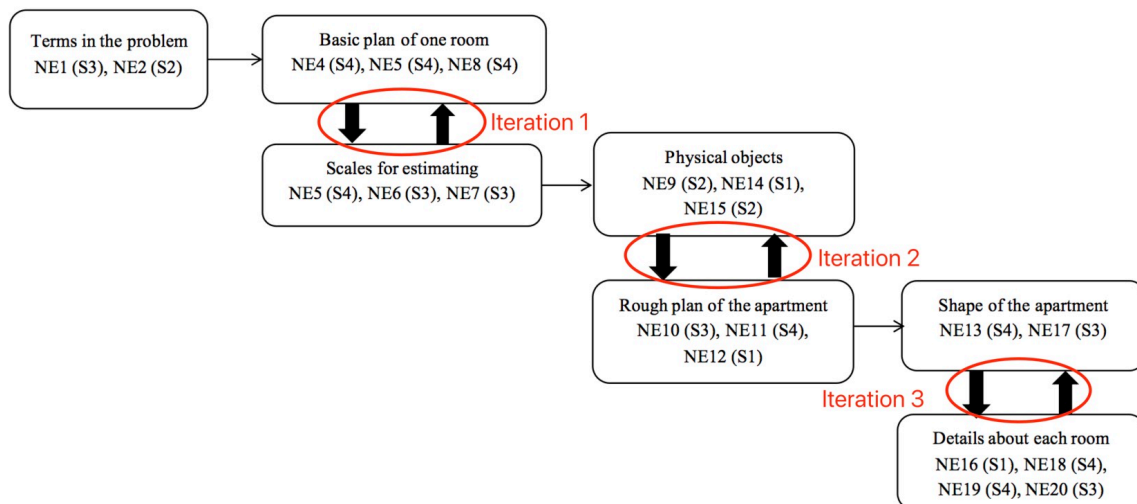7. Negotiation of details regarding area and location of each room (NE18, 19, 20).

Figure 3. Development of group work in terms of NEs for the student group.

Three iterations have been observed in this group's interaction where some topics were revisited. For instance, in iteration 1, there was a revisit of topic initially at NE5 then again at NE8 after the group had gone through NE6 and NE7. The group firstly proposed questions to discuss *how big a toilet should be*; as the group did not seem to have a clear sense connecting '2 m^2' to how big it is in social life. Therefore, they then worked on developing a common scale for estimating the size of the room with a 'brick' which could be seen directly on the ground of the classroom. After that the actual question of '*how big a toilet should be'* was revisited and solved with reference to 'the area of a brick'. Iteration of topics could possibly be one way of describing the group's approach to reaching a solution, negotiating intergroup conflict and solving the given problem. However, a follow-up question could be: How did individual group member contribute to the group process?

The case of S3 and S4 – girl pair. S3 and S4 constituted a dominant pair in this group in a sense that, firstly, the two girls proposed 13 of 18 topics widely from various positions for group discussion, especially S4. Secondly, looking at the iterative circles shown in Figure 2, two of them (circle 1 and circle 3) were fully driven by S3 and S4 in a sense that the shifting topics were facilitated by the two students, and the remaining one (circle 2) was partly driven by the pair. Furthermore, student interaction positioning also showed how the pair interacted

in a coordinated pattern of *initiation-response (evaluation)* when working from NE15 to NE20 which was the period of drawing the apartment.

The case of S2 - Significant change of position. A significant change in S2's positions, from 'initiation' to 'non-interactive', was found from the analysis. His interests in initiating a topic to discuss in the beginning changed to self-talk. To summarise his behaviours based on our analysis, he proposed three topics in NE2, NE9, NE21, which had been discussed by the group: one happened at the beginning (NE2), which addresses the group's attention on clarifying the problem; one happened at the end (NE21), when he tried to judge the solution of the group work; and the third one happened in the middle (NE9), when he wanted to draw his plan in a draft paper but was rejected by the group. From then, S2 started to engage in self-talk until almost the end of the group work.

The case of S1- Evaluator. S1 played his part mostly from the position of evaluation in discussion. He initiated two topics (NE12, NE16) for the group. One of them worth mentioning is the topic of NE12 which directed the group attention to start to draw rather than solely discussing. However, his maintenance of his position on evaluation in student interaction shows that proposing topics for negotiation is not the only way to be involved in group work.

Discussion

This case study investigates the connection between collaborative problem solving and student interaction based on a unique Chinese dataset generated in the Social Essentials of Learning project. The main research question addressed in this paper is: How do students interactively position themselves in the group so as to be involved in collaborative problem solving?

Tracing the flow of negotiative events describes in detail how the group solve the problem which in turn helps to unfold the path of collaborative problem solving in terms of group goals temporally (Figure 3). As was noted before, the group solved the problem by achieving a series of goals including: clarification of problem, developing ideas on room and on scale, operation of physical objects, creation of a tentative plan, creative of a broad plan and agreement on a detailed plan. The revisitation of topics in different periods in terms of goals revealed an iteration process, which also suggests a possible way of solving problem collaboratively. Identification of the topic proposer for every negotiative event shows how students have been involved in the problem solving process. Seven, six, three and two topics were proposed by S4, S3, S2 and S1 respectively, which suggests a difference among students in terms of their involvement in group work.

Analysis on students' interactive positioning corresponding to the flow of negotiative events, provides us with a way of looking at how students position themselves toward each other during discussion, and what positions they were in for every topic. Although most topics (13) were proposed by the person at the initiation position, some of them (5) were proposed by members at other positions which means that entry to topics of negotiative events is not determined by specific positions. For example, S4 proposed a topic for NE11 when she occupied the position of evaluation in that event. Her statements on the remaining area of the apartment challenged S1's initiation of commenting on kitchen ware which led to group discussion. Students appeared to show personal characteristics in terms of what interactive positions they tend to occupy in group discussion. The case of S1 suggests that keeping a specific position among different interactive positions also provides an entry to collaborative problem solving.

Comparison of performance between S2 and the pair of S3 and S4 on either the number of proposed topics or form of interactive positioning gives an indication of possible

dominance within the group. Taking also the iteration of topics during group work into consideration, suggests either a reinforcement of a topic or a dominance of the topic proposer. If the dominance of some group members has played critical roles in organising the group work, deciding the direction, and constructing social interaction within the group, is this an indication of a group form (or norm) for collaborative problem solving? More research is needed in this regard.

This study suggests a mutual relationship between student interactive positioning and group problem solving in terms of collaborative problem solving. This approach helps to understand not only student interaction but also within the context of collaborative problem solving. In addition, the methods used for analysis also provides a possible framework for further research investigating characteristics of positioning as well as connecting between student positioning and the group.

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